DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

SHORE PROTECTION PLANNING AND DESIGN

TECHNICAL REPORT NO. 4

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JUNE 1954

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PREFACE

Vast sums are spent annually in the United States by States, municipalities and private owners for works designed to prevent erosion damage on seacoasts and lake shores. The alarming number of such works which have failed due to structural inadequacy, or which have been allowed to deteriorate through lack of maintenance because they were functionally or economically unsound, testifies to the inadequacy of general technical knowledge in this field.

The Beach Erosion Board and its staff have been engaged in the study of shore erosion problems since 1930 and since then have pursued an intensive program of research and development with a view to improving shore protection techniques. While the Board considers that this is a science still very much in the development stage, it is evident that many errors of the past may be avoided in the future by proper application of knowledge thus far gained. The need for providing a sound basis for shore protection planning and design is evident and it is with that objective that this report has been prepared. It is primarily designed to present in a single publication, techniques currently used in the solution of shore protection problems. It is expected that these techniques will be improved by further research and experience, and to facilitate changes as they become desirable, publication is in loose leaf form. Recipients of the publication are urged to mail the inclosed form to the Resident Member, Beach Erosion Board in order that they may receive changes as issued.

This publication supersedes Special Issue No. 2, Bulletin of the Beach Erosion Board, which was published in March 1953. The report was prepared by the Engineering Division of the Beach Erosion Board under the direct supervision of J. V. Hall, Jr., and under general supervision of Colonel E. E. Gesler, President of the Board until 31 March 1953, and R. O. Eaton, Chief Technical Assistant. The task group initially assigned to the preparation of the report was headed by K. P. Peel, temporarily assigned to the Board for this purpose from the South Pacific Division, Corps of Engineers, and Kenneth Kaplan of the Board's staff. Other staff personnel who participated were R. H. Allen, C. T. Fray, R. L. Harris, W. J. Herron, T. Saville Jr., W. H. Vesper and L. L. Watkins.

Limited distribution of Special Issue No. 2 of the Bulletin resulted in many comments from interested engineers which proved to be helpful in producing this report. Revisions originating from that source, and those directed by the Board, were made by R. A. Jachowski and G. M. Watts. The report was edited for publication by A. C. Rayner and R. L. Rector. Members of the Board at the time the report was approved for publication were: Colonel Leland H. Hewitt, President; Colonel Wendell P. Trower, Colonel Herman W. Schull, Jr., Colonel John U. Allen, Resident Member; Thorndike Saville, Morrough P. O'Brien, and Lorenz G. Straub.

This report is published under authority of Public Law 166, 79th Congress, approved July 31, 1945.

NOTICE TO RECIPIENTS OF TECHNICAL REPORT NO. 4
OF THE BEACH EROSION BOARD

"SHORE PROTECTION PLANNING AND DESIGN"

Corrections, revisions and addenda for Technical Report No. 4 transmitted herewith, are the first revisions to be furnished in accordance with procedure proposed in the preface to the original report published in 1954.

As indicated on the change sheet bound with the revisions new pages are furnished to replace certain pages of the original report. Also listed are minor corrections which should be made on the original pages.

Principal additions to the report include later material available through February 1957 on forecasting of wind-generated waves, generation of wind-waves in shallow water, wave run-up and overtopping, and determination of K' in the modified Iribarren formula for rubble-mound structures.

August 1957

CORRECTIONS, REVISIONS AND ADDENDA FOR
TECHNICAL REPORT NO. 4 OF THE BEACH EROSION BOARD
"SHORE PROTECTION PLANNING AND DESIGN"

(Changes compiled February 1957)

(* indicates reprinted page containing corrected or revised material)

	Page No.	Change
1.	i* , ii*	Replace with revised pages attached.
2.	7, 8*	Replace with revised pages attached.
3.	15*,16,17,18*	Replace with revised pages attached and insert additional pages 16a and 16b
4.	25*,26,27*,28*	Replace with revised pages attached and insert additional pages 28a - 28q.
5.	29	Equation 12, change to read: $E_t/E_{to} = (1/2)(1/n)(b_o/b)$
		3rd line from bottom, change to read:
		"From equation 10, $H/H_o = \sqrt{(E_t/E_{to})(L_o/L)}$, therefore"
		7th line from bottom, change "orthogoanls" to "orthogonals"
6.	31,32*	Replace with revised pages attached.
7.	36	Delete 2d sentence of text immediately under Figure 19, beginning "The dots on the graph of Figure 17 ...".
8.	37, 38*	Replace with revised pages attached.
9.	Between 46 & 47	Insert additional pages 46a - 46d.
10.	48	Figure 38, change title to read: "BREAKER DEPTH INDICES".
11.	49*, 50*	Replace with revised pages attached.
12.	55	Change table heading from TABLE 6 to TABLE 4
		Add title under Figure 39a as: "HYDROGRAPH OF MONTHLY MEAN LEVELS OF THE GREAT LAKES".
13.	89*,90*,91,92*	Replace with revised pages attached and insert additional pages 89a - 89f and 90a - 90o.
14.	96	2d paragraph, 4th line, change "along stockpiles" to read "long stockpiles".
		2d paragraph, 7th line, change "be economic considerations" to read "by economic considerations".
15.	105*,106,107*, 108	Replace with revised pages attached.

February 1957

Page No.	Change
16. 110	Paragraph 3.54, 7th line, change "characteristics" to "characteristic".
17. 123, 124*	Replace with revised pages attached.
18. 133	7th line from bottom of page, change "varifications" to "verifications".
19. 149*,150*, 151*,152*, 153*,154	Replace with revised pages attached.
20. 195*,196	Replace with revised pages attached.
21. 204	2d paragraph, last sentence, insert "both typical and existing" after "breakwaters"
22. 205*, 206	Replace with revised pages attached.
23. 209*,210	Replace with revised pages attached.
24. 213	15th line from top of page and 3rd line from bottom of page, change "equation 12" to read "equation 13"
25. 229*,230,231*, 232*, 233*, 234*	Replace with revised pages attached.
26. 239*, 240	Replace with revised pages attached.
27. A-10 (App.A)	Under "Drift (noun)" change "Drunlin" to "Drumlin"
28. A-30	2d line from top of page, delete "Grad"
29. A-39, A-40*	Replace with revised pages attached.
30 A-49*	Replace with revised page attached.
31. B-11*, (App. B)	Insert additional page, B-11 attached.
32. C-9*, C-10*, C-11*	(App. C) - Replace with revised pages attached, and insert additional page, C-11.
33. D-1*, D-2 (App. D)	Replace with revised pages attached.
34. D-25*, D-26*	Replace with revised pages attached.
35. D-33, D-34*	Replace with revised pages attached.
36. D-38	Plate D-8b, 5th curve from top of plot, change legend from "slope 1:1.2" to "slope 1:2"
37. D-39, D-40*	Replace with revised pages attached.
38. D-41*,D-42*, D-43*	Replace with revised pages attached, and insert additional page, D-43

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SHORE PROTECTION
PLANNING AND DESIGN

INTRODUCTION

This report has been prepared with a view to assembling in a single volume, insofar as practicable, a manual of practice for shore protection. The term "shore protection", as it is used herein, applies primarily to works designed to stabilize seacoasts and shores of large bodies of water where wave action is the principal cause of erosion.

The nature and degree of protection required differ widely at different localities and the proper solution of any specific problem requires a systematic and thorough study. The first requisite for such a study is a clear definition of the problem and the objectives sought; the first factor to be determined in the course of the study is the cause of the problem. Ordinarily there will be more than one method of obtaining the immediate objective. In the study, therefore, the long term effects of each method should be forecast and evaluated, beyond as well as within the problem area. All advantages and effects should be considered in comparing annual costs and benefits to determine the justification of remedial measures.

An attempt has been made to include herein a detailed summary of applicable methods, techniques and useful data pertinent to the solution of shore protection problems. Part I discusses the factors considered important in the analysis of such problems, beginning with the sources of energy and characteristics of material, proceeding with the interaction of these factors and ending with the development of functional plans. Part II presents generally accepted practice in structural design techniques for shore structures. Graphs and tables to facilitate computations and analysis are provided throughout the text and in appendices. Techniques presented herein are generally applicable to the broad scope of shore protection problems but competent engineering judgment is required for determining their application to any specific problem.

As the meanings of terms used in coastal engineering differ from place to place, the reader is advised to make full use of Appendix A (Glossary of Terms).

PART I

FUNCTIONAL PLANNING

CHAPTER 1

PHYSICAL FACTORS

1.1 GENERAL

Beach and shore erosion consists of wearing away of the land by the application of energy to shore materials. Kinetic energy is available at the shore in the form of wind, which acts directly on shore materials. However, the principal method of application of the wind's energy to the shore is through water waves generated by the wind. A considerable portion of the total energy of the wind acting over great expanses of water is thus enabled to reach the shore. Occasionally waves are also generated by other sources of energy, such as earthquakes.

Other physical factors which establish the criteria for planning protective measures are changes in water level, which determine the elevation at which the wave energy will act on the shore. Though ice may be a major factor, especially on the Great Lakes, it should primarily be considered in its structural aspects. Functionally only the fact that ice may reduce wave action at certain locations need be considered.

1.2 WAVE ACTION

1.21 GENERAL - Waves generated in deep water are usually of the type known as oscillatory waves, in which the particles of water making up the wave oscillate in a circular orbit about some mean position (see Figure A-3, Appendix A).

An oscillatory wave is well defined if the wave length, L, the horizontal distance between corresponding points on two successive crests, the wave height, H, the vertical distance to a crest from the preceding trough, the wave period, T, the time for two successive crests to pass a given point, and the depth, d, over which the wave moves, are known. The velocity, C, with which an oscillatory wave progresses is related to period and length by

$$L = CT \tag{1}$$

and to the depth and length by

$$C^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L} \tag{2}$$

where g is the acceleration of gravity. The total energy per unit crest width in an oscillatory wave is given by

$$E_T = \frac{1}{8} \rho g H^2 L (1 - M \frac{H^2}{L^2}) \tag{3}$$

Energy Equation (deep water)

where ρ is the mass density of water and M is a function of d/L (See Tables D-1 and D-2, Appendix D).

The velocity at which energy is propagated, known as the group velocity C_g , is not necessarily the same as the wave velocity C, but bears a known relationship to this velocity,

$$C_g = nC \tag{4}$$

where

$$n = \frac{1}{2}\left[1 + \frac{4\pi \frac{d}{L}}{\sinh \frac{4\pi d}{L}}\right] \tag{5}$$

$d > L_o/2$

In water deeper than one-half the wave length, known as deep water, sinh $4\pi d/L$ becomes very large, and n becomes approximately 1/2. Thus, in deep water, energy moves at one-half the wave speed. In a group of waves, this fact causes individual waves to move forward through the group, and die out at the group front. In water shallower than about 1/25th the wave length, called shallow water, sinh $4\pi d/L$ approaches $4\pi d/L$, n approaches 1 and the group velocity is the same as the wave velocity. In water whose depth is between 1/2 and 1/25 the wave length, called transitional water $(1/2 < n < 1)$, individual waves still move faster than the group they are in but not at twice the speed as in deep water.

$d < \frac{L_o}{25}$

The dependency of wave velocity on water depth in the transitional and shallow water zones (equation 2) causes a reduced velocity for the portion of a wave in shallower water. The waves are thus bent or refracted (see section on wave refraction) with a consequent change in direction of propagation. This water wave refraction is analogous to refraction of light in which light rays change direction in passing through media in which the speed of light differs.

1.22 <u>GROWTH AND DECAY OF WIND WAVES</u> - In an area of water over which the wind is blowing and generating waves, known as a fetch or fetch area, the growth of the waves is governed by at least three factors: the speed of the wind (U); the length of time the wind has been blowing, or duration (t_d); and the length of the fetch (F) in the direction the wind is blowing. In relatively shallow water areas, in addition to these factors, the depth of water in the fetch limits wave growth. Both wave heights and periods increase in a fetch, from the fetch rear where the wind is first significant to the fetch front.

Diagramatically wave growth may be illustrated as in Figure 1. Assuming that a wind of constant speed springs up, at an early stage of wave development, wave heights and periods will change from fetch rear to fetch front roughly as shown by the dashed line in the lower part of Figure 1. When the wind starts blowing, small waves develop throughout the fetch and move forward in the direction of the wind, growing in size as they move forward. No waves enter the fetch at the rear so at this point the wave height and period must be zero. At this stage the wind has been blowing only long enough to generate maximum heights or periods given by the

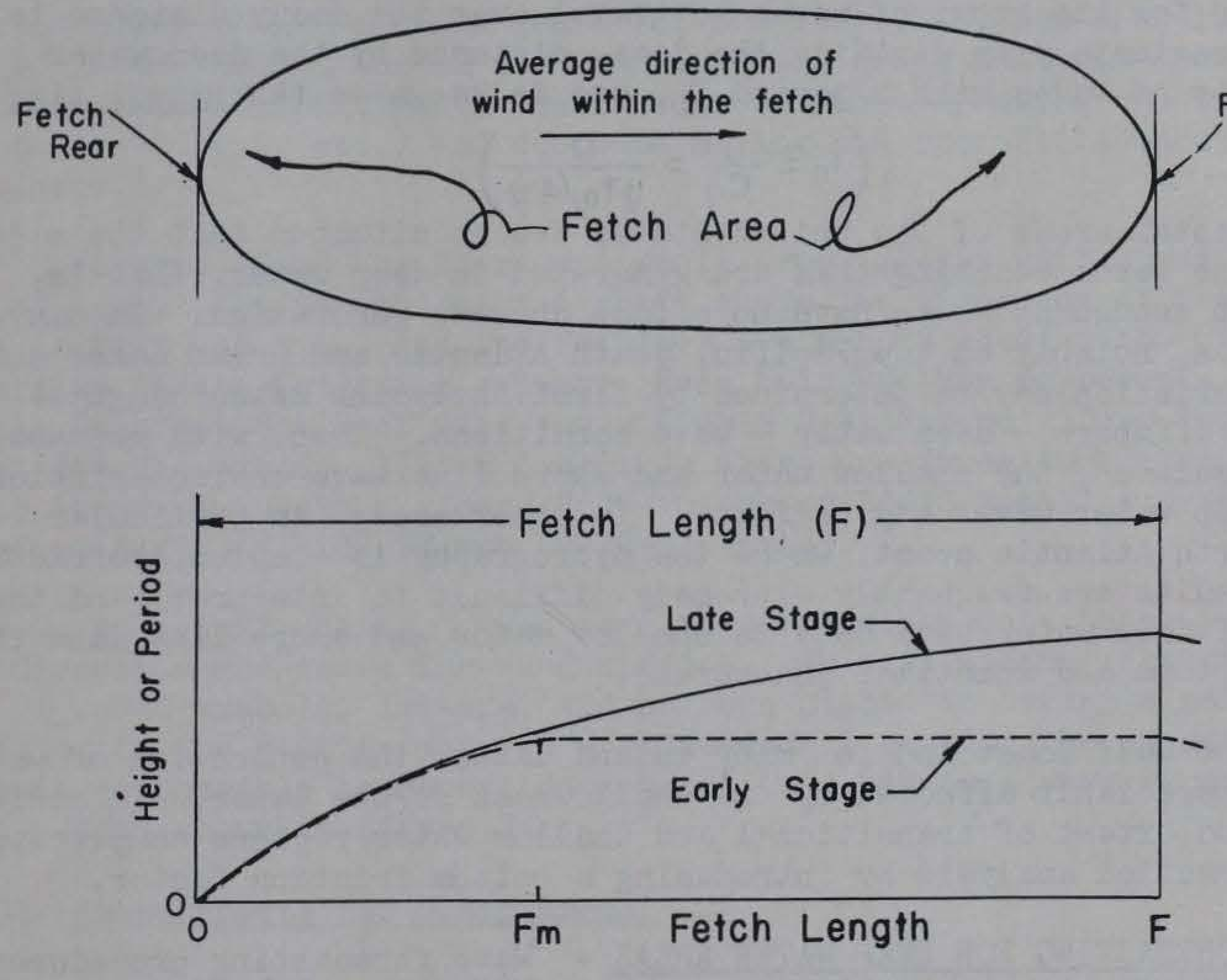
deep water $d > L_o/2$

Transitional water $\frac{L_o}{25} < d < \frac{L_o}{2}$

Shallow water $d \leq \frac{L_o}{25}$

horizontal dashed line in the figure. The waves are limited in height by the duration of the wind. Any fetch length greater than that from 0 to F_m in Figure 1 will not be accompanied by greater wave heights or periods. The distance from 0 to F_m is known as the minimum fetch length F_m corresponding to the least duration of wind - called the minimum duration t_m which will give rise to wave conditions illustrated by the dashed line.

A later stage of wave development is illustrated by the solid line in Figure 1. The wind has been blowing long enough to generate maximum heights or periods given by the intersection of the solid curve with the ordinate representing the fetch front. After this stage is reached, an increase in duration of the wind gives no corresponding increase in wave height or period. Waves start from zero at the fetch rear, increase under the influence of the wind to the fetch front, then leave the generating area. In other words, wave growth is limited by the available fetch. In such a case the least length of time it takes a given wind blowing over a definite fetch length to cause such a steady state condition is known as the minimum duration (t_m). The fetch length involved, in this case the total available fetch, is still known as the minimum fetch corresponding to the minimum duration.



WAVE DEVELOPMENT WITHIN A FETCH

FIGURE I

It should be particularly noted that for the early stage shown the measured fetch F is longer than the minimum fetch F_m corresponding to t_m as given above. In the late stage, the measured duration may be longer than that minimum duration corresponding to the full measured fetch length F.

The distributions shown in Figure 1 are actually simplified distributions, since a spectrum of wave heights and periods is generated in a fetch. These simplified distributions refer to what are known as significant waves, a statistical term which is used to describe the average of the heights and periods of the highest one-third of the waves in a group.

After leaving a fetch, waves travel to a point some distance away - a coast for example - with speeds proportional to their periods (equations 2, 4 and 5). In this decay distance D, the longer period waves move faster than, and consequently arrive at the end of the decay distance before, those with shorter periods. An observer at the end of the decay distance sees a group of wave whose significant period T_d, is longer than the significant period T_f at the fetch front. Therefore, the significant period of waves seems to increase after they leave the fetch.

The heights of waves actually decrease after leaving a fetch, and at the end of the decay distance, the observed significant wave height H_D will be smaller than H_F, the significant wave height at the front of the fetch.

The time for the group of waves to travel over the decay distance is obtained approximately by dividing the decay distance by the deep water group velocity of waves with a period T_D, and is known as the travel time

$$\left(t_D = \frac{D}{C_g} = \frac{D}{gT_D/4\pi}\right)$$

Most coastal areas of the United States are so situated that the major portion of the waves reaching them are generated in deep water, that is, in water deep enough so as to have no effect on wave generation. In many of these areas, notably on the Pacific, South Atlantic and Great Lakes coasts, wave characteristics may be determined by first analyzing meteorological data to find offshore - deep water - wave conditions. Then, with refraction analysis procedures, the shallow water and shore line wave characteristics for these deep water waves may be found. In other areas, in particular along the North Atlantic coast, where the hydrography is complex, refraction procedure results are frequently extremely difficult to interpret, and the conversion of deep water wave data to shallow water and shore line data thus becomes laborious and sometimes inaccurate.

Along the Gulf Coast and in many inland lakes, the generation of waves by wind is appreciably affected by the shallowness of the water. In addition the nature and extent of transitional and shallow water regions complicates ordinary refraction analysis by introducing a bottom friction factor.

1.23 WAVE FORECASTING FOR DEEP WATER AREAS - Wave forecasting procedures may be used to translate the meteorological data into wave data. Actually for both functional planning and structural design the term "hindcasting" might better be applied since it is historical wave data which is used.

However, the techniques are the same for both hindcasting and forecasting, and the latter term is usually used.

To make a forecast it is necessary to delineate a fetch or generating area, to measure its length, and the decay distance, if any, and to know the wind speed and wind duration in the fetch. These determinations may be made in many ways depending on the forecasting area, and the type of meteorological data available. For relatively restricted bodies of water such as lakes, the fetch length is often the distance from the forecasting point to the opposite shore measured along the wind direction. In these cases there is no decay distance, and it is most often necessary to use observational data to determine wind speeds and durations.

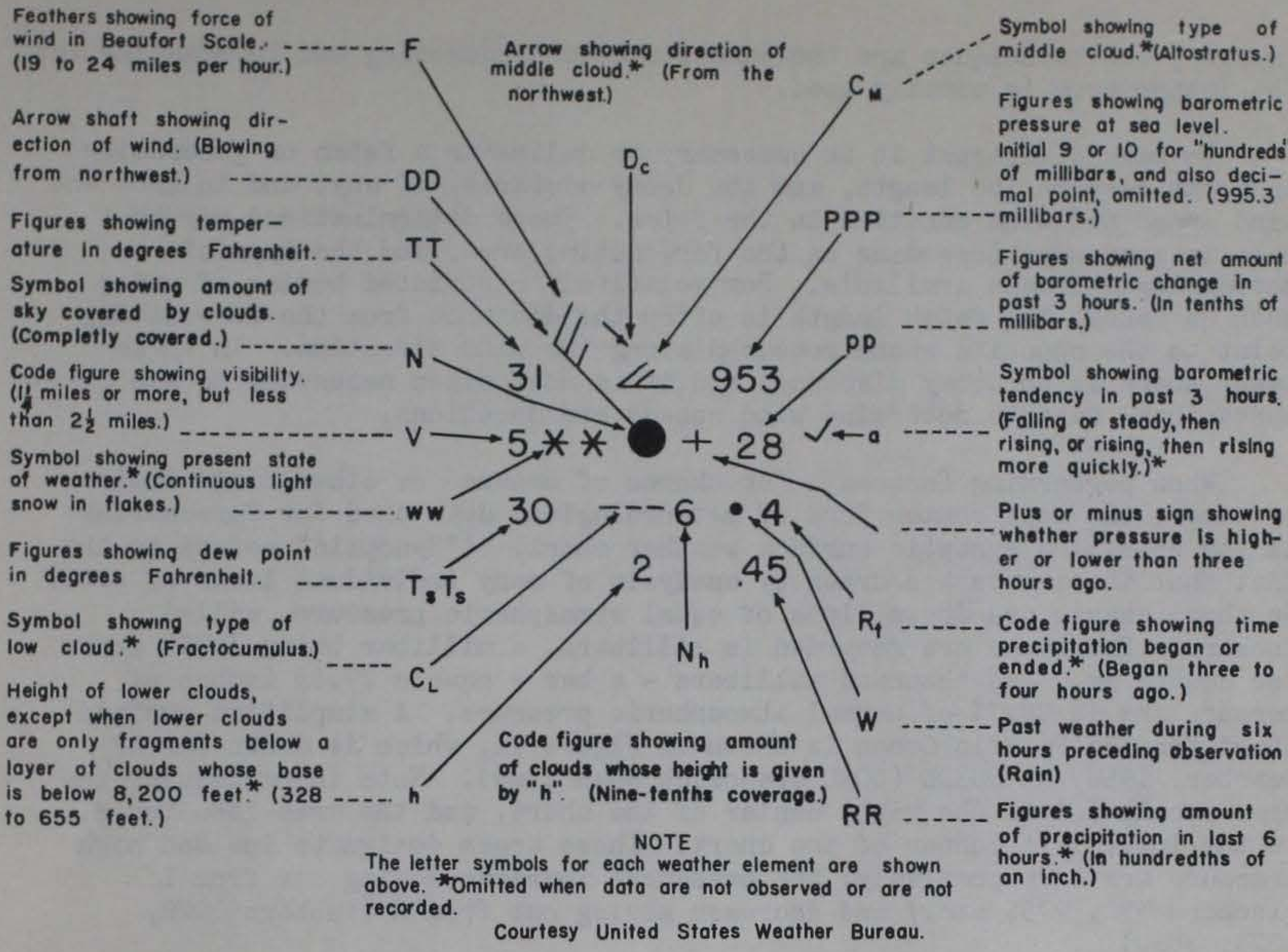
When performing forecasts for shores of oceans or other large bodies of water, the most common form of meteorological data used for forecasting is the so-called synoptic surface weather chart. ("Synoptic" refers to the fact that the charts are drawn by analysis of many individual items of data). On these charts are drawn lines of equal atmospheric pressure, called isobars. Pressures are recorded in millibars, a millibar being 1,000 dynes per square cm. One thousand millibars - a bar - equals 29.53 inches of mercury and is 98.7% of normal atmospheric pressure. A simplified surface chart for the Pacific Ocean is shown on Figure 11, which is drawn for 27 October, 1950, at 0030Z (0030 Greenwich Mean Time). Note in particular the area labelled L in the right center of the chart, and the area labelled H in the lower left corner of the chart. These areas designate low and high pressure areas respectively; the pressures increase moving out from L (isobars 972, 975, etc.) and decrease moving out from H (isobars 1026, 1023, etc.)

Scattered about the chart are small arrow shafts with a varying number of feathers. The direction of a shaft shows the direction of the wind at that station and time, and the feathers are a code showing the wind force according to the following scheme: Each half feather represents one unit on the Beaufort Scale of wind force (Table D-3, Appendix D). Thus near the point 35° N latitude, 135° W longitude there are three such arrows, two of which show a Beaufort scale of 8(39-46 mph or 34-40 knots) and the third a Beaufort Scale of 7(32-38 mph or 28-33 knots).

On an actual chart, much more meteorological data than wind speed and direction are shown for each station. This is accomplished by the use of coded symbols, letters, and numbers placed at definite points in relation to the station dot. A station model, showing the amount of information it is possible to report on a chart (not all is always reported or plotted) is shown in Figure 2.

1.231 Measurements for Ocean Areas

(a) - Wind Speed and Direction - Certain relationships exist between wind and isobaric configurations which permit the use of these charts in wave forecasting for areas bordering oceans. If a line is drawn perpendicular to a set of isobars, pressures at different points on the line would differ,



STATION MODEL
FIGURE 2

(Stewart, 1945)

since each isobar crossed represents a different pressure. The rate of this pressure change toward lower pressures is called the gradient. If no other factors were present, this gradient would cause wind to blow normal to the isobaric pattern. However, other forces are present, in particular the Coriolis force (caused by the rotation of the Earth on its axis, which acts normal to the wind direction), centrifugal force, and frictional force. Without friction, equilibrium between these forces is attained when wind blows along the isobars. If the isobars are straight, only Coriolis force and the force due to the pressure gradient are acting, and the resultant computed wind is called the geostrophic wind. If the isobars are curved, centrifugal forces also act; the resultant wind is called the gradient wind. Frictional forces, however, cause the wind to blow, not parallel to the isobars, but at a slant across them toward the lower pressure region. Near the surface, the angle between the gradient or geostrophic winds and the actual wind is 10 to 15 degrees over sea areas and about 40 degrees over land areas. The spiraling wind pattern thus created is called a cyclonic or anticyclonic pattern depending on whether the wind is blowing about a low pressure area as center, or a high pressure area as center. In the northern hemisphere, cyclonic winds (about a low) rotate counterclockwise. The opposite is true in the southern hemisphere. Friction causes actual wind speeds to be lower than computed gradient or geostrophic wind speeds.

Two graphs are used to facilitate the computation of actual wind speed. The first is used to calculate a geostrophic wind speed assuming the isobaric configuration under consideration is straight. The second is used to find the ratio of the actual to the geostrophic wind speed taking account of both isobar curvature (to convert geostrophic to gradient wind) and friction.

To utilize the first of these (Figure 3) it is necessary to have a measure of the pressure gradient over the area in question. Since most charts are drawn with either a 3-millibar or a 5-millibar spacing, for either of these standard spacings the geographical distance between isobars is adequate as such a measure. To use Figure 3, the distance between isobars on a chart is measured in degrees of latitude (an average spacing over a fetch is ordinarily used), and the latitude position of the fetch is determined. Using the spacing as ordinate and location as **abscissa,** the plotted or interpolated slant line at the intersection of these two values gives the geostrophic wind speed. For example, on Figure 11, a chart with 3-millibar isobar spacing, the average isobar spacing over F_2, located at 37 degrees N latitude, is 0.70 degrees of latitude. Using the scales on the bottom and right of Figure 3, a geostrophic wind of 67 knots is found.

The second graph (Figure 5) relates the ratio $\dfrac{\text{"actual surface wind speed"}}{\text{geostrophic wind speed}}$ to the difference between the sea and air temperature for various radii of isobar curvature measured in degrees latitude. The difference of sea and air temperature here acts as a measure of the friction effect, and the isobaric curvature as a measure of the relationship between gradient and geostrophic wind. Thus, on this graph, two corrections are applied. As an example of its use, reference is made again to fetch F_2, Figure 11, in which the average isobar radius of curvature was found to be about 10 degrees of latitude (mild cyclonic curvature). There was found to be no difference of sea-air temperatures, and the curves of Figure 5 give 0.59 as the ratio of actual surface to geostrophic wind speed. The actual wind speed in this fetch, found by multiplying the 67-knot geostrophic wind by the 0.59 ratio above was about 40 knots.

Note that for this particular storm the sea temperature used (65°) was that given on Figure 4, from the World Atlas of Sea Surface Temperatures (130)* Sea temperatures are also given in the Climatic Atlas(137). The air temperature was taken from the station reports in the area (these reports are not plotted).

The procedures illustrated above are those ordinarily used to determine wind speeds. However, if the speeds so derived differ from the average value of ship reported wind speed in the fetch area by more than one

* Numbers shown in this manner refer to references listed in Appendix C.

GEOSTROPHIC WIND SCALE

$$V_g = \frac{1}{2\Omega\rho\sin\phi}\frac{\Delta p}{\Delta n}$$

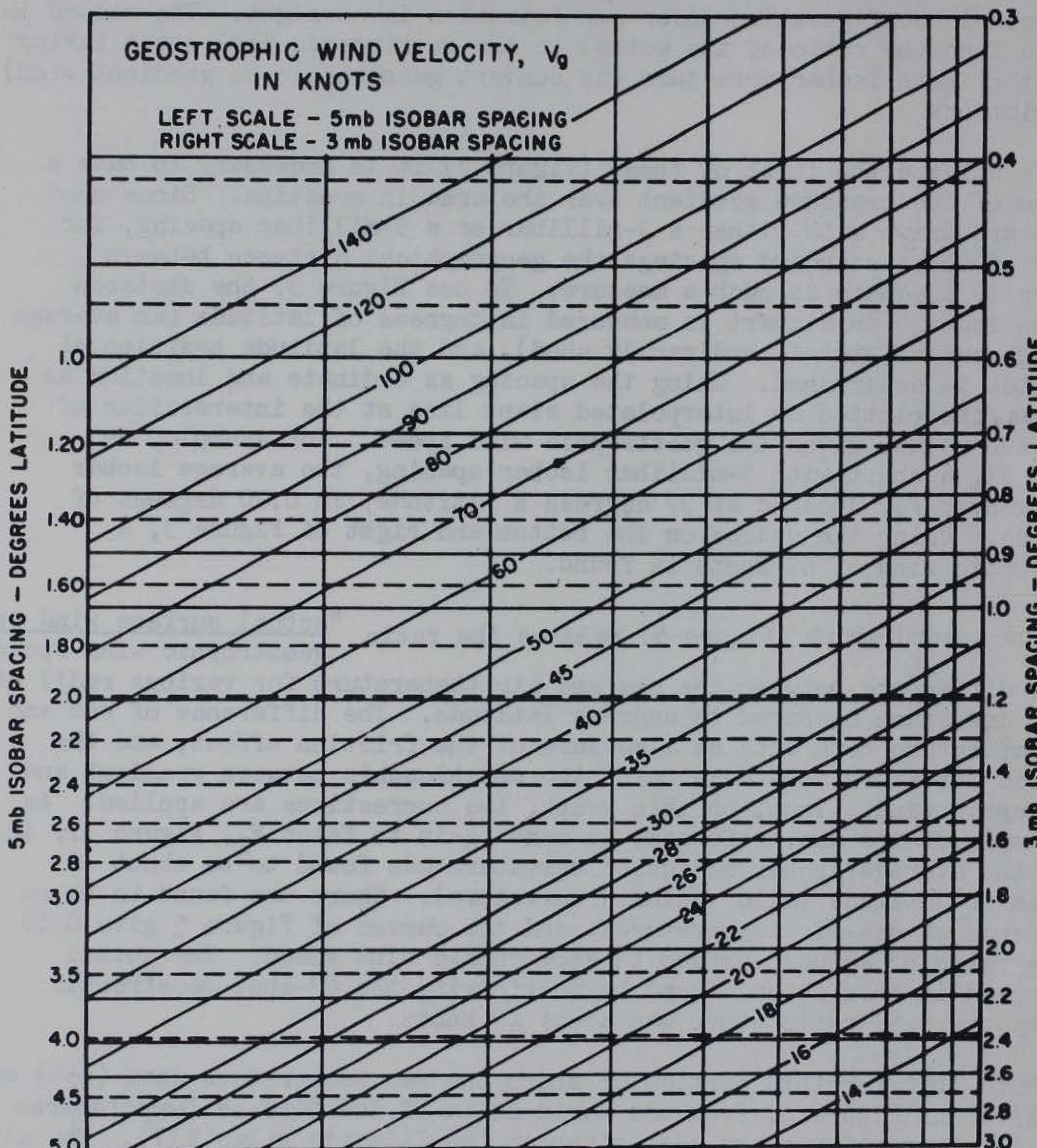
FOR Δp = 5mb & 3mb

Δn = Degrees Latitude

p = 1013.3mb

T = 10°C

$\rho = 1.2 \times 10^{-3}$ gm/cm.^{3}



(Bretschneider, 1952)

FIGURE 3

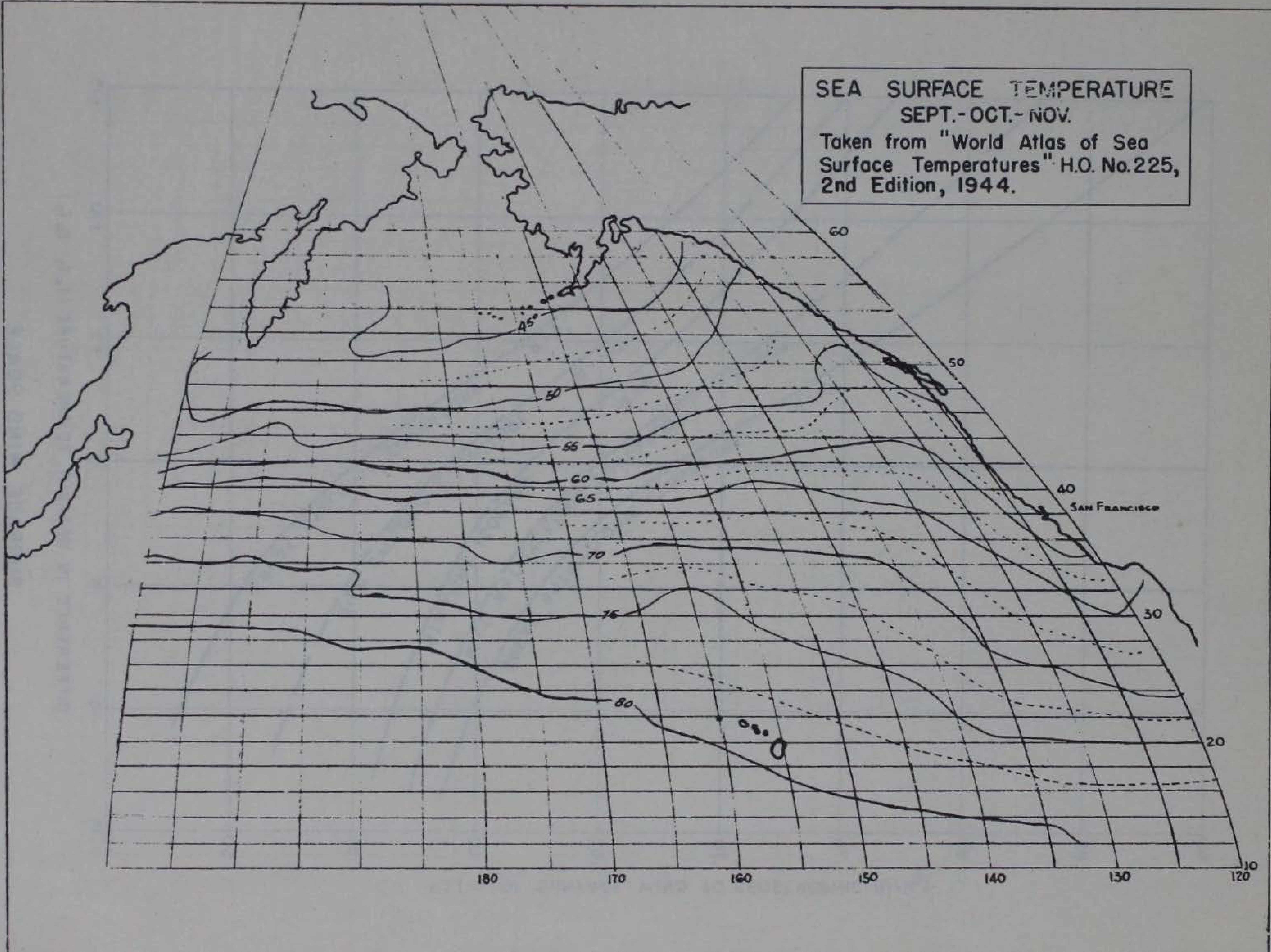
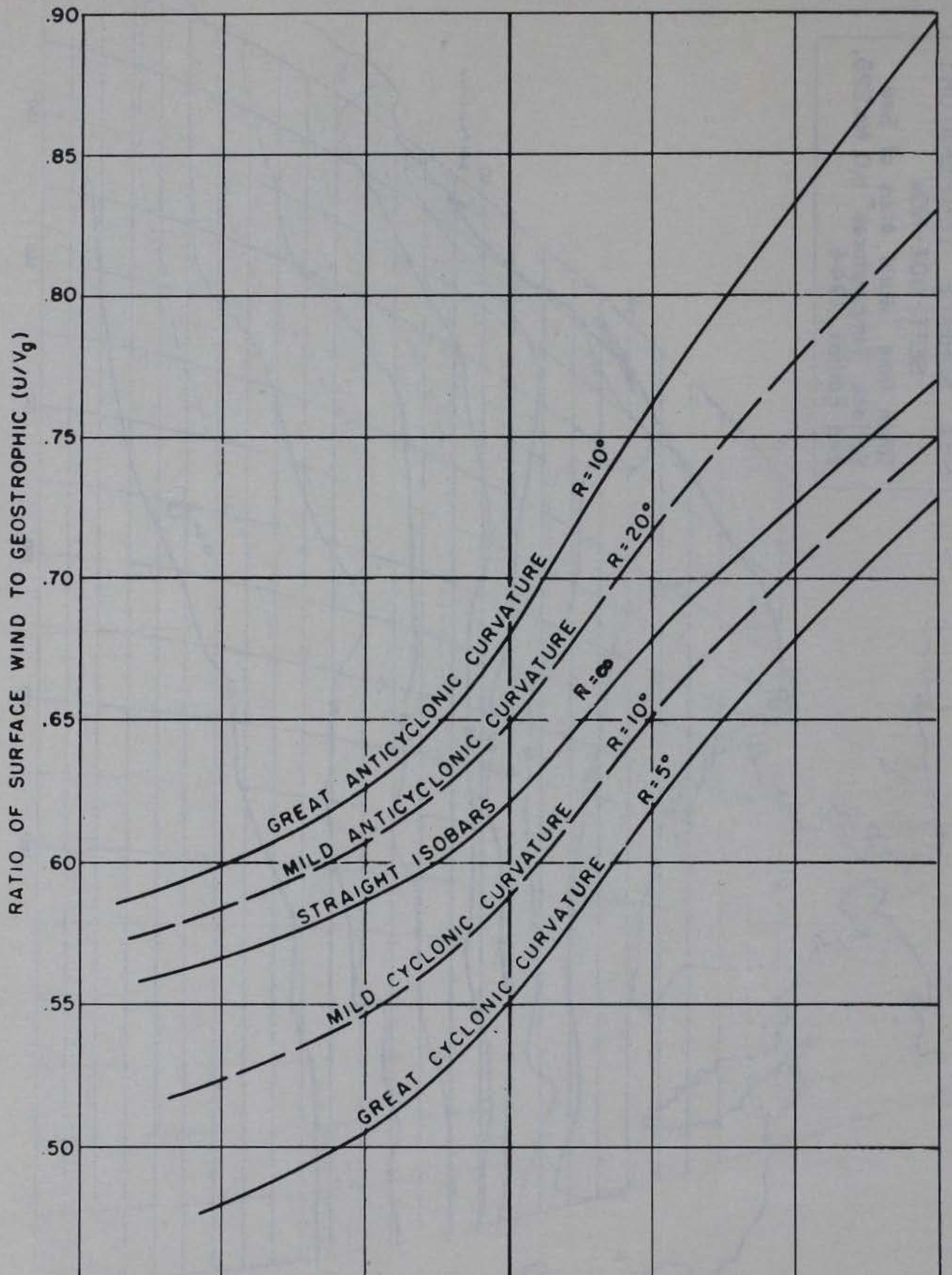

SEA SURFACE TEMPERATURE
SEPT.-OCT.-NOV.
Taken from "World Atlas of Sea Surface Temperatures" H.O. No.225, 2nd Edition, 1944.
45°
50
55
60
65
70
75
80
60
50
40
SAN FRANCISCO
30
20
10
180
170
160
150
140
130
120

FIGURE 4



SURFACE WIND SCALE

FIGURE 5 (Bretschneider, 1952)

Beaufort Scale division the forecaster must judge for himself which of the two values is more likely to represent actual conditions. For example, if there are many ship reports in the immediate fetch area, all in approximate agreement the wind speeds from these may be more reliable than that derived from the isobaric pattern.

(b) <u>Delineating a Fetch</u> - The problem of determining on a surface chart that area which may properly be called a generating area when forecasting for a specific locality, is quite complex. The following factors must be given consideration.

(1) Variability of wave direction: It is possible, with a fair degree of assurance, to obtain the general direction of the wind for a given ocean area. However, it has been shown[2] that waves in a generating area move not only in the direction of the wind, but at various other angles to the wind. For a fetch with relatively straight isobars, those waves moving at 30° to the average wind direction will have heights of 80 to 90 percent of the waves moving in the direction of the average wind. When isobars are curved, it may be expected that this 80 to 90 percent semi-sector will be larger than 30°. It has been common practice to account for this variability by assuming that undiminished waves may move at 30° to the wind direction for a relatively straight isobaric pattern, and at 45° for a curved isobaric pattern.(132)

(2) Isobar spreading: The greater the distance between isobars, the smaller is the wind speed. When the wind speed drops below a certain value, and the decay distance is relatively large, waves generated by this wind will not be significant at the point for which forecasts are being made. The commonly adopted limiting criterion (132) is that for decay distances of 500 miles or more, wind speeds of 20 knots or less may be ignored. Noticeable spreading of isobars is often a sign useful in locating both a fetch front and rear.

(3) Frontal lines: The continually changing isobar patterns represent constantly moving air masses, the characteristics of any two of which usually differ though they may be contiguous. Boundary areas of different air masses are plotted on surface charts, four different symbols being used depending on the type of air mass movement. The symbol ▲▲ , called a cold front, is used to indicate the line on the earth's surface along which a mass of cold air is advancing into a mass of warm air. The symbol , called a warm front, is used to indicate the line along which a mass of warm air is advancing into a mass of cold air. Both symbols on a surface chart represent only the line of intersection of the surface of separation of the two air masses with the earth's surface. The actual surface of separation, called a frontal surface, will be one in which the cold air forms a tongue intruding at low levels into the warm air mass. The symbol , called an occluded front, represents a line along which a cold front having overtaken a warm front and lifted

the warm air from contact with the ground, meets the cold air mass which had been ahead of the now lifted warm air mass. The symbol is called a stationary front, and represents a front not moving at the time of the chart. Pictorial representations of these conditions are shown in Figure 6. It may be seen from the typical isobaric configurations also shown on Figure 6 that at a front line the direction of the isobars often abruptly changes. Since wind direction depends on isobar direction, at a front wind direction often changes abruptly, often enough to limit a fetch at or near a front.

(4) Accuracy: One of the most important considerations in locating and limiting a fetch is the accuracy with which charts are drawn. Especially over ocean areas, the information from which a meteorologist constructs a synoptic chart is usually very limited. Few ships sail outside normal shipping lanes, thus presenting the analyst with wide expanses of ocean for which little data exist. Many isobaric configurations on synoptic charts represent estimates, limited in accuracy by the extent of detailed data available.

(5) Land Masses: It should be noted that land masses often limit the water area available for a fetch. This is true for lakes, bays and land-locked or partly land-locked sea areas.

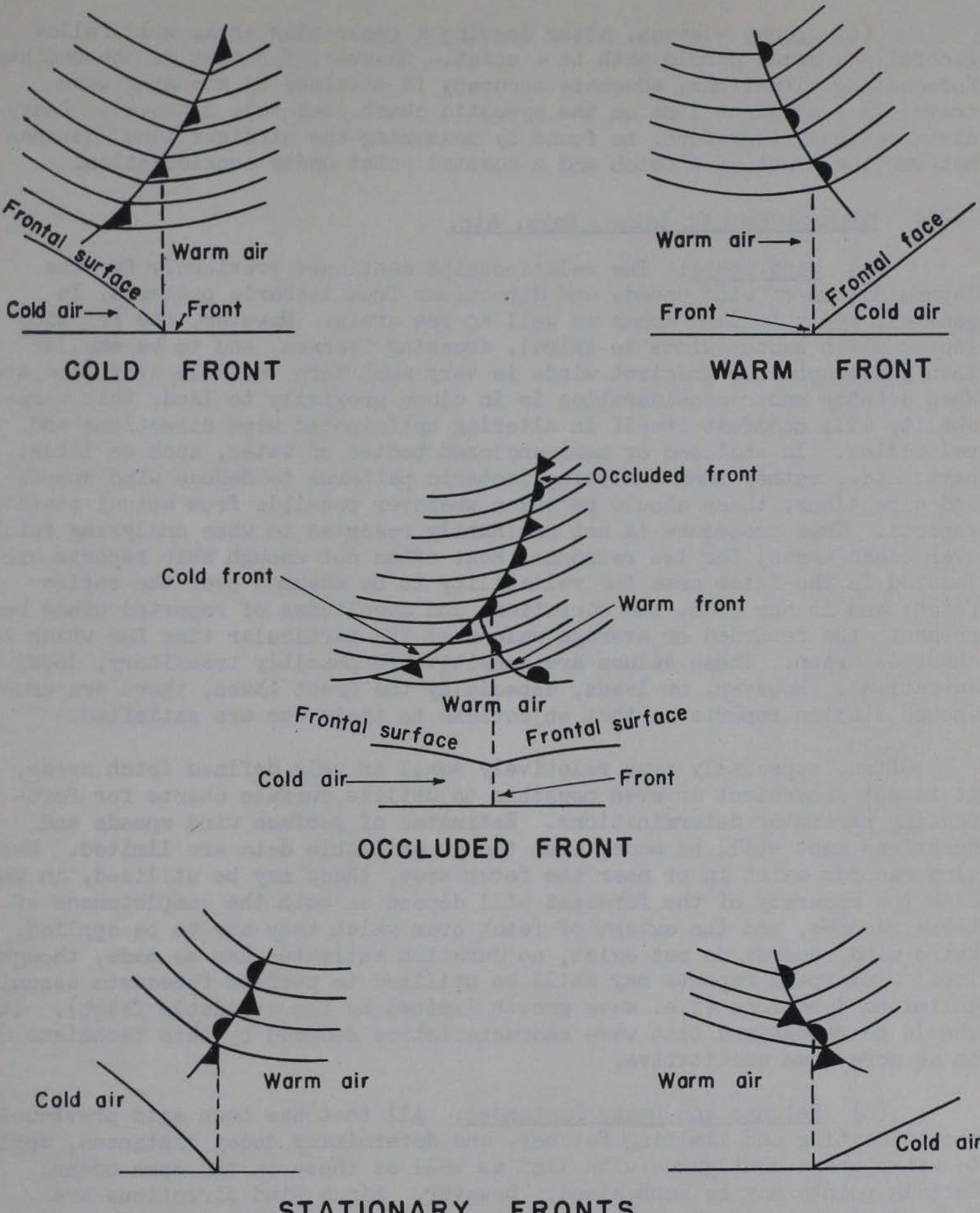
With these considerations in mind certain generalized rules may be adopted to aid in delineating fetch areas.

(1) Depending on the relative locations of the point for which forecasts are to be made and a storm area, waves which are moving at angles of from 15° to 50° with the isobar directions will be significant. For simplicity the somewhat inaccurate rule (see following rule 4) that for the general isobaric pattern - without fronts or significant isobar spreading - directions of approach of 30° with the isobars for relatively straight isobars, and 45° for curved isobars are allowed. To locate either a fetch front or rear, therefore, a straight edge may be rotated about the forecasting area until it cuts the various isobars of a particular possible fetch at about 30° or at about 45° depending on isobar curvature (See Figure 10).

(2) If isobars spread noticeably in either the front or rear of a possible fetch area, there is a good possibility that the fetch front or rear will lie in this area of spreading. With decay distances of more than 500 miles, areas of 20-knot winds can be considered to be fetch boundaries of an area with greater wind speeds. (See Figure 10, isobars 990 and 993).

(3) Wind directions often change so abruptly at front lines that the front line itself may limit the fetch.

(4) Accuracy of the chart limits accuracy of the forecast.



ISOBARIC PATTERNS AND CONDITIONS FOR FRONTAL LINES

FIGURE 6

(c) Decay - Waves, after leaving a generating area, will follow generally a great circle path to a coast. However, for most of the ordinary forecasting situations, adequate accuracy is obtained by assuming wave travel in a straight line on the synoptic chart (see rule 1 above). Decay distances may, therefore, be found by measuring the straight line distance between the front of a fetch and a coastal point under consideration.

1.232 Measurements On Lakes, Bays, Etc.

(a) Wind Speed: The relationships mentioned previously for the determination of wind speeds and directions from isobaric patterns, in general, apply to land areas as well as sea areas. However, the friction factor which causes winds to spiral, crossing isobars, and to be smaller than geostrophic or gradient winds is very much more variable over land areas. When a fetch under consideration is in close proximity to land, this variability will manifest itself in altering anticipated wind directions and velocities. In enclosed or semi-enclosed bodies of water, such as lakes, bays, etc., rather than analyzing isobaric patterns to deduce wind speeds and directions, these should be taken wherever possible from actual station reports. This procedure is not ordinarily resorted to when analyzing fetches over ocean areas, for two reasons: Most often not enough ship reports are located in the fetch area for reliability to be assumed over the entire fetch; and in any case, the directions and magnitudes of reported winds represent the recorded or average values at the particular time for which the chart is drawn. These values are sensitive to possibly transitory, local variations. However, on lakes, especially the Great Lakes, there are usually enough station reports so that objections to their use are satisfied.

Often, especially over relatively small or well defined fetch areas, it is not convenient or even possible to utilize surface charts for forecasting parameter determinations. Estimates of surface wind speeds and durations must still be made, even though available data are limited. Where wind records exist in or near the fetch area, these may be utilized, in which case the accuracy of the forecast will depend on both the completeness of these records, and the extent of fetch over which they are to be applied. Where wind records do not exist, no duration estimates can be made, though local wind speed reports may still be utilized to perform forecasts assuming unlimited durations (i.e. wave growth limited by the available fetch). It should be recognized that wave characteristics deduced by this technique can be no more than qualitative.

(b) Fetches and Decay Distances: All that has been said previously about locating and limiting fetches, and determining decay distances, applies to water areas contiguous with land as well as those in the open ocean. Certain points may be emphasized, however. Since wind directions are determined from actual station reports, it is permissible in general to use the 30° semi-sector rule, where necessary, to limit a possible fetch. That is, waves may be assumed to vary in direction by as much as 30° with direction and still be undiminished in size. It may be expected, though, that most fetches will be limited either at the front or at the rear by a land mass. It should also be kept in mind that decay distances will be most often, relatively small or non-existent.

1.233 Forecasting Techniques for Deep Water Areas (Bretschneider revised, Sverdrup-Munk Method) - Figure 7A shows wave height and period as functions of fetch length and wind speed for deep water and unlimited wind duration using nondimensional parameters. Figure 7 which has been obtained from curves on Figure 7A, is a plot of the forecasting curves presently in use. With it, one may determine the significant wave height (H_F) and significant wave period (T_F) at the front of a fetch, knowing the wind speed in the fetch, and either the duration of the wind or the fetch length. In using it, the actual wind velocity (U), the fetch length (F), the decay distance (D), and the estimated duration (t) of the wind, when a fetch first appears on a chart, are tabulated. (When a fetch first appears on a chart the duration of the wind at the time of the chart may be taken as one-half the time (Z) between charts.) Figure 7 is then entered with the known value of U, on the left if U is in knots, or on the right if in statute miles per hour. This "U" line is then followed across to its intersection with the fetch length (F) line, or the duration (t) line, whichever comes first from the left side of the graph. For example, with a wind of 35 knots or 40 mph, a duration of 10 hours comes before a fetch length of 200 nautical miles. Similarly an F of 80 nautical miles comes before a t of 10 hours. At this point H_F and T_F, the wave height and period at the head of the fetch, may be read off and tabulated. For the case given above with $U = 35$ knots, $t = 10$ hours, and $F = 200$, $H_F = 18$ feet, $T_F = 9.5$ seconds, t_m of course equals 10 hours, and $F_m = 100$ nautical miles. If F had been 80 nautical miles with t still 10 hours, the heights, periods, minimum duration and fetch would be $H_F = 16$ feet, $T_F = 9$ seconds, $t_m = 8.5$ hours and $F_m = 80$ nautical miles. The wave pattern in the first case where the duration is limiting, would correspond to the dashed line in Figure 1, that of the second case, where the fetch is limiting, to the solid line in Figure 1.

For the same fetch on a later chart drawn for a time Z after the first chart, U, F, and D are again tabulated. Using the subscript "2" to refer to forecasting parameters of this second chart, and subscript "1" to refer to those of the first chart, if $U_2 = U_1$ the above procedures should be followed using either $t_2 = t_{m1} + Z$ or F_2. If, however, $U_2 \neq U_1$ certain additional assumptions must be made before using the forecasting curves.

It has been suggested (14), that for purposes of forecasting, a change in wind speed from U_1 to U_2 in a time Z between charts, may be assumed to take place instantaneously at a time Z/2. Waves due to U_1 may then be calculated by assuming that the first chart's minimum duration time has been lengthened by an amount Z/2 or that its minimum fetch has been changed by $\Delta F/2$, where ΔF represents the change in fetch length between weather charts. Since at the assumed abrupt change in wind speed, the energy imparted to the waves by U_1, with a minimum duration $t_{m1} + Z/2$ for a minimum fetch $F_{m1} + \Delta F/2$, does not change, U_2 will begin imparting energy to waves which already contain energy due to U_1.

Plotted on Figure 7 are dotted lines of constant $(H_F \; T_F)^2$ which may be thought of as lines of constant wave energy. (To a first approximation, deep water wave energy is given by $E = \frac{\rho g H^2 L}{8} = \frac{5.12 \, \rho g (HT)^2}{8}$; compare with equation 3). If the energy had been imparted to the waves by U_2 acting alone, these waves would be of length and height given on Figure 7 by the intersection of the U_2 ordinate with the constant energy line (plotted or

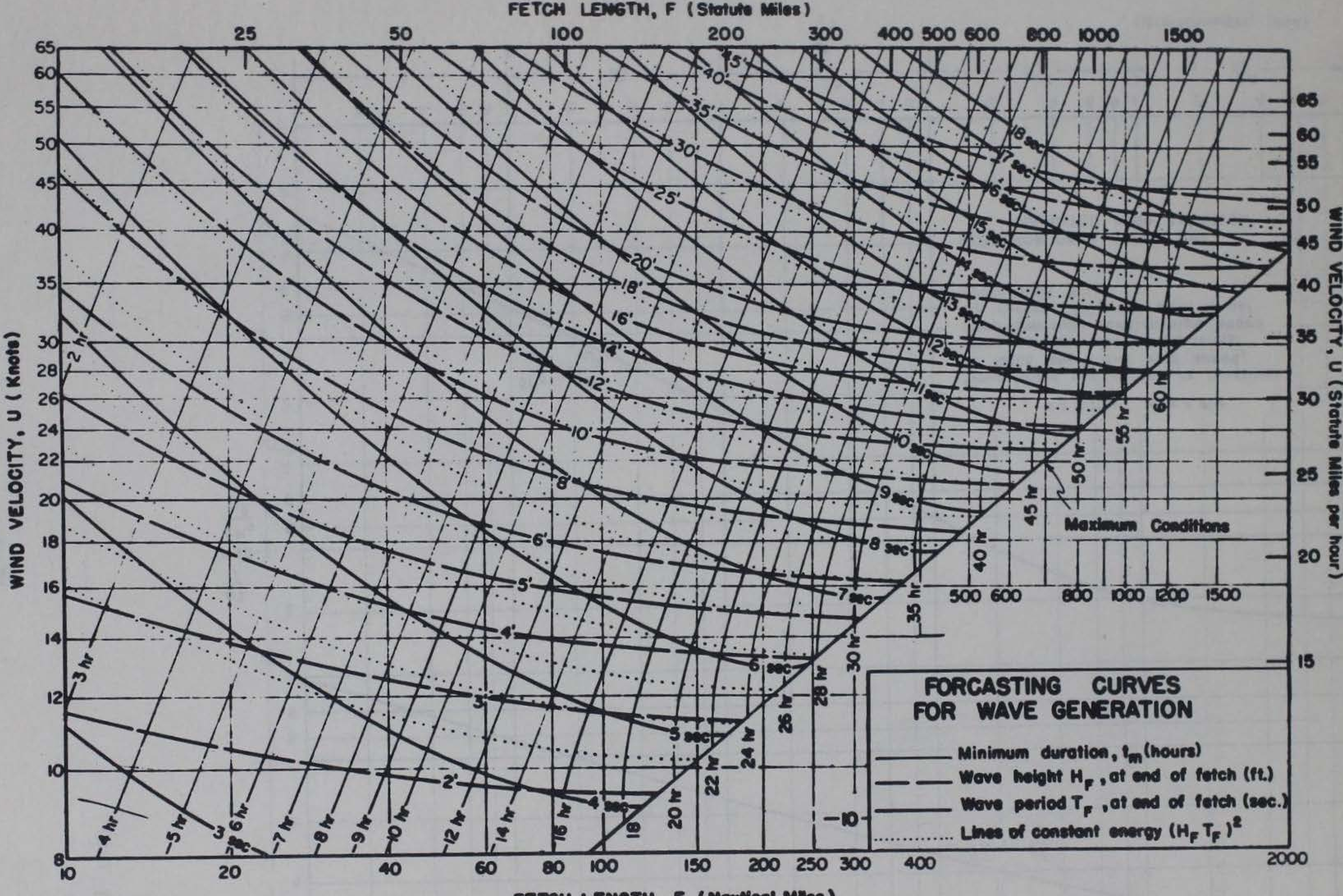
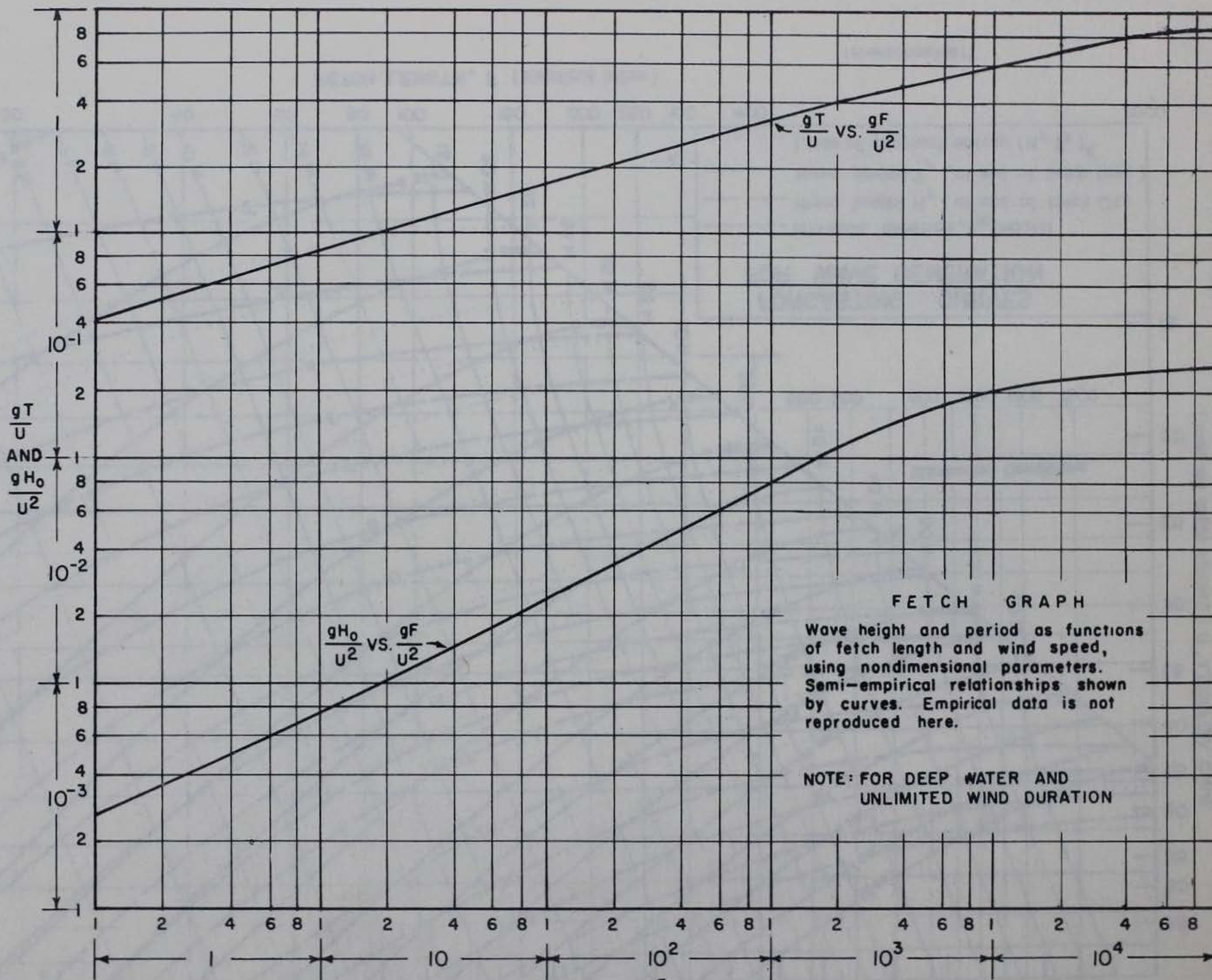


FIGURE 7

(Bretschneider)



(Bretschneider, 1954)

FIGURE 7A

interpolated) corresponding to energy imparted by U_1 with a minimum duration of $t_{m1} + Z/2$ or a minimum fetch $F_{m1} + \Delta F/2$. By increasing the minimum duration at this point by an amount Z/2 or by changing the minimum fetch by an amount $\Delta F/2$ wave conditions under U_2 at the time of the second chart may be approximated.

For example, with $U_1 = 35$ knots, $t_{m1} = 10$ hours, $F_{m1} = 100$ nautical miles, $t_{m1} + Z/2 = 13$ hours; an interpolated (by eye) dotted line would be followed up to the $U_2 = 40$-knot line where the duration = 9.5 hours. To this value, 3 hours is added and H_{F2}, T_{F2}, t_{m2}, and F_{m2} are read off as $H_{F2} = 23$ feet, $T_{F2} = 11.2$ seconds, $t_{m2} = 12.5$ hours, and $F_{m2} = 150$ nautical miles. If the measured fetch F_2 had been less than 150 nautical miles, this length of fetch would limit the growth of waves. Though the preceding discussion would indicate that ΔF should be calculated, in practice this need not be done; the results gotten through calculation of ΔF would be obtained by reading off wave heights at the intersection of U_2 and F_2 if F_2 is limiting. Therefore if F_2 had equalled 130 nautical miles in this case, (less than 150 miles and therefore limiting) at the intersection of $U_2 = 40$ knots, and $F_2 = 130$ nautical miles, $H_{F2} = 23$ feet, $T_{F2} = 10.8$ seconds, $t_{m2} = 11$ hours, and $F_{m2} = 130$ nautical miles. Note this important distinction, t_1 , F_1 and F_2 are measured or determined from the synoptic chart, t_{m1} , t_{m2} , F_{m1} and F_{m2} are calculated by use of Figure 7. Some of the measured and calculated values will be the same, but not all of them.

If the wind velocity U_2 is less than U_1, essentially the same procedures are followed though there are some differences. From the intersection of U_1 and $t_{m1} + Z/2$ a constant energy line is followed to its intersection, if there is one, with either U_2 or F_2 whichever comes first from the left side of the figure. If U_2 comes first Z/2 is added to the duration at this point, and the U_2 ordinate is followed to either this new duration or to F_2 whichever is first from the left side of the chart. (Compare with the preceding paragraph). At this point, H_{F2} , T_{F2} , t_{m2} and F_{m2} inclusive are read off. If the constant energy line had intersected F_2 before U_2, it is only necessary to drop down along the F_2 abscissa to its intersection with U_2, and at this point read H_{F2} , T_{F2} , t_{m2} , and F_{m2}. (This procedure could be used for many cases in which U_2 is greater than U_1). For examples of this procedure see notes 8 and 11 at the end of the typical forecast given later.

The major differences in technique which must be used when U_2 is less than U_1 , occurs when the constant energy from the intersection of U_1 and $t_{m1} + Z/2$ does not intersect either U_2 or F_2. Forecasting theory used here predicts that the waves due to any wind blowing over an unlimited fetch for an unlimited duration will eventually attain a limiting height and period and growth will not increase. On Figure 7 the lower limit of this state is delineated by the line labelled "maximum conditions". To the right of this line, no wave growth takes place. If we may assume that a given wind speed can support waves no higher than occur along this line, then Figure 7 may be utilized in this case by finding the wave conditions which may be supported by the lowering wind speed U_2. A convenient method of doing this is to follow the constant energy line at the intersection of U_1 and $t_{m1} + Z/2$ to the line labelled "maximum conditions" and follow this line to its intersection with U_2. At this point H_F and T_F may be read off.

For example if $U_1 = 30$ knots, $t_{m1} = 10$ hours, $Z = 6$ hours, $U_2 =$ 22 knots and $F_2 = 800$ nautical miles, the constant energy line at the intersection of $U_1 = 30$ and $tm_1 \neq Z/2 = 13$ hours would be followed to the "maximum conditions" line, at a point where $F_2 = 800$ nautical miles, and this line followed to its intersection with $U_2 = 22$ knots. At this point, $H_{F2} = 11.5$ feet, $T_{F2} = 10.3$ seconds, $t_{m2} = 47$ hours, $F_{m2} = 750$ miles.

1.234 Decay Analysis for Deep Water Areas - Figures 8 and 9 are the curves used to find wave characteristics after the waves have left the fetch but are still travelling in deep water. With Figure 8 it is possible to compute the ratios $\frac{\text{Decayed wave height}}{\text{Fetch wave height}} = \frac{H_D}{H_F}$, and $\frac{\text{Decayed wave period}}{\text{Fetch wave period}} = \frac{T_D}{T_F}$ given H_F , T_F , F_m and D (the decay distance). An example of its use is shown on the figure. With Figure 9, it is possible to compute wave travel time between a fetch and a coast, knowing the decayed wave period T_D , and the decay distance D. This information enables the determination of wave times of arrival at the end of the decay distance.

To illustrate the foregoing methods, essentially those of Bretschneider(14) the following short forecast is made (Table 1). Refinements for the analysis of moving fetches are given by Kaplan(67). Synoptic conditions are shown on Figures 10, 11 and 12.

1.235 Significant and Higher Waves - It was noted in section 1.22 that the wave height determined from forecasting or hindcasting procedures is the so-called significant wave height, the average of the one-third higher heights of a given wave group.

It has been found(106) that a linear relationship exists between the significant wave height and the mean wave height of a group. This relationship is

$$H_m = 0.624\ H_s - 0.015 \qquad (6)$$

where H_s = the significant wave height of a wave group (in feet), and
H_m = the mean wave height of the group (in feet)

It was further found that this mean wave height could be related statistically to any other height which may occur more or less frequently. This relationship is represented graphically on Figure 13. The graph can be used to find a wave height more nearly the maximum of a group of waves, say one which will not be exceeded by 95 percent of the waves.

In using Figure 13, the mean wave height of a group of waves should first be found by use of equation 6. The diagonal line corresponding to H_m is followed to its intersection with the vertical line labelled (say) 95 percent. The height which will not be exceeded by 95 percent of the waves in a group whose mean height is H_m is read on the vertical scale. In the example on the graph this height is 5 feet where H_m is 2.5 feet.

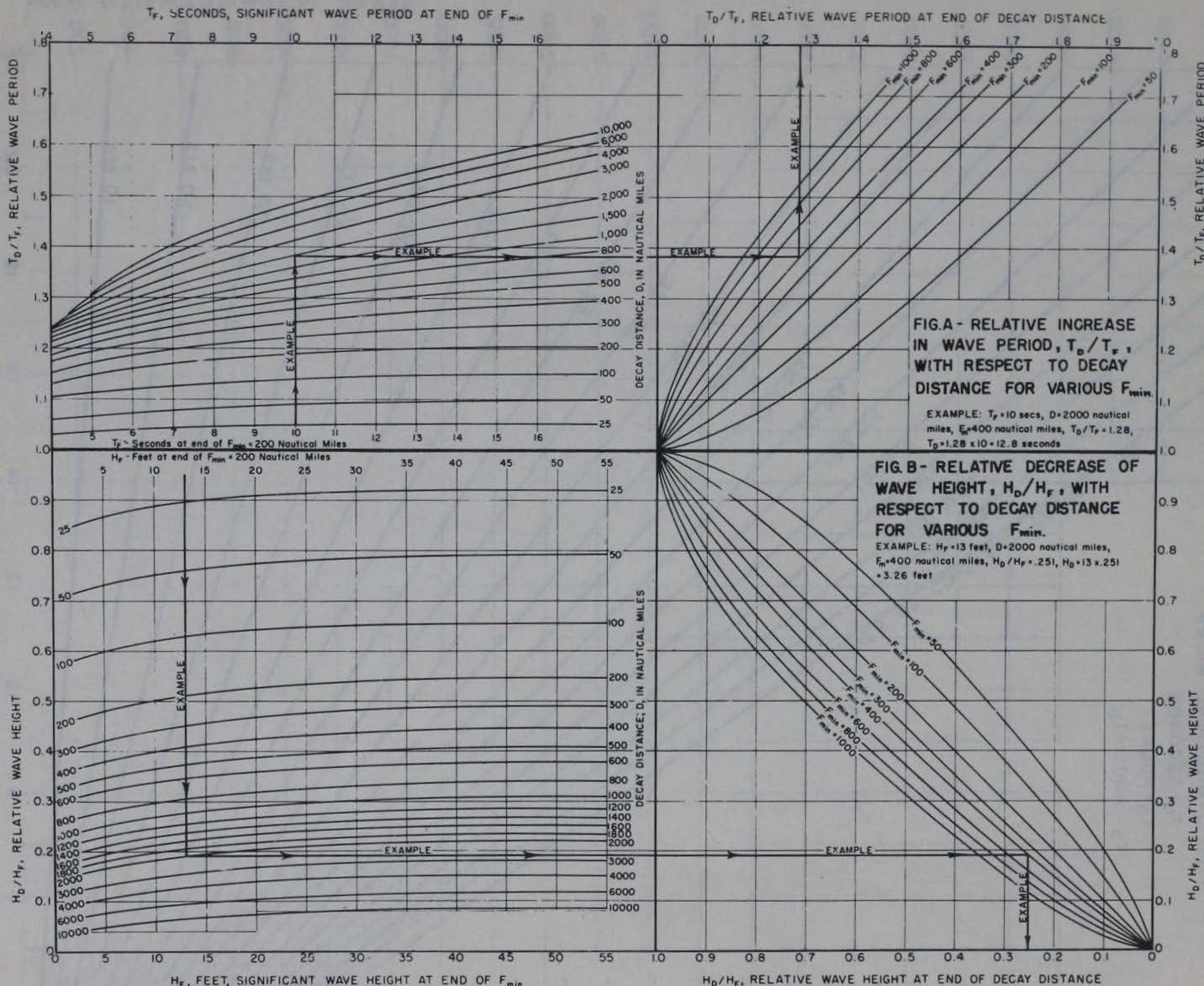


FIGURE 8

DECAY CURVES

(Bretschneider, 1952)

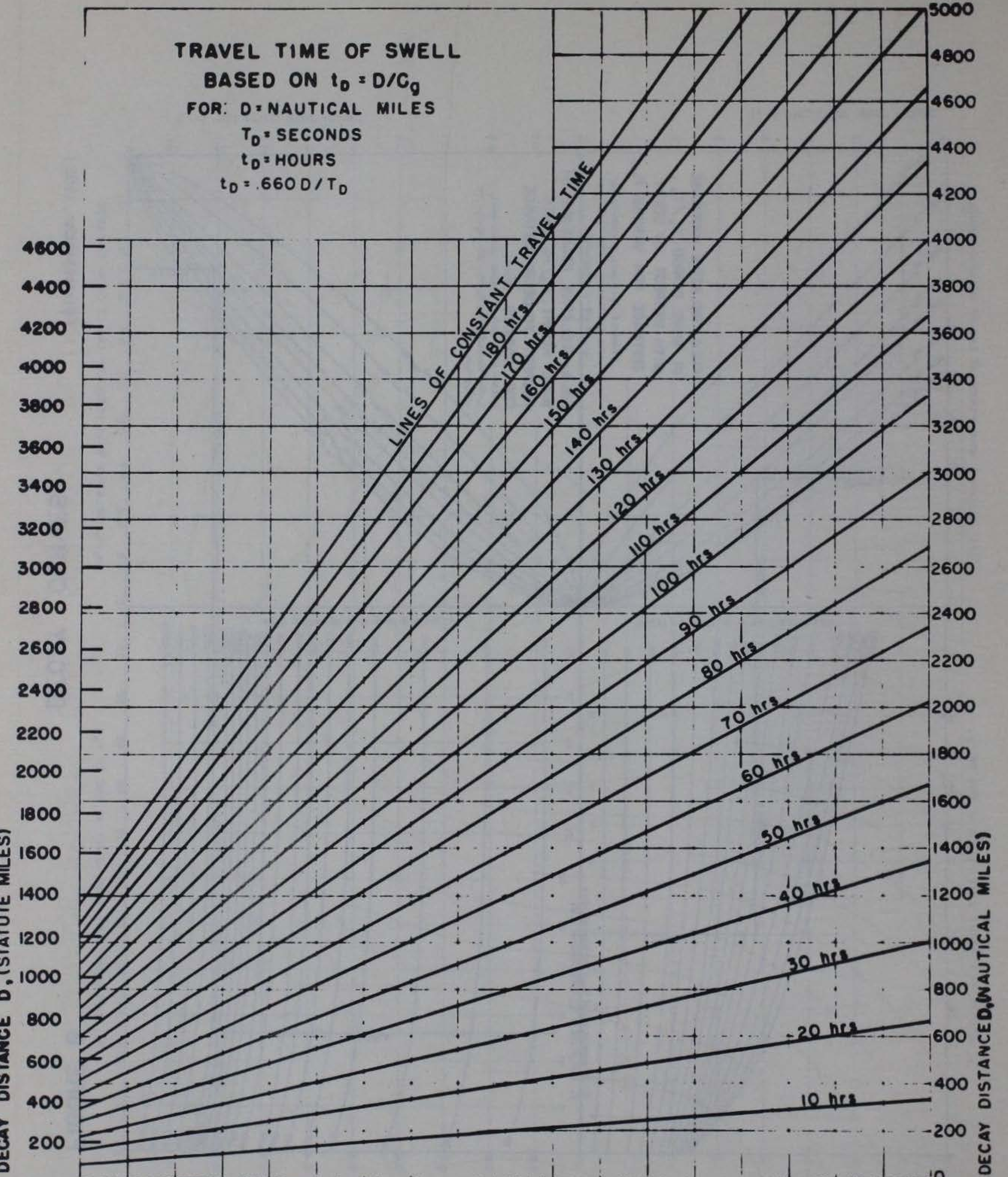
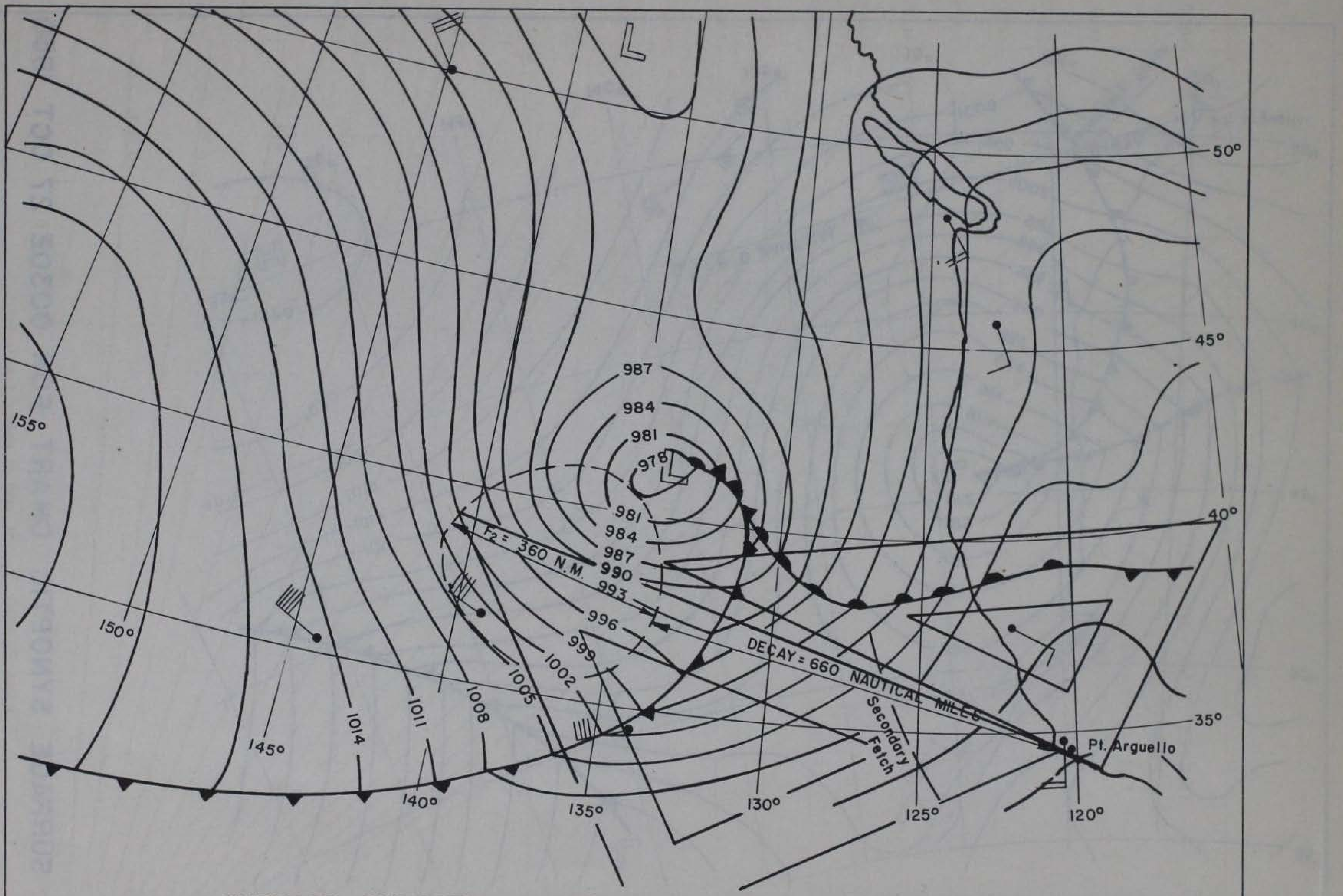
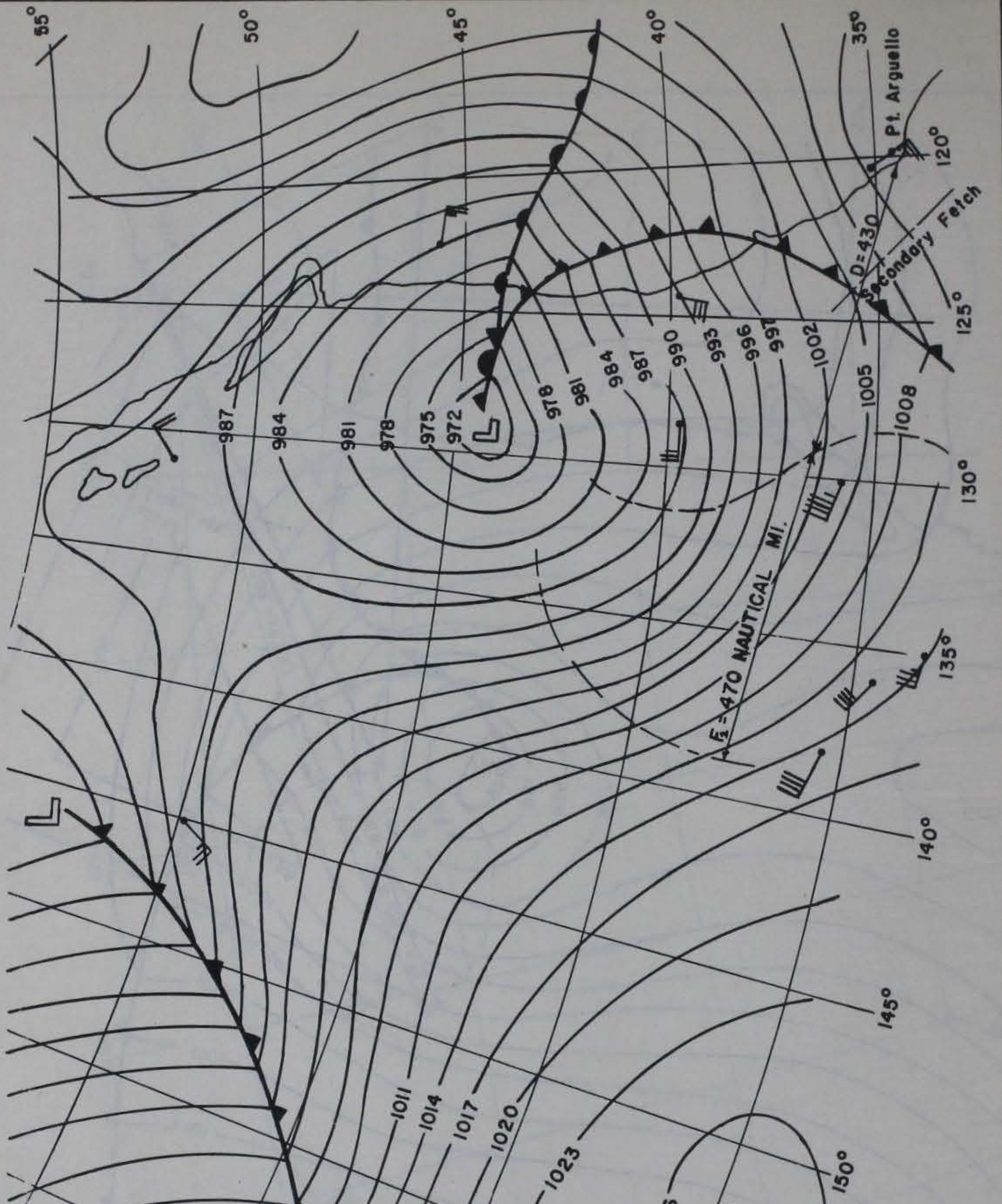


FIGURE 9

(Bretschneider, 1952)

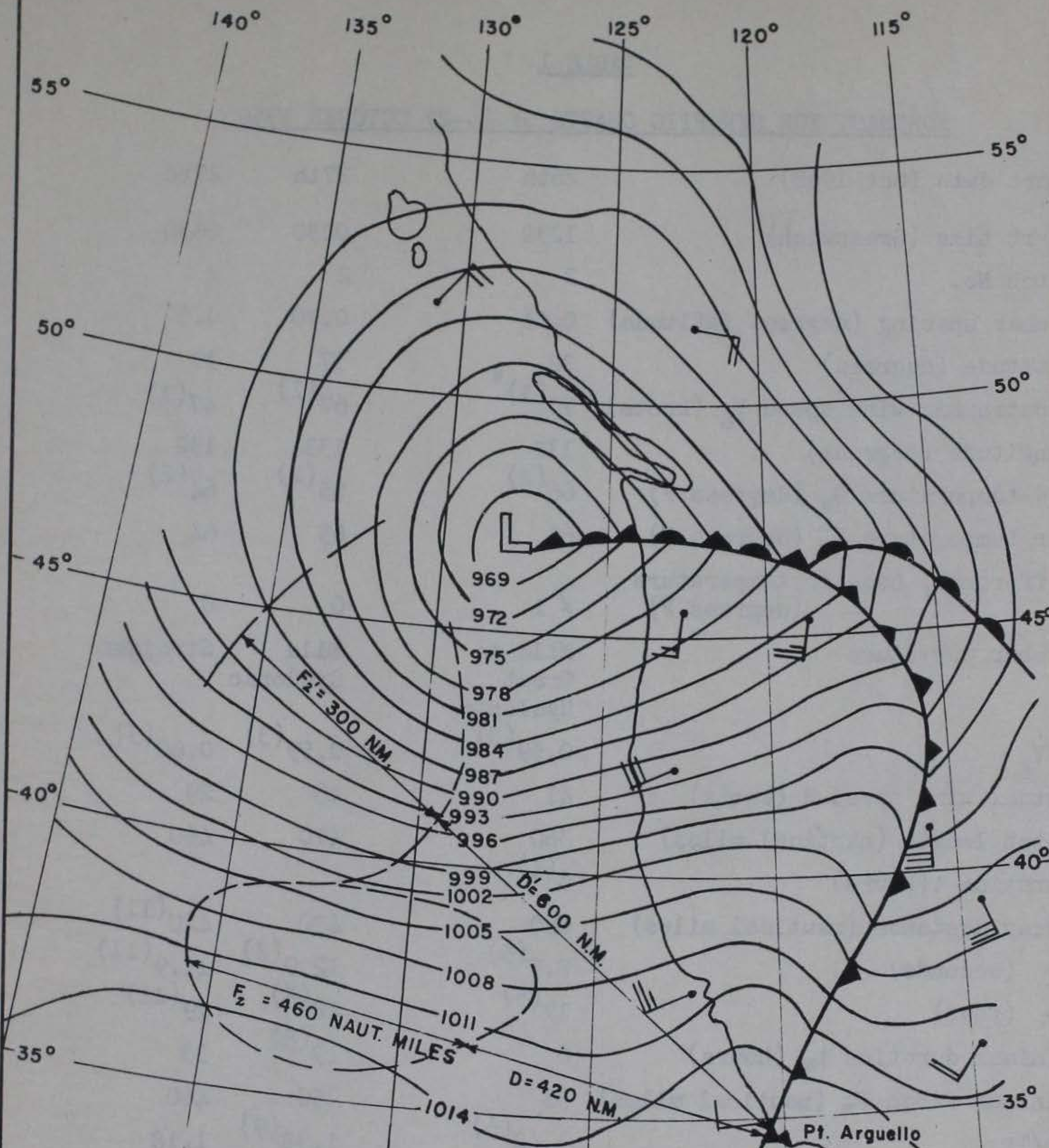


SURFACE SYNOPTIC CHART FOR 1230Z 26 OCT. 1950
FIGURE 10



SURFACE SYNOPTIC CHART FOR 0030Z 27 OCT 1950

FIGURE 11



SURFACE SYNOPTIC CHART FOR 0630Z 27 OCT 1950

FIGURE 12

TABLE 1

FORECAST FOR SYNOPTIC CHARTS OF 26-27 OCTOBER 1950

Chart date (Oct 1950)	26th	27th	27th
Chart time (Greenwich)	1230	0030	0630
Fetch No.	2	2	2
Isobar spacing (degrees latitude)	0.65	0.70	1.0
Latitude (degrees)	38	37	37
Geostrophic wind speed V_g (knots)	70[1]*	67[1]	47[1]
Longitude (degrees)	137	133	132
Sea temperature θ_s (degrees F)	66[2]	65[2]	64[2]
Air temperature θ_a (degrees F)	65	65	64
Difference, sea-air temperature (degrees F)	+ 1	0	0
Isobar curvature	Mild to Great Cyclonic	Mild Cyclonic	Straight
U/V_g	0.59[3]	0.59[3]	0.62[3]
Actual wind speed U (knots)	41	40	29
Fetch length (nautical miles)	360	470	460
Duration t(hours)	6[4]		
Decay distance (nautical miles)	660	430	420[11]
T_F (seconds)	8.9[5]	12.9[8]	11.9[11]
H_F (feet)	17[5]	28[8]	19[11]
Minimum duration t_m (hours)	6	19[8]	33
Minimum fetch F_m (nautical miles)	58	250	460
T_D/T_F	1.48[6]	1.38[9]	1.18
H_D/H_F	0.22[6]	0.47[9]	0.52[9]
T_D (T_o) (seconds)	13	16.5	15
H_D (H_o) (feet)	4	13	11
Travel time (Hours)	33[7]	18[10]	20[10]
Arrival time - Greenwich (Z) time	27 Oct 2130	27 Oct 1830	28 Oct 0230
- Pacific Std. time	27 Oct 1330	27 Oct 1030	27 Oct 1830

* For foot notes see following page.

