ETL-0162

UG470
U5d
no. 0162
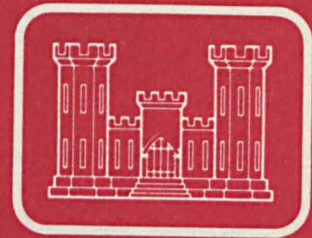
Investigation of discrete function technology for topographic sciences

Frederick W. Rohde

OCTOBER 1978

U.S. ARMY CORPS OF ENGINEERS

ENGINEER TOPOGRAPHIC LABORATORIES

FORT BELVOIR, VIRGINIA 22060

$5^{'}0 2^{5-}7 9^3$

U G-T-
U J-d
ho.c/6 2-

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ETL- 0162	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INVESTIGATION OF DISCRETE FUNCTION TECHNOLOGY FOR TOPOGRAPHIC SCIENCES		5. TYPE OF REPORT & PERIOD COVERED Research Note
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Frederick W. Rohde		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 4A161101A91D, Task 01 Work Unit 0060
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060		12. REPORT DATE October 1978
		13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
Applications of discrete function technology in the topographic sciences with emphasis upon the extraction of cartographic features from images is discussed. Specifically, research was done on the use of various orthogonal sets of functions for decomposing topographic images in spectral components and whether such decompositions are useful for the extraction of cartographic and terrain features. Also, Fourier, Block, Walsh, and Haar transforms of images for machine feature extraction were compared, and the potential of devices that may facilitate direct discrete transforms using electro-optical technology was studied.
(continued)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

20. continued

The report concludes that discrete function technology can be applied to at least three areas of the topographic sciences, namely to image analysis for cartographic and terrain feature extraction, to geopotential representation, and to remote sensing.

CONTENTS

ILLUSTRATIONS

3

INVESTIGATION OF DISCRETE FUNCTION TECHNOLOGY FOR TOPOGRAPHIC SCIENCES

INTRODUCTION ■ The purpose of this Work Unit was to determine the feasibility and potential merits of discrete function technology for applications in

Objectives the topographic sciences, emphasizing the extraction of cartographic features from images. Specific objectives of this study were to

1. Research the use of various orthogonal sets of functions for decomposing topographic images in spectral components.
2. Determine whether such decompositions or transforms are useful for the extraction of cartographic and terrain features.
3. Compare the use of Fourier, Block, Walsh, and Haar transforms of images for machine feature extraction.
4. Research the potential of devices that may facilitate direct discrete transforms using electro-optical technology.
5. Research other applications of discrete functions to the topographic sciences.

Image analysis for the purpose of enhancement, restoration, coding, and data and feature extraction emerged as a new field of research based primarily on the newly

Background developed technologies for television (TV) and digital computers.

In TV, the image of the TV camera is scanned and converted into a time-dependent electrical signal. This signal is transmitted through communication channels to the receiver and restored to the image. To determine optimum channel bandwidth, filter performance, and possible bandwidth compression, the signal would be transformed from the time domain into the frequency domain. For the most part, this transformation is accomplished in communciations by the application of the Fourier transform and is also called the decomposition of the signal in its spectral components. If the spectral components of a signal are known, suitable filters may be designed that reject most other signals and noise and pass only the signal for which the filter is designed. Thus, filters may be used for signal processing and extraction. A TV signal consists of a carrier, video modulation representing the image content, sound modulation, and synchronization signals. The various components of the TV signal are separated, extracted, and processed by filters having special purpose spectral response characteristics. Certain processing functions of a TV set, namely brightness, contrast, hue, tint, may be adjusted by the viewer to his satisfaction. These processing functions are also used in image analysis.

From digital computer technology emerged the simulation of signal processing capabilities and systems. The large storage capacity of digital computers and the

4

feasibility of digitizing images in a way that they can be stored and processed in a computer led to the techniques of digital image processing and visual pattern recognition.

Research was conducted on the discrete function technology as used in the analysis and feature extraction of topographic images, in representing the earth's gravity field, and in remote sensing applications of discrete electromagnetic wave forms. The theory, generation, and properties of potentially useful discrete functions are explained by examples. The applications to topographic sciences are investigated and also demonstrated by selected examples. In addition, various hardware approaches for implementation of discrete function technology are critically examined, and some concepts of machine feature extraction from topographic images are presented.

Scope and Approach

THEORY AND PROPERTIES OF DISCRETE FUNCTIONS

Basic Concepts

■ A rule of correspondence that associates a real or complex number, $y = f(x_1, x_2, ..., x_n)$, with each given set of real or complex numbers $x_1, x_2, . . ., x_n$ is called a function of $x_1, x_2, ..., x_n$. The function can be expressed analytically or by an n-dimensional table. The arguments may assume all real and complex numbers between $-\infty$ and $+\infty$ and/or portions thereof. Many of the functions commonly treated in applied mathematics are continuous and have finite derivatives.

The y-values of discrete function are frequently real and vary only in discrete steps. Many discrete functions of interest assume only the values

$$-1, 0, +1 \text{ or just } 0 \text{ and } 1.$$

If the discrete function assumes only two values, such as 0 and 1 or -1 and +1, the function is also called a binary function. Most discrete functions have integer values and rarely exceed 1000. Discrete functions are not necessarily continuous and possess no derivatives in the normal sense.

Discrete functions used in computers and communication equipment include the output of analog-to-digital converters, digital voltmeters, digital clocks, maximum length code generators, and teletype machines. This report deals with discrete functions as they pertain to the time, the spatial coordinates x and y of a plane, or the three-dimensional space coordinates.

Signals for TV have two space variables and one time variable. The space variables are derived from the image of the TV camera by a scanner in the case of digital TV. The scanner operation may be described by a two-dimensional block pulse function.

