Mississippi Valley Division

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Comparing Methods for Estimating Water Surface Elevation between Gages in the Lower Mississippi River

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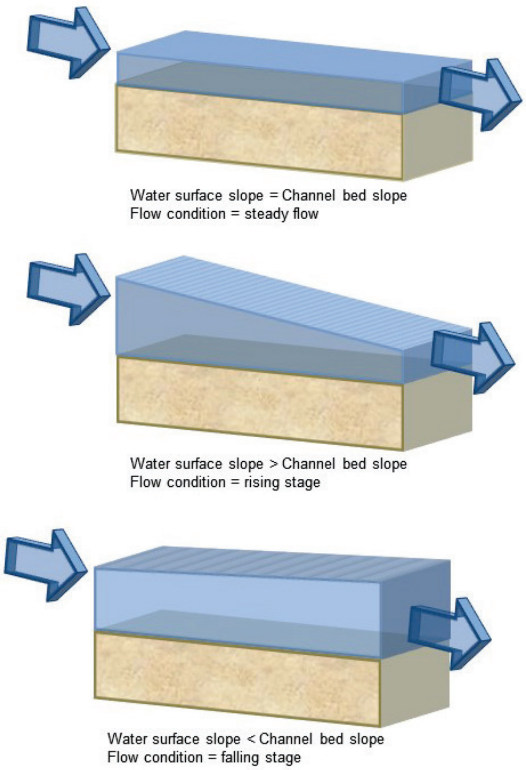

*Graphic adapted from www.meted.ucar.edu

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ABSTRACT: Predicting a water surface elevation (WSElev) at a particular location has a wide range of applications like determining if a levee will overtop or how much a dike notch will increase water flow into a secondary channel. Five existing methods for predicting the water's surface, (1) daily slope, (2) average slope, (3) River Analysis System (RAS) 1D, (4) RAS 2D, and (5) Adaptive Hydraulics modeling system (AdH), were used to predict the Mississippi River's daily water surface from 10 October 2014 to 31 May 2016 at Friar's Point, Greenville, and Natchez gages. The error, calculated as the model-predicted water surface minus the gage-observed water surface, was compared among the methods. The average slope method, using Helena and Fair Landing gages, and the daily slope method, using either Memphis and Helena or Helena and Arkansas City gages, most closely estimated the observed WSElev. The RAS 1D predictions for Friar Point and Greenville produced more accurate estimates than the RAS 2D model and were the only estimates that did not show a pattern of over- or underestimation. When the daily slope method was applied to gages that were farther apart (Memphis and Arkansas City, Arkansas City and Vicksburg, or Vicksburg and Knoxville), the error became greater than most RAS 1D and 2D predictions. The low error and simple calculations of the daily slope and average slope methods using gages $\leq$110 river miles apart make these methods useful for calculating current and historic conditions. The lack of over- or underestimation in the RAS 1D predictions (for locations away from the edges of the model area) make this method a better choice for predicting average WSElevs and a good choice for forecasting future WSElevs.

INTRODUCTION: Knowledge of the WSElev at any given point along a river is important for both engineering and ecological studies. WSElev is used to convert depth to elevation, determine the contour elevation for area, and height for volume calculations. When surveyor's measure ground elevation, they measure depth. To convert depth to elevation, the elevation of the surveying equipment is needed. For bathymetric surveys (surveys of the bed of a waterbody), this elevation is the water surface elevation. Engineering studies utilize WSElev in calculations of navigable area, sediment transport, flood height and flood area predictions. WSElevs are also used to determine suitable depth or elevation for water control structures, drainage canals, pump stations, pipelines, and bridges. Ecological studies use WSElev to study nutrient transport, water quality, depth, area, volume, habitat availability, habitat suitability, and how often the river's main channel connects to the floodplain and its waterbodies (Oliver et al. 2016). Nutrient and water quality rely on water volume, and height (WSElev) is a component of volume calculation. These data can be used to predict species movements and distribution, calculate floodplain storage potential, model nutrient cycling, and prioritize restoration locations. Thus, from an engineering standpoint, WSElev is important to ensure safe ship passage, protect infrastructure, and estimate flood risk. From an ecological standpoint, WSElev is important to ensure sufficient habitat for species persistence (environmental flows), monitor marsh creation, and manage gulf hypoxia.

Slope is a critical factor in predicting a river's WSElev at a certain location. An illustration of the Lower Mississippi River's slope is the low water elevation at different gages. At the Memphis gage, the low water elevation is 181.3 ft[1] (all elevations throughout referenced to the national geodetic