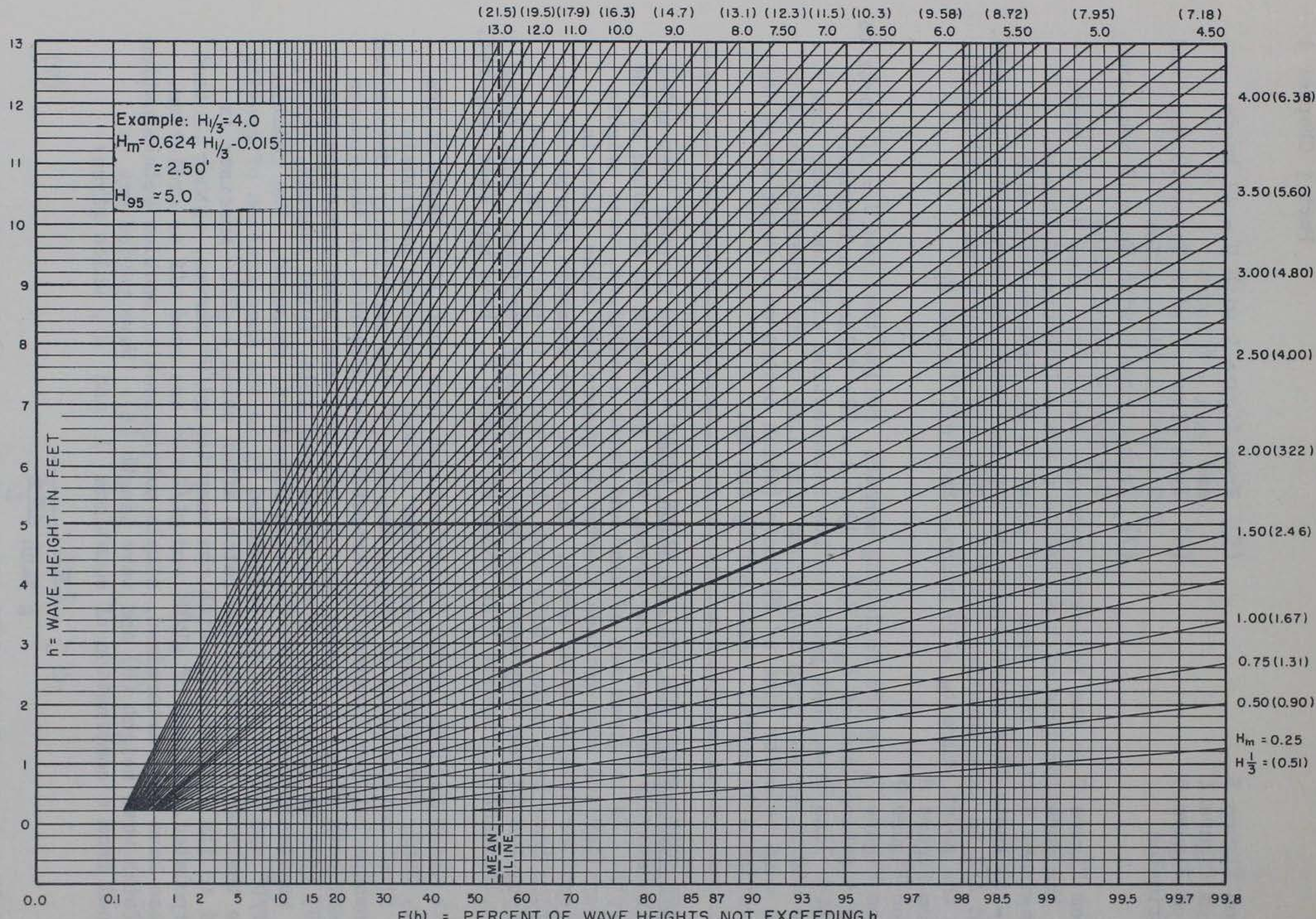
(1) From Figure 3
(2) From Figure 4
(3) From Figure 5
(4) Estimated
(5) From Figure 7 with $U = 41$ knots and $t_{min} = 6$ hours
(6) From Figure 8 with $T_F = 8.9$ sec. and $D = 660$ nautical miles
(7) From Figure 9 with $T_D = 13$ sec. and $D = 660$ nautical miles
(8) From Figure 7 in the following manner. Follow the line $U = 41$ knots to the duration line of $t + Z/2 = 6 + 6 = 12$ hours; follow an imaginary dotted line from that point to its intersection with the $U = 40$ knots line which occurs at a point where the duration = 13 hours. Add $Z/2$ to this duration $(13 + 6 = 19)$, and follow along $U = 40$ knots. to the point where $t_m = 19$ hours. Read off T_F and H_F.
(9) From Figure 8
(10) From Figure 9
(11) From Figure 7, in the following manner. Follow the line $U = 40$ knots to the duration line of $t_m + Z/2 = 19 + 3 = 22$; follow an imaginary dotted line from that point to its intersection with the fetch line $F = 460$ miles and drop down along this line to its intersection with the wind velocity line of 29 knots (which occurs at a point where the duration = 33 hours). Read off T_F and H_F.

1.24 WAVE FORECASTING FOR SHALLOW WATER AREAS - It has already been noted that in relatively shallow water areas, wave generation is affected by the water depth. For a given set of wind and fetch conditions, wave heights will be smaller and wave periods shorter if generation takes place in transitional or shallow water areas. To date, two separate approaches to the problem have been made. Both have shown worth in predicting wave generation, but as yet they have not been adequately verified. This being the case, forecasting curves like those of Figure 7, have not been prepared.

Both these methods permit determinations very much like those of the Sverdrup, Munk, Bretschneider deep water methods presented previously. For the first method (that of Thysse and Schijf) the empirically determined relationships between forecasting parameters are presented as two sheaves of curves, both shown on Figure 14. To utilize them, wind direction and speed (U) must be determined by any means available, (wind records, etc.), and the available fetch (F) in the wind direction, measured. The relationships gF/U^2 and gd/U^2 are then calculated (g is the acceleration of gravity and d is the mean depth over the fetch) after which a gF/U^2 **abscissa is** followed to its intersection with the computed gd/U^2 curves, either plotted or interpolated. Values of gH/U^2 and $gL/2\pi U^2$ are read off as ordinates. Once these have been determined, simple multiplication enables determination of the fetch wave height (H) and wave length (L). Wave period may be determined from a combination of equations 1 and 2 , which gives:

$$T^2 = \frac{2\pi L}{g \operatorname{Tanh} \frac{2\pi d}{L}} \qquad (7)$$

For example, given a fetch of 10 miles, a wind speed of 45 miles per hour and a mean depth in the fetch of 20 feet,



PREDICTION LINES FOR WAVE HEIGHT DISTRIBUTION FUNCTIONS FOR VARIOUS MEAN WAVE HEIGHTS

(SIGNIFICANT WAVE HEIGHT $H_{1/3}$ SHOWN IN PARENTHESES)

(Putz, 1950)

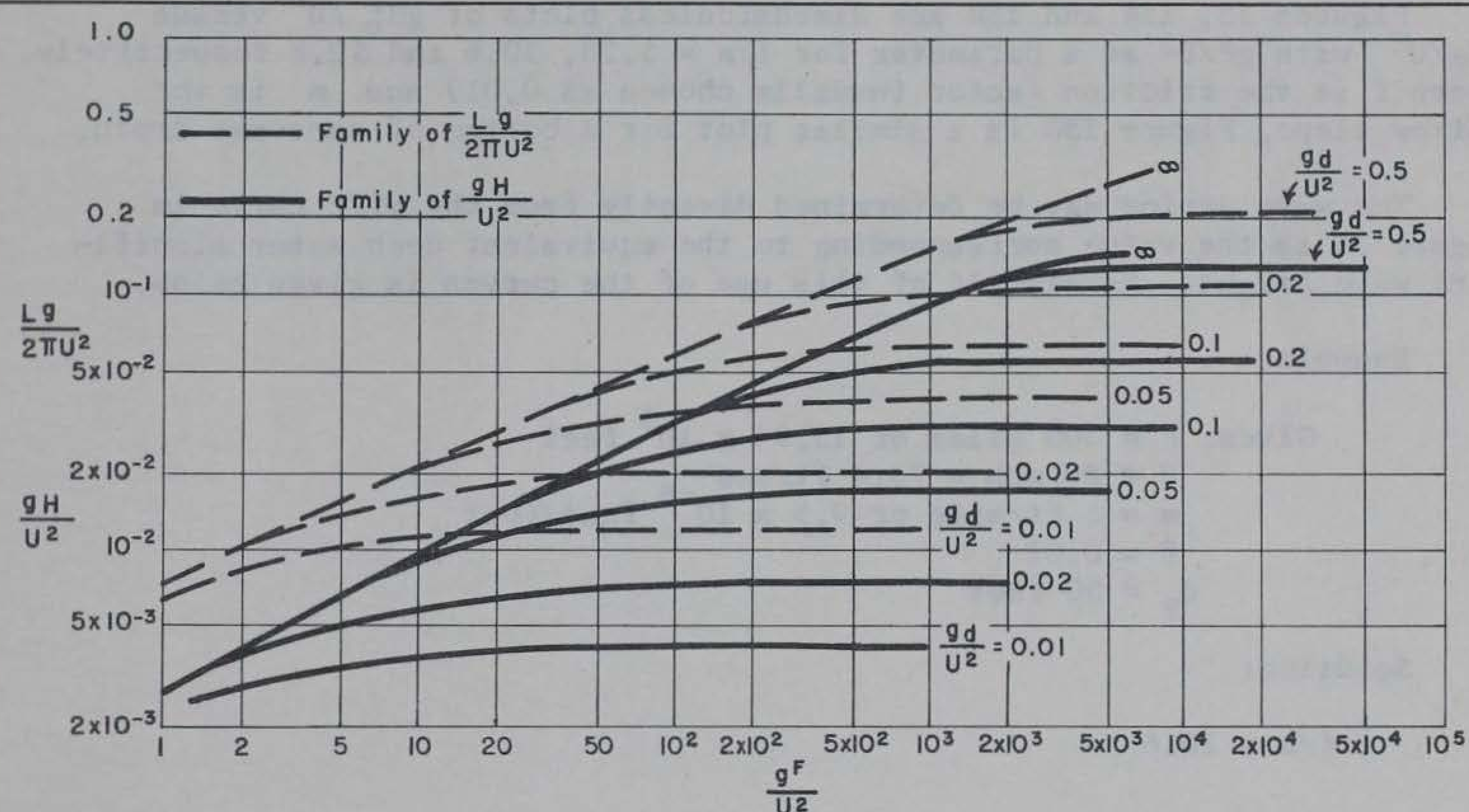
FIGURE 13

$$\frac{gd}{U^2} = 0.148, \quad \frac{gF}{U^2} = 3.90 \times 10^2$$

Then from the curves of Figure 14, (the solid curve is used to read the values of gH/U^2, and the dashed curve is used to read the values of $Lg/2\pi U^2$)

$$\frac{gH}{U^2} = 3.5 \times 10^{-2} \text{ and } \frac{Lg}{2\pi U^2} = 7 \times 10^{-2}$$

from which H = 4.7 feet and L = 59.6 feet. Equation 7 gives the period as 3.5 seconds.



GROWTH OF WAVES IN A LIMITED DEPTH

FIGURE 14

(After Thysse)

The second method (tha+ of Bretschneider)[145] takes bottom friction and percolation in the permeable sea bottom into account. To date as far as is known there has beer no single theoretical development for determining the actual growth of waves generated by winds blowing over relatively shallow water. The numerical method presented is essentially that of successive approximations wherein wave energy is added due to wind stress and subtracted due to bottom friction and percolation. This is done by making use of the deep water forecasting relationships originally developed by Sverdrup and Munk[124] and revised by Bretschneider[141] for determining the energy added due to wind stress. The amount of wave energy lost due to bottom friction and percolation is determined from the relationships developed by Bretschneider and Reid[142]. The resultant wave heights and periods are obtained by combining the above relationships by numerical methods. The basic assumptions applicable to the development of the deep water wave generation relationships[141] as well as the development of the relationships for bottom friction loss[143] and percolation loss[144] apply to the development.

Although there are insufficient wave data to date to verify completely all relationships derived, it is believed that the method will prove satisfactory once sufficient wave data become available to enable accurate evaluation of the constants.

Figures 15, 15A and 15B are dimensionless plots of gH_o'/U^2 versus gd_F/U^2 with gF/U^2 as a parameter for $f/m = 5.28$, 10.6 and 52.8 respectively, where f is the friction factor (usually chosen as 0.01) and m is the bottom slope. Figure 15C is a similar plot for a bottom of constant depth.

B4

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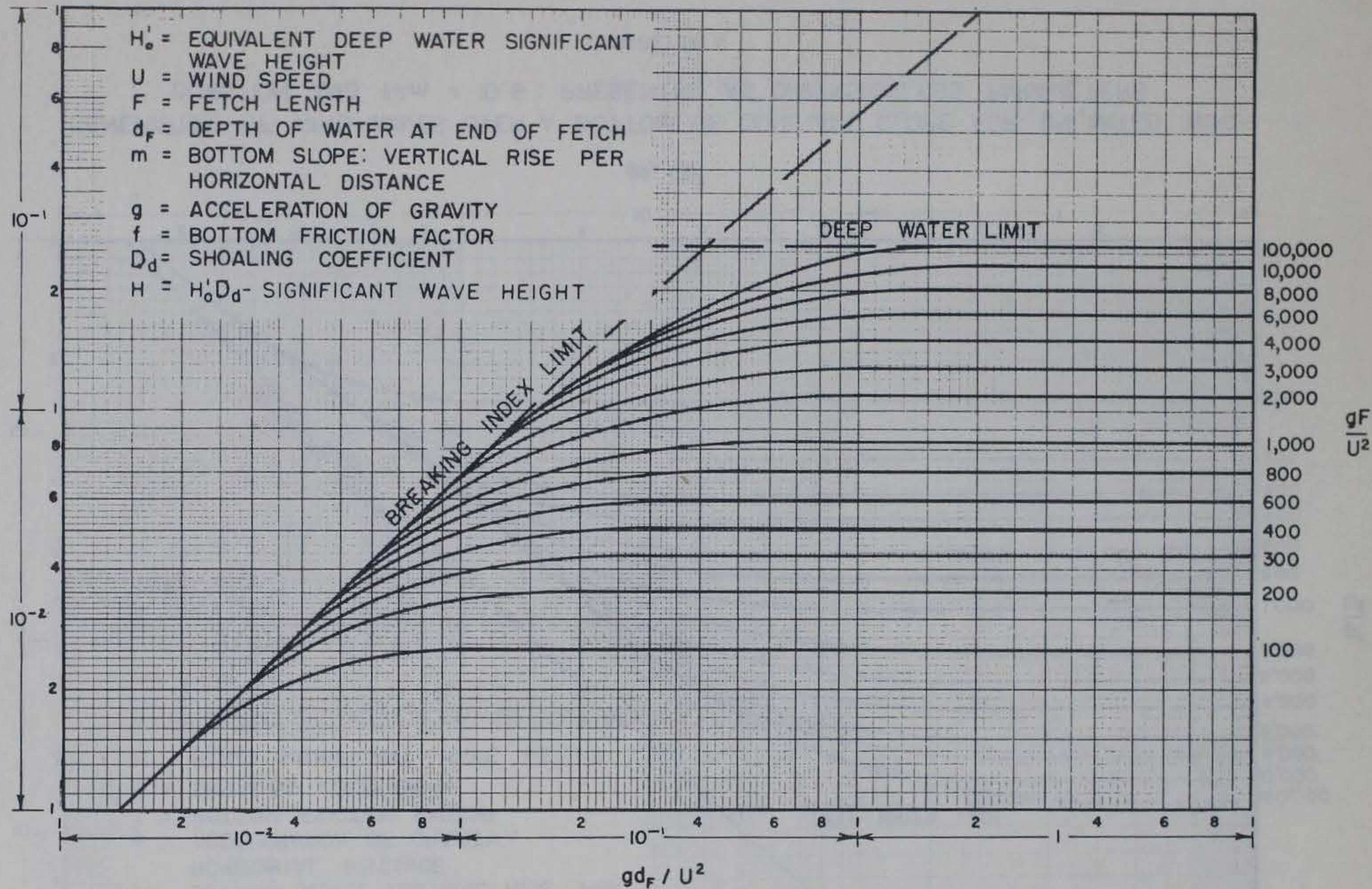
On Page 28 - Replace 4th paragraph with ---

The wave period may be obtained directly from Figure 7 or 7A using the value of the corresponding equivalent deep water significant wave height (H_o'). An example of this use of the curves is given below.

$U^2 \quad (73.4)^2$

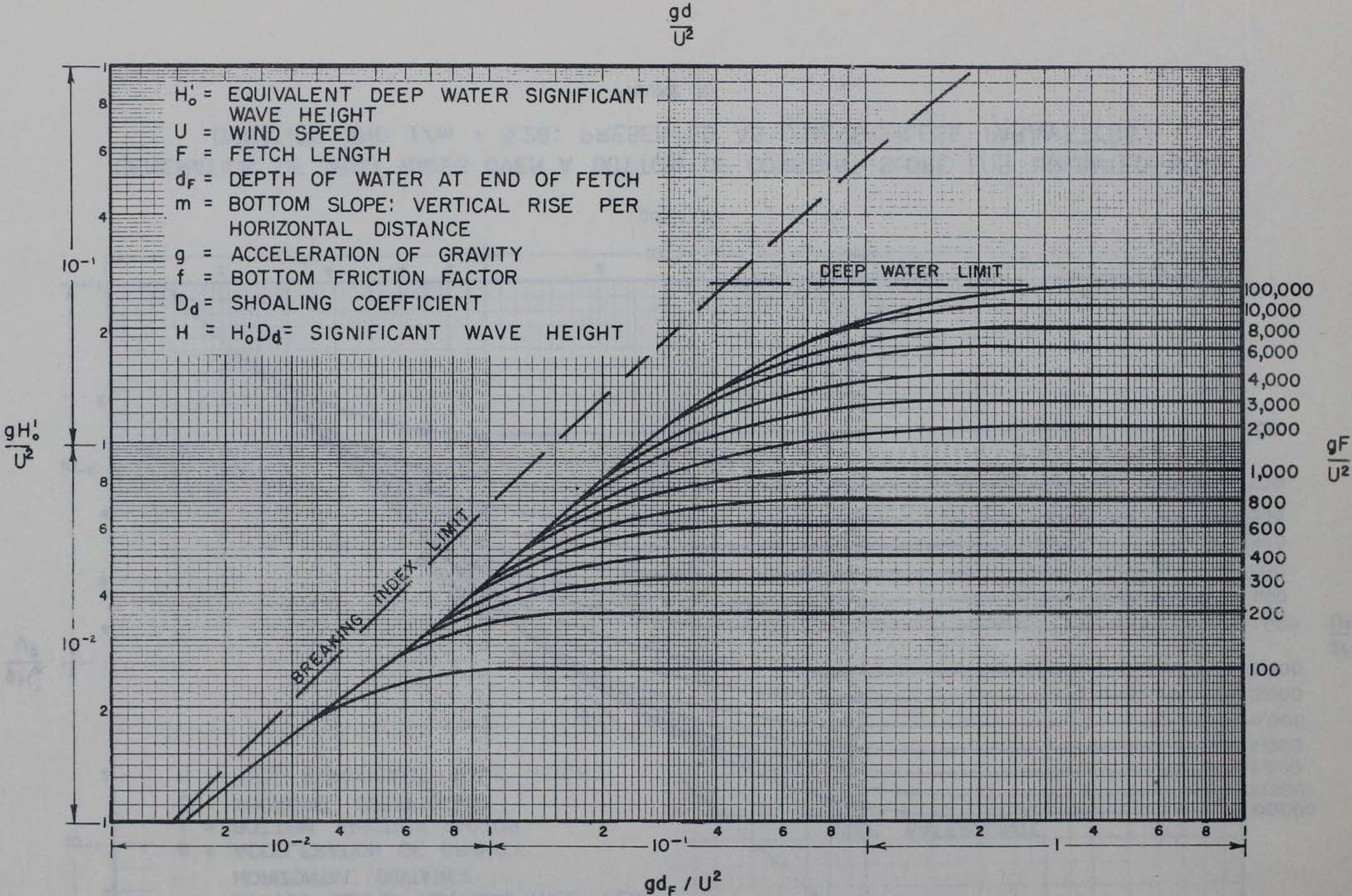
$$\frac{gH_o'}{U^2} = 0.11 \quad \text{Figure 15A}$$

$H_o' = 18.4$ feet



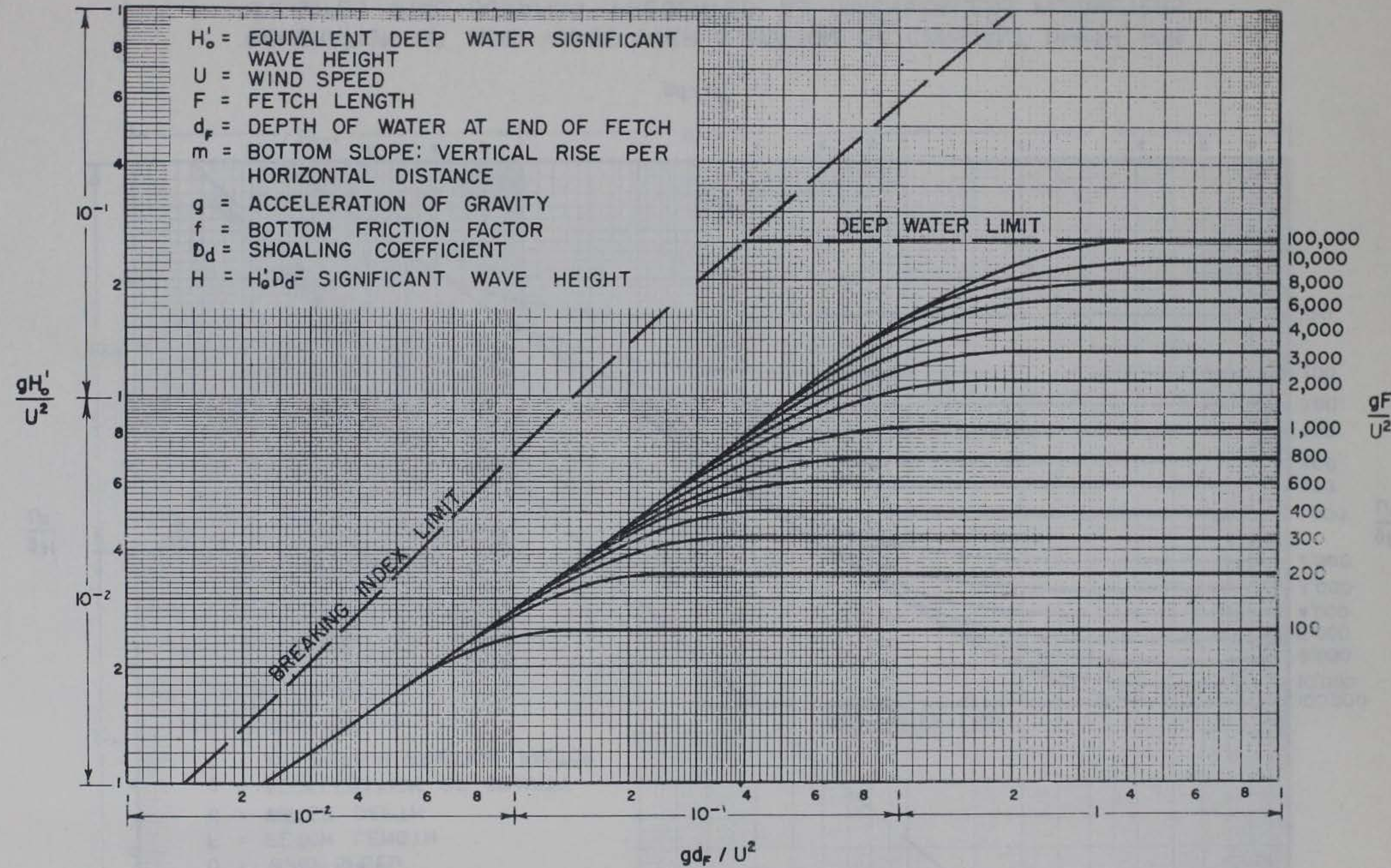
GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT SLOPE FOR UNLIMITED WIND DURATION AND f/m = 5.28: PRESENTED AS DIMENSIONLESS PARAMETERS

FIGURE 15



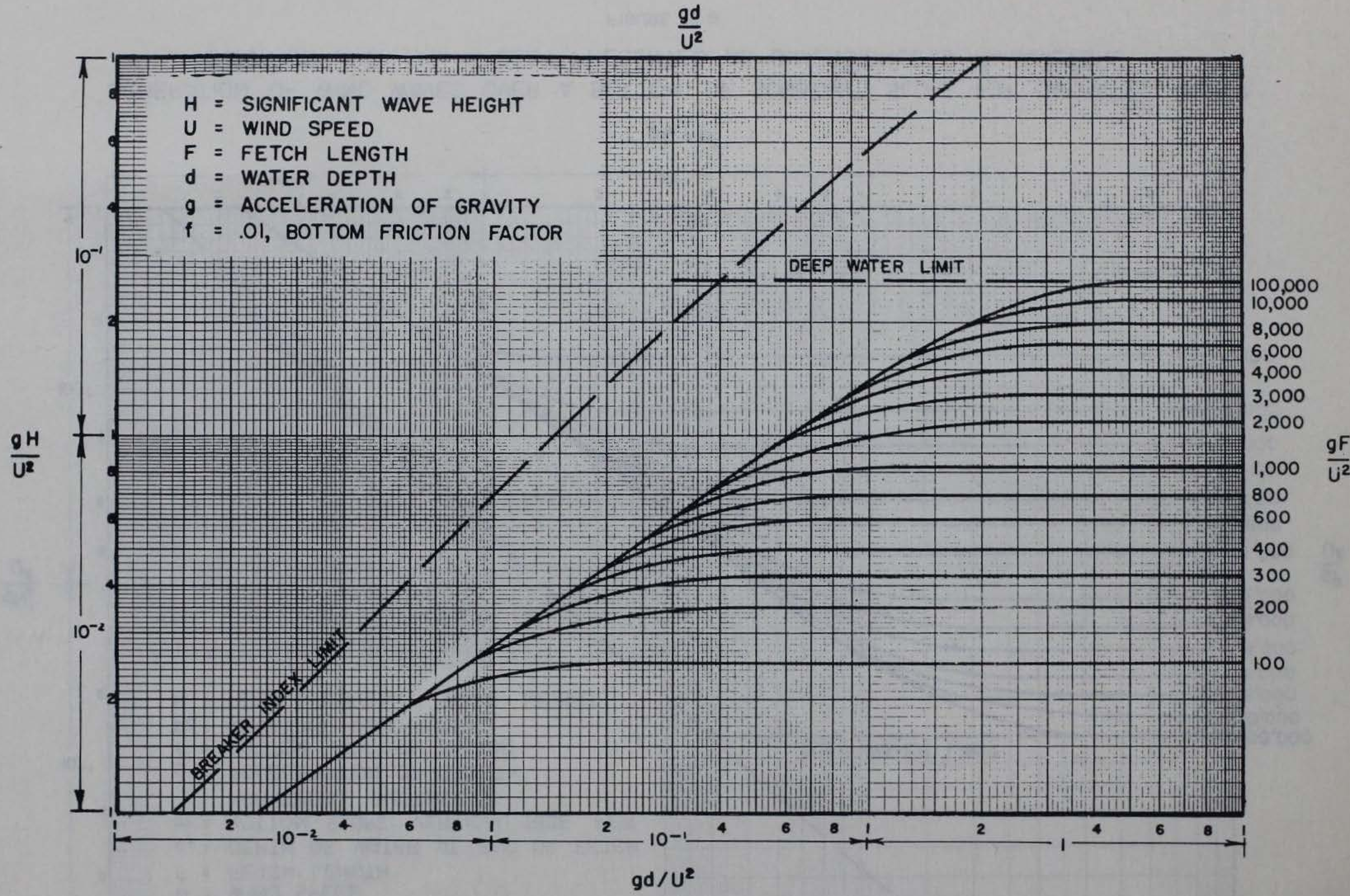
GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT SLOPE FOR UNLIMITED WIND DURATION AND f/m = 10.6 : PRESENTED AS DIMENSIONLESS PARAMETERS

FIGURE 15 A



GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT SLOPE FOR UNLIMITED WIND DURATION AND $f/m = 52.8$: PRESENTED AS DIMENSIONLESS PARAMETERS

FIGURE 15 B



GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT DEPTH FOR UNLIMITED WIND DURATION PRESENTED AS DIMENSIONLESS PARAMETERS

FIGURE 15 C

at $\frac{gH_o'}{U^2} = \frac{gH_o}{U^2} = 0.11$, $\frac{gF}{U^2} = 2000$ Figure 7A

at $\frac{gF}{U^2} = 2000$, $\frac{gT}{U} = 4.0 = \frac{gT'}{U}$ Figure 7A

$T' = 9.1$ seconds equivalent deep water significant wave period

$d/L_o = 50 \div 5.12\ (9.1)^2 = 0.118$

$D_d = H/H_o' = 0.921$ Table D-1

$H = 18.4 \times 0.921 = 16.9$ feet, significant wave height at

On Page 28e - Replace first 8 lines at top of page with ---

With the value of H_o' = 18.4 feet, a wind speed U = 50 mph and fetch length F = 300 miles, use Figure 7 to determine (T') the equivalent deep water significant wave period. In this particular case, the limiting condition will occur, with U = 50 mph and H_o' = 18.4 feet (rather than with F = 300 miles and H_o' = 18.4 feet). Thus

$$d/Lo = 50/5.12\ (10.4)^2 = \frac{50}{554} = 0.09025$$

From Table D-1

$$H/H_o' = 0.9419$$

$H = 18.4 \times 0.9419 = 17.3$ feet, significant height at $d_F = 50$ feet for F = 300 miles, U = 50 mph and f/m = 10.6.

between 20 and 36 knots. Most of the spectral energy is concentrated at the maximum slope of the CCS curves.

To facilitate the practical use of these CCS curves, the ordinate, E_f, is scaled in units having the dimensions of $(\text{feet})^2$. The E_f values are related to the wave energy and determine the height characteristics of the composite wave motion.

a. The fully arisen sea - In a fully arisen sea, all wave periods between zero and infinity are theoretically possible. But practically, the

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wave components with very high and very low periods contribute only such a vanishingly small amount of energy to the total that they can be neglected.

In order to forecast or describe the properties of a fully arisen sea, the horizontal or nearly horizontal parts of the CCS curves can be considered as cut off. Physically, this means that certain high and low period portions of the wave spectrum can be filtered out without any noticeable change of the wave pattern. As a rule, by empirical evidence, about 5% of the total E_f value for the fully developed state of wind-generated sea can be cut off at the upper end of the curves; about 3% can be cut off at the lower end. For example, the total E_f value at the upper end of the curve for a wind speed of 30 knots is 58.5 (feet)^2.

$$\therefore 95\% \text{ of } E_f = 55.6$$

The frequency corresponding to this value of E_f (in this case f = 0.06 or a wave period of 16.7 seconds). is considered as the upper limit of the significant range of periods. Similarly the lower limit of the significant range of periods. is determined as the frequency value corresponding to 3% of E_f. In the case above, 0.03 E_f = 1.76 ft^2 and the corresponding frequency is 0.213, or a wave period of 4.7 seconds. Therefore the significant range of periods for a fully arisen sea generated by a 30-knot wind speed is 4.7 to 16.7 seconds.

Table 1-A shows the theoretical ranges of significant periods for a fully arisen sea for different wind velocities, V, as determined from the wave spectra and the "cut-off" procedure previously given.

For a complete description of the fully arisen sea at different wind speeds, the average "period", T_{av}, the period of the band in the wave spectrum in which most of the spectral energy is concentrated, T_{max}, and the wave height data are given in Table 1-B.

b. The non-fully developed state - In the fully arisen sea the dominating wave pattern covers a relatively wide portion of the significant range of periods in the spectrum. The width and range of this portion of the spectrum depend upon the wind velocity. For non-fully arisen sea it also depends on the fetch length and the duration of wind action.

In the growing stage of the wave pattern it is assumed that the waves pass through three principal stages, before becoming fully arisen sea.

The short period, steep wave components develop first. These remain in a quasi-steady state, always being regenerated after breaking by energy supply from the wind. Once these waves have developed, longer wave trains may arise which cover a spectral band around an average "wave" which travels with the wind speed. If the wind continues to blow, and the fetch and duration are long enough, still larger waves can develop.

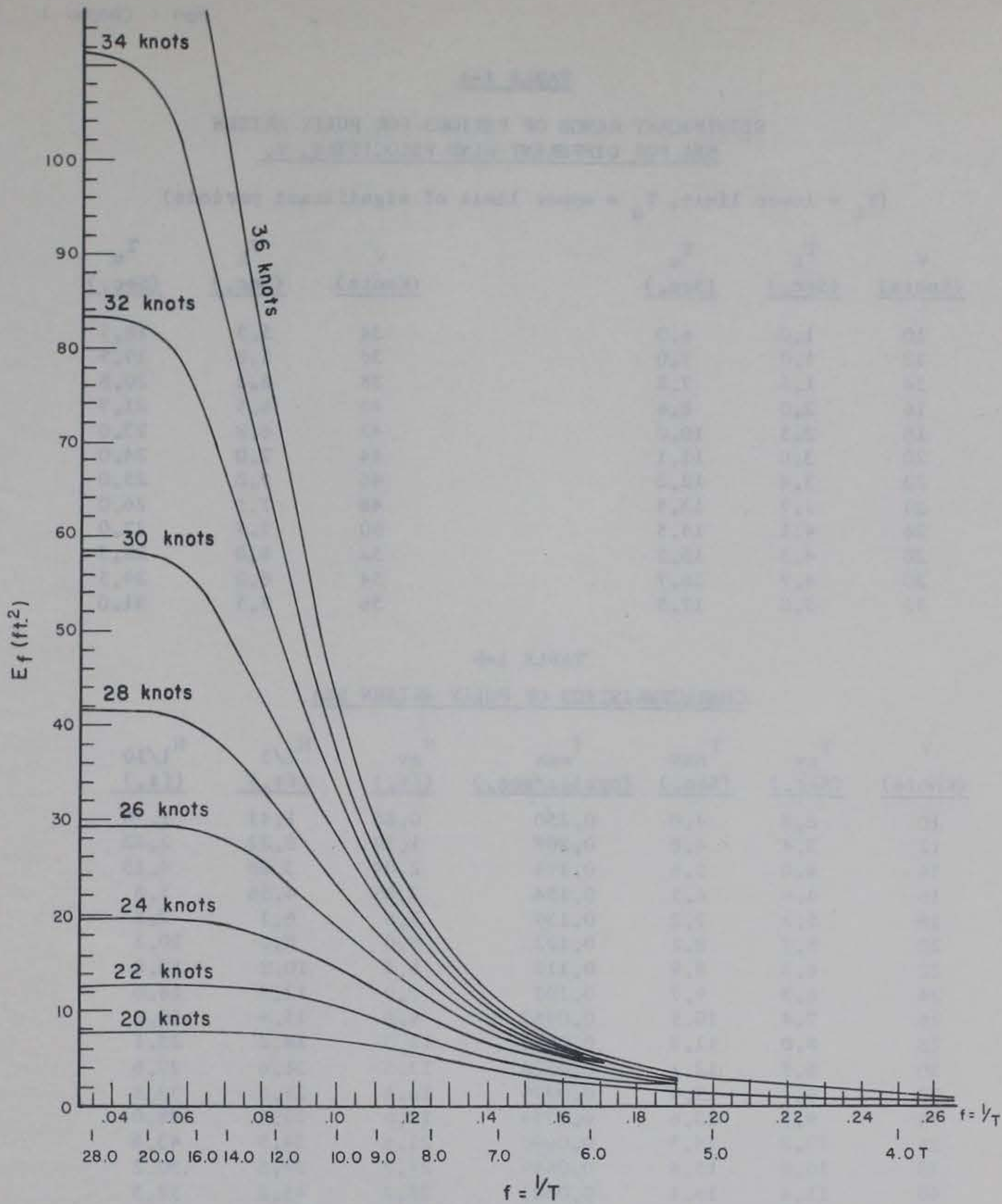


FIGURE 15-D. CO-CUMULATIVE POWER SPECTRA FOR OCEAN WAVES AT WIND VELOCITIES BETWEEN 20 KNOTS AND 36 KNOTS. THE ORDINATE VALUES E_f, ARE PROPORTIONAL TO THE TOTAL WAVE PATTERN AND DETERMINE THE HEIGHT CHARACTERISTICS OF THE SEA.

TABLE 1-A

SIGNIFICANT RANGE OF PERIODS FOR FULLY ARISEN SEA FOR DIFFERENT WIND VELOCITIES, V.

(T_L = lower limit, T_u = upper limit of significant periods)

V (Knots)	T_L (Sec.)	T_u (Sec.)	V (Knots)	T_L (Sec.)	T_u (Sec.)
10	1.0	6.0	34	5.5	18.5
12	1.0	7.0	36	5.8	19.5
14	1.5	7.8	38	6.2	20.8
16	2.0	8.8	40	6.5	21.7
18	2.5	10.0	42	6.8	23.0
20	3.0	11.1	44	7.0	24.0
22	3.4	12.2	46	7.2	25.0
24	3.7	13.5	48	7.5	26.0
26	4.1	14.5	50	7.7	27.0
28	4.5	15.5	52	8.0	28.5
30	4.7	16.7	54	8.2	29.5
32	5.0	17.5	56	8.5	31.0

TABLE 1-B

CHARACTERISTICS OF FULLY ARISEN SEA

V (Knots)	T_{av} (Sec.)	T_{max} (Sec.)	f_{max} (cycles/sec.)	H_{av} (ft.)	$H_{1/3}$ (ft.)	$H_{1/10}$ (ft.)
10	2.8	4.0	0.250	0.88	1.41	1.79
12	3.4	4.8	0.208	1.39	2.22	2.82
14	4.0	5.6	0.179	2.04	3.26	4.15
16	4.6	6.5	0.154	2.85	4.56	5.8
18	5.1	7.2	0.139	3.8	6.1	7.8
20	5.7	8.1	0.123	5.0	8.0	10.2
22	6.3	8.9	0.112	6.4	10.2	12.8
24	6.8	9.7	0.103	7.9	12.5	16.0
26	7.4	10.5	0.0952	9.6	15.4	19.6
28	8.0	11.3	0.0885	11.3	18.2	23.1
30	8.5	12.1	0.0826	13.5	21.6	27.6
32	9.1	12.9	0.0775	16.1	25.8	32.8
34	9.7	13.6	0.0735	18.6	29.8	38.0
36	10.2	14.5	0.0690	21.6	34.5	43.8
38	10.8	15.4	0.0649	24.7	39.5	50.2
40	11.4	16.1	0.0621	28.2	45.2	57.5
42	12.0	17.0	0.0588	31.4	50.1	63.9
44	12.5	17.7	0.0565	36.0	57.7	73.4
46	13.1	18.6	0.0538	39.7	63.5	80.6
48	13.7	19.4	0.0515	44.4	71.0	90.4
50	14.2	20.2	0.0495	48.9	78.0	99.4
52	14.8	21.0	0.0476	54	87	110
54	15.4	21.8	0.0459	59	95	121
56	15.9	22.6	0.0442	64	103	130

$T_{av.}$: average "period" (seconds)

T_{max} : period of most energetic wave in the spectrum

H_{av}, $H_{1/3}$, $H_{1/10}$: Heights of average wave, average height of 1/3 highest waves and average of 1/10 highest waves. (see Table 1-A)

By the assumption of three characteristic "waves" which are equivalent to these three broad spectral bands in the actual wave spectrum, an approximation is given for calculating and describing the state of the sea during wave generation and growth. Relationships between wind and wave characteristics have been established on the basis of energy considerations, which allow calculation of the characteristics of the complex sea for different fetch lengths and wind durations, such as the range of time intervals between succeeding crests at a fixed position, the average "period", the range of wave heights, and the average wave height. Observations have been compared with theoretical results and they support the ideas involved in the theoretical approximation.

The minimum fetch length and minimum wind duration required to maintain a fully arisen sea are shown in Table 1-C for various wind velocities. In the practical case, if either or both of these values are not exceeded by the actual fetch and duration a fully arisen sea will not have developed, and other methods (to follow) must be used to describe the state of the sea.

TABLE 1-C

MINIMUM FETCH AND MINIMUM DURATION OF WIND ACTION NEEDED TO GENERATE A PRACTICALLY FULLY ARISEN SEA FOR DIFFERENT WIND VELOCITIES

V (Knots)	F_m (n. miles)	t_m (hours)	V (Knots)	F_m (n. miles)	t_m (hours)
10	10	2.4	34	420	30
12	18	3.8	36	500	34
14	28	5.2	38	600	38
16	40	6.6	40	710	42
18	55	8.3	42	830	47
20	75	10	44	960	52
22	100	12	46	1100	57
24	130	14	48	1250	63
26	180	17	50	1420	69
28	230	20	52	1610	75
30	280	23	54	1800	81
32	340	27	56	2100	88

Case A: Wave Characteristics Limited by the Wind Duration. A constant wind with mean velocity V (knots) blows over an unlimited fetch F (nautical miles). The energy added to the composite wave motion is the same everywhere so that the waves grow at all localities at the same rate with time, t, (duration, hours). The stage of development of the sea depends only upon the duration of wind action, and is given by the "duration-graphs" in Figures 15E, 15F, and 15G.

Case B: Wave Characteristics Limited by the Length of the Fetch. The wind duration, t, is long enough to produce a steady state but the fetch, F, is limited; the state of the sea then depends only upon the length of the water surface over which the wind has blown. This case is given by the fetch-graphs in Figures 15H, 15I, and 15J.

The intersection points of the CCS curves with the duration or fetch lines, (Figures 15E-J respectively) show the limit of the development of the composite wave motion at the given duration or fetch. Physically, it means that the state of development is limited by a certain maximum amount of total energy which the wave motion can absorb from the wind within the given conditions. The E_f value of the ordinate of each intersection point is a practical measure of the total energy accumulated in the wave motion of the non-fully arisen state, limited either by the fetch or duration.

Under actual conditions, both fetch and duration may be limited, and the E_f value for any given situation, in most cases, will be different for the fetch and duration. In such cases the smaller of the E_f values is taken.

As in the case of a fully arisen sea the wave height characteristics can be computed from the E_f value.

The upper limit of the significant range of periods for a sea not fully arisen is approximately determined from the wave frequency value (f_i) at the intersection of a CCS curve for a given wind speed with the given fetch or duration line, respectively. The wave spectrum may be considered as cut off abruptly at this given minimum frequency, f_i , or maximum period, T_i. The wave components with periods a little longer than T_i, and which are just in the beginning stage of development, have a small amplitude and contribute so little to the total wave energy, that they may be neglected.

When the value f_i, is larger than or equal to the frequency, f_{max}, of the optimum band as given in Table 1-B where $T_{max} = 1/f_{max}$, a small correction may be applied in order to make allowance for the presence of some longer waves. This correction D_f, has been determined empirically from wave observations on limited fetches which indicate that

$$D_f = 0.15\ f_i$$

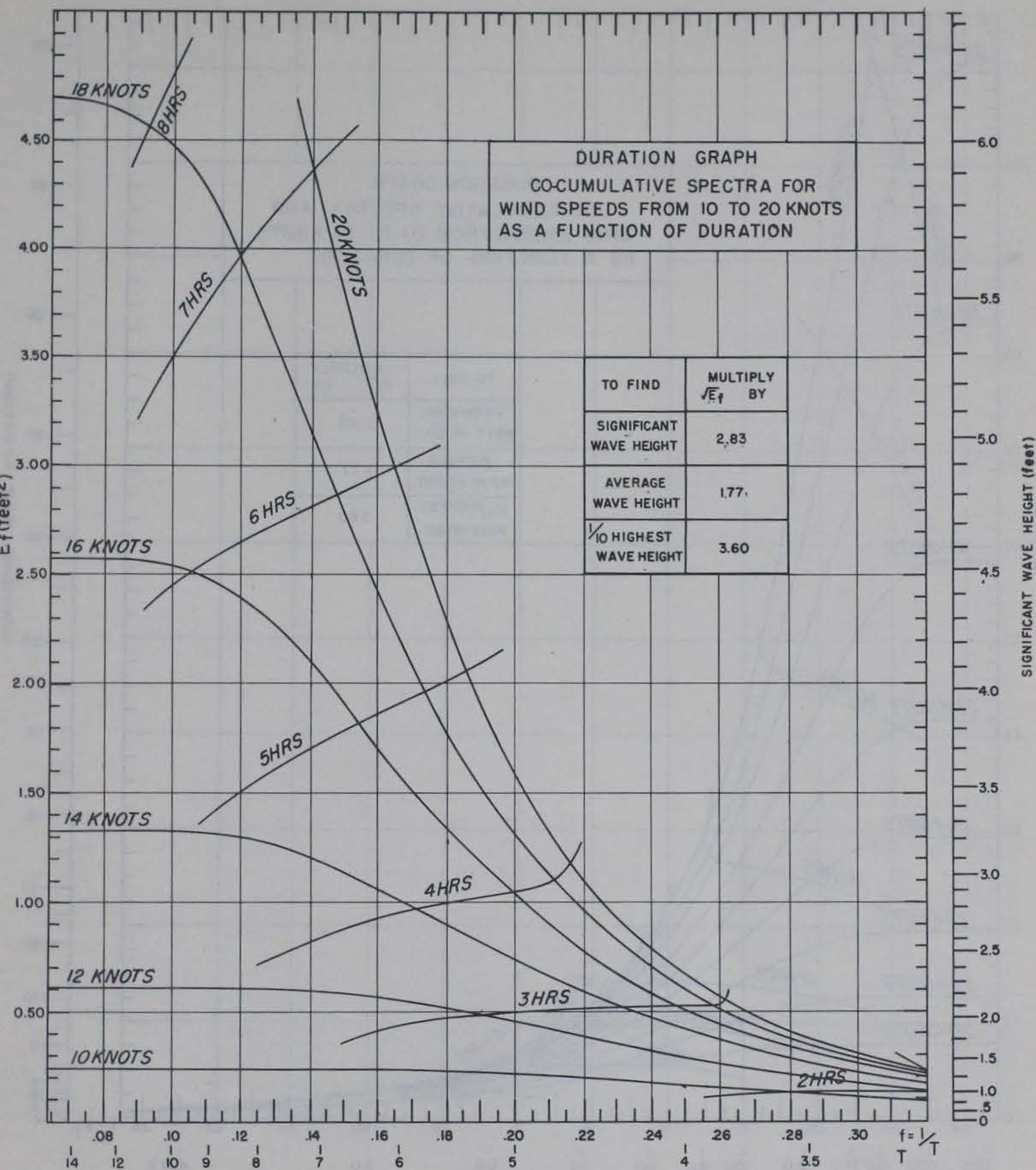
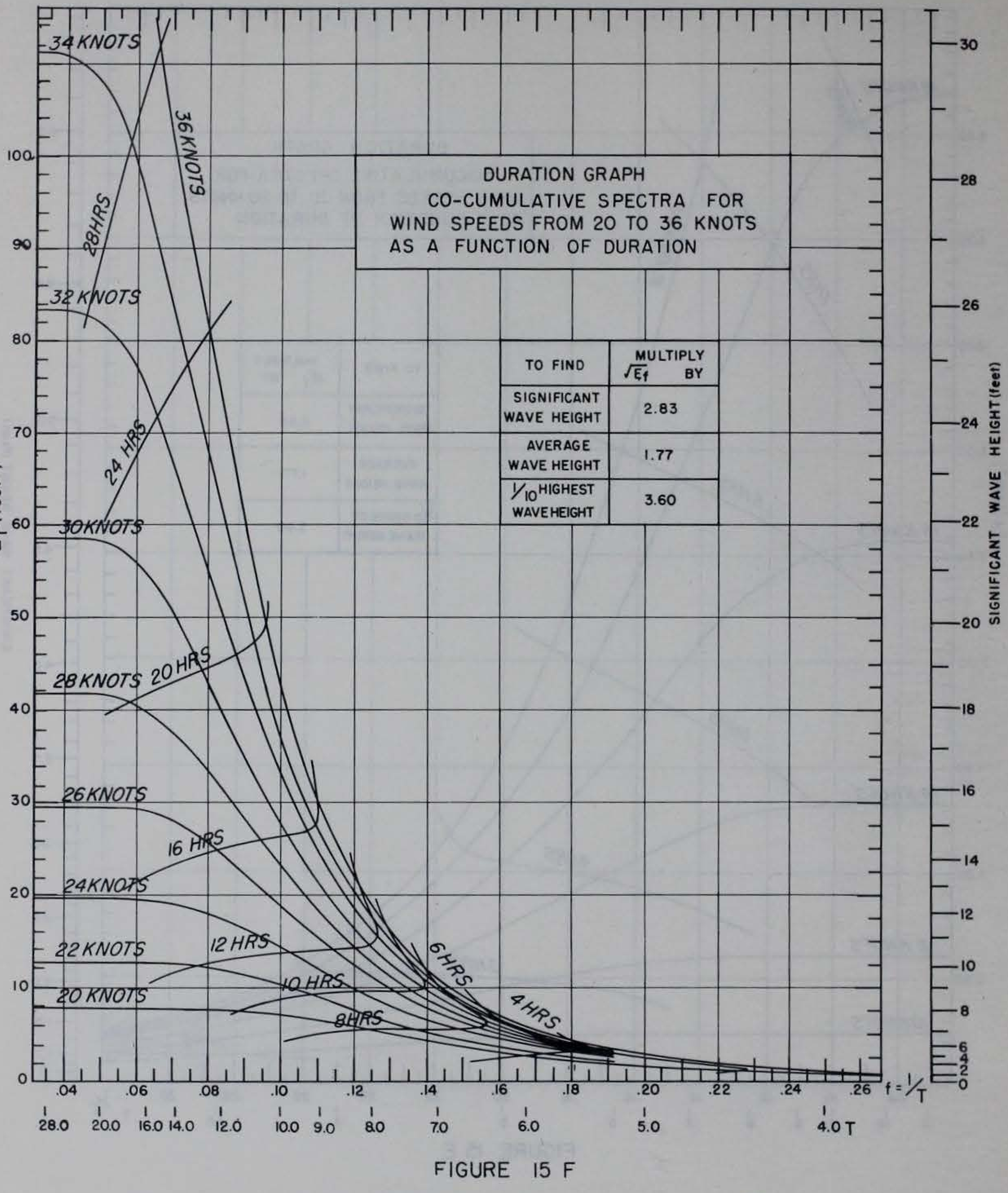

DURATION GRAPH
CO-CUMULATIVE SPECTRA FOR WIND SPEEDS FROM 10 TO 20 KNOTS AS A FUNCTION OF DURATION
TO FIND
MULTIPLY $\sqrt{E_f}$ BY
SIGNIFICANT WAVE HEIGHT
2.83
AVERAGE WAVE HEIGHT
1.77
1/10 HIGHEST WAVE HEIGHT
3.60
18 KNOTS
16 KNOTS
14 KNOTS
12 KNOTS
10 KNOTS
20 KNOTS
8HRS
7HRS
6 HRS
5HRS
4HRS
3HRS
2HRS
E_f (feet²)
SIGNIFICANT WAVE HEIGHT (feet)
$f = 1/T$
T

FIGURE 15 E



TO FIND	MULTIPLY $\sqrt{E_f}$ BY
SIGNIFICANT WAVE HEIGHT	2.83
AVERAGE WAVE HEIGHT	1.77
1/10 HIGHEST WAVE HEIGHT	3.60

FIGURE 15 F

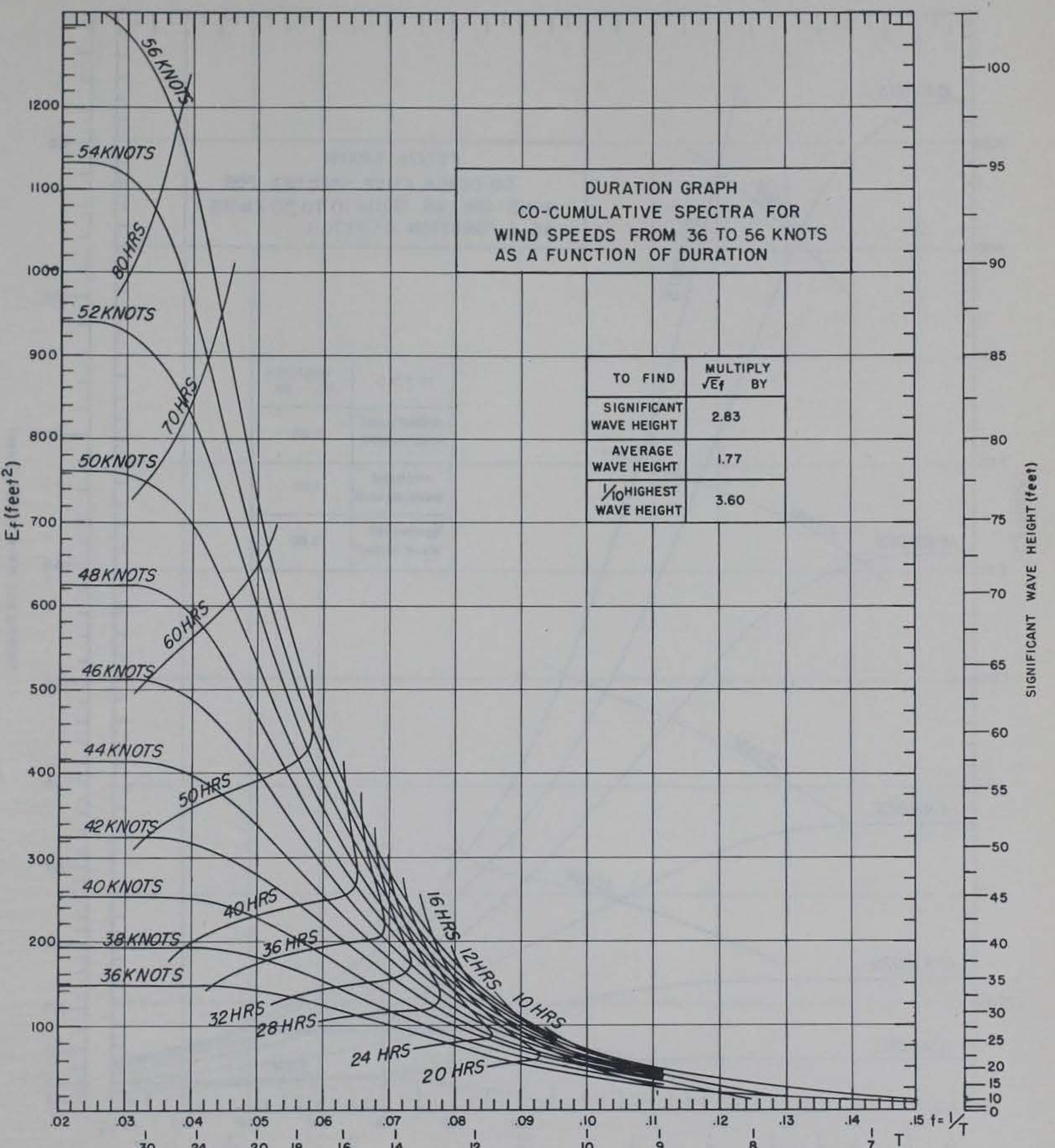
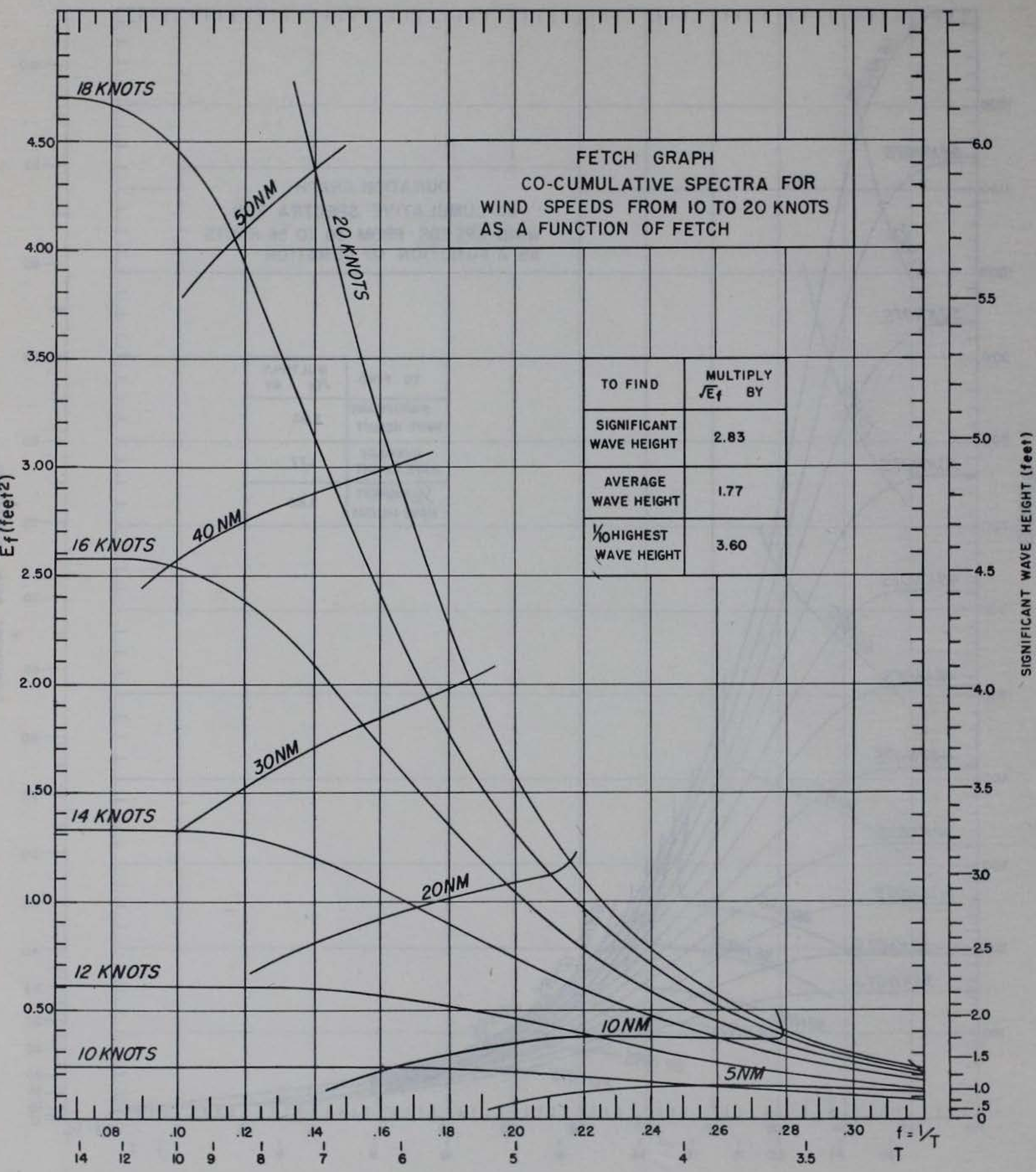
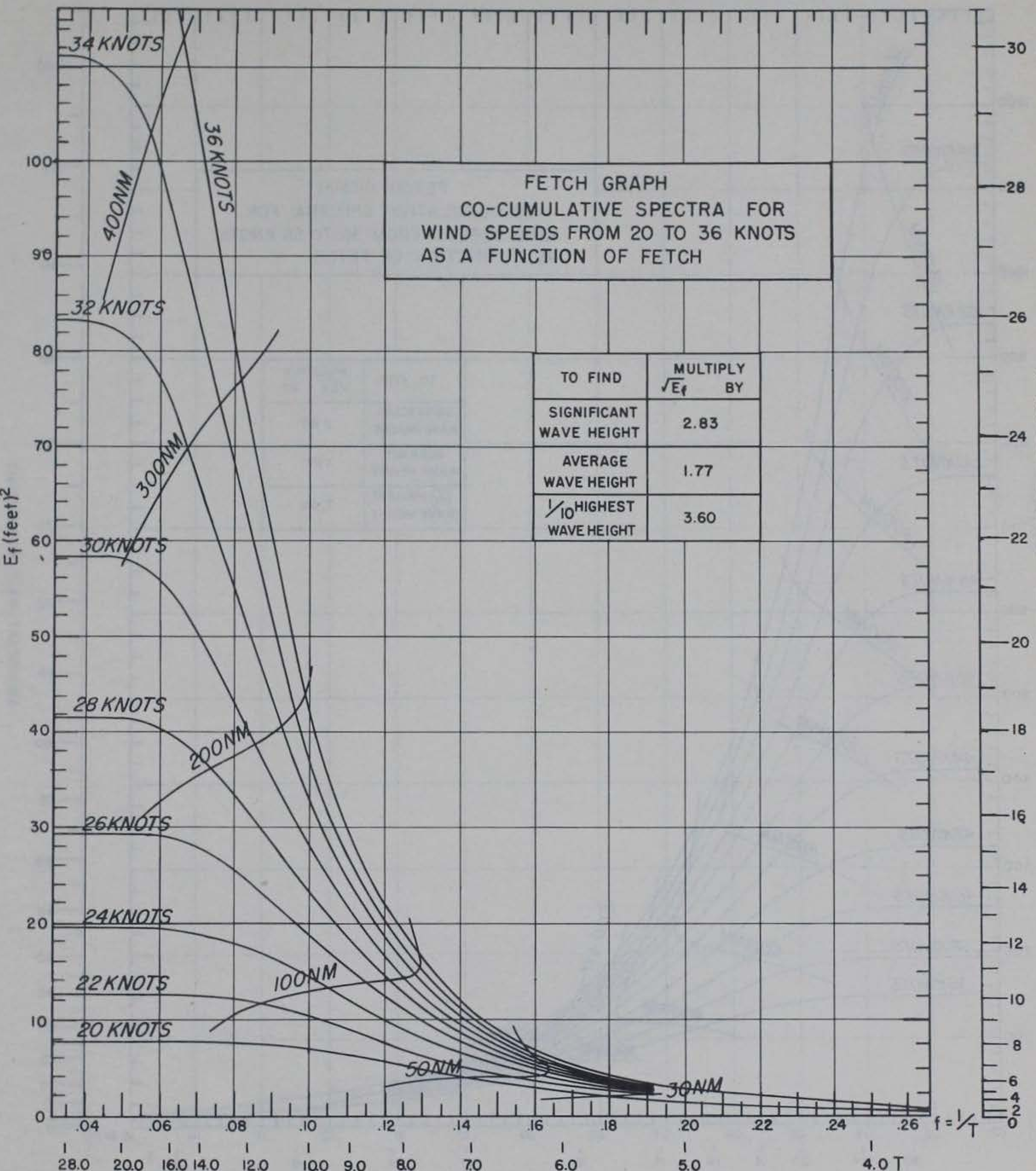

DURATION GRAPH
CO-CUMULATIVE SPECTRA FOR
WIND SPEEDS FROM 36 TO 56 KNOTS
AS A FUNCTION OF DURATION
TO FIND
MULTIPLY √Ef BY
SIGNIFICANT WAVE HEIGHT
2.83
AVERAGE WAVE HEIGHT
1.77
1/10 HIGHEST WAVE HEIGHT
3.60
56 KNOTS
54 KNOTS
52 KNOTS
50 KNOTS
48 KNOTS
46 KNOTS
44 KNOTS
42 KNOTS
40 KNOTS
38 KNOTS
36 KNOTS
80 HRS
70 HRS
60 HRS
50 HRS
40 HRS
36 HRS
32 HRS
28 HRS
24 HRS
20 HRS
16 HRS
12 HRS
10 HRS
Ef (feet²)
SIGNIFICANT WAVE HEIGHT (feet)
f = 1/T
T

FIGURE 15 G



TO FIND	MULTIPLY $\sqrt{E_f}$ BY
SIGNIFICANT WAVE HEIGHT	2.83
AVERAGE WAVE HEIGHT	1.77
1/10 HIGHEST WAVE HEIGHT	3.60

FIGURE 15 H


FETCH GRAPH
CO-CUMULATIVE SPECTRA FOR
WIND SPEEDS FROM 20 TO 36 KNOTS
AS A FUNCTION OF FETCH

TO FIND	MULTIPLY $\sqrt{E_f}$ BY
SIGNIFICANT WAVE HEIGHT	2.83
AVERAGE WAVE HEIGHT	1.77
1/10 HIGHEST WAVE HEIGHT	3.60

34 KNOTS
36 KNOTS
32 KNOTS
30 KNOTS
28 KNOTS
26 KNOTS
24 KNOTS
22 KNOTS
20 KNOTS
400NM
300NM
200NM
100NM
50NM
30NM
E_f (feet)^2
SIGNIFICANT WAVE HEIGHT (feet)
f = 1/T
4.0 T

FIGURE 15 I

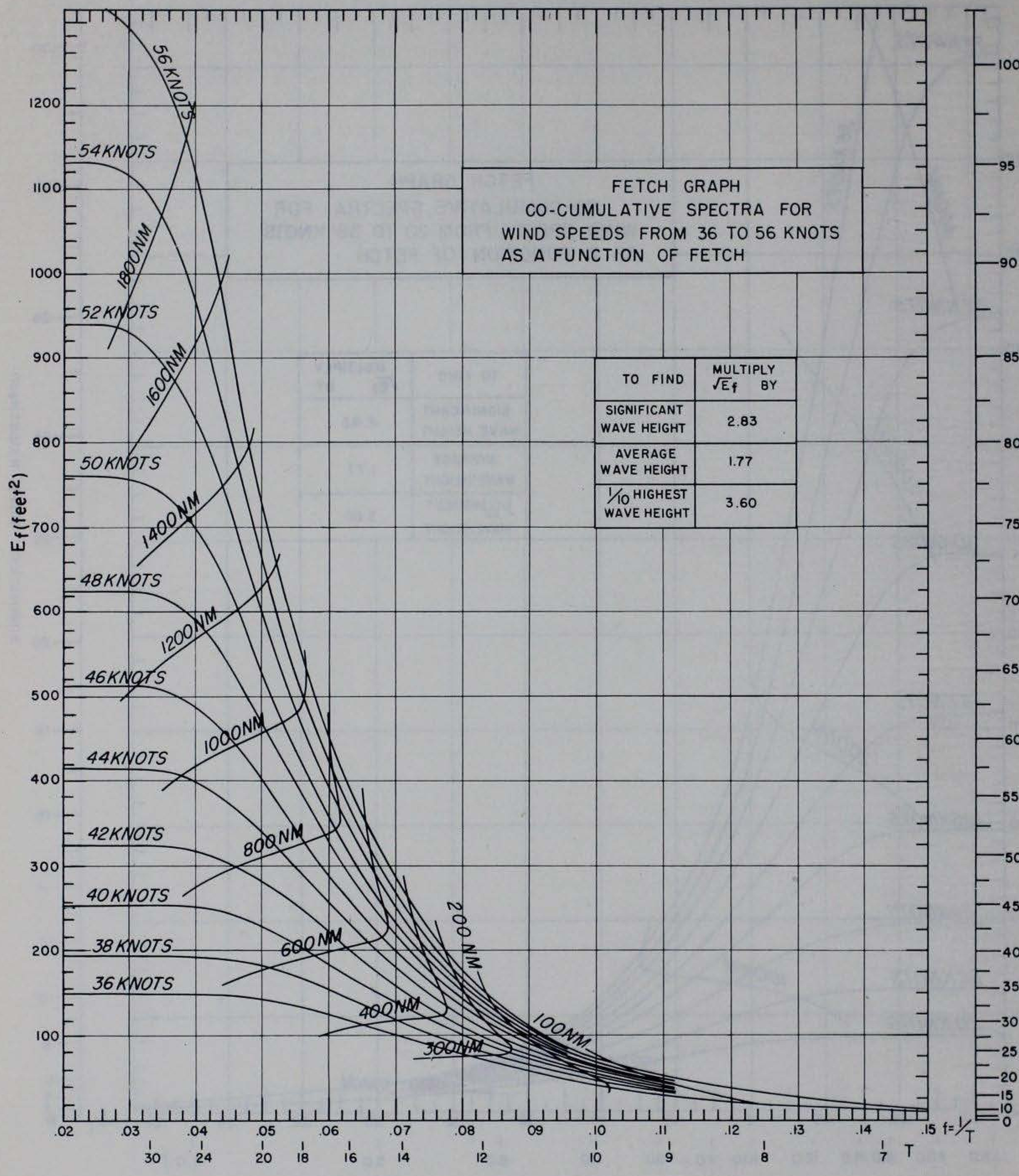

FETCH GRAPH
CO-CUMULATIVE SPECTRA FOR
WIND SPEEDS FROM 36 TO 56 KNOTS
AS A FUNCTION OF FETCH
TO FIND
MULTIPLY $\sqrt{E_f}$ BY
SIGNIFICANT WAVE HEIGHT
2.83
AVERAGE WAVE HEIGHT
1.77
1/10 HIGHEST WAVE HEIGHT
3.60
56 KNOTS
54 KNOTS
52 KNOTS
50 KNOTS
48 KNOTS
46 KNOTS
44 KNOTS
42 KNOTS
40 KNOTS
38 KNOTS
36 KNOTS
1800 NM
1600 NM
1400 NM
1200 NM
1000 NM
800 NM
600 NM
400 NM
300 NM
200 NM
100 NM
E_f (feet^2)
SIGNIFICANT WAVE HEIGHT (feet)
$f = \frac{1}{T}$
T

FIGURE 15 J

In most practical cases, the wave-generating winds will be complicated, and the determination of the parameters "fetch" and "duration" is probably the most subjective factor in the process of wave forecasting. It is difficult to set up strict rules for determining these important parameters of wave forecasting. In practice, innumerable possibilities of different fetch and duration conditions may be encountered by the wave forecaster. It must be left to his experience and good judgment in the interpretation of consecutive synoptic weather maps to use the given graphs and tables in the most effective way.

The corrected upper limit of the significant range of wave periods for a non-fully arisen sea, where $f_i > f_{max}$, is then given by the equation:

$$f_u = f_i - D_f$$

Example: With a wind velocity of 20 knots and a duration of 6 hours, E_f = 2.99 and f_i = 0.165 (T_i = 6.1 seconds) from Figure 15E. Table 1-B shows that f_{max} = 0.124; thus $f_i > f_{max}$ and the correction may be applied. D_f = 0.025, and f_u = 0.165 - 0.025 = 0.14; thus T_u = 7.1 seconds and is used in place of T_i = 6.1 seconds. The lower limit of the significant range of wave periods, T_L, can be determined in the same manner as for a fully arisen sea (i.e., the frequency corresponding to 3 percent of E_f). For this case $f_L \sim$ 0.4 and $T_L \sim 2.5$. Thus the significant range of wave periods for this example is 2.5 to 7.1 seconds. The wave height characteristics are computed from $\sqrt{E_f}$ = 1.73. It follows: H_{av} = 2.1 feet, $H_{1/3}$ = 4.9 feet, $H_{1/10}$ = 6.2 feet.

1.26 WAVES IN TRANSITIONAL AND SHALLOW WATER. - From equation 2, it may be seen that wave velocity varies with the depth of water in which the wave is moving; as the depth decreases, the velocity also decreases. In addition, assuming constant wave period, equation 1 indicates that wave length also varies with water depth. The variation in wave velocity along the crest of a wave moving at an angle to underwater contours, (i.e., that portion of the wave in deeper water is moving faster than the portion in shallow water) causes the crest to bend toward alignment with the contours (see Figure 16). This effect, called refraction, is more or less significant depending on the relation of the water depth to the wave length.

In water deeper than one-half the wave length, the hyperbolic tangent function in the formula

$$C^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L}, \qquad (2)$$

is nearly equal to unity, and equation 2 reduces to

$$C^2 = \frac{gL}{2\pi} \qquad (8)$$

In this equation, the velocity (C) is not dependent on depth; therefore in those regions deeper than one-half the wave length (deep water) refraction is not significant. In the region where water depth is between 1/2 and 1/25 the wave length (transitional water), and in the region where water depth is less than 1/25 the wave length (shallow water) refraction effects are significant. In the first of these, wave velocity must be computed from equation 2 ; in the second, tanh $2\pi d/L$ becomes nearly equal to $2\pi d/L$ and equation 2 reduces to

$$C^2 = gd \qquad \text{- Shallow water - } d < \frac{L_o}{25} \tag{9}$$

Both equations 2 and 9 show a dependence of wave velocity on depth. To a first approximation, equation 3 for total energy in a wave per unit crest width may be written as

$$\left(E_t = \frac{\rho g H^2 L}{8} \right) \tag{10}$$

It has already been noted that wave energy does not propagate at the wave velocity, but at another velocity known as the group velocity (see equations 4 and 5). Put another way, we may say that only a portion of the total wave energy is transmitted forward with the wave form, this portion being given by

$$E = nE_t \tag{11}$$

Transitional water $(1/2 < n < 1)$

where

$$n = \frac{1}{2}\left[1 + \frac{\frac{4\pi d}{L}}{\sinh\frac{4\pi d}{L}}\right] \tag{5}$$

shallow water $n = 1$

which in deep water becomes $n = 1/2$.

1.261 Refraction of Waves. In performing refraction analyses, it is assumed that in a wave advancing toward shore, no energy flows laterally along a wave crest. That is, the transmitted energy remains constant between two lines (called orthogonals) drawn perpendicular to a wave crest as it passes over changing hydrography. The wave energy transmitted forward between any two adjacent orthogonals in deep water is $E_o = (1/2)\, b_o\, E_{to}$, where b_o is the spacing of the orthogonals in deep water. (The subscript "o" always refers to deep water conditions). This energy may be equated to the energy transmitted forward between the same two orthogonals in shallow water ($E = n\, b\, E_t$), where b is the spacing between the orthogonals in shallow water. Therefore $(1/2) b_o E_{t_o} = n\, b\, E_t$ or

$$E_t/E_{to} = (1/2)\,(1/n)\,(b_o/b) \tag{12}$$

From equation 10, $H/H_o = \sqrt{(E_t/E_{to})\,(L_o/L)}$ therefore

equation 12 may be written

$$H/H_o = \underbrace{\overbrace{\sqrt{(1/2)(1/n)\,(L_o/L)}}^{\text{shoaling factor}}}_{\text{Table}}\; \overbrace{\left(\sqrt{b_o/b}\right)}^{\text{Refr. Coef.}} \tag{13}$$

WAVE REFRACTION AT WEST HAMPTON BEACH, L.I., N.Y.
FIGURE 16

The term $\sqrt{(1/2)(1/n)(L_o/L)}$, is known as the shoaling coefficient (H/H_o'). This shoaling coefficient is a function of wave length and depth. It, and various other functions of d/L, eg, $2\pi d/L$, $4\pi d/L$, $\tanh (2\pi d/L)$, $\sinh (4\pi d/L)$, etc. are tabulated in Appendix D (Table D-1 for even increments of d/L_o and Table D-2 for even increments of d/L). For any depth and deep water wave length, the ratio d/L may be determined by a method of successive approximations from the ratio d/L_o .

Equation 13 shows that wave heights in transitional or shallow water may be found, knowing deep water wave heights, if the relative spacing between lines drawn perpendicular to wave crests can be determined. The square root of this relative spacing (b_o/b) is known as the refraction coefficient, usually designated K. It should also be noted that these perpendicular lines, when constructed, will show the direction of movement of the waves to which they are drawn perpendicular.

The lines drawn perpendicular to the wave crests are known as orthogonals. Various methods have been proposed for constructing these lines. The earlier approaches required drawing positions of wave crests, (64) then erecting perpendiculars to them; later approaches eliminate the intermediate wave crest step, permitting the immediate construction of orthogonals themselves (3) (67)(116).

It can be shown (see Appendix E) that the change of direction of an orthogonal as it passes over relatively simple hydrography is

$$\sin \alpha_2 = (C_2/C_1) \sin \alpha_1 \qquad (14)$$

where α_1 is the angle a normal to an orthogonal makes with a contour the orthogonal is passing over,

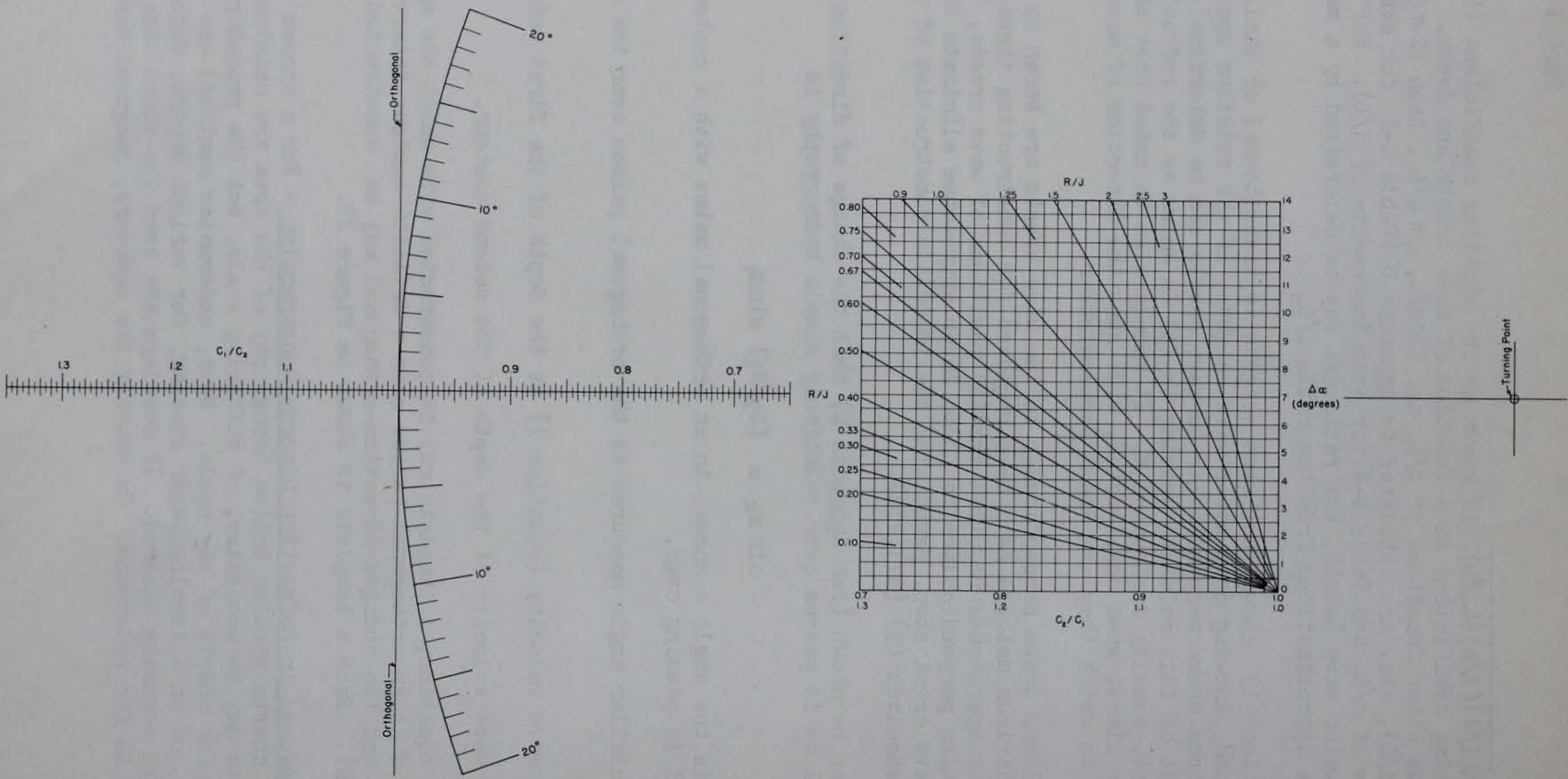
α_2 is a similar angle measured as the orthogonal passes over the next contour,

C_1 is the wave velocity (equation 1) at the depth of the first contour, and

C_2 is the wave velocity at the depth of the second contour.

From this equation, a template may be constructed with which the angular change in α as an orthogonal passes over a definite contour interval may be found, and the changed-direction-orthogonal may be constructed (see Appendix E). Such a template is shown on Figure 17.

Procedures in Refraction Diagram Construction - For a chosen shore location, charts showing bottom topography of the area are obtained. Two or more charts may be necessary, of differing scale, but the procedures are identical for charts of any scale. Next, underwater contours are drawn on the chart, or on a tracing paper overlay, for various depths, depending on the diagram accuracy desired. If overlays are used the shore line should be traced in for reference. In tracing the contours, judgment must be used



REFRACTION TEMPLATE
FIGURE 17

(After Arthur, Munk, & Isaacs, 1952)

in "smoothing out" small irregularities, since bottom features which are comparatively small in respect to the wave length do not affect the wave appreciably.

The range of wave periods to be used is determined by a hindcasting study of historical weather charts or from other historical data relating to wave periods. With the wave period so determined, C_1/C_2 values for each contour interval should be marked between the contours. The method of computing C_1/C_2 is illustrated by Table 2. A tabulation of C_1/C_2 for various contours intervals and wave periods is given in Table D-9 of Appendix D.