Block Pulse Function
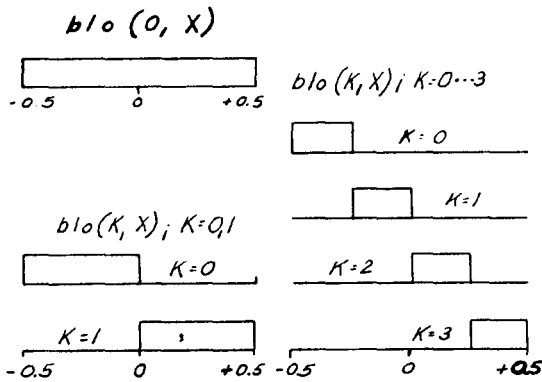
$blo\ (0,\ X)$

$blo\ (K,X)_i\ K\cdot 0\cdots 3$

$blo\ (K,X)_i\ K=0,1$

$blo\ (K,X)_i\ K=0\cdots 7$

Figure 1. Graphic Representation of the
Block Pulse Functions

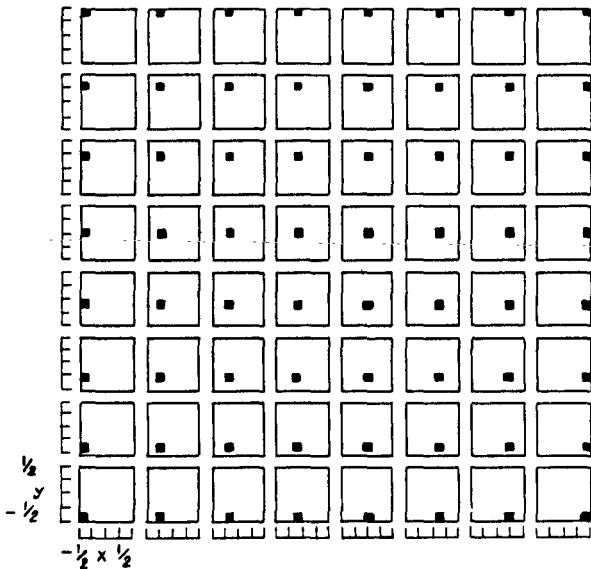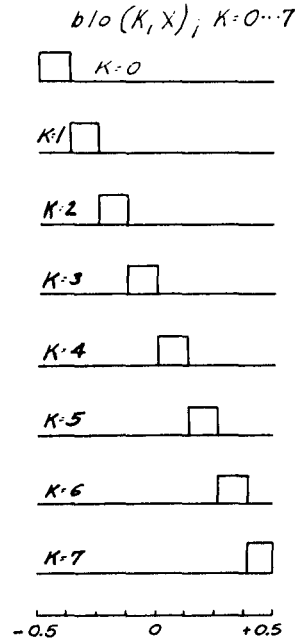$blo(k_1 x)$ for K = 0, K = 0,1
K = 0,1,2,3
and
K = 0,1,2,3,4,5,6,7

The block pulse function blo(k,x) is defined in the interval $-\frac{1}{2} \langle x \langle +\frac{1}{2}$. It has the values 0 and +1. The pulse shifts from left to right within the interval. The parameter K = 0, 1, 2, ...,n defines the number of shifts in equal steps within the interval. Figure 1 shows examples of one-dimensional block pulse functions. Two-dimensional block pulse functions may be written as products blo(K,x) blo(ℓ, y). An example of two-dimensional block functions is shown in figure 2.

Block pulse functions are orthogonal, which is defined as two functions f(i,x) and f(j,x) with the variable x and the parameters i or j in the interval $-\frac{1}{2} \langle x \langle +\frac{1}{2}$ if the integral

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} f(i,x)\ f(j,x)\ dx$$

is zero for $i \neq j$ and non-zero for $i = j$. The functions are normalized and called orthonormal if the integral equals 1 for $i = j$. From figure 1, the integral condition for orthonomaly is verified for block pulse functions.

Figure 2. Two-Dimensional Block Pulse Functions

blo (k,x) blo (ℓ,y)
K = 0,1,2,3,4,5,6,7,
ℓ = 0,1,2,3,4,5,6,7,

6

Sets of orthogonal functions are suitable for decomposing a given function in its spectral components of the orthogonal function. The most commonly known case of spectral decompositions is the Fourier analysis, where periodic functions are decomposed into an infinite series of sine and cosine functions and nonperiodic functions are decomposed into a Fourier integral that represents the continuous spectral density distribution from minus to plus infinity. Correspondingly, the block pulse spectral component of a function f(x) existing between -½ and + ½ may be expressed by

$$B_m = \int_{-\frac{1}{2}}^{+\frac{1}{2}} f(x) \; blo\;(m,x) \; dx$$

If an image of the dimension a x b is being analyzed by a block pulse function, a is scaled to fit into the interval -½ ⟨ x ⟨ ½ and b into -½ ⟨ y ⟨ ½. The two-dimensional spectral component B_{mn} is expressed by

$$B_{mn} = \int_{-\frac{1}{2}}^{+\frac{1}{2}} \int f(x,y) \; blo(m,x) \; blo(n,y) \; dx \; dy$$

where B_{mn} represents the average transparency, or brightness, of the image in the area element designated by the block pulse value blo (m,x) blo(n,y). The spatial arrangement and characteristics of the spectral components and further signal processing may be used to identify and extract topographic terrain features.

