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PARALLEL OPTICAL PROCESSING TO CONVERT ELEVATION DATA TO SLOPE MAPS
PHASE II: PRACTICAL CONSIDERATIONS

February 1975

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The considerations in designing and calibrating a coherent optical data processing system for quantitative slope computation from a transparency with amplitude transmittances as a function of surface terrain elevations are described in this report. Methods of designing the spatial filter and criteria for selecting the optical system components to minimize the errors in determining slopes are described. Procedures for calibrating the input transparency and the optical system are specified.

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PREFACE

The work described in this report was authorized under Project 4A161101A91D and was conducted under the supervision of Mr. Bernard B. Scheps, Chief, Technology Development Branch; Mr. Alphonse C. Elser, Chief, Geographic Information Systems Division; and Dr. Kenneth R. Kothe, Director, Geographic Sciences Laboratory. The author acknowledges the helpful comments of Dr. N. Balasubramanian of the University of Rochester where this phase of the work was started.

This Research Note is the second in a three-part series on "Parallel Optical Processing to Convert Elevation Data to Slope Maps." In "Phase I: Theoretical Analysis" it was established that for a photographic transparency having amplitude transmittances being a functional representation of the elevation information for a particular geographic area, the slope information can be displayed using a one dimensional derivative filter in a coherent optical data processor. This phase of the work discusses the practical considerations required for implementing this technique.

CONTENTS

ILLUSTRATIONS

PARALLEL OPTICAL PROCESSING TO CONVERT ELEVATION DATA TO SLOPE MAPS PHASE II: PRACTICAL CONSIDERATIONS

I. INTRODUCTION

The use of coherent optical systems for parallel processing of two-dimensional information is well known. Various systems have been discussed in the literature employing either one or two lenses for optical image processing.[1] [2] There is an object plane where the input transparency is located and an image plane which displays the processed image. Between these two planes is another plane where the diffraction pattern or Fourier transform of the object is located and modified by a spatial filter to produce a particular image.

Because of its ability to process large quantities of pictorial information faster than digital computers, widespread studies on the applications of coherent processing techniques have been conducted. Only recently has the application of these techniques to photogrammetry been investigated. The general problem of photogrammetry requiring the display of elevation contours is presently being studied; however, minimal research has been conducted using optical processing for generating specialized map products. Specifically, the rapid determination of terrain slopes is an important factor related to military geographic area analysis. The considerations in designing and calibrating a coherent optical processor for the quantitative determination of terrain slopes are described in this report.

This is the second in a series of three reports to consider the use of coherent optical data processing systems for performing slope computations. In the first report, the theoretical aspects of this problem for obtaining terrain slopes were considered by Brooke.[3] The initial impetus originated from an expression derived by Hevenor relating the Fourier spectrum of the terrain elevations to the spectrum of the slopes.[4]

The input to the optical processor is a specially prepared transparency derived from aerial photography. In particular, the amplitude transmittances in the transparency are a

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1 J. W. Goodman, *Introduction to Fourier Optics*, Chapter 7, McGraw Hill, New York, 1968.

2 A. Shulman, *Optical Data Processing*, Chapter 7, Wiley Interscience, New York, 1968.

3 R. K. Brooke, Jr., *Parallel Optical Processing to Convert Elevation Data to Slope Maps, Phase I, Theoretical Analysis*, U.S. Army Engineer Topographic Labs, ETL-RN-74-9, October 1974.

4 R. A. Hevenor, *A Relation Between the Spectrum of the Surface Slopes and the Spectrum of the Surface Elevations and Its Usefulness in the Theory of Electromagnetic Wave Scattering from Rough Surfaces*, U.S. Army Engineer Topographic Labs, ETL-RN-70-2, July 1971.

functional representation of terrain elevations. The Fourier transform plane contains a spatial filter that operates on the diffraction pattern of the transparency to yield the slope information in one direction of the image plane. The operation in the diffraction plane is, therefore, a directional derivative. Rotation of the filter or the transparency provides slope information in other directions. Thus, image intensities at the output are a function of the slopes.