TABLE 2

COMPUTATIONS FOR VALUES OF C_1/C_2

T = 10 seconds

d (1) (Feet)	$\frac{d}{L_0}$ (2)	$\tanh \frac{2\pi d}{L}$ (3)	C_d/C_s (4)	C_s/C_d (5)
6	0.0117	0.268		
			1.40	0.72
12	0.0234	0.374		
			1.21	0.83
18	0.0352	0.453		
			1.14	0.88
24	0.0469	0.516		

Column 1 gives depths corresponding to chart contours. These would extend from 6 feet to a depth equal to $L_0/2$.

Column 2 is column 1 divided by L_0 corresponding to the determined period.

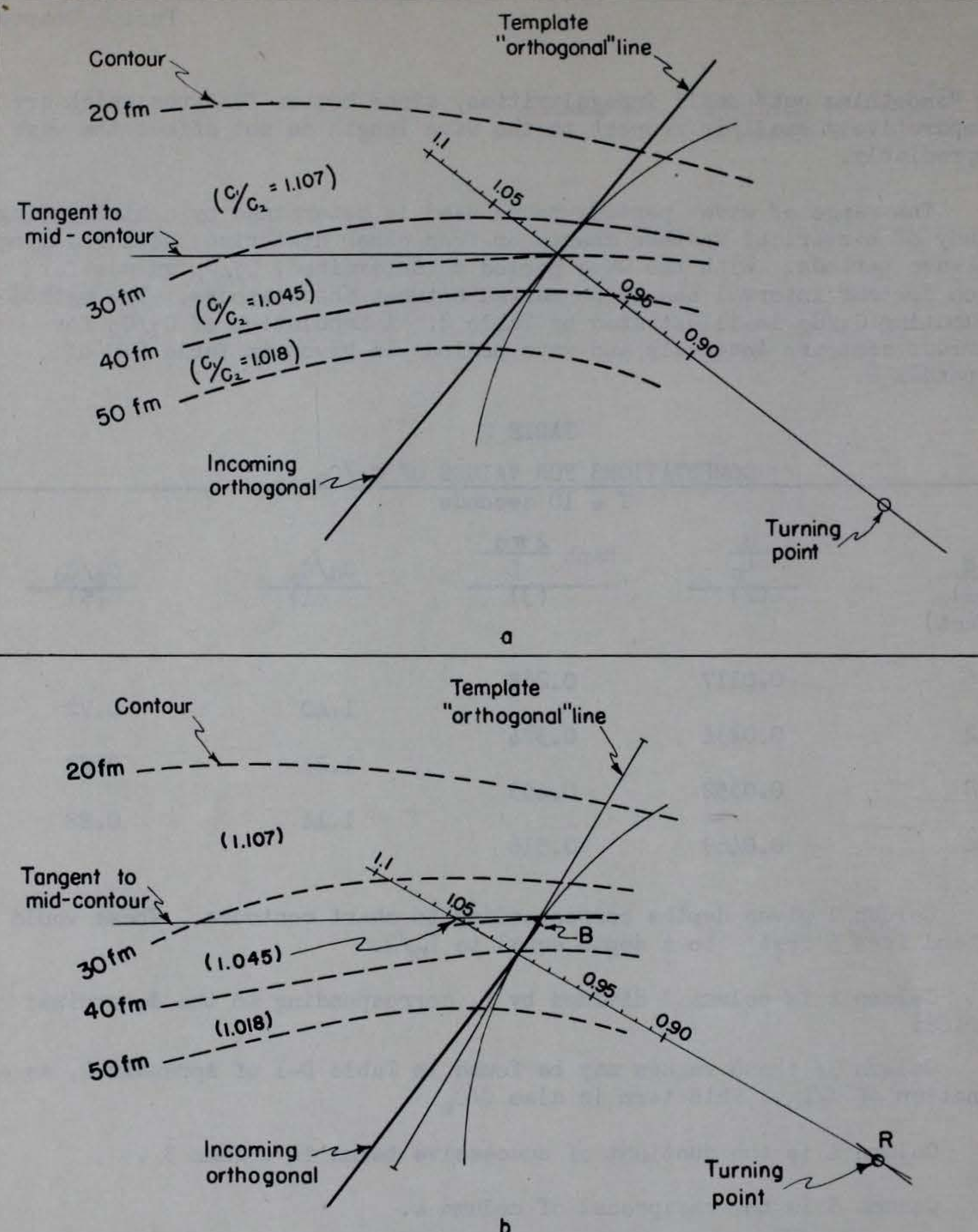
Column 3; these values may be found in Table D-1 of Appendix D, as a function of d/L_0. This term is also C/C_0.

Column 4 is the quotient of successive terms in column 3 .

Column 5 is the reciprocal of column 4.

To construct an orthogonal from deep to shallow water, a deep water direction of wave approach is first selected by a hindcasting study of historical weather charts, by fan diagram analysis or by direct observation. A deep water wave front (crest) is drawn as a straight line perpendicular to this wave direction and suitably spaced parallel lines (orthogonals) are drawn from this wave front in the chosen direction of wave approach. These lines are extended to the first depth contour shoaler than $L_0/2$ where L_0 (in feet) = $5.12T^2$.

$L_0 = 5.12T^2$ when $d \geq \frac{L_0}{2}$



Note: The template has been turned about R until the value $C_1/C_2 = 1.045$ intersects the tangent to the mid-contour. The template "orthogonal" line lies in the direction of the turned orthogonal. This direction is to be laid off at some point "B" on the incoming orthogonal which is equidistant from the two contours along the incoming and outgoing orthogonals.

USE OF THE REFRACTION TEMPLATE
FIGURE 18

Procedures for α less than 80° - Starting with any one orthogonal, the following steps should be taken:

(a) Sketch in a mid-contour between the first two contours to be crossed, extend the orthogonal to this mid-contour, and construct a tangent to the mid-contour at this point;

(b) Lay the line labelled "orthogonal" along the incoming orthogonal with the point marked 1.0 at the intersection of orthogonal and mid-contour (see Figure 18a);

(c) Rotate the template about the "turning point" until the C_1/C_2 value (C_d/C_s) corresponding to the contour interval being crossed intersects the tangent to the mid-contour. The "orthogonal" line now lies in the direction of the turned orthogonal (see Figure 18b);

(d) Place a triangle along the base of the plotter and erect a perpendicular to it so that the intersection of this perpendicular with the incoming orthogonal is equidistant from the two contours measured along the incoming orthogonal and this perpendicular. This line represents the turned orthogonal;

(e) Repeat the process for successive contour intervals.

If the orthogonal is being constructed from shallow to deep water, the same procedure is followed, except that C_s/C_d values are used for C_1/C_2 instead of C_d/C_s .

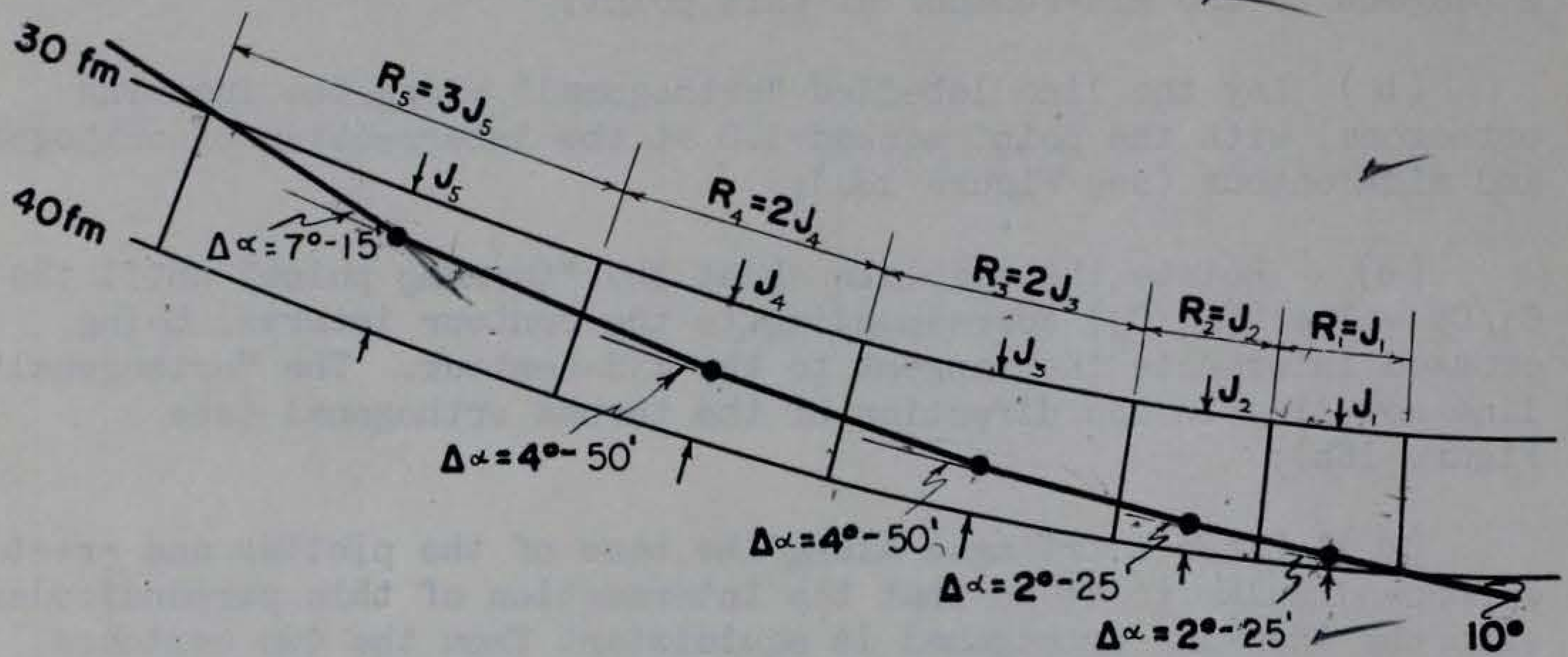
Procedures for α greater than 80° - In any depth of water, when α becomes greater than 80° , the above procedures may not be used. The orthogonal no longer appears to cross the contours, but tends to run parallel to them. In this case the contour interval is crossed in a series of steps. In essence, the whole interval is divided into a series of smaller intervals, at the mid-points of which, orthogonal angle turnings are made.

Referring to Figure 19, an interval to be crossed is divided into segments or boxes, by transverse lines. The spacing, R , of the lines is arbitrarily set as a ratio of the distance, J , between the contours. For the complete interval to be crossed, C_2/C_1 is computed or found from Table D-9 of Appendix D (Note: C_2/C_1 not C_1/C_2).

On the template (Figure 17) is a graph showing orthogonal angle turnings ($\Delta\alpha$) at the center of a box, plotted as a function of the C_2/C_1 value of any contour interval for various values of the R/J ratio, which may be chosen.

The orthogonal is brought into the middle of the box, $\Delta\alpha$ is read from the graph, and the orthogonal turned by that angle. The procedure is repeated for every box until α at a plotted or interpolated contour becomes

J = Distance between contours at turning points, •
R = Distance along orthogonal
T = 12 seconds
L_o = 737 ft.
For contour interval from 40fm to 30fm $C_1/C_2 = 1.045$, $C_2/C_1 = 0.957$



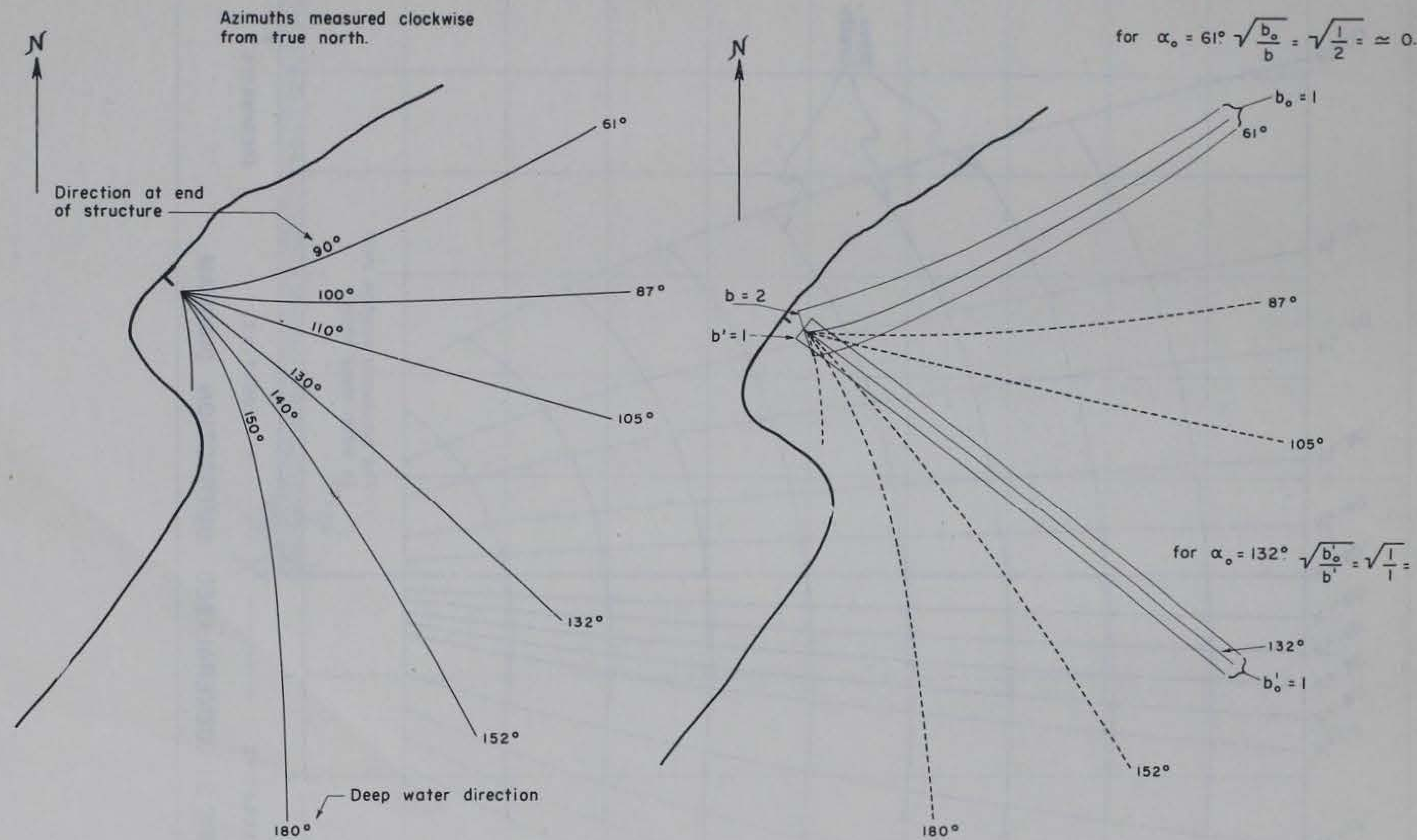
REFRACTION DIAGRAM USING $\frac{R}{J}$ METHOD

FIGURE 19

less than 80°. At this point this method of orthogonal construction must be stopped or error will result. ~~The dots on the graph of Figure 17 are those used to determine values for the example on Figure 19.~~

Refraction Fan Diagrams - It is often convenient, especially where a coastal area is shielded by land features from waves approaching in certain directions, to construct refraction diagrams from shallow toward deep water. In such cases, a sheaf or fan of orthogonals may be projected seaward in direction some 5 or 10 degrees apart. (See Figure 20a) With the deep water directions determined by the individual orthogonals, companion orthogonals may be projected shoreward on either side of the seaward projected ones in order to determine the refraction coefficient for the various directions of wave approach. (See Figure 20b).

Refraction Diagram Limitations - In many cases refraction diagrams provide a reasonably accurate measure of the changes waves undergo on approaching a coast. Quite often they provide the only measure of these changes available. However, the accuracy of data determined from refraction diagrams is limited by the validity of the theory of their construction and the accuracy of depth data on which they are based. The orthogonal direction change equation 14 is derived for the simplest case of straight parallel contours, and although little error is introduced by bringing orthogonals over relatively simple hydrography, it is difficult (94) to carry an orthogonal accurately into shore over complex bottom features. Moreover, the equation is derived for small waves moving over relatively flat slopes. Although



FAN — TYPE REFRACTION DIAGRAM
FIGURE 20a

COMPANION ORTHOGONALS FOR DETERMINING
REFRACTION COEFFICIENTS
FIGURE 20b

USE OF FAN-TYPE REFRACTION DIAGRAM
FIGURE 20

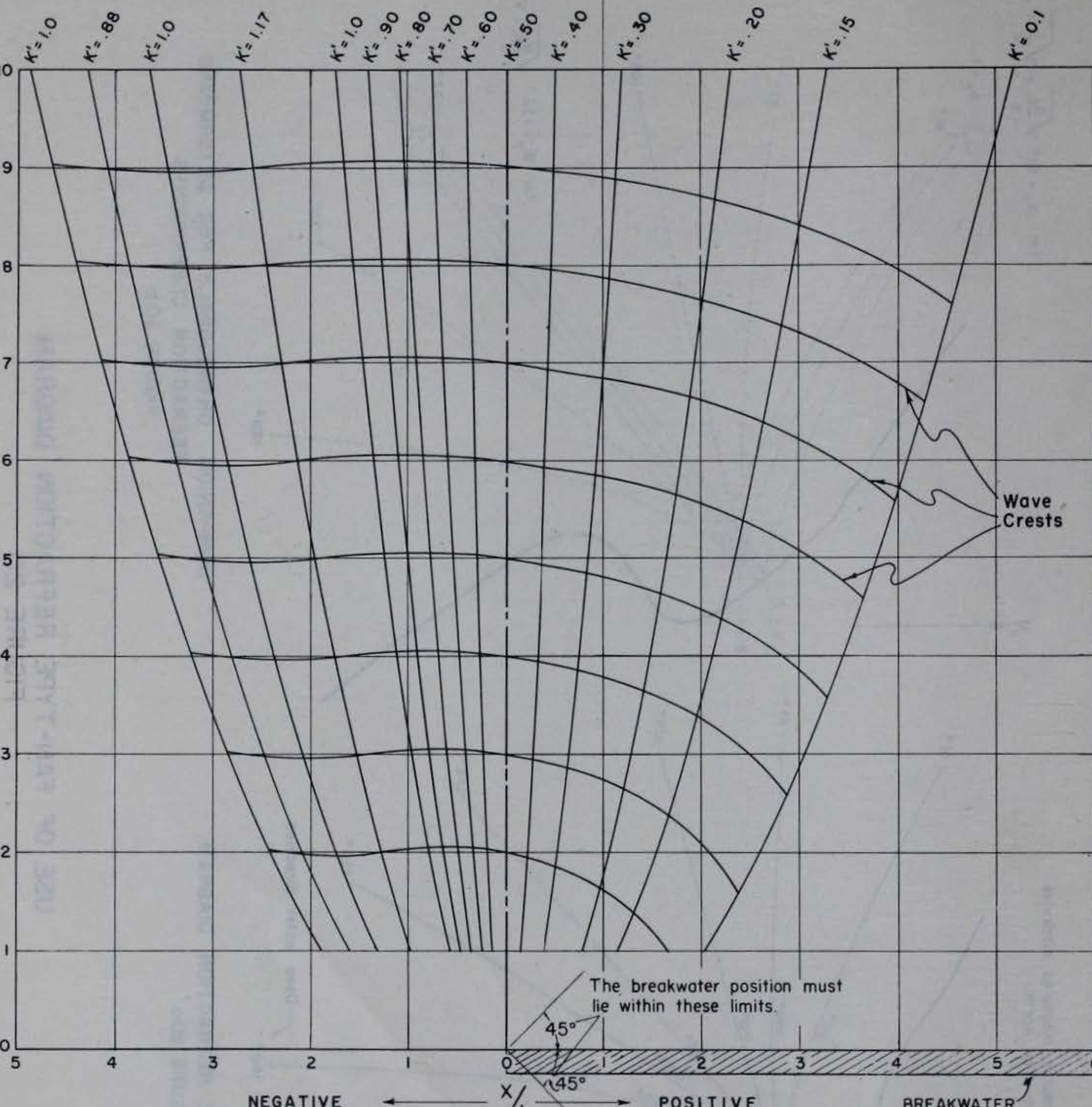


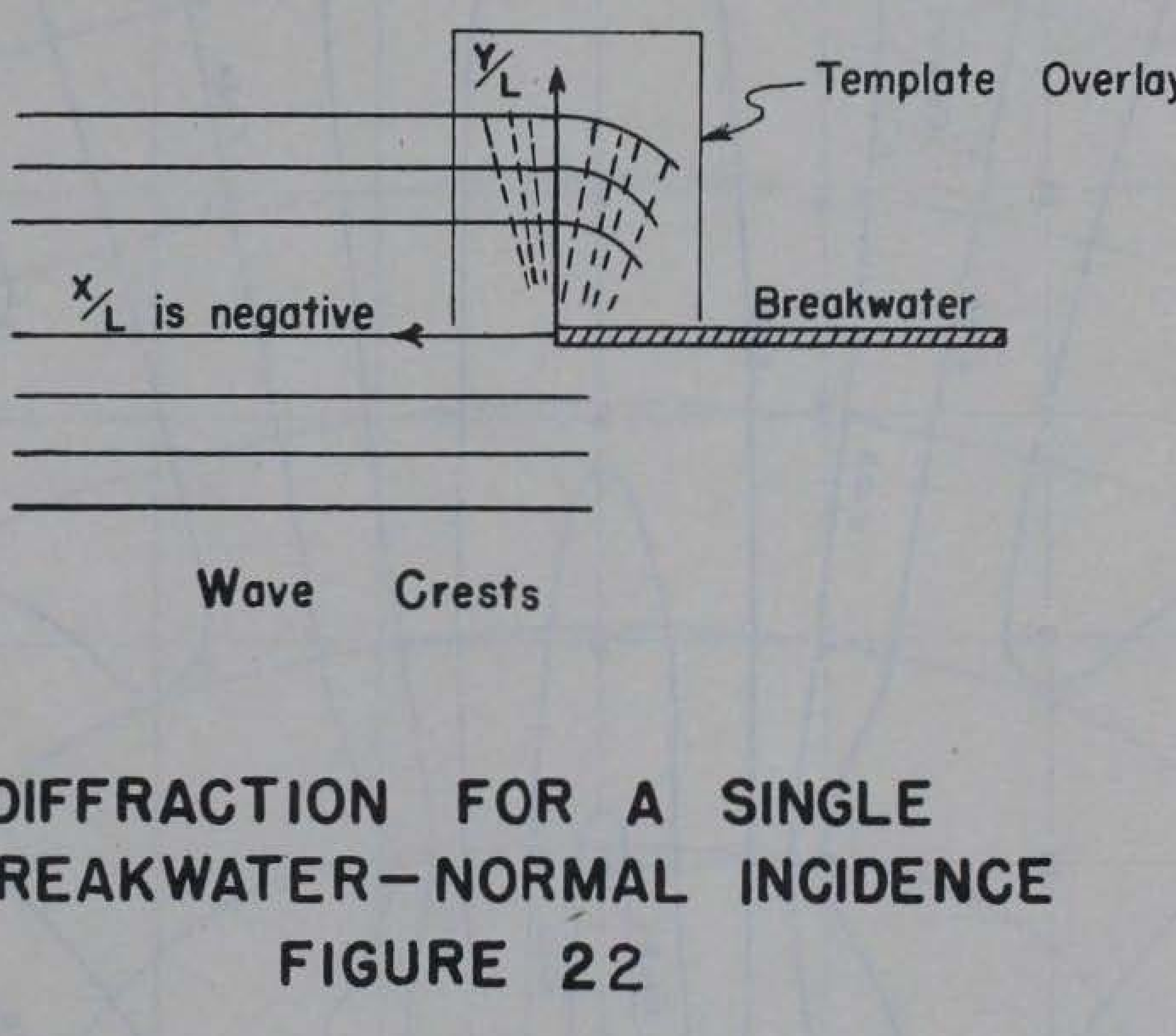
FIGURE 21 - GENERALIZED DIFFRACTION DIAGRAM

no strict limits have been set, strict accuracy cannot be expected where bottom slopes are steeper than 1 on 10. A third limitation is inherent in the assumption that no energy travels laterally along a wave crest. No strict limits have been set, but the accuracy of wave heights derived from orthogonals which bend sharply, is questionable.

1.262 Diffraction of Waves. - Diffraction in water waves is that phenomenon whereby energy is transferred laterally along a wave crest. It is most noticeable where an otherwise regular train of waves is interrupted by a barrier such as a breakwater.

Putnam and Arthur (105) presented experimental data verifying a method of solution proposed by Penny and Price(98) for the behavior of waves after passing a single breakwater. Blue and Johnson(11) have dealt with the problem of the behavior of waves after passing through a gap, as between two breakwater arms.

(a) - Waves Passing a Single Breakwater. The solution to this problem is presented in Appendix E. From this solution an overlay has been prepared (Figure 21) which, for the case of uniform depth shoreward of the breakwater, shows positions of diffracted wave crests and lines of equal wave height reduction. Plotting data used in the construction of this figure is given on Table E-1, Appendix E.



DIFFRACTION FOR A SINGLE BREAKWATER—NORMAL INCIDENCE
FIGURE 22

The diagram of Figure 21(38) is presented in dimensionless form and can therefore be used for any condition of wave period and water depth by scaling the entire figure up or down. The manner of use of the overlay is illustrated in Figure 22. The wave length, L, at the depth d at the breakwater tip must be found by computing the ratio d/L_o (L_o = the deep water

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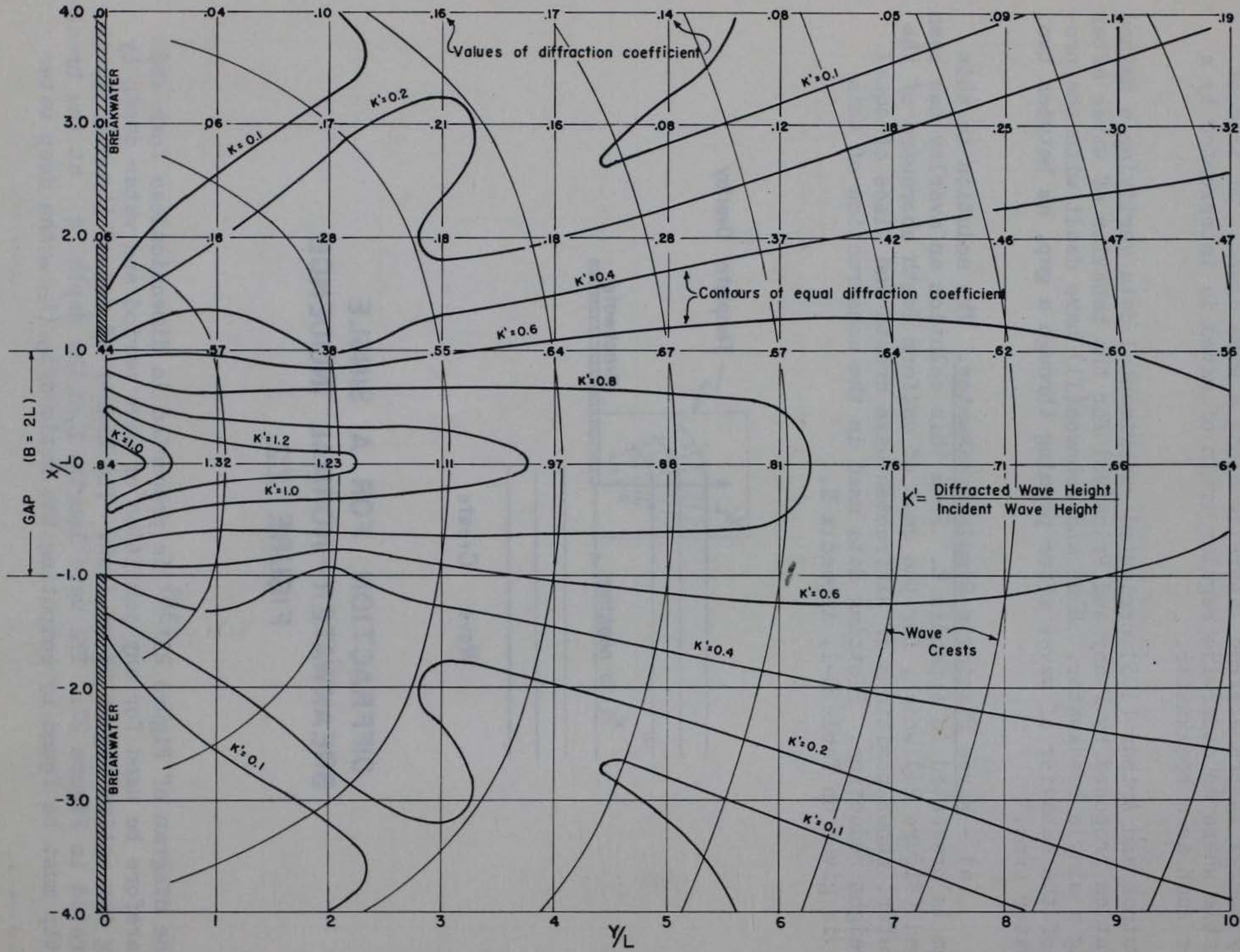


GENERALIZED DIFFRACTION DIAGRAM FOR A BREAKWATER GAP WIDTH OF TWO WAVE LENGTHS (B/L=2)

FIGURE 23

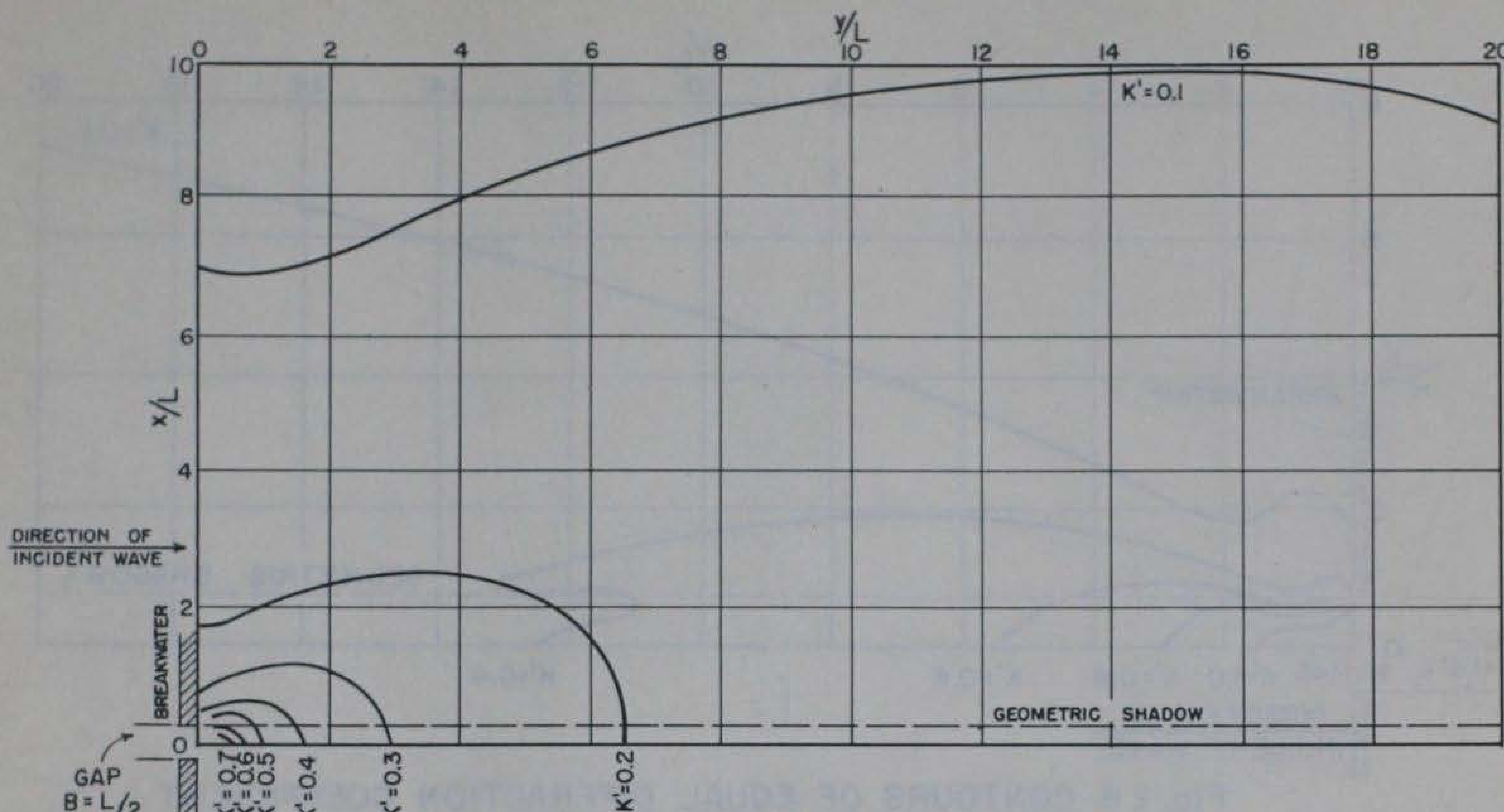


Fig. 24 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 0.5 WAVE LENGTH (B/L = 0.5)

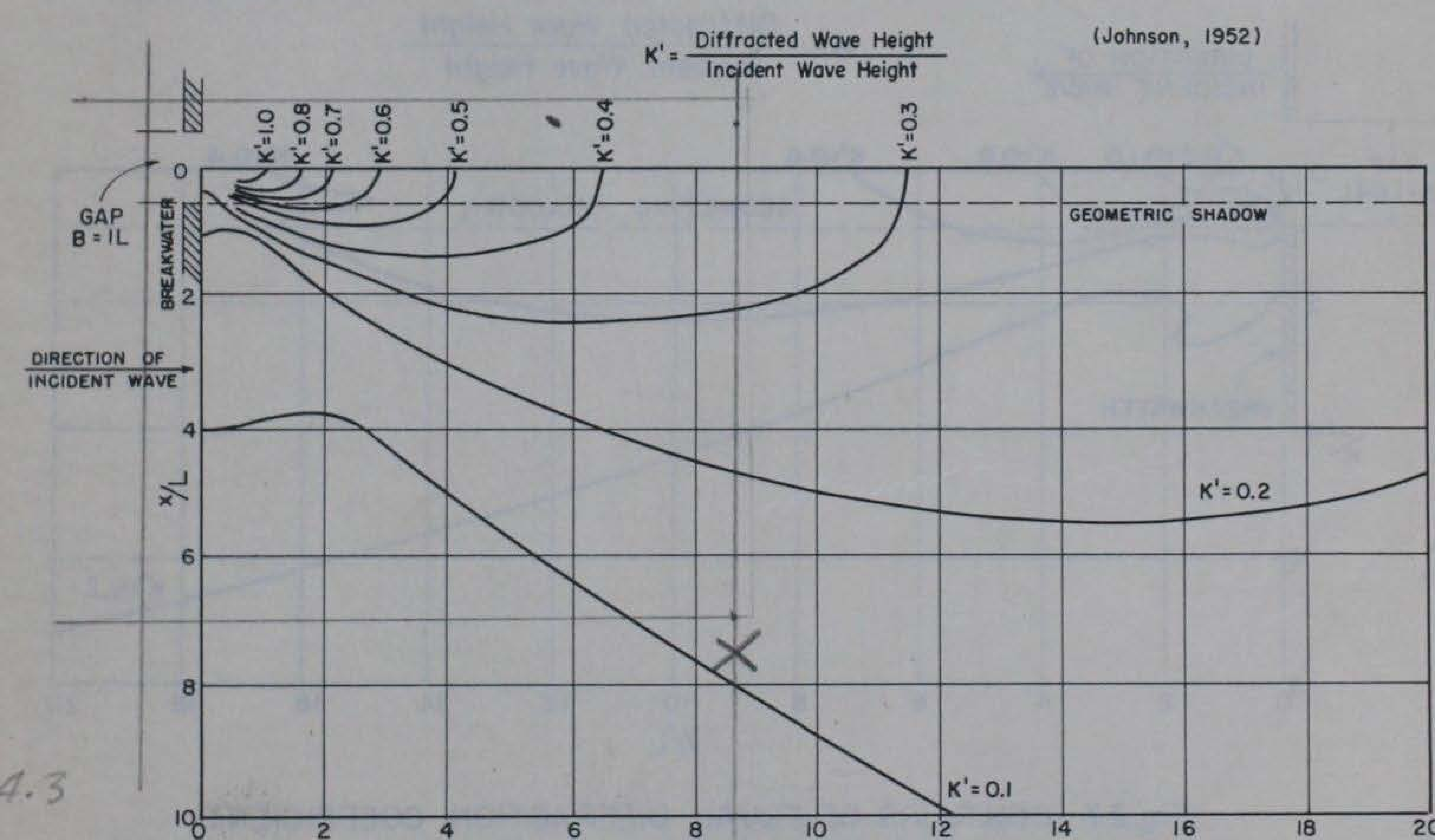


Fig. 25 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 1 WAVE LENGTH (B/L = 1)

DIFFRACTION OF WAVES AT A BREAKWATER GAP

(Johnson, 1952)

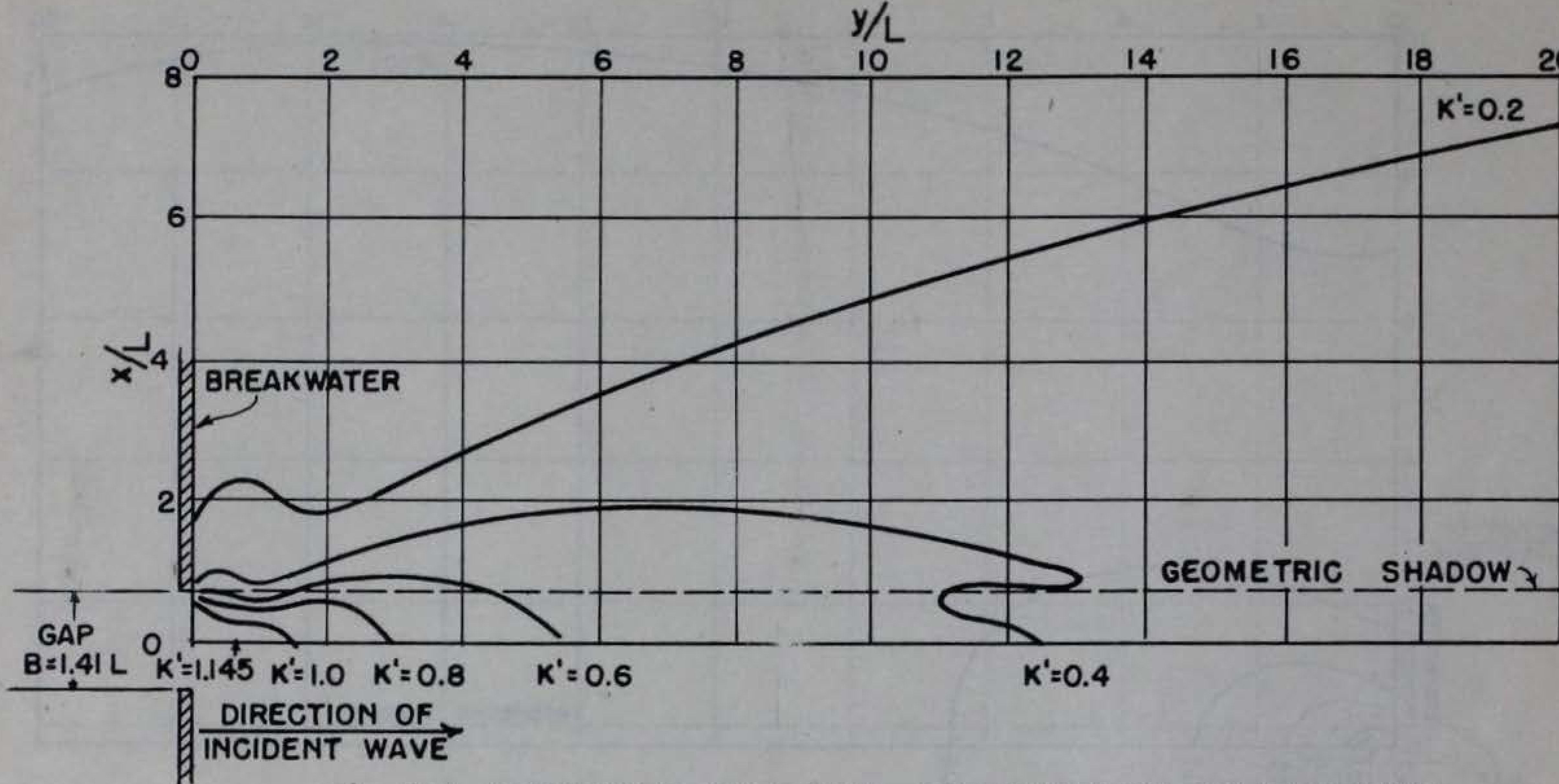


Fig. 26 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 1.41 WAVE LENGTHS (B/L = 1.41)

(Johnson, 1952)

$$K' = \frac{\text{Diffracted Wave Height}}{\text{Incident Wave Height}}$$

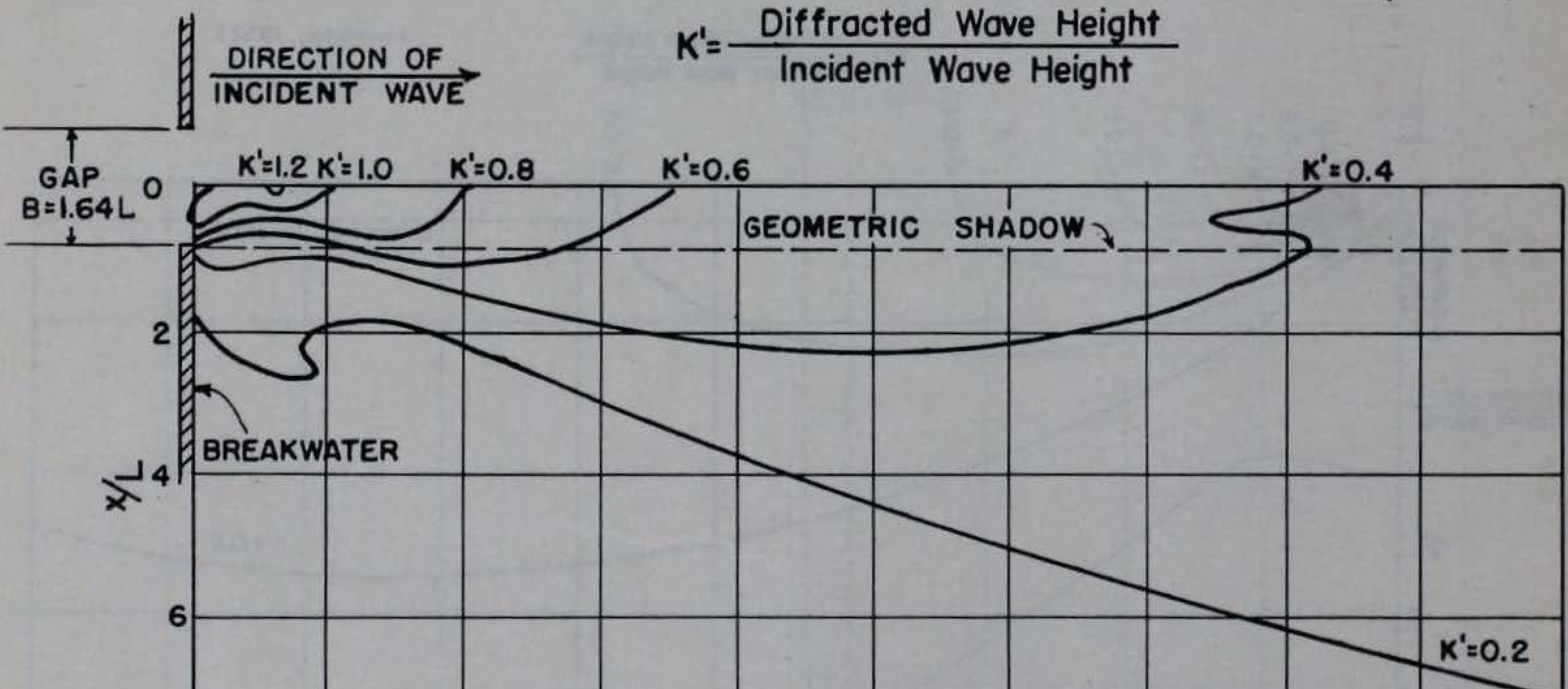


Fig. 27 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 1.64 WAVE LENGTHS (B/L = 1.64)

DIFFRACTION OF WAVES AT A BREAKWATER GAP

(Johnson, 1952)

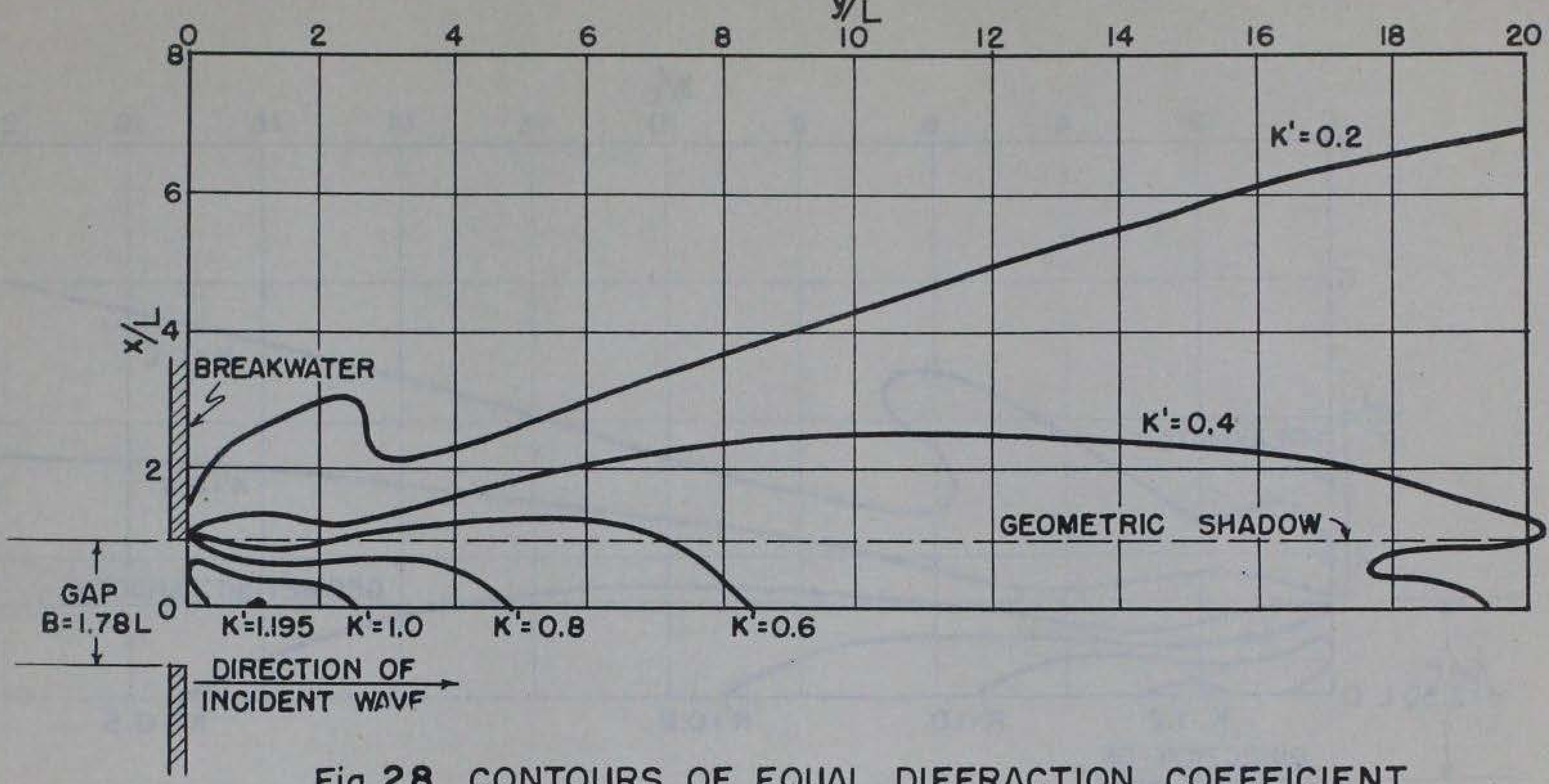


Fig. 28 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT GAP WIDTH = 1.78 WAVE LENGTHS (B/L = 1.78)

(Johnson, 1952)

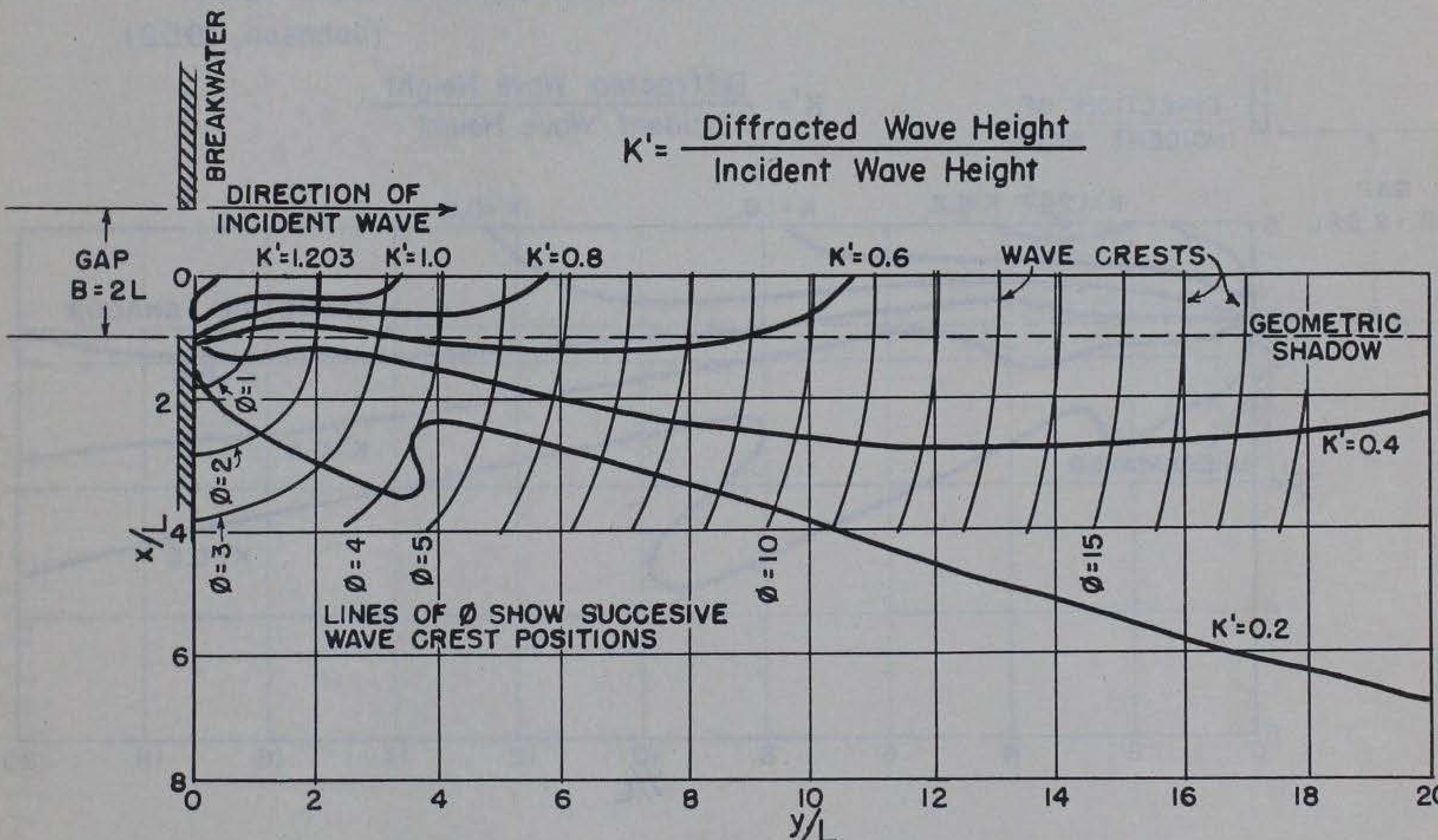


Fig. 29 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT GAP WIDTH = 2 WAVE LENGTHS (B/L = 2)

DIFFRACTION OF WAVES AT A BREAKWATER GAP

(Johnson, 1952)

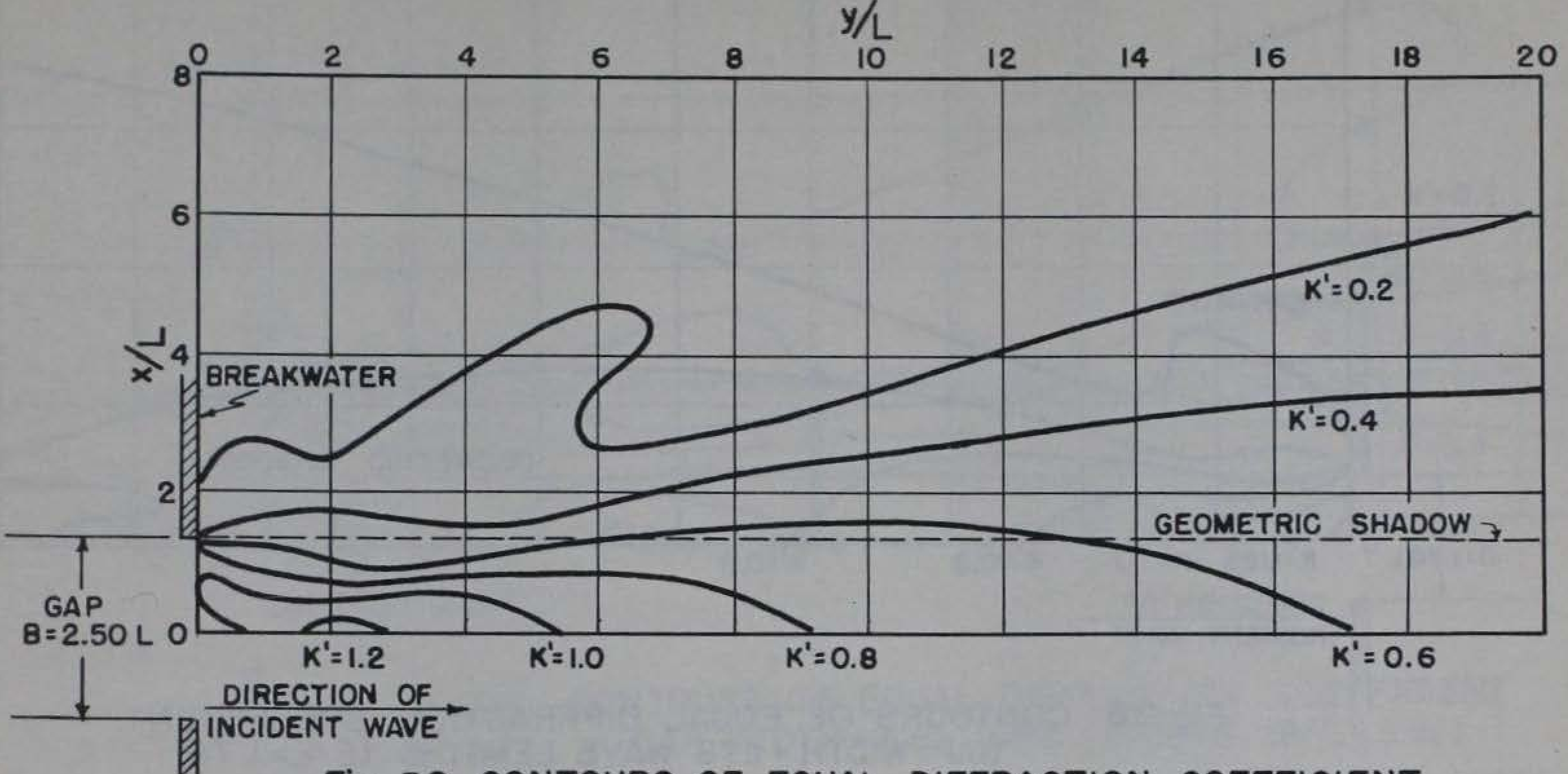


Fig. 30 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 2.50 WAVE LENGTHS (B/L = 2.50)

(Johnson, 1952)

$$K' = \frac{\text{Diffracted Wave Height}}{\text{Incident Wave Height}}$$

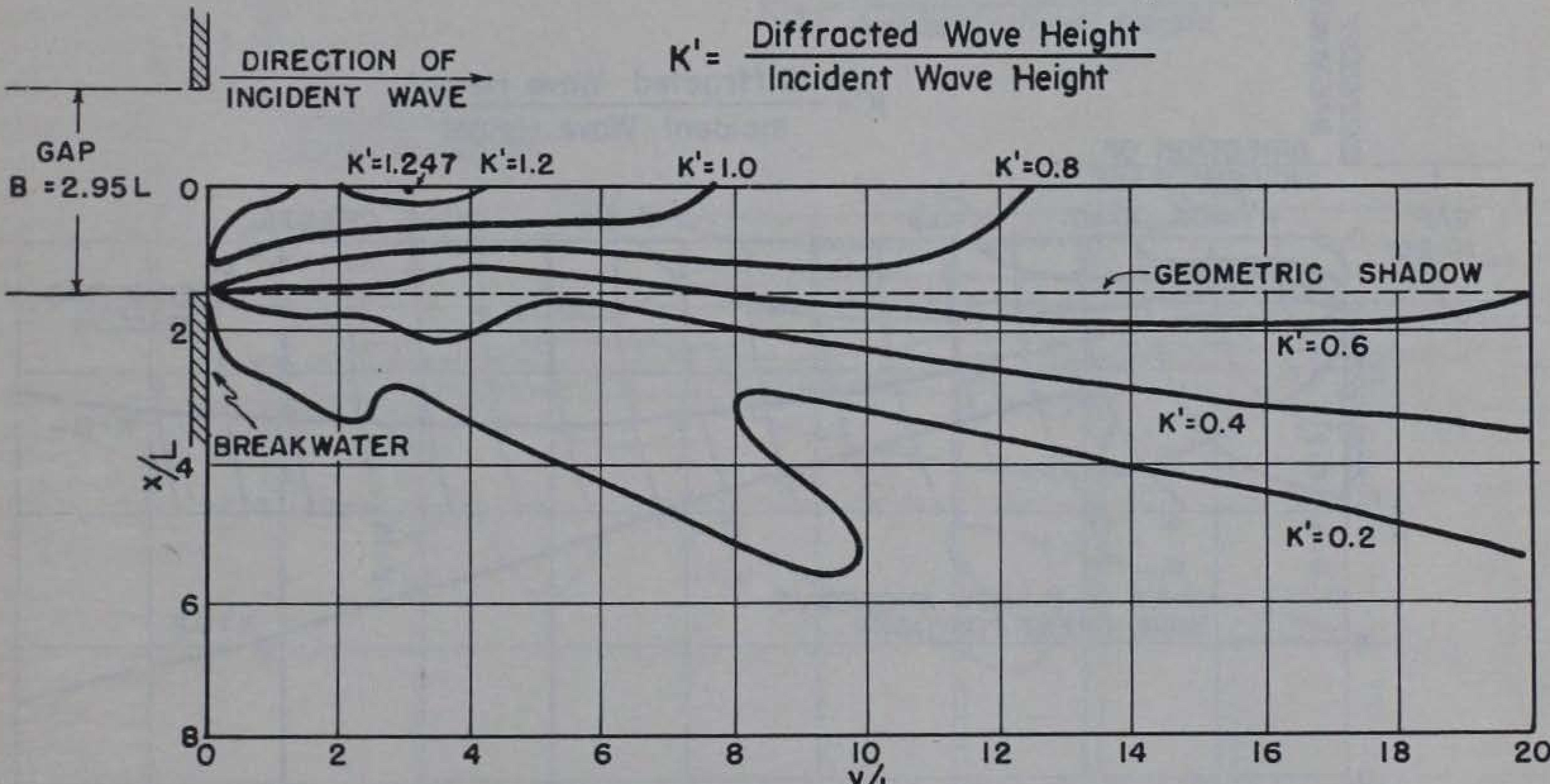


Fig. 31 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 2.95 WAVE LENGTHS (B/L = 2.95)

DIFFRACTION OF WAVES AT A BREAKWATER GAP

(Johnson, 1952)

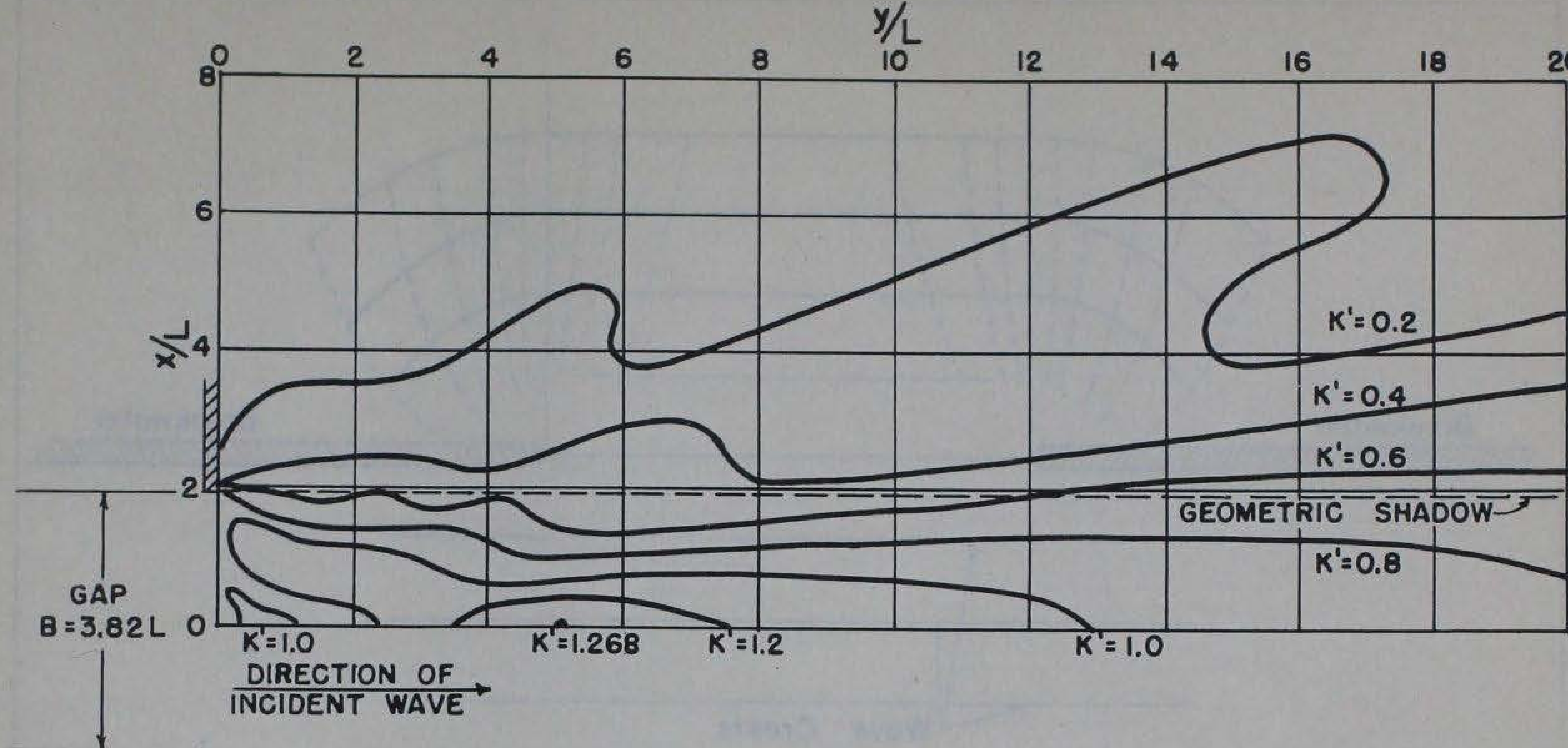
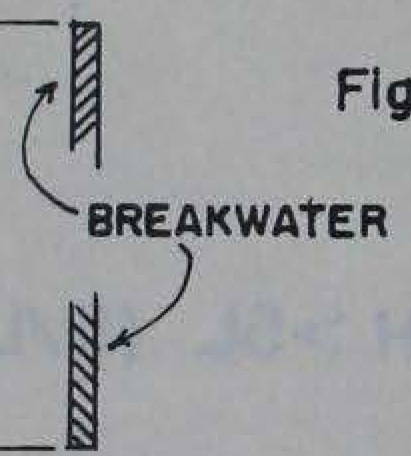


Fig. 32 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 3.82 WAVE LENGTHS (B/L = 3.82)

(Johnson, 1952)



$$K' = \frac{\text{Diffracted Wave Height}}{\text{Incident Wave Height}}$$

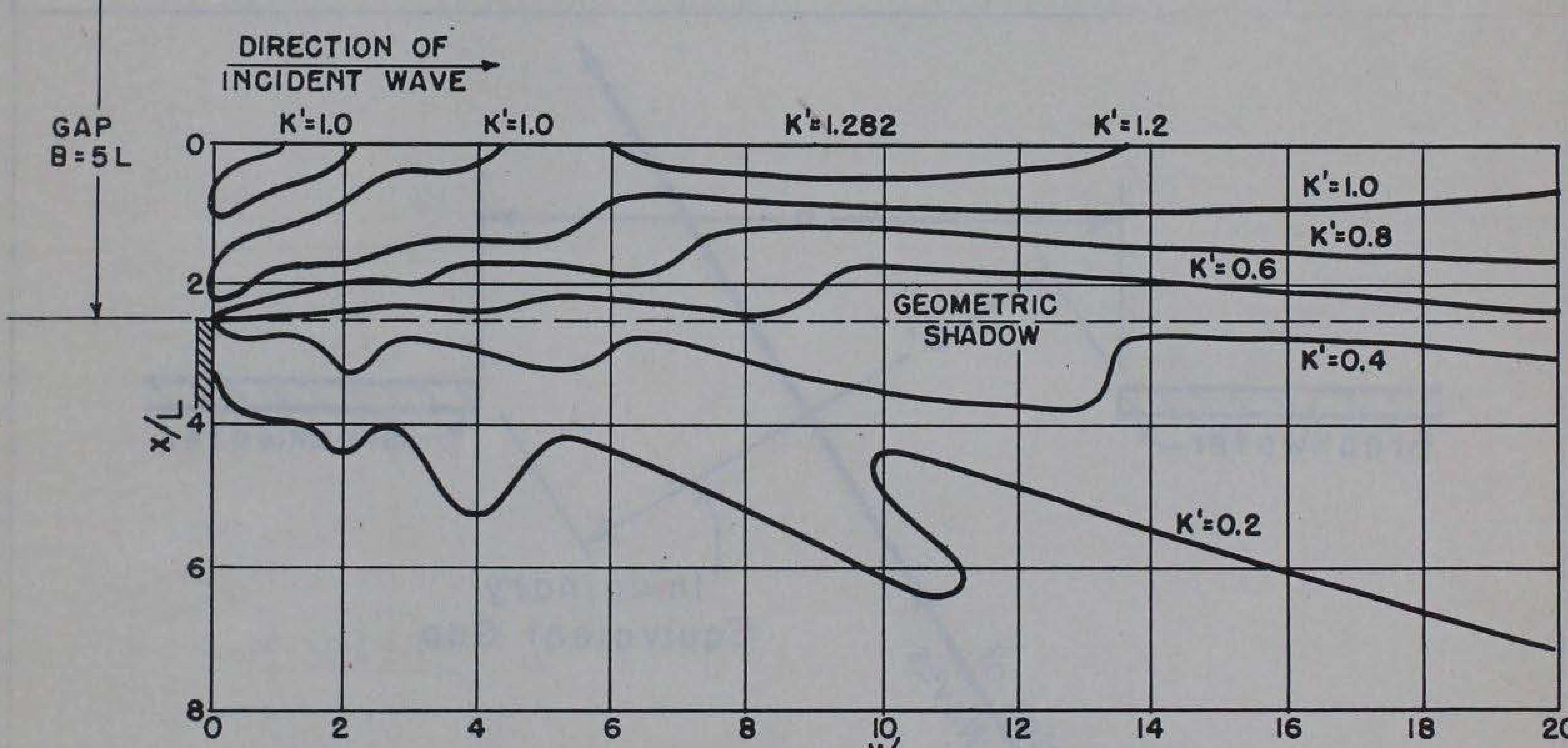
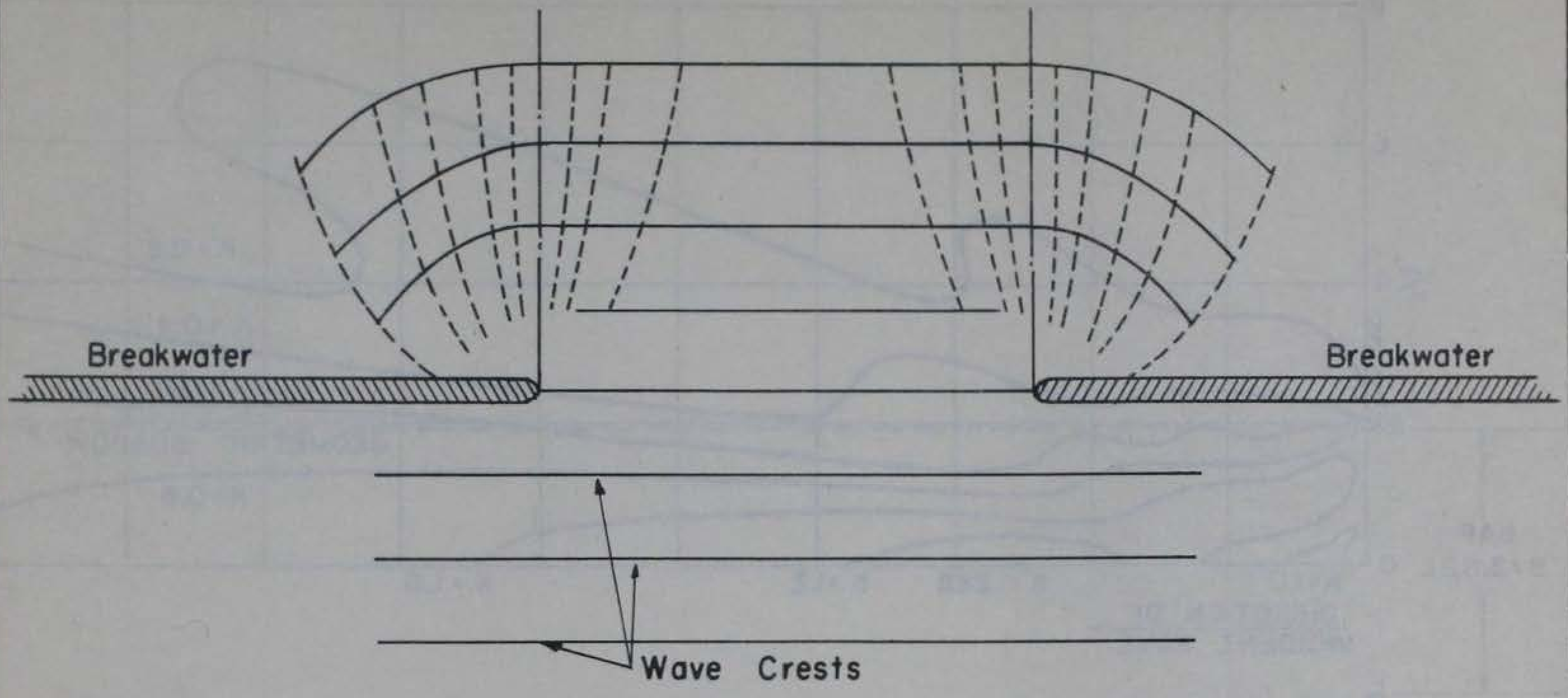


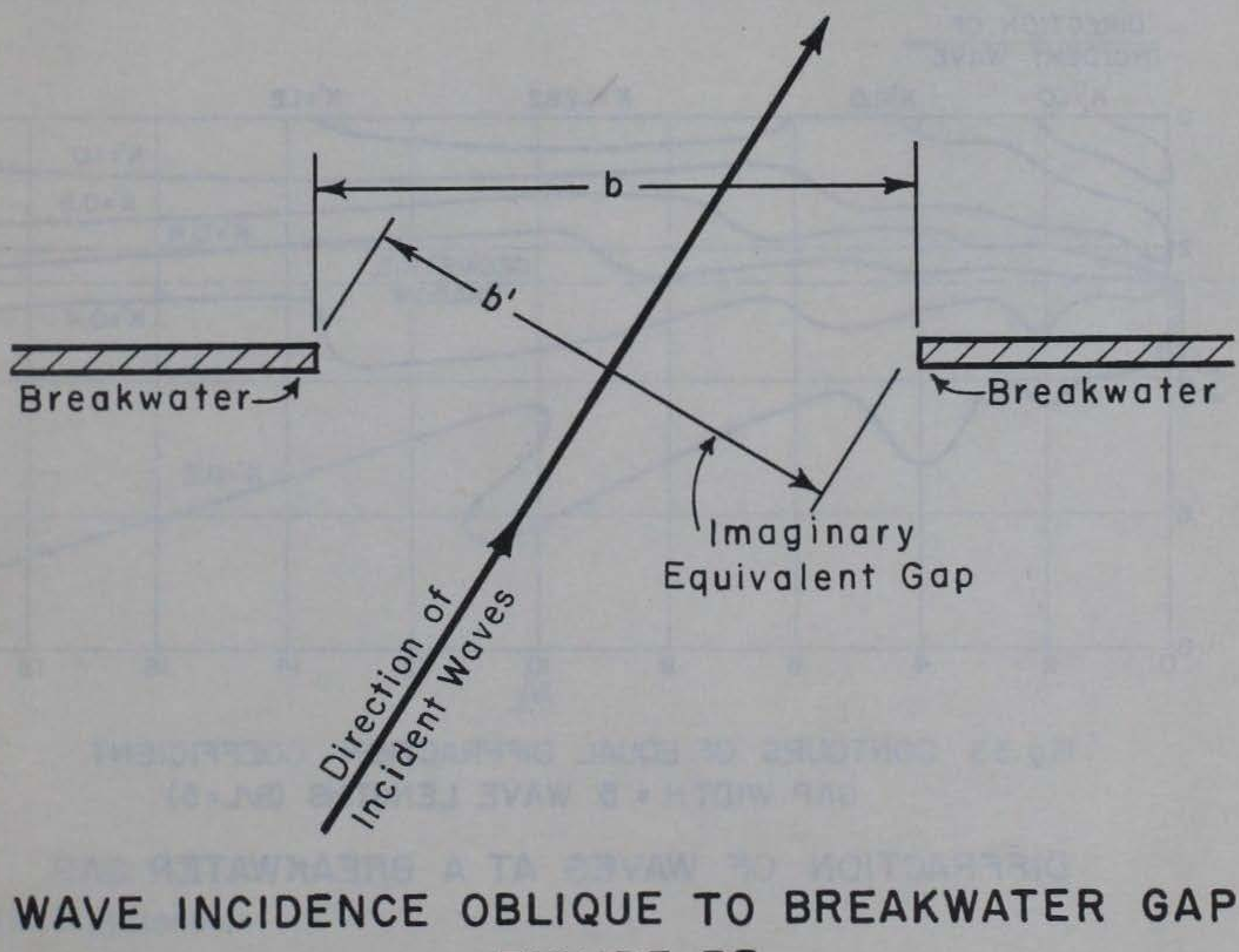
Fig. 33 CONTOURS OF EQUAL DIFFRACTION COEFFICIENT
GAP WIDTH = 5 WAVE LENGTHS (B/L=5)

DIFFRACTION OF WAVES AT A BREAKWATER GAP

(Johnson, 1952)

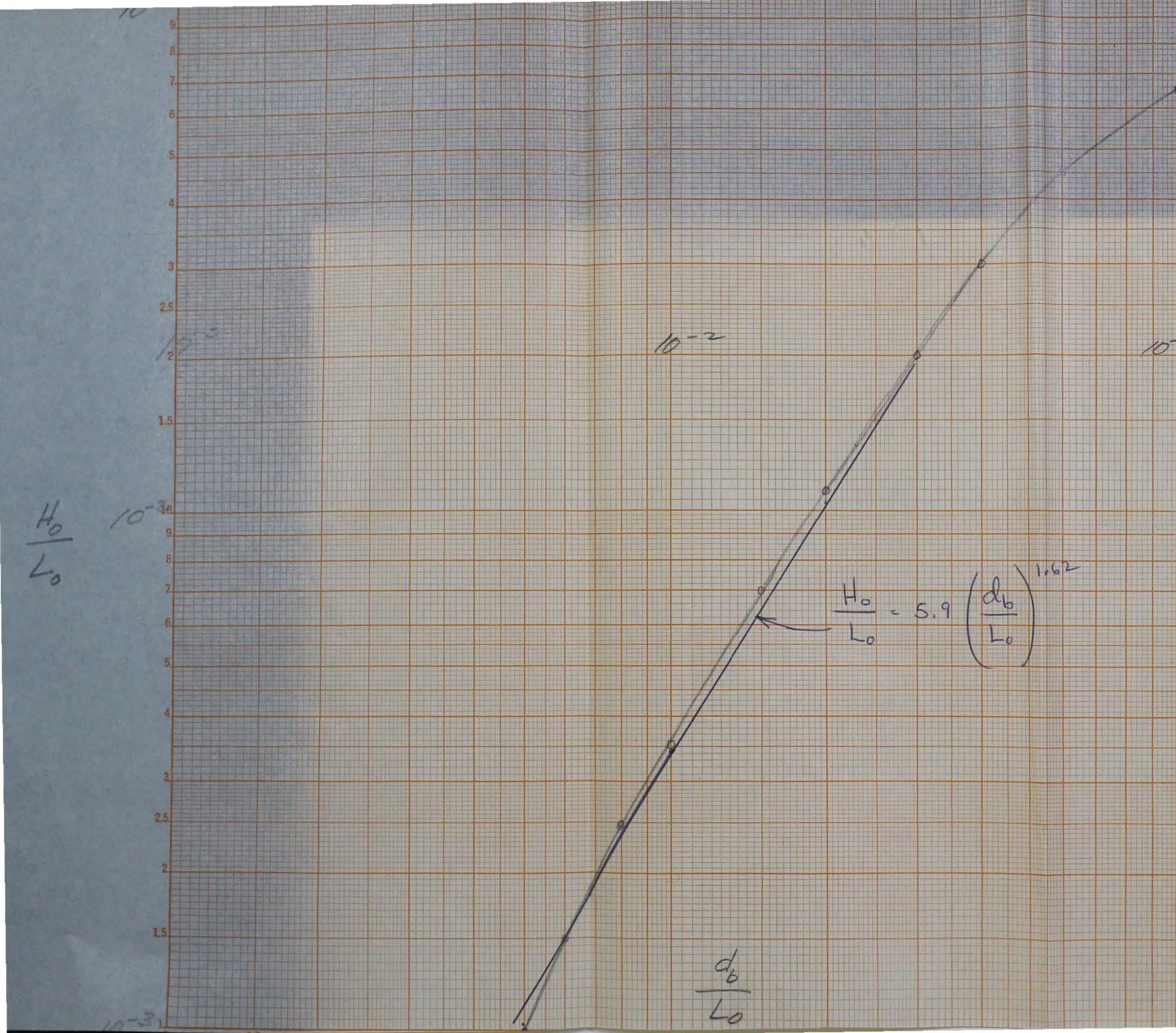


DIFFRACTION FOR A BREAKWATER GAP OF WIDTH >5L (B/L >5)
FIGURE 34



WAVE INCIDENCE OBLIQUE TO BREAKWATER GAP
FIGURE 35

(Johnson, 1952)



This curve was taken from "La Houille Blanche", July - August, 1956, Page 417.

H_o/L_o vs $\frac{d_b}{L_o}$

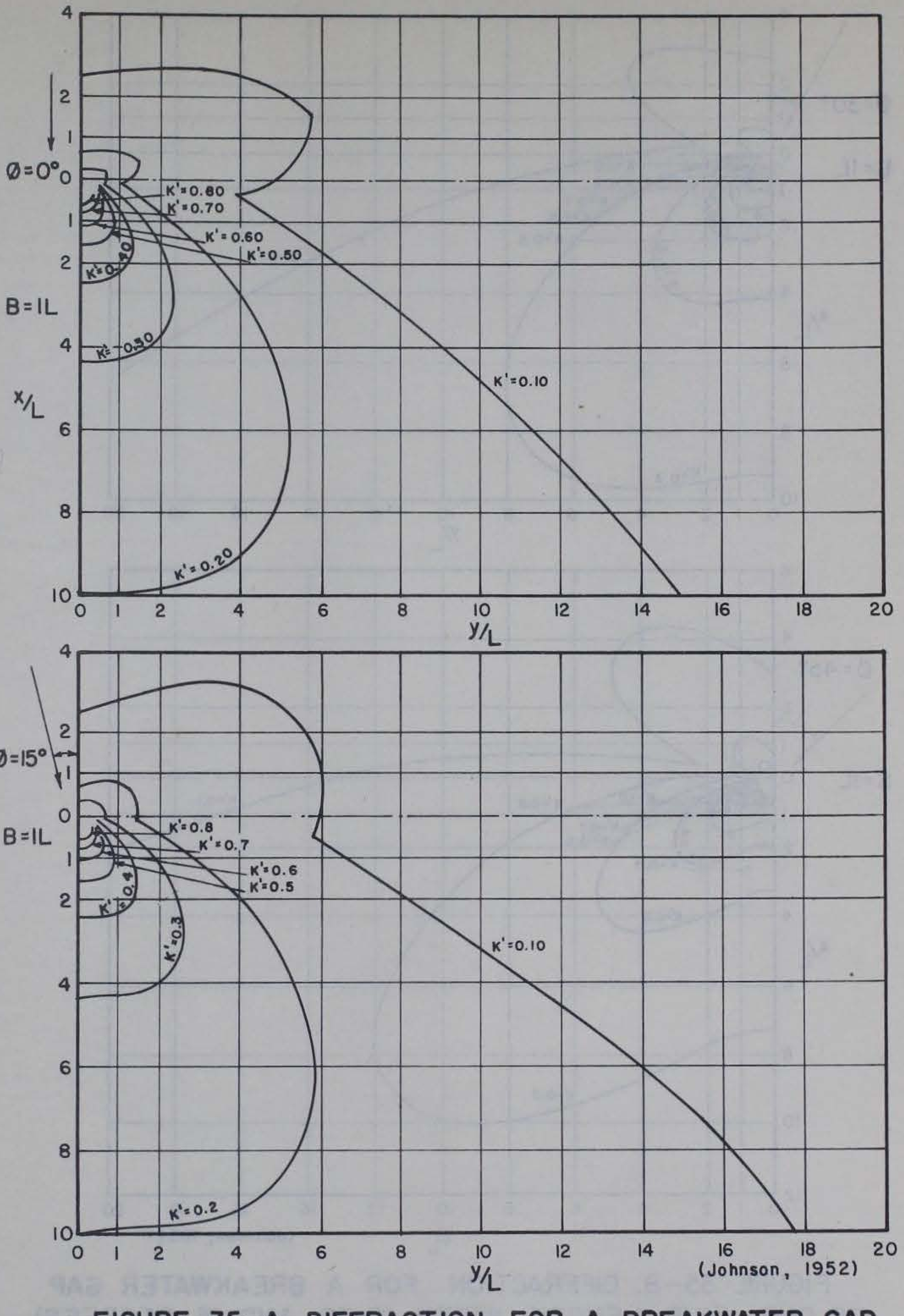


FIGURE 35-A. DIFFRACTION FOR A BREAKWATER GAP OF ONE WAVE LENGTH WIDTH (Ø=0 AND 15 DEGREES).

