MISCELLANEOUS PAPER M-68-7

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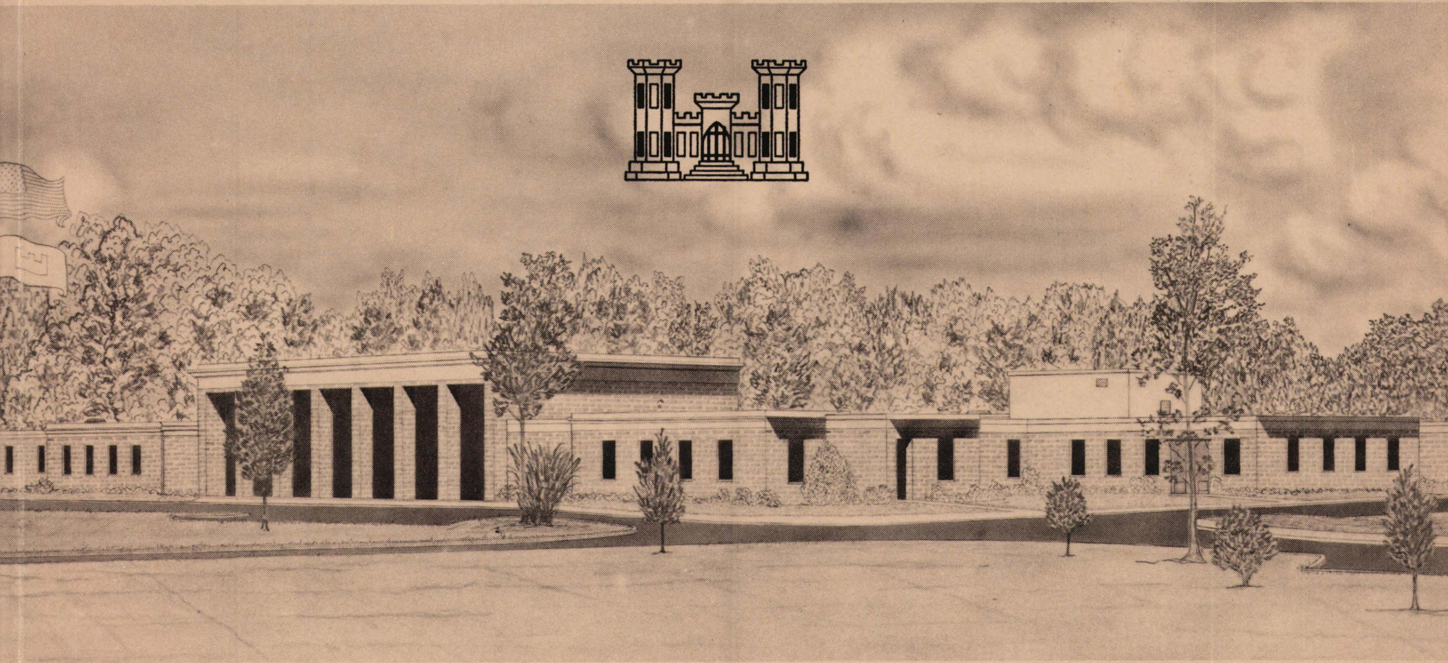

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Sponsored by U. S. Army Materiel Command

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Foreword

This study was conducted at the U. S. Army Engineer Waterways Experiment Station (WES) as a part of the vehicle mobility research program under DA Project 1T014501B52A, "Research in Earth Sciences," Task 01, "Terrain Analysis," under the sponsorship and guidance of the Development Directorate, U. S. Army Materiel Command.

Acknowledgment is made to personnel of the Simulation Laboratory of the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama, for assistance and use of their computing facilities.

The tests were performed by personnel of the Mobility Research Branch, Mobility and Environmental Division, WES, during the period September 1966 to April 1967 under the general supervision of Messrs. W. J. Turnbull, W. G. Shockley, and S. J. Knight, and under the direct supervision of Dr. D. R. Freitag. Messrs. A. S. Lessem and A. J. Green prepared this paper for presentation at the Army Science Conference at West Point in June 1968.

COL John R. Oswalt, Jr., CE, was Director of WES during this study and preparation of this paper. Mr. J. B, Tiffany was Technical Director.

TITLE: A Mathematical Model for Traversal of Rigid Obstacles by a Pneumatic Tire.

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Vicksburg, Mississippi

ABSTRACT:

The mathematical model for a pneumatic tire described in this paper is used to compute the horizontal and vertical forces transmitted through the tire to the vehicle axle to provide realistic force inputs for model studies of vehicle dynamics. The present model is valid for the case of a pneumatic tire traversing nondeforming obstacles with zero slip.

Static load-deformation characteristics and dynamic obstacle-traversal characteristics were obtained in laboratory tests with 9.00-14 tires under several conditions of ply rating and inflation pressure. These data were used to calculate model parameters and to produce time histories of dynamic responses.

Computer implementation of the mathematical model produced force and displacement time histories similar to those obtained during the obstacle-traversal laboratory tests. The model produced the essential features of the waveforms seen in the laboratory and is a valid representation of a pneumatic tire for dynamic analysis of vehicles on nonyielding terrain.

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A MATHEMATICAL MODEL FOR TRAVERSAL OF RIGID OBSTACLES BY A PNEUMATIC TIRE

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PART I: INTRODUCTION

Background

The U. S. Army Engineer Waterways Experiment Station (WES) is active in a program of research in vehicle dynamics, as part of the vehicle mobility research program under the sponsorship and guidance of the Development Directorate, U. S. Army Materiel Command. One of the principal tools used in vehicle dynamics research is the mathematical model, whose main purpose is to predict the dynamic performance of vehicles. Ideally, this prediction takes place in a computing laboratory. The prediction may suggest certain vehicle design modifications, and, in addition, can serve to guide the conduct of field and laboratory tests.

The mathematical model for predicting the performance of a particular vehicle can be organized in several complementary sections. One may be a model of the terrain profile and its capacity for deformation; another, the mechanical interaction of the various members of the vehicle body and suspension system; and a third, the dynamic properties of the traction elements--tires, tracks, or other devices. This paper is concerned with the modeling of traction elements. Heretofore,* in mathematical models of vehicles, pneumatic tires have been represented by a simple, single spring, or a spring and viscous damper. Results have shown that these are inadequate.

Purpose and Scope

The purpose of this work was to develop and verify a mathematical model that simulated the ability of a pneumatic tire to envelop obstacles. The model was to be able to predict both the vertical and horizontal forces transmitted from the terrain through the tire to the vehicle axle. A realistic tire-terrain contact mechanism

* For example, (1), (2).

that considers terrain features and vehicle motions was to be made a part of the model. Only nondeforming terrain was considered. The pneumatic tire model was to be verified by simulating laboratory tests in which a tire was towed over a series of rigid obstacles, and comparing the output with the actual test results.