Walsh functions are another example of a system of orthonormal functions. One-dimensional Walsh functions are expressed by wal (k,x), where k represents the order, and x the variable. These functions assume the two values +1 and -1 only, which makes them comparatively simple as the block pulses and leads to simple equipment. The Walsh functions are defined in the interval -½ ⟨ x ⟨ + ½ and are zero everywhere else. They are global functions, that is, the functions have defined values over the entire interval. The shape of the Walsh function is more complicated than the block pulse function and resembles somewhat the sine and cosine functions. In some cases, the Walsh functions are periodic within the interval; in other cases, they are nonperiodic. The functions wal (k,x) may be defined by the difference equation

Walsh Functions

$$wal \; (2K+p,x) = (-1)^{[K/2]+p}[wal \; (K,2x+\frac{1}{2}) \; +(-1)^{K+p}wal \; (K,2x-\frac{1}{2})]$$

The expression [K/2] means the largest integer less than or equal to K/2.

$$K = 0, 1, 2, 3, ..., n \qquad\qquad p = 0, 1$$

$$and \; wal \; (o,x) = \begin{Bmatrix} 1 \; for \; -\frac{1}{2} ⟨ x ⟨ \frac{1}{2} \\ 0 \; for \; x ⟨ -\frac{1}{2}, \; x ⟩ \frac{1}{2} \end{Bmatrix}$$

7

In the following, the walsh functions wal (1,x) to wal (5,x) are developed using the generating formula:

$$K = 0 \quad p = 1$$
$$wal (1,x) = -wal (0, 2x + \tfrac{1}{2}) + wal (0, 2x - \tfrac{1}{2})$$
$$K = 1 \quad p = 0$$
$$wal (2, x) = wal (1, 2x + \tfrac{1}{2}) - wal (1, 2x - \tfrac{1}{2})$$
$$K = 1 \quad p = 1$$
$$wal (3, x) = -wal (1, 2x + \tfrac{1}{2}) - wal (1, 2x - \tfrac{1}{2})$$
$$K = 2 \quad p = 0$$
$$wal (4,x) = -wal (2, 2x + \tfrac{1}{2}) - wal (2, 2x - \tfrac{1}{2})$$
$$K = 2 \quad p = 1$$
$$wal (5,x) = wal (2, 2x + \tfrac{1}{2}) - wal (2, 2x - \tfrac{1}{2})$$

Figure 3 shows the first 10 Walsh functions wal (K,x). All of these functions meet the orthogonality condition. Therefore, any continuous function defined in the interval $-\tfrac{1}{2} \langle x \langle \tfrac{1}{2}$ can be decomposed into spectral components of a Walsh function.

Two-dimensional Walsh functions are written as products wal (K,x) (wal ℓ,y). Figure 4 shows the two-dimensional Walsh functions from wal (0,x) wal (0,y) to wal (7,x) wal (7,y). The two-dimensional spectral Walsh component W_{mn} of the function f (x,y) is

$$W_{mn} = \int\int_{-\tfrac{1}{2}}^{+\tfrac{1}{2}} f(x,y) \, wal (m,x) \, wal (n,y) \, dx \, dy$$

if f(x,y) is an image function and $-\tfrac{1}{2} \langle x \langle \tfrac{1}{2}$ and $-\tfrac{1}{2} \langle y \langle \tfrac{1}{2}$ is scaled to match the dimensions of the image, the spectral components W_{mn} are representative of the entire image rather than an area element as in the case of spectral block pulse components.
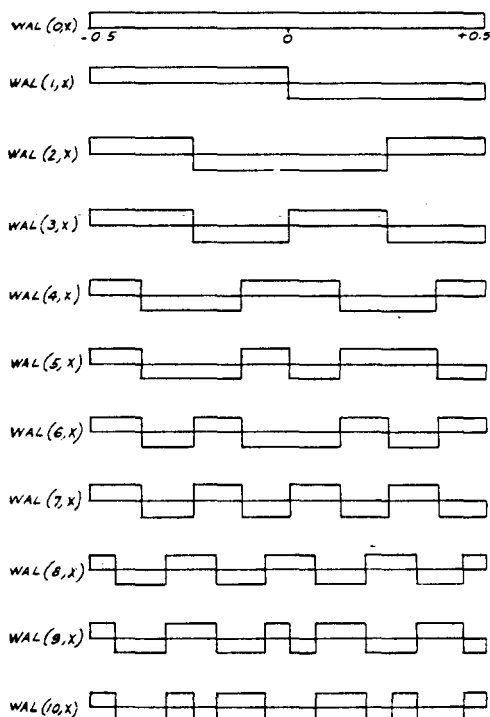

Figure 3. The First 10 Walsh Functions. wal(k,x)
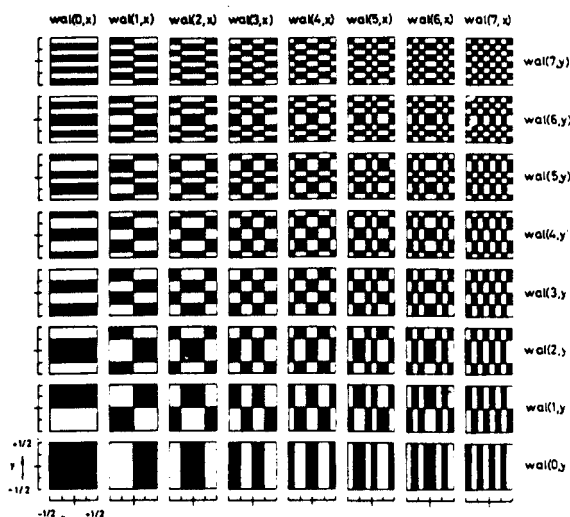K = 0, 1, ...,10

Figure 4. Example of Two-Dimensional Walsh Functions.

8

Haar functions assume the values of +1, 0, and -1. The first few functions are shown in figure 5. The first two Haar functions are identical with the first few

Haar Functions

Walsh functions. The one-dimensional Haar function is expressed by har (K,ℓ,x). It is

$$\text{har}(0, 0, x) = \text{wal}(0, x)$$
$$\text{har}(0, 1, x) = \text{wal}(0, x)$$

The generating process for Haar functions is as follows:

The first two Haar functions are defined as the first two Walsh functions. The function har $(0, 1, x)$ is then squeezed into half the interval and shifted to yield har $(1, 1, x)$ and har $(1, 2, x)$. Each one of the new functions is squeezed again in half of its space and shifted to yield har $(2, 1, x)$, har $(2, 2, x)$, har $(2, 3, x)$, and har $(2, 4, x)$. This process, if continued, generates the set of all Haar functions.