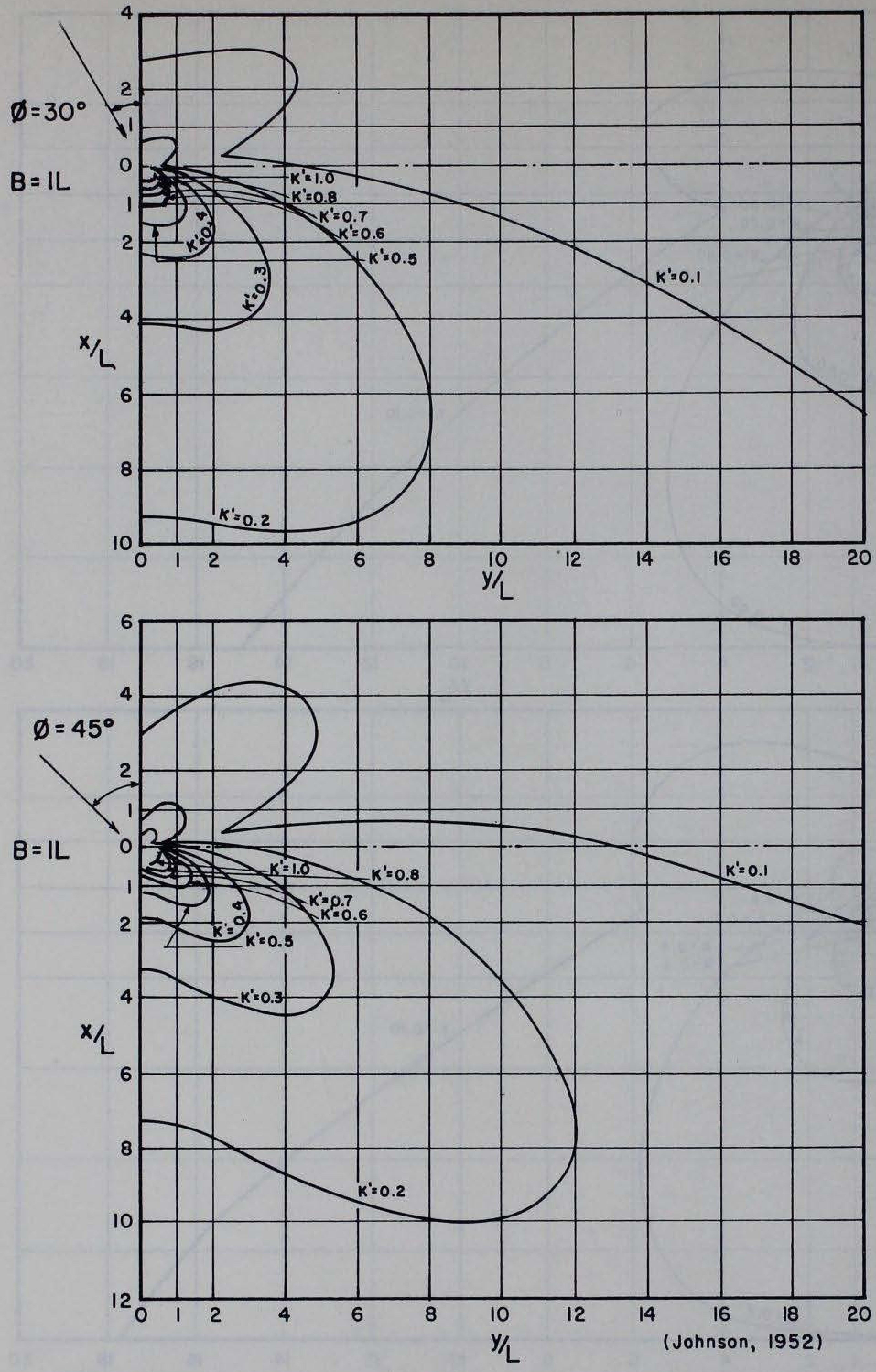


FIGURE 35-B. DIFFRACTION FOR A BREAKWATER GAP OF ONE WAVE LENGTH WIDTH (Ø=30 AND 45 DEGREES).

February 1957

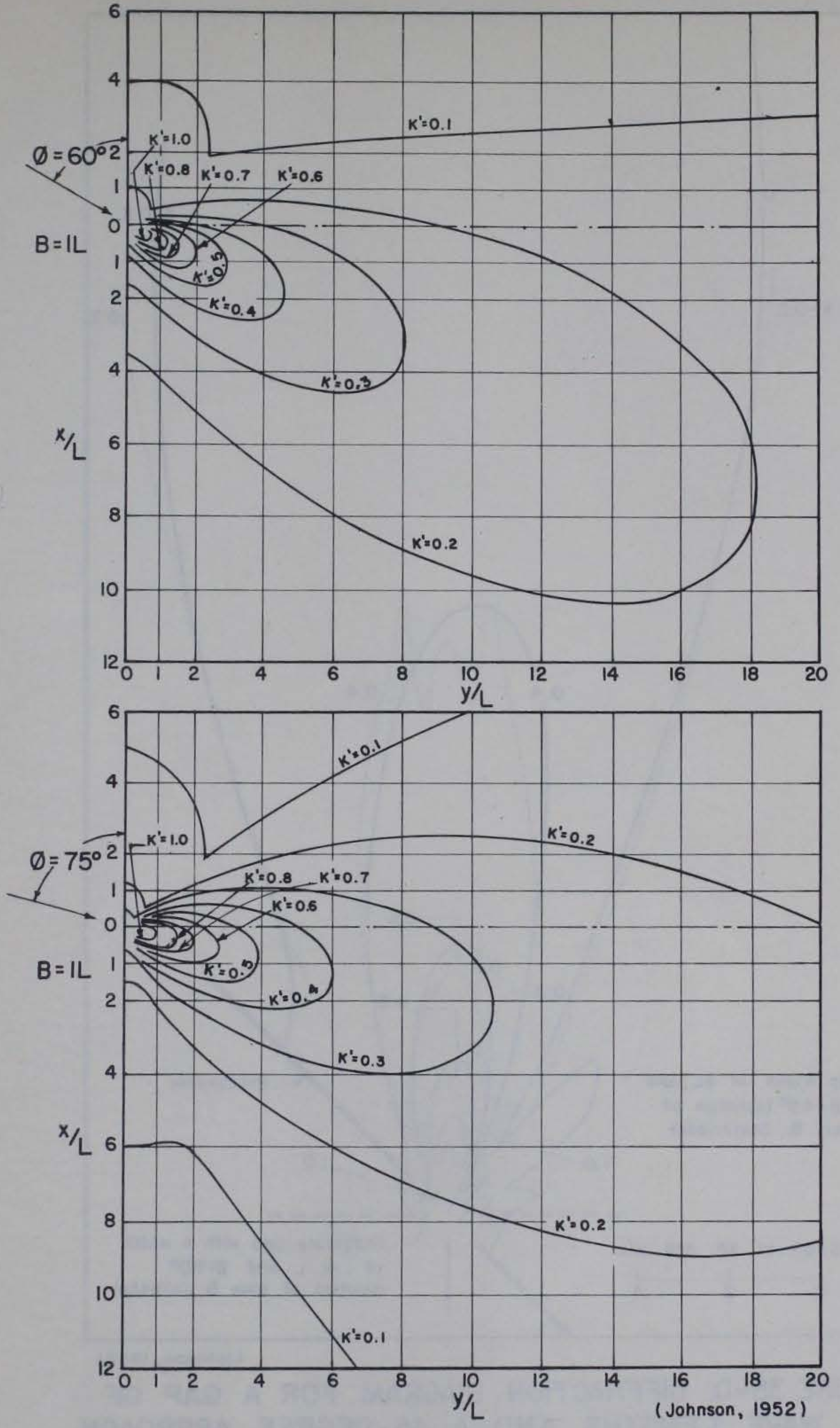
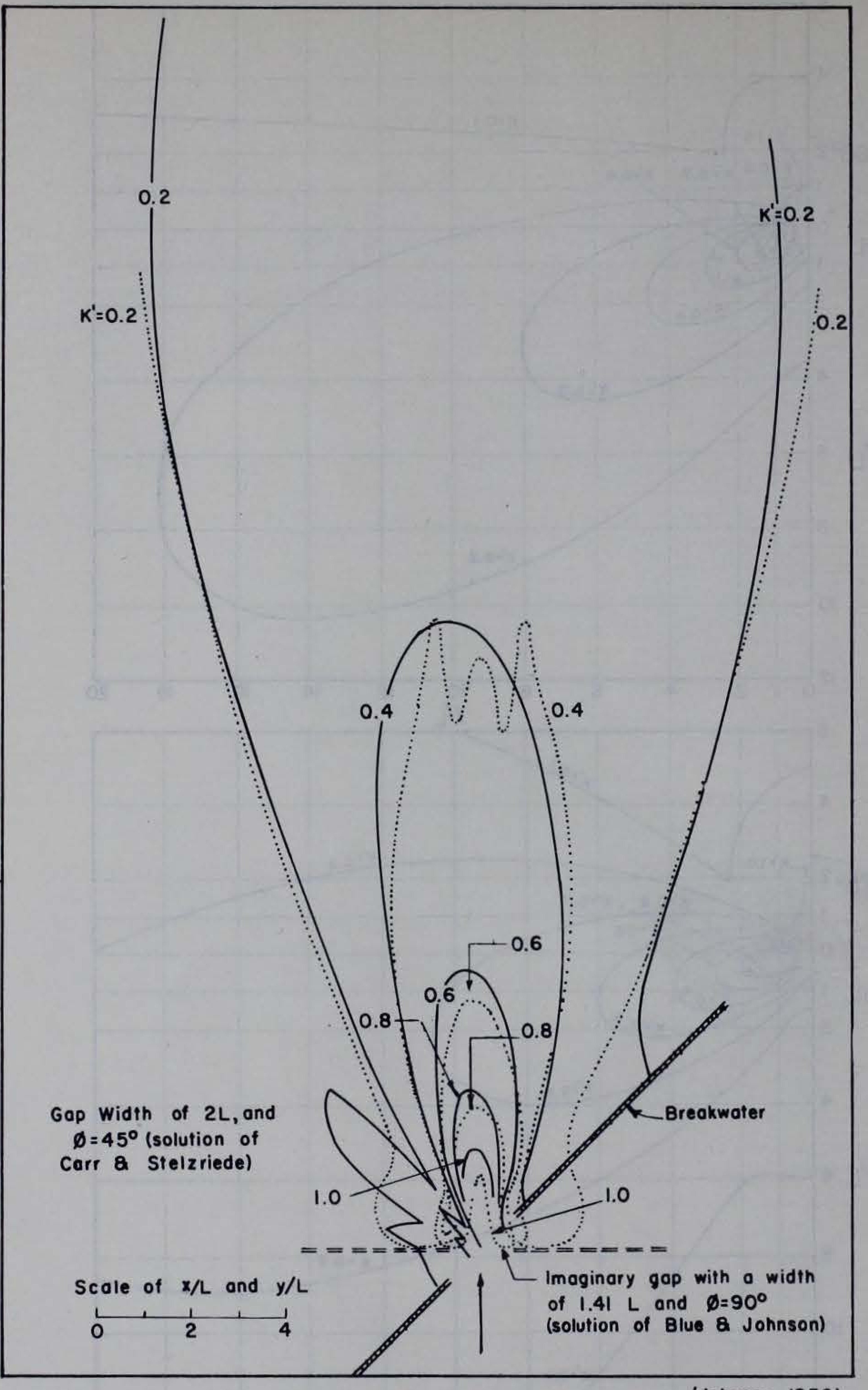
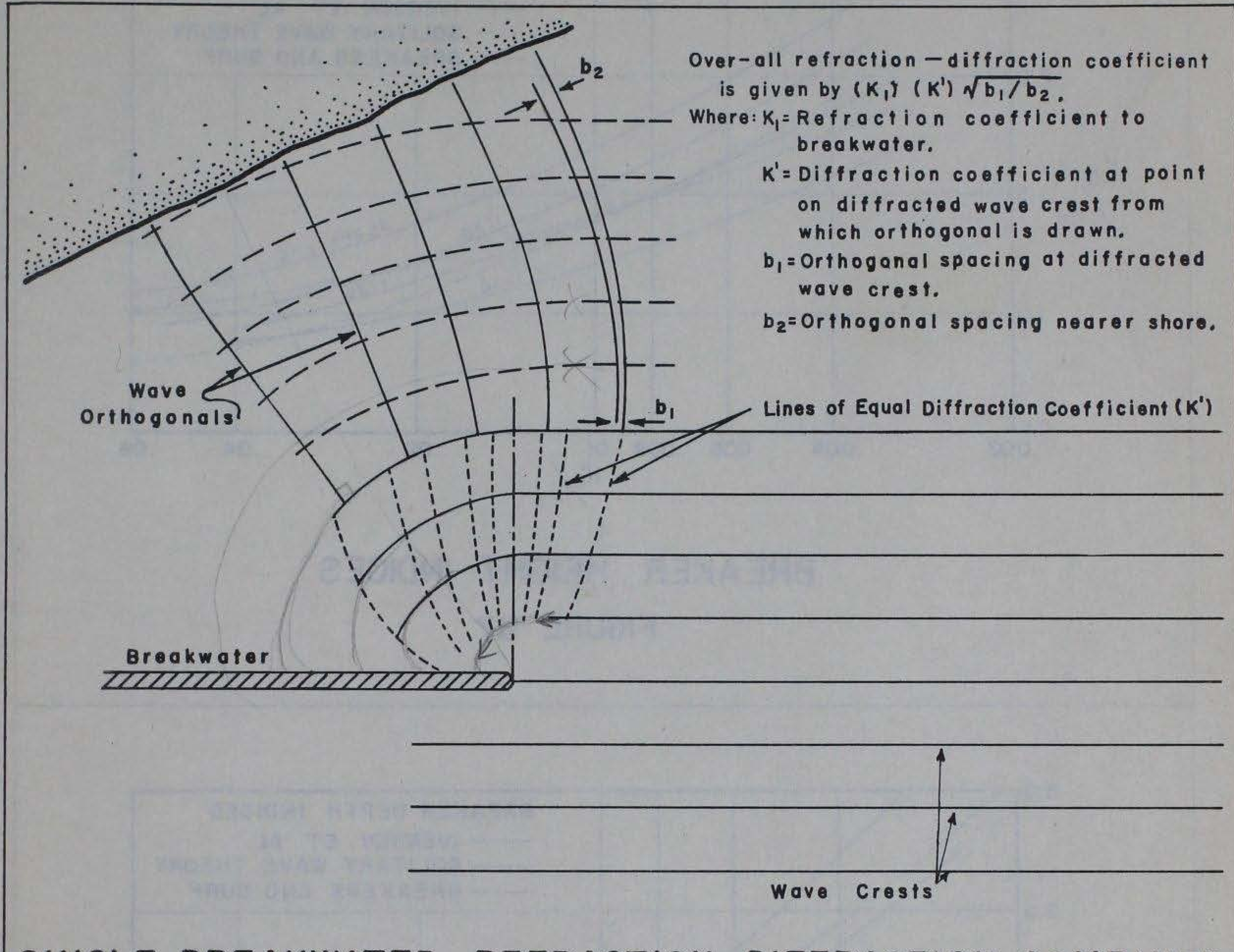


FIGURE 35–C. DIFFRACTION FOR A BREAKWATER GAP OF ONE WAVE LENGTH WIDTH (Ø=60 AND 75 DEGREES).

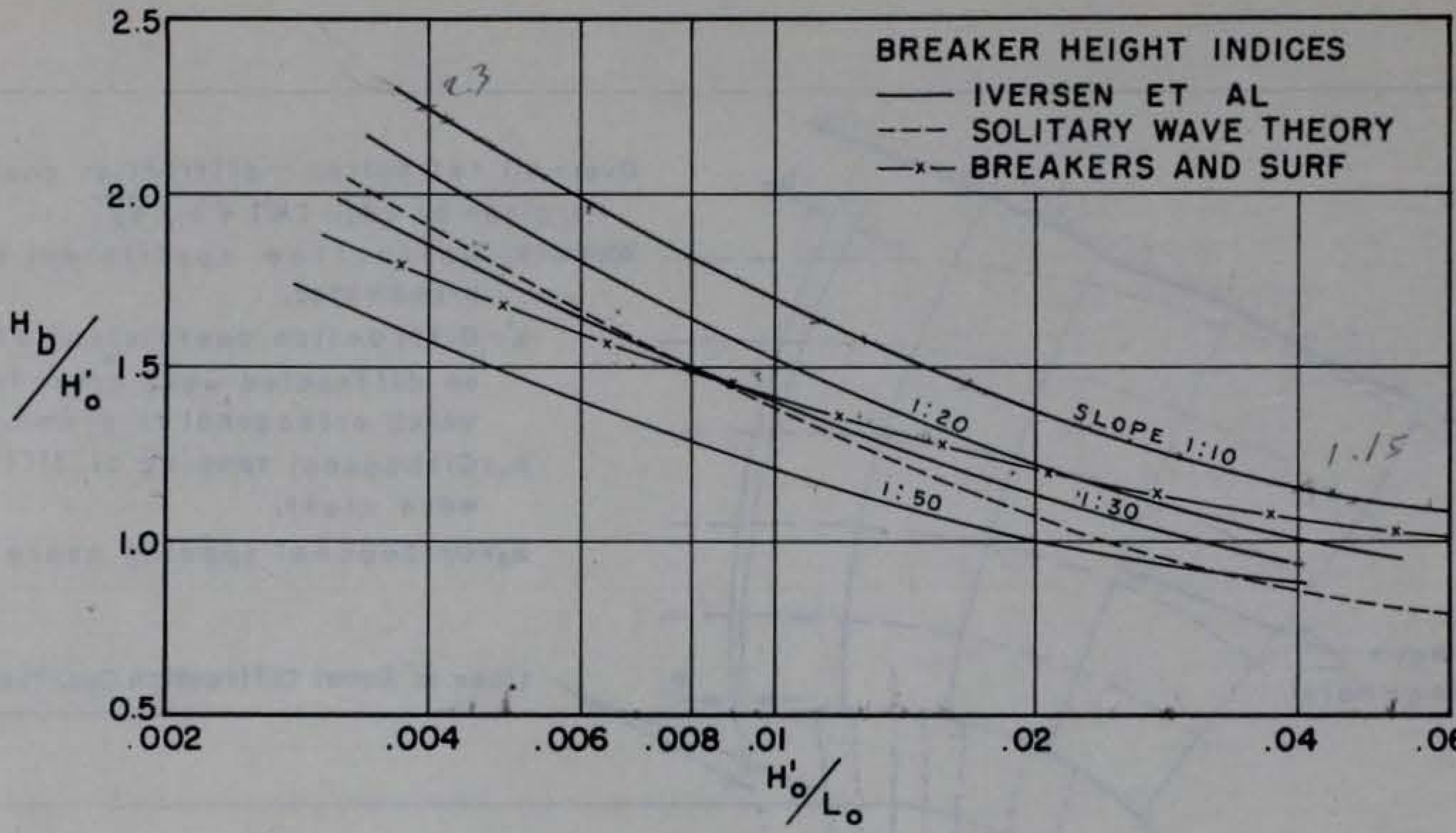


(Johnson, 1952)

FIGURE 35-D. DIFFRACTION DIAGRAM FOR A GAP OF TWO WAVE LENGTHS AND A 45-DEGREE APPROACH COMPARED WITH THAT FOR A GAP WIDTH $\sqrt{2}$ WAVE LENGTHS WITH A 90-DEGREE APPROACH.

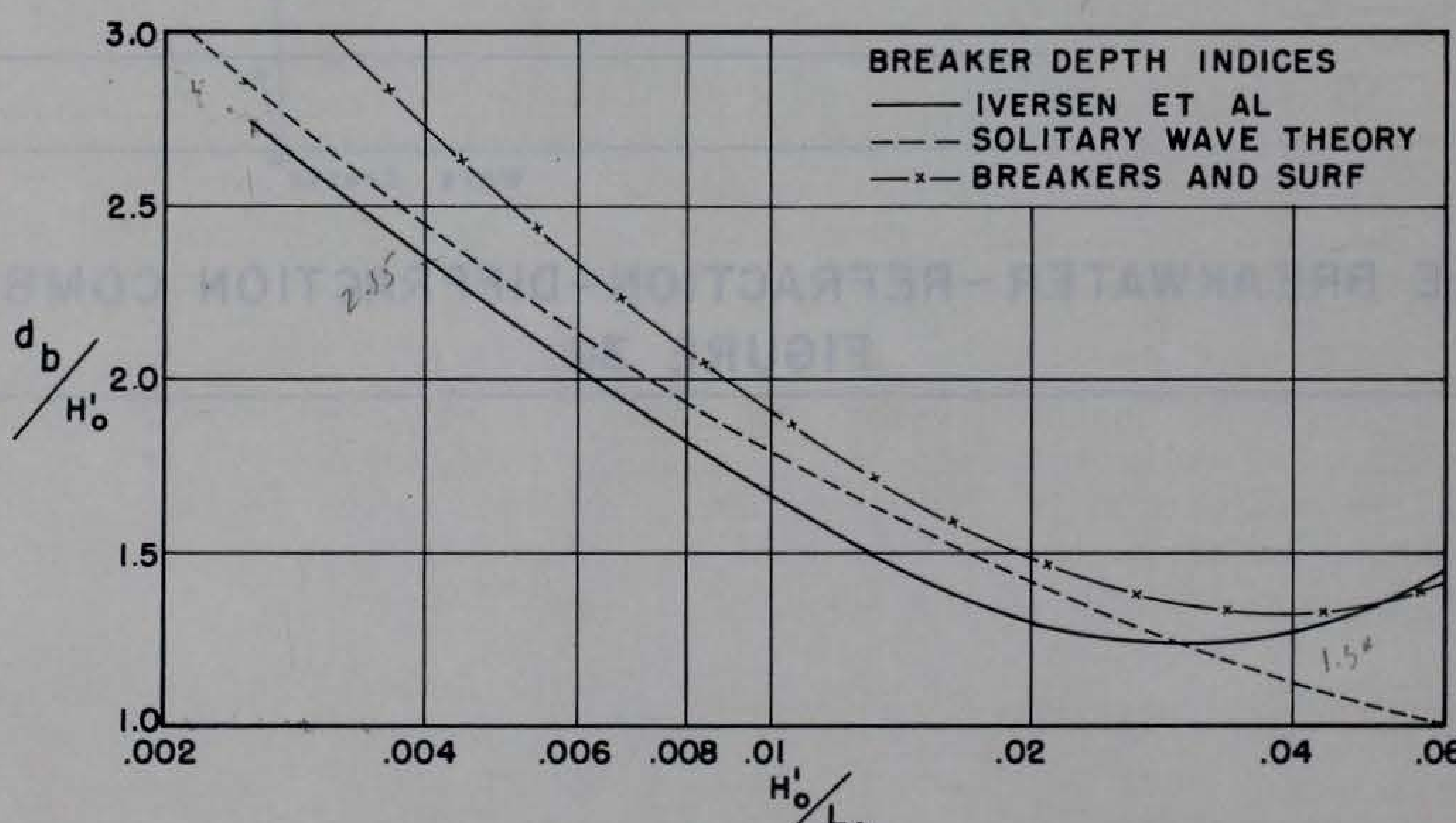


SINGLE BREAKWATER – REFRACTION – DIFFRACTION COMBINED
FIGURE 36



BREAKER HEIGHT INDICES

FIGURE 37



BREAKER ~~HEIGHT~~ DEPTH INDICES

FIGURE 38

wave length), by referring to Table D-1, Appendix D for the corresponding value of d/L and dividing, d, by this ratio. The diagram itself must then be scaled up or down so that the distance from $y/L = 0$, to $y/L = 1$ corresponds to one wave length on the scale of the chart on which the diagram is to be drawn.

The diffraction diagram is then placed so that the $x/L = 0$ ordinate lies in the direction of wave approach, with x/L positive values on the sides of the breakwater toward the protected area in the breakwater's lee. The lines labelled "wave crests" then represent positions of successive diffracted wave crests from the breakwater. The lines labelled K' are lines of equal decreased wave height. For example, along the $K' = 0.20$ line wave heights are two-tenths of their values outside the breakwater.

(b) - Waves Passing A Gap of Width Less Than Five Wave Lengths. - The solution for this problem (Appendix E) is more complex, and it is not possible to construct one diagram for all conditions. A new diagram must be drawn for each different ratio of gap width to wave length. One for a gap width to wave length ratio of 2 (see Appendix E) is shown in Figure 23, which figure also illustrates its use. Figures 24 through 33(65) show lines of equal diffraction coefficient for gap width (B) to wave length (L) ratios of B/L of 0.50, 1, 1.41, 1.64, 1.78, 2, 2.50, 2.95, 3.82 and 5 drawn for a somewhat more complex solution of the diffraction problem than that in Appendix E. In all but Figures 29, for $B/L = 2$, the wave crest lines have been omitted. Wave crest lines are usually only of pictorial use. One-half the diffraction coefficient lines have been eliminated from these figures but the diffraction coefficients are symmetrical about the $x/L = 0$ ordinate, thus the diagrams may be completed by folding the diagram about that ordinate.

(c) - Waves Passing A Gap of Width Greater Than Five Wave Lengths. - Where the breakwater gap width is greater than five wave lengths, the diffraction effects of each wing are essentially independent, and the diagram (Figure 21) for a single breakwater may be used to define the diffraction characteristics in the lee of both wings. (See Figure 34).

(d) - Diffraction at A Gap - Oblique Incidence. - When waves approach at an angle to the centerline of the breakwater, the diffracted wave characteristics differ from those resulting from wave approach in a direction normal to the centerline. Figures 35A, 35B and 35C show the diffraction diagrams for the angles 0, 15, 30, 45, 60 and 75°. Although by using these diagrams somewhat more accurate results may be obtained, an approximate appraisal of diffracted wave characteristics may be made by considering the gap to be as wide as its projection in the direction of incident wave travel as indicated in Figure 35. A comparison of a 45° incident wave using the approximate method and the more exact diagram is shown on Figure 35D.

1.263 Refraction and Diffraction Combined. In the usual case, the bottom seaward and shoreward of a breakwater is not flat, therefore refraction as well as diffraction occurs. Though a unified theory of the two has not yet been devised, an approximate picture of wave changes may be drawn by: (1) constructing a refraction diagram to the breakwater, (2) at this point, constructing a diffraction diagram carrying successive crests 3 or 4 wave lengths shoreward, if possible, and (3) with the wave crest and wave direction indicated by the last shoreward wave crest determined from the diffraction diagram, constructing a new refraction diagram to the breaker line. A typical refraction-diffraction diagram is shown on Figure 36. Note the method of determining combined refraction-diffraction coefficients.

1.264 Breaking Waves. - At a certain point in its advance toward shore a wave becomes unstable, peaks up, and breaks. The water motion changes from a laminar orbital motion to turbulent, white-water conditions. The determination of point of breaking and breaking wave heights is of major importance in the planning and designing of shore protection measures.

Three different sets of curves are currently used to determine breaking depths and wave heights on breaking; one theoretical, and two empirical.(61)(93) The theorectical curve is derived from the analysis of a so-called solitary wave(93) and results in the following equations:

$$\frac{H_b}{H_o'} = \frac{1}{3.3\sqrt[3]{H_o'/L_o}} \tag{15}$$

$$d_b/H_b = 1.28 \tag{16}$$

$$d_b/H_o' = \frac{1.28}{3.3\sqrt[3]{H_o'/L_o}} \tag{17}$$

Graphs drawn from these relations as well as those from the two other studies(60) are presented in Figure 37 and 38. All three curve sets relate the breaker depths d_b and the breaker wave height H_b to deep water wave length L_o and deep water wave height, H_o', which would exist if refraction were ignored. (Knowing the deep water wave height H_o and the refraction coefficient K, H_o' may be determined from $H_o' = K H_o$; i.e., $H = K (H/H_o') H_o = (H/H_o') H_o'$; see section 1.261. The solid line curves include the additional parameter of bottom slope, the effect of which was adequately verified under controlled laboratory conditions.

To use the curves on Figures 37 and 38, H_o' is determined as above, L_o from the known relationship L_o (in feet) $= 5.12T^2$, and the beach slope from hydrographic charts. By computing the ratio H_o'/L_o the ratios H_b/H_o' and d_b/H_o' may be picked off the graphs and, from these, H_b and d_b determined by multipication by H_o'.

1.3 CHANGES IN WATER LEVEL

1.31 TIDES - The tide is the alternate rising and falling of the level of the sea caused by the attractive forces of the sun and moon on the rotating earth. There are usually two high and two low waters in a tidal or lunar day. Tides follow the moon more closely than they do the sun. As the lunar day is about 50 minutes longer tha the solar day, the tides occur on the average 50 minutes later each day. Because of the varying effects of the sun and moon, a diurnal inequality in tides occurs in which, at certain places, there may be little, if any difference between one high water and the succeeding low water of a day but a marked difference in height between the other high water and its succeeding low water. Along the Atlantic coast the two tides each day are of nearly the same height. On the Gulf coast the tides are low but in some instances have a pronounced diurnal inequality. Pacific coast tides compare in height with those on the Atlantic coast but have a decided diurnal inequality (see Appendix A, Figure A-10).

Pertinent data concerning tidal ranges along the sea coasts of the United States are given in the following tables. Spring ranges are shown for areas having approximately equal daily tides and diurnal ranges are shown for areas having a pronounced inequality. Detailed data concerning tidal ranges are given in Tide Tables, U. S. Department of Commerce, Coast and Geodetic Survey. (21)(22).

TABLE 3

TIDAL RANGES

Localities		Approximates Ranges (feet)		
From	To	Mean		Spring
	ATLANTIC OCEAN			
Calais, Maine	W. Quoddy Head	16-19		18-22
W. Quoddy Head	Englishman Bay	12-16		14-18
Englishman Bay	Belfast	10-12		11-14
Belfast, Maine	Provincetown, Mass.	9-10		10-11
Provincetown	Chatham	7-9		8-11
Chatham	Cuttyhunk	3-7		4-8
Cuttyhunk	Saybrook, Conn.	3.5		4.2
Saybrook, Conn.	East River, N.Y.	3.5 - 7		4-9
Montauk Pt., N.Y.	Sandy Hook, N.J.	2-5		2-6
Sandy Hook	Cape May, N.J.	4-5		5-6
Cape May, N. J.	Cape Henry, Va.	3-4		3-5
Cape Henry, Va.	Charleston, S.C.	3-5		3-6
Charleston, S.C.	Savannah, Ga.	5-7		6-8
Savannah, Ga.	Mayport, Fla.	5-7		5-8
Mayport, Fla.	Key West, Fla.	1-5		2-5
	GULF OF MEXICO			
Key West, Fla.	Apalachicola, Fla.	1		2
Apalachicola, Fla.	Atchafalaya Bay, La.		1-2*	
Atchafalaya Bay	Port Isabel, Texas	1		1-2
	PACIFIC OCEAN			
Point Loma, Calif.	Cape Mendocino, Calif.	4		5-6
Cape Mendocino	Siuslaw River, Ore.	4-5		6-7
Siuslaw River	Columbia River	5-6		7-8
Columbia River	Port Townsend, Wash.	5-6		8
Puget Sound, Wash.		7-11		10-14

* Diurnal range

1.32 WIND SET-UP - In most coastal locations the wind may induce a surface current in the general direction of the wind movement, thus causing an increase or decrease in water level above or below that due to tidal action. This results from tangential stresses at the water surface between wind and water and, to a lesser degree, from differences in atmospheric pressure over the water surface. The wind-induced surface current produces a piling up of water at the leeward side and a lowering of water level at the windward side with a return flow along the bottom. This deviation from still water level caused by the wind-driven currents is called wind set-up (or, frequently, wind or storm tide, although it has no connection with primary tidal causes). An example of such set-up is shown for Lake Erie for east and west winds of 10 meters per second in Figure A-11, Appendix A. This effect may be significantly higher than normal tidal action in coastal areas; for example, in Galveston, Texas, where the normal tidal range is about 2 feet, hurricanes caused water levels up to 15 feet in 1900 and to 12.5 feet in 1915. The amount of set-up depends on the wind velocity, the distance over which it blows, and the depth, being greater for lesser depths. This dependence on depth is the reason for the generally greater values of set-up observed along the Gulf coast than on the Atlantic, and along the Atlantic as compared to the Pacific. Set-up may also be increased, particularly in coastal areas, by a funneling effect in converging open mouth bays. There is at present no known method of predicting accurately wind set-up in a coastal area other than by use of frequency charts obtained from a statistical analysis of water level data. Accumulation of much data over many years in some areas (as the North Sea) has led to relatively accurate empirical methods of prediction for specified points - but these formulae are not applicable to places other than the specific points for which they were derived.

The prediction of set-up in landlocked areas presents a somewhat easier problem. Theoretical studies and field and laboratory data have led to the general acceptance of the formula $S = 1.16 \times 10^{-3} (V^2F/D)N \cos \theta$ where: S is the difference in water surface elevations at the windward and leeward sides of the lake in feet,

- V is the wind velocity at an elevation of about 20 feet in miles per hour,
- F is the fetch, or distance the wind blows over the water in miles,
- D is the average depth of the lake in feet ,
- θ is the angle between the wind and the fetch, and
- N is a coefficient dependent on the slope and hydrography of the lake.

Methods for determining N are given by Keulegan(73), but for most areas it may be assumed to be 1. The rise in water level at the leeward end of the lake is usually of more importance than the full set-up between the two ends of the lake. Creager, Justin, and Hinds (36) have shown that the ratio of the rise at the leeward end to the total set-up averages about 0.571. A more precise method of determination of this value has been shown by Saville(112), but in general the departure from the average value does not warrant the additional work involved.

1.33 SEICHES(32)- Seiches are standing waves of relatively long period which occur in lakes, canals, bays and along open sea coasts. Originally the word seiche (pronounced sāsh) designated long-period, free oscillations in lakes. However, common usage now applies this descriptive term to free oscillations of all relatively small bodies of water. The mechanics of seiche generation are not completely understood, although all available evidence proves rather conclusively that lake seiches are the result of a sudden change, or a series of intermittent-periodic changes, in atmospheric pressure and similar changes in wind velocity. Standing waves in canals can be initiated by suddenly adding or substracting appreciable quantities of water. Seiches in bays can be generated by local changes in atmospheric pressure and wind, the same as lake seiches, and by oscillations transmitted through the mouth of the bay from the open sea. Open-sea seiches can be caused by changes in atmospheric pressure and wind, earthquakes and submarine landslides. Standing waves of large amplitude are likely to be generated if the causative force which sets the water basin in motion is periodic in character, especially if the period of this force is the same as, or is in resonance with, the natural or free oscillating period of the basin. Free oscillations have periods which are dependent upon the horizontal and vertical dimensions of the basin, the number of nodes of the standing wave, and friction. Friction can usually be neglected unless the basin is very long and shallow. The period of a true forced-wave oscillation is the same as the period of the causative force. Forced oscillations, however, are usually generated by intermittent external forces and, in this case, the period of the oscillation is determined partly by the period of the external force and partly by the dimensions of the water basin and the mode of oscillation. Oscillations of this type have been called forced seiches(18) to distinguish them from pure seiches in which the oscillations are free. In a closed basin, wave loops must be situated either at the ends (longitudinal seiche) or the sides (transverse seiche).

A closed rectangular basin with length L, depth d, and n the number of nodes, has a natural free oscillating period;

$$T_n = \frac{2L}{n\sqrt{gd}} \qquad (18)$$

The fundamental and maximum period, when n = 1, becomes:

$$T = \frac{2L}{\sqrt{gd}} \qquad (19)$$

Equation 19 is called Merian's formula(123). In a rectangular bay the simplest form of standing wave is one with a node at the opening and the loop at the closed end of the bay. The period of the free oscillation in this case is:

$$T = \frac{4L}{\sqrt{gd}} \qquad (20)$$

(the total length of the bay is occupied by only one-fourth of a wave length.)

Considerable modification is necessary in applying the above simple theory to lake or bay oscillations because of the variations in width and depth along the axes of actual basins. The theory of free oscillations in basins of various particular shapes, has been developed by many workers(18)(19)(48)(52)(54). Defant(37) developed a convenient but rather laborious method of determining the possible periods of free oscillation in lakes of any shape. Defant's method is considered the most useful in engineering work because it permits the computation of periods of oscillation, relative magnitudes of the vertical displacements along chosen axes, and the positions of nodal lines and loops. This method, which is appreciable only to free oscillations, can be used to determine the modes of oscillation of binodal as well as uninodal seiches. The theory for a particular case of forced oscillations was also derived by Defant(37) and is discussed by Sverdrup(123).

1.34 LAKE LEVELS - The Great Lakes have only insignificant tidal variations, but are subject to seasonal and annual changes in water level and to changes in water level caused by wind set-up, barometric pressure variations, and by seiches. The average or normal elevations of the lakes' surfaces vary irregularly from year to year. During the course of each year the surfaces are subject to consistent seasonal rises and falls, reaching their lowest stages during the winter months and attaining their maximum stages during the summer months. Hydrographs of monthly lake levels from the year 1860 to the present are shown in Figure 39(134). Table 4 summarizes certain lake level data.

In addition to the seasonal and annual fluctuations, the lakes are subject to occasional seiches of irregular amount and duration. Sometimes these result from variations in barometric pressure, which may produce changes in water surface elevation ranging from a few inches to several feet.(75) At other times the lakes are affected by wind set-up which raises the level at one end and lowers it at the other end of a lake.(51).

In general, the maximum amounts of these irregular changes in lake level must be determined for each location under consideration. Some idea of the extent of fluctuations which may be expected is given by the following. On Lake Superior a barometric storm in June 1939 caused a surface oscillation at Marquette with a maximum range in surface elevation of 7.4 feet. The largest fluctuations of any of the lakes occur on Lake Erie because of its shallow depth (see Figure A-11, Appendix A). The largest fluctuations on this lake occur at Sandusky, Toledo, and the mouth of Detroit River at the western end of the lake, and at Buffalo Harbor at the eastern end of the lake. At Buffalo Harbor the extreme range is 14.1 feet, the highest level on record being 9.9 feet above low water datum on 7 December 1909 and the lowest being 4.2 feet below that datum on 30 January 1939. The greatest range for any one year was 12.2 feet in 1909, with a high stage of 9.9 feet and a low stage of minus 2.3 feet. The least range for any one year was 5.3 feet above and 0.64 feet below datum. The foregoing are based on U. S. Lake Survey records for the past 50 years.

TABLE 4 - FLUCTUATIONS IN WATER LEVELS - THE GREAT LAKES
PERIOD 1860 - 1952 INCLUSIVE

	Low Water* Datum	Mean Surface Elevation	Change in Stage Between Monthly Means		Difference Between Alltime Monthly Mean		
			Maximum rise	Maximum fall	High	Low	Difference
Lake Superior	601.60	602.22	+0.81	-0.92	604.05	599.98	4.07
Lake Michigan-Huron	578.50	580.55	+0.70	-0.69	583.68	577.35	6.33
Lake Erie	570.50	572.33	+1.57	-0.73	574.70	569.43	5.27
Lake Ontario	244.00	246.03	+1.90	-0.95	249.29	242.68	6.61

Elevations are in feet above mean tide at New York, 1935 datum.

*Low water datum is the plane to which depths in Federal navigation improvements and on Lake Survey charts are referred.

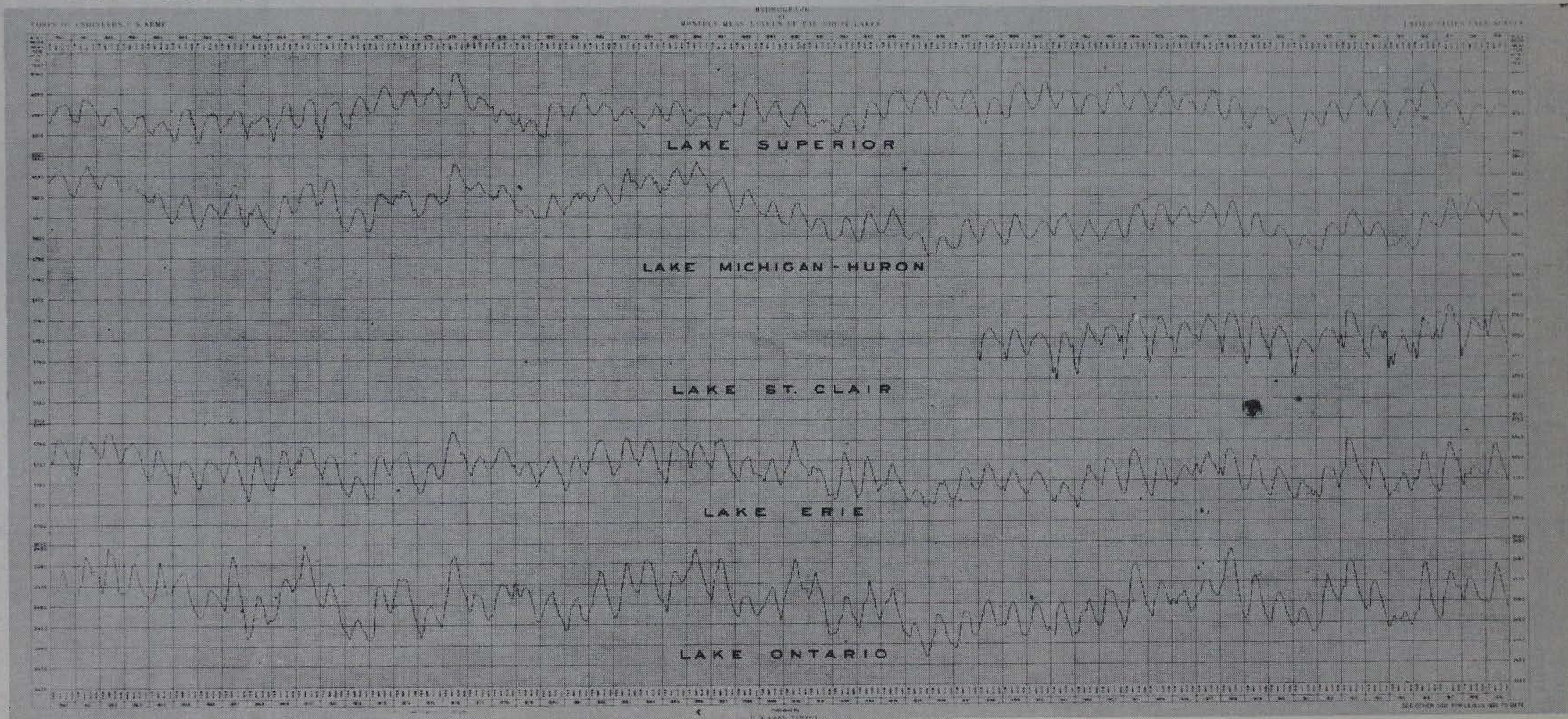


Fig. 39a

Hydrograph of Monthly Mean Levels of the Great Lakes

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CORPS OF ENGINEERS, U. S. ARMY

HYDROGRAPH
OF
MONTHLY MEAN LEVELS OF THE GREAT LAKES

UNITED STATES LAKE SURVEY

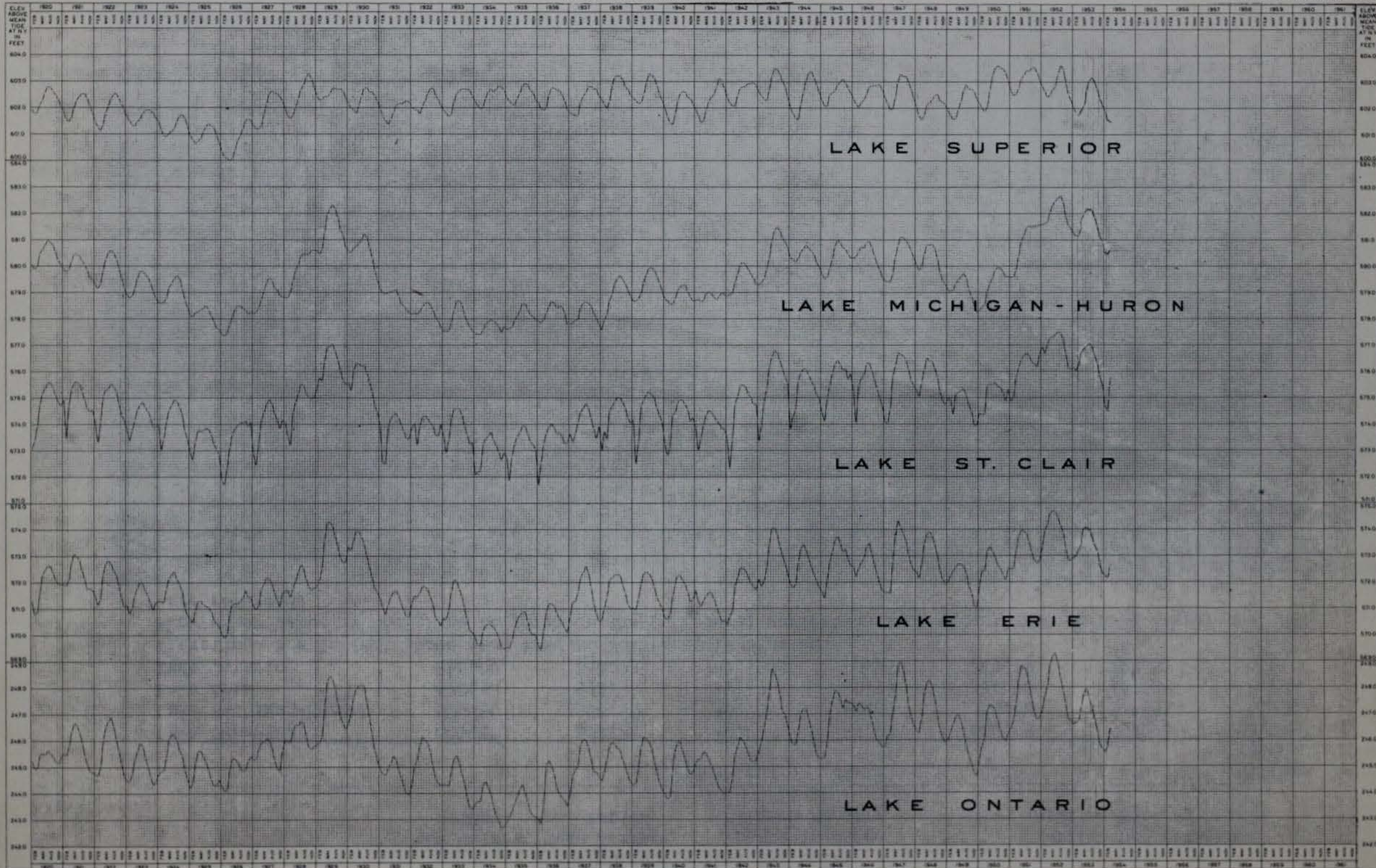

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Fig. 39b

1.4 CHARACTERISTICS OF BEACH MATERIALS

The beach, the nearshore and in some cases the offshore area to considerable depth is included in the littoral zone. The composition of the material therein may be defined by analyzing samples of surface material. Customary procedure is to obtain samples of the top 2 inches of material at the mean water line and along the profile at 5-foot or 1 fathom depth intervals seaward as far as regular sorting pattern along the profile is indicated. Any sampling device that will secure a representative sample without loss of fines is suitable. For sandy material, 1/2 to 1-pint samples are usually adequate. Spacing of sampling profiles depends upon regularity of the shore and slopes, and on long straight beaches profiles quite widely spaced will usually provide adequate definition of the distribution of materials in the littoral zone.

The physical properties of the littoral material include size (usually in millimeters) of the individual grains, the shape and roundness of the grains, mineral composition, porosity, and permeability. Of these, size and mineral composition are the most important and generally will be the only properties examined in detail.

Since the size distribution is of primary concern, all samples should be subjected to mechanical analysis. Sieving or settling velocity methods are equally acceptable for this purpose. Uniform size classifications (based on Casagrande classification) approved by the Corps of Engineers, March, 1953 are given in the following table.

TABLE 5 - STANDARD SIZE CLASSIFICATIONS*

Name	Grain size limits (diameters)
Cobbles	Above 3 inches (above 76mm)
Coarse gravel	3/4 inch to 3 inches (19 mm to 76 mm)
Fine gravel	** No. 4 sieve to 3/4 inch (4.76 mm to 19mm)
Coarse sand	No. 10 sieve to No. 4 sieve (2.0 mm to 4.7 mm)
Medium sand	No. 40 sieve to No. 10 sieve (0.42mm to 2.0 mm)
Fine sand	No. 200 sieve to No. 40 sieve (0.074 mm to 0.42 mm)
Silt or clay	Below No. 200 sieve (below 0.074 mm)

* Corps of Engineers Uniform Soil Classification
** U. S. Standard Sieve Size

Preparatory to mechanical analysis, the samples are washed free of salt, dried and quartered down to about 50 grams for sieving, or to 5-7 grams if a settling velocity tube is used. After determining the relative percentages by weight of the grade sizes contained in the material, cumulative size distribution curves are constructed (76) from which the grain diameter (in millimeters) at the first quartile (Q_1), third quartile (Q_3), and the median are read. Twenty-five percent of the sample by weight has a grain diameter larger than the diameter of the first quartile and seventy-five percent larger than the diameter of the third quartile. The median is the point at which 50 percent of the material has a larger grain diameter and 50 percent smaller.

The three statistical parameters employed to express the characteristics of the size distribution are: the median diameter, the coefficient of sorting, and the skewness. All of these parameters are derived from the cumulative size distribution curves.

The median diameter (M_d) is the mid-point of the grain size diameters in millimeters contained in the sample, and is read directly from the 50 percent figure on the cumulative size distribution curve. Fifty percent of the total weight of the sample is composed of particles with a diameter greater than, and fifty percent is smaller than the median diameter.

The coefficient of sorting (S_o) is a measure of the spread in grade sizes represented in the sample of the littoral material. It is determined by the formula $S_o = \sqrt{Q_1/Q_3}$. If there is perfect sorting the value of S_o would be unity. A value of 1.25 is indicative of good sorting in the beach material, and 1.45 for material from the nearshore bottom.

The skewness (S_k) is a measure of the degree of symmetry of the size distribution with respect to the median. It is derived from the formula $S_k = Q_1 \, Q_3/M_d^2$. If the value of the skewness is unity, the point of maximum sorting coincides with the median diameter, if the value is greater than unity the maximum sorting lies on the fine side of the median diameter, and if it is less than unity the maximum sorting lies on the coarse side of the median diameter.

The greater the value of the median diameter the coarser is the material, the larger the value for the coefficient of sorting the more poorly sorted is the material, and the more the value for skewness diverges from unity the more unsymmetrical is the size distribution curve. Small values for both S_o and S_k indicate the material is in adjustment with its environment. A large value for S_o and a small value for S_k indicate the material is spread through many grade sizes. A small value for S_o and a large value for S_k indicate that the material ranges through many grade sizes and that one set of environmental factors is dominant, though traces of others are still retained. Large values for both S_o and S_k indicate that the sediment is completely out of adjustment with its environment.

If the source of material is uncertain, petrographic analysis may provide evidence by comparison of mineral content on the beach and at possible sources. Samples should be about 250 cc. or about 1/2 pint in size. This size sample will permit both a sieve analysis and microscopic examination for mineral content.

The first step in the microscopic examination for mineral content is to separate the "heavy minerals", (minerals with a specific gravity greater than 2.85), from the quartz and feldspar. This is accomplished by panning, by use of heavy liquids, by use of electromagnets, or by some special method or device. The minor accessories, or so-called "heavy minerals", even though present in very small amounts (tenth of 1 percent or less) are the important elements in determining source sands. The next step involves the identification of minerals and determination of the frequency of their occurrence. In this the percentage of total heavy minerals in the sample is computed. The heavy minerals are then subdivided to a quantity sufficient for mounting on a microscopic slide. After identification, the frequencies are determined by actual count or estimation and recorded for comparative purposes.

The abundance of a mineral species can be expressed in percent by number or in percent by weight. The frequency in percent by number is obtained by counting the mineral grains in carefully spaced fields under the microscope. To determine the frequency by weight, the relative percentage of the mineral species in closely sized fractions and the weight of the total separate is first determined, and then the weight percentage is computed from the relative percentage. Corrections for size and specific gravity must be made to make the values obtained comparable with the percent by number. Mineral frequencies in the different grade sizes of a sample are not alike, hence, in the comparative study of many samples it is necessary to study the same grade size in all samples or to study the same relative size within the distribution curves.

The variations in the mineral composition of a suite of samples within an area are related to the physical conditions of deposition. The reasons for the variation in the frequency of a given mineral species may be due to contamination by the addition of locally derived species to the littoral drift; by selective breakage and abrasion, since all minerals are not equally resistant to these two processes tending to eliminate the mineral during transportation; and by selective sorting by littoral processes. An increase in the percent of the frequency of occurrence of a mineral does not mean an absolute increase but may mean only a relative increase which is the complement of the decrease of another mineral, since the total must always add up to 100 percent. The probable error in the value for the frequency of occurrences of a mineral species is greatest for the rare constituents and lowest for the abundant species.

Complete descriptions of laboratory procedures in making mechanical and petrographic analyses of sands are given by Krumbein and Pettijohn(76).

Table 6 lists the fifty most common detrital minerals of sands. The most common minerals in beach sands are capitalized. Light minerals are underlined; all others are the heavy minerals.

TABLE 6

DETRITAL MINERALS IN SANDS (Pettijohn, 100)

1.	Actinolite - tremolite	26.	HORNBLENDE
2.	Anatase	27.	HYPERSTHENE - ENSTATITE
3.	ANDALUSITE	28.	ILMENITE
4.	APATITE	29.	KYANITE
5.	AUGITE	30.	LEUCOXENE
6.	Barite	31.	Limonite
7.	Biotite	32.	MAGNETITE
8.	Brookite	33.	Monazite
9.	CALCITE	34.	Muscovite
10.	Cassiterite	35.	Olivine
11.	Chalcedony	36.	RUTILE
12.	Chloritoid	37.	Serpentine
13.	CHLORITE	38.	Siderite
14.	Clinozoisite	39.	Sillimanite
15.	Collophane	40.	Spinel
16.	Cordierite	41.	SPHENE
17.	Corundum	42.	STAUROLITE
18.	DIOPSIDE	43.	Topaz
19.	DOLOMITE	44.	TOURMALINE
20.	Dumortierite	45.	Vesuvianite
21.	EPIDOTE	46.	Xenotime
22.	Fluorite	47.	ZIRCON
23.	GARNET	48.	ZOISITE
24.	Glauconite	49.	FELDSPAR
25.	Hematite	50.	QUARTZ

CHAPTER 2

LITTORAL PROCESSES

2.1 GENERAL - The existence of an erosion problem is prima facie evidence that one of two conditions exist. The first is that the water level and the land have not become adjusted in terms of shore slope, as in the case of exceptionally high lake levels or storm tides, and upland material is being eroded to establish equilibrium slopes. The second and more common condition, which may exist concurrently with that cited above, is that material being removed from the area exceeds that being supplied. This material, designated "littoral drift" may be defined as the material moved in the littoral zone under the influence of waves and currents. The term is sometimes considered as the movement of the material as well as the material itself, however, in a strict sense the term littoral drift should be restricted in meaning to the material, and the term littoral transport used where movement is meant.

A stable shore line is one in which the supply of material to the area under consideration is approximately equal to losses of material from the area. On an accreting shore line the supply of material exceeds the losses, and the reverse is true of an eroding shore line. Accordingly, the need for protective works and the choice of type of protective works to be provided are dependent on the net balance between supply and loss of material.

Analysis of an erosion problem preparatory to functional planning of remedial measures requires that conclusions be reached from the most reliable data available concerning the nature of the littoral drift and littoral transport within the problem area. The information required can be placed in three basic categories:

a. Sources and characteristics of littoral materials

b. Modes and direction of littoral transport

c. Rates of supply and loss of material

It will not always be possible to reach well supported conclusions for these categories; nevertheless careful investigation will provide knowledge in place of conjecture in many cases.

2.2 SOURCES AND CHARACTERISTICS OF MATERIALS

2.21 GENERAL - The characteristics of materials found in littoral zones have been discussed in Chapter 1, Section 1.4. The three main natural sources of material to any beach segment are: (a) material moving into the area by natural littoral transport from adjacent beach areas; (b) contributions by streams; and (c) contributions through erosion of coastal formations, other than beaches, exposed to wave attack. In addition, there may occasionally be some long range net movement of material onshore apart from normal seasonal or other periodic fluctuations. The latter might occur, for example, with permanent or semipermanent changes in water level. Considering coasts as a whole, maintenance of beaches must be attained at the expense of erosion of the land mass. For any individual segment of beach, the largest source of material moving into the area is generally littoral drift eroded from the adjoining updrift segment unless some major sediment bearing stream enters the segment in question or cliff or dune erosion is sufficiently rapid to provide appreciable supply.

Caution should be exercised when determining the source, as the material on any one beach may be the product of several source areas or it may be obtained from only one of several possible sources. A study of the beach environment, the relative availability of material in the possible source area, the agents of erosion that are active, and the conditions favorable to transport material from the source areas to the beach site will generally indicate the source or sources of supply. Petrographic analysis of samples of the littoral material and of samples of possible source materials may establish a correlation between the mineral content of the littoral material and that of a source area. The correlation might be established through a similarity in the frequency of occurrence of a particular mineral or mineral suite, or by identifying a specific mineral or variety of a mineral unique to the littoral material and to the material from one of the source areas.

2.22 CONTRIBUTIONS BY STREAMS - The amounts of the various contributions to the littoral supply by sand carrying streams can be determined approximately by these general methods; (a) direct measurements, (b) studies of terrestrial sedimentation, and (c) computation of the sediment carrying capacity of the streams. To date, the only method upon which any great degree of reliance can be placed is that of direct measurements.

Direct measurements may be made with considerable accuracy under certain conditions. Delta measurements by successive hydrographic surveys are adequate to determine the amount contributed by those streams which carry sediment to the ocean or lake only during flash floods lasting a relatively short period of time. Similar comparative surveys are adequate to determine the amount contributed by streams which bring material to the shore continuously, or over an extended period of time, where those streams terminate in navigable channels or other natural settling basins. Some correction may be necessary to account for sediment deposited outside of the channels or basin area, for material moving out of the area between surveys by natural littoral transport, and for material removed from the area artificially during maintenance of navigation channels and basins.

Should investigation show that the principal sources of beach-building materials are the drainage basins tributary to the shore under consideration, a detailed study of the geology of these basins may be required if direct measurements are impracticable. Such a study should include data on hydrology, physiography, petrology, and sedimentology, and the sediment supply deduced from measured or estimated rates of terrestrial sedimentation.

For mountainous watersheads, the Forest Service has developed empirical methods for estimating the sedimentation rate(1). The determination was for a specific area of known geologic characteristics, but the results are applicable for other areas if corrections are made for several variables, i.e., vegetation, hydrology, rock type, etc.

Even if there are excellent data on terrestrial sedimentation rates, it may be very difficult to estimate how much material reaches the shore. The measurement of losses must be indirect. If the streams are degrading or appear to be at grade, it can be assumed that all source material ultimately reaches the shore. But if the streams are aggrading, deposition rates along the channel must be estimated and losses subtracted from the total sedimentation to determine the net sediment supply to the beaches.

The method generally employed in the computation of the sediment carrying capacity of a stream is the direct measurement of the wash-load of a river by suspended-load sampling(35). This method is expensive and time consuming, taking from 1 to 10 years of continuous observation to predict the wash-load of a river, depending on the regularity of its flows. A different method(40) (41) has been developed which is based on grain size of the bed materials. Its basic concept is that bed material always moves according to the capacity of the stream. The capacity rates for bed material may be computed by formulas which were developed to permit the prediction of the individual bed-load rates of the different bed components in terms of stream discharge. The solutions are laborious, but are not difficult to follow. However, the method would probably be used only if a determination of the stream carrying capacity was of great importance and could not be determined by direct measurements or historic records.

2.23 <u>CONTRIBUTIONS BY EROSION OF COASTAL FORMATIONS</u> - Eroding coastal formations are the last major source of beach material. Along the Great Lakes this is a major source, whereas, along much of the sea coast it is of comparative unimportance. As long as a beach berm is maintained between the formation and the action of the waves, the formation contributes negligibly to the littoral supply. At some locations, littoral transport has been interrupted by artificial barriers and the ocean has turned to the upland for its supply, causing serious recessions of the coast line. The amount of such contribution can only be estimated through comparative surveys, sub-division plot maps, property surveys, and statements of long-time residents of an area. The formations frequently contain much material too fine to remain on the beach. The proportion of beach material supplied out of total material eroded may be determined by mechanical analysis of a composite sample. Each stratum should be represented in proportion to its thickness. In the Great Lakes area, rises in lake level may allow waves to attack bluffs, which are generally of a very friable material. This causes recession of the shore line and contributes to the supply of beach material. Where erosion of coastal formations is important, a geological study may be required. The extent of field work and investigation will depend on the importance of erosion of coastal formations as a source of littoral material.

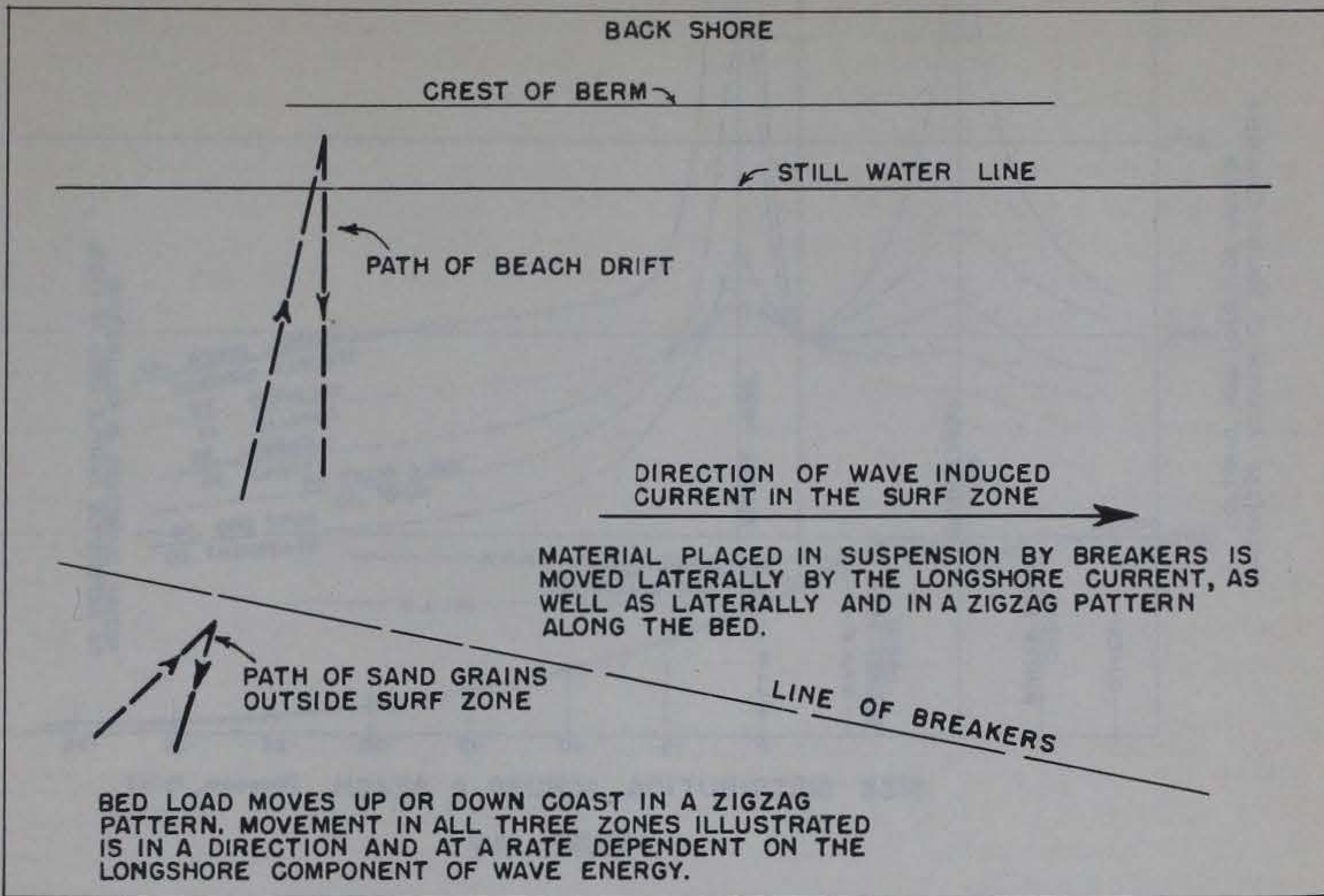
2.3 MODES OF LITTORAL TRANSPORT - Waves and currents supply the necessary forces to move the littoral materials. The mechanics of littoral transport are not precisely known, but it may be generally stated that littoral material is moved by one of three basic modes of transport: (a) Material, known as "beach drift", moved along the foreshore in a more or less scalloped path due to uprush and backwash of obliquely approaching waves; (b) Material moved principally in suspension in the surf zone by littoral currents and the turbulence of breaking waves; (c) Material moved close to the bottom by sliding, rolling, and saltation, collectively known as "bed load", within and seaward of the surf zone by the oscillating currents of passing waves. Significant bottom movement has been observed in depths exceeding 100 feet on exposed sea coasts. Figure 40 illustrates the three basic modes of transport, and Figure 41 (111) depicts trends of proportionate bed load transport related to wave energy and steepness from laboratory experiments.

Regardless of the mode of transport, the direction and rate of littoral drift depend primarily upon the direction and energy of waves approaching the shore. Exceptions exist on short reaches of shore adjoining tidal inlets where the tidal currents pattern may be dominant.

2.31 DEPTHS AT WHICH MATERIAL MOVES - Wherever sandy beaches exist or where the surf zone and nearshore bottom are composed of sand, the grain size of material along a shore profile decreases generally as the water depth increases until depths are reached where normal wave currents are incapable of moving bed material. The coarsest material is usually found in the surf zone in the vicinity of the plunge point of waves, though in protracted periods of mild wave action the foreshore and surf zone material may be nearly equal in size.

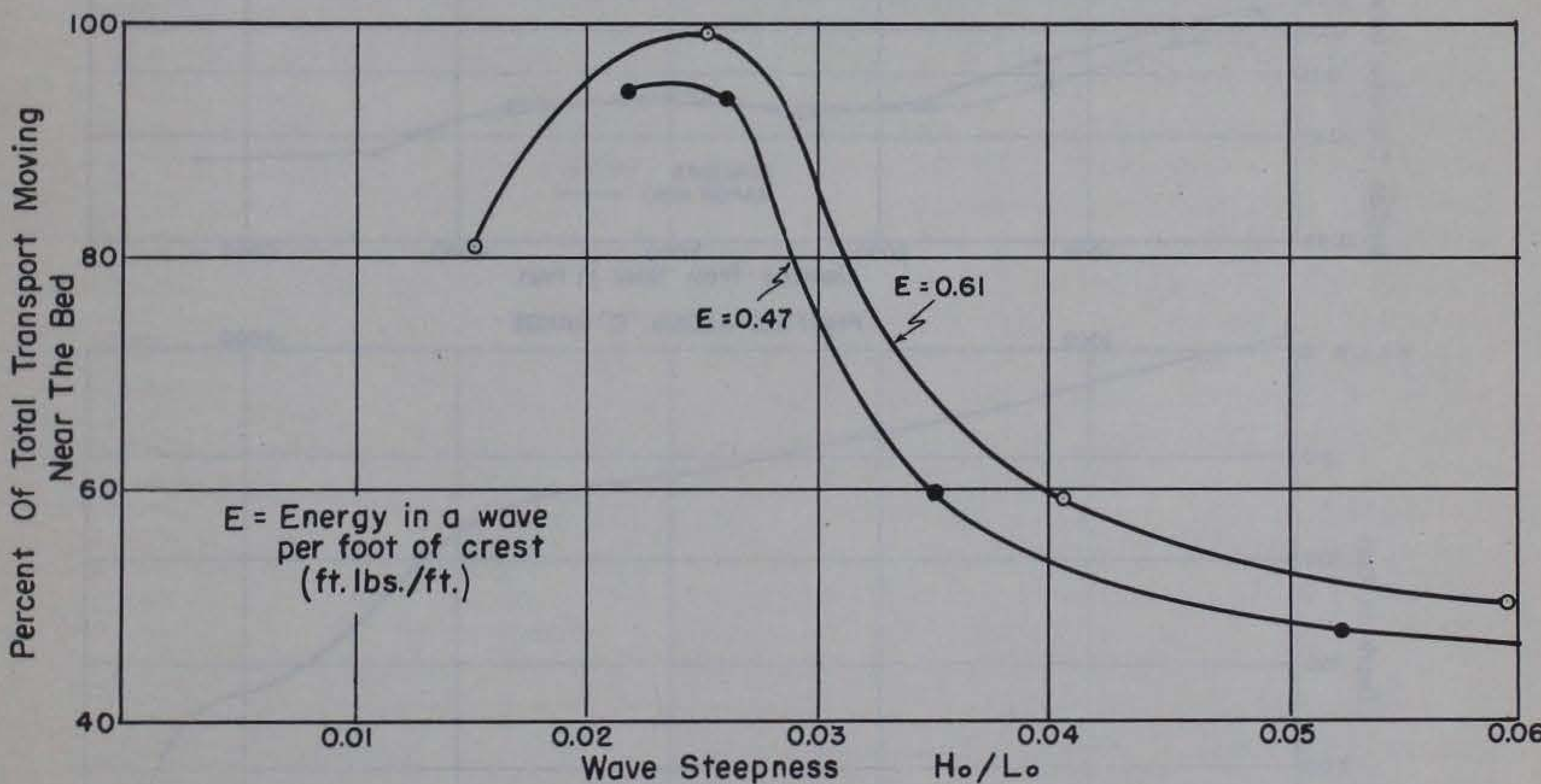
This gradation of grain sizes along the beach profile is due to the process of the slope sorting of beach materials which is illustrated in Figures 42 and 43. Slope sorting is ascribed generally to the differential velocity in the oscillating wave currents in shallow water. Whereas, the velocity of water particles is theoretically uniform about the orbital path for particles in deep water, moving in the direction of wave propagation while the crest is passing and in the opposite direction with passage of the trough, the uniformity ceases when the wave begins to feel the bottom. Deformation takes the form of steepening the crests, shortening them in relation to lengths of the troughs, so that the particles move forward with the wave crest in less time than they return with the trough. Figure 44 illustrates laboratory measurements of particle movement in water of various depths. The effect of this phenomenon is to transport coarser particles of bed material shoreward. Where the slope becomes sufficiently steep, gravity counterbalances the current effect and an equilibrium condition results. Currents introduced by reflected waves likewise affect the balance of forces in this region near the shore.

Because of slope sorting, the material in littoral transport moves generally within a depth range compatible with its size or resistance to transport. The actual path or rate of transport of individual particles or groups of particles cannot be stated from present knowledge. It is known that foreshore and nearshore slopes are related to the grain size



LATERAL MOVEMENT OF LITTORAL DRIFT

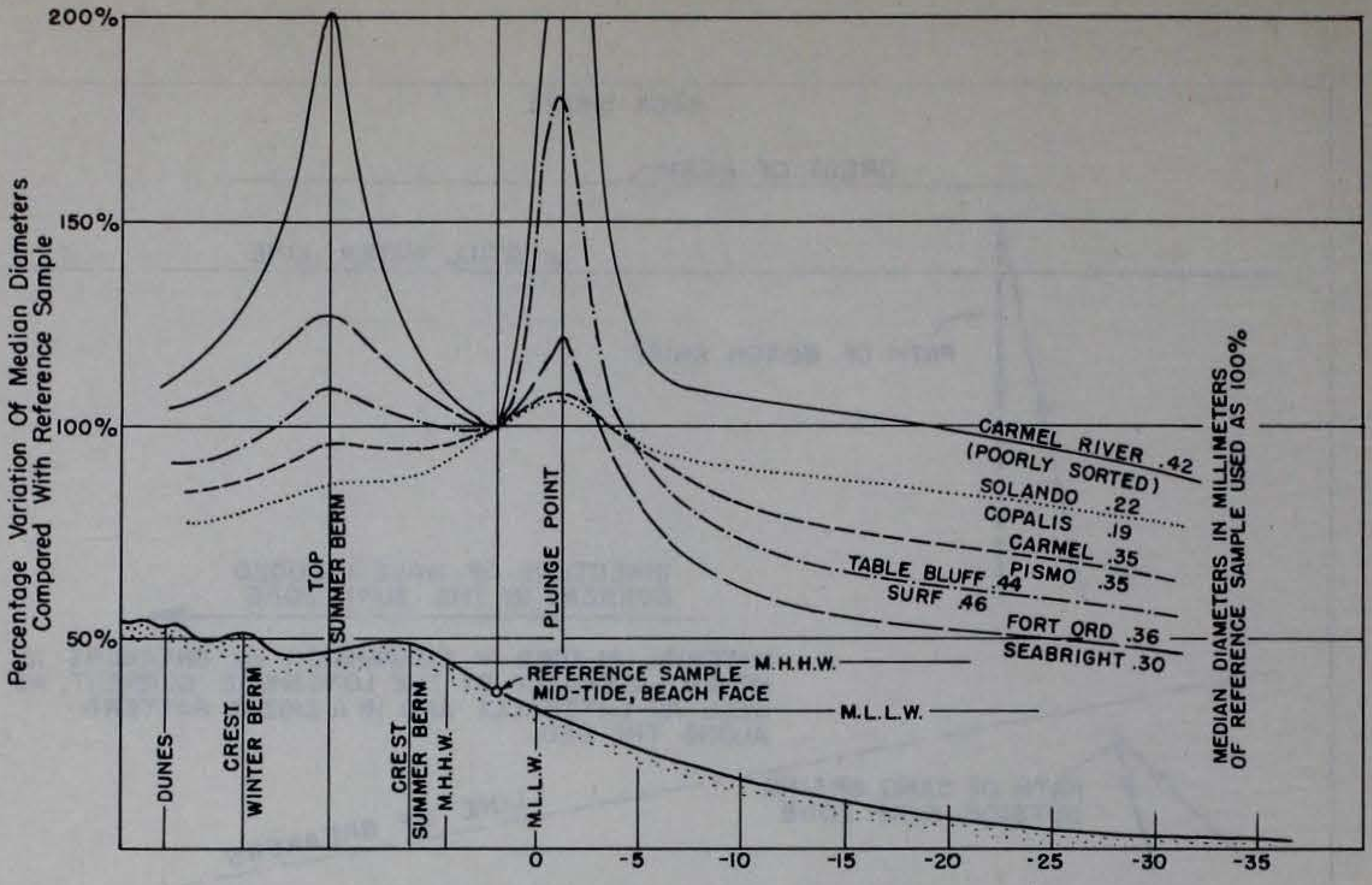
FIGURE 40



LABORATORY RESULTS INDICATING TRENDS OF PROPORTIONATE BED LOAD AND SUSPENDED LOAD TRANSPORT RELATED TO WAVE ENERGY AND WAVE STEEPNESS.

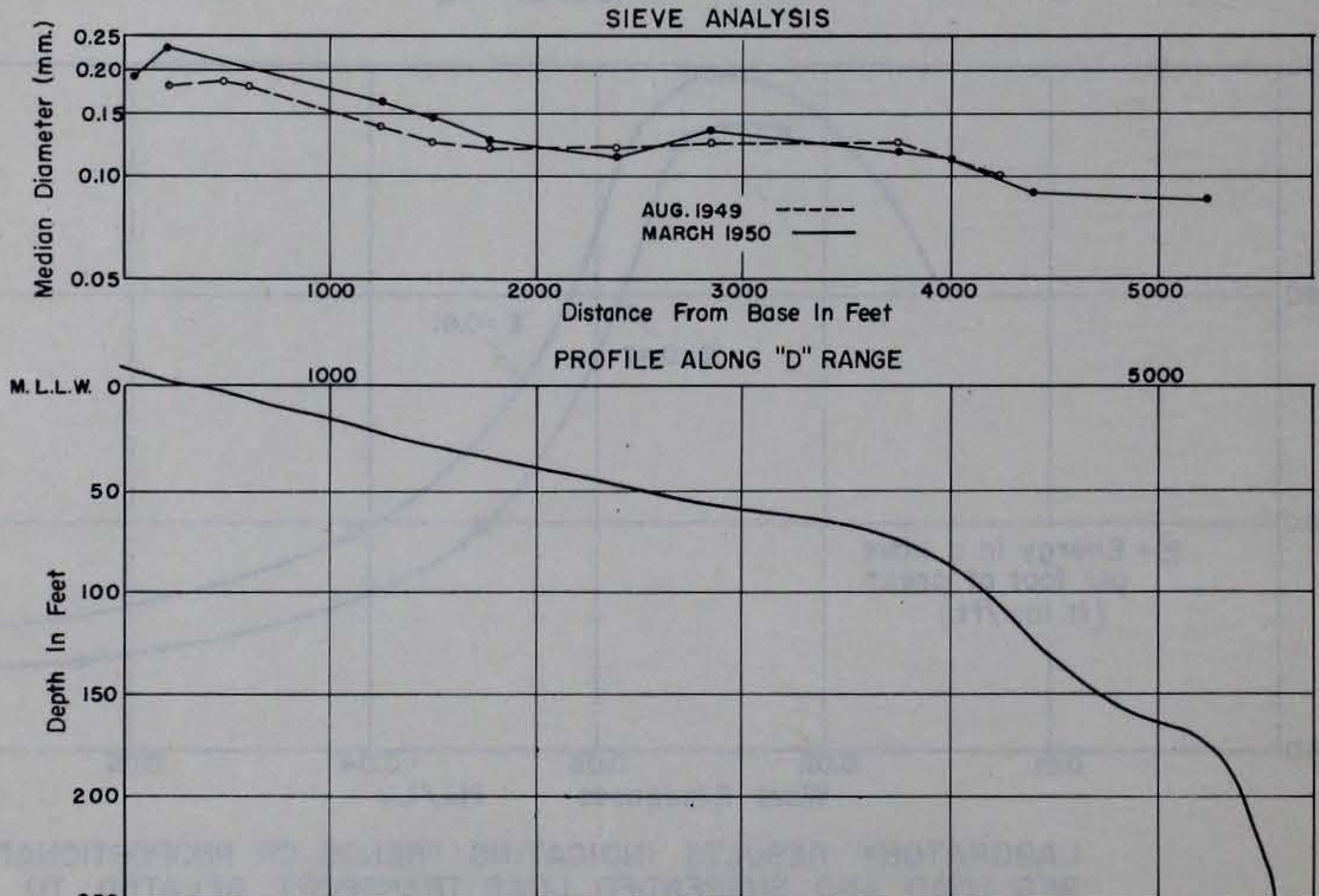
(Saville, 1950)

FIGURE 41



SIZE DISTRIBUTION ACROSS A BEACH (Bascom, 1951)

FIGURE 42



SEASONAL VARIATION OF SIZE DISTRIBUTION IN SEDIMENTS ALONG "D" RANGE OFF LA JOLLA, CALIFORNIA (Inman, 1953)

FIGURE 43

material (see Figure 45) of which they are formed, however, this relationship is not the same at all localities since it is also influenced by water level variability, wave exposure, and ground water level. Although particle density and shape are factors in transportability, median grain size is a satisfactory parameter for evaluating generally the transportability of littoral material.

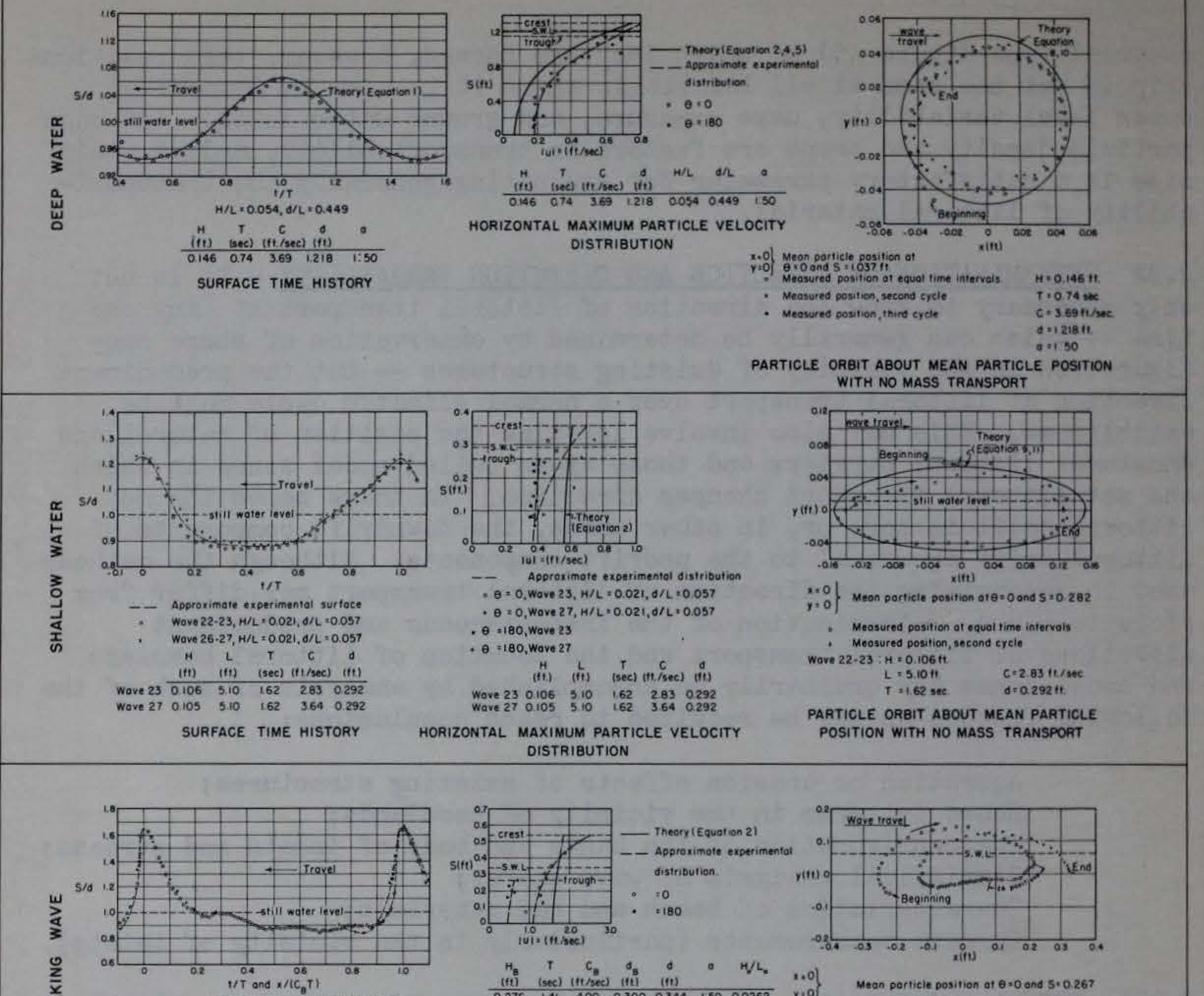
2.32 DETERMINATION OF DIRECTION AND DIRECTION VARIABILITY - It is not only necessary to know the direction of littoral transport at any one time -- which can generally be determined by observation of shore configuration in the vicinity of existing structures -- but the predominant direction of littoral transport over a normal climatic cycle must be established. This may also involve locating the position of natural and unnatural littoral barriers and those areas called nodal zones in which the net littoral transport changes direction. In these zones the net littoral drift is zero or, in other words, the downdrift components of littoral drift are equal to the updrift components. Although the methods used in determining the direction of littoral transport may differ from place to place, determination of the instantaneous and predominant directions of littoral transport and the location of littoral barriers and nodal zones may ordinarily be accomplished by analysis of such of the following factors as may be required to reach conclusions:

Accretion or erosion effects of existing structures;
Shore patterns in the vicinity of headlands;
The configuration of the banks and beds of inlets and streams;
Statistical analysis of wave energy;
Characteristics of beach and bed materials;
Current measurements (particularly in the vicinity of inlets).

2.321 Effects of Existing Structures - This provides the most reliable means of determining littoral transport characteristics, and will ordinarily outweigh all other evidence. The use of existing structures to determine the direction of littoral transport is illustrated in the series of aerial photographs, Figures 46 to 51, inclusive. Considering the evidence presented by groins, Figures 46 and 47, the condition of the beach at the time of inspection probably indicates the direction of littoral transport during the immediately preceding period. To determine the predominant direction of littoral transport requires ovservations at regular intervals over a period of at least one year to avoid misinterpretation due to seasonal effect.

Considering the evidence presented by breakwaters and entrance jetties, Figures 48, 49, and 50, the quantities involved are generally large enough so that the condition observed at any one particular time probably is indicative of the predominant direction of littoral transport, with incremental changes showing the short term direction variability.

2.322 Evidence at Headlands - The evidence presented by headlands as to the direction of littoral transport is not ordinarily as clear as



Legend

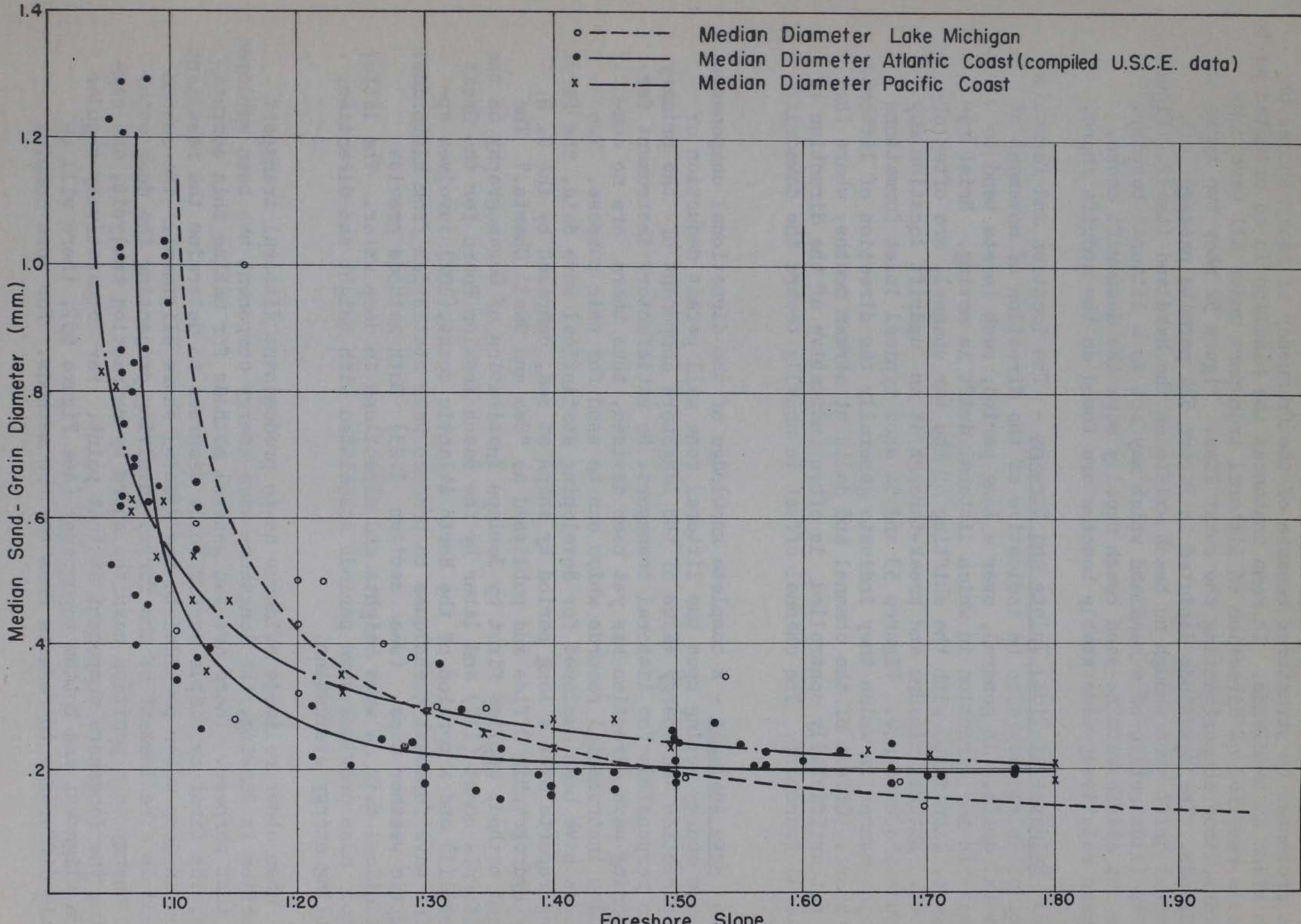
- S = elevation of the particle above the bottom (ft.), in this case the surface particle.
- d = still water depth. (ft.)
- H = wave height. (ft.)
- L = wave length. (ft.)
- t = time. (sec.)
- T = wave period. (sec.)
- u = horizontal particle velocity, (ft. per sec.)
- θ = angular position of particle in its orbit measured counterclockwise and where there is no mass transport, (degrees)
- x = horizontal position of particle in its mean horizontal position. (ft.)
- C = wave velocity of propagation. (ft./sec.)
- y = verticle position of particle in its mean horizontal position.
- α = beach slope.
- B = subscript – refers to breaking conditions.