PART II: TEST PROGRAM

The laboratory program consisted of two series of tests: one, to obtain quasi-static load-deflection characteristics for pneumatic tires under several conditions of ply rating (PR) and inflation pressure; the other, to generate dynamic performance data for a pneumatic tire towed across a series of rigid obstacles, under different conditions of load, inflation pressure, and speed.

Load-Deflection Tests

The tires used in the load-deflection tests were mounted on a rigid axle-hub assembly and loaded by raising a motor-driven platform against the stationary tire, at various speeds. Quasi-static load-deflection curves were obtained for 9.00-14, 2-, 4-, and 8-PR, treadless pneumatic tires at inflation pressures of 0, 10, 20, 30, and 40 psi. A representative set of curves is shown in fig. 1. The curves represent several cycles of loading and unloading and show pronounced hysteresis. It became apparent early in the test series that there was no significant difference between the load-deflection curve recorded at the slowest vertical speed (0.022 in./sec) of the platform and that recorded at the fastest speed (0.143 in./sec), so all subsequent tests were conducted at the fastest platform speed.

Rigid-Obstacle Tests

Under several combinations of load, inflation pressure, and speed, a pneumatic tire was towed over a series of rigid obstacles on a nondeformable base. The test carriage and a portion of the obstacle course are shown in fig. 2. Tests were conducted using a 9.00-14, 2-PR tire inflated to 10, 20, and 30 psi. The tire was loaded to 500, 1000, and 1500 lb and was towed at speeds of 1, 2, and 3 ft/sec. Vertical axle acceleration, vertical displacement, and horizontal restraining force were recorded.

PART III: MATHEMATICAL MODEL

The model that has been used often in vehicle dynamics studies to represent a pneumatic tire is shown in fig. 3. The spring element represents the ability of the tire to recover its original shape in the absence of deforming forces; the dashpot element (when it is included) represents the dissipation of energy in the tire by means of viscous damping. Forces are transmitted through the tire to the axle in the vertical direction only. Contact between tire and terrain takes place at a single point.

The basic idea for the tire model described herein is to represent the tire by many spring elements, as shown in fig. 4. This accounts for the physically significant contributions of horizontal forces transmitted to the axle. It was anticipated that a model composed of only spring elements would be adequate for high inflation pressures and low speeds, since accompanying dashpot elements, although important to a truly realistic representation of tire compliance, were not included in the present model. It may be necessary to include dashpot elements in the model to achieve the desired degree of model accuracy for low inflation pressures and high speeds.

When the pneumatic tire is represented by many spring elements, the displacement forcing function for one spring element need not be the same as that for the neighboring spring. This feature permits small-size terrain features with abrupt changes in slope to be "enveloped" by the model. As each spring element is deflected, the spring force produced and transmitted to the vehicle axle is determined. Knowledge of the magnitude and location of this force permits a force vector to be computed and resolved into horizontal and vertical components. The contributions of the individual elements are summed vectorially and used as inputs to the mathematical model that represents the vehicle dynamic characteristics.

To convert the basic idea for the tire model into practical form, procedures were developed for division of the pneumatic tire into segments, calculation of spring coefficients for the segments, and conversion of terrain profiles into separate displacement forcing functions for the segments.

Division into Segments

As a vehicle traverses irregular, obstacle-ridden terrain, the tires contact the ground through "contact areas" that change in size as the tire-vehicle system goes through dynamic displacements. That portion of the tire carcass that undergoes flexing and deformation at any instant is identified as the "active region" (see fig. 5). The active region was assumed constant in size and large enough to include most anticipated fluctuations in contact area. In reality, the size of this active region varies with time just as does the tire contact area. As the tire rotates, different portions of the carcass are swept into, and out of, this active region. Thus, instead of focusing attention on a particular portion of the tire as it rotates about the axle, a nonrotating area between the terrain and axle is monitored. It is this nonrotating active region that is divided into segments (see fig. 5). These segments are fixed in position with respect to one another, and may be regarded as boundaries through which the physical tire passes during rotation.

Calculation of Segment Spring Coefficients

The load-deflection curves shown in fig. 1 represent conditions for which large-scale carcass flexing occurs, so that as more load is applied, a larger area of carcass is involved in restraining this load. The load-deflection curves can be regarded as the result of joint contributions of many segments acting together. Thus, from these curves the small-scale flexural properties of any one segment in the active region can be inferred.

A concept of "effective radial deflection" was developed as an aid in calculating segment spring coefficients. In this concept, illustrated in fig. 6, the actual segment deflection encountered during a load-deflection test is replaced by a fictitious uniform radial deflection, Δ . The effective radial deflection of each segment varies as the deflection varies at the vertical reference position (VRP).

The flexure property of the tire carcass contained within each segment boundary is represented by a linear spring with coefficient K that is the same for all segments. The spring is positioned along the radial center line of each segment. With the location of each segment with respect to the VRP known, the magnitude and line of action of the force due to deflection of each spring can be determined. In the experimental setup for recording quasi-static load-deflection curves, the load cell registered the total vertical force exerted by the deflected tire carcass. The link between the analytic representation of the tire and the physical reality of its load-deflection curve can be made by using an equation of static equilibrium as follows:

$$F = 2 \sum_{i=1}^{n} K \Delta_i \cos \phi_i \tag{1}$$

where

F = Load cell reading, lb
K = Segment spring coefficient, lb/in.
Δ_i = Effective radial deflection, i^{th} segment, in.
ϕ_i = Force direction angle, i^{th} segment, radians
n = Number of segments each side of VRP.

To apply this analytic representation to a specific tire, one point on the load-deflection curve for that tire must be used to calculate K in equation 1. If a value of carcass deflection at the VRP, δ , is selected (for instance, $\delta = 1$ in.), and if the load cell output corresponding to this selection, F_1 , is read from a load-deflection graph, then K can be computed as:

$$K = \frac{F_1}{2 \sum_{i=1}^{n} \Delta_i \cos \phi_i} \tag{2}$$

In this equation, the values of Δ_i corresponding to $\delta = 1$ in. are read from a graph (fig. 7) of effective radial deflection versus carcass deflection, and the values of $\cos \phi_i$ are fixed by section geometry.