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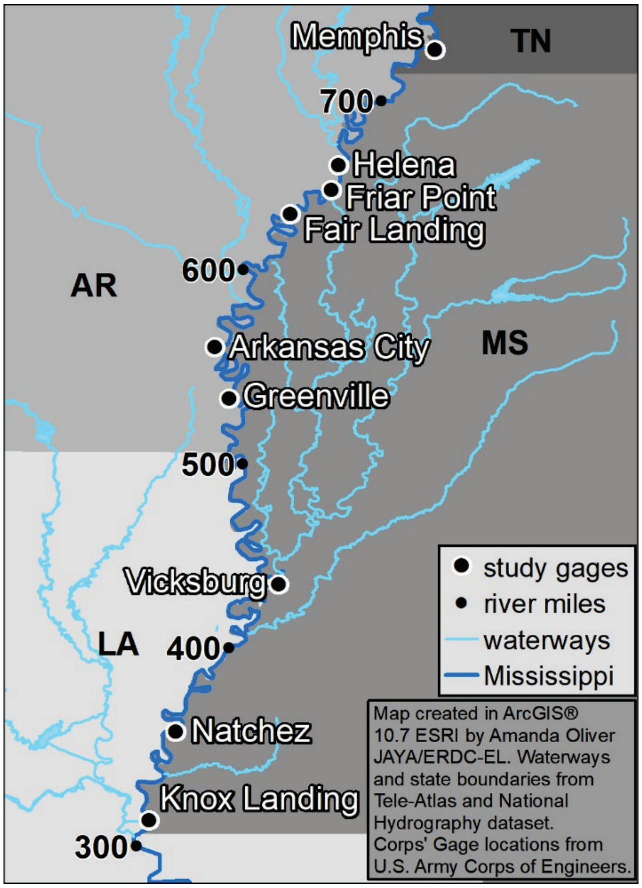
[1] For a full list of the spelled-out forms of the units of measure and unit conversions used in this document, please refer to US Government Publishing Office Style Manual, 31st ed. (Washington, DC: US Government Publishing Office 2016), 248–52 and 345–7, respectively. https://www.govinfo.gov/content/pkg/GPO-STYLEMANUAL-2016/pdf/GPO-STYLEMANUAL-2016.pdf.

vertical datum, NGVD29) while at the Knox Landing gage, it is 14.8 ft (Figure 1). The slope of a river's water surface experiences short-term (snowmelt, rainfall, reservoir release, vegetative growth) and long-term (riverbed erosion, sediment deposition, lateral river migration) changes and is not consistent from headwater to delta. In the Mississippi River, short-term changes in slope are primarily due to watershed rainfall and reservoir releases while lateral migration from cutoffs and shifts in riverbed elevation and composition have the greatest influence over the long term.



Map created in ArcGIS® 10.7 ESRI by Amanda Oliver JAYA/ERDC-EL. Waterways and state boundaries from Tele-Atlas and National Hydrography dataset. Corps' Gage locations from U.S. Army Corps of Engineers.

Figure 1. Study area including gages used, river miles, and state lines.

To calculate WSElev for a specified area and time, gage data are used to calculate both slope and location specific WSElev. Because gage data form the basis of the calculation, the gage's period of record and position relative to other gages and major river features are important considerations. If tributary inputs do not interfere, the closest gages upstream and downstream of the project area are typically chosen to reduce the effects of changing riverbed slope and bed composition. The time period of gage data utilized to estimate slope should be informed by stage discharge trends (also referred to as flow analysis). Discharge is the volume of water flowing past a location over a specified amount of time, in the United States typically measured in cubic feet per second (cfs). From 1940 to 2015, the WSElev at the Helena (H) gage decreased by ~5 to 10 ft at lower discharges (i.e., 190,000, 400,000, and 650,000 cfs) while the Vicksburg gage WSElev increased by ~5 ft at discharges at or above 400,000 cfs (Biedenharn et al. 2017). The relative changes between the two gages would affect both slope and WSElev. Observing the change in WSElev with constant discharge is desired for geomorphic studies but complicates short-term ecological studies. For short-term studies, the time period of gage data used to predict WSElev should encompass the discharge range of interest while minimizing the effects of changing riverbed slope. This can be done by estimating WSElev slope over relatively short time frames, 5–10 yr, or by estimating from an equal number of years before and after the study period.

Numerous methods are available for predicting WSElev. In this study, the following methods were compared: average slope and daily slope, which can predict WSElev for current and past dates, and RAS 1D and 2D version 5.0.1 and AdH version 4.6, which are generally used to predict WSElev for future dates. These methods vary by data needs, setup, and computational requirements, as well as their underlying mathematical procedures. To test the accuracy of each method, methods used gage data from of Memphis (M), Helena (H), Arkansas City (A), Vicksburg (V), and Knox Landing (K) (and additional gages excluding prediction gages for calibration) to predict WSElev at Friar Point (Fr), Greenville (Gr), and Natchez (Na) gages (Figure 1) from 10 October 2014 to 31 May 2016 resulting in 11 combinations of methods and gages. These gages were chosen because they were between other major gages, outside tidal effects, and within various project areas. Predictions were compared to the observed 0800 Central Time (CT) gage readings. The results of this study provide guidance for choosing an appropriate method to predict WSElev. In addition, the estimated error can be applied to uncertainty analyses.

METHODS

Average Slope. Average slope method calculates the water surface at a particular location using the 1962 river miles[2], the nearest gage, and an average water surface slope between two gages, preferably within a specified discharge range. This method is useful when suitable gage data are not available for the entire period of analysis. This method was used to estimate WSElev at Friar Point for the Island 63 geomorphology study. Since these data were already complete, they were included in this study as an example of this method. A stage discharge analysis at Helena indicated a decreasing WSElev from 1970 to present for the discharge range of interest, 190,000–650,000 cfs (Biedenharn et al. 2017). Therefore a slope of 0.000093 ft/mi. was calculated using Helena and Fair Landing daily WSElev from a reduced time period (1998 to 2001). Gage data were no longer collected at Fair Landing after 2001, so using 1998 to 2001 data minimizes the effect of decreasing WSElev while ensuring sufficient gage data across the discharge range of interest. WSElev at Friar Point was calculated as

$$Helena\ WSElev\ ft\ - [(Helena\ RM - Friar\ Pt.RM) * 0.000093\ \frac{ft}{mi}]$$ (2,3)

Daily Slope. The daily slope method calculates the water surface for a location using the observed water surface at two gages and the river miles of the gages and location. Unlike the average slope method, this method accounts for daily fluctuations in water surface slope. The general hypothesis

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[2] River miles determined in 1962 were used for this study. It is likely that each river mile is no longer 5,280 ft apart. Any discrepancy in river miles is captured in the error for the various methods.