Equations 1, 4, 5, 8 and 10 are from Stokes' second approximation.
Equation 2 is from Stokes' first approximation.
Equations 9 and 11 are reduced forms of Stokes' second approximation.

TIME HISTORIES, PARTICLE VELOCITY DISTRIBUTIONS AND ORBITAL PATHS, OF WAVES IN DEEP WATER, IN SHALLOW WATER, AND AT BREAKING

FIGURE 44

(Morison and Crooke, 1953)



RELATIONSHIP BETWEEN GRAIN SIZE AND FORESHORE SLOPE
FIGURE 45

that presented by structures because of the frequency of rocky shores on both sides of headlands. In some instances the headland is so oriented as to cause a reversal of direction of littoral transport under all wave conditions, thus compartmenting the coast line. Figure 52 show two types of headlands. The headlands depicted in Figure 52a permits passage of littoral drift even though no beach exists on the headland itself. Figure 52b is illustrative of a headland which may act as a littoral barrier. Wave cut cliffs with no sand beach usually mark the downdrift shore, whereas relatively wide stable beaches are found on the updrift shore.

2.323 Evidence at Tidal Inlets and Streams - The location and formation of tidal inlets may also be indicative of the direction of movement of littoral drift. In general, over a long period, such inlets tend to migrate in the direction in which littoral drift is moving. Brief reversals, associated with the shifting of the bar channel, are often observed. Natural closure and break-through at an updrift location may confuse the evidence. Figures 53 and 54 show typical inlet formations and the manner in which they indicate generally the direction of littoral transport. Offset of the channel and delta at stream mouths, where they are not artificially controlled, is often indicative of the direction of littoral transport. The channel offset is usually toward the downdrift side.

2.324 Wave Analysis - A complete knowledge of the directional components of wave energy acting upon the littoral zone will permit deduction of the direction and energy value of the longshore component of the primary force responsible for littoral transport. No satisfactory instrument for measuring wave direction has yet been devised, thus there are no continuing instrumental records which can be used for this purpose. Two methods have been employed for developing statistical wave data, the first being reports over a long period by ships at sea, compiled by the U. S. Navy Hydrographic Office and published as "Sea and Swell Charts." The second method, applied first by Scripps Institution of Oceanography on the California coast (117) and later by the Beach Erosion Board for the Great Lakes(113) and a portion of the North Atlantic coast,(102) involves applying wave forecast techniques to produce wave statistics from historical synoptic weather maps. (see section 1.23) Both methods provide statistical data on wave heights and directions in deep water. The latter method also provides wave periods associated with height and direction, enabling energy evaluation.

When other reliable evidence as to predominant littoral transport direction is lacking, the longshore wave energy component has been employed for that purpose. There are two general methods for making this computation, the first by simple vector force diagrams to determine the resultant force in deep water parallel to the general shore alignment. The second method is a refinement of the first, involving projecting the deep water wave energy to a position near the shore by refraction analysis, and computing the longshore component at that point. For comparatively regular shore alignment and bottom topography (see Figure 55), there will be little difference in results from the two methods. For more complex

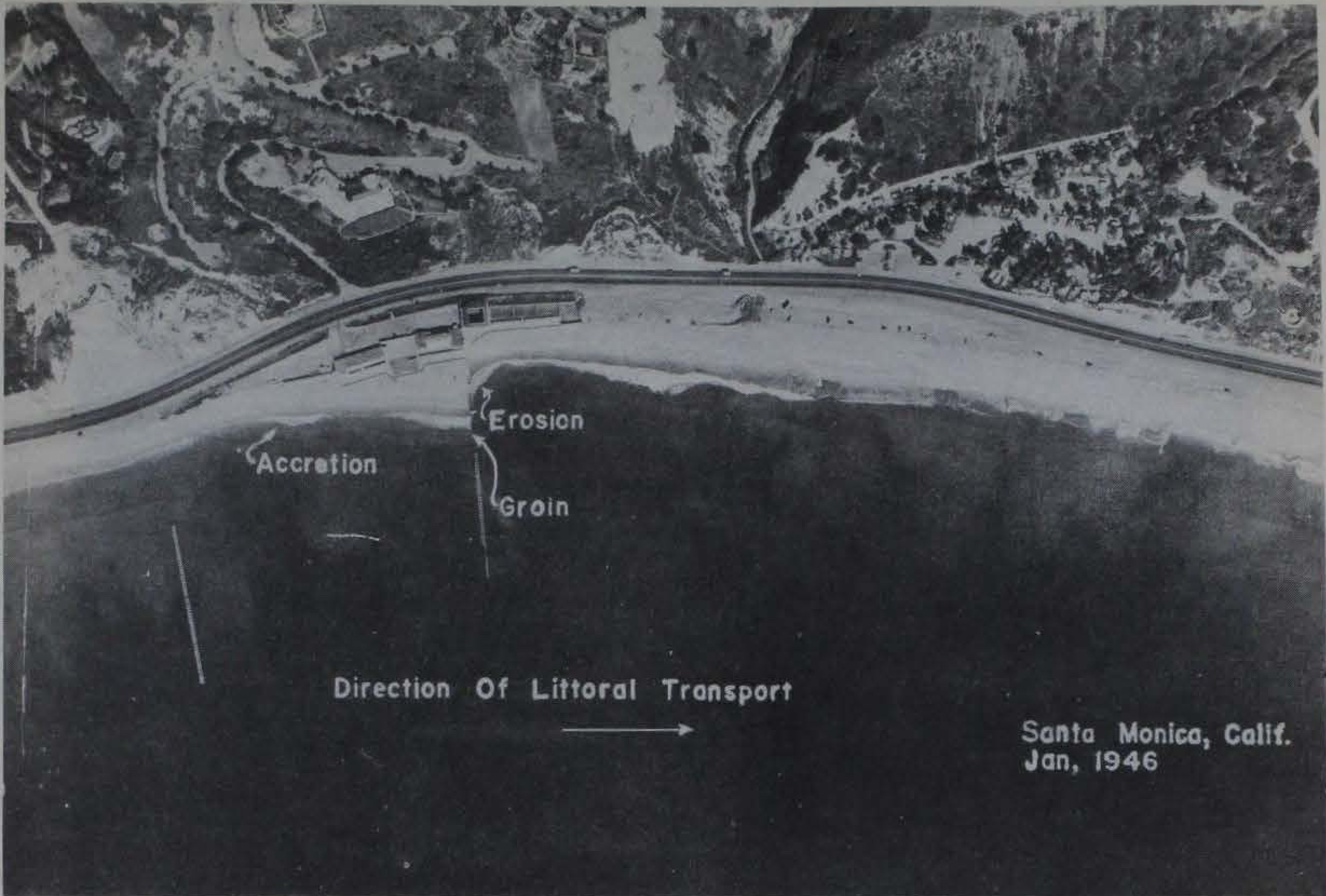


FIGURE 46. EFFECT OF SINGLE GROIN

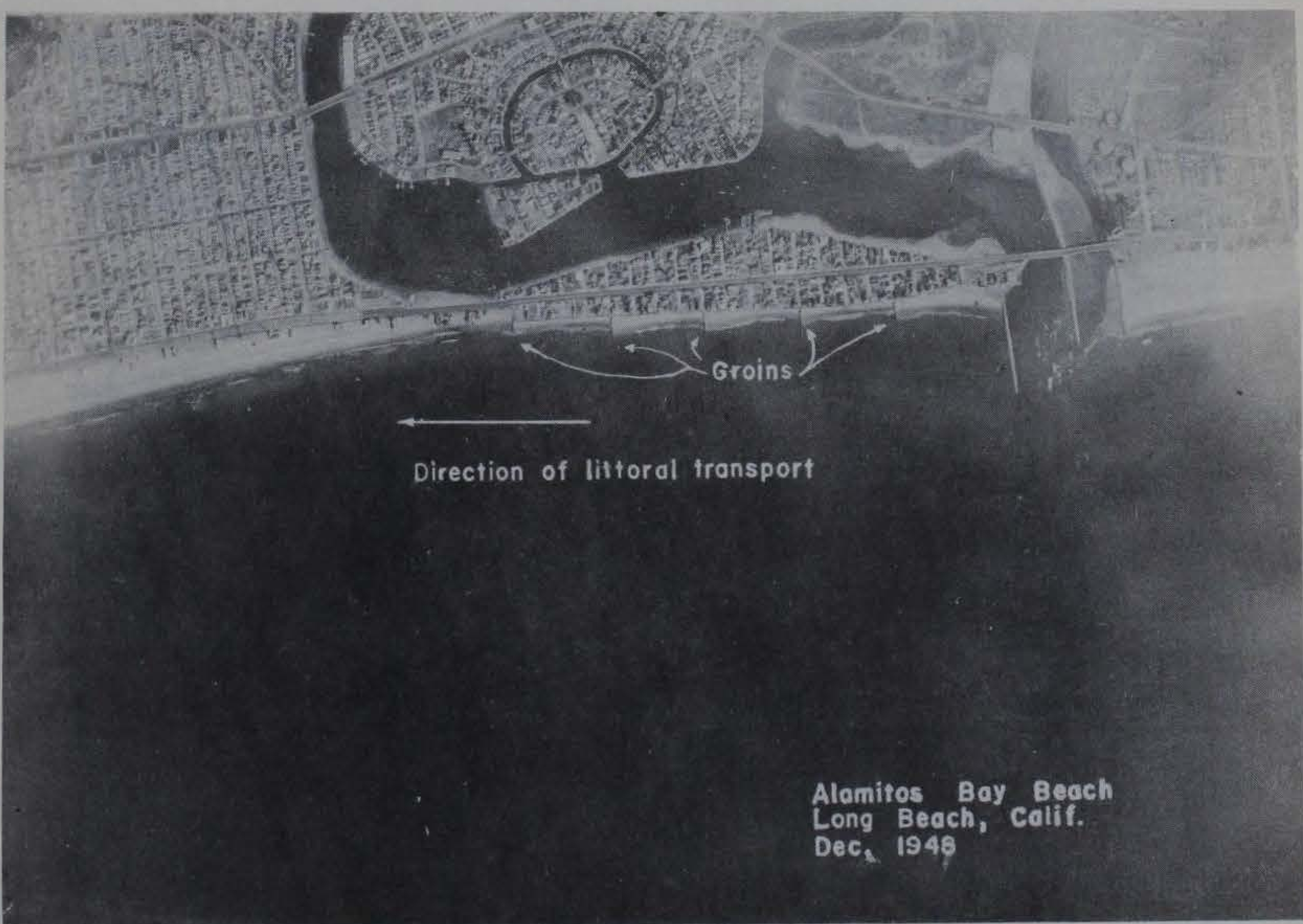


FIGURE 47. EFFECT OF A SERIES OF GROINS

FIGURE 48 EFFECT OF OFFSHORE BREAKWATER

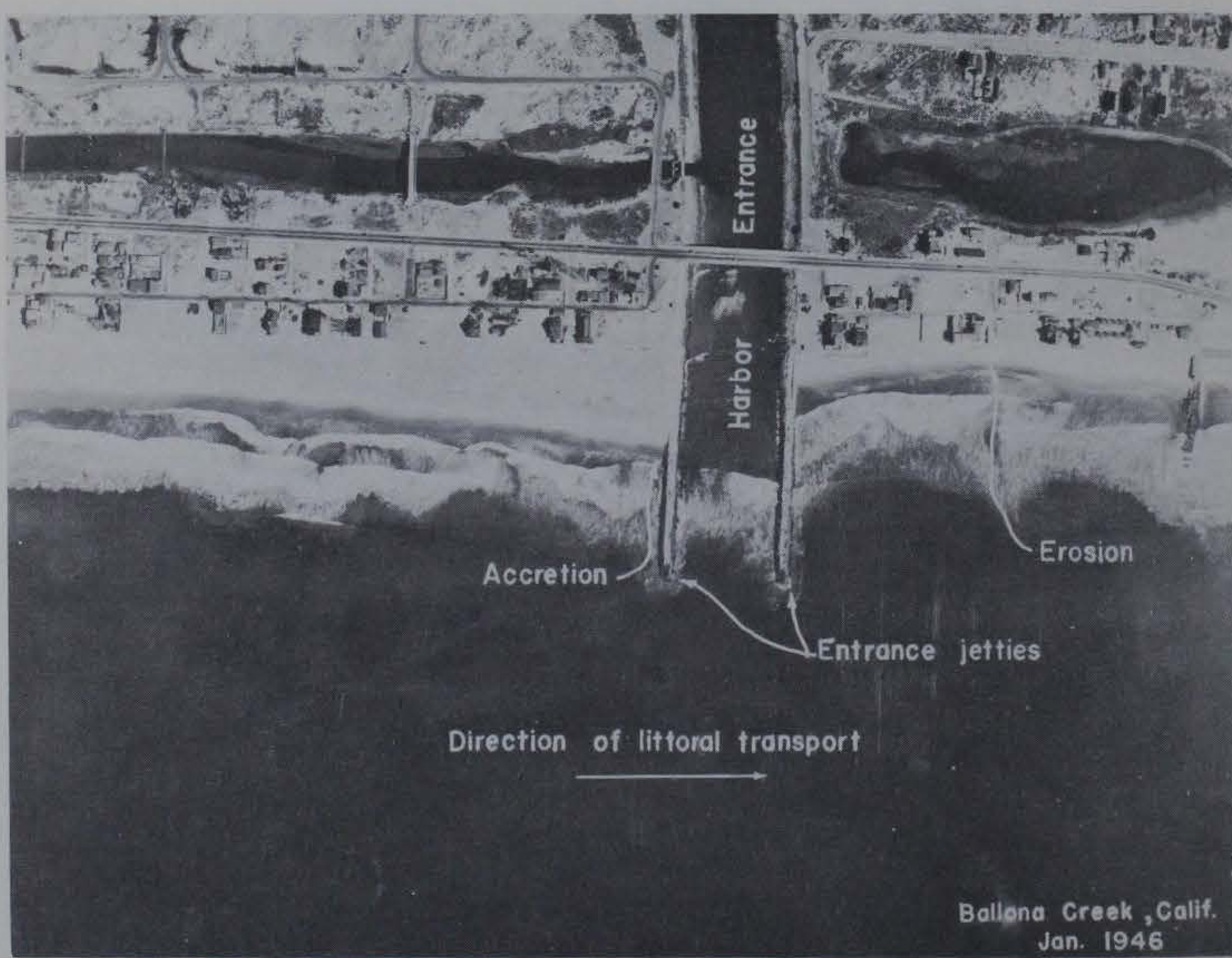


FIGURE 49 EFFECT OF ENTRANCE JETTIES

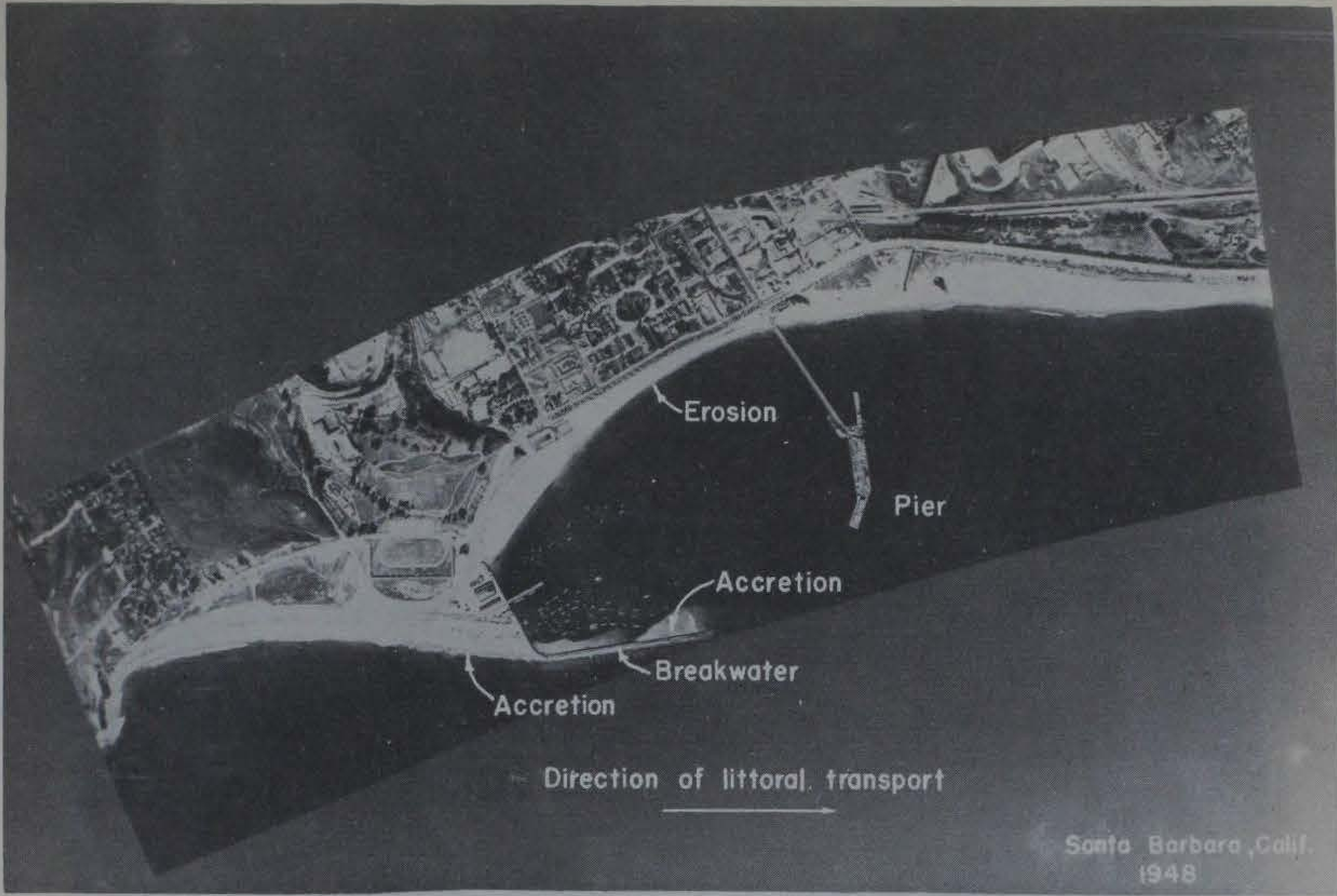


FIGURE 50 EFFECT OF SHORE CONNECTED BREAKWATER

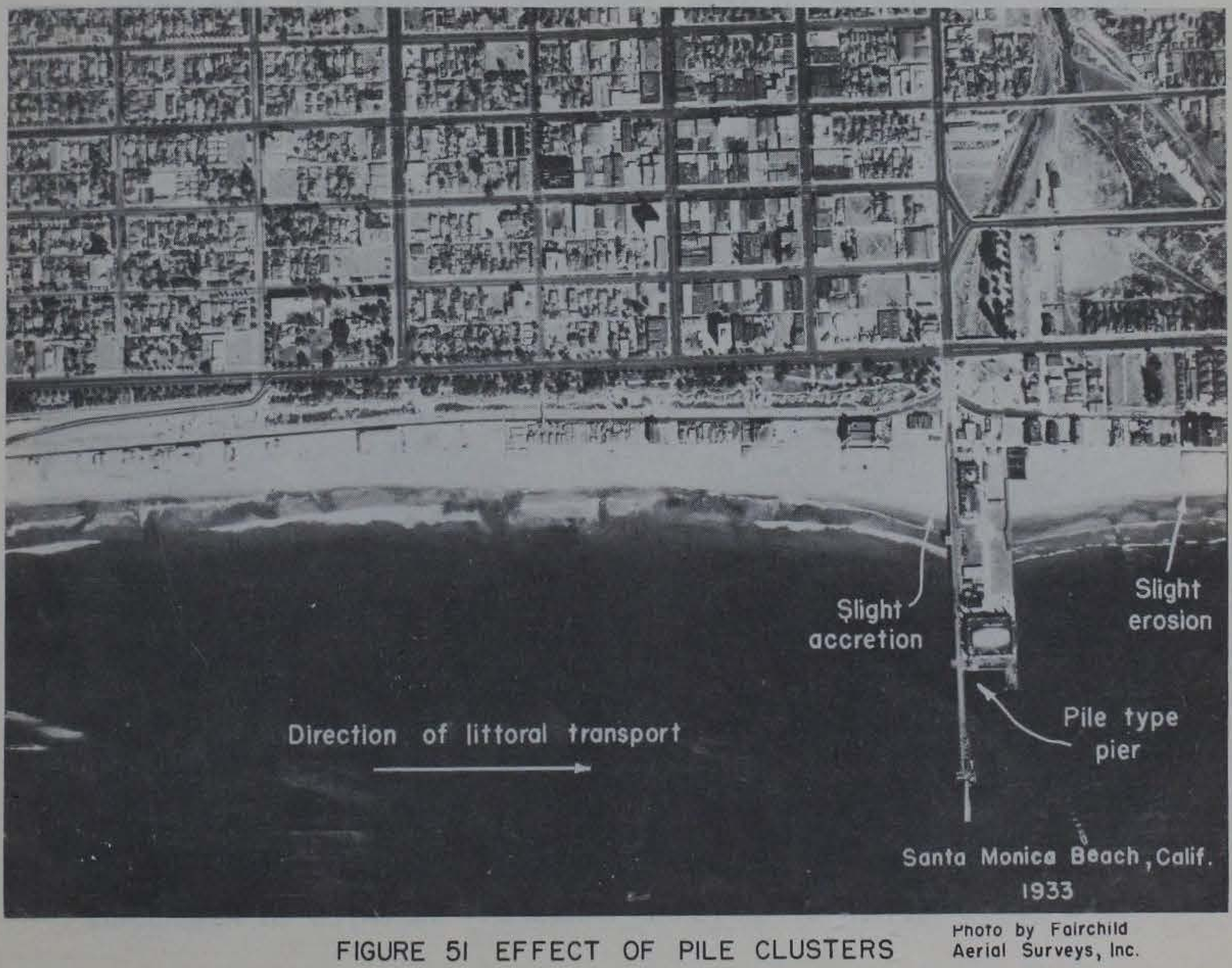


FIGURE 51 EFFECT OF PILE CLUSTERS

Photo by Fairchild Aerial Surveys, Inc.

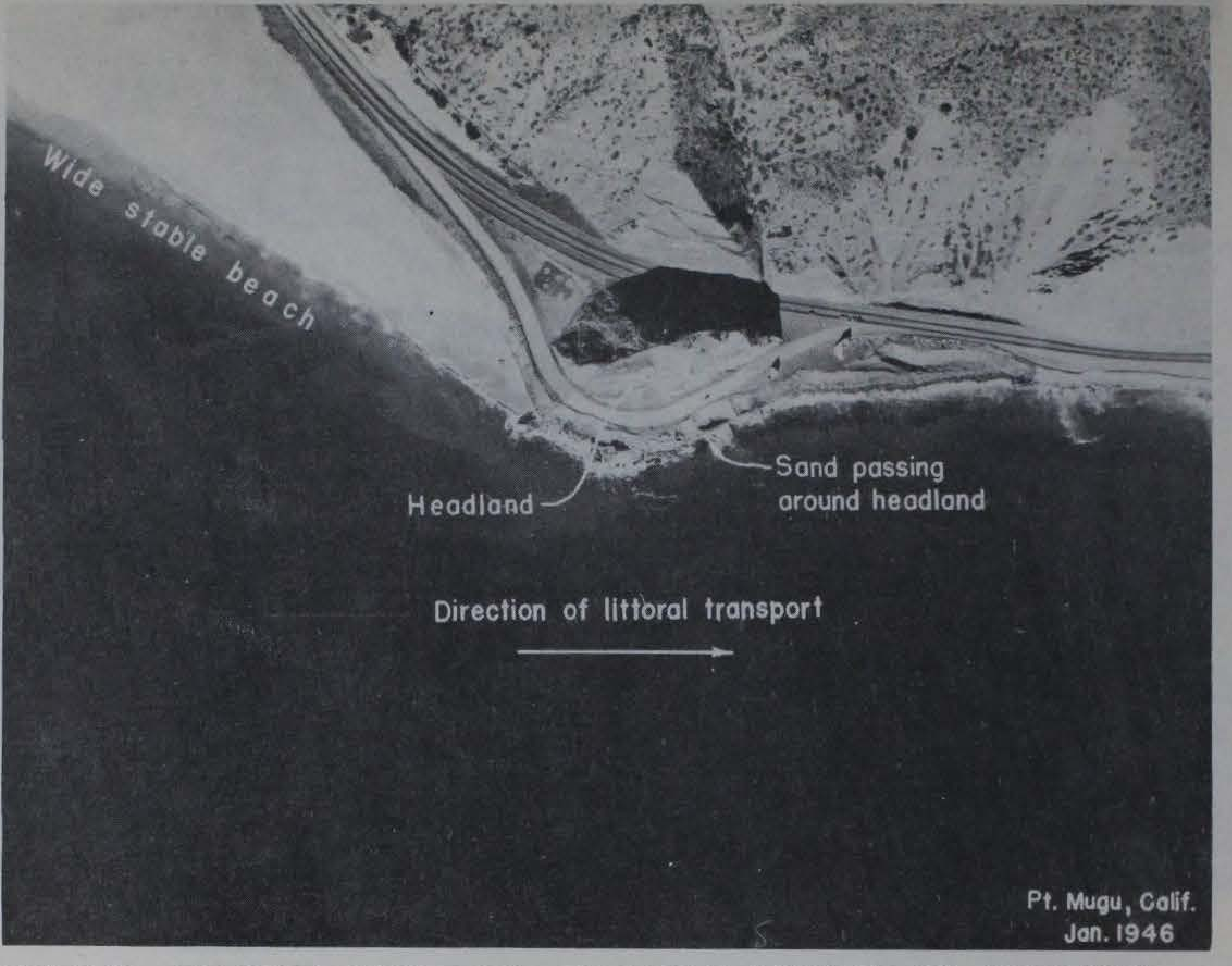


FIGURE 52a DIRECTION OF LITTORAL TRANSPORT INDICATED BY HEADLANDS

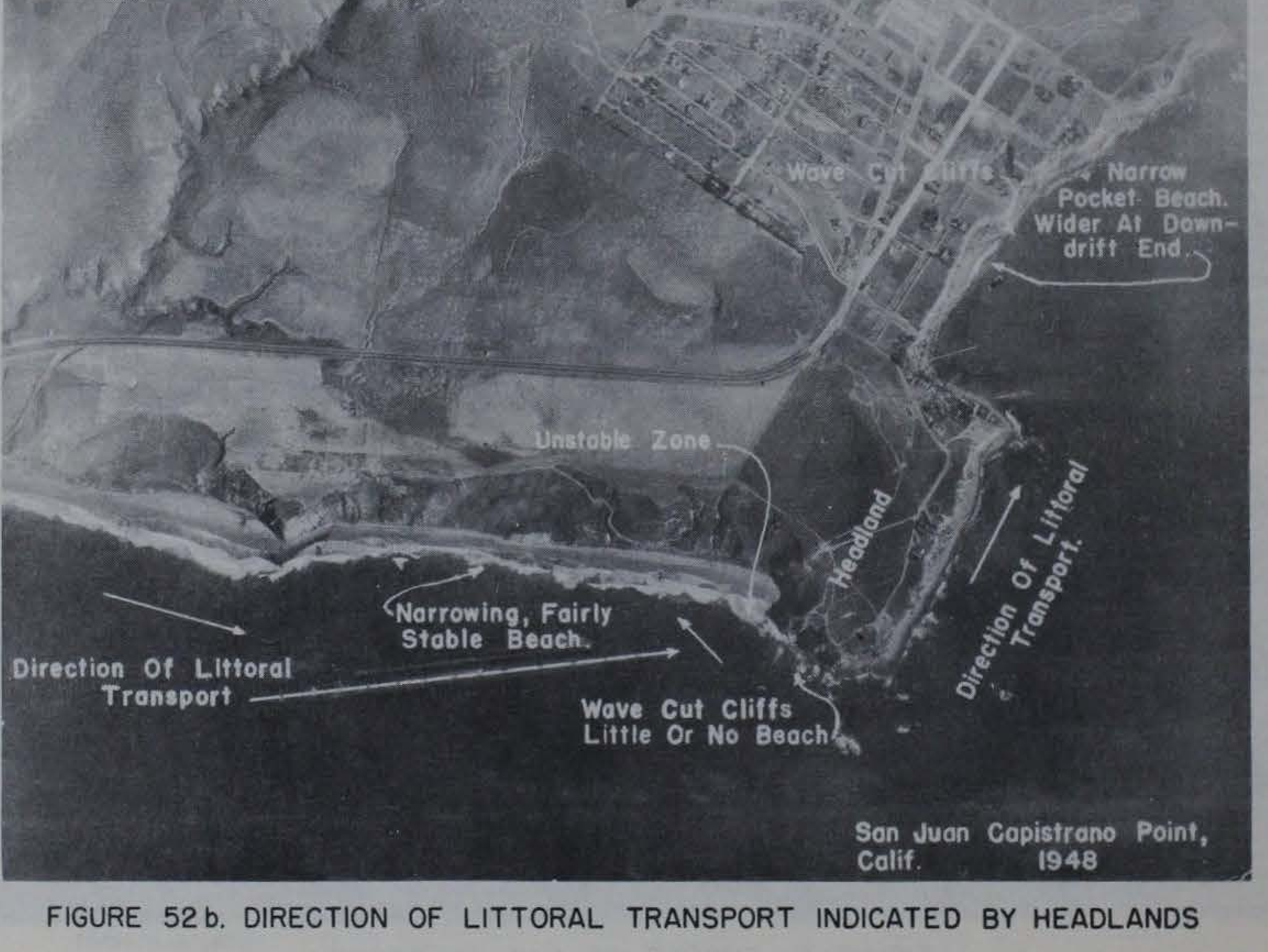


FIGURE 52b. DIRECTION OF LITTORAL TRANSPORT INDICATED BY HEADLANDS

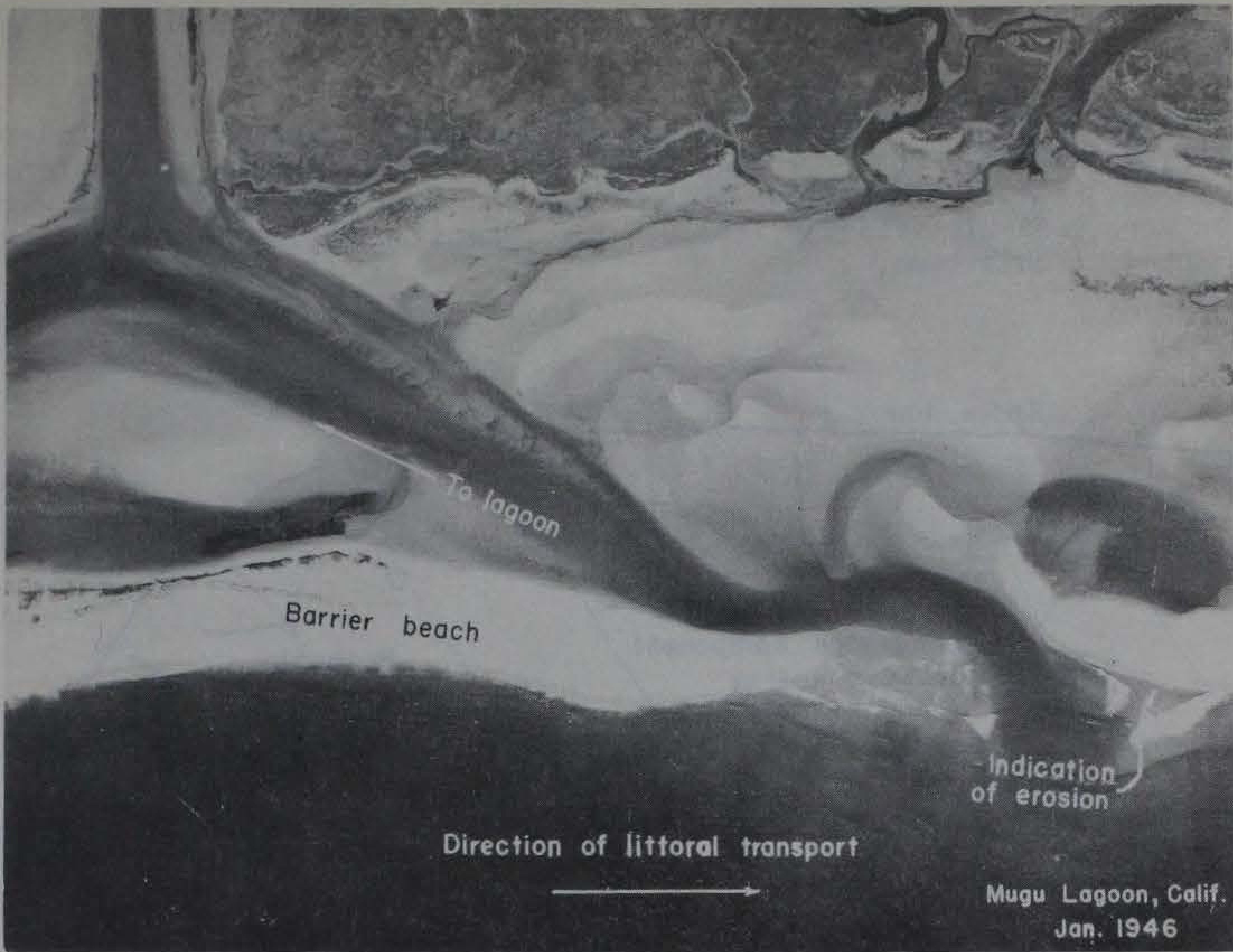


FIGURE 53 TIDAL INLET TO BAY OR LAGOON

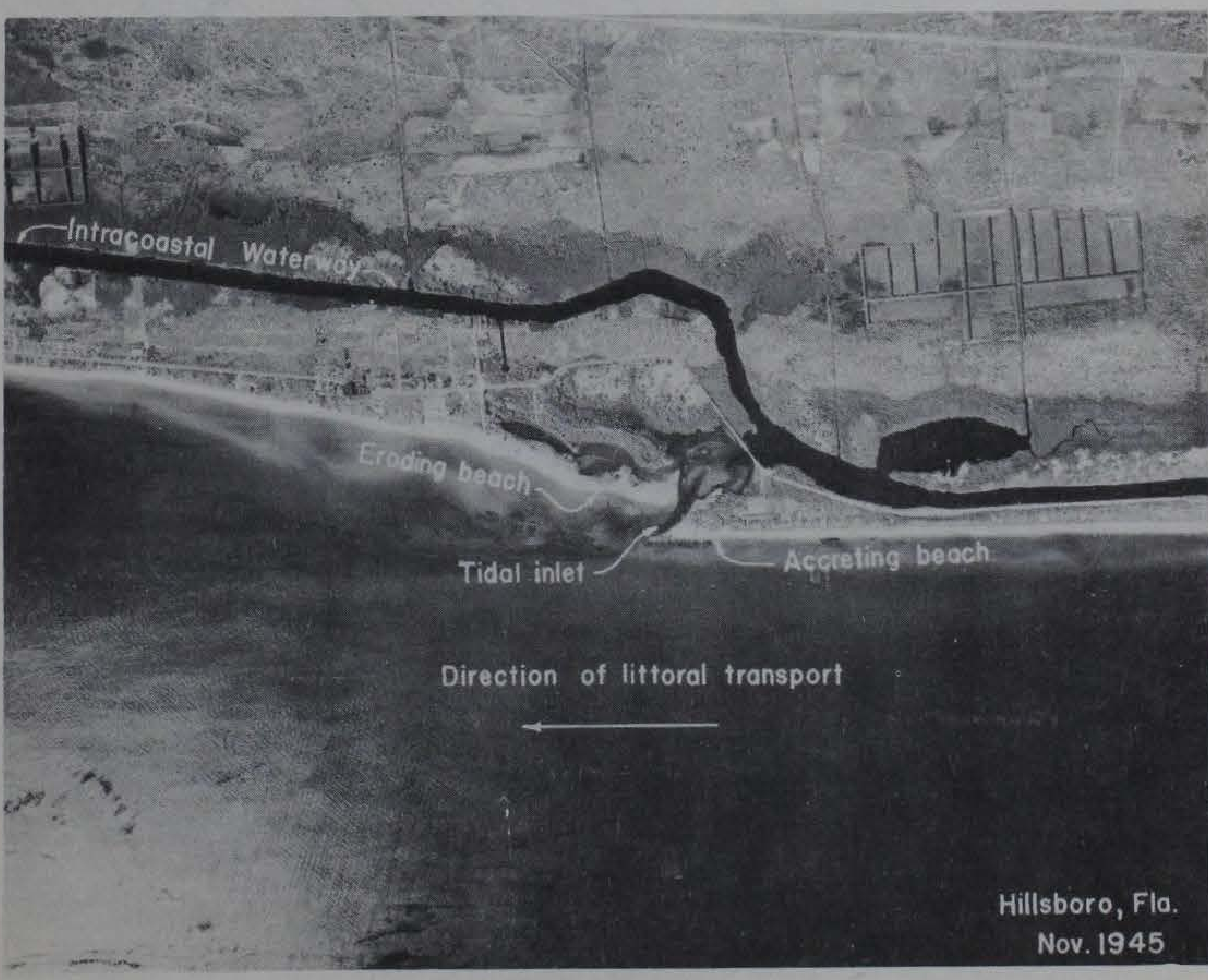


FIGURE 54 TIDAL INLET THROUGH A BARRIER BEACH

topography, local inconsistencies are usually found. There is at present no proven technique for employing refraction analysis to determine transport direction, and as results of this method are sometimes in conflict with other more reliable evidence, its use for general practice does not appear warranted at this time.

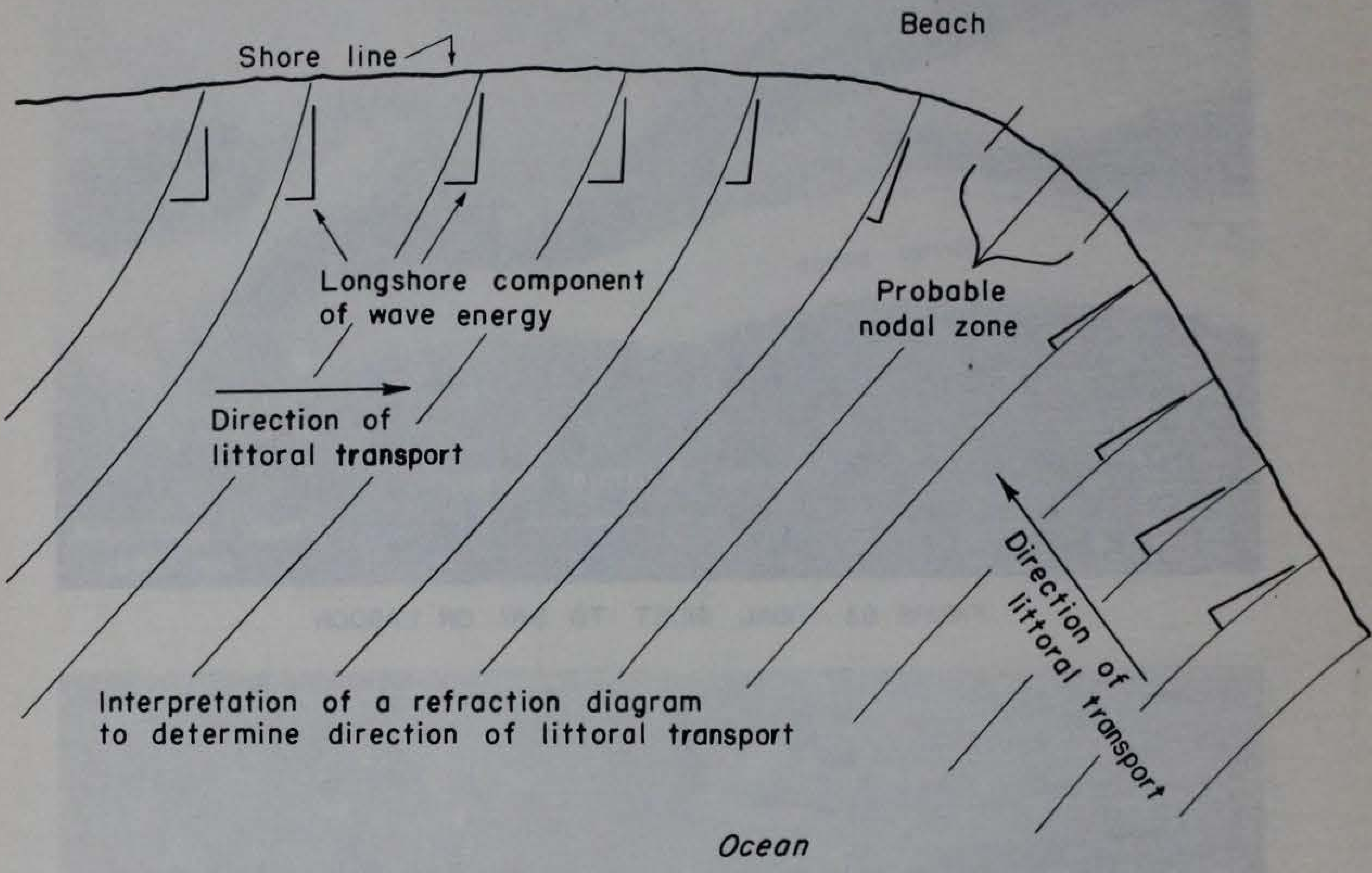


FIG. 55 — REFRACTION DIAGRAM-WAVE CONDITIONS BY ORTHOGONALS

2.325 Variations in Material Characteristics - A comparison of the median diameters of a series of samples taken in the same manner along a coast frequently indicates a progressive trend in the variation of the grain size of the material. In ordinary cases of sediment transport (upland) the median grain size will decrease with distance from the source, thus indicating the predominant transport direction.

Because of the wide variation in grain size along beach and nearshore slopes; the effect of exposure and underwater topography upon slope sorting; and the disruptive effects of varied sea conditions which may occur just preceding or during a sampling program, such evidence is not a reliable indication of predominant littoral transport.

A progressive variation in the total heavy mineral content in a series of samples taken along a coast may indicate the direction of littoral transport since these are less subject to frequent dislocation. Heavy minerals have a tendency to remain on the beaches, while the light minerals often are carried out beyond the plunge zone. When this occurs, the relative percentage in total heavy mineral content contained in the samples will increase with distance from the source of the light minerals. A progressive decrease in the frequency of a particular mineral may also indicate the direction of littoral transport; the frequency decreasing with distance from the source area.

2.326 Current Measurements - Measurements of littoral current may sometimes give an indication of the direction of littoral transport, but they require much time and are frequently unreliable. They must be made at frequent intervals over a full year to be of value, and if reversals in direction and wind velocity variations are observed, they cannot be evaluated in terms of littoral drift. The most common methods used to obtain the direction and velocity of the currents are by use of floats outside the breaker zone and fluorescein inside the breaker zone.

Fluorescein is a yellowish-red crystalline compound which receives its name from the brilliant yellowish-green fluorescence of its alkaline solutions. Fluorescein can be purchased at moderate cost from most chemical firms in quantities of one pound or more. A common method of employing fluorescein is to place a handful of dry sand in a paper towel, or other substance, which would disintegrate readily, with a heaping teaspoon of fluorescein crystals. The entire mass is twisted within the towel and tossed seaward into the breaker zone. As the towel disintegrates and the crystals dissolve, a small patch of water is dyed an easily distinguishable brilliant yellowish-green. The movement of this patch of colored water in a longshore direction can be traced from the beach, noting the distance traveled and the time required for the travel. All measurements should be to the center of the colored area, which will gradually disperse until no longer distinguishable. From the distance moved and the time elapsed during the movement, the velocity of the current can be computed. Time permitting, 3 readings should be taken at each station and averaged. For the best effect, these readings should be taken twice a day at regular intervals along the beach under study, with concurrent measurements of wave height, period and direction. As the wind affects the shallow water currents, its velocity and direction should also be recorded.

Current measurements seaward of the breaker zone are generally made with floats. In general, these react still more erratically than does the fluorescein. In the low current velocities common to this area, wind conditions have considerable bearing on the floats unless great care is taken. Although many types of floats have been used, most of them follow the same general pattern of movement. Floats should be low in the water and designed to offer the least wind resistance and the maximum water resistance possible. The floats should be released in sequence along the entire length of a profile, spaced at regular intervals from 200 to 400 feet. Each float should have a distinctive color flag or marker to permit its identification. Locations of the floats at regular time intervals may be made by transit intersection. These measurements are generally continued through one or more tidal cycles. Because of the short time interval, this type of current measurement does not represent the seasonal changes and can be used only in connection with other observations.

Two general types of subsurface floats are used, the rod float and the vaned float. The rod float, of uniform dimensions, gives an approximation of the integrated values over the depth covered by the float. The vaned float gives an approximation of the current velocities at the depth of the vanes. Floats of too great length will drag on the bottom and give improper readings. Floats of comparatively short length record only surface currents. The float lengths should be such as to permit selection of the proper length float for the range of depths along a profile. This selection of proper float lengths is based on experience at the site after a trial run. One design for a vaned float is shown in Figure 56. Pegram current meters are used on extensive studies to measure bottom currents. These are not generally used in the determination of direction of littoral transport because the information gained is seldom commensurate with the cost of the operation. Balls of slightly greater density than the water have been used to roll along the bottom dragging a very light line and float to make their location.

2.33 <u>RATES OF LITTORAL TRANSPORT</u> - The rate of littoral transport is as important as the direction of movement of littoral drift in the functional and structural design of shore protective structures. The rate of littoral transport can only be measured accurately at a substantially complete artificial littoral barrier. At such barriers this rate can be computed by measuring either the accretion at the updrift side of the barrier or the erosion at the downdrift side. Accretions can be measured at partial barriers, but no method has been devised to determine what proportion of the total littoral drift is trapped by each partial barrier. Until some such method is devised, the measurement of material trapped by groins or short jetties is a most inadequate way of determining the rate of movement of this drift. Natural littoral barriers are of little use in determining the rate of littoral transport because over geologic time the beaches either updrift or downdrift from these barriers tend to reach a condition of stability where the sand supply equals the sand losses.

Typical examples of essentially complete, substantially complete, and temporary artificial barriers are shown in Figures 57, 58, and 59. In these examples, and in all similar cases, the rate of littoral transport is determined by measuring the amount of accretion or erosion occurring during a known period of time. To compensate for seasonal changes, surveys should be taken at about the same time each year. To compensate for annual fluctuations, the period of time between surveys should be extended as conditions permit. The rate of movement should be expressed in amount of drift per unit time, usually a year.

Where the rate of littoral transport is to be established at a littoral barrier, the base surveys should be extended a sufficient distance updrift and downdrift from the barrier to include the entire accretion and erosion

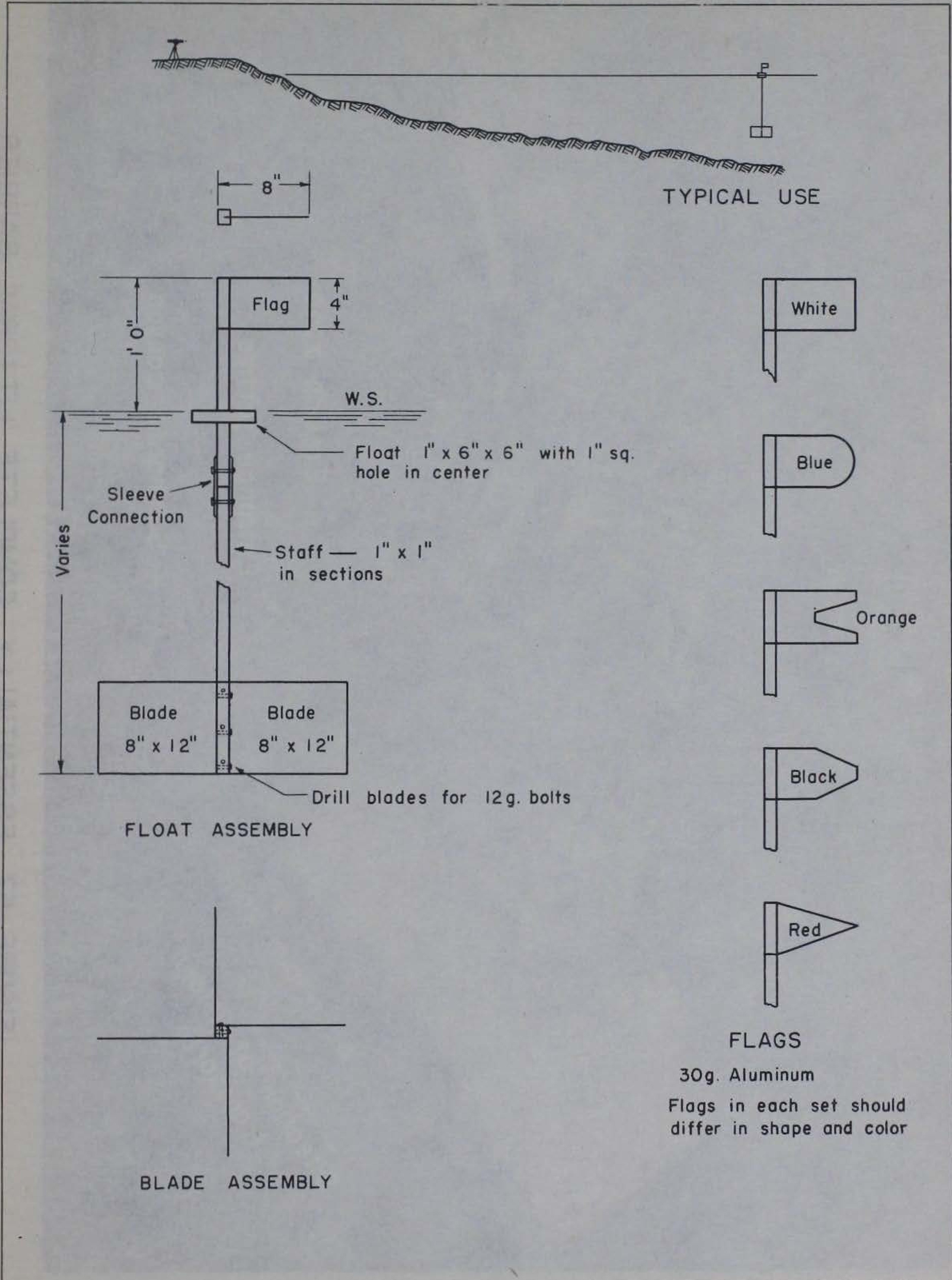


FIG. 56 TYPICAL VANE FLOAT ASSEMBLY

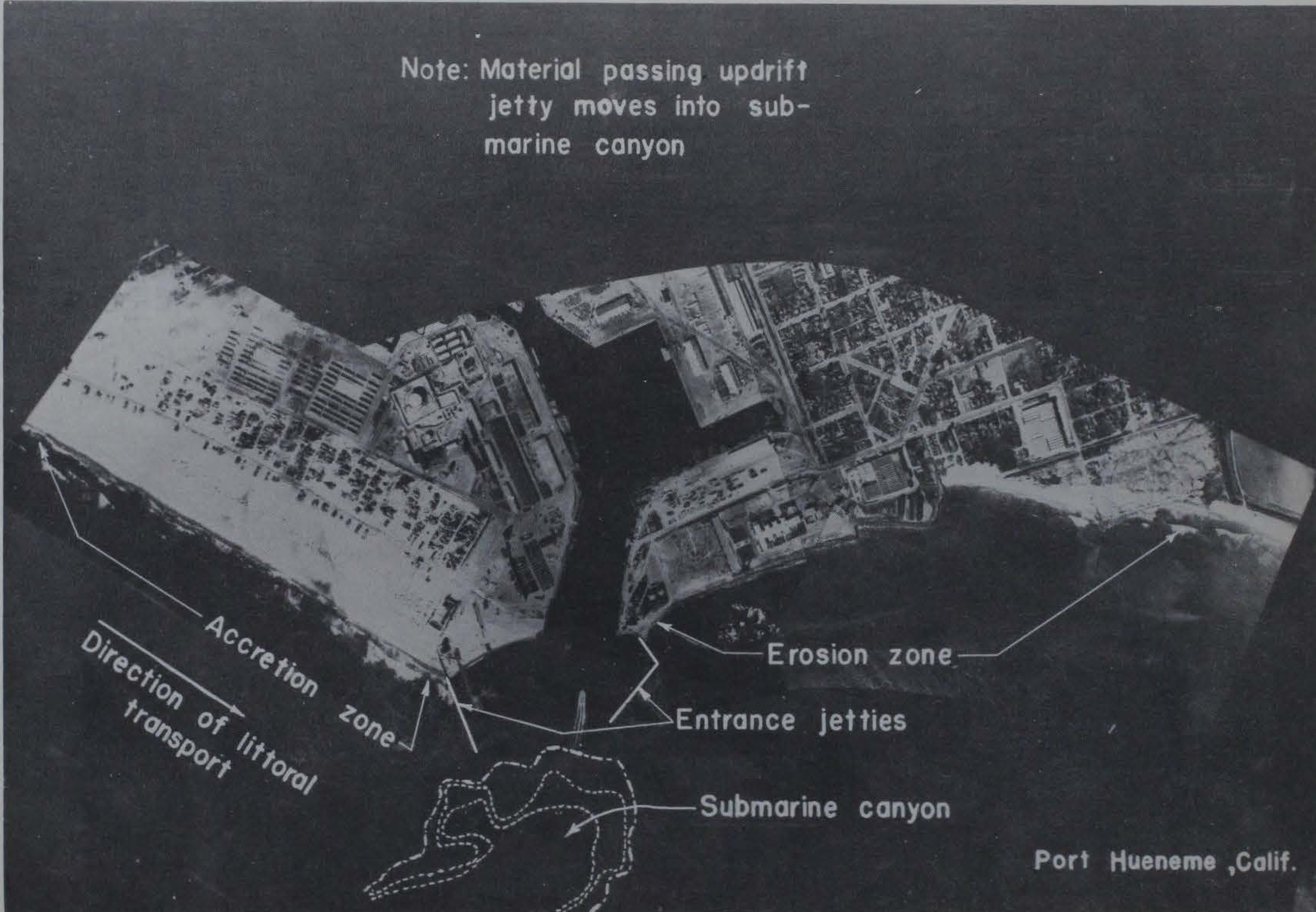


FIGURE 57 ESSENTIALLY COMPLETE LITTORAL BARRIER

FIGURE 58. SUBSTANTIALLY COMPLETE LITTORAL BARRIER

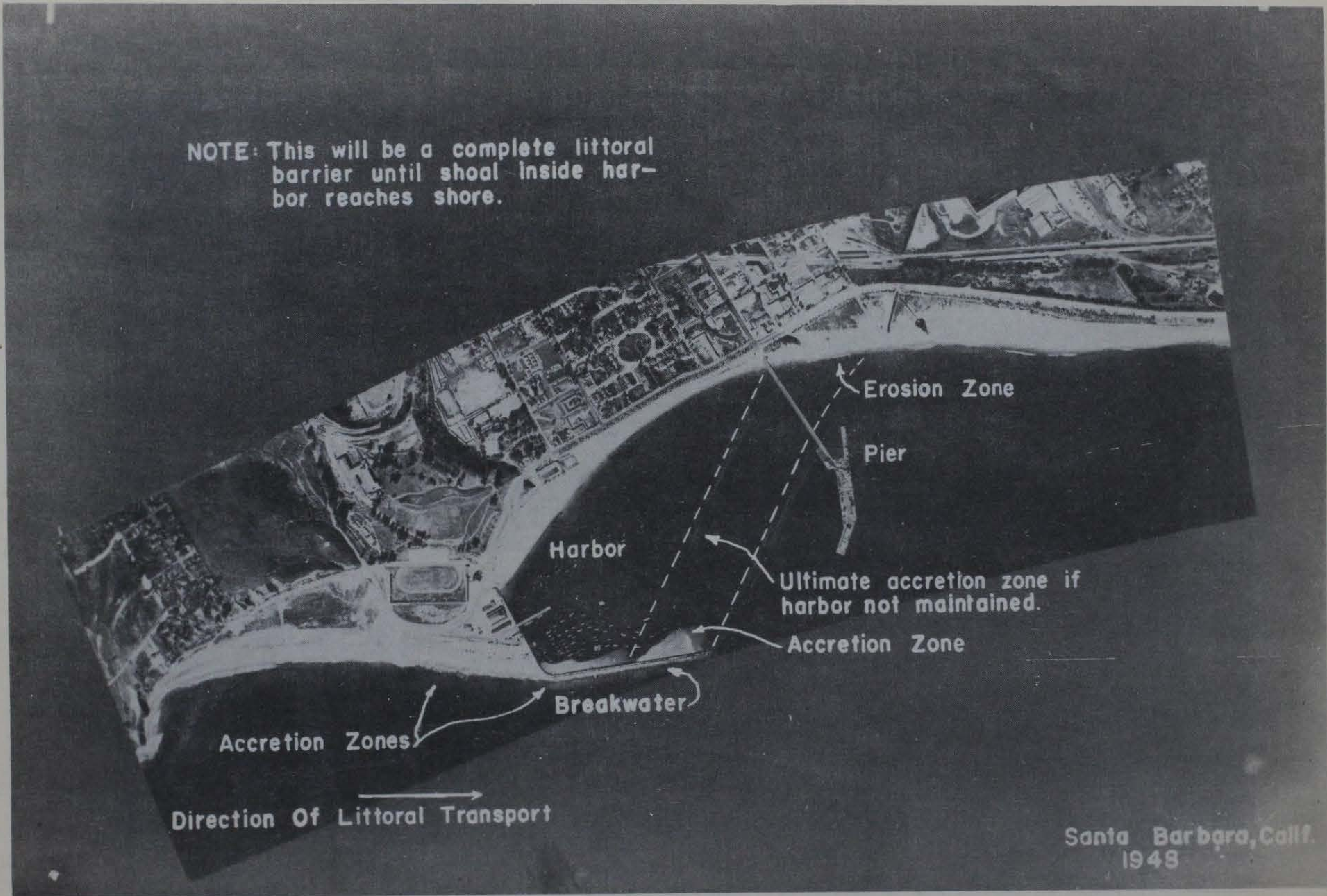


FIGURE 59. TEMPORARY LITTORAL BARRIER

zones at the end of the study period. Where erosion is anticipated, the base line should be referenced to points at a considerable distance from the ocean as recessions of the shore line of 1,000 feet or more are not uncommon. Profiles are run from the base line seaward at least to the 30-foot depth contour, although extension of the profiles to greater depths may be required in areas of severe exposure or in the vicinity of submarine canyons. Profiles should be spaced in conformity with accuracy desired and the degree of regularity of the area.(114) Profiles may be run by any standard hydrographic survey method. Where the amount of such work to be done is large and continuing, echo sounding equipment and amphibious vehicles are advantageous. Care should be taken to insure accurate vertical control and measurement as small uncompensating vertical errors result in large quantity errors.

Measurement of accumulations on the updrift side of jetties or long groins provides a good basis for estimating the rate of littoral transport. Depth of water to which the structures extend, and the character of material trapped, must be considered in evaluating the impoundment rate in comparison with total littoral drift. Short groins provide a poor means for measuring the rate of littoral transport, because the amount of material trapped is usually a small and indeterminable part of the entire quantity of littoral drift. Shoaling rates in entrance channels may provide an estimate of littoral drift in locations where maintenance dredging is done frequently. However, this method can seldom be used because of the difficulty of separating shoaling caused by reversals in the direction of littoral transport from that caused by the predominant direction.

2.34 <u>LOSSES OF LITTORAL MATERIAL</u> - Principal avenues of loss of littoral material from a specific beach area include (a) movement of material laterally out of the area; (b) movement of material offshore into water of sufficient depth that it is lost to the littoral supply; (c) loss of material into submarine canyons; and (d) loss of material inland. Loss of material by abrasion of sand has been found of slight importance.(83)

2.341 <u>Losses by Longshore Transport</u> - The movement of materials out of the area is measured by the net rate of transport at the downdrift end of the beach segment under study. It may be that this loss can be measured directly as outlined previously. If it cannot be measured directly at a particular location, it may be possible to estimate it by considering the rates of transport at the two closest known points above and below where the rate has been established or can be measured directly. At best this is a rough estimate as the unknown factors of added supply and losses throughout the area must also be taken into consideration.

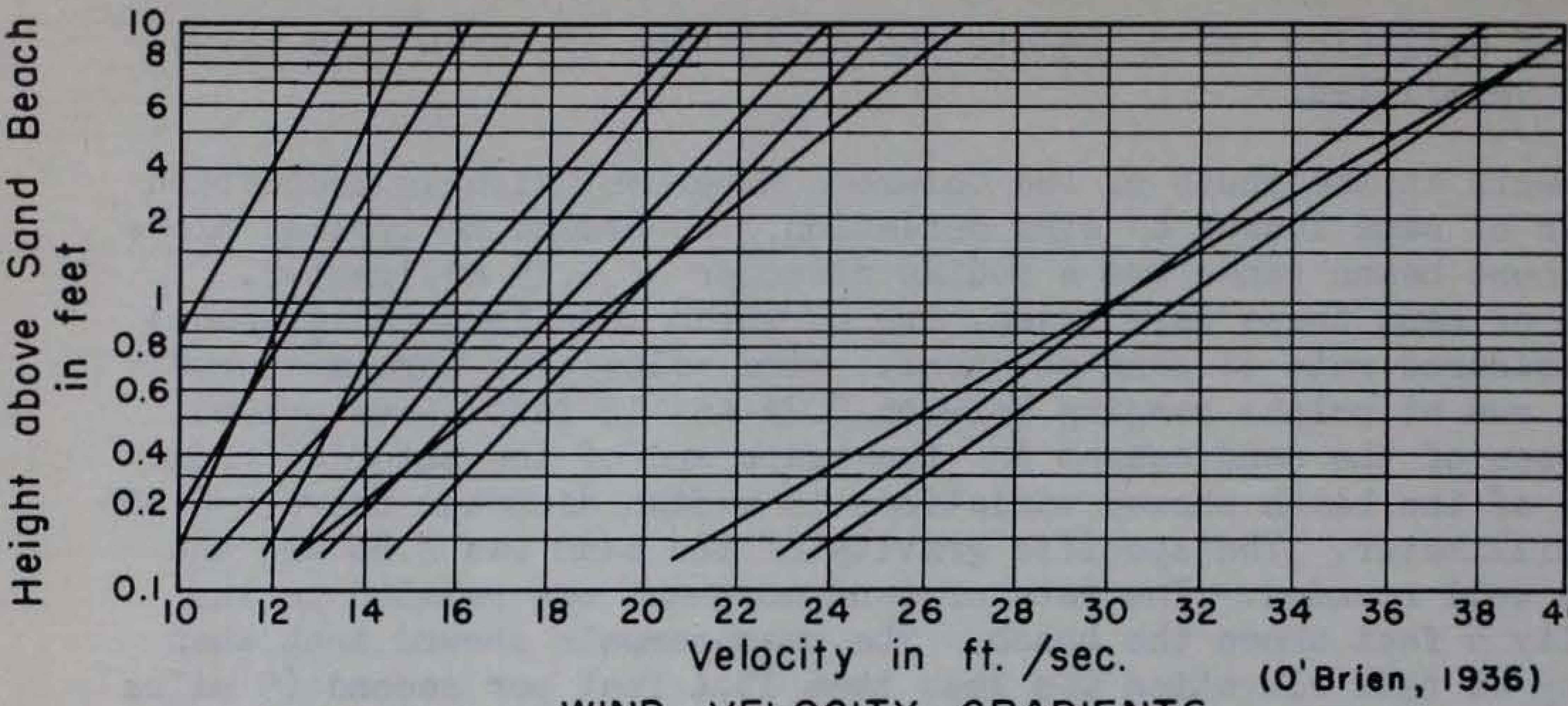
2.342 <u>Movement Offshore</u> - It has been observed that changes in the shore profile occur with changes in water depth or in wave characteristics. Profile adjustment due to change in water depth is relatively slow, and therefore, minor with respect to single tidal cycles. Measurable change has been detected when comparisons are made with water level expressed as daily mean sea level. The profile adjustment due to change in wave characteristics is rapid, though usually temporary. A single storm of a few hours duration may cause a major change in profile. Profile changes may be ascribed primarily to the onshore or offshore shifting of beach and bottom material. Generally, the shift is offshore as the water level rises or the waves steepen, and onshore as the water level lowers or the waves are flatter. Laboratory studies indicate that the critical wave steepness defining the boundary between onshore and offshore movement is in the order of $H_o/L_o = 0.025$. This has not yet been confirmed in nature. The continual onshore and offshore shifting of material results in a longshore movement of material at a sluggish rate and may be classed as a type of transport, the relative importance of which is unknown at this time.

The quantity of material lost to the offshore depths cannot in itself be determined in the light of present knowledge. It is possible that as information on material sorting with respect to slope and wave characteristics is developed, equations may be evolved by which this possibly important avenue of material loss may be evaluated. At present, it can only be assumed as the amount of loss remaining after all known losses have been subtracted. As the rate of material supply into an area is increased to exceed the transport capacity out of the area, or as the transport capacity along a beach decreases, either sediments accumulate along the coast line or losses occur to the offshore depths. As these deposits reduce the depth, the beach slope assumes a profile governed by the littoral forces and the beach material. Assuming material characteristics to remain constant in gradation, the profile of equilibrium would be reached when all of the beach material has been sorted roughly. Each size gradation assumes its characteristic slope, depending on the wave competence, between minimum and maximum depth limits governed by the material size gradation. Continued excess supply would advance the bern seaward without appreciable change in profile, causing deposit of sediments in greater depths.

2.343 <u>Losses in Submarine Canyons</u> - The existence of a submarine canyon in the littoral zone provides a respository for important losses of material into the offshore depths. When combined with a jetty or breakwater, the submarine canyon may constitute an essentially complete littoral barrier by drawing off all material passing around the jetty or breakwater. Comparative surveys have been in insufficient detail to enable determination of the extent of the losses of littoral material into a submarine canyon.

2.344 <u>Losses by Deflation</u> - As a beach widens and the expanse of permanently dry sand increases, the losses by deflation (removal by wind action) increase, generally resulting in the development of a dune belt immediately behind the beach. Rates of loss by deflation are generally difficult to determine. In some instances loss can be determined by measurement of the changes in dune size between successive surveys. Such a measurement would generally be more costly than the information would be worth unless the problem of dune control must be considered as well as the losses of material from the beach. In general, losses by deflation are not an important factor in the design of shore structures. However, because of the aspects of dune control, some attempts have been made to devise means of measuring the amount of deflation and to relate the quantities of beach sand moved to the wind velocities.

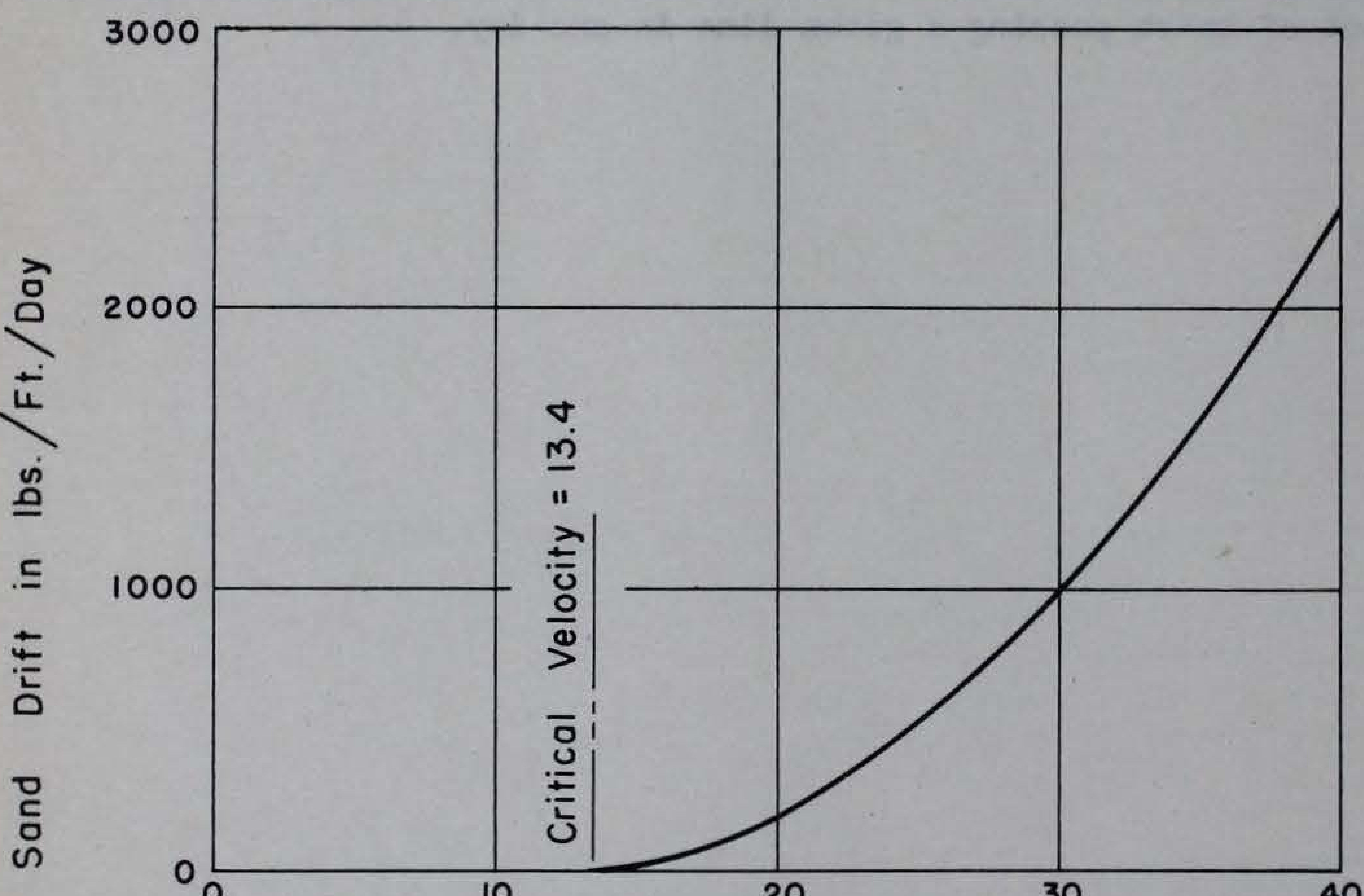
Experiments at the mouth of the Columbia River(96) give an indication of the order of sand losses by wind deflation. According to typical sieve analyses, these beach sands had a median diameter of 0.19 millimeter. Three types of sand traps were tried, two of which were in good agreement as to the measured rate of sand movement. Wind velocities were measured during each run at points ranging between 0.25 and 12 feet above ground. Sieve analyses of the sand caught in the traps and of the material from the surface of the beach showed variations in median diameter between 0.165 and 0.216 millimeter. The specific gravity of the sand was 2.65 and the grains were well rounded. The rate of sand movement was related to the wind velocity 5 feet above the beach. The measurements showed that when the velocity at this elevation was less than 13.4 feet per second (9 miles per hour) no movement of sand occurred, but movement was general at this velocity and above. Figure 60 shows wind velocity gradients taken during typical runs. Figure 61 shows the relation between wind velocity and rate of sand movement. The rate of movement is in terms of the number of pounds per linear foot of beach passing a given line in one day.



(O'Brien, 1936)

WIND VELOCITY GRADIENTS

FIGURE – 60



RELATION BETWEEN WIND VELOCITY AND RATE OF SAND MOVEMENT

(O'Brien, 1936)

FIGURE – 61

CHAPTER 3

PLANNING ANALYSIS

3.1 GENERAL - In selecting the shape, size, and location of works the objective should be to design an engineering work which will accomplish the desired results most economically and with full consideration of its effects on adjacent shore lines. The cost of maintenance, as well as interest on and amortization of the first cost, must always be evaluated. If any plan considered would result in elongating or preventing the elongation of the existing problem area, the economic effect of each such consequence should likewise be evaluated. A convenient yardstick for comparing various plans is the total cost per year per foot of shore protected. The following sections describe the most common engineering solutions now used to meet functional requirements, and give guides for their application.

3.2 SEAWALLS, BULKHEADS AND REVETMENTS

3.21 FUNCTIONS - Seawalls, bulkheads and revetments are structures placed parallel, or nearly parallel, to the shore line, separating a land area from a water area. The primary purpose of a bulkhead is to retain or prevent sliding of the land, with the secondary purpose of affording protection to the fill against damage by wave action. The primary purpose of a seawall or revetment is to protect the land and upland structures from damage by wave forces, with incidental functions as a retaining wall or bulkhead. There are no really sharp distinctions between the three structures, and many cases exist where the same type of structure in different localities bear different names. Thus, it is difficult to say whether a stone or concrete facing designed to protect a vertical scarp is a seawall or a revetment, and often just as difficult to determine whether a retaining wall subject to wave action should be termed a seawall or bulkhead. All these structures, however, have one feature in common, in that they separate land and water areas, and are generally used where it is necessary to maintain the shore in an advanced position relative to that of adjacent shores, where there is a scant supply of littoral material,to the area and little or no protective beach, as along an eroding bluff, or where it is desired to maintain a depth of water along the shore line as for a wharf.

3.22 LIMITATIONS - These structures afford protection only to the land immediately behind them, and none to adjacent areas up or down coast. When built on a receding shore line, the recession will continue downdrift. In addition, any tendency for loss of beach material in front of such a structure may well be intensified; where it is desired to maintain a beach in the immediate vicinity, companion works may be necessary.

3.23 FUNCTIONAL PLANNING OF THE STRUCTURE - The planning of these structures is a relatively simple process, since their functions are restricted to the maintenance of fixed boundaries. The features which must be analyzed

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in adequately planning such a structure are: its use and its overall shape, its location with respect to the shore line, its length, its height, and often the ground level in front of the wall.

3.24 USE OF THE STRUCTURE - The shape to be chosen must be determined by consideration of desired collateral uses. Face profile shapes may roughly be classed as: vertical or nearly vertical face, sloping face, convex curved face, concave curved and re-entrant face, or stepped face. Each silhouette has certain functional applications and so may be used in comination with any other if diverse functional criteria are to be met.

A vertical or nearly vertical face structure lends itself to use as a quay wall or landing place, where other shapes need to be provided with additional work to be so adapted. In addition, especially where a relatively light structure is required, a vertical face (of sheet pile, for example) may often be constructed more quickly and more cheaply than any other type. This may be an important consideration where emergency protection is needed. Against wave attack, and specifically in regard to reduction of overtopping, a vertical face is more effective than any but the concave curved and re-entrant face.

A backward sloping or convex curved face is the least effective of all types against wave attack for a given height of structure. It is, however, more adaptable to use as emergency protection, (sand bag, or dumped stone mounds, for example) than the other types. Actually the use of such a face type should be restricted to those areas in which wave overtopping is not a problem, or where esthetic, emergency, or structural considerations prohibit the use of other shapes.

Concave curved or re-entrant faced structures are the most effective in reducing wave overtopping to a minimum. Where the structure's crest is to be used (for a roadway or promenade for example), a wall so designed will be of the most desirable shape for protecting the crest. This is especially true if the beach in the vicinity is narrow or entirely absent, or if the water level is over the structures base.

A stepped face provides the most ready access to beach areas from protected areas and in addition acts to disrupt the scouring action of the wave backwash.

3.25 LOCATION OF STRUCTURE WITH RESPECT TO SHORE LINE - In general, a seawall or bulkhead would be constructed along that line landward of which further recession of the shore line is not to be permitted. Where an areas is to be reclaimed, a wall may be constructed along the seaward edge of the reclaimed area. (A seawall constructed in the water, isolated from shore, becomes an offshore breakwater).

3.26 LENGTH OF STRUCTURE - A seawall, bulkhead or revetment protects no more than the land and improvements immediately behind it. No protection is afforded either to upcoast or downcoast areas as in the case with beach fills. It must be emphasized that in the usual case where erosion may be expected to occur at either end of a structure, wing walls or tie-ins to adjacent land features must be provided to prevent flanking and possible progressive failure of the structure from the ends. Short term beach changes due to storms, as well as seasonal and annual changes must be considered. It must be remembered that changes updrift from such a structure will continue unabated after the wall is built, and that downdrift, these changes will be, if anything, intensified.

3.27 HEIGHT OF STRUCTURE - Seawalls, bulkheads and revetments can be built to such a height that no water would overtop the crest of the structure regardless of wave attack, though it is frequently not economically feasible to do so. Wave run-up and overtopping criteria on which the height of structure should be based are not completely definitive at the present time (1956); however, some tests to determine the relation of wave run-up and quantity of overtopping water to various wave parameters have been carried out and others are presently under way at several laboratories. The results of tests conducted to data are presented in the following two sections. These data will be supplemented as additional test results become available.

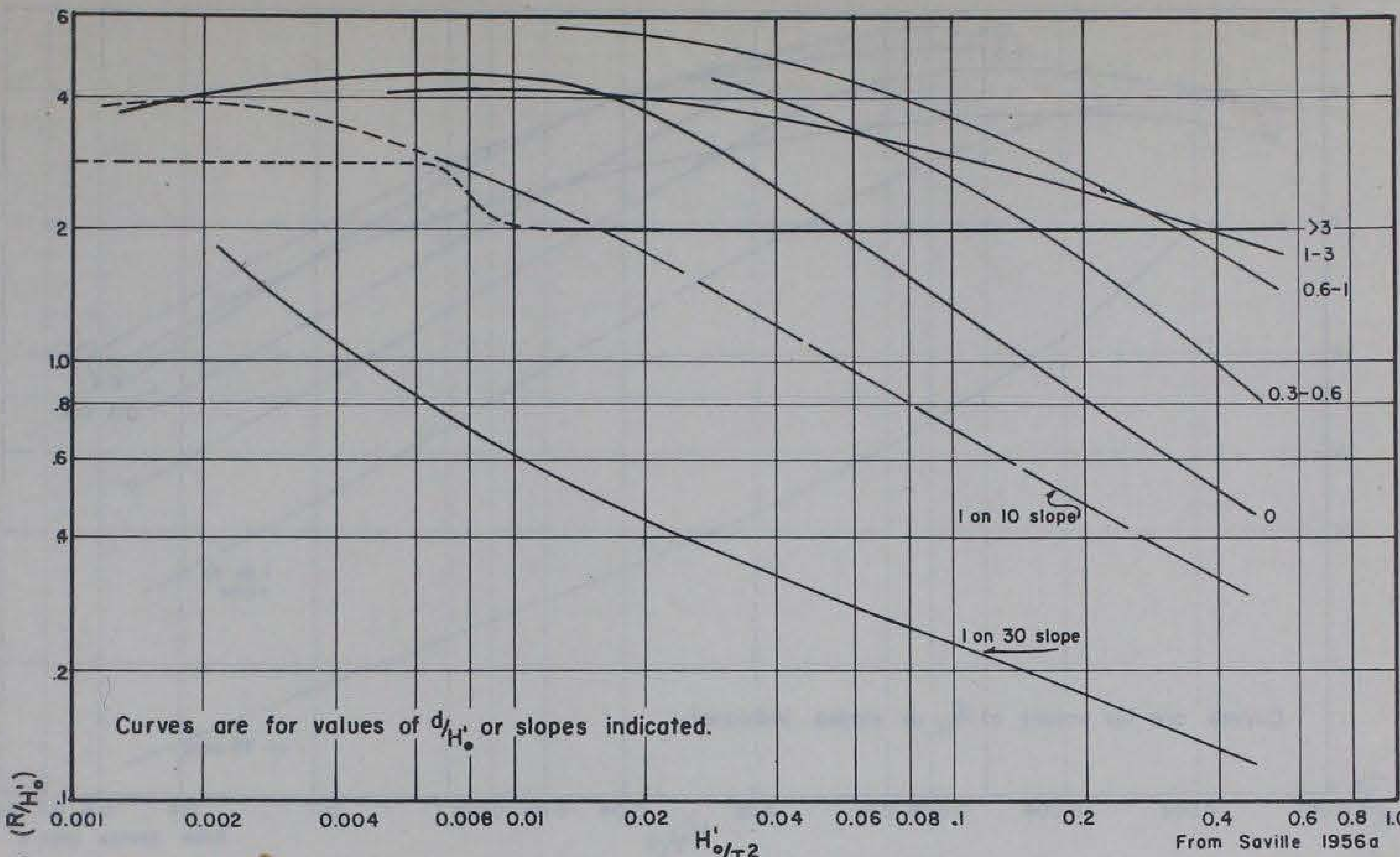
3.271 Wave Run-up[150] - The vertical height to which water from a breaking wave will run up on a given structure determines the top elevation to which the structure must be built to prevent wave overtopping and resultant flooding on the landward side and to prevent possible structural damage by rear-face erosion. Crest elevation has usually been determined by applying a rather arbitrarily selected ratio of run-up to wave height (R/H). A value of R/H equal to 1.5 appears to be that most frequently used, although values as low as 0.9 and as high as 2 have been suggested for particular designs. It has been generally recognized that the correct value would depend on the structure characteristics (i.e., shape and roughness), the water depth at its toe, and the wave characteristics, but sufficient data have not been available to accurately determine these relationships. The value of 1.5 is the one which is given for general use, for structures in, or landward of, the breaker zone. In addition to the need for data for the more conventional types of shore structures, the increasingly frequent use of a beach of artificially placed fill as a protective structure has led to the need for data on relatively gentle slopes (i.e., 1 on 10 to 1 on 30) to enable an adequate design of beach berm elevations.

The curves in Figures 61A to 61J show relative run-up (the ratio of vertical rise, R, of water on the structure face above still water level to the deep-water wave height H_o') plotted against the ratio of deep-water wave height to the square of the wave period. The latter value (H_o'/T^2) is directly proportional to the dimensionless value of deep-water steepness (H_o'/L_o). (Actually $H_o'/L_o = H_o'/(gT^2/2\pi)$, and so $H_o'/T^2 = gH_o'/2\pi L_o = 5.12\ H_o'/L_o$.) However, the term H_o'/T^2 is somewhat easier to use, since the wave period is usually directly available (either from forecasts or from measurements), while the wave length generally must be computed. For the various structures tested the curves on each plot are identified as falling within various ranges of depth-wave height ratio values, where the depth (d) indicates the depth at the toe of the structure in relation to the still water level. A very definite relationship is apparent throughout, with the run-up increasing with depth at the structure until a depth-height ratio of between about 1 and 3 is reached, and then apparently decreasing somewhat. Actually the apparent decrease as the larger depths are reached appears generally quite small (particularly in the range of greatest interest of values of H_o'/T^2 greater than about 0.05) and may in fact be due only to the difference in scatter of the various points. The initial increase in run-up with depth may be explained by the location of the breaking wave. When the wave breaks offshore from the structure it acts over a relatively large distance of gentle beach slope before reaching the actual structure. It may be thought of then as essentially acting over a gentler (total combined) slope than when it breaks more nearly on the structure itself — and consequently has a somewhat lower run-up. In most cases the curves derived for the 1 on 10 slope and 1 on 30 slope are also shown to aid in comparison. These curves show the run-up relation which would exist for the (1 on 10 and 1 on 30) beach slopes were the test structure not in place. Were the structure depth to be decreased still further (i.e., the structure toe, placed above the still water level), the resulting run-up curves would approach still more nearly the curve for the 1 on 10 slope. The depth-wave height ratio (d/H_o') ranges chosen were purely arbitrary.

The curves for the smooth slopes were based on a large number of observations, and are felt to be fairly accurate; the curves for the other structures (vertical, curved, recurved, stepped, riprap) are based on a relatively small number of observations and consequently should be regarded more as indications only. To help in interpreting these latter curves, the actual observed points have been plotted on the figures, although they have been omitted from the smooth slope figures. Reference to the original publication[150] may be made if determination of point location is desired. In addition, all data for the non-smooth slope structures and some for the smooth slopes are tabulated in reference 151.