Once K is calculated, this value can be used in equation 1 to define the analytical load-deflection curve. Thus, any other δ may be selected and corresponding Δ_i obtained from the effective radial deflection graph. These values of Δ_i, together with the known values of $\cos \phi_i$, are put into equation 1 to compute the load corresponding to the selected deflection. Used in this manner, equation 1 produces a concave-upward load-deflection curve typical of pneumatic tires. In fig. 8 an experimental load-deflection curve is compared to several points produced by equation 1 using a K value determined at $\delta = 1$ in. This value was chosen on the ascending portion of the experimental load-deflection curve. The use of a linear, constant-coefficient spring as representative of the flexing of a segment of tire carcass appears to be reasonable. The basically hysteretic characteristic of the deflecting tire has been ignored for the present.

Segment Displacement Functions

With the rotating tire mathematically represented by a stationary segmented active region, the nondeformable terrain profile encountered by the tire is represented as a displacement function that traverses the active region. The segments are deflected, in sequence, by the displacement function. These sequential deflections are illustrated in fig. 9. The time of application and the deflection amplitude are different for each segment. The application times shown in the figure are calculated using dimensions of the active region and number of segments.

The computer implementation of the model must provide for generation of a terrain profile function. This function must be shifted in time to account for its sequential encounters with the segments, and it must be changed in amplitude to account for dynamic motions of the tire-vehicle system. Computer capacity allocated to the tire model determines how accurately these requirements are met.

PART IV: APPLICATION TO TRAVERSAL OF RIGID OBSTACLES

Obstacle Test Simulation

The tire model was combined with a model for a test carriage, and the overall model was used to predict the responses recorded in the obstacle tests. Study of the physical structure of the test carriage suggested the composite tire-carriage model shown in fig. 10.

The model of the pneumatic tire was organized with the active region divided into 10 segments, and the segment spring coefficients were calculated for several inflation pressures.

Numerical values for the carriage spring and damping coefficients were obtained by repeating the obstacle tests using a rigid aluminum wheel, thus revealing the dynamic properties of the carriage alone. The values obtained were reasonably independent of carriage speed.

In the writing of the equations of motion, it was assumed that (a) the obstacle does not deform, (b) there is no slip between tire and obstacle under towed conditions, (c) no forces are generated parallel to the wheel axle, (d) the carriage towing speed remains constant, and (e) the pneumatic wheel load remains constant. The nomenclature used in the equations is as follows:

- B_H Horizontal carriage damping coefficient, lb/in./sec
- B_V Vertical carriage damping coefficient, lb/in./sec
- F_o Pneumatic load applied to tire in excess of, or in opposition to, deadweight load
- g Acceleration of gravity
- H_{ri} Threshold height of i^{th} segment in equivalent rigid wheel, in.
- K Segment spring coefficient, lb/in.
- K_H Horizontal carriage spring coefficient, lb/in.
- m Inertial mass of test carriage, lb sec^2/in.
- x Horizontal axle displacement, in.
- Y_i Vertical obstacle height beneath i^{th} segment, in.
- z Vertical axle displacement, in.
- δ Center-line static deflection, in.
- Δ_i Deflection of i^{th} segment,
- ϕ_i Location angle for i^{th} segment

The equations of motion are as follows:

$$m\ddot{z} = \sum_{i=1}^{10} K \Delta_i \cos \phi_i - B_V \dot{z} - g \left(m \pm \frac{F_o}{g}\right) \tag{3}$$

$$m\ddot{x} = \sum_{i=1}^{10} K \Delta_i \sin \phi_i - B_H \dot{x} - K_H x \tag{4}$$

where

$$\Delta_i = \begin{cases} Y_i - H_{ri} - z \ , & Y_i - H_{ri} - z \geq 0 \\ 0 \ , & Y_i - H_{ri} - z < 0 \end{cases} \tag{5}$$

Equation 3 is used to calculate the resultant vertical force, equation 4 the resultant horizontal force, and equation 5 the deflection of each segment. This segment deflection, Δ_i , is permitted to have positive values only; negative values are replaced by zero. This corresponds to an assumption that the tire may only be compressed by the obstacle and not stretched by it.

The rigid-wheel threshold height, H_{ri} , is illustrated in fig. 11. Each H_{ri} gives the height of the i^{th} segment contact point above the ground when the tire has no static deflection. The H_{ri} concept serves two purposes: first, to permit the tire model to display realistic static deflections by invoking as many segments as needed to restrain an applied load, and second, to modify the height of obstacle displacement functions as required by the height of the contact point of each segment.

Computer Implementation

The tire-carriage model was run on a general-purpose analog computer. The overall organization of the computing elements is shown in block-diagram form in fig. 12. The segmented spring computer diagram is shown in fig. 13; the vertical and horizontal computer diagram for the carriage dynamics, in fig. 14.

The generation of segment obstacle functions required the shifting process discussed previously under the heading "Segment Displacement Functions." This was accomplished by using the track-hold capability of the analog computer. The shifting interval controls the accuracy of the quantizing of the input obstacle function. Hence, a trade off between allocated computer space and desired quantizing accuracy is generally necessary for this type of problem. A chain of 34 track-hold amplifier pairs was used in this study; the organization is shown in fig. 15. A representative computer output is shown in fig. 16. The upper trace shows the time history of obstacle heights as seen by the active region of the tire.

Comparison of Computed and Observed Results

Some computed dynamic motions and their laboratory counterparts are compared in fig. 17. This figure shows responses to traversal of a 2- by 8-in. rigid rectangular obstacle. The particular responses shown are the horizontal restraining force exerted on the axle by the carriage and the vertical displacement of the axle.

The comparison of computed and observed responses was essentially qualitative. This was consistent with the use of the crude linear representation of the test carriage. The most desired feature of the composite tire-carriage model, the ability to reproduce the basic features of the response wave shapes as seen in the laboratory, was realized; the composite model, through the segmented representation of the pneumatic tire, was capable of producing responses with realistic wave shapes. The positively and negatively directed horizontal restraining forces and the vertical axle displacements were displayed with basically correct form. If a point-contact model had been used, the response time would have been identical with the time for the point to traverse the obstacle. The multiple-contact model under discussion produces responses whose durations are realistically extended beyond the corresponding obstacle durations. A point-contact model would have been incapable of producing the horizontal forces; the multiple-contact model does so in a natural way.

A quantitative comparison of the computed and observed responses revealed several discrepancies that were related to the simple linear model of the test carriage and to the finite number of segments in the tire model. These can be overcome by more refined modeling (3).

PART V: SUMMARY

The data shown indicate that a mathematical representation of a pneumatic tire, in terms of the deflections of many radial segments, successfully displays the essential feature of horizontal and vertical forces transmitted through the tire. It is indicated further that the segmented tire model enables realistic predictions to be made of the displacement and force time histories for a pneumatic tire towed over a rigid obstacle.