II. ANALYSIS

1. **Ideal Derivative Filter.** A two-lens coherent optical data processing system is defined below and shown in Figure 1. The object transparency is located in the front focal plane (x,y) of the first lens, L1. Its back focal plane (ξ, η) coincides with the front focal plane of a second lens, L2. Finally, the image plane (x',y') is the reflection of the coordinates or the object plane; i.e., the image is inverted with respect to the object. For simplicity, we let both lenses have the same focal length, f. If the focal lengths of the lenses were unequal, a different scale image would result. The scale factor is defined as the ratio of the focal length of the second lens to the first one. In the case considered, it is unity or no change in scale.

The subsequent analysis will now be done on a one-dimensional basis in order to derive the functional form of the spatial filter that will perform a directional derivative.

If an object transparency, T(x), is illuminated with a quasi-monochromatic plane wave of wavelength, λ, its Fourier transform, $\widetilde{T}(\nu_x)$, where $\nu_x = \xi/\lambda f$, is displayed in the (ξ, η) plane. The effect of the second lens is to perform a Fourier transform on $\widetilde{T}(\nu_x)$ to get the image, $T(x')$.

Now, we derive the transmittance distribution of the derivative filter. We write

$$\frac{d}{dx} T(x) = \frac{d}{dx} \mathfrak{F}^{-1} [\widetilde{T}(\nu_x)] \tag{1}$$

where $\mathfrak{F}^{-1}$ is an inverse Fourier transform operator. Equation (1) can be rewritten as

$$\frac{d}{dx} T(x) = \frac{1}{2\pi} \frac{d}{dx} \int_{-\infty}^{+\infty} \widetilde{T}(\nu_x) e^{i\nu_x x} d\nu_x$$

$$\frac{d}{dx} T(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} i\nu_x \widetilde{T}(\nu_x) e^{i\nu_x x} d\nu_x$$

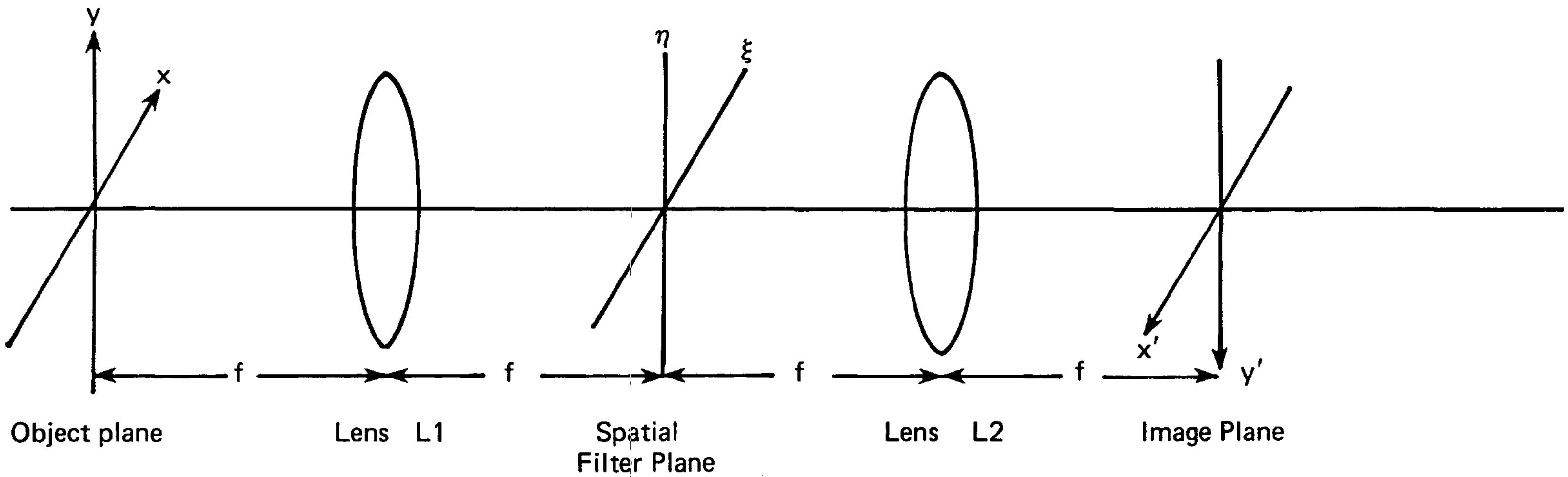

Figure 1. A Two-lens Coherent Optical Data Processing System.