[3] Per rivergages.com, Helena RM 663.1, Friar Point RM 652.5, Arkansas City RM 554.1

is that the greater the distance between gages the less accurate and precise the water surface prediction. Multiple locations and sets of gages were used to test this prediction and develop error estimates. For Friar Point, water surface was estimated using all pairwise combinations of Memphis (M), Helena (H), and Arkansas City (A). Greenville was estimated using Arkansas City and Vicksburg (V). Natchez was estimated using Vicksburg and Knox Landing (K). For example, with Friar Point as the prediction location, WSElev can be calculated using Helena and Arkansas City gages as

$$\mathit{Helena\ WSElev\ ft}\ -\left(\mathit{Helena\ RM}-\mathit{Friar\ Pt.\ RM}\right)\ *\ \left(\frac{\mathit{Helena\ WSElev}-\mathit{Ark\ City\ WSElev\ ft}}{\mathit{Helena\ RM}-\mathit{Ark\ City\ RM}}\right)$$

River Analysis System (RAS). The United States Army Corps of Engineers (USACE), Hydrologic Engineering Center's RAS 5.0.1 is a hydraulic modeling software designed to model WSElev, velocity, sediment transport, and water quality for single and networked natural or constructed channels, overbank/floodplain areas, levee-protected areas and others (Brunner 2020a and 2020b). RAS 1D and 2D models use hydrodynamic equations to compute WSElev at specific locations and times. Unlike 1D, RAS 2D utilizes an Implicit Finite Volume algorithm (Brunner 2020a). This algorithm allows for larger computational time-steps and improves stability and robustness. The 1D and 2D models can be run separately or in combination (Brunner 2020a). In RAS 1D, cross sections are spaced at user-defined intervals, and each cross section is used to determine the inputs for the hydrodynamic equations that compute WSElev for the entire channel (Brunner 2020b). With RAS 2D, the user defines the modeling area, which is then divided into a mesh of variable shape and size cells linked to an underlying elevation model (Brunner 2020b). In RAS 1D, WSElevs are predicted at each cross section. In 2D, elevations are predicted for the center of each mesh cell.

RAS 1D. For this study, an existing, calibrated 1D unsteady flow model was used with established cross sections to represent the riverbed of the Lower Mississippi River and some tributaries (Figure 2) (Howe, n.d.; Lewis et al. 2018). The model was calibrated with an hourly simulation from 2002 to 2011, and calibration was statistically weighted to the more recent flow events. Flow (as discharge, cfs) and WSElev data were acquired for the prediction period (October 2014–May 2016) as model inputs. The upstream model extent was the Mississippi River at Memphis, TN, and flow data were readily available for this gage location (Figure 2). The downstream extent was Knox Landing, LA, and WSElev data were readily available for this gage location (Figure 1). Inputs for local runoff to the mainstem Mississippi River were captured using the National Weather Service hydrologic model output. The RAS 1D model also included flow data for major tributaries within the model domain: the Arkansas, White, and Yazoo Rivers, provided by the USACE Little Rock and Vicksburg Districts offices (Howe, n.d.; Lewis et al. 2018).

The RAS 1D initial conditions were calculated by entering a flow value into the unsteady flow file for each river. A steady state profile, accounting for the dendritic nature of the system, was then computed, which generated flows and WSElevs for each cross section (initialization step). After the system was initialized, the same starter flow values for each river were used to begin the unsteady flow computations. The model simulation was run from October 2014 to May 2016 to encompass the time period of observed data.

Adaptive Hydraulics Modeling System (AdH). Water surface elevations were also modeled with AdH developed by the US Army Engineer Research and Development Center (ERDC), Coastal and Hydraulics Laboratory (Figure 2). AdH represents the riverbed and floodplain using a variable resolution triangular mesh where every triangle vertex has a unique elevation. The model is adaptive because it can vary triangle shape and size as well as time-step during the simulation (ERDC-CHL 2019). AdH can compute sediment transport and hydrodynamics in two-dimensions and utilizes semi-empirical methods to approximate the third dimension. Inputs can include elevation data, hydrologic (e.g., WSElev, discharge) and sediment data. Outputs are primarily shear stress, WSElev, and velocity but can also include discharge, flow direction, riverbed elevation change, and riverbed grain size distribution. Spatial and temporal output resolution is determined by the user (ERDC-CHL 2019). Although AdH incorporates considerably more detail than RAS, it cannot model all the factors (e.g., small scale variations in sediment, roughness, turbidity) that may influence the outputs it predicts. Thus, as with RAS, modeling experience and engineering judgement were also important in setting up the model.

This study used an AdH model of the Island 63 (I63) reach that was under development to create hydraulic datasets and investigate sediment response with computational runs averaging one day on an ERDC supercomputer (Howe 2017; May 2020). Although the model was under development, the WSElev hydrodynamics were considered sufficiently calibrated using gage data and 2011/2014 high water marks. The model extends approximately 15 to 20 river miles upstream and downstream of I63 (Figure 2) and laterally from levee to levee. The I63 reach occurs on a river bend and contains numerous structures. Flows around river bends and structures are highly impacted by 3D physics. Thus, the bend-way correction known as vorticity (a semi-empirical correction) was added as a model parameter to create a quasi-3D component in the I63 AdH (Howe 2017). Although vorticity does not account for all 3D processes, it accounts for the most significant 3D factors while allowing the use of a 2D model. This saves significant time, data storage, and monetary costs as compared to a fully 3D numerical or physical model (ERDC-CHL 2019). In general, a full 3D model would take 1.5 times longer to run and require 10% increase in engineering staff compared to a 2D model[4]. Inputs included flow (upstream boundary) and WSElev (downstream boundary) obtained from an unsteady flow RAS 1D model; material type (silt, clay, sand, gravel, concrete); roughness coefficients; and elevation data from 2004 and 2005 bathymetry, lidar, and river structure design specifications (Howe 2017). Due to AdH complexities and computational time constraints, only Friar Point WSElevs were predicted.