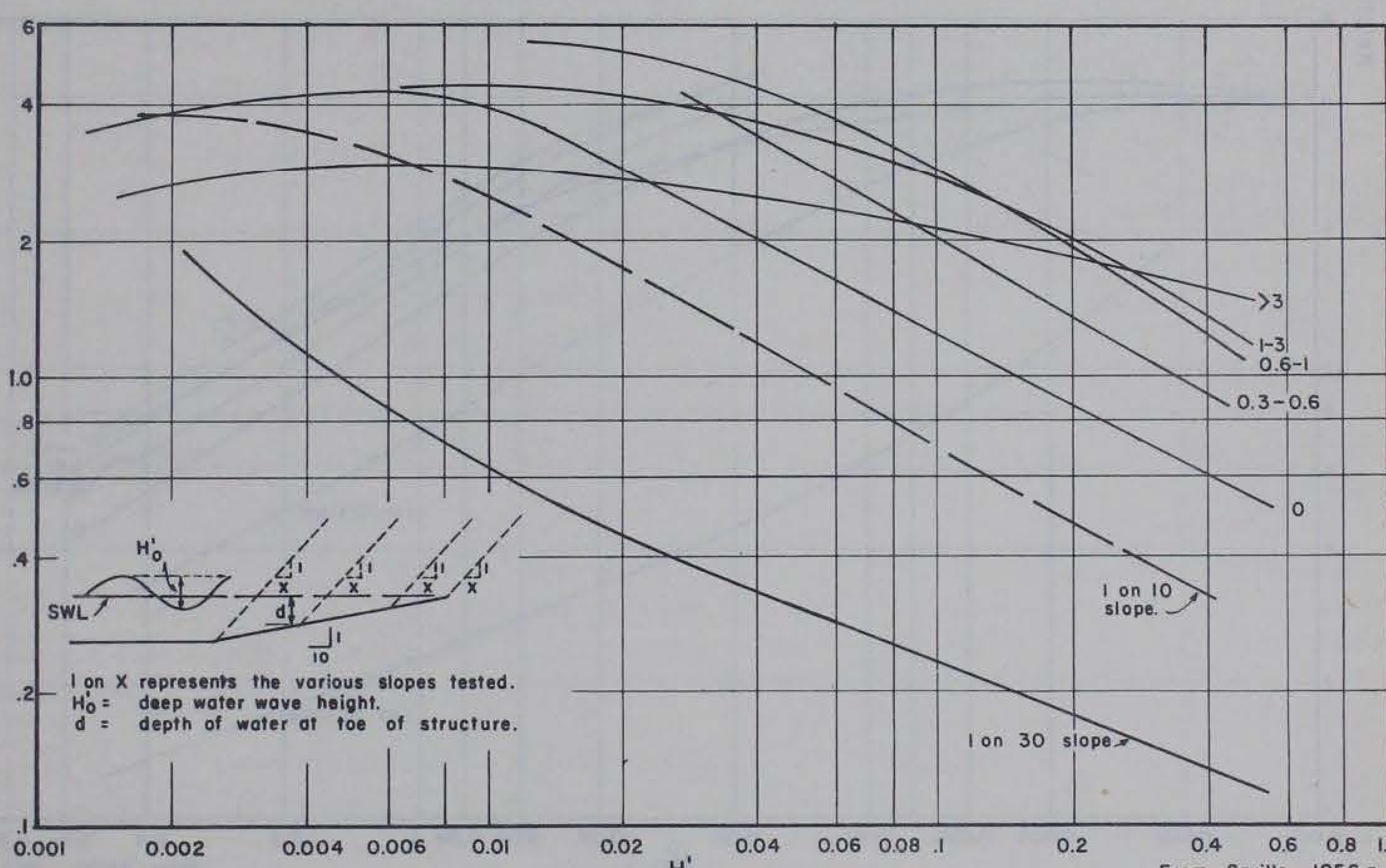
The data for the smooth slopes for the particular case of deep water at the structure toe (i.e. $d/H_o' >$ about 3) have been combined by Saville[152] with additional data obtained by Granthem[153] to give somewhat more easily used curves (for particular application to dam faces in inland reservoirs where deep-water conditions usually obtain). These curves are shown in Figure 61K.

In interpreting and using the data presented above, the designer must bear in mind that data were obtained for <u>impermeable slopes only</u>, with the exception of the single riprap test (Figure 61J), which involved only a single layer of riprap over an impermeable 1 on $1\frac{1}{2}$ slope. In this test the riprap layer should be regarded almost purely as increasing the roughness factor rather than the permeability. Observations indicate that walls of relatively large permeability, as for riprap or rubble-mound structures, generally serve to decrease the amount of run-up considerably. Tests presently (1956) under way at the Beach Erosion Board laboratory, however, are indicating that the permeability does have to be relatively large, and that sand beaches, for example, behave (as far as run-up considerations are concerned) essentially the same as solid impermeable structures. The run-up



1 On 1½ Slope, With 1 On 10 Beach Slope From Toe Of Structure.

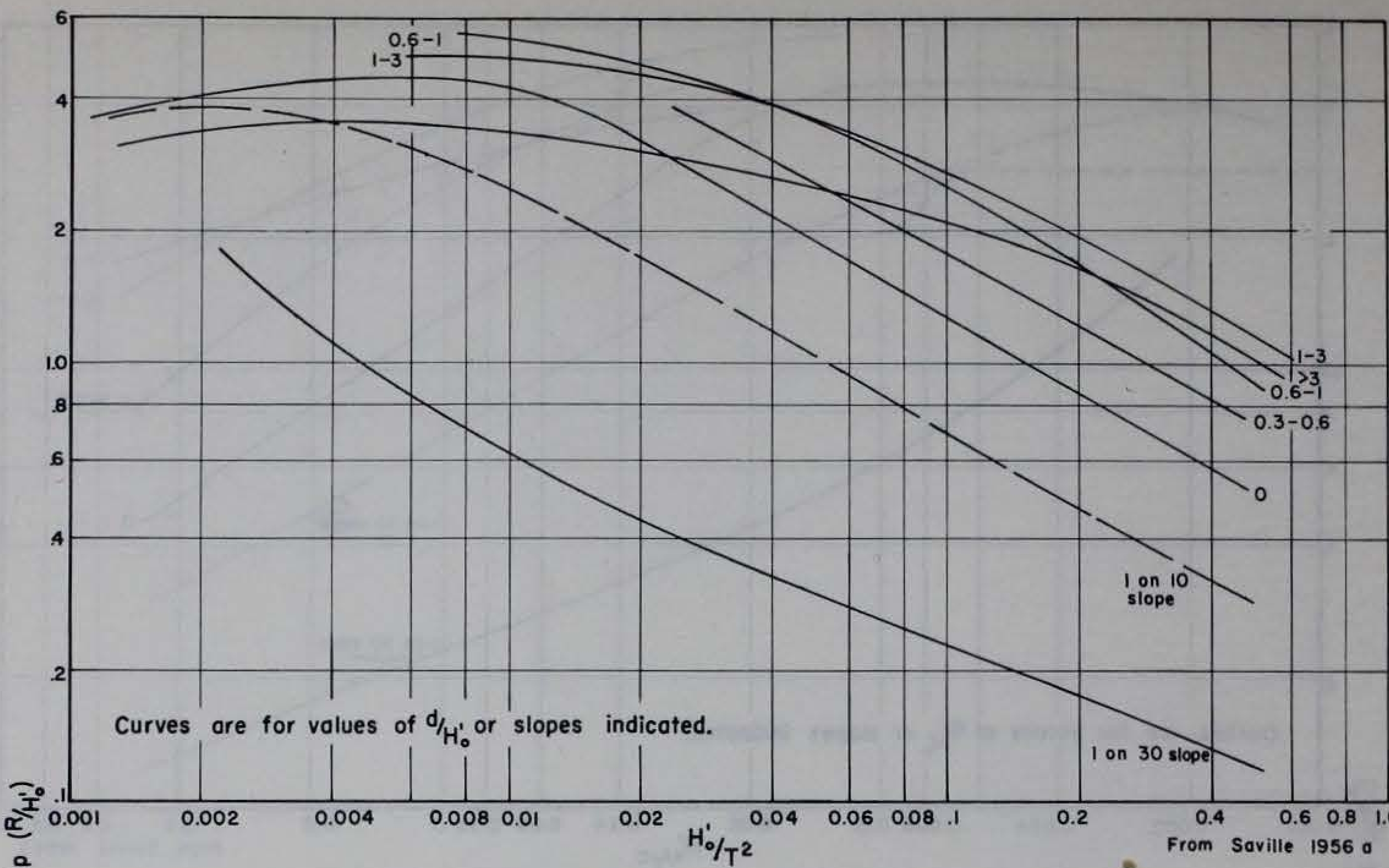
FIGURE 61-A



1 On 2¼ Slope, With 1 On 10 Beach Slope From Toe Of Structure.

FIGURE 61-B

RUN-UP AS A FUNCTION OF STRUCTURE DEPTH AND WAVE STEEPNESS.



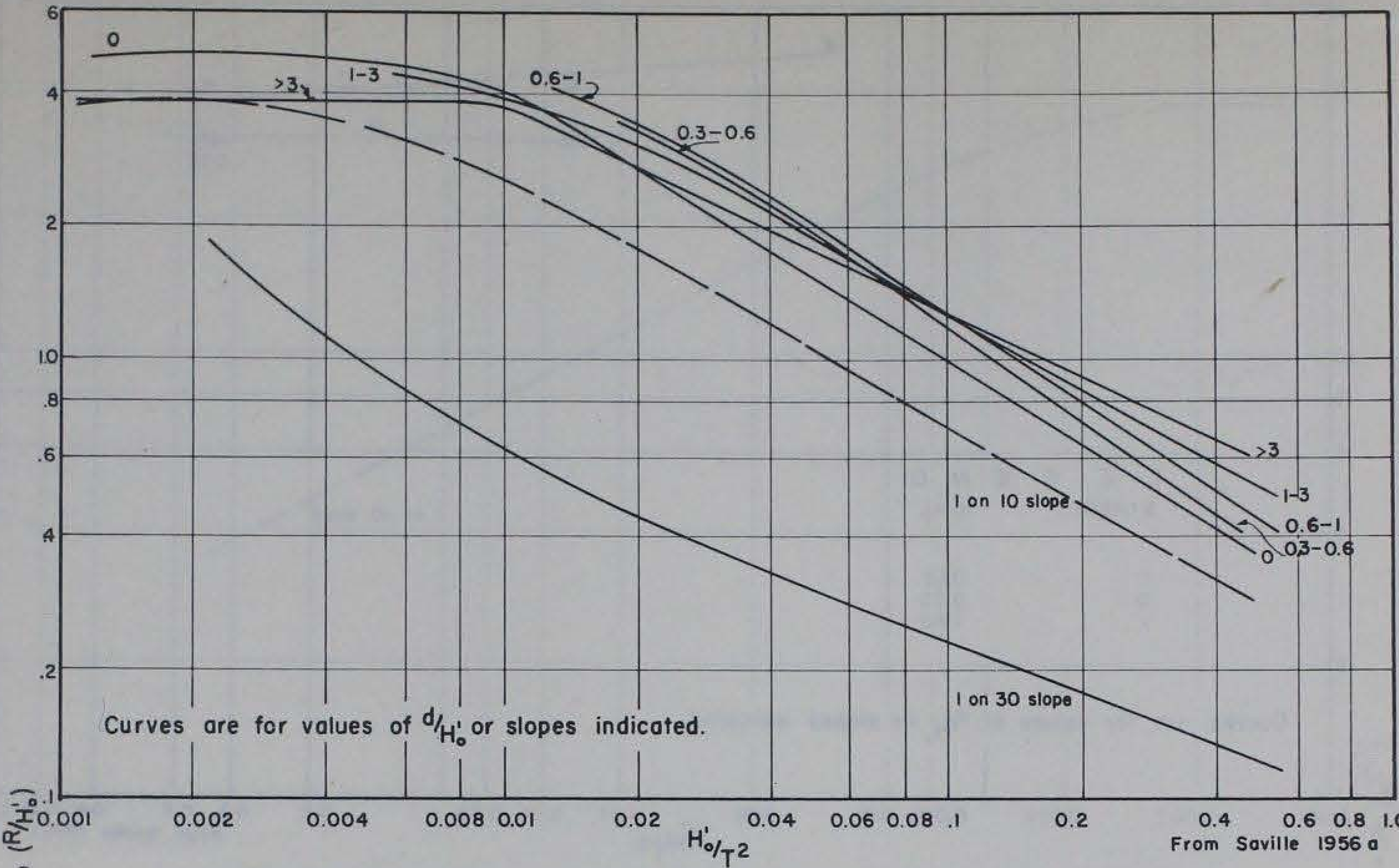
1 On 3 Slope, With 1 On 10 Beach Slope From Toe Of Structure.

FIGURE 61-C

1 On 4 Slope, With 1 On 10 Beach Slope From Toe Of Structure.

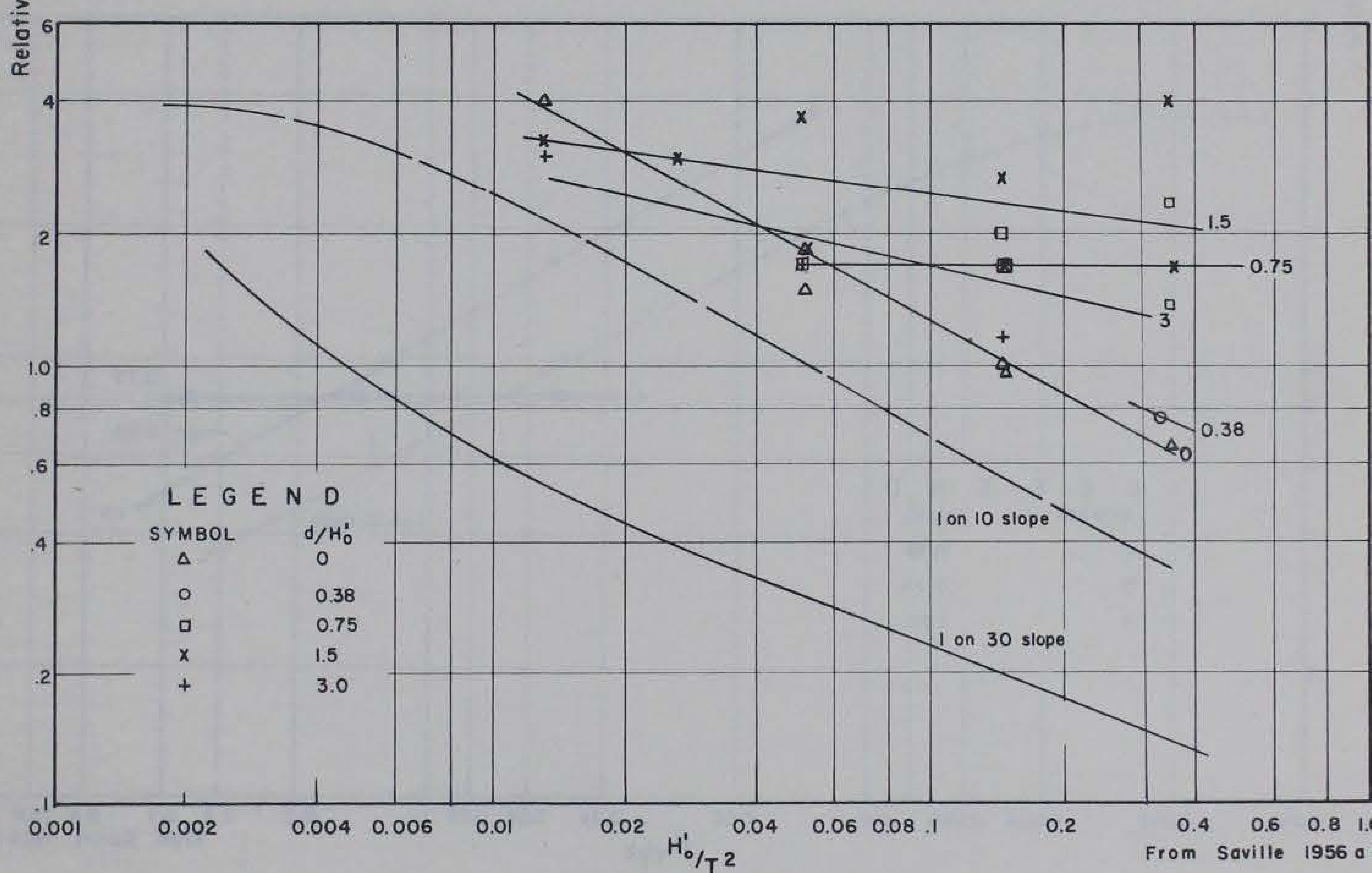
FIGURE 61-D

RUN-UP AS A FUNCTION OF STRUCTURE DEPTH AND WAVE STEEPNESS.



I On 6 Slope, With I On 10 Beach Slope From Toe Of Structure.

FIGURE 61-E



Vertical Wall With I On 10 Beach Slope From Toe Of Structure.

FIGURE 61-F

RUN-UP AS A FUNCTION OF STRUCTURE DEPTH AND WAVE STEEPNESS.

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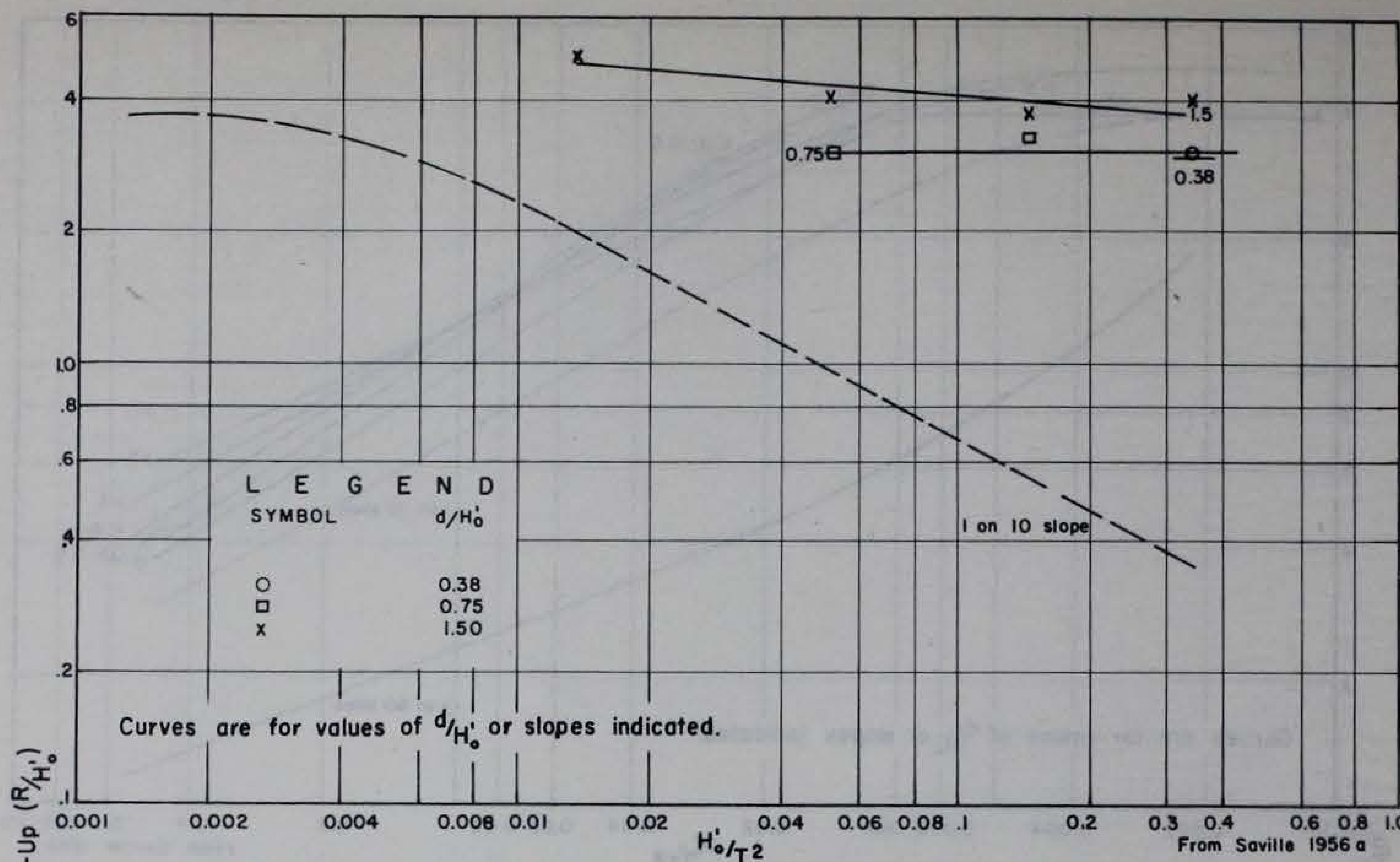
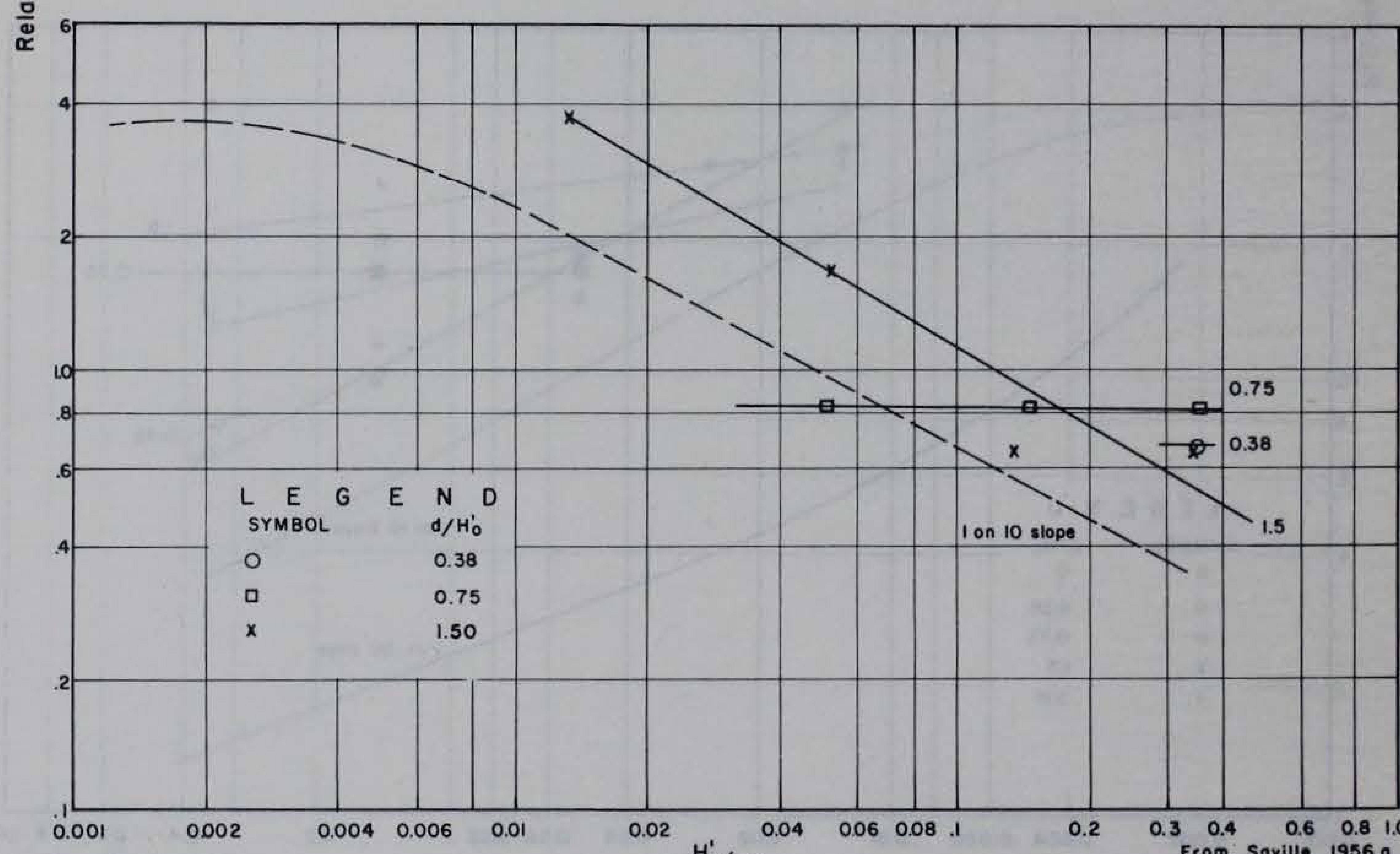


Curved Wall-Galveston Type, With I On 10 Beach Slope From Toe Of Structure,

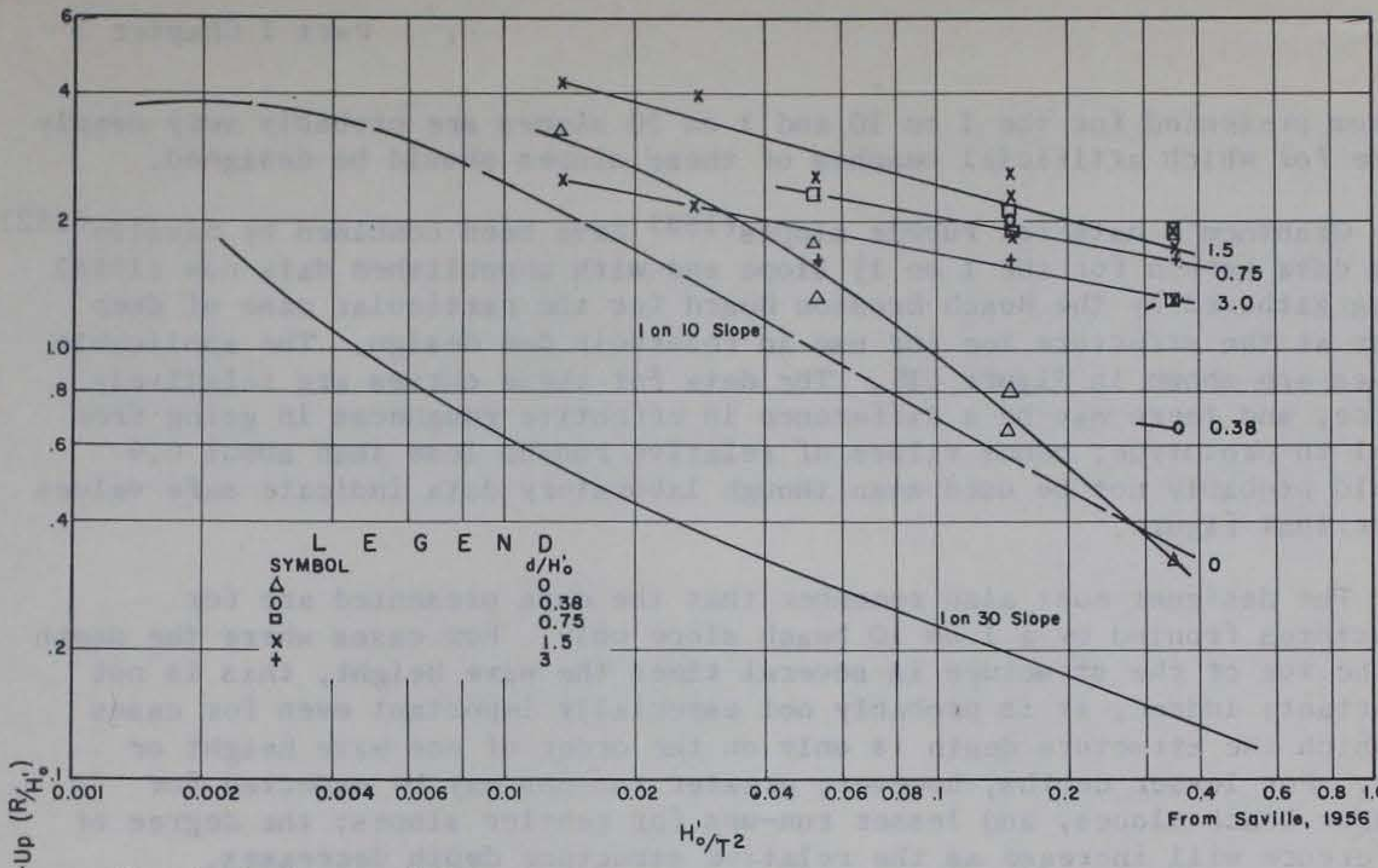
FIGURE 61-G



Recurved Wall—Galveston Type, With Top Recurvature Added

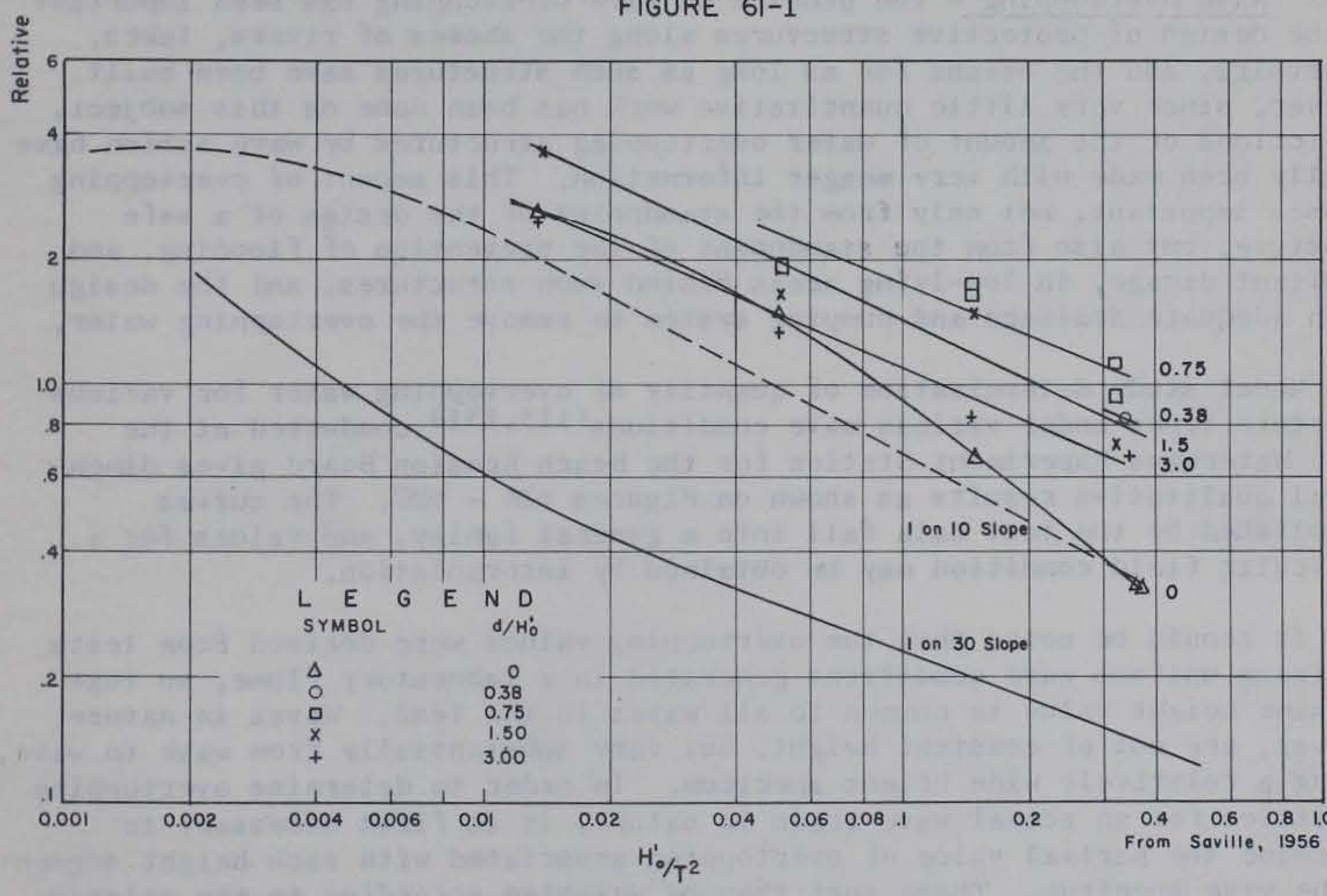
FIGURE 61-H

RUN-UP AS A FUNCTION OF STRUCTURE DEPTH AND WAVE STEEPNESS.



Step-faced Wall I on $1\frac{1}{2}$ Slope With I on 10 Beach Slope From Toe of Structure.

FIGURE 61-I



Riprap Covered Wall, I on $1\frac{1}{2}$ Slope With I on 10 Beach Slope From Toe of Structure.

FIGURE 61-J

RUN-UP AS A FUNCTION OF STRUCTURE DEPTH AND WAVE STEEPNESS.

curves presented for the 1 on 10 and 1 on 30 slopes are probably very nearly those for which artificial beaches of these slopes should be designed.

Granthem's data for rubble slopes(153) have been combined by Saville(152) with data herein for the 1 on $1\frac{1}{2}$ slope and with unpublished data now (1956) being gathered by the Beach Erosion Board for the particular case of deep water at the structure toe for use in reservoir dam design. The applicable curves are shown in Figure 61K. The data for these curves are relatively scarce, and there may be a difference in effective roughness in going from model to prototype; hence values of relative run-up less than about 0.9 should probably not be used even though laboratory data indicate safe values under that figure.

The designer must also remember that the data presented are for structures fronted by a 1 on 10 beach slope only. For cases where the depth at the toe of the structure is several times the wave height, this is not important; indeed, it is probably not especially important even for cases in which the structure depth is only on the order of one wave height or more. For lesser depths, however, greater run-ups may be expected for steeper beach slopes, and lesser run-ups for gentler slopes; the degree of difference will increase as the relative structure depth decreases.

3.272 Wave Overtopping - The problem on wave overtopping has been important in the design of protective structures along the shores of rivers, lakes, reservoirs, and the oceans for as long as such structures have been built. However, since very little quantitative work has been done on this subject, predictions of the amount of water overtopping structures by wave action have usually been made with very meager information. This amount of overtopping becomes important, not only from the standpoint of the design of a safe structure, but also from the standpoint of the prevention of flooding, and resultant damage, in low-lying areas behind such structures, and the design of an adequate drainage and pumping system to remove the overtopping water.

Model study determination of quantity of overtopping water for various structure types under various wave conditions(115,151) conducted at the U. S. Waterways Experiment Station for the Beach Erosion Board gives dimensional qualitative results as shown on Figures 62A - 62G. The curves established by the test data fall into a general family, and values for a particular field condition may be obtained by interpolation.

It should be noted that the overtopping values were derived from tests utilizing uniform wave conditions generated in a laboratory flume, so that the same height value is common to all waves in the test. Waves in nature, however, are not of constant height, but vary substantially from wave to wave, having a relatively wide height spectrum. In order to determine overtopping quantities for an actual wave train in nature, it is first necessary to determine the partial value of overtopping associated with each height segment in the wave spectrum. These must then be weighted according to the relative frequency of occurrence of the particular height (as given by statistical analysis of wave frequency in Figure 13, for example) and then combined in order to get the final value of overtopping associated with a wave train of

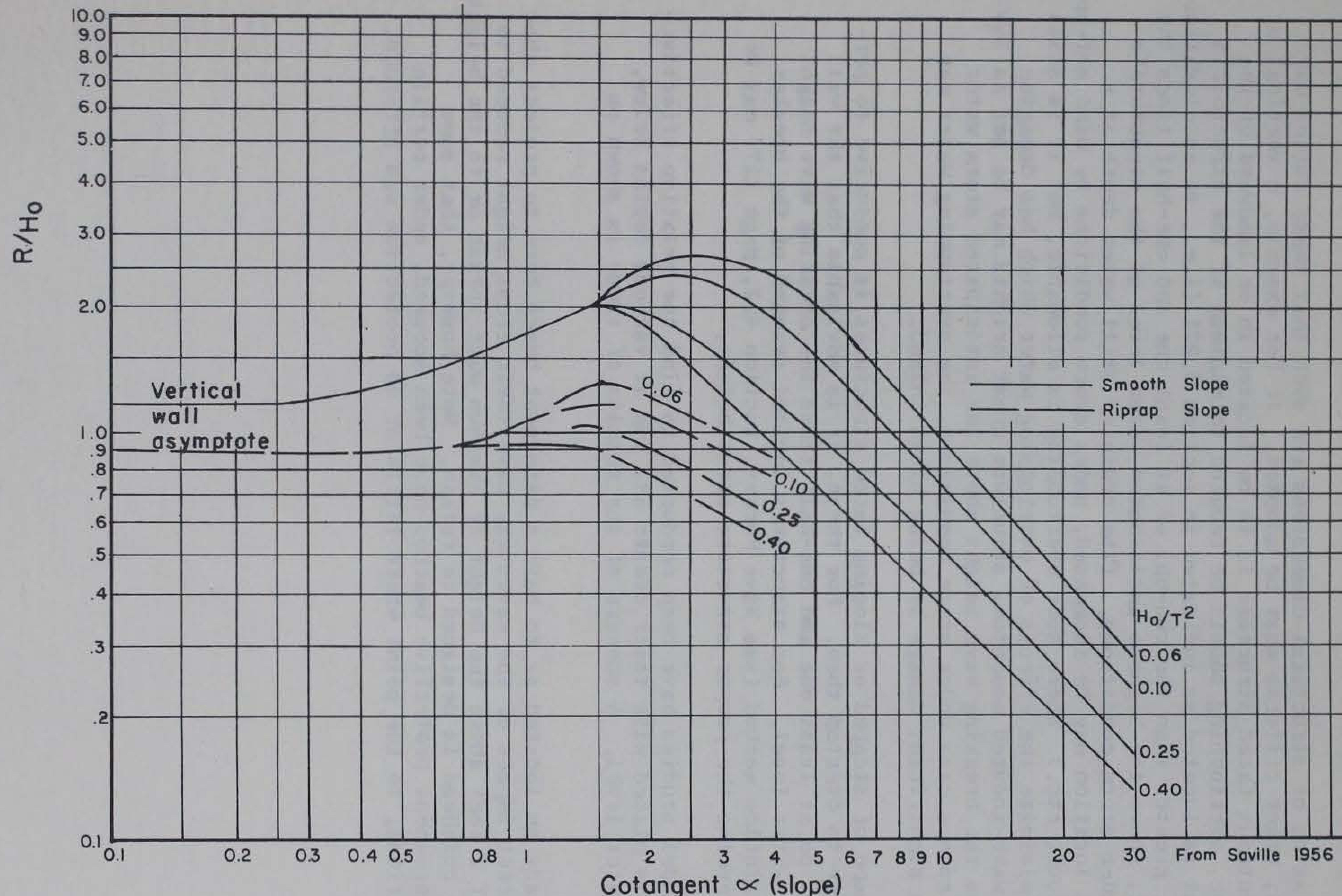


FIGURE 61 K. RELATIVE WAVE RUN-UP (FOR DEEP WATER AT TOE OF SLOPE - $\frac{d}{H_0} > 3$), FOR SPECIFIC $\frac{H_0}{T^2}$ VALUES.

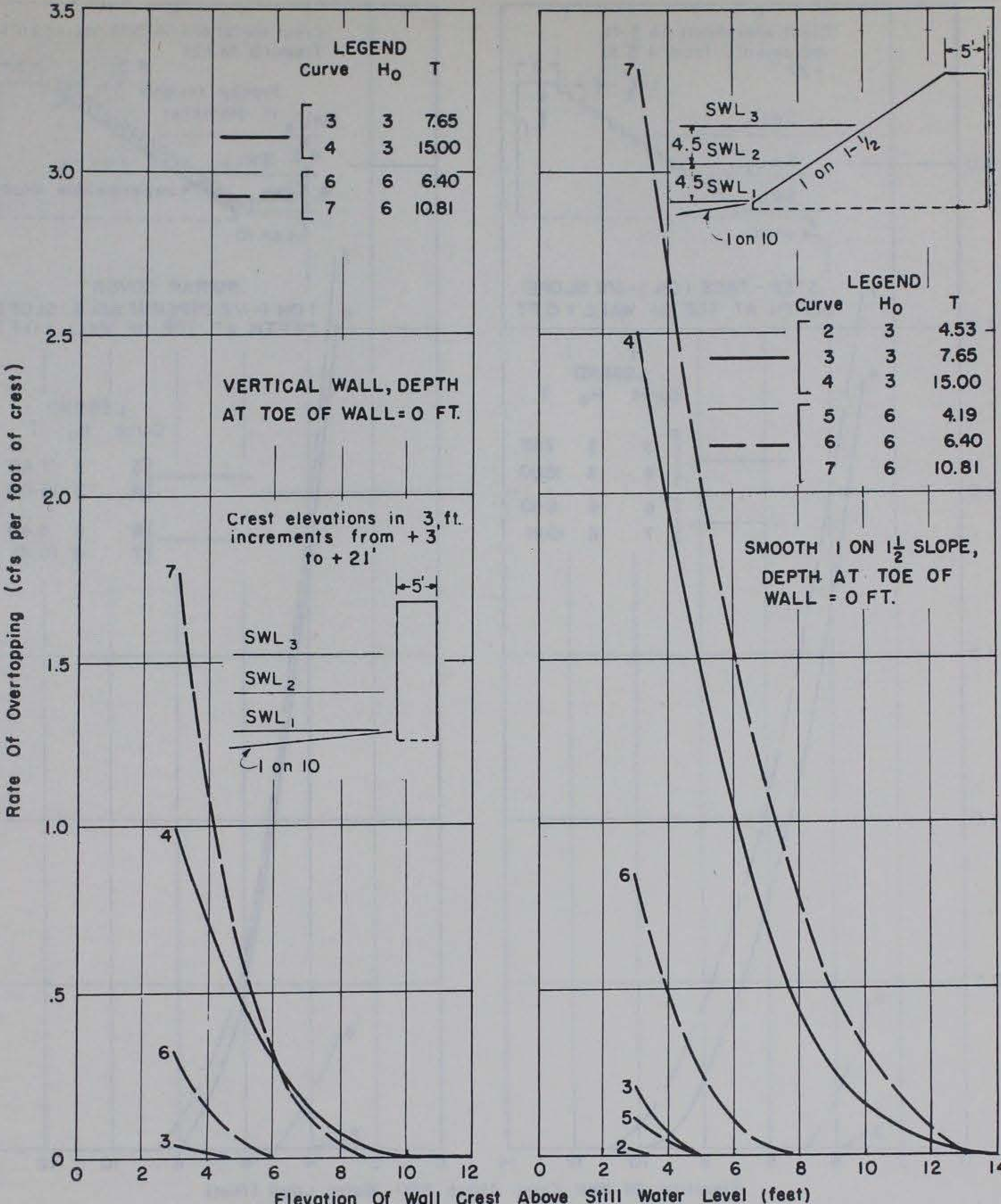
given significant height.

Where waves or structural dimensions are such that these curves may not be used, other criteria must be adopted. If for example, a vertical or curved re-entrant faced structure is to be located in or landward of the breaker zone, overtopping should be reduced to minimum if the structure's crest height is located as indicated in section 3.271 (i.e., to an elevation equal to or greater than the run-up) or at least one and one-half times the wave height, above the highest anticipated water level at the structure's location under storm conditions. (The ordinary still water depth at a structure's location may be increased, under storm conditions by wind set-up, seiches, scour, etc.) Where some overtopping is allowable, but it is still desired to minimize the effects of overtopping water which has damaging horizontal wave-induced momentum, structure crest heights may be set as low as seven-tenths the breaking wave height above the anticipated storm water level. In such a case under storm conditions, the overtopping water may still cause significant damage by being blown inland.

The shape of stepped or sloping faced structures is conducive to permitting water to overtop them. For these, it is advisable that the wall crest height be at least one and one-half times the breaking wave height above storm water level. For structures located seaward of the breaker zone, the Sainflou method (see Wave Forces, section 4.2, page 117) may be used to determine the proper structure crest height.

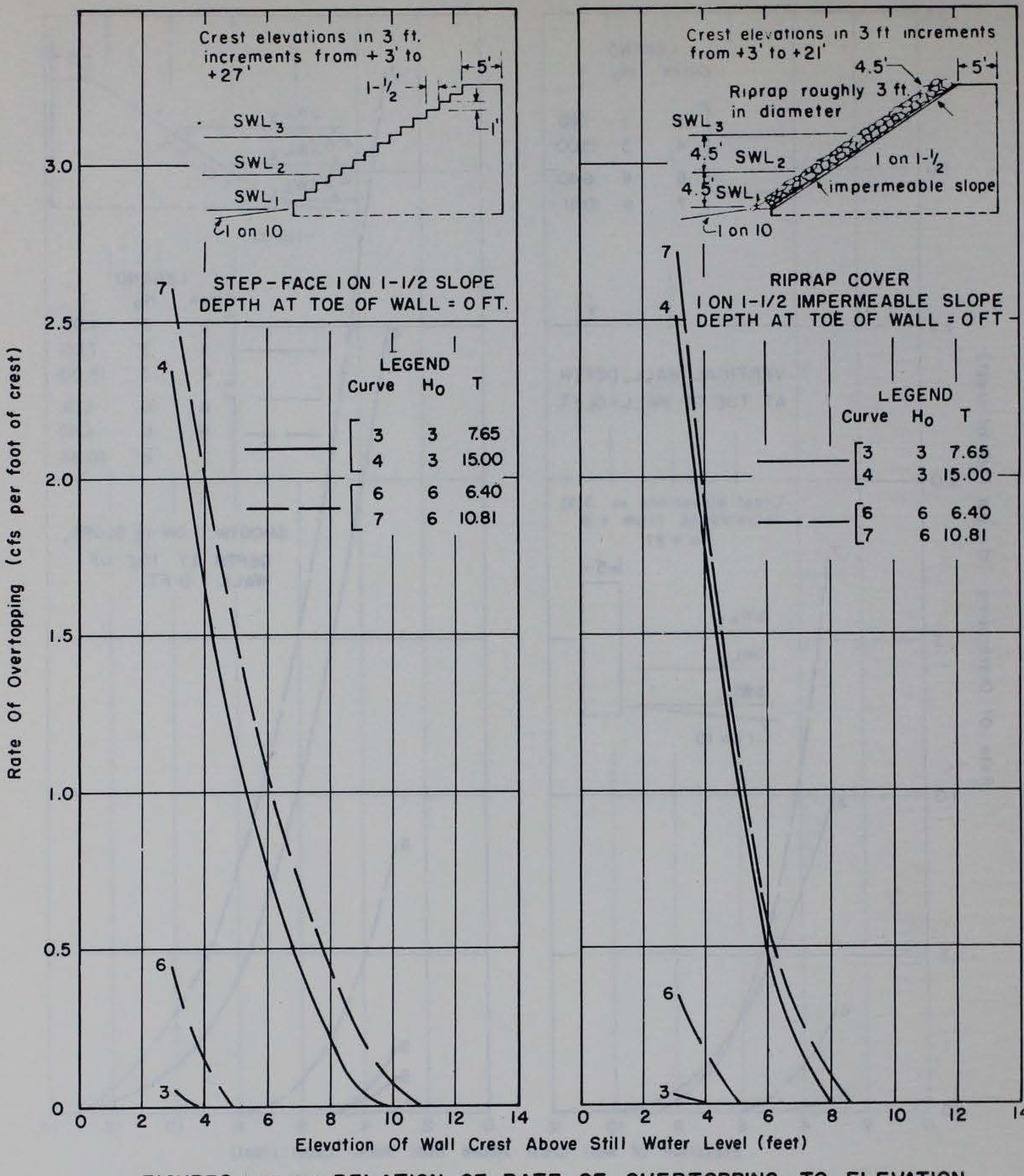
Some model studies have been conducted to find the relative effectiveness of walls placed with their crests at, and at various depths below, the still water level. A summary of the results of these is shown on Figure 63.

Bulkheads so located as to have a permanent beach berm to protect them from the direct impact of the waves may have their crest height reduced to a minimum of 2 feet above the height of maximum wave uprush or to the height of fill the bulkhead is designed to retain. Note however, that some seemingly permanent protective beaches have been scoured, under certain storm conditions, to the point where little or no protection was afforded.

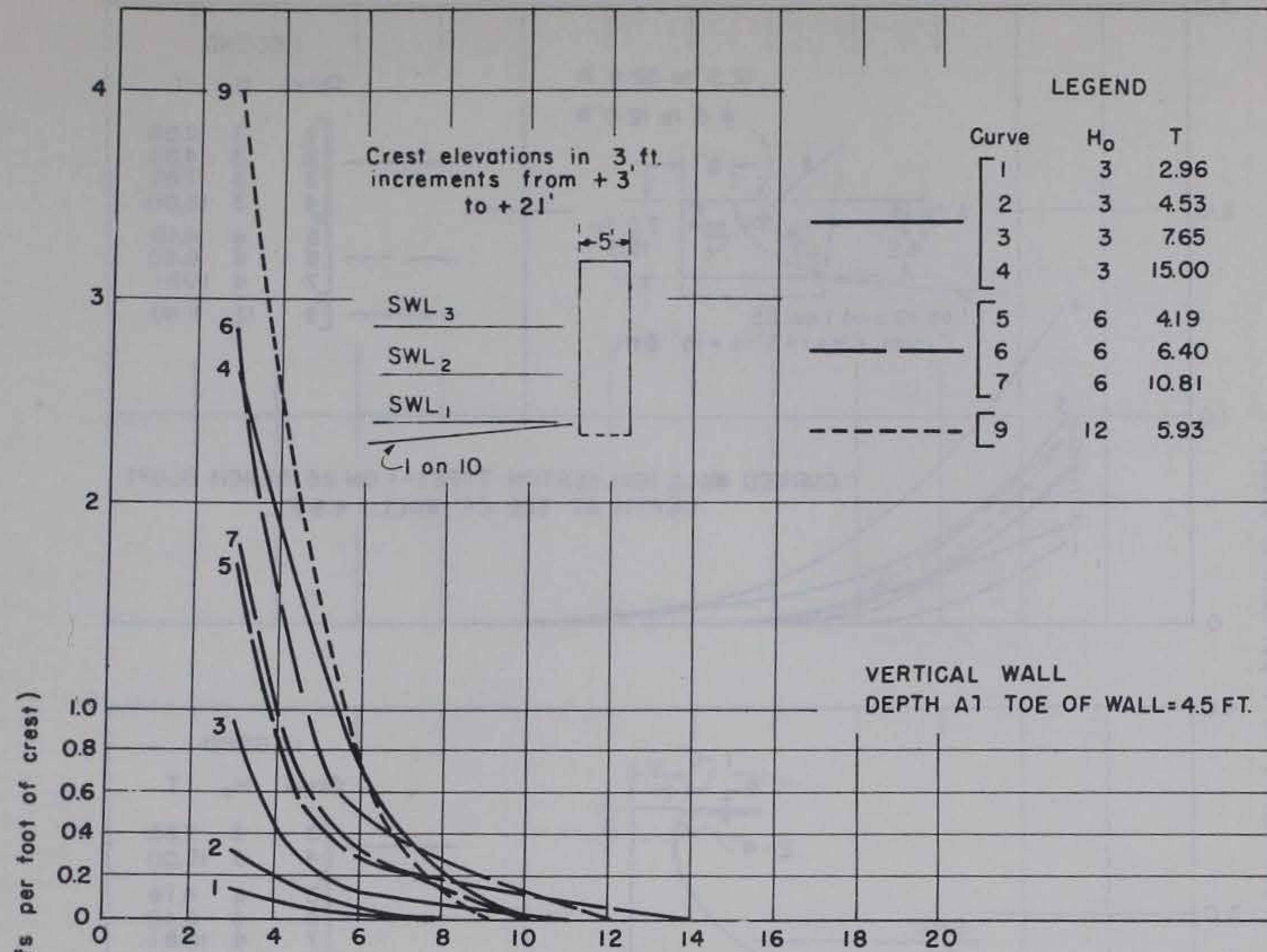


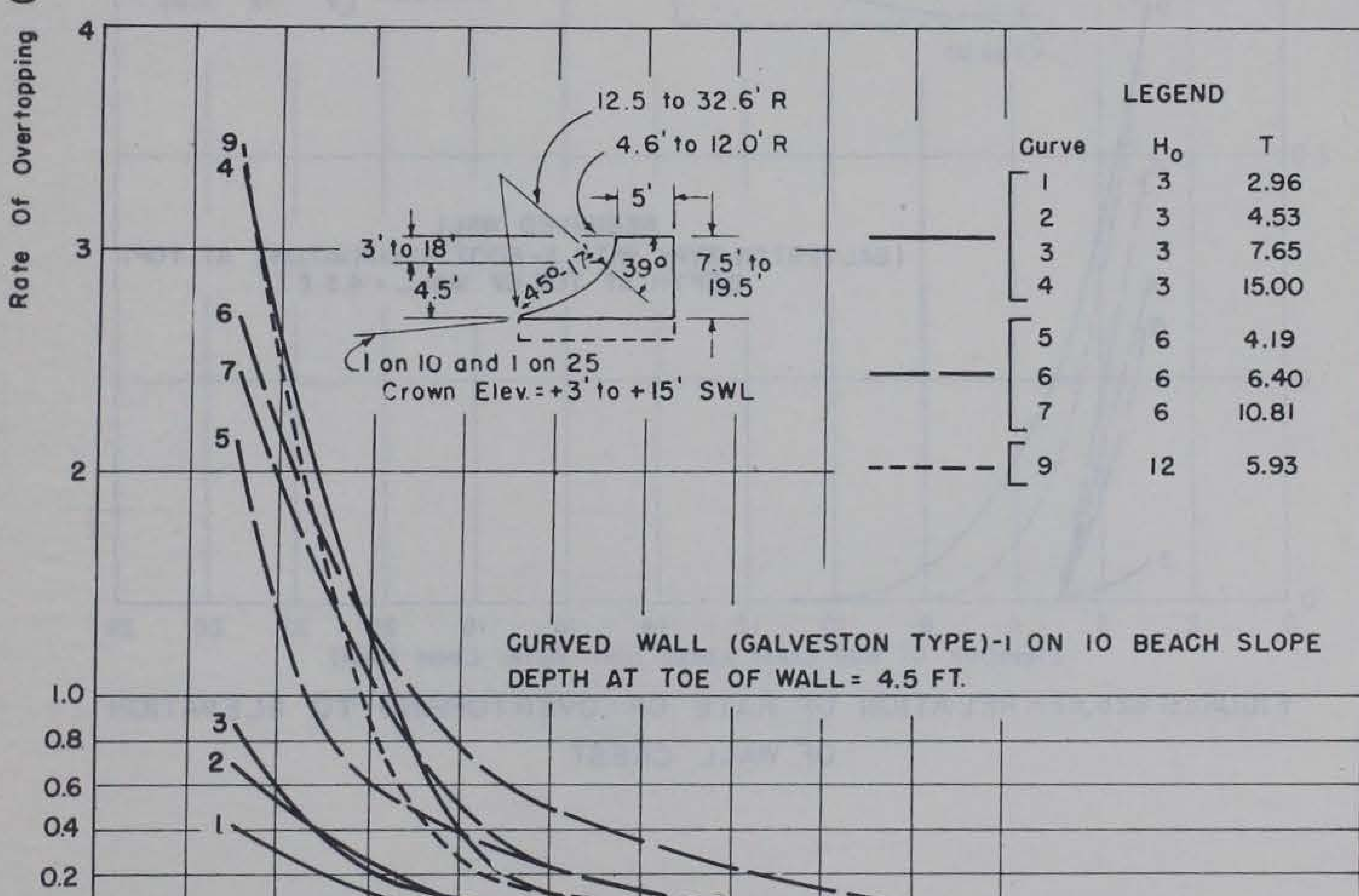
FIGURES 62 A, 62 B RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

February 1957



FIGURES 62C, 62D RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST





FIGURES 62 E, 62 F RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

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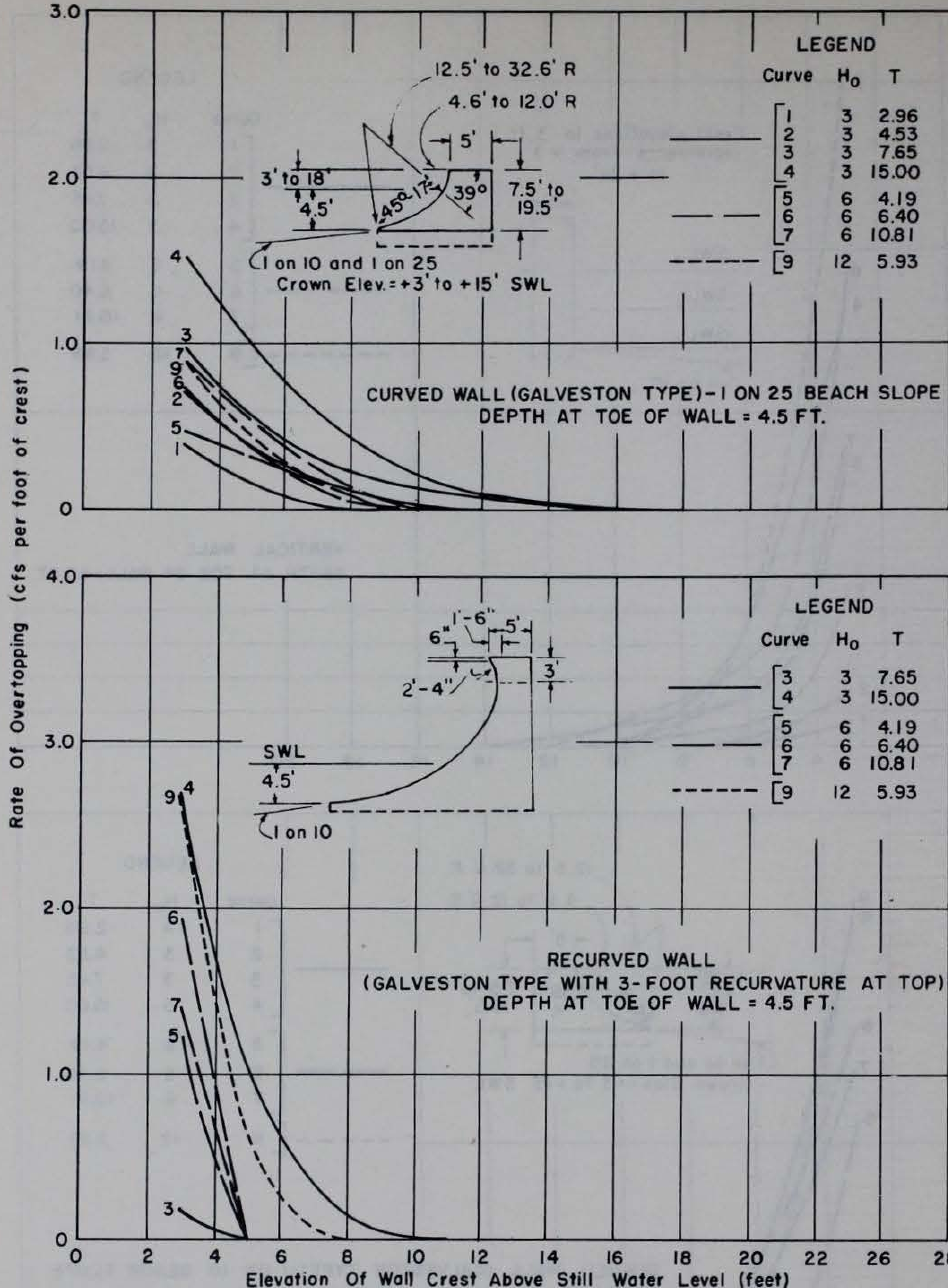

FIGURES 62G, 62H RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

February 1957

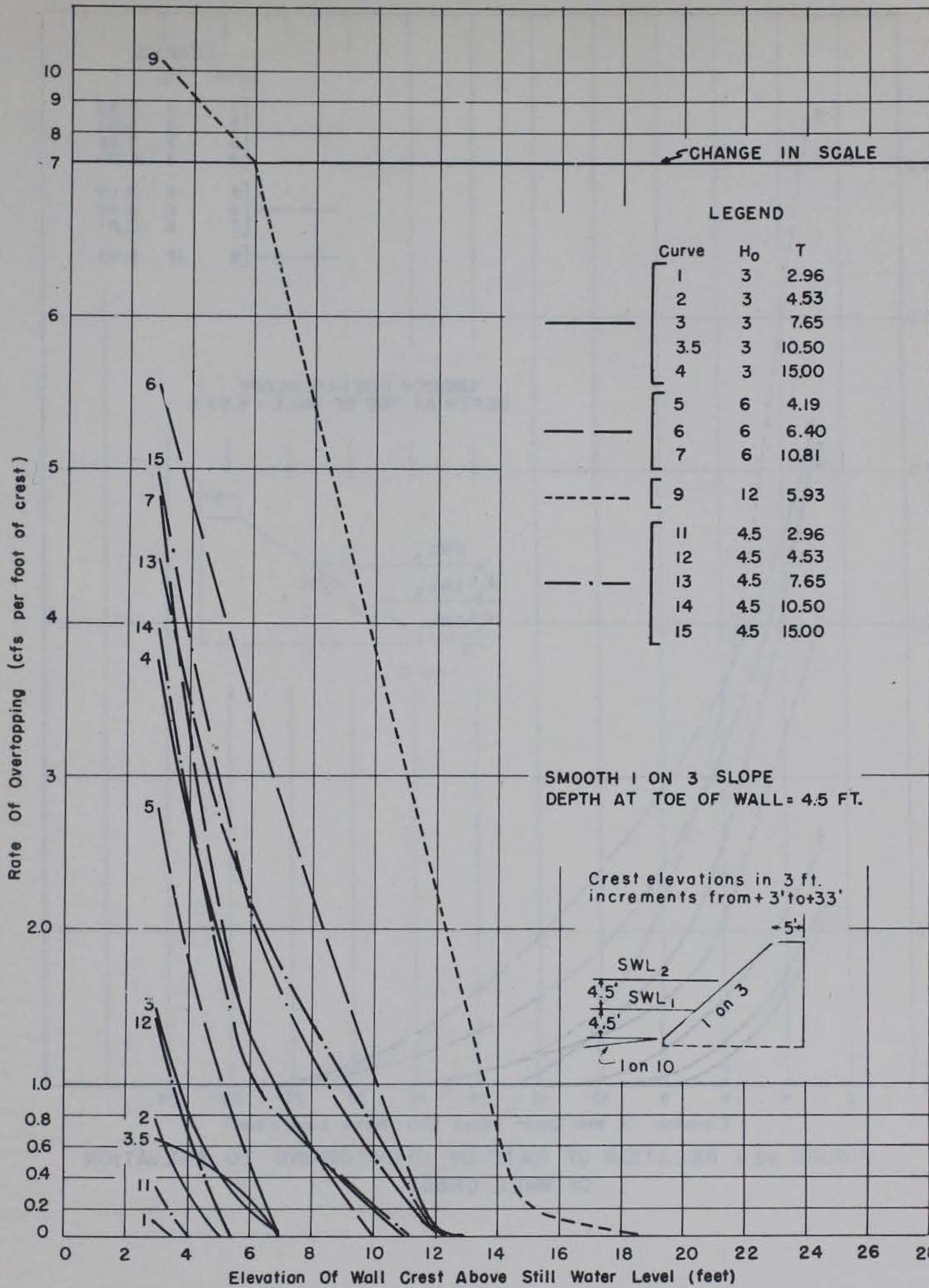

FIGURE 62 I RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

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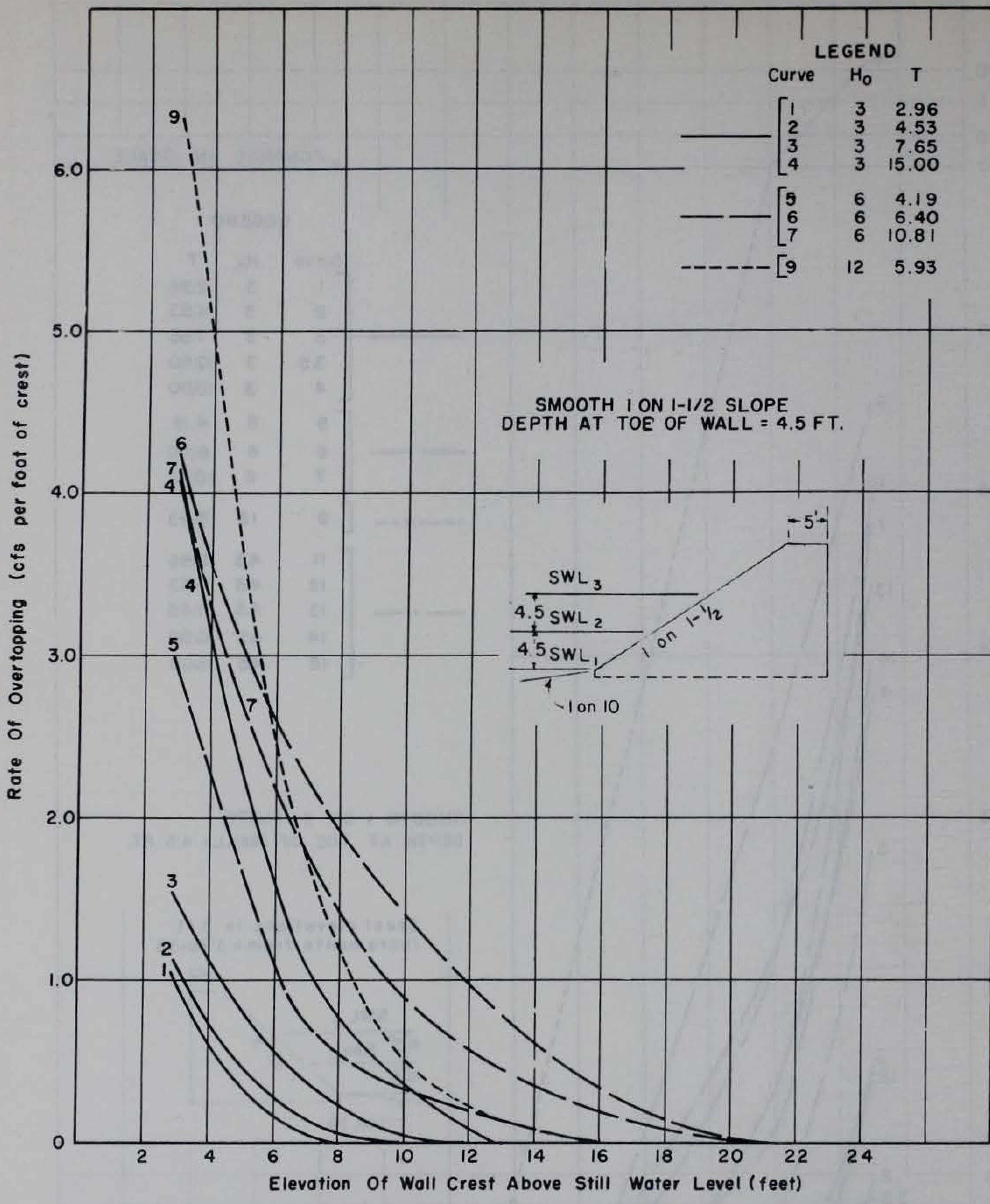

FIGURE 62 J RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

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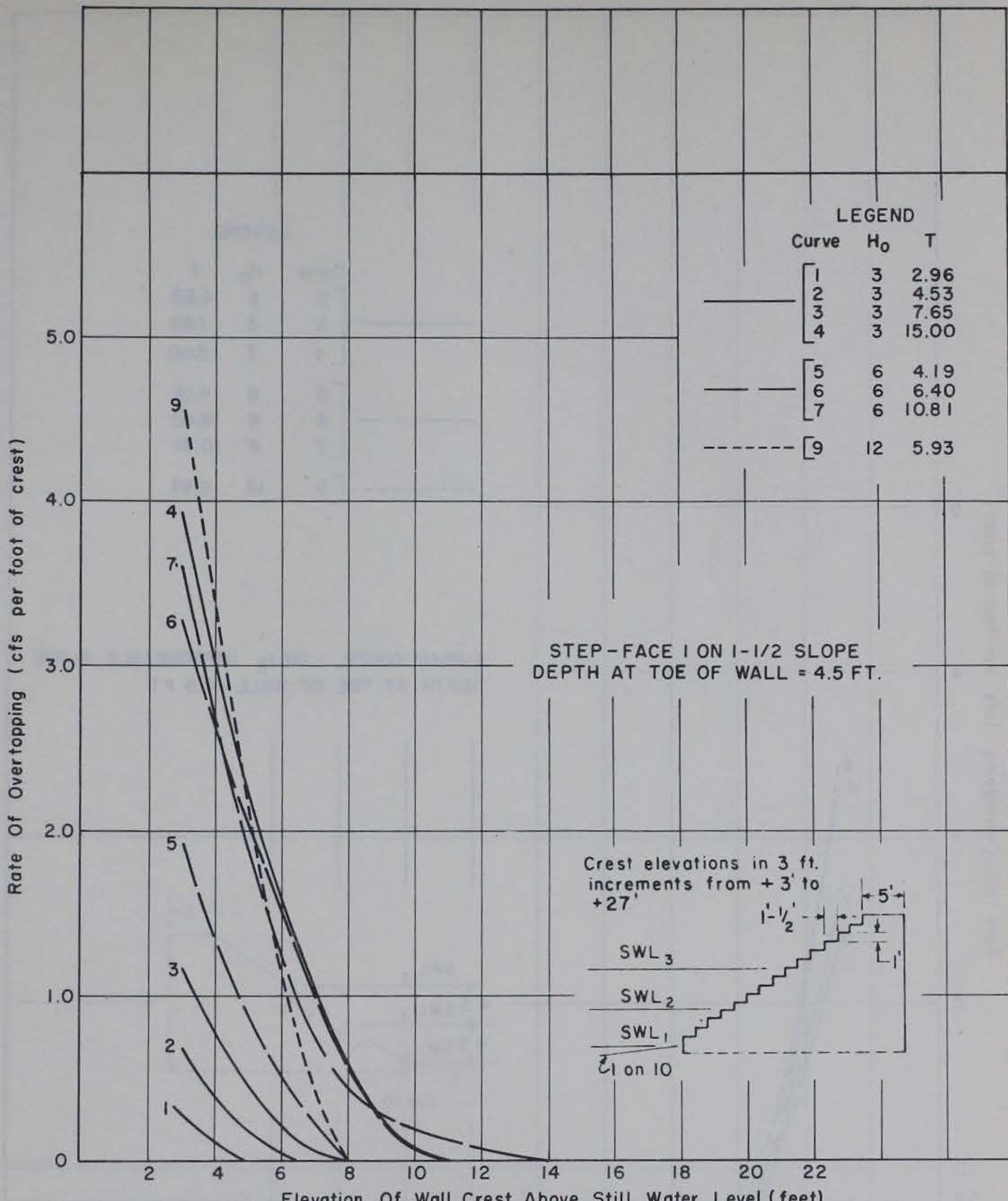

FIGURE 62 K RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

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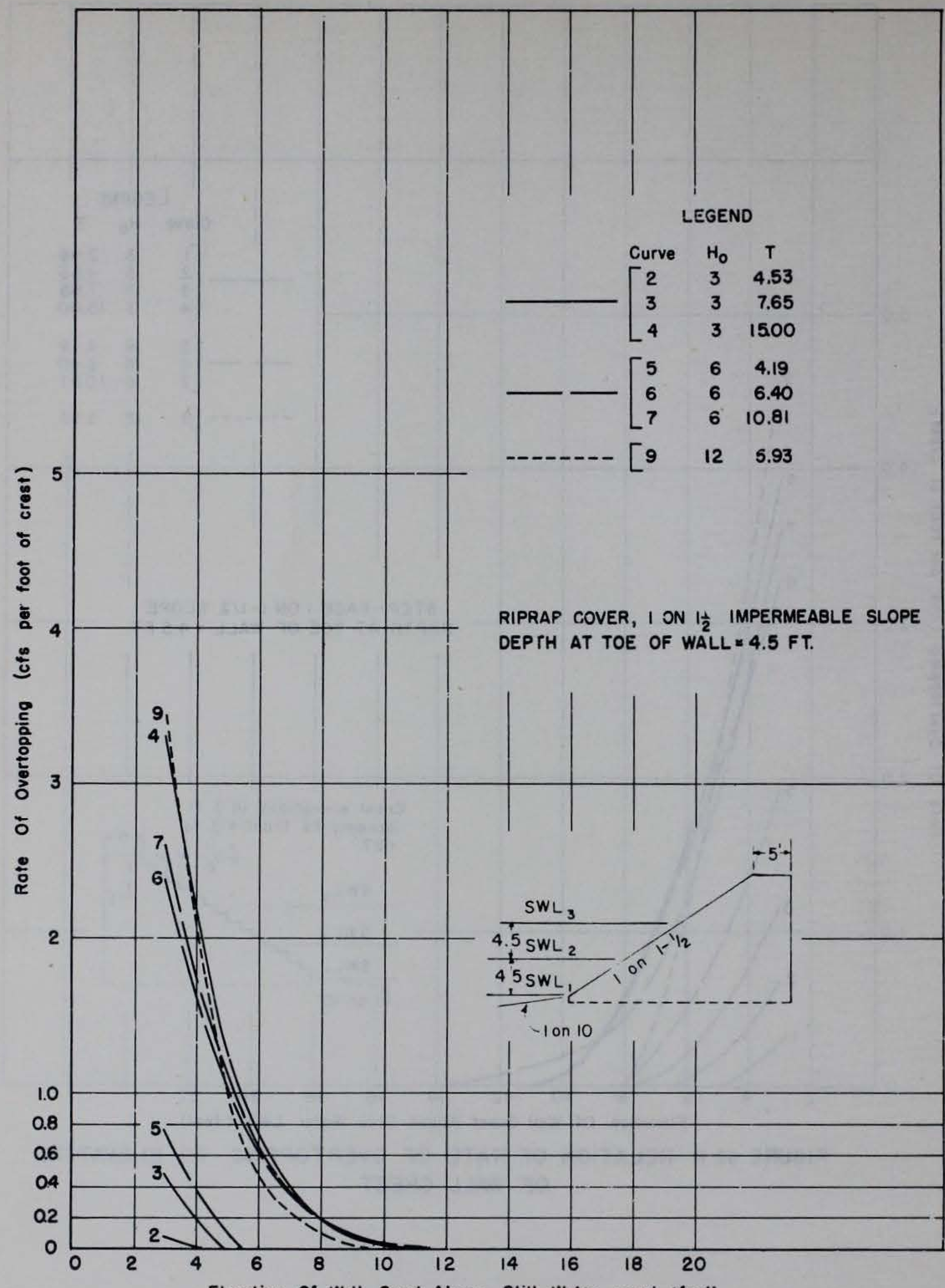


FIGURE 62 L RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

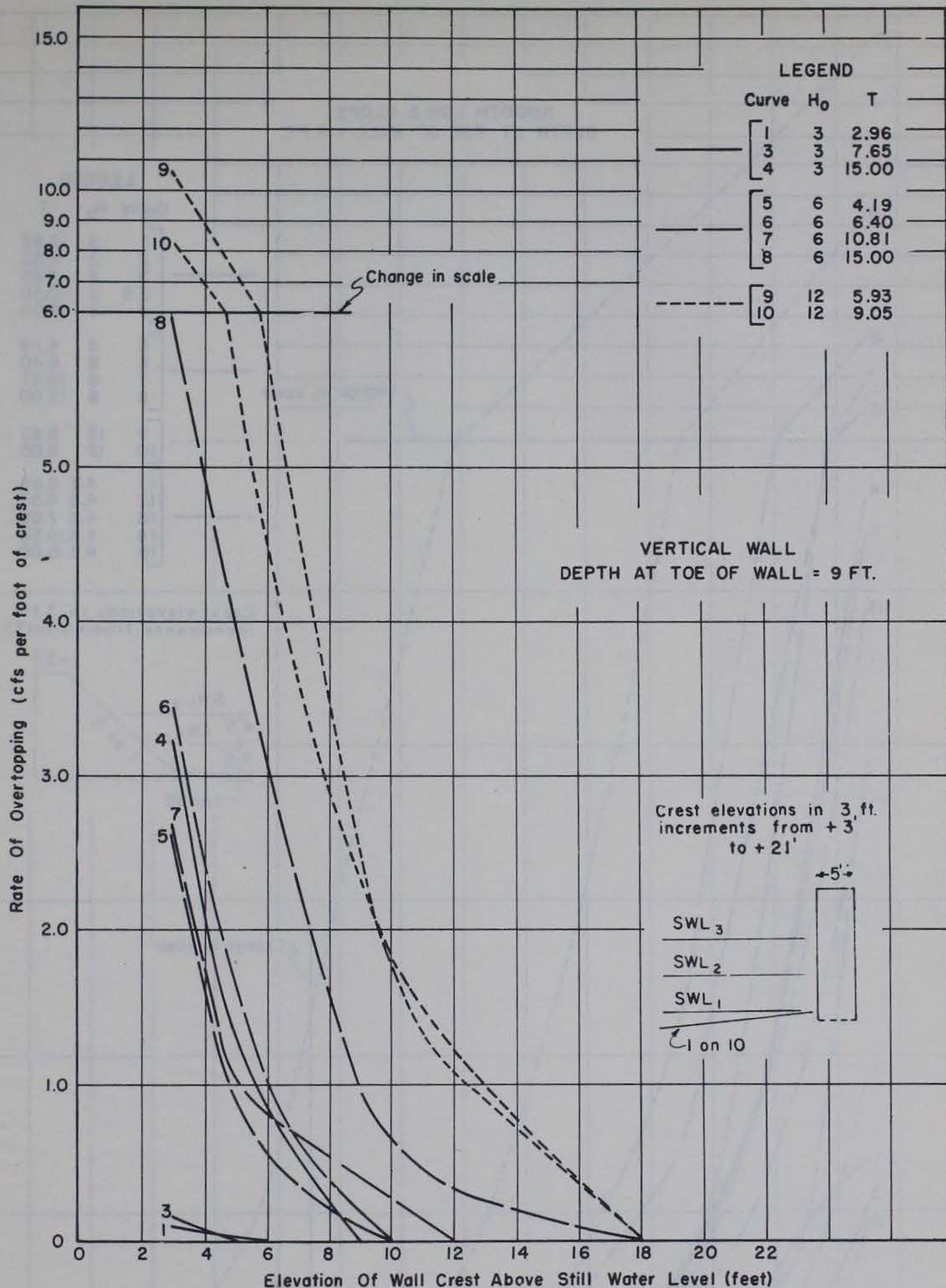
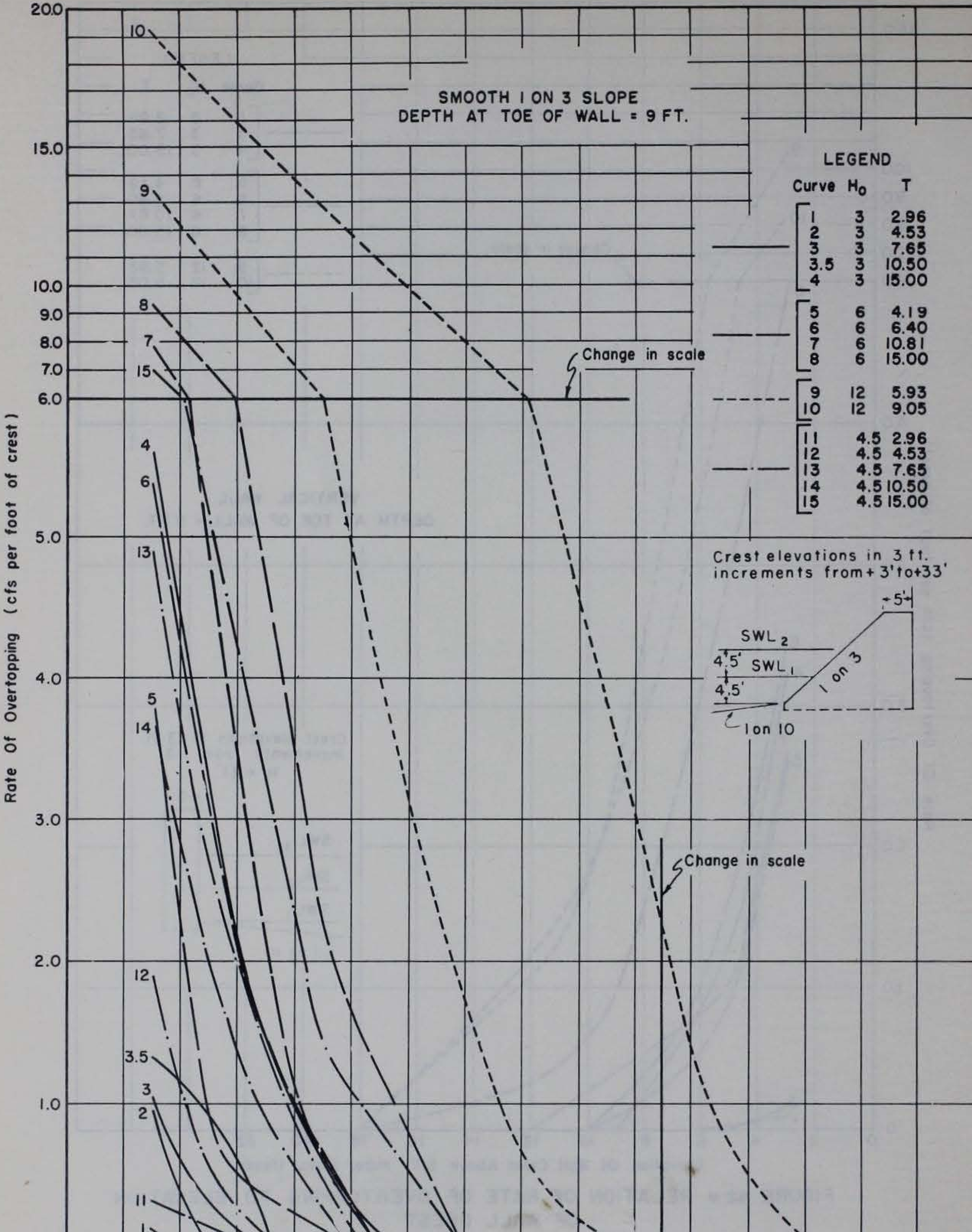


FIGURE 62 M RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

February 1957



Curve	H_0	T
1	3	2.96
2	3	4.53
3	3	7.65
3.5	3	10.50
4	3	15.00
5	6	4.19
6	6	6.40
7	6	10.81
8	6	15.00
9	12	5.93
10	12	9.05
11	4.5	2.96
12	4.5	4.53
13	4.5	7.65
14	4.5	10.50
15	4.5	15.00

FIGURE 62 N RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

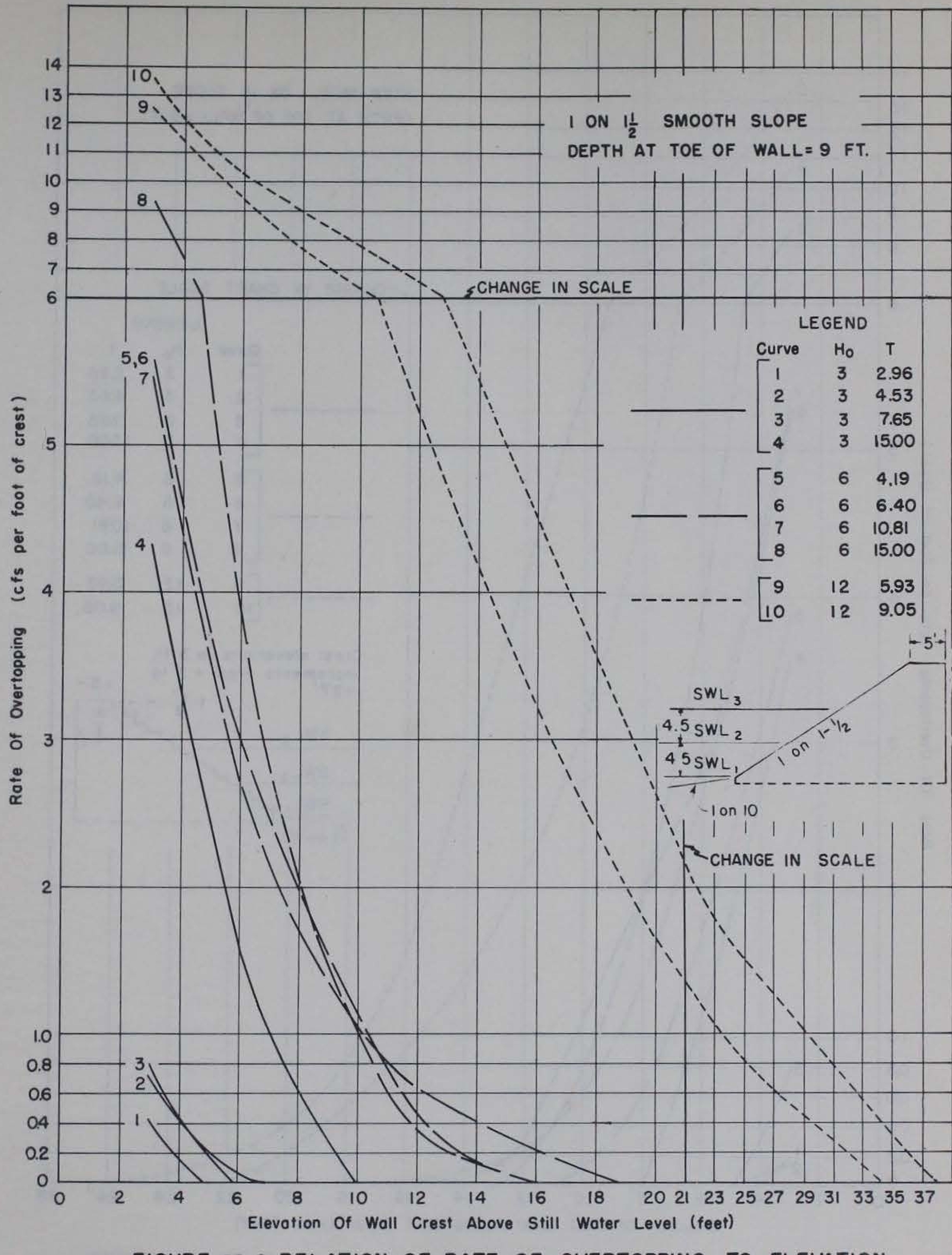


FIGURE 62 O RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

February 1957

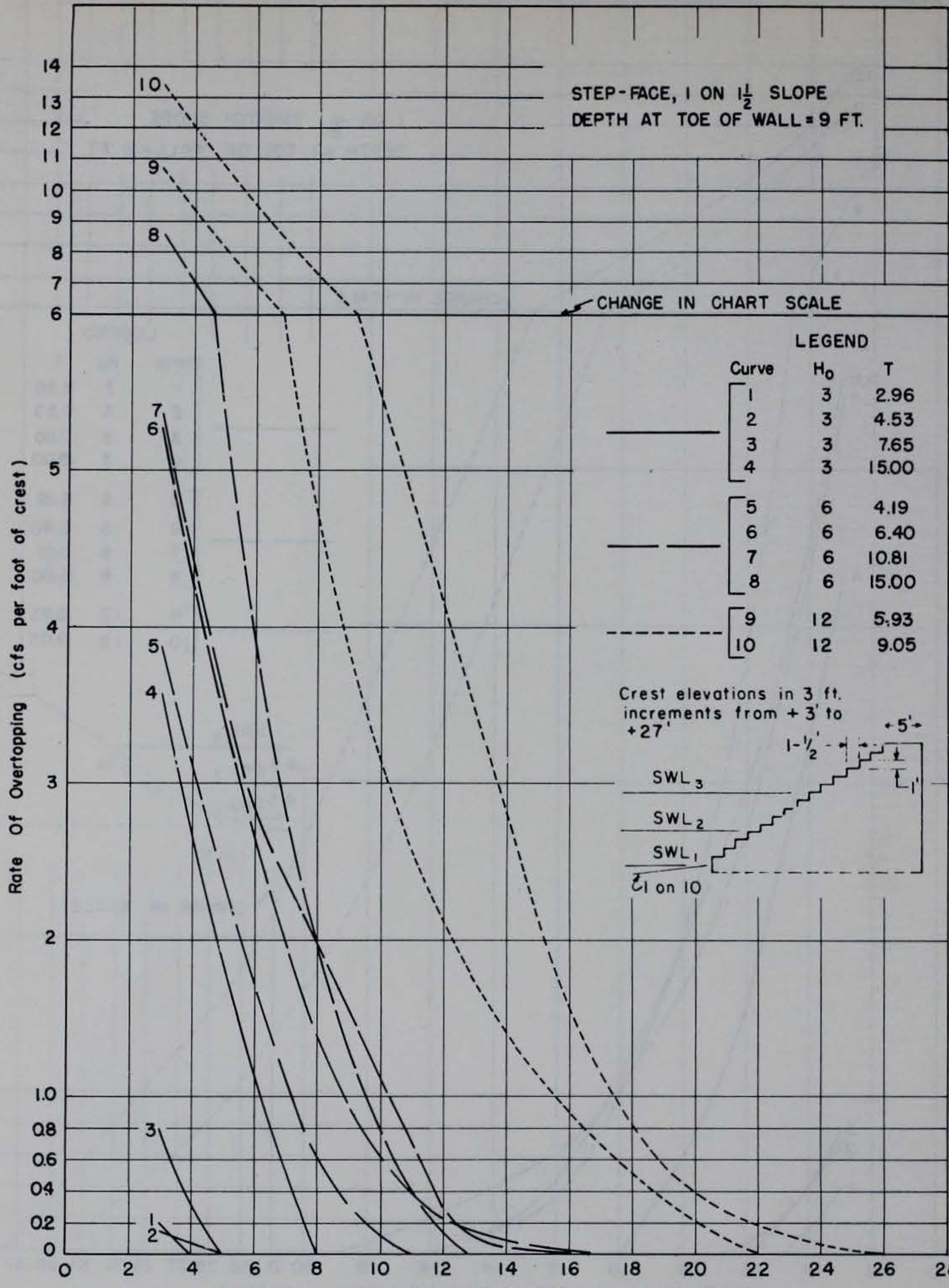


FIGURE 62 P RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

February 1957

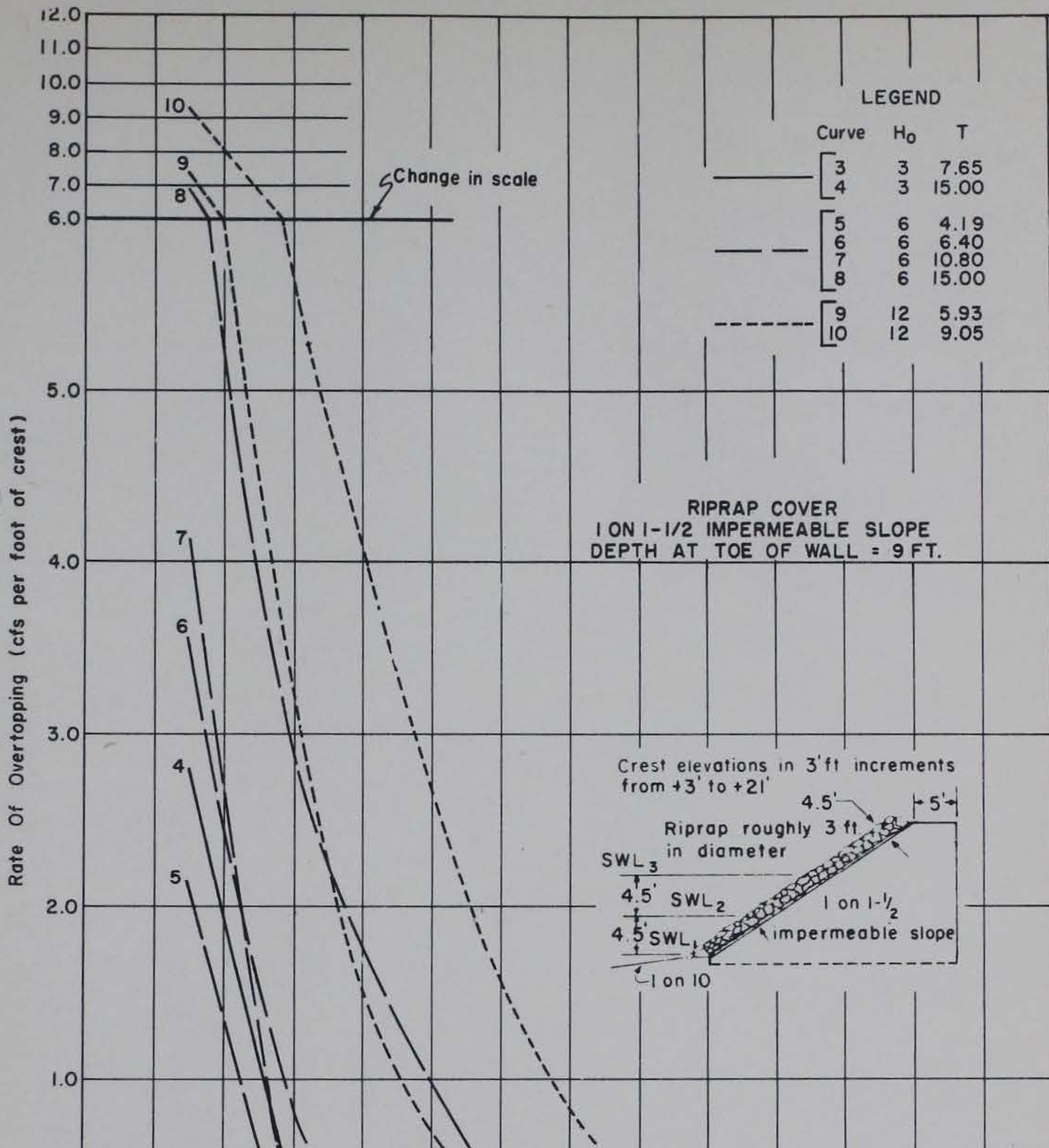


FIGURE 62 Q RELATION OF RATE OF OVERTOPPING TO ELEVATION OF WALL CREST

February 1957

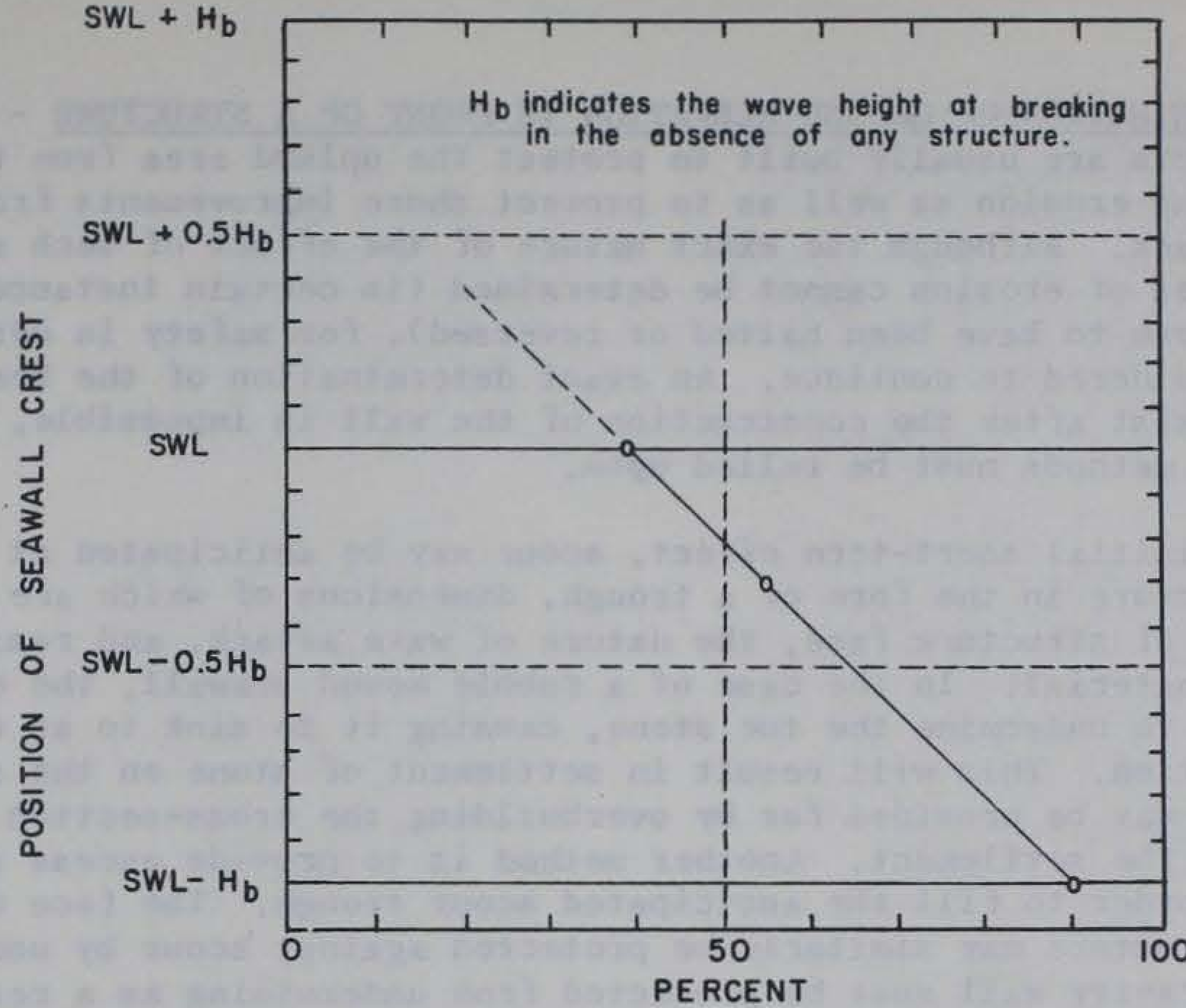


FIGURE 63 A. PERCENT WAVE HEIGHT TRANSMISSION OF STRUCTURE IN THE BREAKER ZONE.