The success of the segmented wheel model suggests that the energy dissipation properties of the tire can be segmented in the same manner as the deflection properties can be segmented, that the model can be refined and enlarged to properly represent a powered wheel traversing deformable obstacles, and that representation of the compliance of other types of traction elements (e.g. tracks) by a realistic series of segments may be possible.

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Lessem, A. S., "Dynamics of Wheeled Vehicles; Report 1, A Mathematical Model for the Traversal of Rigid Obstacles by a Pneumatic Tire," U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Mississippi. (Publication pending.)

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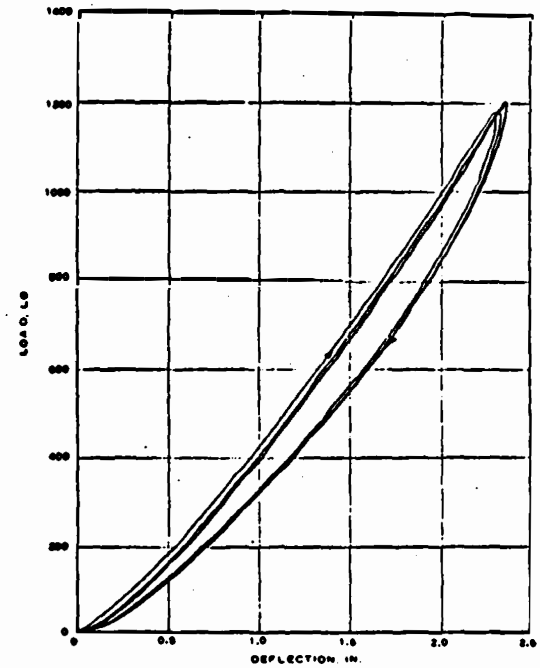


Fig. 1. Representative load-deflection curves; 9.00-14, 8-PR tire, 10-psi inflation pressure