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[4] ERDC-CHL personal communication, June 2020.

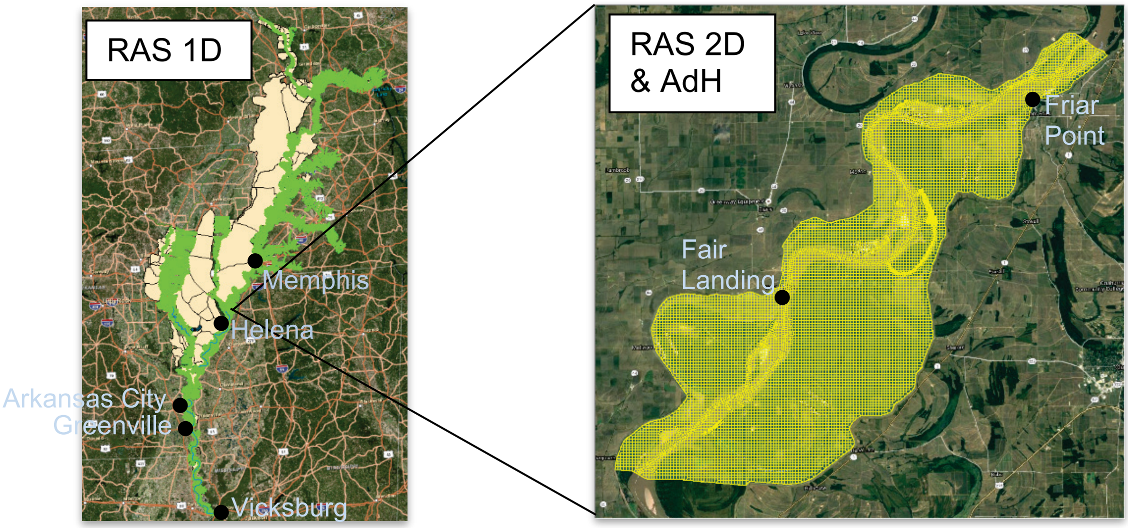

Figure 2. Example of a River Analysis System 1D (RAS 1D) and RAS 2D model. The RAS 2D extent is the same as the Adaptive Hydraulics (AdH) Model System extent used to predict WSElev. The gages used in this study contained within these examples are also listed.

RAS 2D. A fully finite-volume computational RAS 2D model was developed by the Hydrologic Engineering Center, Institute for Water Resources, for the same domain as the AdH model (Figure 2). Using the same inputs (elevation data and longitudinal extent) as the AdH model, the RAS 2D model was set up to simulate the same events to compare WSElev predictive ability. Unlike AdH, which predicts outputs at each triangle vertex, RAS 2D organizes the hydraulic data and the cells' relationships to surrounding cells in a single point at the center of each mesh cell (ERDC-CHL 2019; Brunner 2020a).

RESEARCH QUESTIONS: Are there any significant differences in the precision and accuracy of predicted WSElev among the different methods and gages? In other words, (1) Did the mean absolute error differ among methods, indicating one method was more precise than another? or (2) Did the predictions significantly over- or underestimate the observed WSElev?

To investigate the error distributions, minimum, maximum, median, and mean absolute error (ABSerror) were calculated (Table 1). Error was calculated as the predicted WSElev minus the 0800 CT WSElev observed at the gage. A method that perfectly estimated the observed WSElev would have an error of zero. Because no method is perfect, datasets composed of positive and negative errors were produced. When calculating the mean, positive and negative errors cancel out; thus, for statistical analyses ABSerror was used. ABSerror does not represent the central tendency (average error). Median error was reported in tables to represent the central tendency. For all methods used to predict Friar Point WSElev, histograms were created of the percentage of the errors that occurred in 1.5 ft intervals for each method and gage combination to illustrate the error distributions.

To address the question of which method and gages most closely predicted the observed WSElev, a one-way analysis of variance (ANOVA) with subsequent Tukey-Kramer pairwise comparisons was used to evaluate the ABSerror differences among the 11 combinations of methods and gages (Chai and Draxler 2014). The ANOVA tests for an ABSerror difference among the 11 combinations. The subsequent 55 Tukey-Kramer comparisons test for differences between each possible combination. As the number of statistical tests increases (comparing RAS 1D to RAS 2D, RAS 2D to AdH, AdH to RAS 1D, etc.), the chance of a statistical test indicating a difference when there is no real difference increases (Type 1 error). The Tukey-Kramer comparisons control the Type 1 error rate due to multiple tests with unequal sample sizes (Spence et al. 1992). The null hypothesis was that there would be no difference in ABSerror between method and gage combination. These analyses were done using Microsoft Excel 365 and Real Statistics Resource Pack software (Zaiontz 2020).