Hence,

$$\mathfrak{F}\left[\frac{d}{dx}\,T(x)\right] = i\nu_x\,\widetilde{T}(\nu_x) \tag{2}$$

Equation (2) states that when a spatial filter of complex amplitude transmittance, $i\nu_x$, is placed in the Fourier transform plane the first derivative of the object is displayed.[5] [6] This equation was also discussed by Brooke in Phase I of this series of reports.

Since $i\nu_x$ is a complex filter, it can be decomposed into an amplitude and phase filter, $A(\nu_x)$ and $\theta\,(\nu_x)$ respectively; therefore,

$$i\nu_x = A(\nu_x)\,\theta\,(\nu_x)$$

Both the amplitude and phase functions are given as

$$A\,(\nu_x) = |\nu_x| \tag{3}$$

$$\theta\,(\nu_x) = \text{sgn}\,(\nu_x) \tag{4}$$

where

$$\text{sgn}\,(\nu_x) = \begin{cases} -1 & -\infty < \nu_x < 0 \\ 0 & \nu_x = 0 \\ 1 & 0 < \nu_x < \infty \end{cases}$$

These functions are plotted in Figure 2.

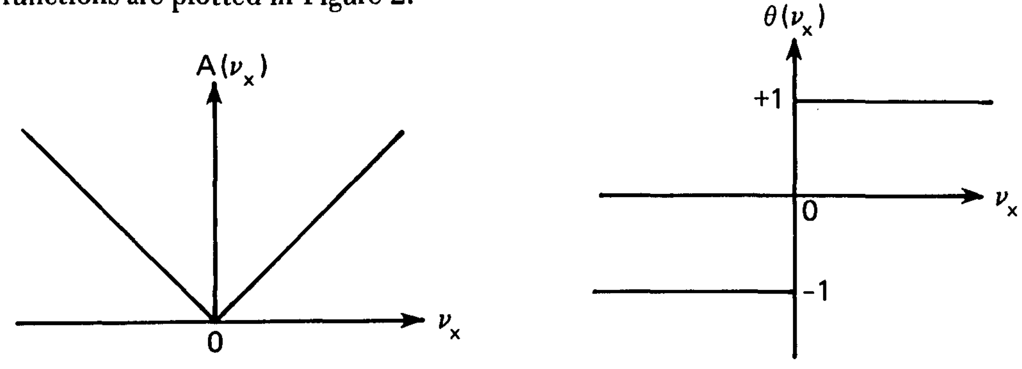

Figure 2. Decomposition of the Complex Filter, $i\nu_x$, into Amplitude and Phase Filters.

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[5] R. L. Cody, *A Comparison of Various Coherent Optical Filtering Operations,* Arnold Engineering Development Center, AEDC-TR-71-137, June 1971.

[6] R. G. Eguchi, and F. P. Carlson, "Linear Vector Operations in Coherent Optical Data Processing Systems," *Applied Optics,* Vol. 9, No. 3, March 1970, pp. 687-694.

It may be seen that $A(\nu_x)$ is a linear amplitude filter and $\theta(\nu_x)$ is a half-wave phase plate.

The reason for the one-dimensional analysis should be more apparent now, since $\theta(\nu_x)$ is also one dimensional. As a result, the optical system displays the directional derivative of the object function. In order to display slopes in other directions, either the input transparency or the spatial filter must be rotated.

2. **Optical System Configuration.** A practical design of a coherent optical data processor which requires a complex spatial filter is shown in Figure 3. This particular system provides separate planes for locating the amplitude and phase filters. The purpose of lenses L2 and L3 is to image the phase filter onto the amplitude filter to ensure that the two filters are effectively in the same Fourier transform plane. In contrast to a two-lens coherent optical data processor, this system enables more accurate superpositioning of the spatial filters in an optical plane.

3. **Filter Realization.** In general, the amplitude filter must be produced on a photosensitive material. However, the requirement that the filter be opaque at $\nu_x = 0$ is impossible to achieve.