Fig. 2. Test carriage and portion of obstacle course

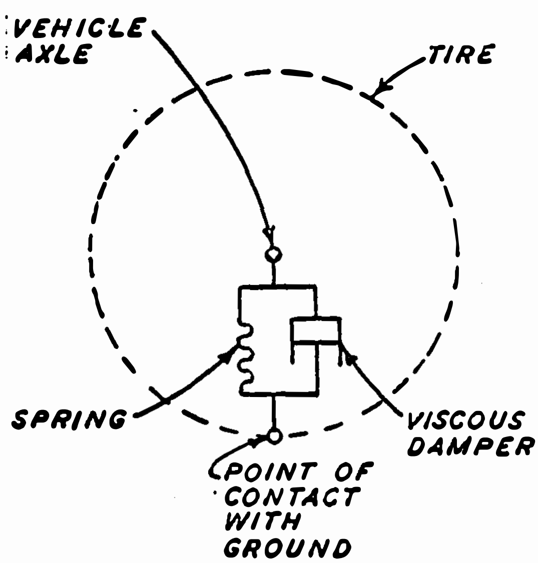


Fig. 3. Point-contact model

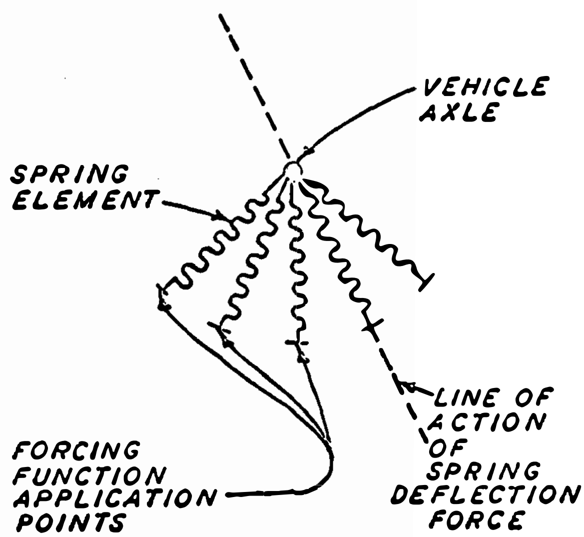


Fig. 4. Extended-contact model

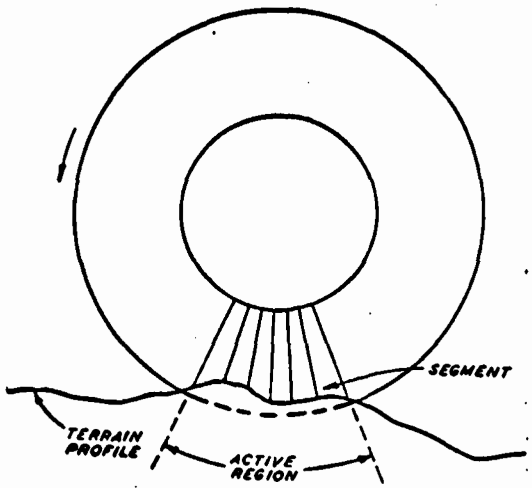


Fig. 5. Segmented, nonrotating, active region

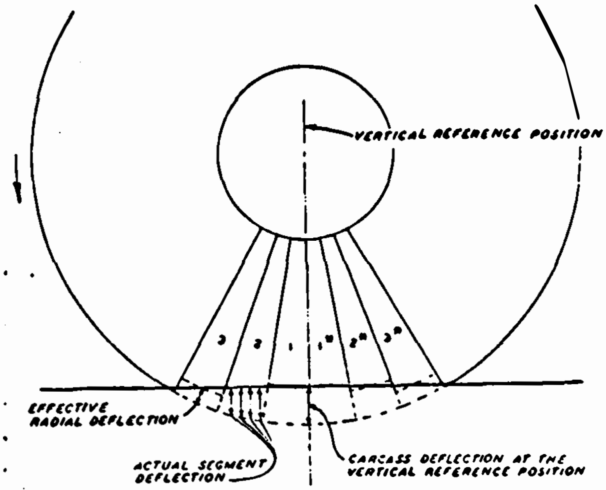


Fig. 6. Illustration of concept of effective radial deflection

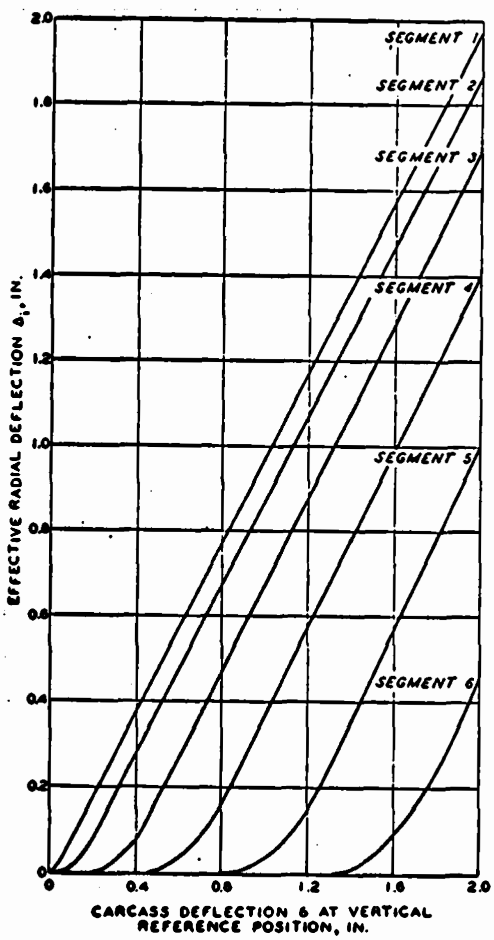


Fig. 7. Representative radial deflection curves

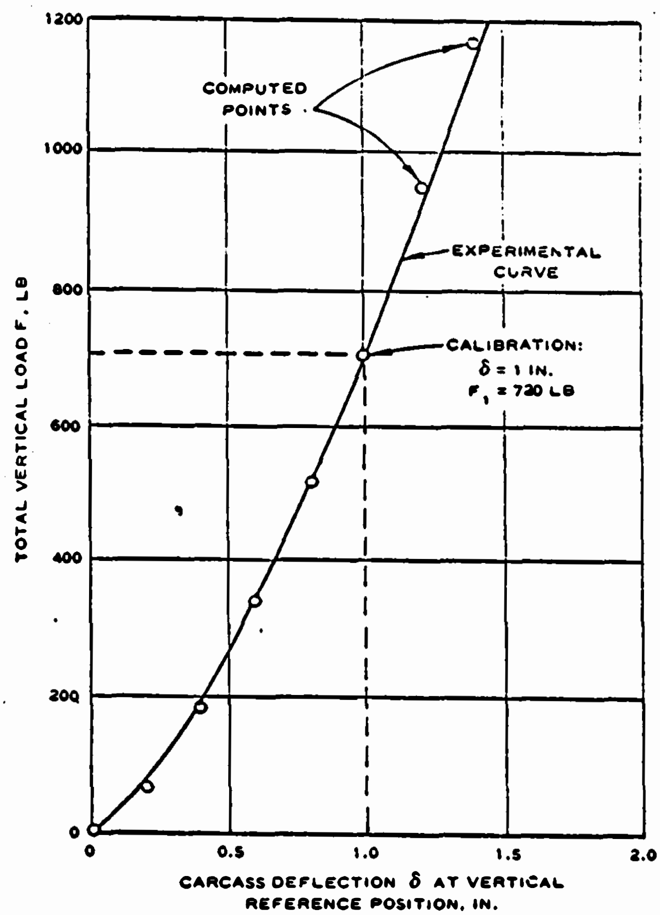


Fig. 8. Experimental load-deflection curve

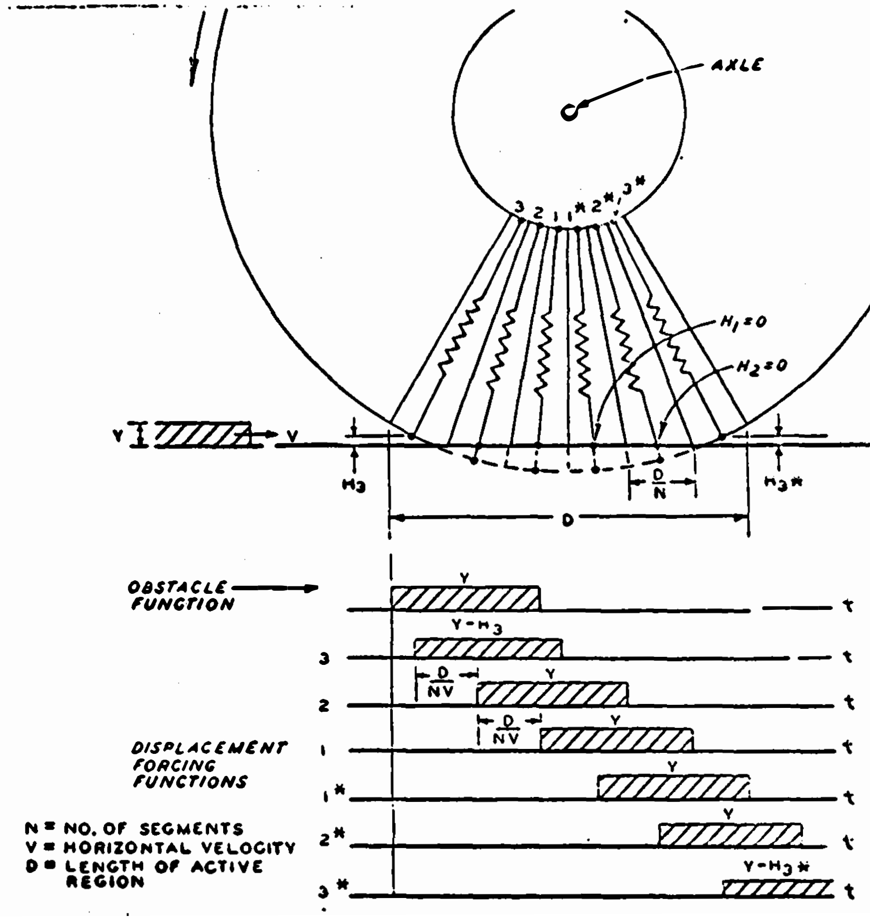


Fig. 9. Sequential deflections

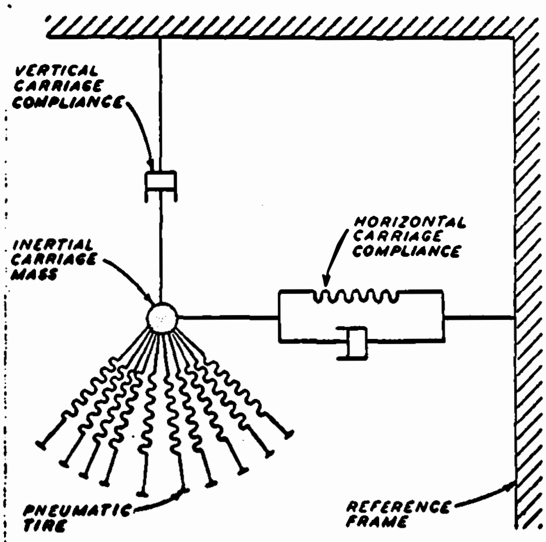