To examine whether the method over- or underestimated the observed WSElev, the predicted and observed WSElev for each day were plotted, and a regression line fit through the data. This line was statistically compared to an ideal line where observed equals predicted (Figure 3A, Table 2) by comparing the regression line's *y*-intercept (a) and slope (b) to the ideal line's *y*-intercept of zero and slope of one. Justification for this can be illustrated mathematically utilizing the equation for a regression:

$$y = a + b * x$$
$$OBS = a + b * PRED$$
$$OBS = 0 + 1 * PRED$$
$$OBS = PRED$$

Theil's partial inequality coefficients (Smith and Rose 1995) were computed on the error to look for trends in over- or underestimation. The three Theil's coefficients (U_{bias}, U_{b-1}, and U_{error}) each represent a percentage of the variation in the error (variance) and thus total to 100. The higher the coefficient the greater amount of variance explained by the coefficient. Theil's U_{bias} represents the proportion of the variance due to consistent over- or underestimation at all WSElevs. Although counterintuitive, Figure 3B illustrates consistent overestimation and thus high U_{bias}. For example, on 1 October 2015, the predicted WSElev was 146.9 ft, and the observed was 142.3 ft. Plotting these values on the *x* and *y* axes of Figure 3B, the point falls below the ideal line where observed equals predicted WSElev. Theil's U_{b-1} represents the proportion of the variance due to a trend in over- or underestimation (i.e., a slope > or <1). For example, as Figure 3C shows, a method may overestimate the water surface when the water is low but estimates improve with higher WSElev, high U_{b-1} (Fig. 3C). Theil's U_{error} is the proportion of unexplained variance generally shown by a large point cloud spaced evenly around the ideal line (i.e., stochasticity) (Figure 3D) (Piñeiro et al. 2008). These analyses were completed using Statistical Analysis Software (SAS 2015).

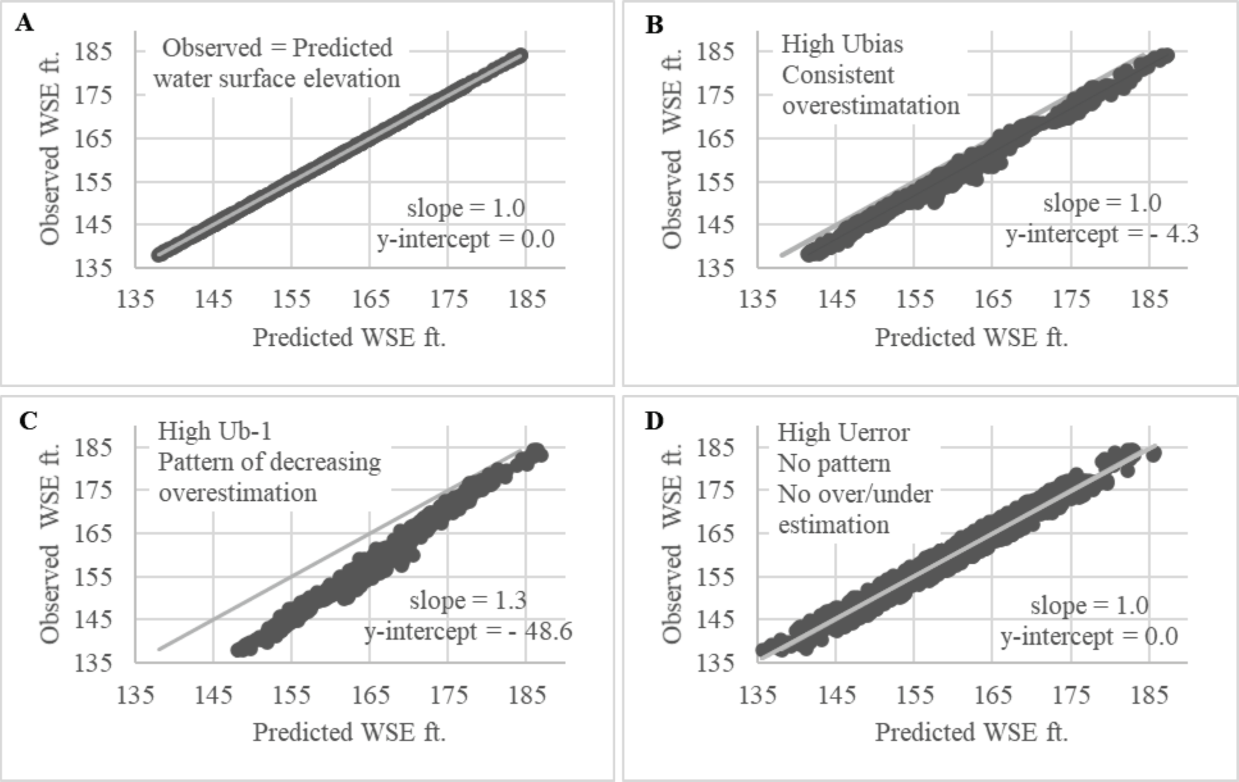

Figure 3. Idealized plots with Obs. = 0 + 1*Pred line to illustrate regression and Theil's coefficients. *A*: A perfect dataset where the observed WSElev equals the predicted. *B*: Predicted WSElev are consistently higher than observed leading to high U_{bias} and illustrating overestimation. *C*: High U_{b-1} where estimation improves with higher WSElevs. *D*: High Uerror, no pattern in the error; thus predictions are accurate but not particularly precise.

RESULTS: There were approximately 600 recorded WSElev readings from October 2014 to May 2016 for Friar Point, Greenville, and Natchez gages; occasionally, readings were not recorded, making the number of readings (sample sizes) slightly different. During the period of analysis, WSElevs at Friar Point, Greenville, and Natchez ranged over 40 ft and were within 16 ft or less of record maximum and minimum for each gage. The error for all methods (predicted minus observed WSElev) ranged from −6.4 to 8.2 ft (Figure 4). The ANOVA indicated that there is a less than 0.1% chance that the ABSerror between the 11 method gage combinations is the same; some WSElev methods produced better WSElev estimates than others (one-way ANOVA, $F_{(10, 6551)} = 543$, $p < 0.001$) (Table 1). The regression models indicated that in most cases, there was a trend in error with increasing WSElev. In other words, except for Fr RAS 1D, Fr RAS 2D, and Gr RAS 1D, the regressions indicated there is a greater than 95% chance that methods under- or overestimated the actual WSElev (nonzero *y*-intercept (a), $p < 0.05$) (Table 2).