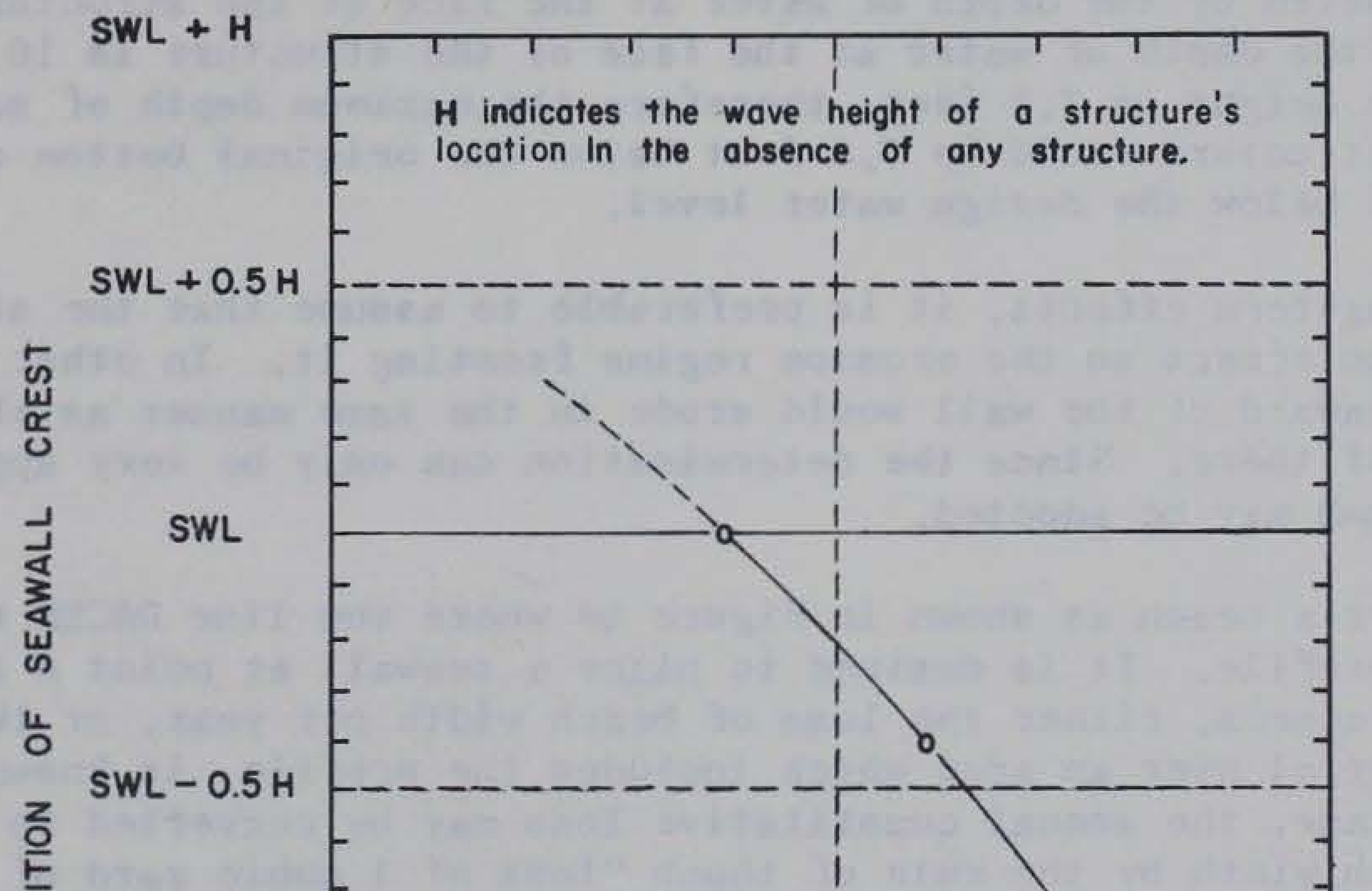


FIGURE 63 B. PERCENT WAVE HEIGHT TRANSMISSION OF STRUCTURE SEAWARD OF THE BREAKER ZONE.

PERCENT WAVE HEIGHT TRANSMISSION

FIGURE 63

(From Kaplan, 1952)

3.28 DETERMINATION OF GROUND ELEVATION IN FRONT OF A STRUCTURE - Seawalls and revetments are usually built to protect the upland area from the effects of continuing erosion as well as to protect shore improvements from damage by wave attack. Although the exact nature of the effect of such a wall on the processes of erosion cannot be determined (in certain instances these processes seem to have been halted or reversed), for safety in design they must be considered to continue. An exact determination of the beach profile that will exist after the construction of the wall is impossible, therefore, approximate methods must be relied upon.

As an initial short-term effect, scour may be anticipated at the toe of the structure in the form of a trough, dimensions of which are governed by the type of structure face, the nature of wave attack, and resistance of the bed material. In the case of a rubble mound seawall, the effect of scour is to undermine the toe stone, causing it to sink to an ultimately stable position. This will result in settlement of stone on the seaward face, which may be provided for by overbuilding the cross-section to provide for the settlement. Another method is to provide excess stone at the toe in order to fill the anticipated scour trough. The face of a vertical structure may similarly be protected against scour by use of stone. A gravity wall must be protected from undermining as a result of scour by providing impermeable cut-off walls at the base. As a rule of thumb method, the maximum depth of scour trough below the natural bed,may be approximated as being equal to the height of the maximum unbroken wave that can be supported by the depth of water at the face of the structure. For example: if the depth of water at the face of the structure is 10 feet, the maximum wave height is 7.8 feet, therefore the maximum depth of scour at the toe of the structure would be 7.8 feet below the original bottom or a total of 17.8 feet below the design water level.

For long-term effects, it is preferable to assume that the structure would have no effect on the erosion regime fronting it. In other words, the beach seaward of the wall would erode in the same manner as if the wall were not there. Since the determination can only be very approximate, rules of thumb may be adopted.

Consider a beach as shown in Figure 64 where the line DACEB represents an average profile. It is desired to place a seawall at point A as shown. From prior records, either the loss of beach width per year, or the annual loss of material over an area which includes the profile, is known. In the latter case, the annual quantitative loss may be converted to an annual loss of beach width by the rule of thumb "loss of 1 cubic yard of beach material is equivalent to loss of 1 square foot of beach area at the berm".

In general, beach slopes are fairly steep shoreward of a depth of from 0 to 10 feet, and fairly flat seaward of that depth. Analysis of beach profiles on eroding beaches would indicate that it may reasonably be assumed that the beach seaward of a depth of 30 feet will remain

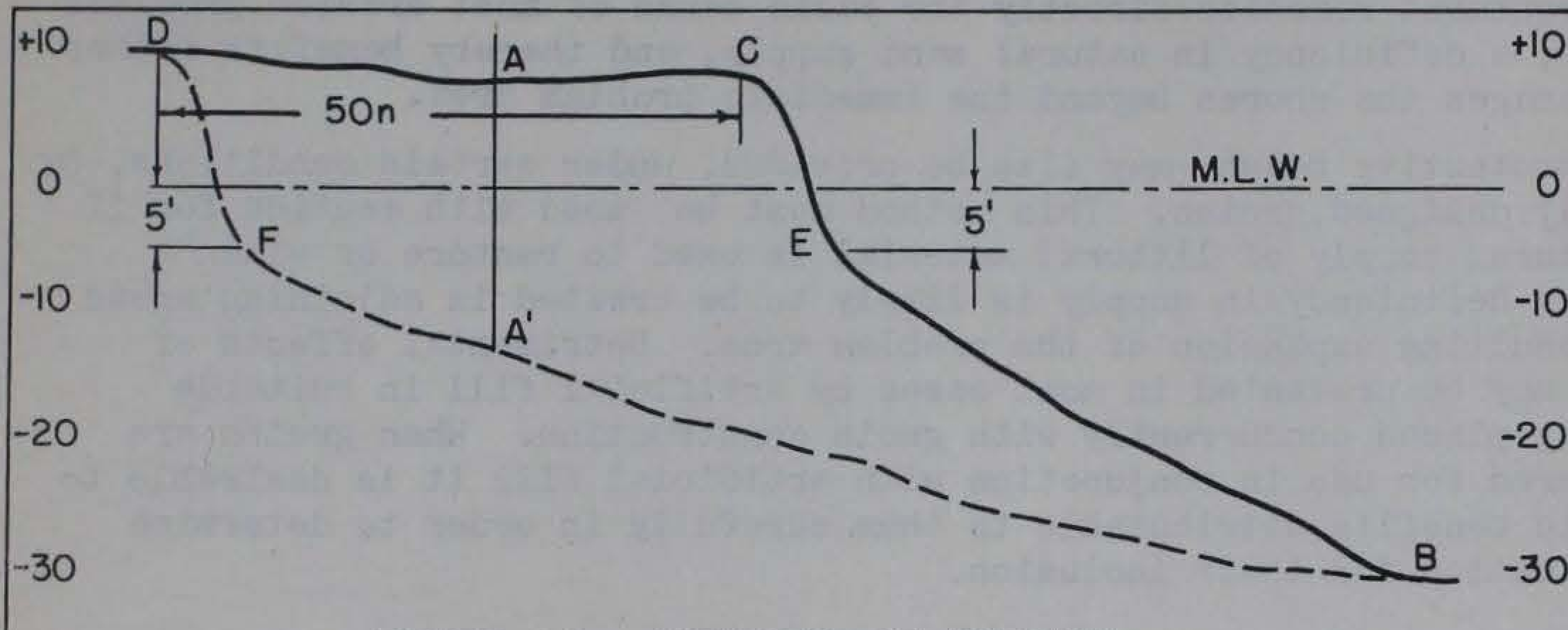


FIGURE 64 - EFFECTS OF EROSION

essentially unchanged, that the point of slope break will remain at about the same elevation, and that the profile of the beach shoreward of the point of break in slope will remain essentially unchanged, accordingly the ultimate depth at the wall may be estimated as follows:

(a) - In Figure 64 let B represent the beach at a water depth of 30 feet, E the beach at the point of slope break (say) at a depth of 5 feet, and C the present position of the berm crest. If at A it it is desired to build a structure whose economic life is (say) 50 years, and it is found that n is the annual loss of beach width at the berm, then in 50 years without the wall this berm will retreat a distance 50 n to point D;

(b) - From D to the elevation of point E draw a profile DF parallel to CE, and connect points B and F. This line DFB will represent the approximate profile of beach after 50 years, without the presence of the wall. The new beach elevation at the wall's location will be approximated by Point A'. Similar calculations may be made for anticipated short time beach depredations caused, for example, by storms.

3.3 PROTECTIVE BEACHES

3.31 FUNCTIONS - Beaches are the most effective means of dissipating wave energy, and, when they can be maintained to adequate dimensions, afford the most effective protection for the adjoining upland. In an erosion problem it is nearly always advisable to investigate the feasibility of artificially providing and maintaining an adequate beach in addition to any other remedial measures considered. When conditions are suitable for artificial nourishment, long reaches of shore may be protected by this method at relatively low cost as compared with other adequate defensive structures. An equally important advantage is that this treatment remedies directly the basic cause of most erosion problems that is, a deficiency in natural sand supply, and thereby benefits rather than damages the shores beyond the immediate problem area.

A protective beach may also be provided, under certain conditions, by properly designed groins. This method must be used with caution for if the natural supply of littoral material is used to restore or widen a beach, a deficiency in supply is likely to be created in adjoining areas with resulting expansion of the problem area. Detrimental effects of groins may be prevented in most cases by artificial fill in suitable quantity placed concurrently with groin construction. When groins are considered for use in conjunction with artificial fill it is desirable to evaluate benefits attributable to them carefully in order to determine justification for their inclusion.

3.32 LIMITATIONS - An obvious limitation in providing a protective beach with or without groins, is the availability of suitable material for the purpose. Also, this method is usually quite costly on a unit length basis when applied to short segments of shore. The latter is not necessarily a limitation if, by artificial nourishment, the enlargement of a problem area can be prevented. Perhaps, the most important limitation is that encountered in financing a shore protection method designed to provide protection beyond the immediate problem area.

3.33 PLANNING CRITERIA - In planning a protective beach the first task is to determine the predominant direction of littoral transport and to determine quantitatively the deficiency in material supply in the problem area. This deficiency is the rate of loss of beach material and is the rate at which the material supply must be increased to balance the transport capacity of littoral forces so that no net loss will occur. If there is no natural supply available, as may be the case on shores downdrift from a major littoral barrier, the deficiency in supply will be equal to the full rate of littoral transport. If the problem area is part of a continuous and unobstructed sandy beach, it is likely that the deficiency will be relatively small compared with the transport rate. Comparison of surveys over a long period of time is the only accurate means of determining the rate of nourishment required to maintain stability of the shore. Since surveys in suitable detail for volumetric measurement are rarely available at problem areas, approximations computed from changes in the shore position determined by air photo or any other suitable records are often necessary. For such approximations a rule of thumb equation wherein one square foot of surface area equals one cubic yard of beach material appears to provide acceptable values on exposed seacoasts. For less exposed shores this ratio would probably result in volumetric estimates somewhat in excess of the true figure and would thus produce conservative values. The various methods by which the predominant direction of littoral movement may be determined are outlined in Section 2.32 (Littoral Drift).

Having established the direction and magnitude of the forces that will operate on a proposed fill the next problem to be encountered is that of selecting a suitable beach material. Adequate criteria have not yet been established for evaluating the qualities of beach materials. However, some information is now available pertaining to the sorting of beach sands and the relation of grain size to beach slope which may be applied in selecting materials for artificial nourishment. When sand is deposited on a shore the waves operating in the area immediately start a sorting action on the surface layer of the fill moving the finer particles seaward, leaving the coarser material shoreward of the plunge point. This sorting action continues until a layer of coarse particles compatible with the wave spectrum of the area armors the beach and renders it relatively stable. However, if the armor is broken due to a storm, the underlying material is again subjected to the sorting process. In view of this sorting process beach fill materials containing substantial proportions of material other than sand may be used with the assurance that natural processes will clean the sand and make it an entirely suitable material for nourishment. This has been confirmed by experience with the fills at Anaheim Bay, California, and Palm Beach, Florida, both of which contained foreign matter.

Material of coarser characteristics may be expected to produce a steeper than normal beach. Material finer than that occupying the natural beach will, when exposed on the surface, move seaward to a depth compatible with its size. Almost any source of borrow near the shore will produce some material of proper beach size. Since the source of artificial nourishment will control the cost to a major degree, evaluation of material characteristics is an important factor in economic design. At present such evaluation must be made largely on a basis of experience at other localities.

The cyclic change in water level and wave pattern will ultimately establish the beach crest height. The foreshore and nearshore slopes will affect wave behavior and thus influence the natural beach crest height. If the beach fill is placed to an elevation lower than the natural crest height, a ridge will subsequently develop along the crest. Concurrent high water stage and high waves will overtop the crest and cause ponding and temporary flooding of the backshore. Such flooding, if undesirable, may be avoided by fixing the berm height slightly above the natúral beach crest height. If there is an existing beach at the site, the natural crest height can be determined therefrom. Otherwise determination must be made on a basis of comparison with other sites possessing similar exposure characteristics and beach material. There is at present no acceptable theoretical basis for predicting beach crest height.

Criteria for specifying berm width depends upon a number of factors. If the purpose of the fill is to restore an eroded beach damaged by a major storm, where inadequate natural nourishment is not a factor in the problem, the width may be determined by the protective width which experience had demonstrated to be required. Where the beach fill is to serve as a stockpile, to be periodically replenished, the berm width should be sufficient to provide for expected recession during the intervals between replenishment. It is generally considered that the toe of fill of a stockpile beach should not extend to such depth that transport of any material forming the surface of the fill would be retarded. There are no firm specifications for this limiting depth at present but available data indicate, that depths of twenty feet below low water datum on seacoasts and twelve feet on the Great Lakes may be used safely. It is obvious that the initial overall slope of any beach fill must be steeper than that of the natural shore area upon which it is placed. Subsequent behavior of the slope depends principally upon the characteristics of the fill material. In ordinary practice the initial fill slope is designed parallel to the local or comparable natural beach slope above low water datum, and thence slopes of 1:20 to 1:30 from low water datum to intersection with the existing bottom. It is undesirable and unnecessary to grade beach slopes artificially below the berm crest for they will be naturally shaped by wave action.

The length of a stockpile beach may vary greatly depending upon local conditions. Lengths from a few hundred feet to a mile have been employed successfully. Since the updrift end of a stockpile beach will be depleted first, long stockpiles are usually most suitable where a bulkhead or seawall exists to protect the backshore as erosion progresses along the stockpile. In general, dimensions of the stockpile will be governed primarily by economic considerations involving comparisons of costs for varying replenishment intervals.

3.34 <u>SAND DUNES</u> - Sand dunes perform the function of beach fills in preventing waves from reaching the upland, but are located landward of the beach itself. A belt of sand dunes will provide effective protection to improve upland property as long as the tops of the dunes remain above the limit of wave uprush. At those locations which have an adequate natural supply of sand and which are subject to inundation by storm tides and high seas, a belt of sand dunes may provide more effective protection, and at lower cost, then either a bulkhead, seawall or revetment.

3.4 GROINS

3.41 <u>DEFINITION</u> - A groin is a shore protective structure devised to provide, build or widen a protective beach by trapping littoral drift or to retard loss of an existing beach. It is usually perpendicular to the shore, extending from a point landward of possible shore line recession into the water a sufficient distance to stabilize the shore line at a desired location. Groins are relatively narrow in width (measured parallel to the shore line) and may vary in length from less than 100 feet to several hundred feet.

Groins may be classified as permeable or impermeable, high or low, and fixed or adjustable. They may be constructed of timber, steel, stone, concrete, or other materials, or combinations thereof. Impermeable groins have a solid or nearly solid structure which prevents littoral drift passing through the structure. Permeable groins have openings through the structure of sufficient size to permit passage of appreciable quantities of littoral drift. Some permeable stone groins may become impermeable with heavy marine growth. A series of groins acting together to protect a long section of shore line is commonly called a groin field. "Groin system" is a preferable term.

Groins differ from jetties in that jetties generally are larger with more massive component parts, and are used primarily to direct and confine the stream or tidal flow at the mouth of a river or entrance to a bay and prevent littoral drift from shoaling the channel. In some sections of the country groins are commonly referred to as jetties or piers.

3.42 <u>GROIN OPERATION</u> - A groin interposes a total or partial barrier to littoral drift moving in that part of the littoral zone between the seaward end of the groin and the limit of wave uprush. The extent to which the littoral drift is so modified depends on the height, length, and permeability of the groin. The manner in which a groin operates to modify the rate of littoral drift is approximately the same whether it operates singly or as one of a system, provided spacing between adjacent groins is adequate. However, under some conditions a single groin or the updrift groin of a system, may have a somewhat smaller capacity than the other individual groins of the system.

A typical groin is illustrated in Figure 65. In this figure, the groin extends from some distance landward of the top of berm to the 6-foot depth contour. The net direction of wave attack, as typified by the orthogonals shown, is such as to cause a net movement of littoral drift. The shore line and 6-foot depth contour are represented by <u>e a i</u> and <u>g c h</u> respectively, occurring in a state of nature prior to the construction of the groin <u>j c</u>. Prior to the construction of the groin the offshore beach slope had stabilized between <u>a</u> and <u>c</u> in a manner dependent on the character of the beach material and the type of wave attack, as shown by the profile <u>d a k c</u>.

The groin acts as a partial dam to intercept a portion of the normal drift. As material accumulates on the updrift side, supply to the downdrift shore is correspondingly reduced and the latter shore recedes. This results in a progressively steepening slope on the updrift side and a flattening slope on the downdrift side, since both slopes must reach a common elevation at or near the end of the groin. Since the grain size of the beach material normally increases to establish a steeper than normal slope, the residual accreted material is probably by selective process the coarser fraction of the material that was in transport.

When the accreted slope reaches ultimate steepness for the coarser fraction of the material available, impoundment ceases and all littoral drift passes the groin. If the groin is sufficiently high that no material may pass over it, all transport must be in depths beyond the end of the

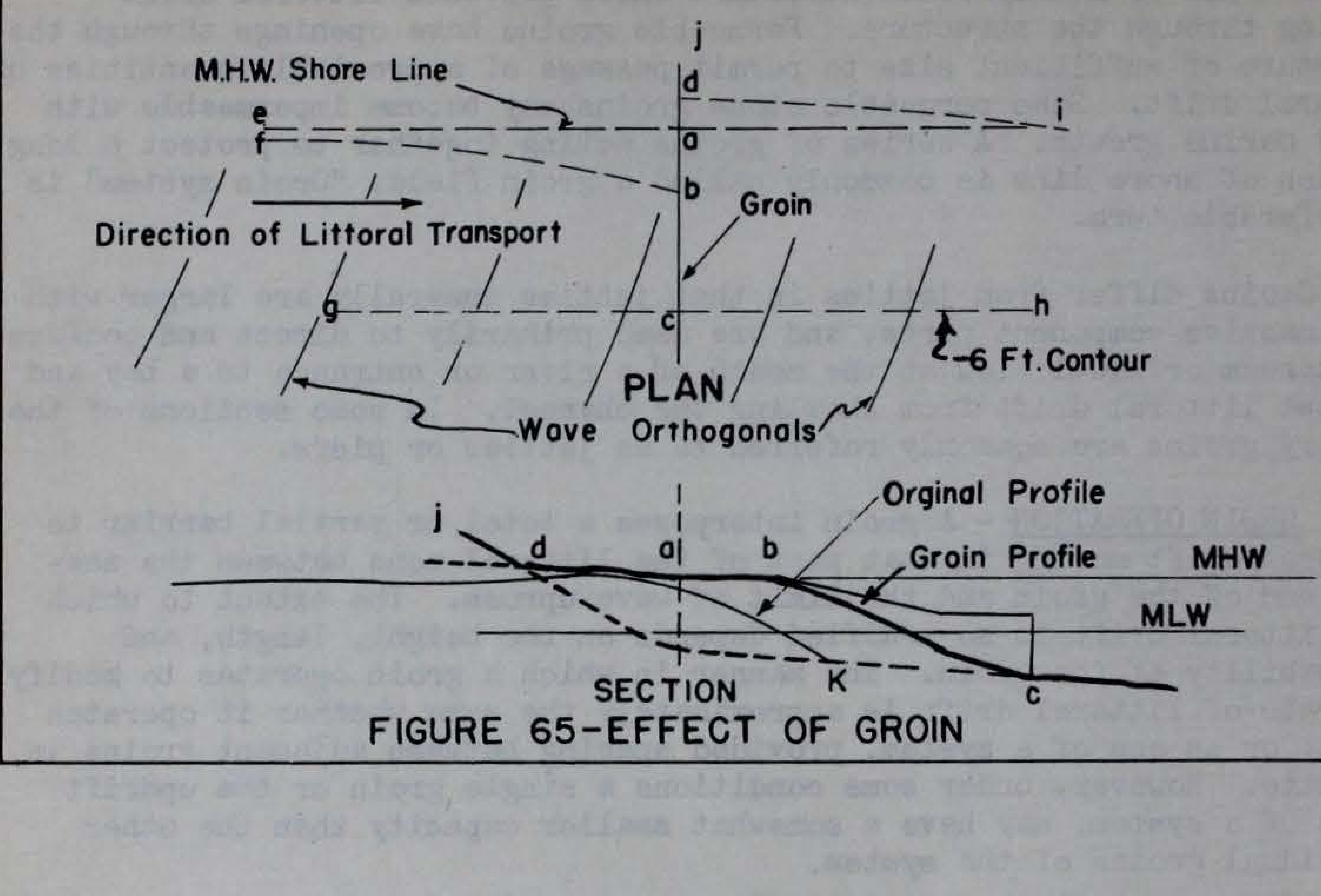


FIGURE 65-EFFECT OF GROIN

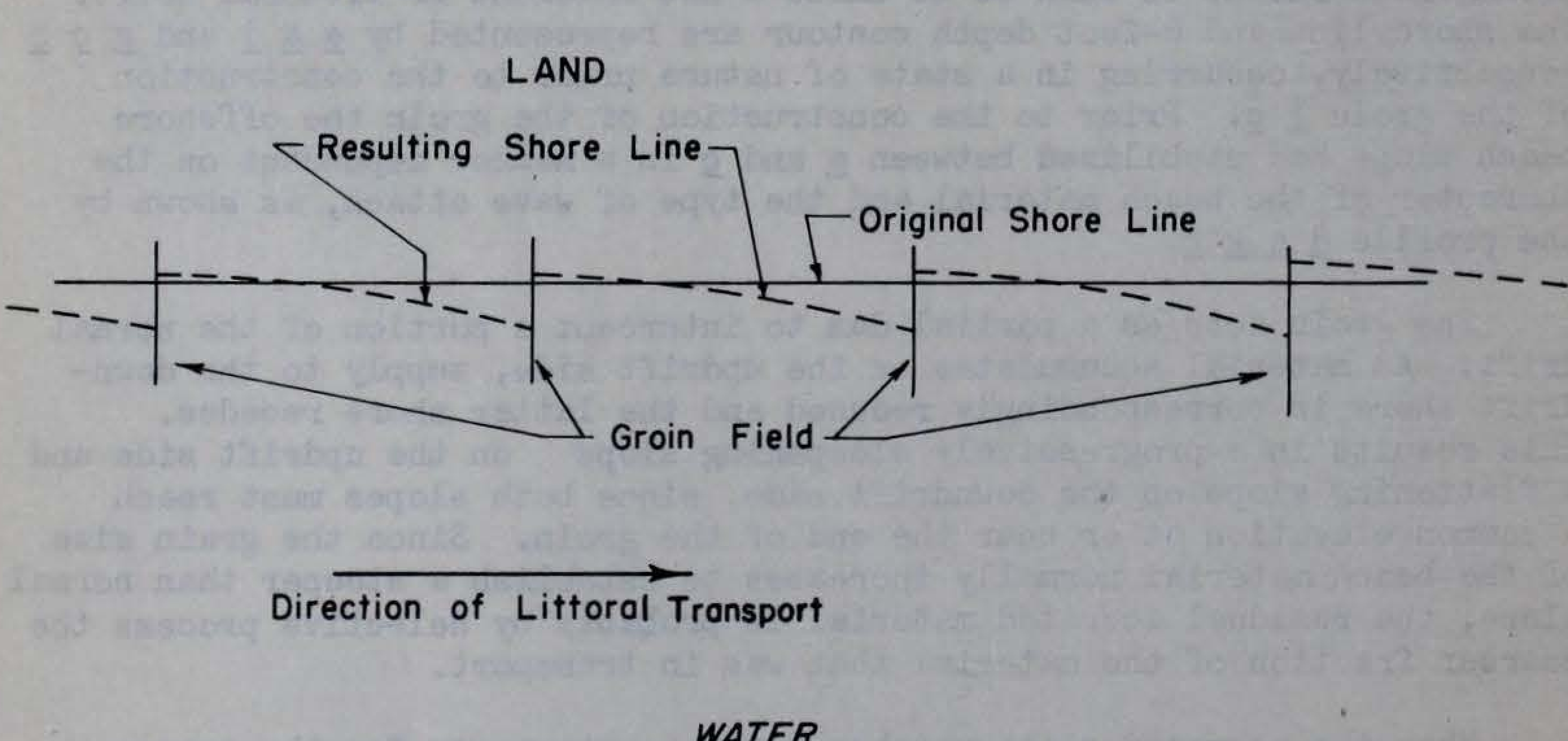


FIGURE 66-EFFECT OF GROIN SYSTEM

groin. Because of the nature of transporting currents the material in transit does not move directly shoreward after passing the groin, and transport characteristics do not become normal for some distance on the downdrift side of the groin. Thus a groin system too closely spaced would divert littoral transport offshore rather than create a widened beach.

The accretion fillet on the updrift side of the groin creates a departure from normal shore alignment, tending toward a stable alignment normal to the resultant of wave attack. The impounding capacity of the groin is thus dependent upon both the stability slope and stability alignment of the accretion fillet. These in turn depend upon characteristics of the littoral material and the direction of wave attack.

Figure 66 shows the general configuration of the shore line to be expected for a system of two or more groins. It assumes a well established net littoral transport in one direction.

3.43 PURPOSE - Under some conditions and subject to definite limitations imposed on their use, groins may be used to:

(a) Stabilize a beach, subject to intermittent periods of advance and recession;

(b) Provide upland protection through prevention of removal of a protective beach;

(c) Reduce the rate of littoral transport out of an area by reorienting a section of the shore line to an alignment more nearly perpendicular to the major direction of wave approach;

(d) Build or widen a beach by trapping littoral material;

(e) Prevent loss of material out of an area by compartmenting the beach;

(f) Prevent advance of a downdrift area by acting as a littoral barrier; and

(g) Stabilize a specific area by reducing the rate of loss from the area.

All of the foregoing ends are attained through modification of the rate of littoral transport. Also, all are forms of shore protection.

3.44 LIMITATIONS ON THE USE OF GROINS - Because of their limitations, the use of the groin as a major protective feature should be decided upon only after careful consideration of the problem and the many factors involved. Principal factors to be considered are:

(a) The extent to which the downdrift beach will be damaged if groins are used;

(b) The economic justification for groins in comparison with stabilization by nourishment alone;

(c) The adequacy of natural sand supply to insure that groins will function as desired;

(d) The adequacy of shore anchorage of the groins to prevent flanking as a result of downdrift erosion;

(e) In providing protection by groins, it must be remembered that the alignment of the shore adjacent to a groin is dependent upon the direction of wave attack and will vary. If minor fluctuations of the shore cannot be allowed a solution other than the use of groins must be found; and

(f) Where the supply of littoral material is insufficient to permit the withdrawal from the littoral stream of enough material to fill the groin or groin field without damage to downdrift areas, artificial fill may be required to fill the groin or groin system and thus permit the natural littoral supply to pass without interruption.

3.45 <u>TYPES OF GROINS</u>

3.451 <u>Permeable Groins</u> - Many types of permeable groins have been employed in efforts to avoid the abrupt offset in shore alignment which normally occurs at impermeable groins. The primary effect of permeability is to reduce the impounding capacity, which can ordinarily be accomplished at less cost with a properly designed impermeable groin. An important disadvantage of permeable groins is their relative ineffectiveness in holding a beach under storm conditions. As a means of river bank control where sediment transport results from ordinary current flow their value is well established. (Such structures are normally called permeable dikes.) Where wave action is the principal cause of transportation, permeable groins are unlikely to prove fully satisfactory as a shore protection measure.

3.452 <u>High and Low Groins</u> - The amount of sand passing a groin depends to some extent on the height of the groin. Groins based on a headland or reef, or at the entrance to a bay or inlet where it may be either unnecessary or undesirable to maintain a sand supply downdrift of the groin, may be built to such a height as to block completely the passage of material moving in that part of the littoral zone covered by the groins. Where it is necessary to maintain a sand supply downdrift of a groin, it may be built to such a height as to allow overtopping by storm waves, or by waves at high tide. Such low groins serve the same purpose as that intended by designers of permeable groins.

3.453 <u>Adjustable Groins</u> - The great majority of groins are fixed or permanently built structures. In England, the Case and Du-Plat-Taylor adjustable groins have been used with reported success. These groins are essentially adjustable batter boards between piles, with a raising and lowering device so that the groin can be maintained at a fixed height (usually one to two feet) above the sand level, allowing a considerable part of the sand to pass over the groin and maintain the downdrift beach. Adjustable groins are reported to be particularly useful where an attempt is being made to widen a beach with a minimum of erosion damage to the downdrift area. However, they are effective only where there is an adequate supply of littoral material.

3.46 DIMENSIONS OF GROINS - The width and side slope of a groin depend on wave forces to be withstood, the type of groin, the materials with which it is constructed, and the construction methods used. These features are considered under structural design. The length, profile and spacing are important considerations with respect to functional success.

The length of a groin is determined by the depths in the offshore area and the extent to which it is desired to intercept the littoral stream. The length should be such as to interrupt such a part of the littoral drift as will supply enough materials to create the desired stabilization of the shore line or the desired accretion of new beach areas. Care must be exercised that these ends are attained without a corresponding damage to downdrift areas. For functional design purposes, a groin may be considered in three sections;

(a) The horizontal shore section;
(b) The intermediate sloped section; and
(c) The outer section

3.461 The Horizontal Shore Section - This section would extend from the desired location of the crest of berm as far landward as is required to anchor the groin to prevent flanking. The height of the shore section depends on the degree to which it is desirable for sand to overtop the groin and replenish the downdrift beach. The minimum height of the groin is the height of the desired berm, which is usually the height of maximum high water that occurs frequently plus the height of normal wave uprush. The maximum height of groin to retain all sand reaching the area (a high groin) is the height of maximum wave uprush during all but the least frequent storms. This section is horizontal or sloped slightly seaward, paralleling the existing beach profile or the desired slope, in case a wider beach is desired or a new beach is to be built.

3.462 The Intermediate Sloped Section - The intermediate section would extend between the shore section and the more or less level outer section. This part of the groin should approximately parallel the slope of the foreshore the groin is expected to maintain. The elevation at the lower end of the slope will usually be determined by the construction methods used, the degree to which it is desirable to obstruct the movement of the material, or the requirements of swimmers or navigation interests.

3.463. The Outer Section - The outer section includes all of the groin extending seaward of the sloped section. With most types of groins, this section is horizontal at as low an elevation as is consistent with economy of construction, since it will be higher than the design updrift slope in any case. The length of the outer section will depend upon the design slope of the updrift beach.

3.464 Spacing of Groins - The spacing of groins in a continuous system is a function of the length of the groin and the expected alignment of the accretion fillet. The length and spacing must be so correlated that when the groin is filled to capacity the fillet of material on the updrift side of each groin will reach to the base of the adjacent updrift groin with sufficient margin of safety to maintain the minimum beach width desired or prevent flanking of the updrift groin. Figure 67 shows the desirable resulting shore line if groins are properly spaced. The solid line shows the shore line as it may develop when erosion is a maximum at the toe of the updrift groin. The erosion shown occurs while the updrift groin is filling. At the time of maximum recession, the solid line is nearly normal to the direction of the resultant of wave approach and the triangle of recession "a" is approximately equal to the triangle of accretion "b". The dashed line m-n shows the stabilized shore line which will obtain after material passes the updrift groin to fill the area between groins and in turn commences to pass the downdrift groin. It will be noted that the fillet of sand between groins tends to become and remain perpendicular to the predominant direction of wave attack. This alignment may be quite stable after equilibrium is reached. However, if there is a marked variation in the direction and intensity of wave attack either seasonally or as a result of prolonged storms, there will be a corresponding variation in the alignment and slope of the shore between groins. In areas where there is a periodic reversal in the direction of littoral transport an area of accretion may form on both sides of a groin as shown in Figure 68. Between groins the fillet may actually oscillate from one groin to the other as shown by the dashed lines, or may form a U-shaped beach somewhere in between depending on the rate of supply of littoral material. With regular reversals in littoral drift, the maximum line of recession would probably be somewhat as shown by the solid line, with the triangular area "a" plus triangular area "c" approximately equal to the circular segment "b". The extent of probable beach recession must be taken into account in establishing the length of the horizontal shore section of groin and in estimating the minimum width of beach that may be built by the groin system.

3.465 Length of Groin - Before the total length of a groin, and the position of the shore line adjacent to a groin can be determined, it is necessary to predict the ultimate stabilized beach profile on each side of the groin. The steps involved for a typical groin are:

(a) Determine the original beach profile in the vicinity of the groin;

(b) Determine the conditions of littoral transport;

(c) Plot a refraction diagram for the mean wave condition, i.e. the wave condition which would produce the predominant direction and net rate of littoral drift;

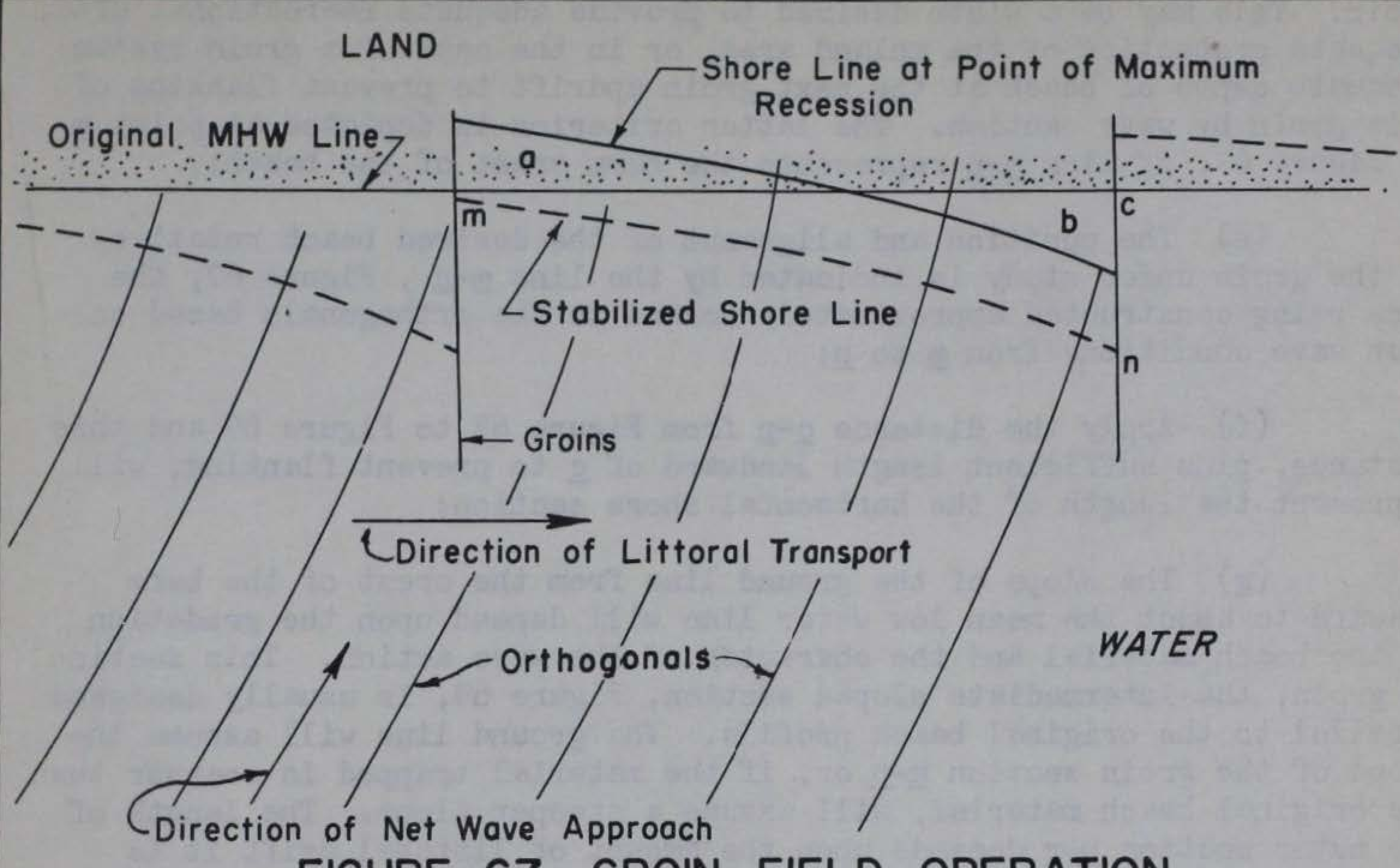


FIGURE 67 GROIN FIELD OPERATION

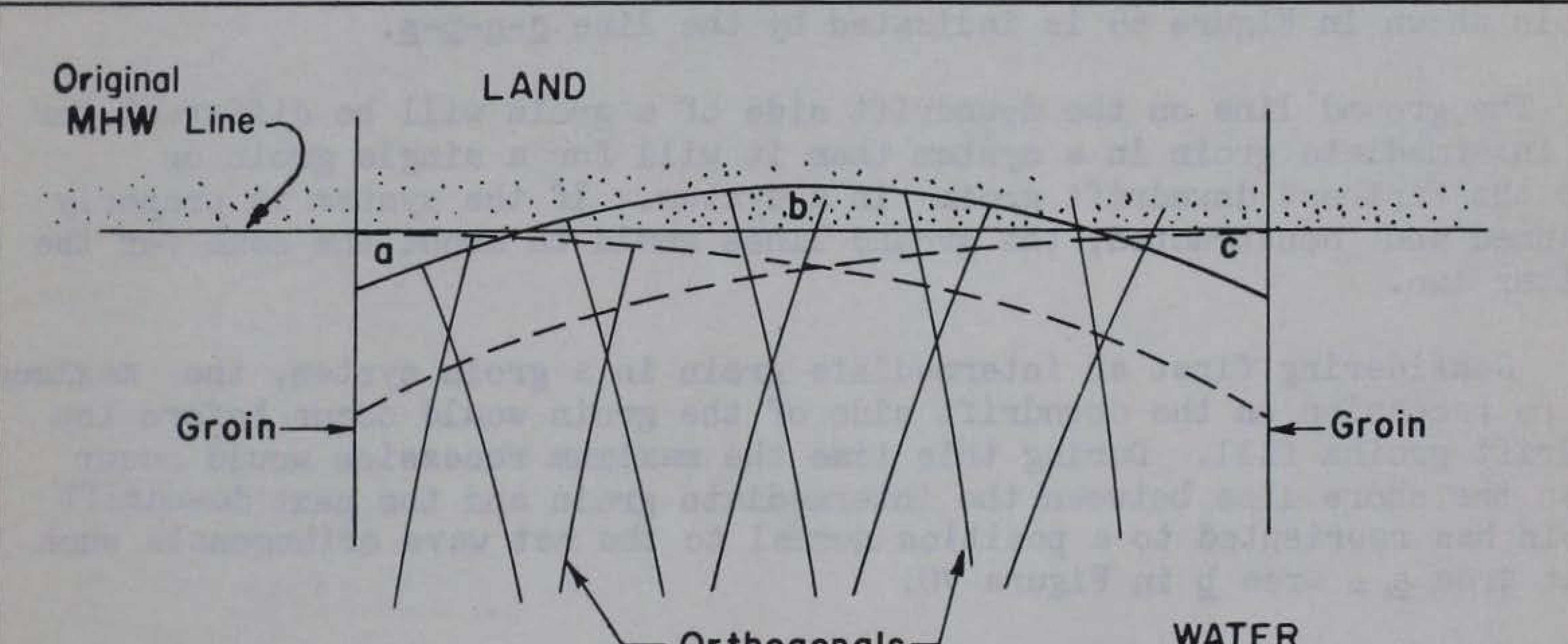


FIGURE 68 GROIN FIELD OPERATION WITH REVERSAL OF TRANSPORT

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(d) Determine the minimum width beach desired updrift of the groin. This may be a width desired to provide adequate recreational area, adequate protection of the upland area, or in the case of a groin system, adequate depth of beach at the next groin updrift to prevent flanking of this groin by wave action. The latter criterion is depicted at point m on Figure 67, if line m-n represents the berm crest of the beach;

(e) The position and alignment of the desired beach relative to the groin under study is indicated by the line m-n , Figure 67, the line being constructed approximately normal to the orthogonals based on mean wave conditions from m to n;

(f) Apply the distance c-n from Figure 67 to Figure 69 and this distance, plus sufficient length landward of c to prevent flanking, will represent the length of the horizontal shore section;

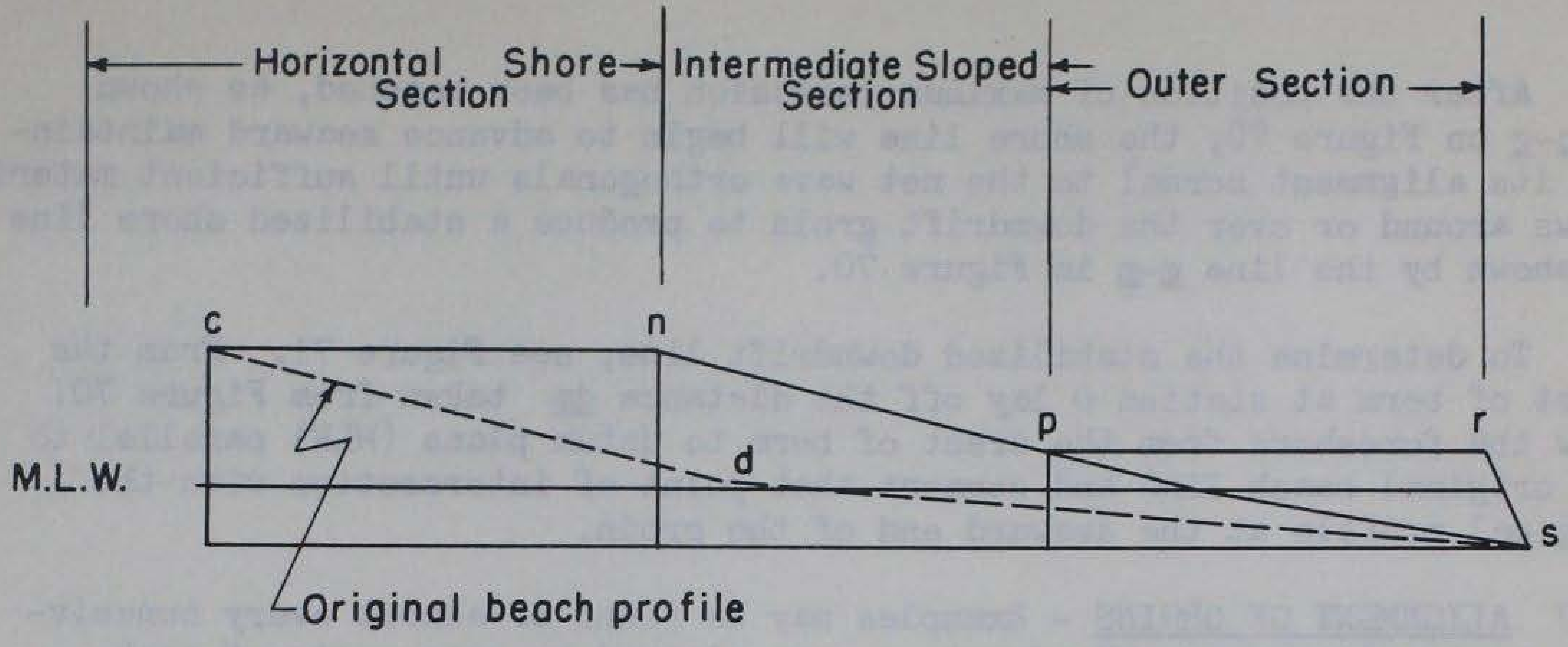
(g) The slope of the ground line from the crest of the berm seaward to about the mean low water line will depend upon the gradation of the beach material and the character of the wave action. This section of groin, the intermediate sloped section, Figure 69, is usually designed parallel to the original beach profile. The ground line will assume the slope of the groin section n-p or, if the material trapped is coarser than the original beach material, will assume a steeper slope. The length of the outer section p-r depends upon the amount of littoral drift it is desired to intercept. It should extend to sufficient depth so that the new profile p-s will intercept the old profile c-d-s within the toe of the groin;

(h) The final ground line on the updrift side of the typical groin shown in Figure 69 is indicated by the line c-n-p-s.

The ground line on the downdrift side of a groin will be different for an intermediate groin in a system than it will for a single groin or for the farthest downdrift groin in a system. If the system is properly planned and constructed, the ground lines would be about the same for the latter two.

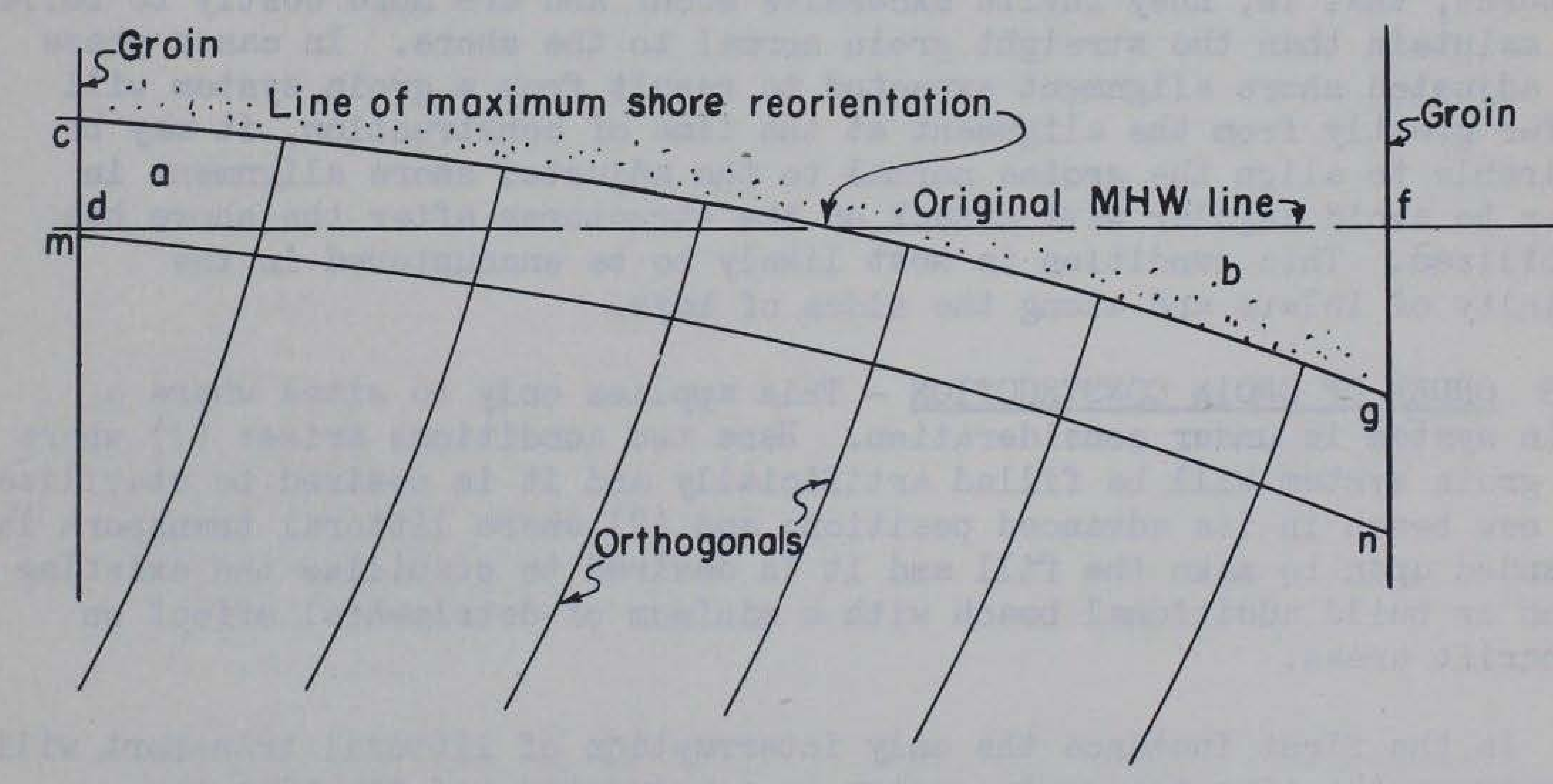
Considering first an intermediate groin in a groin system, the maximum shore recession on the downdrift side of the groin would occur before the updrift groins fill. During this time the maximum recession would occur when the shore line between the intermediate groin and the next downdrift groin has reoriented to a position normal to the net wave orthogonals such that area a = area b in Figure 70.

To determine the maximum recession of the downdrift shore line, draw the proposed groin on the original beach profile as in Figure 71. From the crest of berm at station 0, lay off distance cd taken from Figure 70. Draw the foreshore from crest of berm to datum plane (MLW) parallel to the original beach slope, and connect that point of intersection with the original profile at the seaward end of the groin.



TYPICAL GROIN PROFILE

FIGURE 69



RECESSION AND ACCRETION BETWEEN GROINS

FIGURE 70

After the position of maximum recession has been reached, as shown by c-g on Figure 70, the shore line will begin to advance seaward maintaining its alignment normal to the net wave orthogonals until sufficient material flows around or over the downdrift groin to produce a stabilized shore line as shown by the line m-n in Figure 70.

To determine the stabilized downdrift line, see Figure 71. From the crest of berm at station 0 lay off the distance dm taken from Figure 70. Draw the foreshore from the crest of berm to datum plane (MLW) parallel to the original beach line and connect that point of intersection with the original profile at the seaward end of the groin.

3.47 ALIGNMENT OF GROINS - Examples may be found of almost every conceivable groin alignment, and advantages are claimed by proponents of each type. Based on the theory of groin operation, which establishes the depth to which the groin extends as the critical factor affecting its impounding capacity, maximum economy in cost is achieved with a straight groin normal to the shore line. Various modifications such as a "T" or "L" head are usually designed with the primary purpose of limiting recession on the downdrift side of a groin. While these may achieve the intended purpose in some cases, the zone of maximum recession is often simply shifted to a point some distance away from the groin (on the downdrift side) and benefits are thus limited. Storm waves will normally produce greater scour at the extremities of "T" or "L" head structures than at the end of a straight groin normal to the shore, delaying the return to normal profile after storm conditions have abated.

Curved, hooked or angle groins have been employed for the same purposes as the "T" or "L" head types, and have the same objectionable features, that is, they invite excessive scour and are more costly to build and maintain than the straight groin normal to the shore. In cases where the adjusted shore alignment expected to result from a groin system will differ greatly from the alignment at the time of construction, it may be desirable to align the groins normal to the adjusted shore alignment in order to avoid angular wave attack on the structures after the shore has stabilized. This condition is most likely to be encountered in the vicinity of inlets and along the sides of bays.

3.48 ORDER OF GROIN CONSTRUCTION - This applies only to sites where a groin system is under consideration. Here two conditions arise: (1) where the groin system will be filled artificially and it is desired to stabilize the new beach in its advanced position; and (2) where littoral transport is depended upon to make the fill and it is desired to stabilize the existing beach or build additional beach with a minimum of detrimental effect on downdrift areas.

In the first instance the only interruption of littoral transport will be between the time the groin system is constructed and the time the artificial fill is made. In the interests of economy, the fill is normally placed at one time, especially if it is being accomplished by hydraulic dredge. Accordingly to reduce the time interval between groin construction

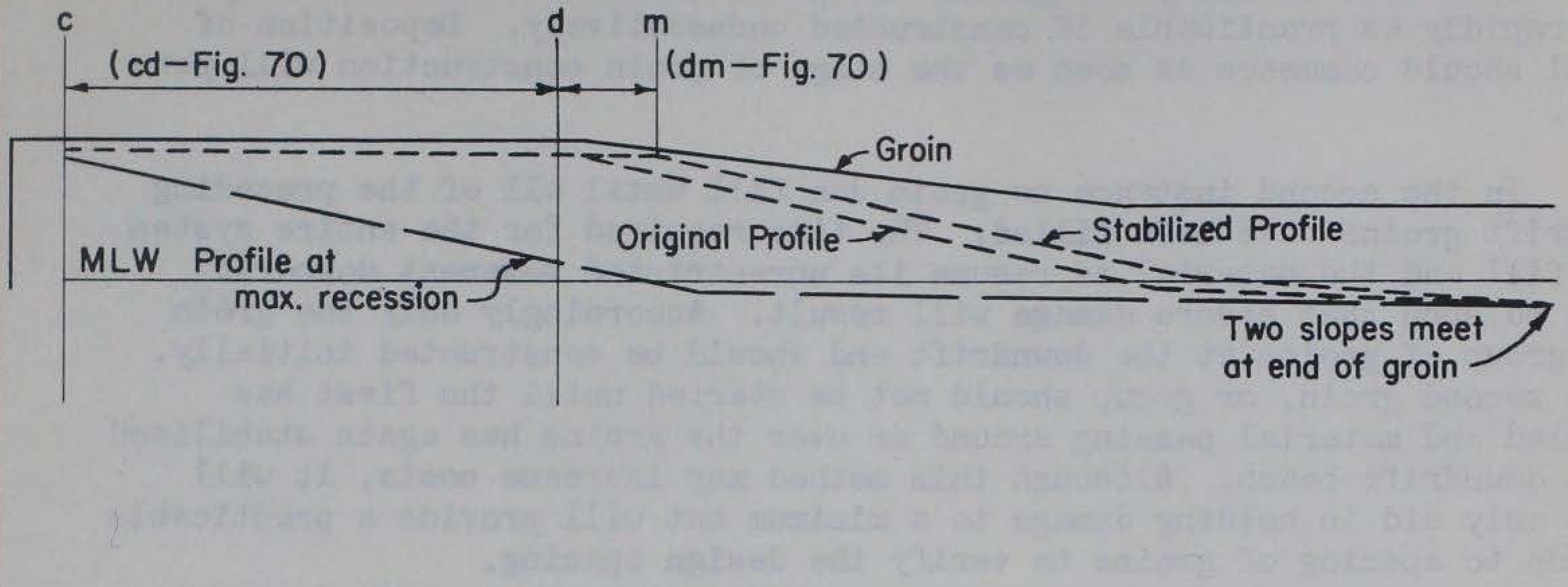


FIGURE 71 PROFILE ON DOWNDRIFT SIDE OF INTERMEDIATE GROIN

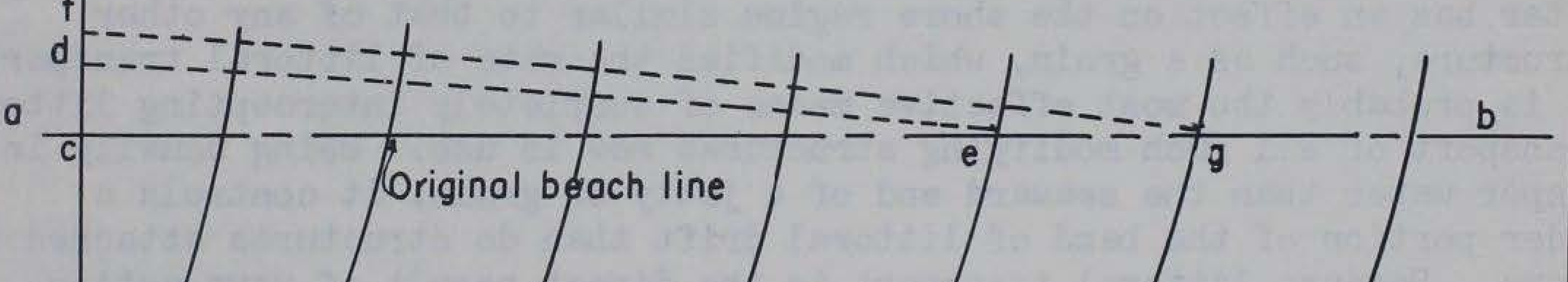


FIGURE 72. RECESSION-DOWNDRIFT OF A SINGLE GROIN

and deposition of fill, all groins should be constructed concurrently or as rapidly as practicable if constructed consecutively. Deposition of fill should commence as soon as the stage of groin construction will permit.

In the second instance no groin can fill until all of the preceding updrift groins have been filled. The time required for the entire system to fill and the material to resume its unrestricted movement downdrift may be such that severe damage will result. Accordingly only the groin or group of groins at the downdrift end should be constructed initially. The second groin, or group should not be started until the first has filled and material passing around or over the groins has again stabilized the downdrift beach. Although this method may increase costs, it will not only aid in holding damage to a minimum but will provide a practicable guide to spacing of groins to verify the design spacing.

3.5 OFFSHORE BREAKWATERS

3.51 USE - An offshore breakwater differs from other breakwaters in that it is generally parallel to and is not connected with the shore. This type of structure seldom has been constructed solely for shore protection because of its comparatively high first cost and the difficulty of minor maintenance, if required. However, under some specific conditions, this cost may be justified.

3.52 EFFECT ON THE SHORE LINE - The effects of an offshore breakwater on the shore line are partially illustrated by Figure 73. An offshore breakwater has an effect on the shore regime similar to that of any other structure, such as a groin, which modifies the rate of littoral transport. It is probably the most effective means of completely intercepting littoral transport of all such modifying structures now in use. Being usually in deeper water than the seaward end of a jetty or groin, it controls a wider portion of the band of littoral drift than do structures attached to shore. Because littoral transport is the direct result of wave action, the extent to which the breakwater intercepts the littoral drift is directly proportional to the extent of interception of wave action by the breakwater.

The effect on the shore line of an offshore breakwater is typified by the 2,000-foot structure at Santa Monica, California. This breakwater was constructed parallel to and 2,000 feet distant from shore in approximately 27 feet of water. Figure 73 shows, as dashed lines, contours of the accretion that existed behind the breakwater after 11 years.

3.53 OPERATION OF AN OFFSHORE BREAKWATER - An offshore breakwater at first tends to cause sand to deposit in its lee by damping the wave action in that area, thus reducing or eliminating the forces responsible for transport. Diffraction causes some wave action within the geometric shadow,

FIGURE 73 EFFECT ON SHORE LINE OF OFFSHORE BREAKWATER

but such wave action is much less than that which would exist in the area in the absence of the breakwater. The typical diffraction diagram drawn on Figure 73 shows that wave heights within the breakwater's geometric shadow are less than one-half the wave heights outside the breakwater.

As sand is deposited, a shore salient is formed in the still water behind the breakwater. This salient shore alignment acts as a groin, which causes the updrift shore line to advance. Concomitant with the advancement of the shore line which brings the zone of littoral transport closer to the breakwater, is an increase of the efficiency of the breakwater in acting as a sand trap. The salient is extended more rapidly and thus becomes increasingly efficient as a groin.

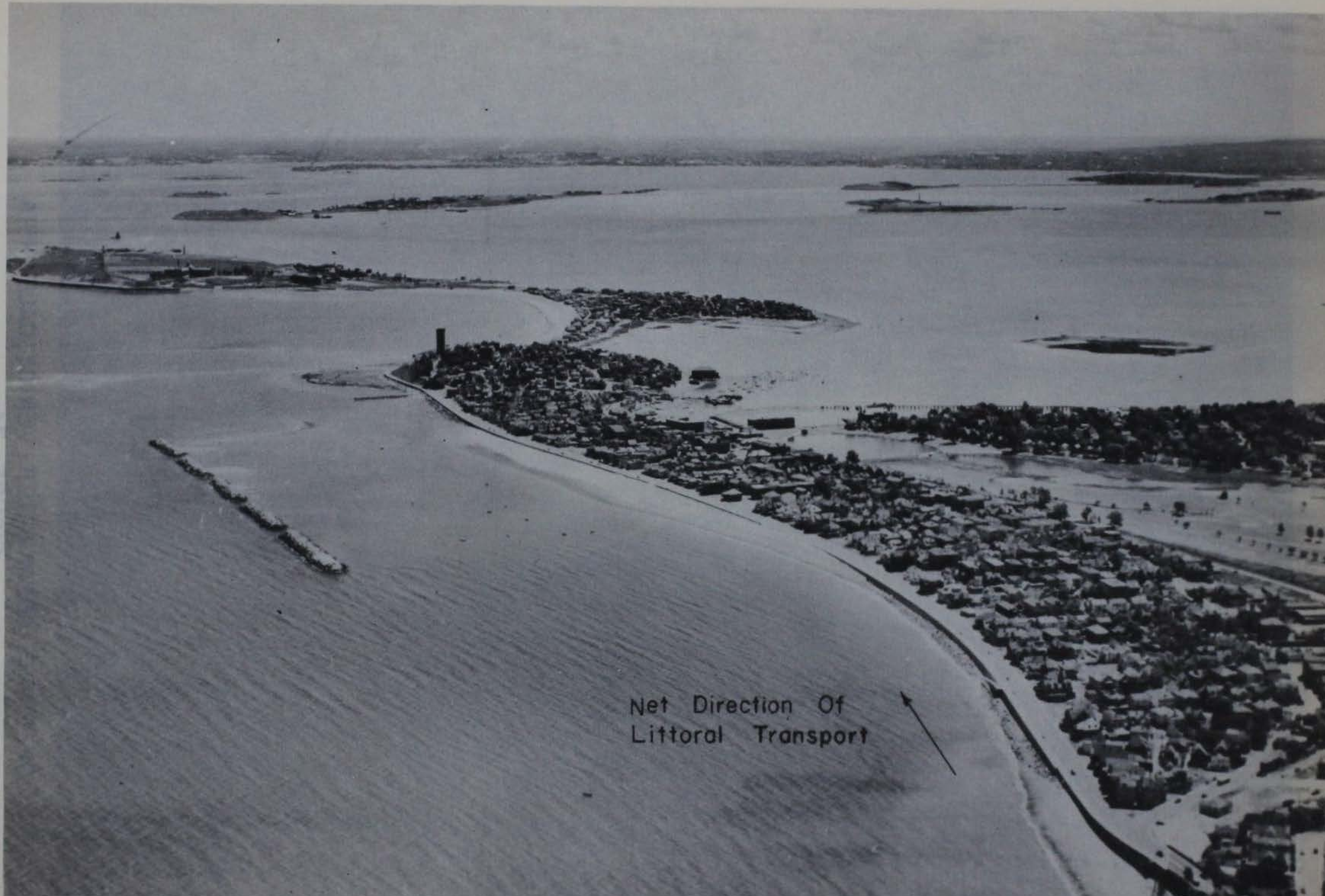
If the breakwater is of sufficient length in relation to its distance from the shore to act as a complete littoral barrier, the sand depositing action may continue until a tombolo is formed with the breakwater at its apex. Such a tombolo accretion is shown on Figure 74, an aerial photograph of the offshore breakwater at Venice, California.

The precise shape of the deposit is difficult to determine. In general there will be accretion updrift from the breakwater and erosion downdrift. The area immediately behind the breakwater customarily assumes a form convex seaward. It has been found for complete barriers that a large percentage of the total accumulation collects in the breakwater lee during the first year and that the ratio of material in the lee of the structure to total material trapped decreases steadily until such time that the trap is filled and littoral drift begins moving around the structure. A shore line profile for a complete barrier may be roughly approximated by drawing the high water line so that the area in square feet between this line and the original high water line is equal to the anticipated accretion in cubic yards at the time for which the profile is to be drawn.

3.54 <u>OFFSHORE BREAKWATERS IN SERIES</u> - It is not necessary to build offshore breakwaters as a single unit. A series of relatively short structures will have the same general effect as a single one, but the efficiency of the series as a sand trap will be decreased; a condition which is sometimes desirable. The tendency for a tombolo to form will be decreased. Figure 75 is an aerial photograph taken in 1949 of the breakwater set off Winthrop Beach, Massachusetts constructed in 1931-1933. The characteristic convex accretion in the breakwater lee is evident, as is also the erosion zone downdrift. Note that here shoals formed from the breakwaters extending landward, indicating that these breakwaters are in the littoral movement band.

3.55 <u>HEIGHT OF AN OFFSHORE BREAKWATER</u> - One of the factors determining the effectiveness of an offshore breakwater as a sand trap is its height in relation to the wave action at the site. That structure which can, except for diffraction at its ends, completely eliminate wave action in its lee will, if long enough, function as a complete littoral barrier. In this sense, then, the most efficient type of breakwater is one whose crest extends to such a height that no significant overtopping by waves takes place.

OFFSHORE BREAKWATER; VENICE, CALIFORNIA
FIGURE 74



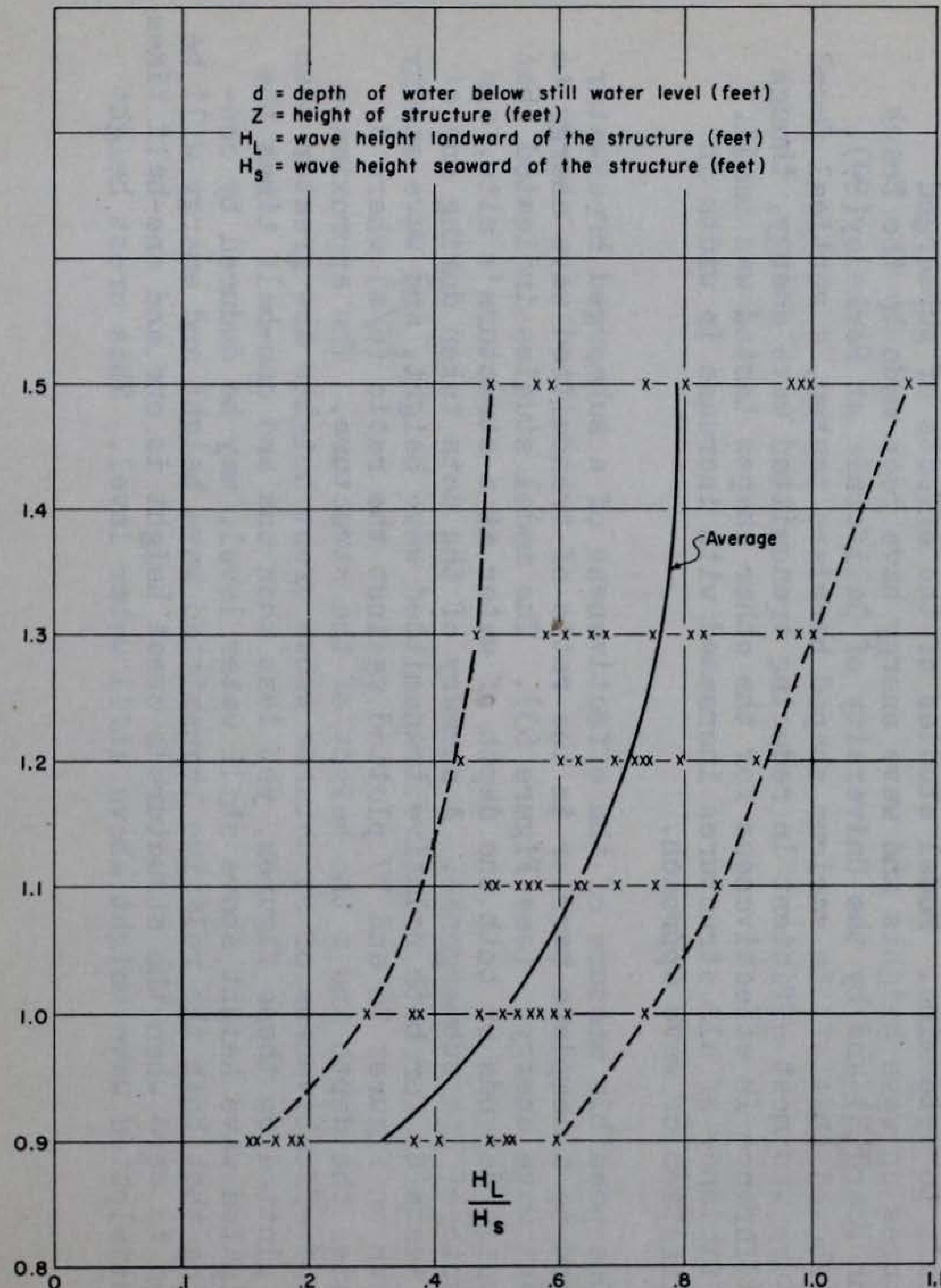
OFFSHORE BREAKWATER; WINTHROP BEACH, MASSACHUSETTS
FIGURE 75

It has been common practice to assume that essentially no wave overtopping takes place, in the case of rubble mound structures, the most common type in this country, if the structures' crest is one and one-half the design wave height above the design still water level, and this criterion is suggested as a measure of the effectiveness of such a breakwater as a littoral barrier. That is, if a breakwater's crest is one and one-half times the anticipated wave height above still water level, it should be considered to be as complete a littoral barrier for a given length of structure as could be built at the site. (Recent model studies give indication that a breakwater crest height of one wave height above still water datum may be adequate, but until further verification can be made of this point, for conservative design, the one and one-half criterion is suggested.)

It has already been noted, in the case of offshore breakwaters in series, that it may be desirable to build a structure so that it is not completely effective as a littoral barrier. This may be accomplished by construction of the breakwater to a height less than that sufficient to prevent overtopping. Subject to adequate marking or other means of preventing a navigation hazard, such partial barriers need not extend above low water. The maintenance requirements of a submerged breakwater are often so much less than those for a high breakwater that, on an economic basis alone, serious consideration should be given to such a structure where only a partial barrier is desired. (Some submerged offshore breakwaters are built primarily to retain a beach fill. They function primarily as bulkheads, though submerged, and may be treated as such.)