A filter, $F(\nu_x)$, which is a good approximation to the ideal derivative filter, is given below:

$$F(\nu_x) = |\nu_x| \operatorname{sgn}(\nu_x) \left[\operatorname{rect}(\xi \mid b) - \operatorname{rect}(\xi \mid a)\right] \qquad b > a$$

or

$$F(\nu_x) = i\nu_x \left[\operatorname{rect}(\xi \mid b) - \operatorname{rect}(\xi \mid a)\right] \tag{5}$$

The function, $\operatorname{rect}(\xi \mid a)$, is a one-dimensional dc block; i.e., it is a strip of width 2a. The function, $\operatorname{rect}(\xi \mid b)$, truncates the complex filter between -b and +b. A diagram of the amplitude filter is shown in Figure 4. The phase filter is the same as in Figure 2. Because the filter requires a finite value of the minimum transmittance, τ_{min}, it is physically realizable.[7]

We now consider the intensity in the image plane of the optical system using the filter, $F(\nu_x)$. The image amplitude, $U(\nu_x)$, in the Fourier transform plane after passing through the spatial filter is given as

$$U(\nu_x) = \widetilde{T}(\nu_x) F(\nu_x) \tag{6}$$

[7] R. L. Cody, *A Comparison of Various Coherent Optical Filtering Operations*, Arnold Engineering Development Center, AEDC-TR-71-137, June 1971.

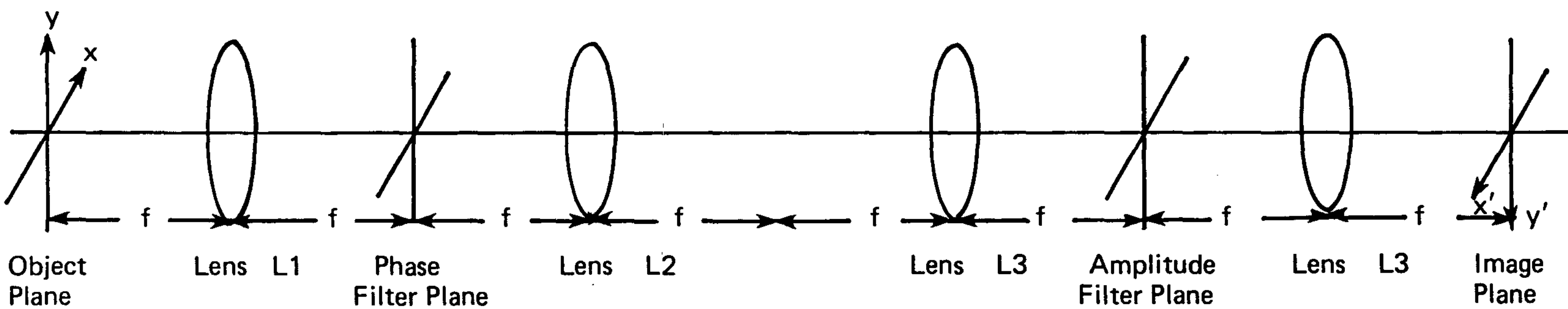

Figure 3. Practical Design for a Coherent Optical Data Processing System Performing Complex Spatial Filtering Operations.

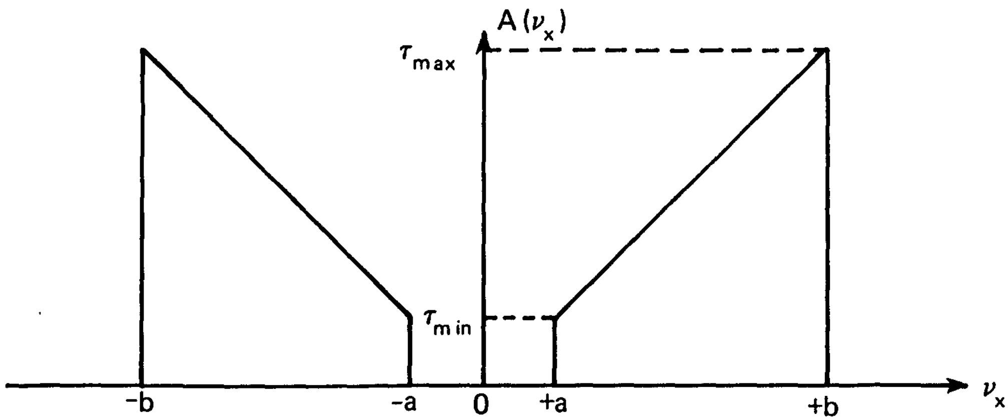

Figure 4. Amplitude Filter for Approximating the Derivative of an Input Transparency to a Coherent Optical Data Processing System.