Entirely new sets of discrete orthogonal functions f_n can be generated by using Walsh functions, and by using the Haar process of squeezing to one-half and shifting. For example,
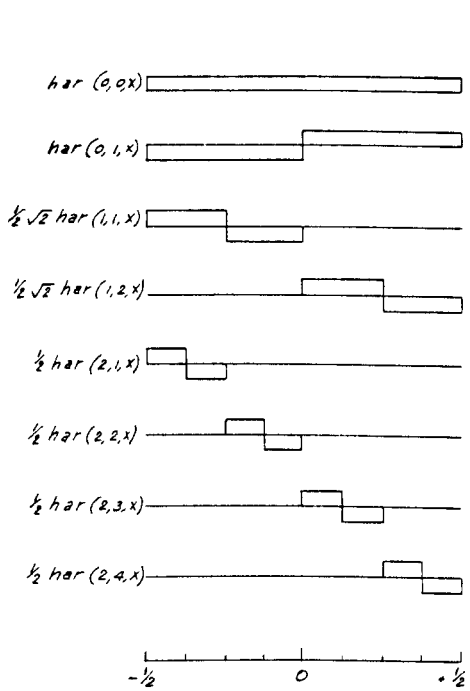
wal $(0, x) = f_1(0, 0, x)$	or	wal $(D, x) = f_2(0, 0, x)$
wal $(1, x) = f_1(0, 1, x)$		wal $(1, x) = f_2(0, 1, x)$
wal $(2, x) = f_1(0, 2, x)$		wal $(2, x) = f_2(0, 2, x)$
$f_1(1, 1, x)$		wal $(3, x) = f_2(0, 3, x)$
$f_1(1, 2, x)$		$f_2(1, 1, x)$
$f_1(2, 1, x)$		$f_2(1, 2, x)$
$f_1(2, 2, x)$		$f_2(2, 1, x)$
$f_1(2, 3, x)$		$f_2(2, 2, x)$
$f_1(2, 4, x)$		$f_2(2, 3, x)$
		$f_2(2, 4, x)$

Figure 5. Examples of Haar Functions.

Two-dimensional Haar and f_n functions are expressed as products

$$\text{har}(m, i, x) \ \text{har}(n, j, y) \ \text{or}$$
$$f_n(m, i, x) \ f_n(n, j, y) \ \text{respectively.}$$

As with block pulse and Walsh functions, a spectral decomposition can occur with the image function decomposing into a series of Haar or f_n components.

A matrix M is called an orthogonal matrix if the inverse M^{-1} and the transposed M are proportional to each other. A Hadamard matrix is an orthogonal matrix

Hadamard Matrices

with elements +1 and -1 only. It is convenient to write instead of +1 and -1 simply + and -. The following are the Hadamard matrices of the rank 1 through 4:

9

$$H_1 = |+|$$

$$H_2 = \begin{vmatrix} + & + \\ + & - \end{vmatrix}$$

$$H_{41} = \begin{vmatrix} + & + & + & + \\ - & - & + & + \\ - & + & + & - \\ + & - & + & - \end{vmatrix}$$

$$H_{42} = \begin{vmatrix} + & + & + & - \\ + & + & - & + \\ + & - & + & + \\ - & + & + & + \end{vmatrix}$$

Hadamard matrices of higher rank can be generated by Kronecker products. This process is explained by the following example:

$$H_2 \otimes H_2 = \begin{vmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{vmatrix} = \begin{vmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{vmatrix}$$

Because the rows and columns of the Hadamard matrices are orthogonal sets, they can be used to decompose, for example, a two-dimensional image function into its spectral components. The orthogonal condition for the rows of an Hadamard matrix having N^2 elements is

$$\sum_{K=1}^{N} a_{ik}\, a_{jk} = N\sigma_{ij} = \begin{cases} 0 & \text{for } i \neq j \\ N & \text{for } i = j \end{cases}$$

The row elements a_{ik}, $K = 1, 2, \ldots, N$ may be considered to represent a discrete function that assumes the values 1 and -1 within the interval $0 < x < N$. The discrete function may be expressed by a (i,x) where

$$a(i,x) = a_{i1} \text{ for } 0 < x < 1$$
$$a(i,x) = a_{i2} \text{ for } 1 < x < 2$$

The same procedure may be applied for the column elements a_{nK} $i = 1, 2, 3, \ldots, N$ to produce a discrete function $a(k,y)$.

The corresponding two-dimensional spectral components for $f(x,y)$ are expressed by