Table 1. Error comparisons among the different methods with Tukey-Kramer pairwise comparisons where different letters indicate a greater than 95% chance ($p < 0.05$) groups are different. Distance in river miles[1] between the gages used to create the estimates when applicable. Methods are ordered from low to high ABSerror. Methods include average slope method (AvgSlope), daily slope method (SM), RAS 1D and 2D, and AdH. These methods use Helena (H), Fair Landing (FL), Arkansas City (A), Memphis (M), Vicksburg (V), and Knox Landing (K) gages to predict WSElev at Friar Point (Fr), Greenville (Gr), and Natchez (Na) gages.

Gage	Method	Distance River Mile[1]	Median	ABSerror	Min.	Max.	Tukey-Kramer Comparisons
Fr	AvgSlope_H&FL	31	−0.28	0.41	−1.61	0.44	A
Fr	SM_H&A	109	0.42	0.46	−0.71	1.13	A
Fr	SM_M&H	71	−0.25	0.48	−1.61	0.64	A
Fr	RAS 1D		−0.13	0.98	−3.68	4.02	B
Gr	RAS 1D		0.39	1.02	−3.34	3.73	BC
Fr	RAS 2D		0.23	1.16	−3.85	4.95	CD
Na	RAS 1D		0.29	1.19	−2.95	5.87	D
Gr	SM_A&V	119	1.89	1.83	−1.04	4.04	E
Na	SM_V&K	122	1.98	1.98	0.07	3.56	F
Fr	AdH		−0.45	2.23	−6.38	8.25	G
Fr	SM_M&A	180	2.97	2.85	0.26	7.1	H

[1] River miles used were determined in 1962 and are the standard for the Lower Mississippi River. Distance between gages may differ due to changes in river length since 1962.

The daily slope method using Helena and Arkansas City or Memphis and Helena, and the average slope method most closely estimated the observed WSElev (Table 1, group A). There is a greater than 96% chance these method and gage pairings' ABSerrors are the same ($p > 0.96$) as indicated by the Tukey-Kramer pairwise comparisons. For these methods, the difference between the predicted WSElev and observed WSElev (error) ranged from −1.6 to 1.1 ft (Figure 4). The error range along with a y-intercept greater than zero (2.85 to 6.78) and slope less than 1 (0.96 to 0.98) suggest these methods underestimate at low WSElevs and overestimate at high WSElevs (Table 2). The Theil's coefficients indicate the strength of this trend varies between methods. For the daily slope using Memphis and Helena gages, the trend of underestimation to overestimation explains the greatest proportion of the error; U_{b-1} explains 59% of the variation in the error (Table 2, Figure 4). When Helena and Arkansas City gages are used, the U_{bias} of 0.51 indicates the error can be explained by consistent overestimation for most WSElev (Table 2, Figure 4). The average slope method has a negative median, suggesting this method tends to underestimate but the Theil's coefficients are nearly equal, suggesting there is no major pattern to the error in relation to the observed WSElev (Tables 1 and 2).

Table 2. Regression analysis of RAS 1D and 2D, AdH, daily slope method (SM) and average slope method (AvgSlope) using Helena (H), Fair Landing (FL), Memphis (M), Arkansas City (A), Vicksburg (V), and Knox Landing (K) to predict WSElev. WSElevs were predicted at and compared to observed at Friar Point, Greenville, and Natchez gages.

Gage	Method	Regression y = a+bx	R^2	Test H_0: a = 0 *y*-intercept = 0	Test H_0: b = 1 Slope = 1	Theil's		
						U_{bias}	U_{b-1}	U_{error}
Friar Point	RAS 1D	OBS = −1.29 + 1.01(PRED)	0.987	$F_{1,593} = 3.0$, p = 0.0822	$F_{1,593} = 2.9$, p = 0.0901	0.0008	0.0048	0.9944
	RAS 2D	OBS = 1.52 + 0.99(PRED)	0.982	$F_{1,593} = 3.1$, p = 0.0777	$F_{1,593} = 4.2$, p = 0.0399	0.0273	0.0069	0.9658
	AdH	OBS = −24.08 + 1.15(PRED)	0.956	$F_{1,590} = 218.8$, p<0.0001	$F_{1,590} = 217.3$, p <.0001	0.0021	0.2686	0.7293
	AvgSlope_H&FL	OBS = 4.82 + 0.97(PRED)	0.999	$F_{1,593} = 966.7$, p<0.0001	$F_{1,593} = 837.2$, p<0.0001	0.3996	0.3514	0.2490
	SM_MtoH	OBS = 6.78 + 0.96(PRED)	0.999	$F_{1,593} = 2521.8$, p<0.0001	$F_{1,593} = 2308.8$, p<0.0001	0.2528	0.5945	0.1527
	SM_H&A	OBS = 2.85 + 0.98(PRED)	0.999	$F_{1,593} = 319.3$, p<0.0001	$F_{1,593} = 409.1$, p<0.0001	0.5095	0.2003	0.2902
	SM_M&A	OBS = −12.06 + 1.06(PRED)	0.993	$F_{1,593} = 401.8$, p<0.0001	$F_{1,593} = 235.1$, p<0.0001	0.8653	0.0382	0.0965
Greenville	RAS 1D	OBS = −0.38 + 1.00(PRED)	0.989	$F_{1,598} = 0.6$, p=0.4240	$F_{1,598} = 0.0$, p=0.9863	0.0915	0.0000	0.9085
	SM_A&V	OBS = −6.60 + 1.04(PRED)	0.998	$F_{1,598} = 1011.1$, p<0.0001	$F_{1,598} = 534.7$, p<0.0001	0.8720	0.0604	0.0676
Natchez	RAS 1D	OBS = −5.23 + 1.09(PRED)	0.993	$F_{1,598} = 678.5$, p<0.0001	$F_{1,598} = 588.8$, p<0.0001	0.0978	0.4476	0.4546
	SM_V&K	OBS = −2.73 + 1.01(PRED)	0.999	$F_{1,598} = 1061.1$, p<0.0001	$F_{1,598} = 83.7$, p<0.0001	0.9489	0.0063	0.0448