Substituting equation (5) into (6) we get

$$U(\nu_x) = \widetilde{T}(\nu_x)\, i\nu_x\, [\mathrm{rect}\,(\xi \mid b) - \mathrm{rect}\,(\xi \mid a)]$$

In the image plane we get

$$U(x') = \mathfrak{F}\,[\widetilde{T}(\nu_x)] \oplus \mathfrak{F}\,[F(\nu_x)]$$

where $\oplus$ denotes the convolution operation.

Therefore,

$$U(x') = \frac{d}{dx'}\, T(x') \oplus H(x')$$

where $H(x') = 2b \text{ sinc } \dfrac{kbx'}{f} - 2a \text{ sinc } \dfrac{kax'}{f}$.

Finally, the intensity in the image plane is given as

$$I(x') = \left| \frac{d}{dx'}\, T(x') \oplus H(x') \right|^2 \tag{7}$$

In equation (7), $H(x')$ represents the impulse response of the optical system. The resultant image is, therefore, the square of the convolution of the derivative of the input function with the impulse response of the system. This means that the coherent optical system displays both the negative and positive slopes as being positive.

4. **Filter Design Parameters.** In order that the differentiation system will perform optimally, the various filter parameters must be properly controlled. These parameters

are listed below:

1. The width of the dc blocking strip for the amplitude filter.
2. The limiting width of the amplitude filter.
3. The slope of the amplitude filter.
4. The size of the phase filter.
5. The method by which the phase delay will be controlled.

Qualitatively, the width of the dc blocking strip, 2a, should be as small as possible, since the strip causes loss of low frequency detail in the image plane.

The limiting width of the amplitude filter, 2b, is governed by the cutoff frequency, ν_o, of the coherent transfer function of the input transparency. The reason for this constraint is that no information is contained in frequencies greater than the cutoff frequency. Therefore, one must have prior knowledge of the transfer function of the input transparency.

It is also wise to match ν_o with the cutoff frequency of the Fourier transform lens, ν'_o. This insures that the noise power spectrum of the transparency greater than ν_o will not be passed.

We can now write the relationship between the lens parameters and the value of the cutoff frequencies as

$$\nu_o = \nu'_o = \frac{d}{2\lambda f}$$

where d is the lens diameter, and f is the lens focal length.

After determining the above parameters, the halfwidth, b, of the amplitude filter can be found. Remembering that $\nu = \xi/\lambda f$, we can substitute b for ξ, and get

$$\nu_o = \frac{b}{\lambda f}$$

Therefore,

$$b = \nu_o \lambda f$$

The system parameters are also used to determine the slope of the linear amplitude filter. Equation (3) can be written as

$$A(\nu_x) = |P\xi|$$

where

$$P = \frac{\tau_{max} - \tau_{min}}{b - a}$$

The last consideration is the design of the phase filter. In order to get a phase shift of $\lambda/2$ over half the Fourier transform plane, a dielectric material which delays the light waves by π radians with respect to air is coated on one-half of a glass plate. The size of the plate must be equal to, or greater than, the limiting width of the amplitude filter.

5. **Filter Production.** The experimental details for the production of the amplitude and phase filters will not be discussed in great length. Most methods used to produce each filter type are based on the same principles. Hence, only the general theory will be discussed.

Amplitude filters having particular transmittance distributions are produced on photographic film. It is first necessary to calibrate the nonlinear response of the transmittance as a function of exposure. Using the calibration or film characteristic curve, the proper exposure as a function of distance on the film is determined so that the desired transmittance as a function of distance is obtained. Most methods which have been described in the literature are based on controlling the exposing time to produce a particular transmittance function. Some of these methods are listed below:

1. Exposing film through a uniformly illuminated rectangular mask where one side of the mask moves during the exposure. To control the exposure, the velocity of the mask edge is varied.[8]

2. Exposing film through a uniformly illuminated variable area sliding mask where the incremental area as a function of slide direction controls the exposure. Some systems reverse this arrangement letting the mask remain stationary and having the film move.[9] [10]

[8] R. L. Cody, *A Comparison of Various Coherent Optical Filtering Operations*, Arnold Engineering Development Center AEDC-TR-71-137, June 1971.