$$M_{iK} = \int\int_{O}^{N} f(x,y)\, a(i,x)\, a(k,y)\, dx\, dy.$$

A shift register consists of n-consecutive binary storage elements called stages and of a feedback network. Figure 6 shows a shift register with 10 stages. The binary values (e.g. two discrete voltages) that can be stored in each stage may be defined as value zero (0) and value one (+1). The feedback network feeds the values of some designated stages into a modulo 2 adder. Depending on the input values of the adder, the output of the adder will assume the value 0 or +1. The output of the adder is applied to the input of the 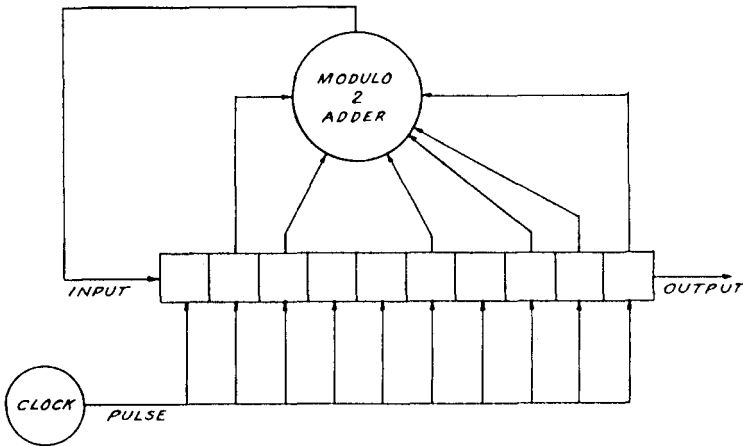shift register. The shift register is operated by applying a clock pulse to each stage, which causes the value of each stage to be shifted to the next stage and the output of the feedback network to be shifted to the first storage of the shift register. The output of the shift register is then a discrete sequence of 0 and 1. The output series may consist of up to $2^n - 1$ bits for a n-stage register, depending on the section of the feedback network. The word bit refers to the output of the shift register which is generated by a one-clock pulse. Practically all discrete functions including multivalue functions may be generated by a suitable set of shift registers, including switchable feedback connections and appropriate logic circuitry. This capability can be used to generate many reference signals, in addition to orthogonal sequences, that can be used for correlation with signals derived from images.

Discrete Functions by Shift Registers

MODULO 2 ADDER

INPUT

OUTPUT

CLOCK   PULSE

Figure 6. 10-Stage Shift Register with Feed Back Network

APPLYING DISCRETE FUNCTION TECHNOLOGY TO TOPOGRAPHIC SCIENCES

■ Discrete function technology can be used in three areas of the topographic sciences: (1) in aerial photographs of topographic terrain, (2) in representing the earth's gravity field by Walsh functions, and (3) in using discrete electro-magnetic wave forms for remote sensing. Discrete functions can be electronically generated by logic switching circuitry using integrated components. The switching speed of these components (TTL and ECL) is typically a few nanoseconds and is expected to increase by the factor 100.[1] Therefore, discrete functions may be generated at a tremendous rate, e.g. up to several millions per second.

---

[1]Anon. "High Performance Josephson Circuits Described by IBM Researchers,"*IEEE Computer*, vol.11, No. 5, p. 102, May, 1978.

The area elements, or pixels, of a black-and-white photo are characterized by their gray shades or degrees of transparency. Hence, the image can be described as a two-dimensional gray shade function expressed, for example, in Cartesian or polar coordinates. Topographic and terrain features recorded by a photographic plate or film are recognizable by their shape, contours, and gray shade distribution. Because each feature has its characteristic spectral components for every set of orthogonal functions, spectral components of features may be useful for recognition, identification, and extraction of cartographic features.

Filters for Two-Dimensional Image Signals

Two-valued discrete functions are particularly suited for development of spectral components because they can easily be generated with off-the-shelf components. Furthermore, electro-optical devices are now being developed that can be used for optical generation of two-dimensional discrete functions. The one value 0, or -1 is facilitated by dark or opaque area elements, and the other value +1 by light or transparent area elements. Four types of electro-optical filters for generating discrete functions are described.

Nematic liquid crystal filters. Liquid crystals are organic substances whose state of matter is between the solid crystaline state and the isotropic liquid state. The liquid crystals are classified into various groups. One group, for electro-optical devices is the nematic liquid crystals. In the nematic liquid crystals, the molecules are rod shaped and arrange themselves parallel in volume elements containing typically 10^4 to 10^5 molecules. The volume elements have characteristics similar to small anisotropic crystals. For electro-optical applications, many of the volume elements are oriented uniformly by putting the nematic liquid crystal substance between two plane glass plates, which are kept at a distance of 10 to 100 micrometers by supports, such as films, threads, etc. The inner sides of the glass plates are plated with transparent electrodes ($Sn\,O_2$, $In\,O_3$) that make it possible to effect the liquid crystal between the plates by electrical fields. The transparent electrodes can be arranged in a pattern of parallel stripes isolated by very small gaps from each other. To make an optical filter, the glass plates are arranged so that the stripes of the one plate (x-direction) are perpendicular to the stripes of the other plate (y-direction). By applying voltages on crossed stripes, the area in between can be made either transparent or opaque. By applying the values of discrete functions electronically to the x-stripes and y-stripes, optical filters representing two-dimensional orthogonal discrete functions are realized.

Two effects of nematic liquid crystals can be exploited for the realization of optical filters. One effect, called the Dynamic Scattering (DS), requires non-polarized monocromatic light for illumination. The other effect, called Deformation Aufgerichteter Phasen (DAP) requires polarized monochromatic light for illumination. The contrast ratio betweeen transparency and opaqueness for the DS effect is 1:50, and for the DAP effect, 1:1000. The total response time, that is the time needed for switching from one pattern to another, requires approximately 3 microseconds for the DS effect and approximately 6 nanoseconds for the DAP effect.

Photoferroelectric (PFE) ceramics. Nonvolatile images can be stored in phase lead lanthanum zirconate titanate (PLZT) ceramic by using near-ultraviolet(UV) light and switching the ferroelectric polarization through a portion of the hysteresis loop. The images are stored as spatial distribution of scattering centers and as surface deformations. Erasure of stored images is achieved by using near-UV light and switching the ferroelectric polarization back to its initial state. In addition to the image storage capability, PFE ceramics may be used for devices that generate optical filters consisting of patterns of transparent and opaque areas. These filters could be used in a similar way as liquid crystal filters. The advantage of PFE-PLZT ceramics is that they are solid state materials and can be made in sheets as thick as 0.5 millimeters. However, certain problems may arise because of the slow total response time and possible UV absorption in the image material.