Fig. 10. Composite tire-carriage model

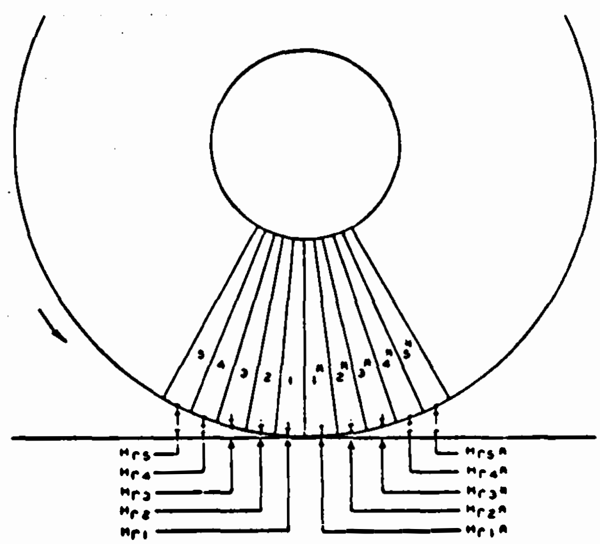


Fig. 11. Rigid-wheel threshold height

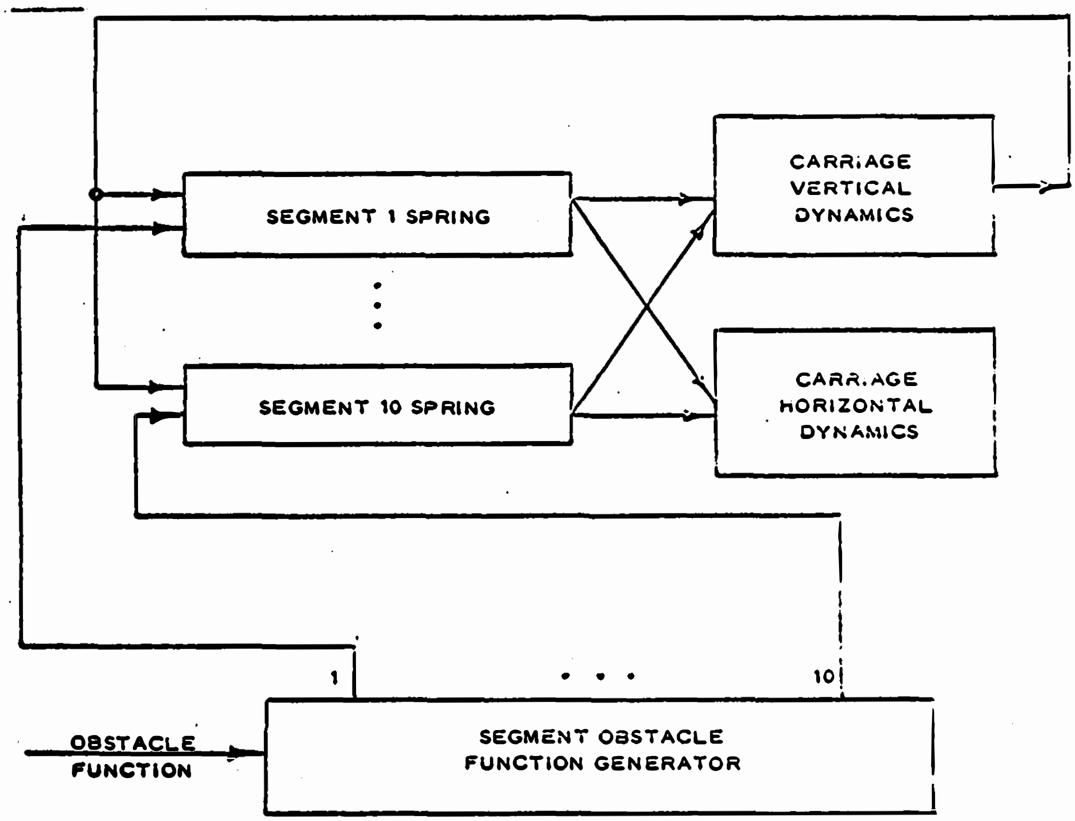


Fig. 12. Organization of computing elements

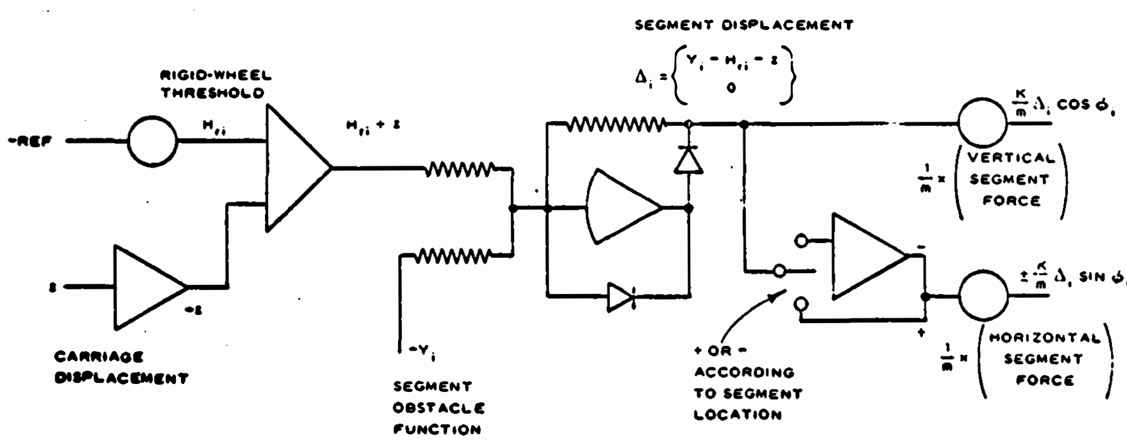


Fig. 13. Segmented spring computer diagram

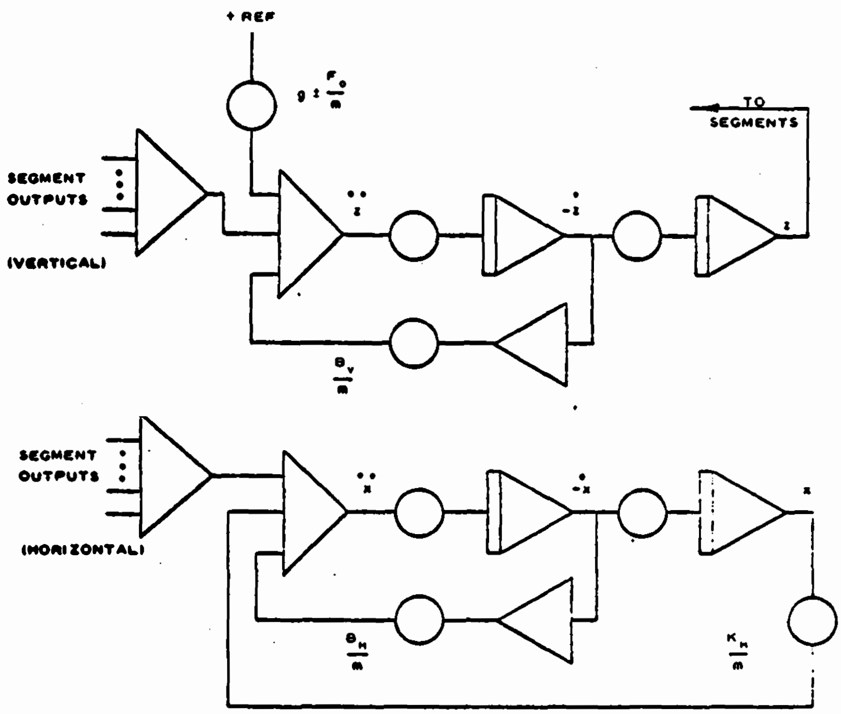


Fig. 14. Computer diagram for carriage dynamics

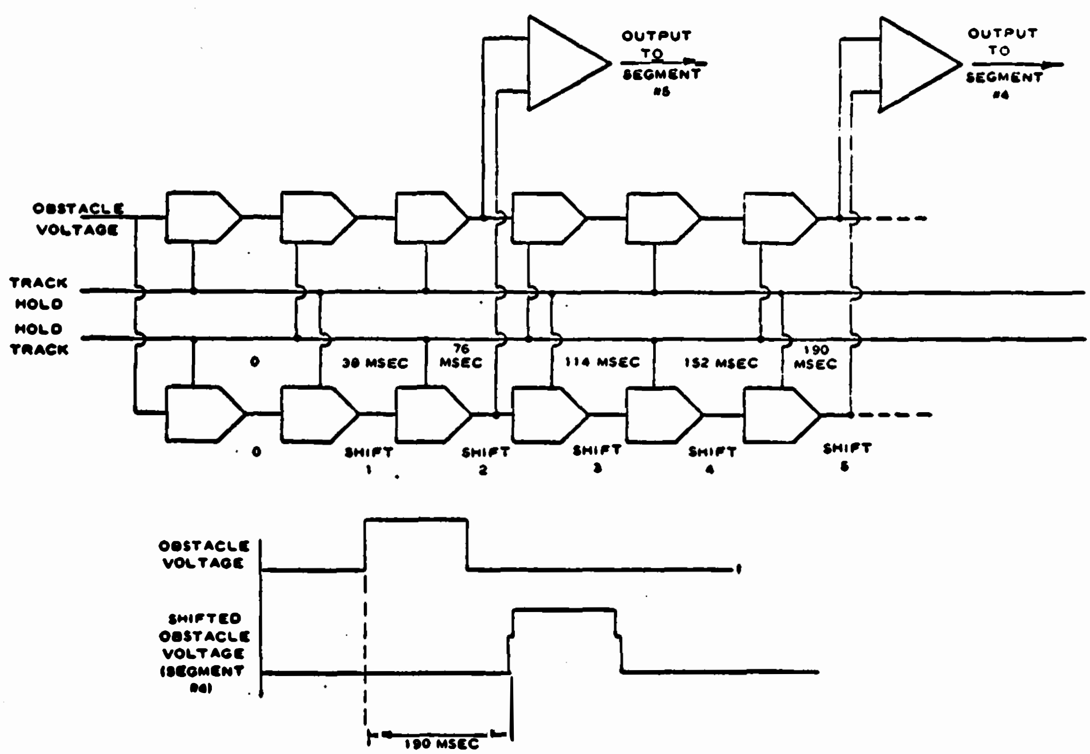


Fig. 15. Organization of track-hold chain

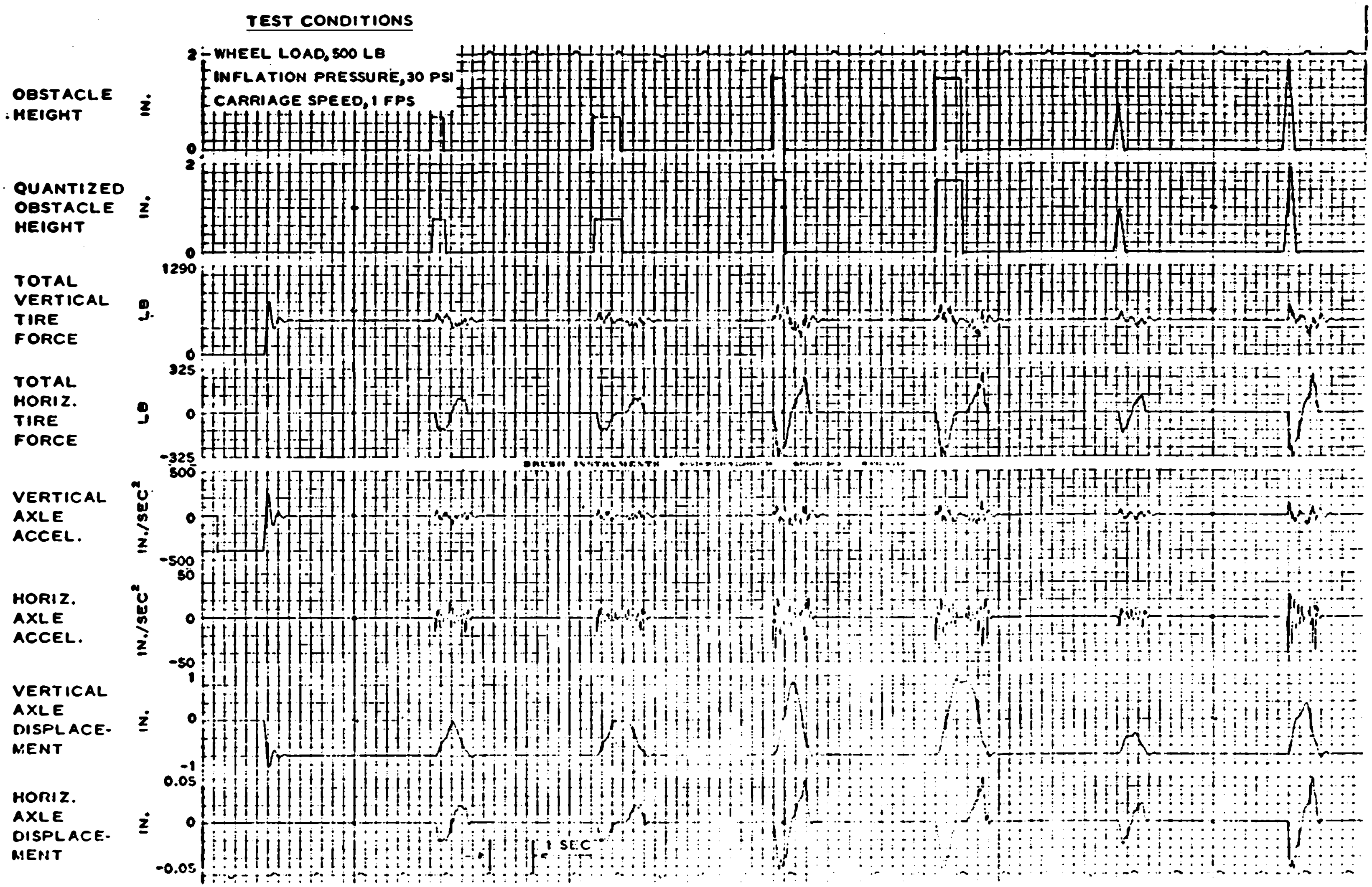
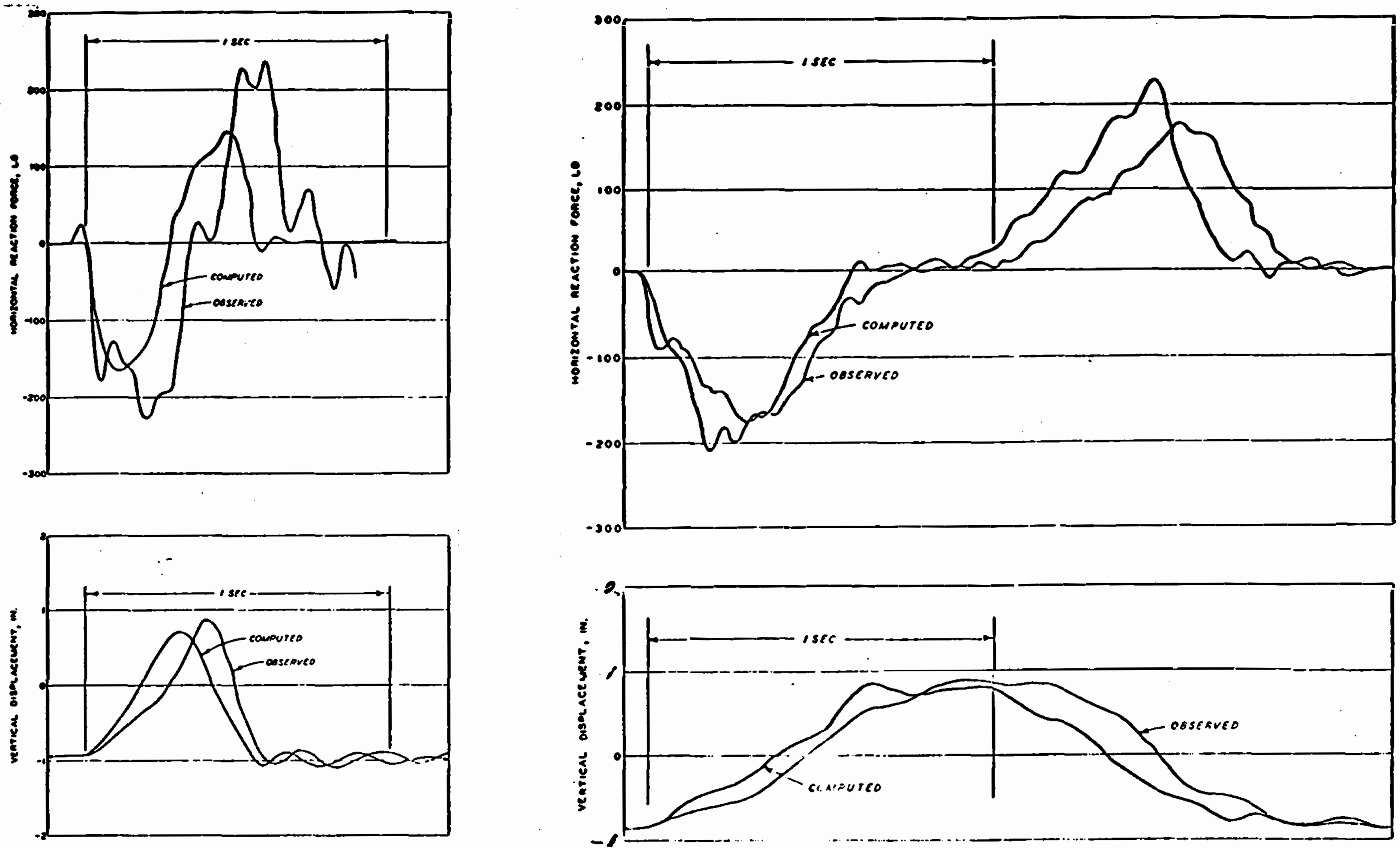


Fig. 16. Representative computed results



a. 1000-lb load, 20-psi inflation pressure, 3-fps carriage speed

b. 500 lb load, 30-psi inflation pressure, 1-fps carriage speed

Fig. 17. Comparison of empirical and analytical results (traversal of 2- by 8-in. rigid obstacle)

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13. ABSTRACT

The mathematical model for a pneumatic tire described in this paper is used to compute the horizontal and vertical forces transmitted through the tire to the vehicle axle to provide realistic force inputs for model studies of vehicle dynamics. The present model is valid for the case of a pneumatic tire traversing nondeforming obstacles with zero slip. Static load-deformation characteristics and dynamic obstacle-traversal characteristics were obtained in laboratory test with 9.00-14 tires under several conditions of ply rating and inflation pressure. These data were used to calculate model parameters and to produce time histories of dynamic responses. Computer implementation of the mathematical model produced force and displacement time histories similar to those obtained during the obstacle-traversal laboratory tests. The model produced the essential features of the waveforms seen in the laboratory and is a valid representation of a pneumatic tire for dynamic analysis of vehicles on nonyielding terrain.

14. KEY WORDS	LINK A		LINK B		LINK C	
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