[9] R. A. Sprague, and B. J. Thompson, "Quantitative Visualization of Large Variation Phase Objects," *Applied Optics*, Vol. 7, No. 7, July 1972, p. 1,472.

[10] F. Scott, "The Production of Variable Transmission Sinusoidal Patterns and Other Images," *Photographic Science and Engineering*, Vol. 9, No. 2, March-April 1965, pp. 86-90.

3. Exposing film on apparatus controlled by computers such as microdensitometers.[11] [12]

Phase filters are made by depositing a dielectric coating on glass. The thickness of the dielectric coating is controlled by vacuum evaporation techniques. These filters are available from various optical manufacturers and thin film coating facilities.

6. Calibration of the Input Transparency. In order to extract quantitatively the slope values in the optical data processor, the input transparency must be properly calibrated so that its transmittances are a known function of the surface elevations. The elevation data base is obtained from map compilation systems, such as the UNAMACE, and stored on magnetic tape. Output of this data base onto photographic film may be accomplished using the UNAMACE or DICOMED equipment.

The relationship between the transmittances, T, and the elevations, Z, should have a form which simplifies the computation of slopes in the coherent optical data processor. Slope computation can be greatly facilitated if this relationship is given as

$$T(x,y) = [Z(mx, my)]^{1/2} \tag{8}$$

where m is the scale factor relating the coordinates of the ground to the coordinates of the transparency.

Equation (8) compensates for the fact that the coherent processor displays an image that is the square of the derivative of the transmittance function of the input transparency. By first changing the coordinate system of equation (8) to conform with the image plane of the coherent processor and substituting a one-dimensional form of this equation into equation (7), we get

$$I(x') = \left| \frac{d}{dx'} [Z(mx')]^{1/2} \oplus H(x') \right|^2 \tag{9}$$

As a result, the value of the slopes are directly related to the image intensities.

In order for equation (8) to be valid, the elevation data base must go through a series of transformations similar to those described in the previous section when producing amplitude filters having particular transmittance distributions. Therefore, the conversion of surface elevations to exposures must compensate for the film response to

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[11] A. W. Lohman, and D. P. Paris, "Computer Generated Spatial Filters for Coherent Optical Data Processing," *Applied Optics*, Vol. 7, No. 4, April 1968, pp. 651-655.

[12] K. Campbell, G. W. Wecksung, and C. R. Mansfield, "Spatial Filtering by Digital Holography," *Optical Engineering*, Vol. 13, No. 3, May-June 1974, p. 175.

get the desired relationship between elevations and transmittances of the input transparency. Specifically, preprocessing of the surface elevation data is required for the transformation to the output exposures. This can be done, for example, under computer control of the DICOMED display.

7. **Calibration of the Optical System.** The final requirement for quantitative determination of the slope values is the calibration of the coherent optical data processing system. In particular, the image intensities as measured by a detector must be calibrated.

The preceding analysis demonstrated that if the input transparency is calibrated according to equation (8), then the image intensities are a direct measure of the terrain surface slopes. Calibration of the optical processor, therefore, requires that a special target transparency with known values of amplitude transmittance as a function of distance (transmittance profile) be input to the optical system and the measured image intensities be correlated with the values of the slopes. The target transparency should be designed to calibrate not only the image intensities but also the positional accuracies of the optical system. This target should have the following form:

1. It should have regions containing transmittance profiles having constant slopes to ensure that image intensities are reliably correlated with the slopes. These regions should be large enough so that calibration of the slopes is not dependent on positional accuracy.

2. It should have a region containing a continuous sloping transmittance profile to test the positional accuracy of a scanning detector.

These particular transmittance profiles can be produced by the methods cited in the section II, para. 5, describing production of the spatial filters.

III. DISCUSSION

Although a directional derivative can be displayed, the impulse response of the coherent imaging system causes a significant loss in resolution. Unfortunately, the reduction in resolution must be tolerated owing to the effects of diffraction; i.e., the finite size of the spatial filter. The effect of the optical system is an important consideration; however, the components that ultimately determine the system resolution are the detection optics. To be compatible with the coherent imaging system, the detection optics must have a resolution equal, at least, to the imaging system.