Figure 7 shows an optical configuration for spatial spectral analysis. The parallel light beam generated by lens 1 is modulated by the image. A filter function is then applied by the filter to the modulated light beam. Lens 2 focuses the output of the filter onto a photo cell. The output of the photo cell is a measure of the spectral component of the image in terms of the filter function. The filters may be realized by liquid crystals or PFE ceramics.
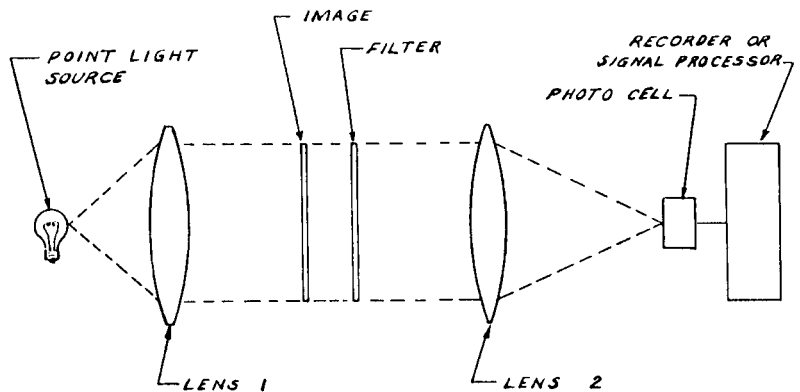

Figure 7. Optical Configuration of Spatial Spectral Image Analyzer.

Plasma filters. A new type of display technology called plasma display technology will provide a capability to generate two-dimensional light distributions that represent discrete functions. Thin sheets of glass are spaced about 0.1 millimeter apart to form a cavity for special gaseous mixtures. Rows of x and y conducting address lines are deposited on this glass. The x lines are on the one side of the cavity; the y lines are on the other side and perpendicular to the x lines. A thin dielectric glass film is placed directly over those lines, and a continuous charge of a.c. voltage is applied across the cavity by the x and y lines. To start a discharge in the gas at any point where an x and y line intersect, an additional voltage pulse is superimposed that is large enough to trigger the gas discharge at this point. The gas discharge appears as a light dot. The discharge can be terminated by another

electrical pulse that cancels the a.c. voltage at the line intersection. The dots can be arranged in patterns of two-dimensional discrete functions, such as Walsh functions. This light distribution can be directly applied to an image to produce corresponding transforms or correlations. The transforms can be recorded electrically. The advantage of this approach is that the light source and optical matrix are combined in one single device. Such a matrix may be called an active optical filter. Figure 8 shows an experimental setup using a plasma filter that could be used for pattern recognition and feature extraction.
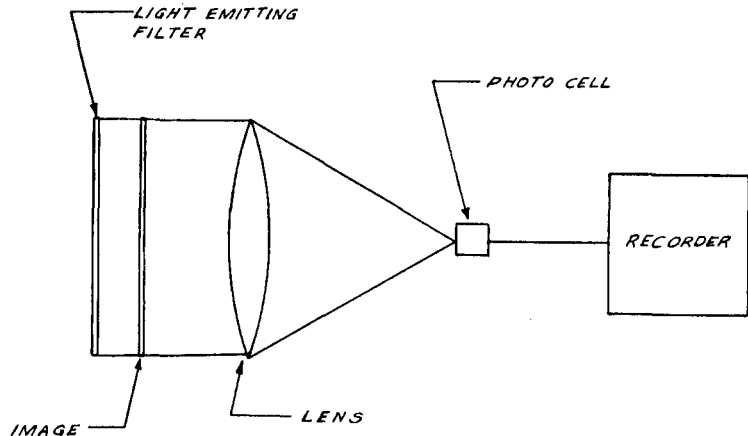

Figure 8. Principle Configuration of an Electro-Optical Spatial Spectrum Analyzer.

LED arrays as filters. Light emitting diodes (LED) may be considered as building blocks for active optical filters. They have the advantage of high reliability and very short switching times. The size of LEDs is still relatively large (about 1 mm^2), and the problem of manufacturing large two-dimensional high resolution matrices has not yet been solved.

Representation of the Earth's Gravity Field by Walsh Functions

Discrete functions, such as Walsh functions, can also be used to construct a binary sample function (BSF) model of the geopotential and to recover geoid and geopotential parameters from satellite altimeter data. A comparison of the performance of the BSF model with the performance of the spherical harmonics sample function (SHSF) model shows that the BSF model is better equipped to recover small features, typically less than 10 arc degrees, efficiently and accurately on a side.

A new algorithm for rapid spectral analysis of satellite altimeter data has been developed by R.D. Brown[2] that reduces the computer time by at least one order of magnitude. The algorithm is based on Walsh functions and computes the spectral components of the measured data. The algorithm is also capable

[2]Richard D. Brown, "Geopotential Modeling by Binary Sample Functions," Catholic University, Washington, D.C., Doctoral dissertation, 1974.

of handling irregularly spaced data. The resulting spectrum retains the same general shape, amplitude, and location of peaks as the spectrum derived from the least-square method. If N is the number of samples, t_1 the required processing time for the Walsh function algorithm, and t_2 the processing time for the least square algorithm, the ratio t_1/t_2 is $\ln N/N\ln 2$. For 1,000 sample points, the time t is only 0.01 t_2, which means that the Walsh process is 100 times faster than the least-square process for this sample number.

Applying Discrete Function Technology to Remote Sensing

Currently, generators can be built that can produce electromagnetic discrete wave forms with pulse widths as small as 1 nanosecond. However, the trend is toward shorter pulse widths down to 100 picoseconds or even to 10 picoseconds. These wave forms can be transmitted directly by an array of radiators that serves as an antenna system. The generator and antenna systems are designed into one unit. Because the current that is fed into the antenna elements is two valued, the electrical field in the wave zone will consist of positive and negative Dirac pulses.