The RAS 1D regression line for predicted and observed WSElev for Greenville most closely matches the ideal line *y*-intercept (a) and slope (b) (p = 0.4, 0.99). In other words, although the individual predictions (error ranging from −3.3 to 3.7 ft, group B) are less precise than the average slope using H and FL and slope method using H and A or M and H, a line fit through the estimated WSElevs closely matches the ideal line where observed equals predicted (the error averages out) (Tables 1 and 2). When predicting Friar Point, the RAS 2D has greater error (group C) and its regression line's slope deviates more from the ideal line than the RAS 1D (Tables 1 and 2). The AdH and daily slope method using Memphis and Arkansas City, Arkansas City and Vicksburg, and Vicksburg and Knox Landing have a greater error range and/or considerable over/underestimation.

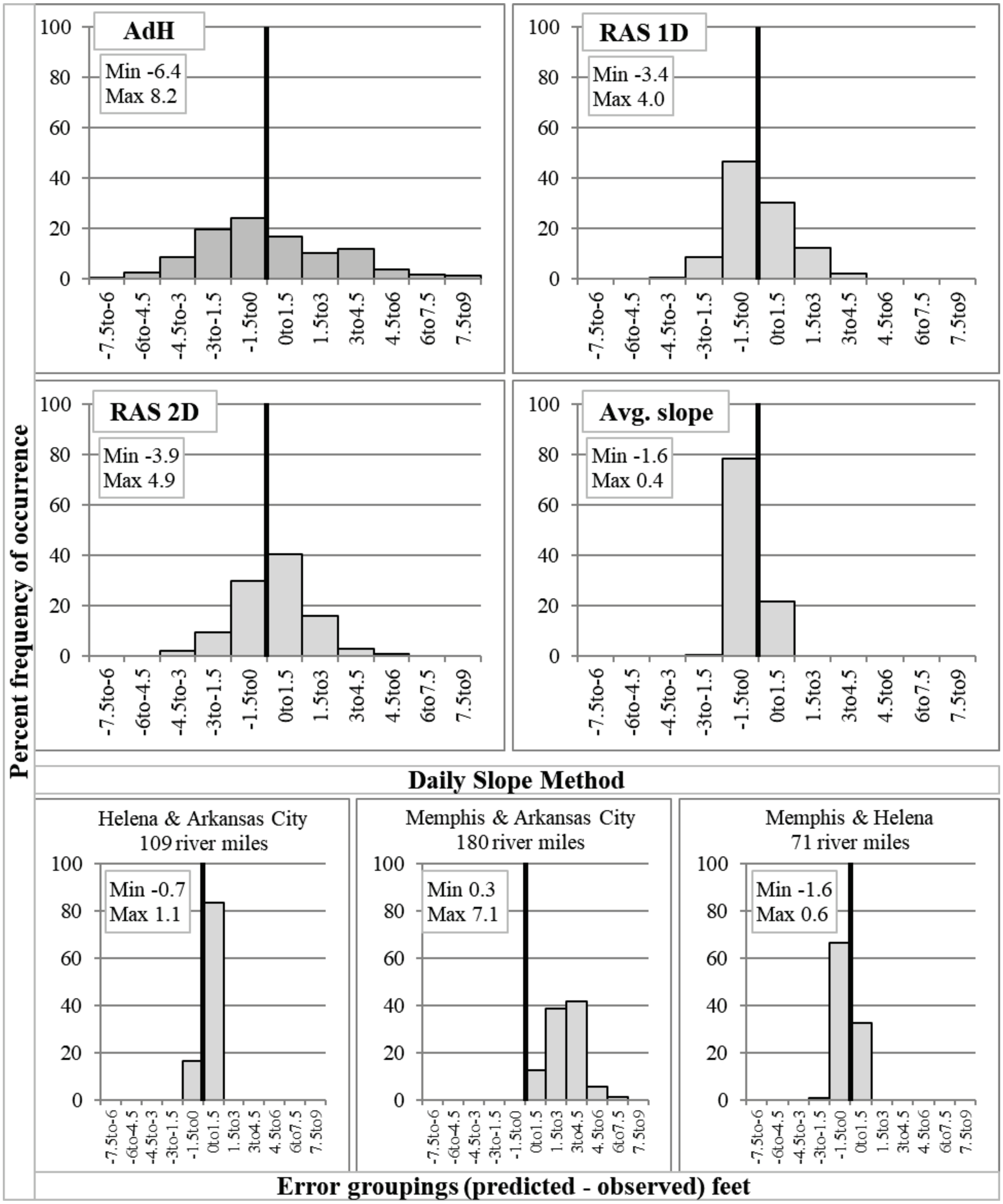

Figure 4. The percentage of errors that fell within each 1.5 ft grouping. For the daily slope using Helena and Arkansas City, 98 of 595 errors (16.5%) were within −1.5 to 0 ft of the observed 0800 CT Friar Point gage readings. If the slope, daily average, RAS or AdH methods predicted the water surface at Friar Point gage perfectly, the error would be zero (*black line*). The minimum and maximum provide the error range for each combination of method and gage.