The accuracy of the system is also affected by the type of sloping terrain exhibited in the imagery. High sloping terrain, or edges, will cause ringing (Gibb's phenomenon) which, in turn, will cause errors in extracting slope values in the neighborhood of these locations. These problems also apply to input imagery produced by digital means. As the contrast between adjacent points decreases, the errors caused by ringing will decrease. An excellent discussion of systematic errors caused by ringing in coherent imaging systems is provided by Parrent and Thompson.[13]

The fact that this system detects the squared modulus of the input amplitude transmittance causes ambiguity in the interpretation of negative and positive slopes. When using this slope map in conjunction with a contour map of a given area, the sign of the slope is interpretable. However, the deliniation of local slope reversals (adjacent regions where slopes are of equal magnitude, but opposite in sign) would be very difficult to realize in a manner that could be formatted for easy application.

A final consideration is the effect of phase information present in the input transparency. Any systematic or random phase distributions, such as thickness variations, will be visualized in the image plane.[14] To eliminate this, the input transparency must be immersed in a liquid gate.

These effects are all possibly degrading to the intended use. How significant the degradation will be in a practical sense requires experimentation.

IV. OTHER APPLICATIONS

The operation of performing a derivative by optical means has other applications, in addition to the one described above. Of particular importance is the relationship between this system and other ongoing efforts at ETL in the area of image quality evaluation.

It is well known in the field of photographic image quality analysis that the first derivative of the transmittance profile of an image on film produced by a perfect edge is the line-spread function of the photo-optical system (film/camera combination) used to produce the edge. Thus, the optical processor described in this report has the ability to rapidly display the square of the line-spread function of a photo-optical system given the necessary input imagery on a photographic transparency. With proper calibration of a scanning photometer, the profile of the line spread function can be easily determined. The usefulness of the line spread function is based on the fact that other important image quality parameters, e.g., resolution and acutance, arise from it. More fundamentally, it is the spread function of the system which determines the image, i.e., the image function

[13] G. B. Parrent, and B. J. Thompson, *Physical Optics Notebook*, Chapter 15, Society of Photo-Optical Instrumentation Engineers, Rendondo Beach, California, 1969.

[14] R. A. Sprague, and B. J. Thompson, "Quantitative Visualization of Large Variation Phase Objects," *Applied Optics*, Vol. 7, No. 7, July 1972, pp. 1,469 - 1,478.

is the convolution of the object function with the point spread function of the photo-optical system.

V. CONCLUSIONS

It is concluded that:

1. Quantitative values of the terrain slopes may be determined with careful calibration of a coherent optical data processing system and the input transparency.

2. Most random noise errors in determining the terrain slopes using coherent optical data processing may be minimized by the proper design of the spatial filter and the judicious selection of the optical system components.

3. Other errors, due to ringing, phase shift, etc., may require experimental determination of their practical effect on the output.

4. A device so designed, in addition to its special application to slope computation may have a more general application to the study of image quality factors of edge analysis and line spread functions.

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GLOSSARY OF SYMBOLS

Term	Meaning
(x,y)	Object transparency plane coordinates
(x',y')	Image plane coordinates
(ξ,η)	Fourier transform plane coordinates
(ν_x,ν_y)	Spatial frequencies in lines per millimeter corresponding to the coordinates in the Fourier transform plane
λ	Wavelength of light
f	Lens focal length
d	Lens diameter
$T(x,y)$	Amplitude transmittance function of the object transparency
$T(x',y')$	Amplitude transmittance function of the unfiltered image
$\widetilde{T}(\nu_x,\nu_y)$	Amplitude transmittance function of the Fourier transform of the object transparency amplitude transmittance function
$\mathfrak{F}$	Fourier transform operation
$\mathfrak{F}^{-1}$	Inverse Fourier transform operation
$A(\nu_x)$	Amplitude transmittance function of the spatial filter
$\theta(\nu_x)$	~~Phase function of the spatial filter~~
$F(\nu_x)$	Approximating spatial filter function
$H(x')$	Impulse response of the optical system in one direction
$I(x')$	Transmittance function of the filtered image
$Z(x,y)$	Surface terrain elevation function