Some work has been done by F.T. Ulaby[3] at the University of Kansas in which a sinusoidal radar spectrometer was focused on cropfields. The received scattered energy was analyzed as a function of field roughness, soil moisture content, crop type, and crop height. The radar used a sinusoidal carrier of 4 to 18 GHz (gigahertz), with a FM (frequency modulation) of approximately 400 MHz (megahertz). Two of Ulaby's conclusions are

1. Although soil moisture can be sensed through vegetation, the sensitivity of radar backscatter to soil moisture is quite dependent on vegetation characteristics and sensor parameters.

2. Spectral response curves indicate that lower frequencies provide more information on soil moisture because of inherently better penetrating ability. Angles near zenith are necessary to accurately estimate soil moisture.

One class of nonsinusoidal waveforms yields more information than sinusoids when used in topographic radar. These waveforms are sequences of short pulses, particularly positive and negative Dirac pulses, where an amplitude reversal can be easily distinguished from any phase shift even in a noisy environment. This effect also exists for sinusoidal waves, but it cannot be exploited because an amplitude reversed sinusoidal function looks like the original function shifted by a half period.

---

[3]F.T. Ulaby, *The Effects of Soil Moisture and Plant Morphology on the Radar Backscatterer from Vegetation*, University of Kansas, July, 1974.

Special Walsh functions that generate two-valued Dirac pulses have been used experimentally for mine detection, detection of metallic and nonmetallic pipes underground, reflectors in noisy environment, etc. Therefore, ground truth data can be expected to be more accurately obtained by using discrete function radar rather than by using sinusoidal pulsed radar.

FUTURE RESEARCH AND DEVELOPMENT

■ In the preceeding section, discrete function technology was shown to have a potential for applications in the topographic sciences. In this section, some concepts are presented that may be solutions to topographic problems that are now significant to the military.

Figure 9 shows the block diagram of a concept for image analysis and feature extraction using an active optical filter (the filter is simultaneously the light source) and a two-dimensional sensing array as receiver. The image is placed between the filter and sensing array. Depending on the resolution of the filter and the sensing array spectral, one may obtain decompositions of the entire image or sections thereof in great detail.

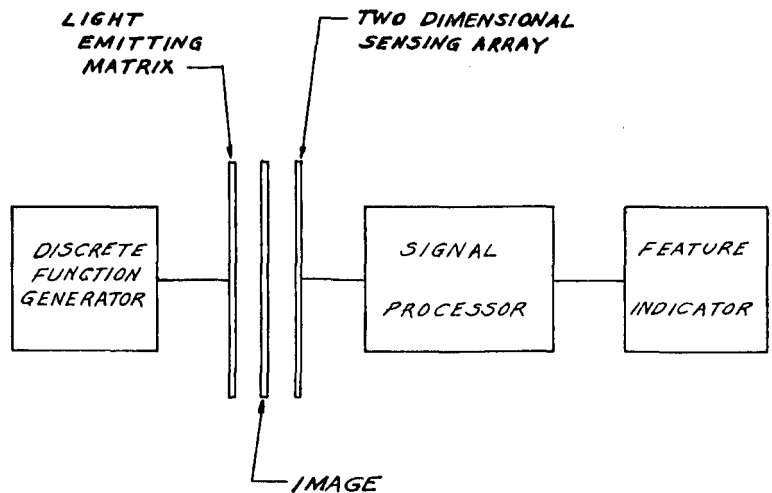**Cartographic Feature Extraction and Signal Processing**

Figure 9. Block Diagram of A Concept for Image Analyzer and Feature Extractor.

Because many discrete patterns may be applied, many characteristic spectra may be obtained. The required function generators are feasible today. However, large active filters and sensing arrays are in the early stages of development. Furthermore, a detailed analysis of cartographic features is required for suitable two-dimensional discrete functions that produce strong responses to specific features in terms of spectral lines and correlation. This research should include the study of topographic features as they appear under various sun angles for clear, partially cloudy, and cloudy skies. Also, the angle of the object relative to the image plane must be considered, as well as possible shadows.

16

So far, only rectangular matrices have been considered as filters. One idea is to include other curvilinear coordinate systems as matrices to express discrete function patterns. Figure 10 shows two-dimensional Walsh functions in polar coordinates.
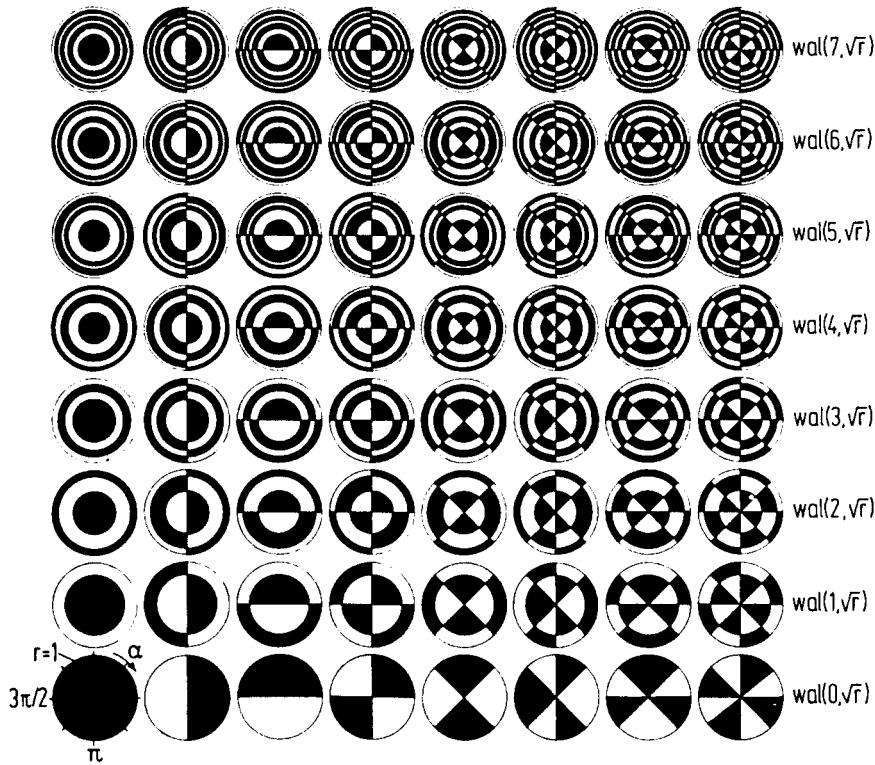
Figure 10. Examples of Polar Walsh Functions.