DISCUSSION: The close estimation of the observed WSElev by the daily slope method using Helena and Arkansas City, or Memphis and Helena gages and the average slope method make these methods good predictors of individual WSElevs. Slope calculations using observations for the time of interest were more precise than the models in this analysis. The strong performance of the average slope method indicates that an observed data record at a location of interest is very valuable even if it covers a much shorter time period. The good predictive ability of Memphis and Helena gages also indicates that predictive gages do not necessarily need to be upstream and downstream of the prediction location. The closeness with which these methods predict individual WSElevs makes them useful for calculating riverbed elevation for bathymetric surveys, determining when a water body connected to the river or estimating surface area because they would introduce the least amount of additional error (−1.6 to 1.1 ft). The gages used in these estimations were approximately 109, 71, and 31 river miles apart, respectively (Table 1). The poorer predictive ability using A and V (119 river miles), V and K (122 river miles), and M and A (180 river miles) suggests that the daily slope method's predictive ability declines when the gages are >110 river miles apart. This increased error could be due to (1) the distance used in calculations becoming increasingly different from actual distance due to the use of 1962 river miles, (2) tributary effects, (3) changes in riverbed grain size and material (Gaines and Priestas 2016) or other factors influencing river variability. It is tempting to apply this decrease in predictive ability with distance universally. However, two distant gages with homogenous river in between may perform just as well as two close gages on a highly variable river. For predicting WSElev in areas with distant gages, variable roughness, and tributary effects, a RAS 1D model may be a better predictor, although error is greater (−3.9 to 5.9 ft). Alternatively, the average slope method using data from discontinued gages is a good option. Estimating an average slope from Helena and Fair Landing WSElev data from 1998 to 2001 produced good estimates of 2014–2016 WSElevs at Friar Point. The quality of predictions will likely decrease as the years between slope and prediction data increase.

When determining an average WSElev, for example flood height at a certain discharge, the RAS 1D method may be the best choice. The Fr and Gr RAS 1D accuracy would result in the error canceling out over an average of a sufficient number of WSElev estimates resulting in a better prediction. This could be important when predicting flood heights or determining structure elevations. Although the Fr RAS 2D incorporated more elevation data into the model, it was not a better predictor of WSElev compared to the Fr RAS 1D. This suggests that the additional calibration, data, and/or mathematical complexity required of the RAS 2D does not improve WSElev results. The even more complex AdH produced poorer WSElev estimates. This may be due to the models being in development at the time of this project. For both the RAS 2D and AdH, additional calibration and extending the model extent to a downstream flow gage would likely improve the accuracy of water surface elevation estimates.

The error ranges provided for all methods can be used to indicate how far from the true value a predicted value is likely to be. For example, the true value of a RAS 1D estimated water surface of 30 ft at Greenville may be 26.7 to 33.7 ft. This water surface range can be used for uncertainty estimates, or the lower and upper limits chosen for project design parameters. Alternatively, the error range could provide a minimum possible 26.7 ft or maximum possible value 33.7 and used to calculate minimum available aquatic habitat or the start point for a bridge deck height. The data used for these analyses were all located on the Lower Mississippi River, an alluvial, turbid, gently sloping, and meandering river. If these methods are replicated on rivers with different characteristics, the predictive ability of the various methods may change.

Long-term gage data are extremely important to these analyses and many other engineering and ecological investigations. Studley (2001) says it well: “Real-time gage-height and streamflow data are important to the Nation for planning and decisions related to agriculture, industry, urban water supplies, navigation, riverine and riparian habitat, and flood-hazard identification. Long-term records from gaging stations are a cornerstone for national, regional, and local efforts to understand the Nation's water resources. Any gaps in those long-term records of streamflow and lake levels can mean a gap in the essential understanding and management of water resources.” Missing gage data can be interpolated as outlined in this technical note, but this interpolation introduces error that increases with distance between gages. Thus, a loss of gage data increases risk to infrastructure planning, flood height estimation, and habitat analyses. Continued maintenance of gaging infrastructure and widespread data access is therefore a critical task.

SUMMARY

- The daily slope method and average slope method using gages <110 river miles apart produce a close estimate of the WSElev with errors ranging from −1.6 to 1.1 ft.
- The daily and average slope methods can be used for historical stage duration calculations.
- The error calculated in this study can be used to calculate upper and lower bounds. For example, Dike 3 at 668.1 R (R = right descending bank) with an elevation of 163.1 ft has water flowing over it ~287 to 302 days in 2015 using SM_H&A and its error range.
- Water surface slope developed from shorter-term dataset gages closely estimated observed WSElev.
- Using up-to-date river miles may further improve average slope and daily slope method water surface predictions.
- The RAS 1D method is useful for determining average WSElev because it does not over- or underestimate the WSElev.
- The RAS 1D method would be a good method to predict future stage duration.
- The RAS 2D did not produce better WSElev estimates. RAS 2D had greater error than RAS 1D. The model performance could likely be improved with more thorough calibration.
- The AdH model WSElevs had the greatest error range. This could be due to the model being in development and would likely improve with more thorough calibration.

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