Another idea is to combine the photo detector function of the sensing array with the filter function of the matrix. This may be relatively easy to accomplish for two-valued filter functions. One value would be represented by transparency, and the other value by opaqueness. In this case, the opaqueness may be realized by disabling the corresponding photo cells of the array utilizing suitable bias voltages. The image is then simply projected on the array. This also gives the possibility of magnifying sections of the image, which would be equivalent to an increased resolution of the system.

The success of feature extraction depends to a large extent upon the response time of the components for electro-optical filters and electronic processing devices. If as many as 1 to 100 million signal processing procedures can be ex-

ecuted in 1 second, the probability to extract topographic features may be high. Therefore, the final limination is presumably the speed by which the image can be moved through the system for image analysis and feature extraction.

Geodesy

The Binary Sampling Model is a local function model that is particularly well suited for computers. The strictly local nature of the functions also allows local geoid solutions to be obtained without regard to the lack of data in areas immediately outside the local solution region. This capability should be explored when the geopotential of limited areas of high military importance are concerned. Because discrete functions may also be expressed in terms of surface spherical coordinates, the advantages of expressing the earth's gravity field by these functions should be examined. Figure 11 shows Walsh functions in spherical coordinates.

Remote Sensing

Very little information is available on radar type instrumentation using discrete wave forms. The research performed in this area is primarily concerned with underwater remote sensing and detection, usually supported by the Navy. However, a remote airborne sensing radar operating with discrete function wave forms would provide more and better vegetation and terrain information than sinusoidal pulsed radar.[4] Plans are being made to perform experiments with discrete-pulsed radar to investigate the return from vegetation and terrain.
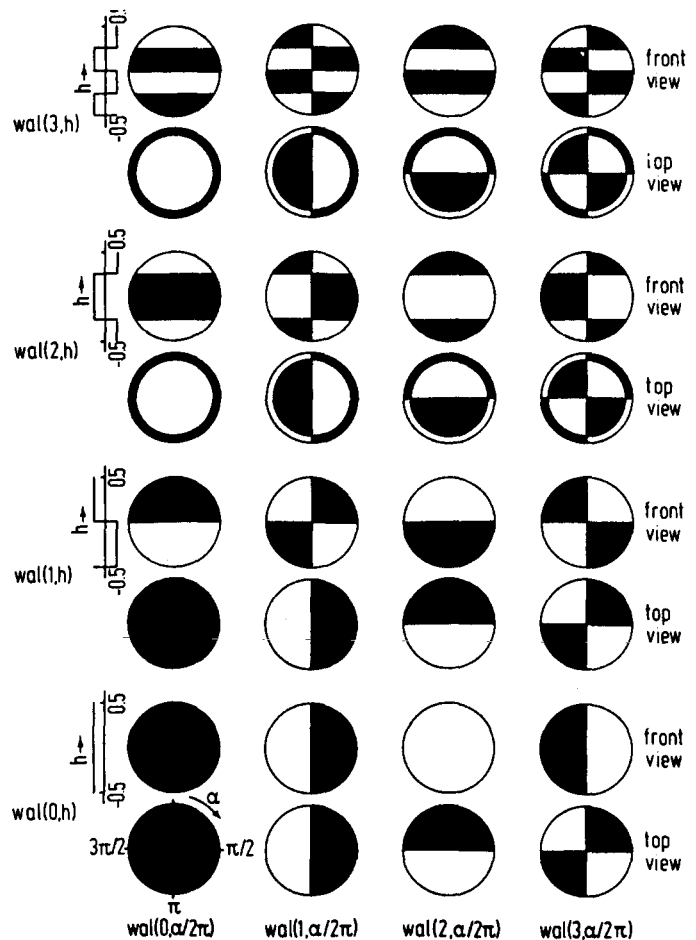

Figure 11. Examples of Spherical Walsh Functions.

---

[4]H. Harmuth, personal communication, Catholic University, Washington, D.C., 1978.

18

CONCLUSIONS ■ On the basis of this study, the following conclusions are
listed:

1. Discrete function technology can be applied to at least three areas of the topographic sciences, namely to image analysis for cartographic and terrain feature extraction, to geopotential representation, and to remote sensing.

2. Many new families of discrete functions can be derived from the Hadamard matrices by means of the Haar process and by shift registers. These functions may be used for spectral decomposition of cartographic features, if the set of functions is orthogonal, or for correlation.

3. Currently, hardware components are available to verify concepts of experimentation, which includes high speed switching circuitry, nematic liquid crystals, PFE-PLZT ceramics, plasma filters, and LED and CCD arrays.

4. High speed image analyzers and feature extractors would require component response times of approximately 1 nanosecond to complete up to approximately 30 images per second.

5. Very little is known at this time about specific spectral decompositions of topographic features.

6. In situ signal processing can be accomplished by using remote sensing array cameras combined with discrete function processors and discrete function signal transmissions.

7. Reduction of satellite altimeter data can be accomplished very efficiently by using a binary sampling function model.

8. Local portions of the geopotential may be advantageously expressed by the Walsh functions.

9. Topographic terrain radar will provide more ground truth information when operated with discrete wave forms rather than with sinusoidal wave forms.