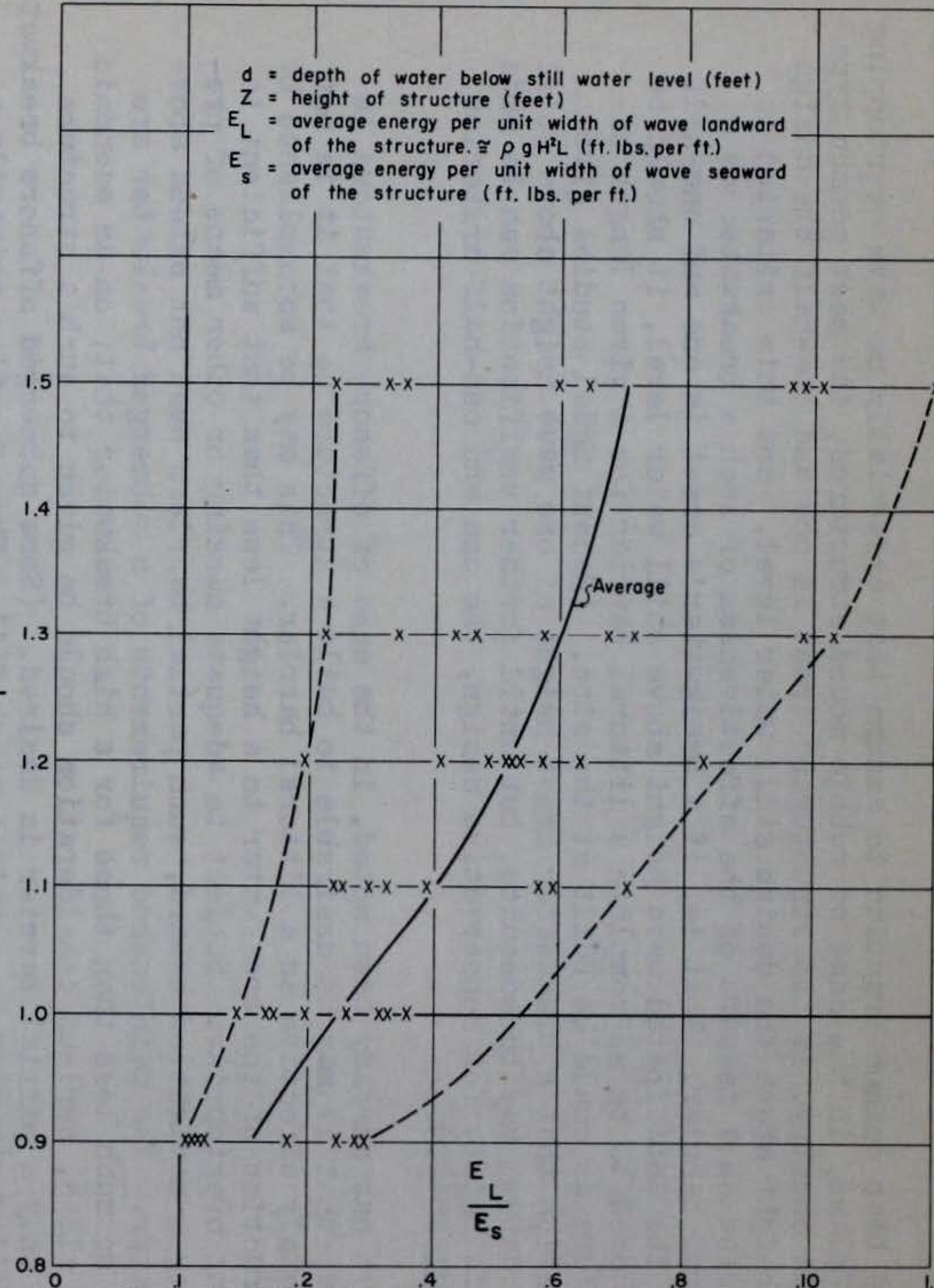
The relative effectiveness of partial and complete barriers is difficult to determine. Model studies of the effects of submerged structures on wave heights and wave energy have been made by the Beach Erosion Board(8) and by the University of California at Berkeley(89). These showed that of the various shaped barriers tested, a vertical faced structure was most efficient in reducing transmitted wave energy, though the difference in effectiveness for the other shapes tested was small. The efficiency of all structures increased with increase in width in the direction of wave approach.

One possible measure of the effectiveness of a submerged breakwater compared to a complete barrier is the ratio of transmitted wave energy to incident wave energy. (see Figure 63). The model studies indicated that this ratio depends on both the depth of water at a structure's site, and the depth of its submergence. A summary of the data taken during one of these tests(8) for both relative transmitted wave height, and wave energy is shown on Figures 76 and 77 plotted against the ratio (d/z) where d indicates the depth and z the height of the structure. The approximate range of effectiveness of structures whose crest heights are greater than those plotted on these figures, yet less than one and one-half times the anticipated wave height above still water level, may be deduced by considering that both the relative transmitted wave height and energy will be reduced to zero when the structure's crest height is one and one-half times the anticipated wave height above still water level. This crest height



AVERAGE EFFECT OF STRUCTURE ON WAVE HEIGHT RATIO WITH SUBMERGENCE

FIGURE 76



RATIO OF ENERGY LANDWARD TO SEAWARD FOR VARIED SUBMERGENCE OF STRUCTURES

FIGURE 77

should then be converted to a d/z ratio and the envelop curves extended to $H_L/H_s = 0$ or $E_I/E_s = 0$ at this value of d/z. For example, a structure founded in 20 feet of water at high tide with a design wave height of 10 feet, to be completely effective, should have its crest 15 feet above the high water line. In this case, $d = 20$ feet, $z = d + 15 = 35$ feet, and H_L/H_s or $E_I/E_s \cong 0$ at $d/z = 20/35 = 0.57$.

3.56 SITING OFFSHORE BREAKWATERS FOR SHORE STABILIZATION - Insofar as is known, there is no case in which offshore breakwaters have been designed with the purpose of stabilizing a shore in a predetermined position and thereafter permitting unimpeded littoral transport to the downdrift shore. It is theoretically possible to accomplish this by the proper combination of siting and structure dimensions. Design criteria for this purpose have not been established. Although they may be used with confidence where circumstances require substantially complete littoral barriers, their use in lieu of groins as a stabilization measure is not recommended in the present state of knowledge.

PART II

STRUCTURAL DESIGN

CHAPTER 4

PHYSICAL FACTORS

4.1 WAVE HEIGHTS

4.11 DETERMINATION OF DESIGN WAVE HEIGHT AND DIRECTION - If economically feasible, any structure exposed to wave action should be designed to withstand the effects of the highest wave to be expected at the structure's location. Visual observations of storm waves are usually unreliable. Direct measurement is costly and time consuming. General procedures for developing the height and direction of the design wave by use of refraction diagrams are described in the following paragraphs.

4.12 SIGNIFICANT DESIGN WAVE - STRUCTURE OUTSIDE THE BREAKER ZONE - From the structure's intended location, draw a set of "refraction fans", for various wave periods in increments of about 2 seconds that might be expected at the site, and determine refraction coefficients by the method given in Chapter I. Tabulate the refraction coefficients so determined for the various wave periods chosen and for each deep water direction of approach. The statistical wave data derived from synoptic weather charts or other sources should then be reviewed to determine if the directions and periods for which the refraction coefficients are large may be expected to recur with reasonable frequency.

That combination of deep water wave height and refraction coefficient which gives the highest wave height at the structure's location determines the design wave direction of approach and period. The inshore height so determined is the significant design wave height. A typical example of such an analysis is shown in Table 9.

In the following example, although the highest deep water waves approached from directions ranging from W to NW, the refraction study indicated that higher waves inshore may be expected from more southerly directions.

It must be remembered that the accuracy with which refraction procedures may be applied in the determination of design wave characteristics decreases as the underwater contours over which the refraction diagrams must be drawn become more complex. Under highly complex bottom conditions direct observations may be required.

TABLE 9

DETERMINATION OF SIGNIFICANT DESIGN WAVE HEIGHTS

Direction (1)	Significant Deep Water Wave Height (2) (feet)	Period (3) (seconds)	Refraction Coefficient* (4)	Refracted Wave Height to Nearest Half Foot (5) (feet)
NW	15	8	0.10	1.5
		10	0.07	1.0
		12	0.04	0.5
WNW	12	8	0.15	2.0
		10	0.12	1.5
		12	0.09	1.0
W	10	10	0.30	3.0
		12	0.31	3.0
		14	0.20	2.0
		16	0.25	2.5
WSW	10	10	0.60	6.0**
		12	0.50	5.0
		14	0.35	3.5
		16	0.35	3.5
SW	8	12	0.72	6.0**
		14	0.59	4.5
		16	0.40	3.0

* This refraction coefficient is equal to $\sqrt{b_o/b}$ (H/H_o')

** Adopted as the significant design wave height

Columns 1, 2, and 3 are taken from the statistical wave data as determined from synoptic weather charts

Column 4 is determined from the relative distances between two adjacent orthogonals in deep water and in shallow water

Column 5 is the product of columns 2 and 4

4.2 WAVE FORCES

4.21 GENERAL - In an analysis of the forces exerted on structures by waves, a division should be made between the action of non-breaking waves, breaking waves, and broken waves. Pressures due to non-breaking waves will be essentially hydrostatic. Broken and breaking waves on the other hand, exert additional pressures due to the dynamic effects of the turbulent water in motion and the compression of entrapped air pockets. These pressures may be much greater than those due entirely to hydrostatic forces. Therefore structures located in an area in which storm waves may break, should be designed to withstand much greater forces and moments than those structures which would be attacked only by non-breaking waves.

4.22 DETERMINATION OF BREAKER DEPTH AND HEIGHTS - Given deep water wave conditions, depths at which waves will break may be found by using the full line curves (those labelled "Iversen") of Figure 38. However, if the depth of breaking as found from this curve is less than the anticipated depth of water at a structure, it should not immediately be assumed that these waves would not break on the structure. Storm waves are rarely so regular that the depth of breaking may be precisely determined. Their heights and lengths may be extremely variable, especially if the generating area is not far removed from the structure. The curve of Figure 38 probably represents the lower limit of a range of breaker depths which may actually be found in nature for any one wave condition. For safety, it should be assumed that waves may break in depths greater than those given by the curve, up to the point at which the actual depth (d) at the structure is 1.5 times the depth of breaking (d_b) as found from Figure 38.

For structures located in shallow water, when deep water wave conditions are not known or when the design wave determined from deep water conditions would break before reaching the structure, the height of the maximum wave which would break on the structure can be found approximately from the relationship $d_b = 1.3H_b$.

4.23 NON-BREAKING WAVES - Ordinarily, a shore structure would be so located that storm waves would break in the depth in which the structure is founded. However, in protected regions or in areas where the available fetch is limited, non-breaking wave conditions may occur. The most commonly used method for the determination of pressure due to these waves in that of M. Sainflou(110).

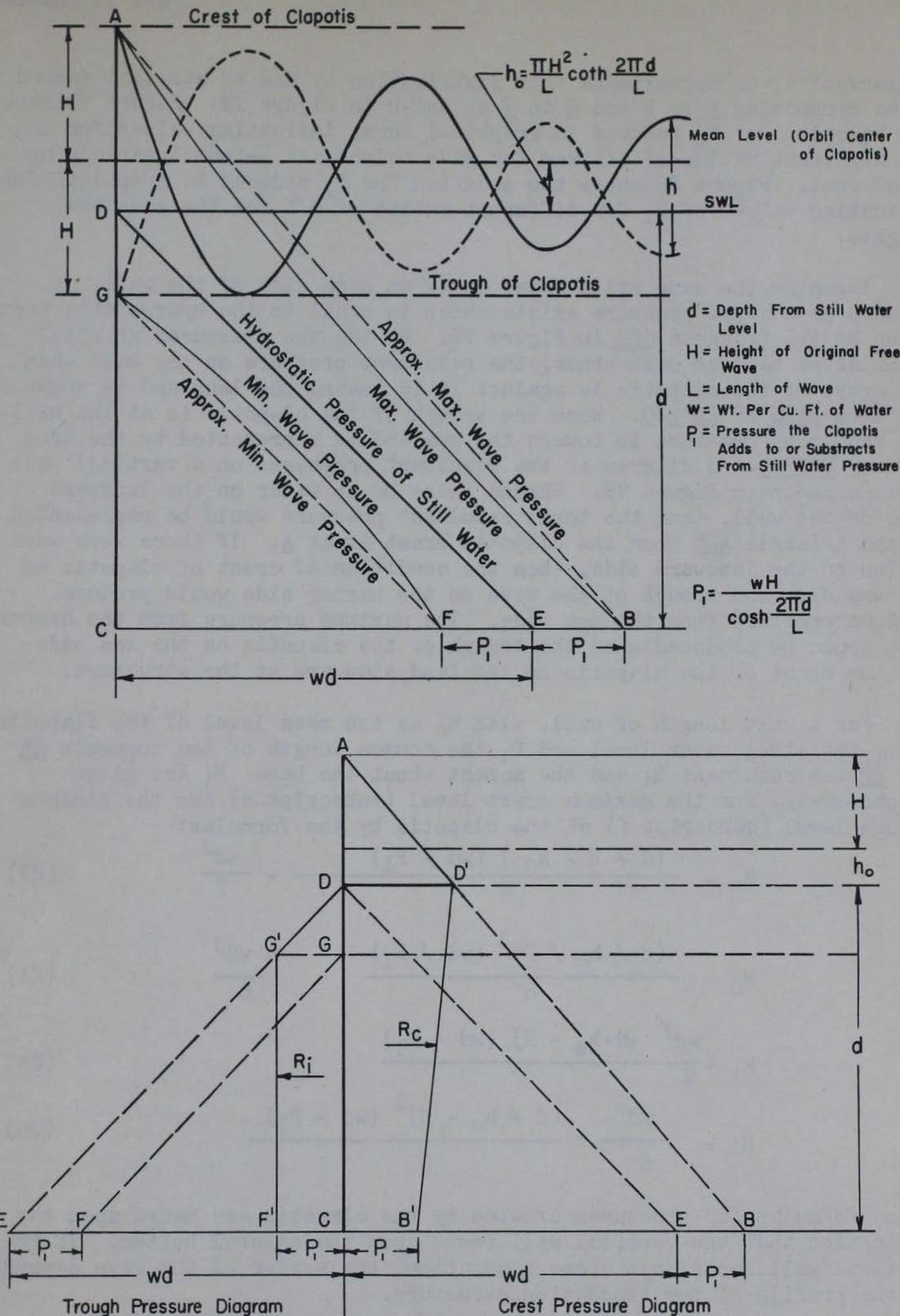
4.231 Sainflou Method(110): Forces Due to Non-breaking Waves - If a wave of length L and height H strikes the vertical face of the wall AC (Figure 78), a standing wave or clapotis will be set up. The point A is the maximum elevation of the crest, and point G is the minimum elevation of the trough of the clapotis. The mean level or orbit center is above the still water level D a distance,

$$h_o = (\pi H^2/L) \coth (2 \pi d/L) \tag{21}$$

and DA is equal to $H + h_o$, while DG equals $H - h_o$. The hydrostatic pressure (wd) at the base C of the wall is scaled out from C and plotted as E. The triangle CDE is the hydrostatic pressure distribution against the wall due to water at still water level. As the surface of the clapotis moves above or below still water it will increase or decrease the hydrostatic pressure at the base of the wall by the amount P_1. This change in pressure is

$$P_1 = \frac{wH}{\cosh \frac{2 \pi d}{L}} \tag{22}$$

Plotting P_1 in both plus and minus directions from point E gives points B and F as the maximum and minimum pressures, respectively, at the base caused by the clapotis against the sea face of the wall. The solid curved lines labelled maximum wave pressure and minimum wave pressure denote the pressure distributions computed by theoretically exact formulas. These curved lines are so close to a straight line that it is permissible, and



CLAPOTIS ON VERTICAL WALL

FIGURE 78

conservative, to approximate this distribution by use of straight dashed lines connecting A to B and G to F as shown in Figure 78. Figure 79 shows the equation for h_o reduced to graphical form, indicating values for Lh_o for different ratios of d/L and for wave heights at 5-foot intervals up to 40 feet. Figure 80 shows the equation for P_1 reduced to graphical form, indicating values of P_1 for different ratios of d/L for the same wave heights.

Assuming the same still water level on each side of the wall, an outward or seaward pressure exists which is equal to the hydrostatic pressures shown by the triangle CDE in Figure 78. As the two pressures at still water level balance each other, the resultant pressure on the wall when the crest of the clapotis is against it is toward the land and is shown by the area ABED or AD'B'C. When the trough of the clapotis is at the wall the resultant pressure is toward the sea and is represented by the area DEFG or DG'F'C. A diagram of the resultant pressures on a vertical wall is also shown in Figure 78. Should there be no water on the landward side of the wall, then the total resultant pressure would be represented by the triangle ACB when the clapotis crest is at A. If there were wave action on the landward side, then the condition of crest of clapotis on the sea side and trough of the wave on the harbor side would produce maximum pressure from the sea side. The maximum pressure from the harbor side would be produced when the trough of the clapotis on the sea side and the crest of the clapotis on the land side are at the structure.

For a unit length of wall, with h_o as the mean level of the clapotis above the still water level and P_1 the common length of the segments EB and EF the resultant R_i and the moment about the base M_i are given respectively, for the maximum crest level (subscript e) and the minimum trough level (subscript i) of the clapotis by the formulas:

$$R_e = \frac{(d + H + h_o)(wd + P_1)}{2} - \frac{wd^2}{2} \qquad (23)$$

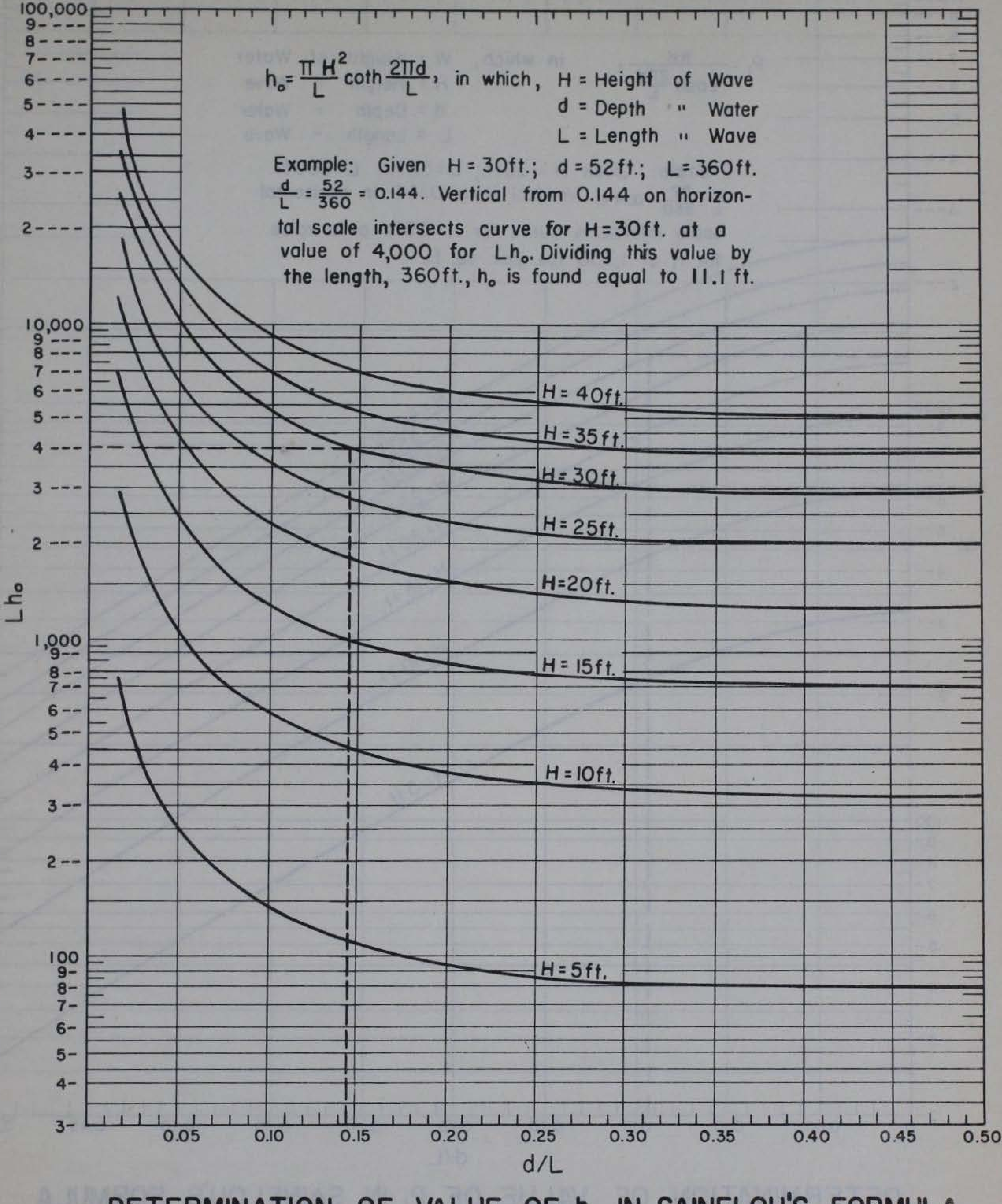
$$M_e = \frac{(d + h_o + H)^2 (wd + P_1)}{6} - \frac{wd^3}{6} \qquad (24)$$

$$R_i = \frac{wd^2}{2} - \frac{(d + h_o - H)(wd - P_1)}{2} \qquad (25)$$

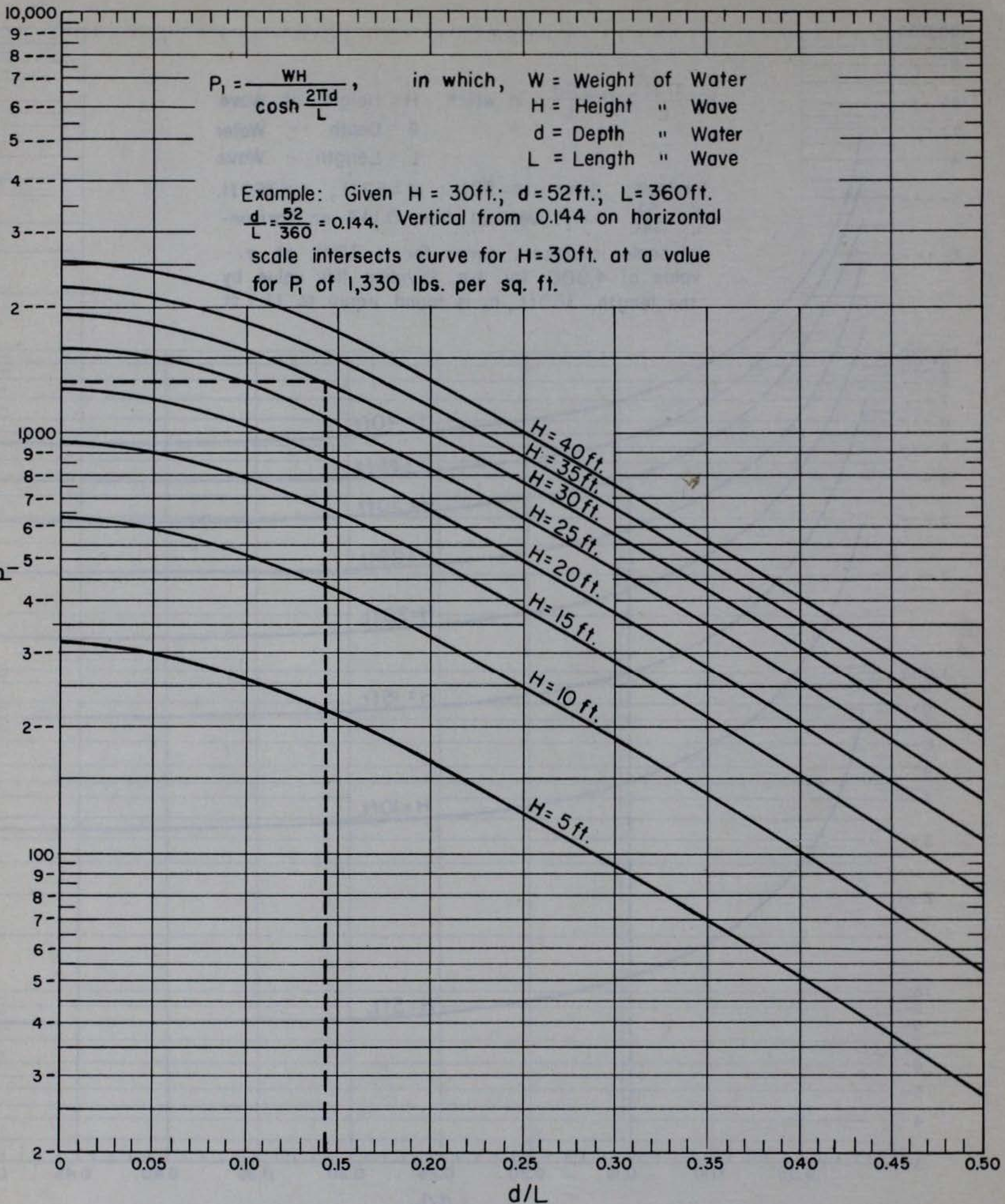
$$M_i = \frac{wd^3}{6} - \frac{(d + h_o - H)^2 (wd - P_1)}{6} \qquad (26)$$

These formulas for pressures created by the clapotis are based upon the assumption that the vertical wall rests upon the natural bottom. If the vertical wall rests on a stone foundation, the action of the wave depends on the profile of the foundation structure.

4.232 Wall of Low Height - If the height of the wall is less than the



DETERMINATION OF VALUE OF h_o IN SAINFLOU'S FORMULA
FIGURE 79



DETERMINATION OF VALUE OF P_1 IN SAINFLOU'S FORMULA

FIGURE 80

predicted wave height at the wall, forces may be approximated by drawing the force polygons as if the wall were higher than the impinging wave, then analyzing only that portion below the wall crest. Thus in Figure 81 the force due to a wave crest at the wall is computed from the area AFBSC.

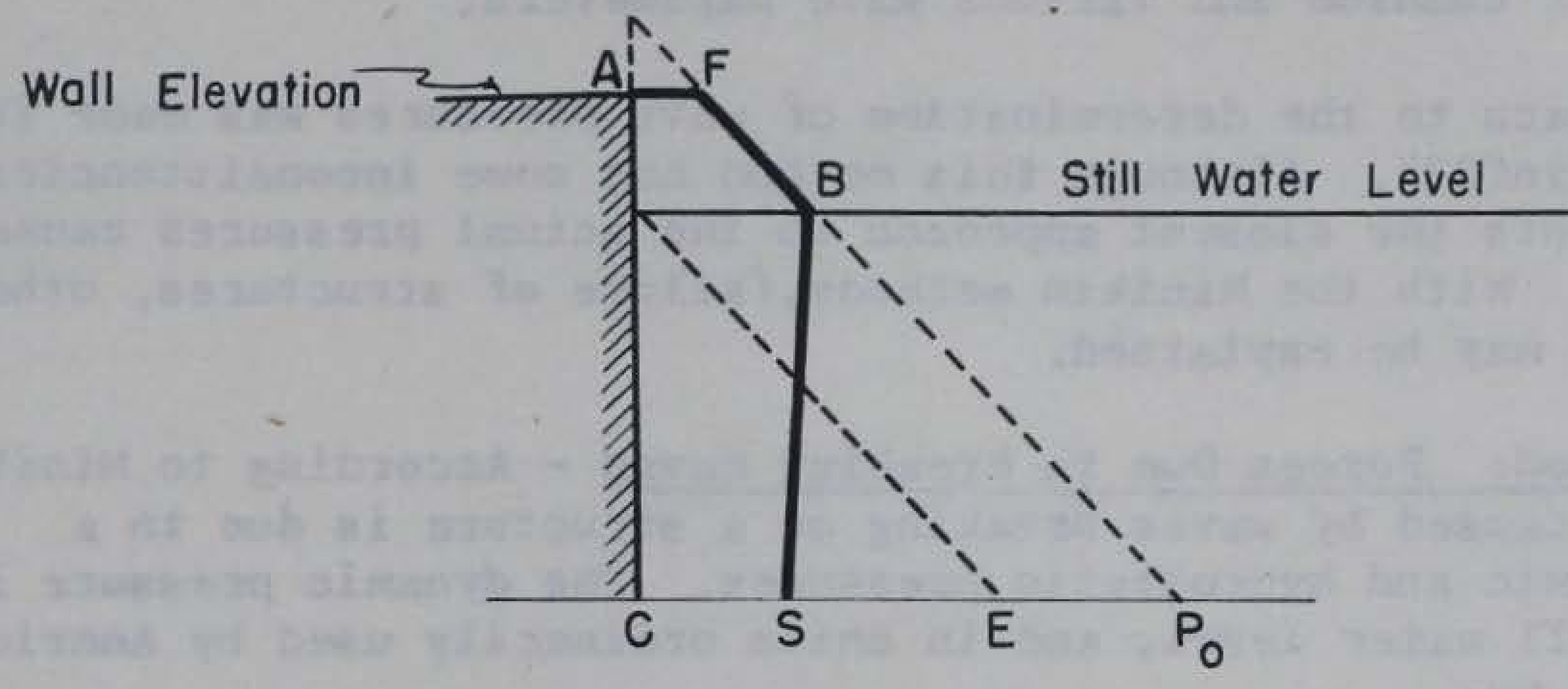


FIGURE 81 PRESSURE ON WALLS OF LOW HEIGHT

4.24 WAVES BREAKING ON A STRUCTURE - Ordinarily bulkheads and seawalls are so located that storm waves may break directly on them. Even those structures which are located landward of the low water shore line may be exposed to the action of breaking waves at times of high water.

There have been three major attempts to correlate the high pressures known to exist with measurable wave parameters. In 1934, D.A. Molitor(88) published a suggested solution of this problem, using a semi-theoretical approach and making use of the observations of D. D. Gaillard(42) taken on Lake Superior in 1904 and spring dynamometer readings taken during a storm at Toronto in 1915. Unfortunately since publication of the paper, pressures have been observed far in excess of those predicted by the Molitor equations and structures have failed which according to the Molitor equations were adequately designed.

Essentially the Molitor wave pressure solution formed an envelope of the dynamometer readings taken in 1915. The measurements were taken throughout the storm and the maxima at various elevations recorded. Thus, though his equations purport to give a pressure curve for a single impinging wave, they really give a curve representing the maximum pressure recorded from many waves. This would ordinarily lead to conservative results but the range of wave parameter variables was too restricted for the results of these measurements to be applied to general wave conditions and the dynamometer equipment used would not measure the impact by pressures involved.

In 1939, R. A. Bagnold(5) reported on measurements of shock pressures due to breaking waves recorded under model conditions. Pressures so recorded were greatly in excess of any prior predicted ones. Bagnold found that for these "shock pressures", a correlation could be established between the magnitude of the peak pressure and the thickness of a cushion of air entrapped by waves breaking on a structure. Unfortunately these experiments were interrupted and no further relationship has been established between the thickness of the air cushion and various wave parameters.

The last approach to the determination of wave pressures was made in 1946 by R. R. Minikin(87). Although this method has some inconsistencies, it probably represents the closest approach to the actual pressures caused by a breaking wave. With the Minikin methods,failure of structures, otherwise unpredictable, may be explained.

4.241 Minikin Method: Forces Due to Breaking Waves - According to Minikin, the total pressure caused by waves breaking on a structure is due to a combination of dynamic and hydrostatic pressures. The dynamic pressure is concentrated at still water level, and in units ordinarily used by American engineers, is given by:

$$P_m = \frac{101\, H_b w}{L_D} \times \frac{d}{D} \,(D + d) \qquad (27)$$

where H_b - the height of wave just breaking on the structure (in feet)
w - the unit weight of the water (in pounds per cubic foot)
d - the depth of water at the structure (in feet)
D and L_D = deeper water depth and wave length respectively (in feet)
Based on descriptive passages in his paper, values found for D and L_D by the following method should approximate those he considered.

(It should be noted that when plotting d/D versus wave pressures, the wave pressures become very large as d/D approaches 1.0. It is recommended that the application of the Minikin Method for determination of pressures due to waves breaking on a structure, be used with reservation when the slope in front of the structure is flatter than about 1 on 20.)

Given a wave period, the deep water wave length L_o (in feet) may be found from

$$L_o = 5.12\, T^2 \qquad (28)$$

With the ratio d/L_o, the wave length at the structure L_d at depth d may be determined from the d/L value taken from Table D-1 of Appendix D (which in this case is d/L_d). Knowing the slope S before the structure, D may be determined from

$$D = d + L_d\, S \qquad (29)$$

L_D may then be determined, again from Table D-1 of Appendix D, by computing the ratio D/L_o, tabulating the corresponding value of d/L (which in this case is D/L_D), and dividing D by this ratio. The hydrostatic pressure on the seaward side at still water level (subscript "s"), and at the depth d (subscript "d") are given by

$$P_s = \frac{wH_b}{2} \quad \text{and} \quad P_d = w\,(d + H_b/2) \qquad (30)$$

The pressure curve may be plotted by considering that the dynamic pressure is concentrated at still water level and falls away rapidly along a parabolic curve to zero at a distance $H_b/2$ above and below still water level. To this is added the hydrostatic pressure on the seaward side plotted in a triangular area from $H_b/2$ above still water level to the bottom. Hydrostatic pressures on the landward side plotted in a triangular area from still water level to the bottom must be subtracted if such pressures obtain. This construction is shown in Figure 82, the dashed and dotted lines indicating the separate effects of the dynamic and hydrostatic pressures, and the solid line the combined pressures, for the case where hydrostatic pressures are on each side of the wall.

The resultant wave thrust or force per linear foot of structure may be determined from the area of this diagram and is given by

$$R = \frac{P_m H_b}{3} + P_s\left(d + \frac{H_b}{4}\right) \tag{31}$$

The resultant overturning moment about the ground line before the wall, is the sum of the moments of the individual areas and is given by

$$M = \frac{P_m H_b}{3} d + \frac{P_s d^2}{2} + \frac{P_s H_b}{4}\left(d + \frac{H_b}{6}\right) \tag{32}$$

Similar computations may be made if there is no water on the land side, in which case the thrust per linear foot of the structure is

$$R = \frac{P_m H_b}{3} + \frac{P_d}{2}\left(d + \frac{H_b}{2}\right) \tag{33}$$

and the moment about the ground line is

$$M = \frac{P_m H_b}{3} d + \frac{P_d}{6}\left(d + \frac{H_b}{2}\right)^2 \tag{34}$$

The combined pressures for this case are indicated by a dot-dash line on Figure 82.

4.25 <u>WAVES BREAKING SEAWARD OF A STRUCTURE</u> - Certain protective structures may be so located that even under severe storm and tide conditions waves will break before striking them. For example, walls built landward of the ordinary high water shore line may, because of scour or wind-set-up, have a significant depth of water against them during a storm. To date no studies have been made to relate forces due to broken waves to various wave parameters, and it is necessary to make certain simplifying assumptions to determine approximate design forces.

Assume that immediately after breaking, the water mass in a wave moves forward with the velocity of wave propagation just before breaking; that is, upon breaking, the water particle motion changes from oscillatory to translatory. This turbulent slug of water then moves up to and over the

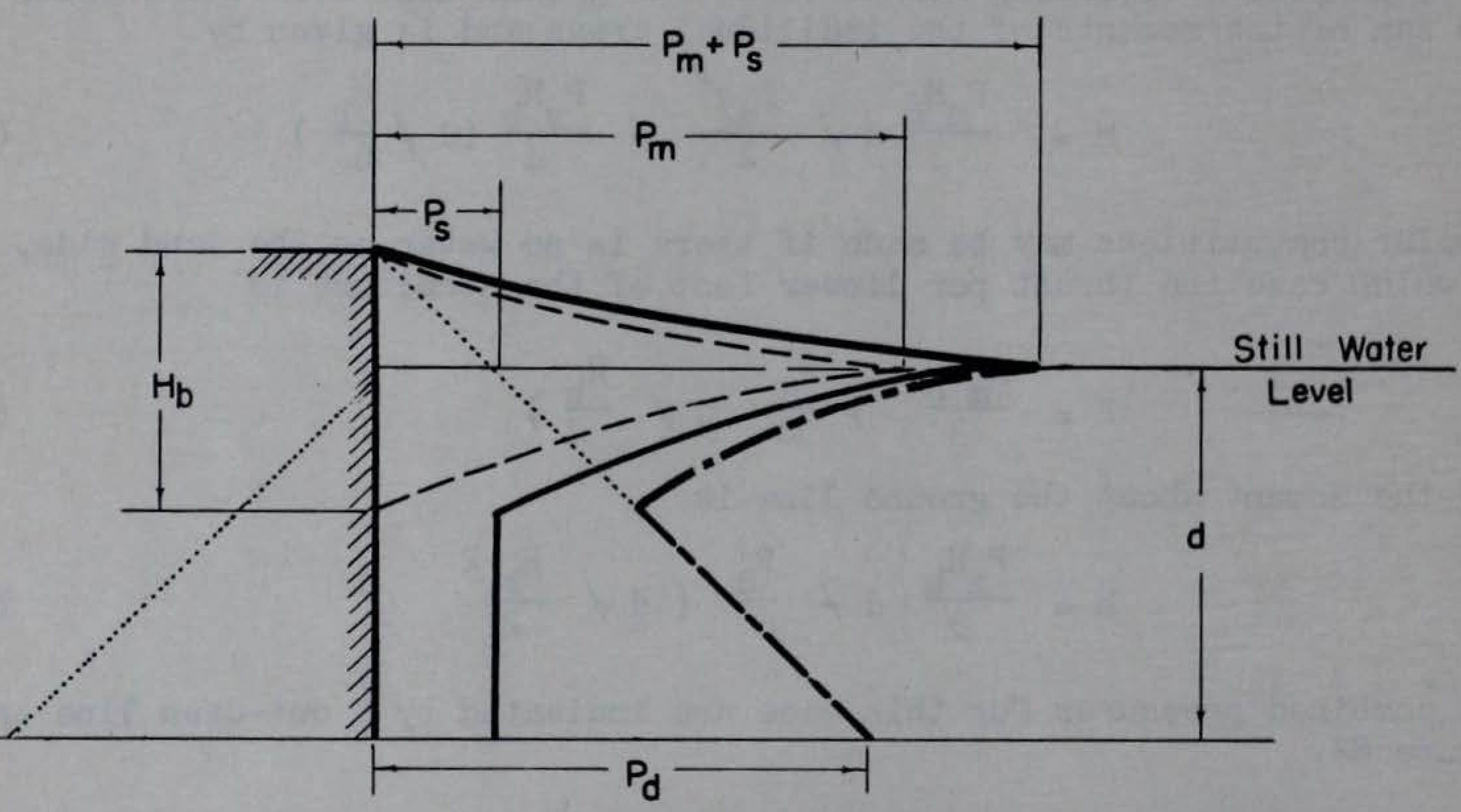


FIGURE 82 MINIKIN WAVE PRESSURE DIAGRAM

shore line. Dividing the inshore area after the wave breaks into two parts, seaward and landward of the shore line, for a conservative estimate of wave forces, assume that neither the wave height nor the wave velocity decreases from the breaking point to the shore line, and that after passing the shore line, the wave will run up to twice its height at breaking, decreasing linearly in velocity to zero at this point.

It has been found from model tests, that upon breaking, approximately 70 percent of the full breaking wave height (H_b) is above the still water level. Using for the velocity of wave propagation (C), the approximate relationship $C = \sqrt{gd_b}$ where g is the acceleration of gravity and d_b is the breaking wave depth, wave pressures on a wall may be approximated.

These pressures will be partly dynamic and partly static. For walls located seaward of the shore line, the dynamic part will be:

$$P_m = \frac{wC^2}{2g} = \frac{wd_b}{2} \qquad (35)$$

where w is the unit weight of water. The static part will vary from zero at a height h_c (the height of that portion of the breaking wave above still water level, $h_c = 0.7\ H_b$) to a maximum at the wall base. This maximum will be given by:

$$P_s = w(d + h_c) \qquad (36)$$

where d is the depth of water at the structure.

Assuming that the dynamic pressure is uniformly distributed from the still water level to a height h_c above the still water level, the total wave thrust will be:

$$\begin{aligned} R &= R_m + R_s \\ &= P_m(h_c) + P_s\left(\frac{d + h_c}{2}\right) \qquad (37) \\ &= \frac{wd_b(h_c)}{2} + \frac{w}{2}(d + h_c)^2 \end{aligned}$$

The overturning moment about the ground line at the seaward face of the structure will be:

$$\begin{aligned} M &= M_m + M_s \\ &= R_m\left(d + \frac{h_c}{2}\right) + R_s\left(\frac{d + h_c}{3}\right) \qquad (38) \\ &= \frac{w\, d_b h_c}{2}\left(d + \frac{h_c}{2}\right) + \frac{w}{6}(d + h_c)^3 \end{aligned}$$

The pressure diagram for this case is shown on Figure 83.

For walls located landward of the shore line, the velocity v' of the water mass at the structure at any point between the shore line and the point of maximum assumed uprush may be approximated by:

$$v' = C(1 - \frac{x_1}{x_2}) \tag{39}$$

and the wave height (h') above the ground surface by:

$$h' = h_c \, (1 - \frac{x_1}{x_2}) \tag{40}$$

where x_1 = the distance from the shore line to the structure
x_2 = the distance from the shore line to the limit of wave uprush; $x_2 = 2H_b \cot \alpha$
α = the angle of the beach slope with the horizontal.

An analysis similar to that for structures located seaward of the shore line gives for dynamic and static pressures:

$$\text{Dynamic pressure } P_m = \frac{w\,v'^2}{2g} = \frac{wd_b}{2}(1 - \frac{x_1}{x_2})^2 \tag{41}$$

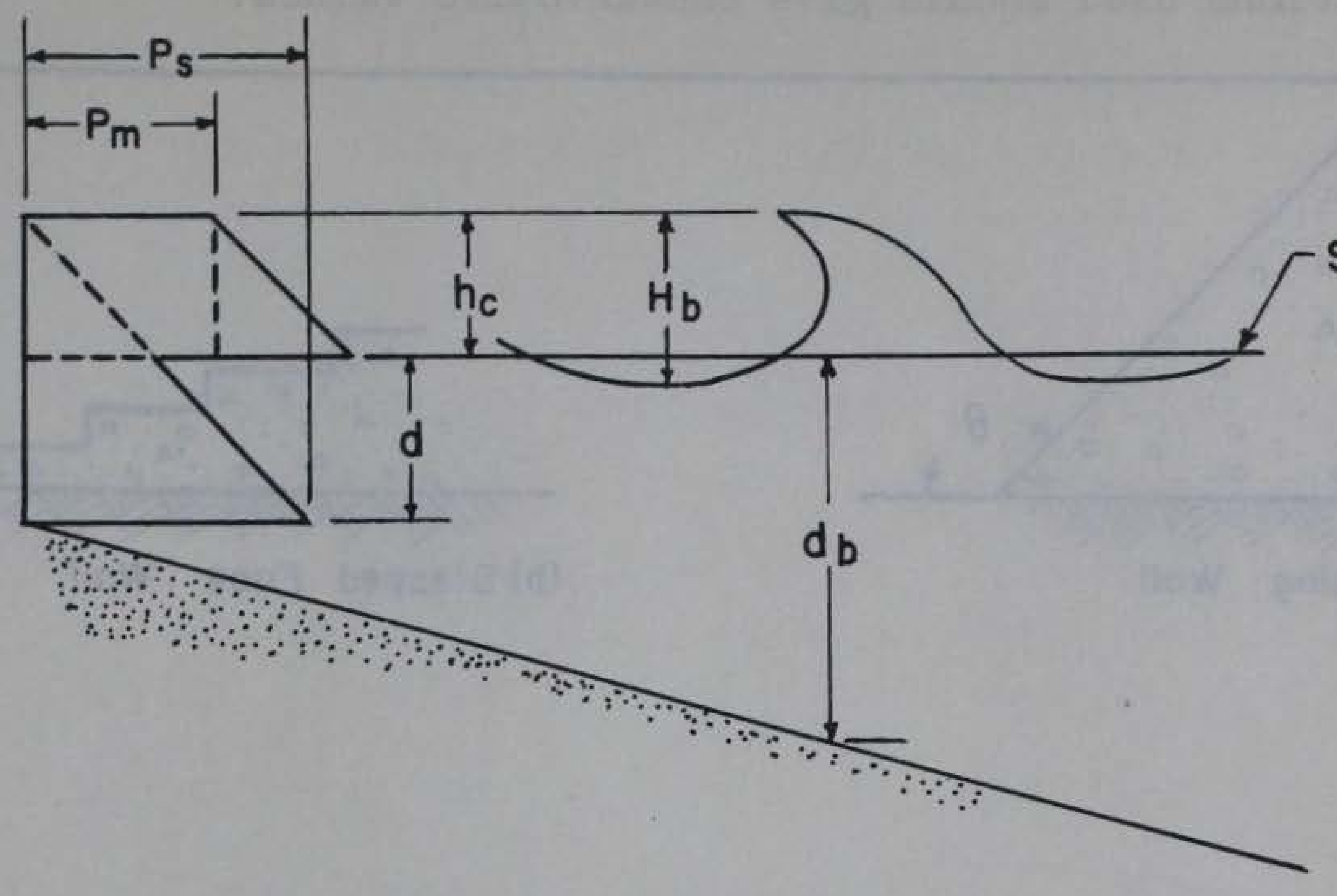
$$\text{Static pressure } P_s = w\,h' = w\,h_c(1 - \frac{x_1}{x_2}) \tag{42}$$

and for the wave thrusts and moments:

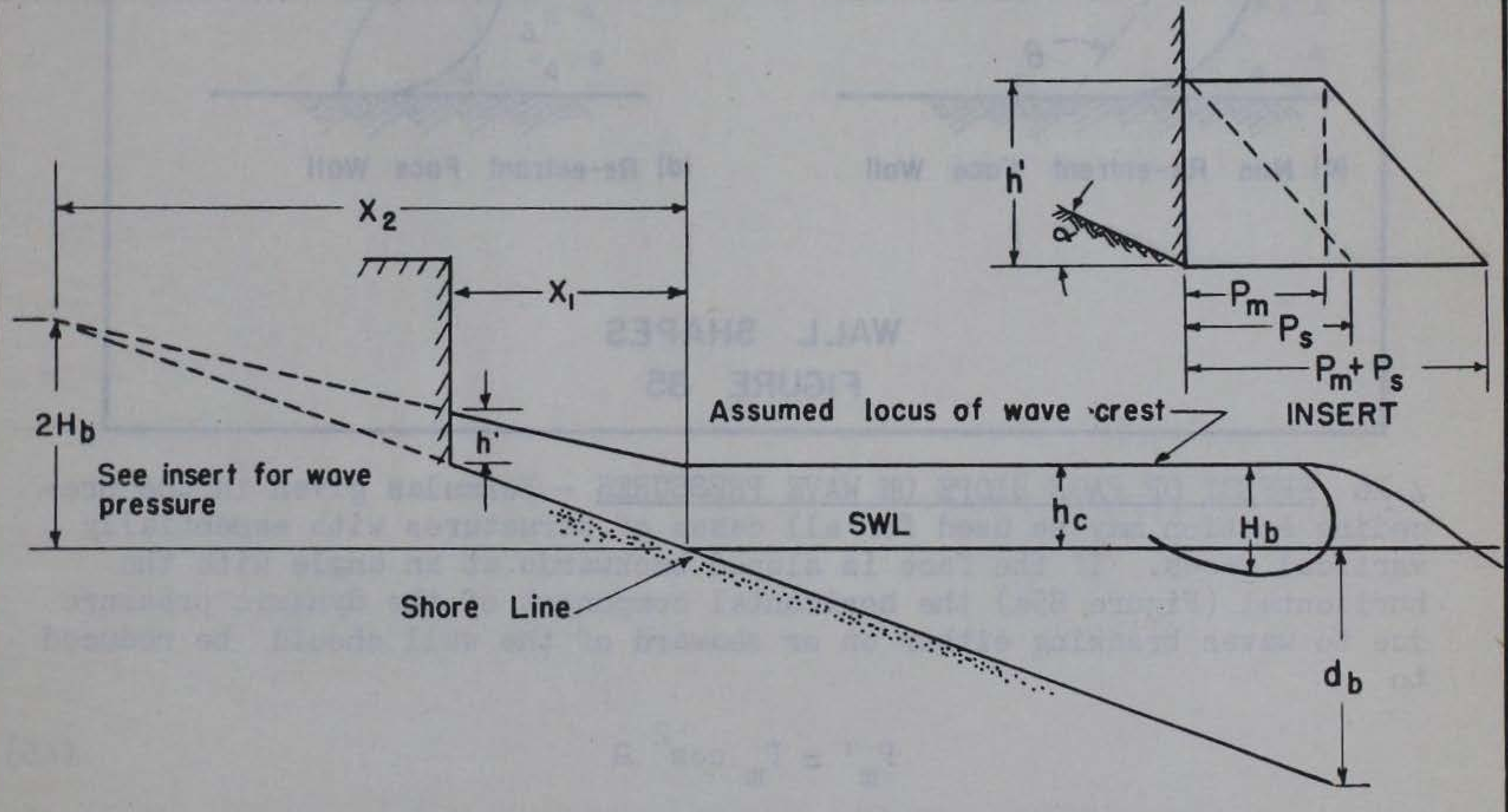
$$\begin{aligned} \text{Thrust} \quad R &= R_m + R_s \\ &= P_m\,h' + \frac{P_s\,h'}{2} \\ &= \frac{w\,d_b h_c}{2}(1 - \frac{x_1}{x_2})^3 + \frac{w\,h_c^2}{2}(1 - \frac{x_1}{x_2})^2 \end{aligned} \tag{43}$$

$$\begin{aligned} \text{Moment} \quad M &= M_m + M_s \\ &= R_m \frac{h'}{2} + R_s \frac{h'}{3} \\ &= \frac{w\,d_b h_c^2}{4}(1 - \frac{x_1}{x_2})^4 + \frac{w\,h_c^3}{6}(1 - \frac{x_1}{x_2})^3 \end{aligned} \tag{44}$$

It must be remembered that pressures, forces and moments computed on the above basis will be approximations. The wave behavior assumed is a simplified one. Especially in the case of structures located landward of the shore line, where the run-up criterion adopted was a fixed function of

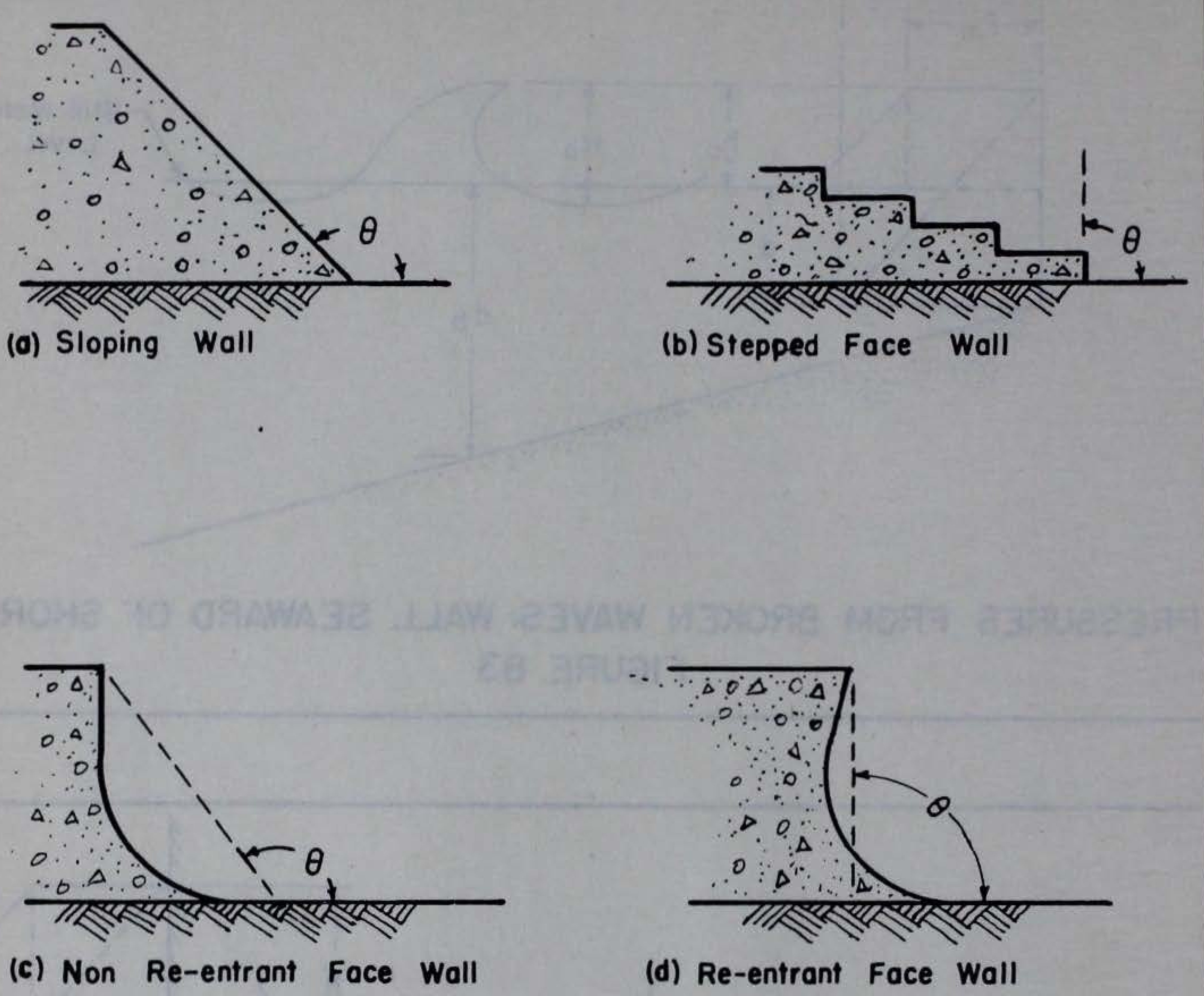


WAVE PRESSURES FROM BROKEN WAVES: WALL SEAWARD OF SHORE LINE
FIGURE 83



WAVE PRESSURES FROM BROKEN WAVES: WALL LANDWARD OF SHORE LINE
FIGURE 84

the wave height alone, the preceding equations will not be exact. However, the assumptions used should give conservative values.



WALL SHAPES
FIGURE 85

4.26 EFFECT OF FACE SLOPE ON WAVE PRESSURES - Formulas given in the preceding section may be used for all cases of structures with essentially vertical faces. If the face is sloped backwards at an angle with the horizontal (Figure 85a) the horizontal component of the dynamic pressure due to waves breaking either on or seaward of the wall should be reduced to

$$P_m' = P_m \cos^2 \theta \tag{45}$$

The vertical component of this wave force may be neglected in stability computations. Forces on stepped face structures (Figure 85b) may, for design calculations, be computed as if the face were vertical, since it is probable that dynamic pressures of the same order as those computed for vertical walls would exist. Forces on curved non-re-entrant face structures (Figure 85c) may be calculated by using a line from the top to the bottom of the face to determine an average slope. Re-entrant curved face walls (Figure 85d) may be considered as vertical.

4.27 WAVE FORCES ON RUBBLE MOUND STRUCTURES(58)(49) - Until recently, the design of rubble mound structures was largely based on experience and general knowledge of a particular site's conditions. Efforts to rationalize the design of these structures have been made. These entail the observation of failures and the determination of constants to be applied to various parameters in an attempt to explain the failures. Because of the empirical nature of the formulas developed, they are generally expressed in terms of the size stone required to withstand design wave conditions. These formulas have been partially substantiated in model studies. However, they still are only a guide and cannot be blindly substituted for experience.

In 1938, Iribarren(58) presented formulas for the design of rubble mound structures, These formulas permitted calculation of sea side slopes and weights of individual stones above the water surface. For the calculation of the weight of cap rock, assuming the effective coefficient of friction of rock on rock to be 1

$$W_s = \frac{H^3 K w_r}{(\cos\alpha - \sin\alpha)^3 (w_r - 1)^3} \qquad (46)$$

where

- w_r - unit weight of the rock in metric tons per cubic meter
- α - the angle the slope makes with the horizontal (A slope is usually referred to as $1/\cot\alpha$)
- H - wave height (in meters)
- W - weight of stone (in kilograms)
- K - an empirically determined coefficient for all unevaluated variables (15 for natural rubble, 19 for artificial blocks)

In this equation however, K is not dimensionless, and it is not possible to conduct small scale verifications of the formula. It has been modified by Hudson(55) using the same assumptions and force diagram as Iribarren to obtain

$$W = \frac{K' w S_f^3 S_r \mu^3 H^3}{(\mu\cos\alpha - \sin\alpha)^3 (S_r - S_f)^3} \qquad (47)$$

in which the additional symbols are:

- w - unit weight of fresh water
- S_f - specific gravity of the fluid in which the breakwater is located
- S_r - the specific gravity of the rock

μ - effective coefficient of friction rock on rock, - 1.05
K' - a variable dimensionless empirically determined coefficient, values of which are determined by Plate D-10, Appendix D;

The curves of Plate D-10, Appendix D, show that K' varies appreciably with side slope. However, for any one slope, an average value for K' may be assumed. Table D-10, Appendix D shows the average values of K' which may be used for various side slopes.

4.271 <u>Equations for the Weight of Above Surface Stones</u> - Equation 47 may be reduced to

$$W = \frac{78.8\ K'\ S_r\ H^3}{(1.05 \cos \alpha - \sin \alpha)^3\ (S_r - 1.03)^3} \qquad (47a)$$

if the breakwater is located in sea water; and to

$$W = \frac{72.0\ K'\ S_r\ H^3}{(1.05 \cos \alpha - \sin \alpha)^3\ (S_r - 1)^3} \qquad (47b)$$

if it is located in fresh water. The wave height H to be used is that height which would exist in the absence of the breakwater. This wave height at the location of the breakwater may be related to a deep water wave height H_o by

$$H = H_o\ (H/H'_o)\ K_r \qquad (48)$$

where

H_o - the deep water wave height
K_r - the refraction coefficient at the breakwater's depth (determined from refraction diagrams)
(H/H'_o) - the shoaling coefficient, values of which are tabulated in Appendix D (H'_o is the wave height which would exist in deep water if the wave at depth d is unaffected by refraction).

4.272 <u>Equation for Weights of Sub-Surface Stones</u> - The only method presently in use for the determination of sub-surface stone weights is that suggested by Iribarren and Nogales(59). In this method, a hypothetical wave height H' whose maximum orbital velocity is the same as that which exists at the depth d is determined. This value is

$$H' = \frac{\pi\ H_s^{\ 2}}{L_o\ (\sinh \frac{2\pi d}{L})^2} \qquad (49)$$

where H_s, the height of a wave steepened by the breakwater, is determined by extending equation 48 to points over the breakwater slope. This hypothetical wave height may then be substituted in equation 47a or 47b with K' = 0.015 or 0.019 (as determined from Iribarren).

4.273 Limitations - The methods of determining stable weights of above surface and sub-surface stones are not compatible. The hypothetical wave height to be determined for substitution in equations 47a or 47b is one in which the wave steepening effect of the breakwater face has been included. If however, these equations are utilized to find above surface stone weights with the empirically determined coefficients listed in Table D-10 or plotted on Plate D-10 of Appendix D, the wave height which must be used is that which would exist in the absence of the breakwater, since this is the height used in determining the coefficient. Near the water surface then, the hypothetical wave height H', and the steepened wave height H_s will exceed the wave height values used to determine the variable coefficient K'. Iribarren and Nogales, in generalizing their original equation to include sub-surface stones suggested using, for above surface stones, the wave height H_s and the constant coefficient developed by observing prototype structures. It would seem, however, that this could lead to results that are not conservative, for breakwater face slopes flatter than 1 on 2. The empirically determined K', for these slopes is greater than the 0.015 value given by Iribarren and Nogales, though this is partially compensated for, since H_s is larger than H used to determine the coefficient.

It has also been suggested, that to make use of equations 47a and 47b with the empirically determined coefficients for determining underwater stone sizes, another hypothetical wave height may be determined in which the steepening effect of the breakwater is not taken into account. By this means a hypothetical wave height given by:

$$H' = H \frac{\cosh \frac{2\pi}{L}(d + z)}{\cosh \frac{2\pi d}{L}} \tag{50}$$

is substituted in equations 47a or 47b to find stable stone sizes at a depth z below the surface. Here "d" is the water depth at the toe of the structure and z is measured negatively downward from the surface.

Neither of these wave height approaches, that using H_s for above surface stones and H' as given by equation 49 for sub-surface stones, or that using H for above surface stones and H' as given by equation 50 for sub-surface stones is entirely satisfactory. Both approaches, in using a hypothetical wave height to extend the use of equations 47a or 47b to the determination of underwater stone weights, for simplicity, ignore the basic hydrodynamic differences between the above and sub-surface water areas. Actually few verifications of any kind exist for formulas for the determination of stable underwater stone sizes. Iribarren and Nogales(59) detail one prototype observation in confirmation of their treatment for which the predicted and observed breakwater profiles are in fairly good agreement. No systematic study equivalent to that by Hudson for above surface stable slopes has been carried out for sub-surface conditions.

Even for above surface determinations there are limitations on the use of equations 47a or 47b with the empirically determined coefficients. Many prototype problems are encountered in which the d/L ratios to be dealt with are smaller than those used to determine the values of the coefficients (see Plate D-10, Appendix D). While the values listed in Table D-10 in themselves representing a range of actual conditions, they may be utilized for part of this range. How far this extrapolation may be carried and still give valid results has not been determined.

4.274 Applications - Above surface slopes determined from equations 47a or 47b should, for safety, be carried down to at least one wave height below the surface. The height to which a breakwater crest should be carried varies with the degree of protection desired. Recent model studies have indicated that crest heights of one wave height above the water surface are adequate for most desired protection, though it has been common practice to carry breakwater crests to heights of 1 1/4 to 1 1/2 design wave heights above water surface due to anticipated variability in actual wave conditions. These same studies indicate that a direction of wave approach other than normal to a structure does not appreciably increase the stability of the structure. Therefore, no matter what the direction of wave approach, the preceding criteria for mound stability should be used.(33).

4.275 Charts and Tables - The following charts and tables have been included in Appendix D to facilitate the solution of the basic equation.

Table D-10 lists average values of K' which may be used for various side slopes.

Table D-11 lists values of angle α, the slope ratio, and $(1.05 \cos\alpha - \sin\alpha)^3$ for slopes ranging from 1 on 1 to 1 on 10.

Plates D-5 through D-9 are curves showing the variation of W/K' from equation 47a for sea water, with wave height H (or H'). Values of the functions have been computed for slopes of 1 on 1 1/4 to 1 on 10 with stone density being used as a parameter.

Plate D-10 shows the variation of K' with d/L for slopes ranging 1 on 1 1/4 to 1 on 3. Note that to use plates D-5 through D-9 for slopes flatter than 1 on 3, values of K' must be assumed.

4.28 FORCES ON PILES - There have been many approaches to the problem of designing for wave action on piling(82)(91)(92)(94)(110), of which one has been extensively verified. It(92) was developed and partially verified in 1950. According to that theory, the moment at any point on a pile is given by

$$M_z = \rho \frac{H^2L^2D}{T^2}\Big[+ C_DK_2 \cos^2\theta + \frac{\pi D}{4H} C_MK_1 \sin\theta - (d + z)\left[+ C_DK_3 \cos^2\theta + \frac{\pi D}{4H} C_mK_4 \sin\theta\right]\Big] \qquad (51)$$

where

$$K_1 = \frac{\frac{2\pi d}{L}\sinh\frac{2\pi d}{L} - \frac{2\pi(d+z)}{L}\sinh\frac{2\pi(d+z)}{L} - \cosh\frac{2\pi d}{L} + \cosh\frac{2\pi(d+z)}{L}}{2\sinh\frac{2\pi d}{L}} \tag{52}$$

$$K_2 = \left[\frac{\frac{1}{2}\left(\frac{4\pi d}{L}\right)^2 - \frac{1}{2}\left(\frac{4\pi(d+Z)}{L}\right)^2 + \frac{4\pi d}{L}\sinh\frac{4\pi d}{L} - \frac{4\pi(d+Z)}{L}\sinh\frac{4\pi(d+Z)}{L}}{64\left(\sinh\frac{2\pi d}{L}\right)^2}\right] - \left[\frac{\cosh\frac{4\pi d}{L} - \cosh\frac{4\pi(d+Z)}{L}}{64\left(\sinh\frac{2\pi d}{L}\right)^2}\right] \tag{53}$$

$$K_3 = \frac{4\pi}{L}\left[\frac{\frac{4\pi d}{L} - \frac{4\pi(d+Z)}{L} + \sinh\frac{4\pi d}{L} - \sinh\frac{4\pi(d+Z)}{L}}{64\left(\sinh\frac{2\pi d}{L}\right)^2}\right] \tag{54}$$

$$K_4 = \frac{2\pi}{L}\left[\frac{\sinh\frac{2\pi d}{L} - \sinh\frac{2\pi(d+Z)}{L}}{2\sinh\frac{2\pi d}{L}}\right] \tag{55}$$

C_D = coefficient of drag

C_M = coefficient of mass

The significance of other symbols are shown on Figure 86. The angular particle position (β) at which the maximum moment will occur is given by

$$\beta = \sin^{-1}\left[\frac{\pi}{8}\ \frac{C_M}{C_D}\ \frac{D}{H}\ \frac{\left[K_1 - (d + Z)\,K_4\right]}{\left[K_2 - (d + Z)\,K_3\right]}\right] \tag{56}$$

This total moment is composed of a moment due to form drag caused by shear at the pile surface, and a moment due to the acceleration force on the displaced volume of fluid including the virtual mass effect. The virtual mass effect may be caused by an apparent increase of the displaced mass of the fluid when an object is accelerated in a fluid as compared to in a vacuum. Thus, there is an apparent increase in displaced volume without an actual increase in the mass of the pile. The two effects of the force are illustrated in Figure 86.

There are certain limitations to this approach. The theory was developed assuming that the wave form is trochoidal and that the partial velocities and accelerations are sinusoidal. For these conditions to hold, water

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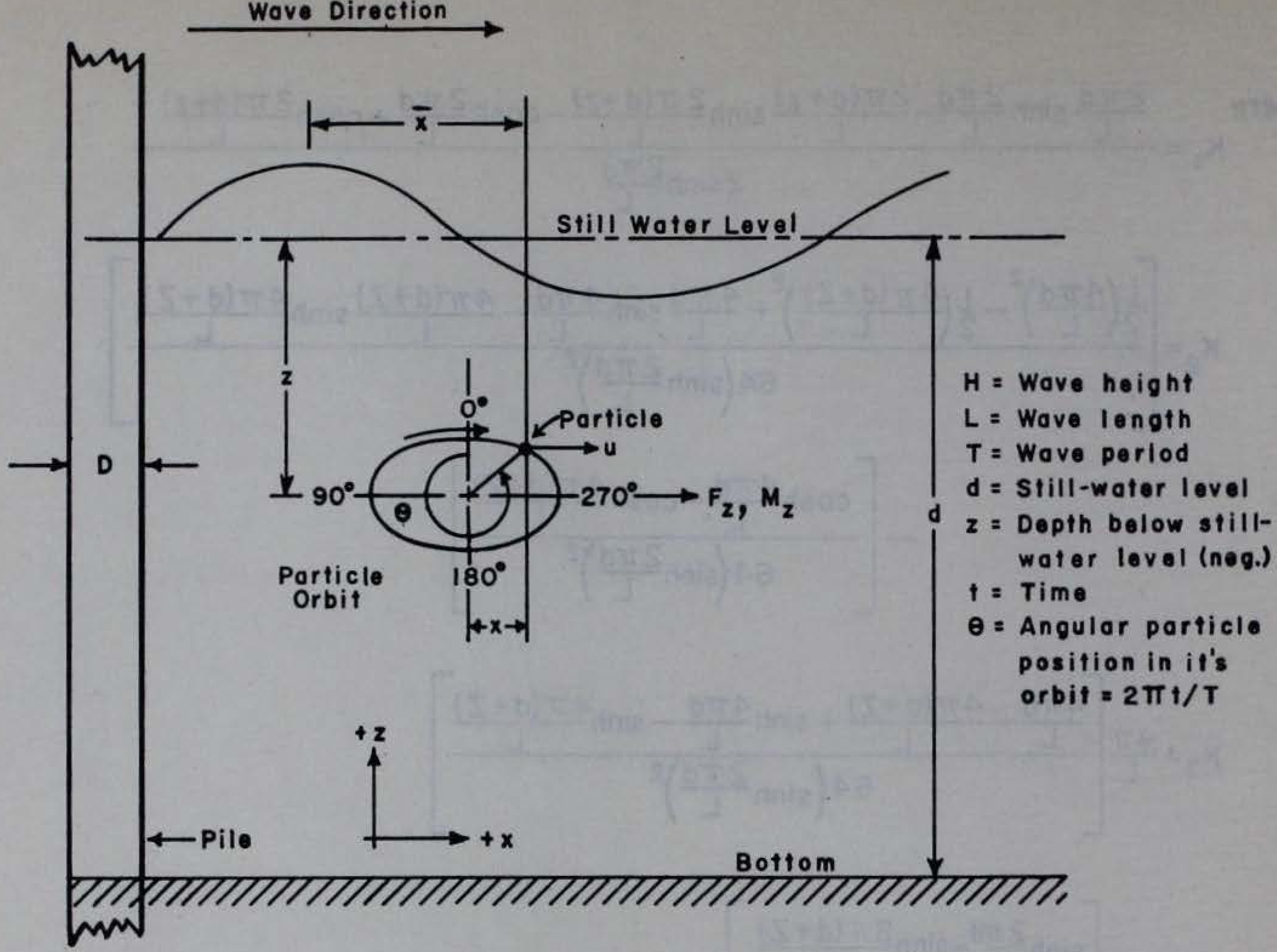


FIGURE 86A
SCHEMATIC DIAGRAM OF PARTICLE MOTION

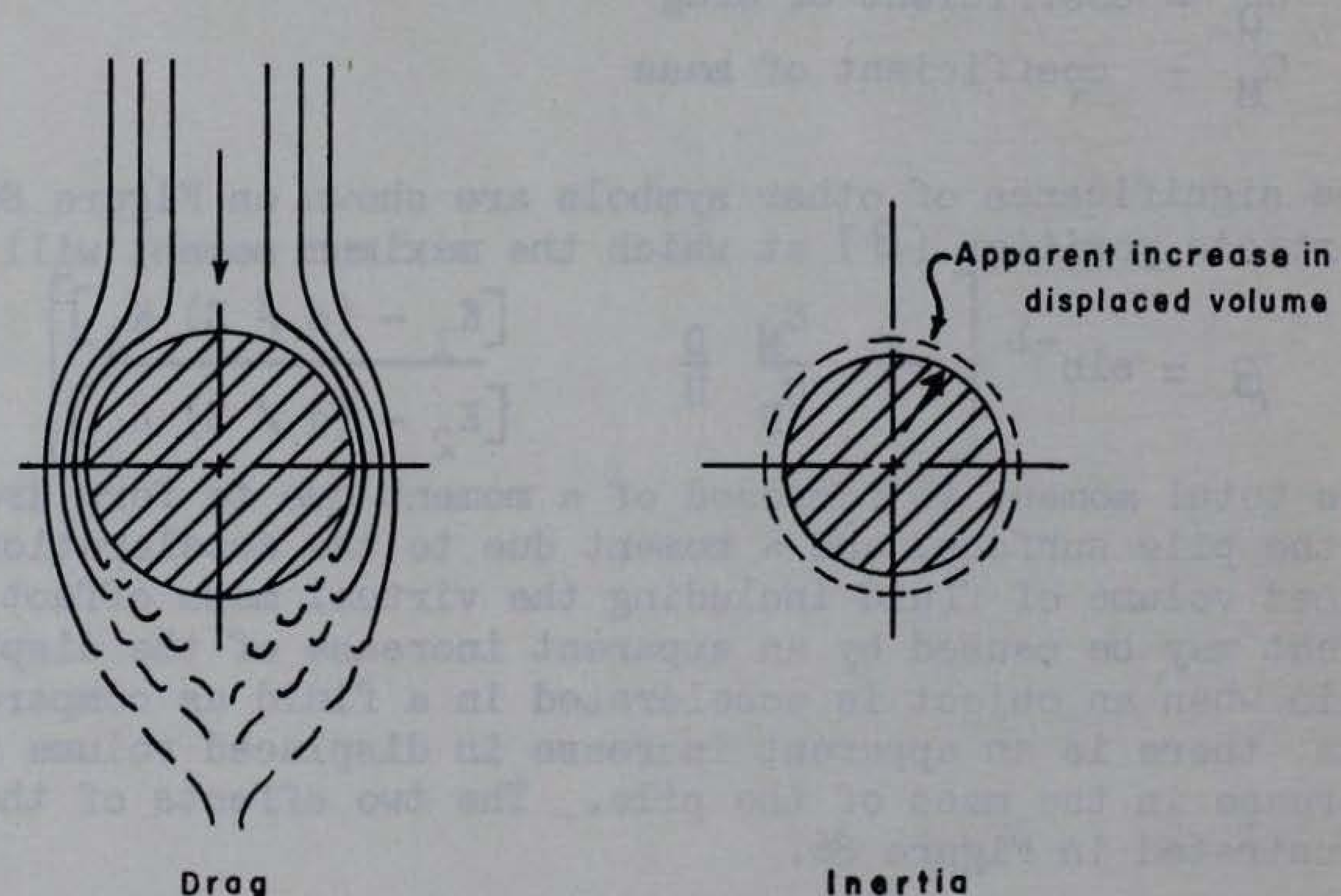


FIGURE 86B
FORCE COMPONENTS

FIGURE 86 FORCES ON PILES

(Morrison, 1951)

depth must be large compared to wave length, and wave height must be small compared to wave length. Though most deep water waves fall in this category, in shallow water, only those of small steepness do. The theory does not apply to shallow water waves of large steepness, and therefore piling near the point of wave breaking may not be so analyzed.

In addition, the coefficients C_M and C_D have not been conclusively determined for ocean conditions. These coefficients are empirical and can only be evaluated from measurements or by some relationship of oscillatory flow to steady flow, or by the extension of model study values. The relationship between β, C_M and C_D developed from model studies are shown in Figure 87.

The means of application of these relationships are illustrated in the following sample calculation.

4.281 Sample Calculation - Find the total moment about the bottom of a pile under the following conditions:

$H = 10$ ft. $\quad T = 10$ sec.
$d = 100$ ft. $\quad D = 1.5$ ft.
Calculations ($Z = 100$ ft.)
$L_o = 5.12T^2 = 512$ ft. (Subscript $_o$ designates deep water values)

Therefore

$$d/L_o = 0.196$$

From appendix D, Table D-1

$$d/L = 0.221$$

and

$$L = 452 \text{ ft.}$$
$$H/L = 0.02$$
$$D/H = 0.15$$

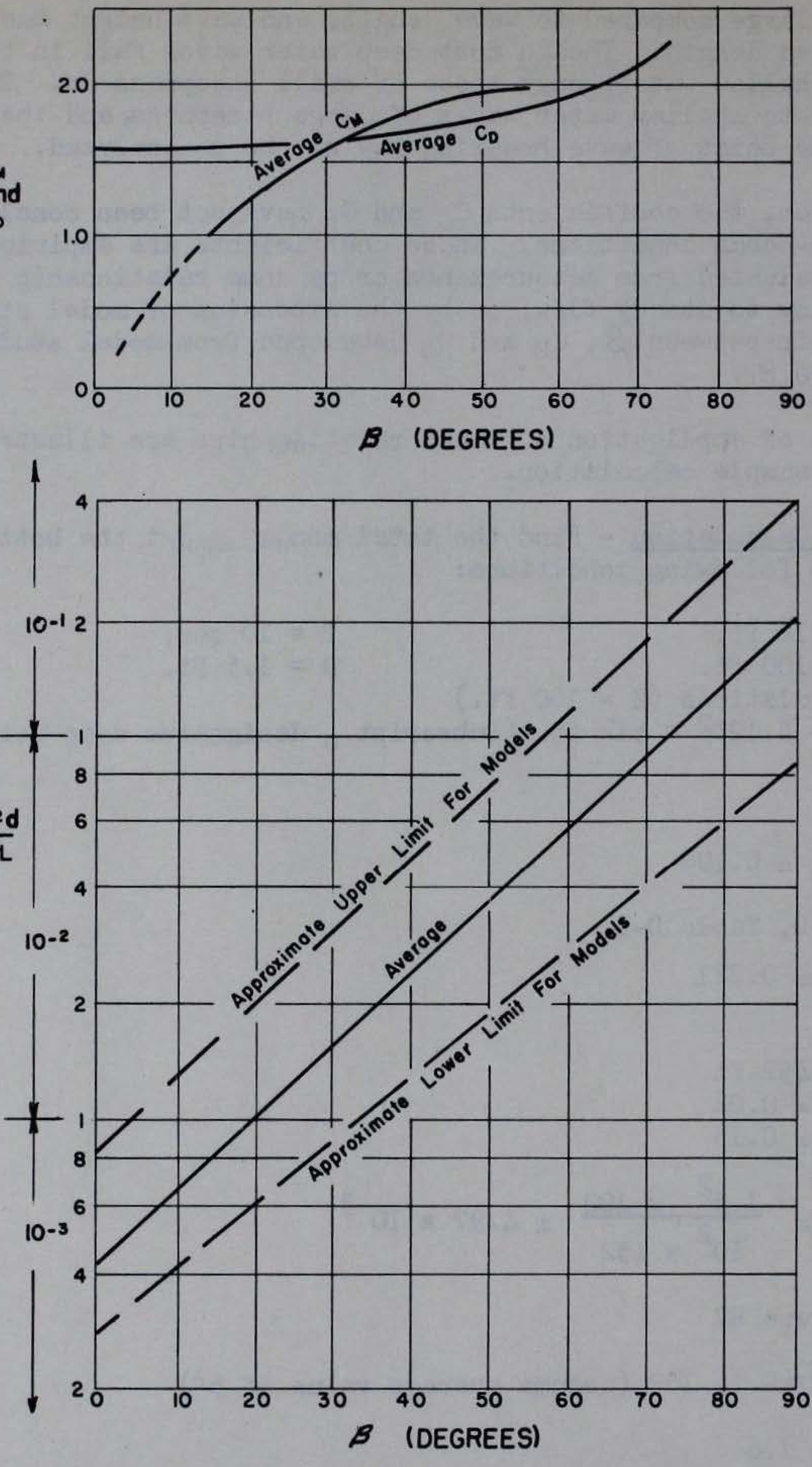
$$\frac{D^2 d}{H^2 L} = \frac{1.5^2 \times 100}{10^2 \times 452} = 4.97 \times 10^{-3}$$

Hence from Figure 87

$$\beta \approx 0 \text{ to } 15° \text{ (assume average value of } 5°)$$

Therefore $C_D = 1.6$

$$C_M = 2.0 \text{ (use theoretical value of 2.0)}$$



RELATIONSHIPS OF ANGULAR POSITION OF MAXIMUM TOTAL MOMENT FROM MODEL STUDIES

(Morison, 1951)

FIGURE 87

By use of Appendix D, Table D-1

$$K_1 = \frac{\frac{2\pi d}{L}\sinh\frac{2\pi d}{L} - \frac{2\pi(d+Z)}{L}\sinh\frac{2\pi(d+Z)}{L} - \cosh\frac{2\pi d}{L} + \cosh\frac{2\pi(d+Z)}{L}}{2\sinh\frac{2\pi d}{L}} \tag{52}$$

$$= \frac{1.389\ (1.880) - 2.129 + 1.000}{2\ (1.880)} = 0.395$$

$$K_2 = \frac{\frac{1}{2}\left(\frac{4\pi d}{L}\right)^2 - \frac{1}{2}\left(\frac{4\pi(d+z)}{L}\right) + \frac{4\pi d}{L}\left(\sinh\frac{4\pi d}{L}\right) - \frac{4\pi(d+z)}{L}\sinh\frac{4\pi(d+z)}{L} - \cosh\frac{4\pi d}{L} + \cosh\frac{4\pi(d+z)}{L}}{64\left(\sinh\frac{2\pi d}{L}\right)^2}$$

$$= \frac{\frac{1}{2}(2.777)^2 + 2.777(8.006) - 8.068 + 1}{64(1.880)^2} = 0.0837 \tag{53}$$

From equations 54 and 55

$$(d + Z)\ K_3 = 0$$

$$(d + Z)\ K_4 = 0$$

From equation 56

$$\beta = \sin^{-1}\left[\frac{\pi}{8}\ \frac{1.5}{10}\ \ \frac{1.0\ (0.395)}{1.6\ (0.0837)}\right] = \sin^{-1}(0.1737) = 10°$$

Equation 51 reduces to

$$M_Z = \rho\frac{H^2L^2D}{T^2}\left[\pm C_D K_2 \cos^2\theta + \frac{\pi D}{4H} K_1 C_M \sin\theta\right]$$

$$= 2.0\frac{(10)^2(452)^2(1.5)}{(10)^2}\left[+1.6(0.0837)(0.9848)^2 + \frac{\pi}{4}\frac{1.5}{10}(0.395)(2.0)(0.1737)\right]$$

$$= 612{,}000\ (0.1294 + 0.0162) = 89{,}000 \text{ ft. lbs.}$$

4.3 EARTH FORCES[23]

4.31 ACTIVE FORCES - The horizontal component of the active earth force is evaluated from the general wedge theory formula:

$$P = \frac{wh^2}{2} \tan^2 \frac{1}{2}(90 - \phi) \tag{57}$$

where P = the horizontal component of the lateral force
w = the unit weight of fill
ϕ = the internal angle of friction of the material.

Equation 57 is used only for vertical walls with substantially horizontal backfill. The structure is assumed to be non-rigid to the extent that an extremely small rotational movement, necessary to produce the internal friction of the backfill, can occur. Values for $\tan^2 (90-\phi)/2$ for various values of ϕ are given in Table D-12, Appendix D.

If the wall is vertical but the fill slopes at an angle p to the horizontal, the complete equation is

$$P = \frac{wh^2}{2} \cos p \frac{\cos p - \sqrt{\cos^2 p - \cos^2 \phi}}{\cos p + \sqrt{\cos^2 p - \cos^2 \phi}} \tag{58}$$

For structures having a uniform back batter and fills with uniform slope , the general wedge theory, equation 59 may be used to evaluate the magnitude and direction of earth force. This formula takes into account the friction along the surface of the wall (see Figure 88).

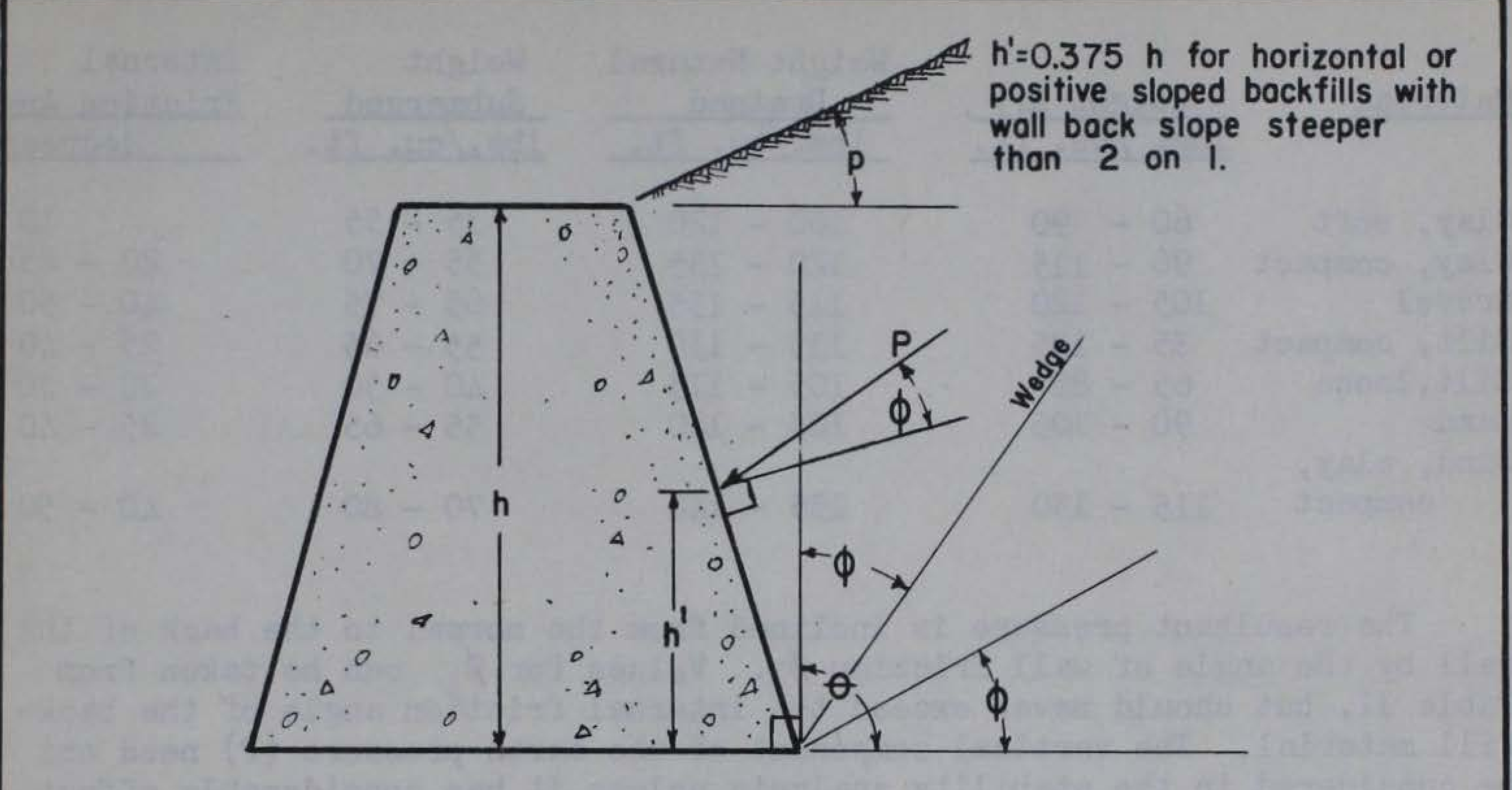
$$P = \frac{1}{2} \left[\frac{\sin(\theta - \phi)}{(1 + N)\sin\theta}\right]^2 \frac{wh^2}{\sin(\phi_1 + \theta)} \tag{59}$$

in which

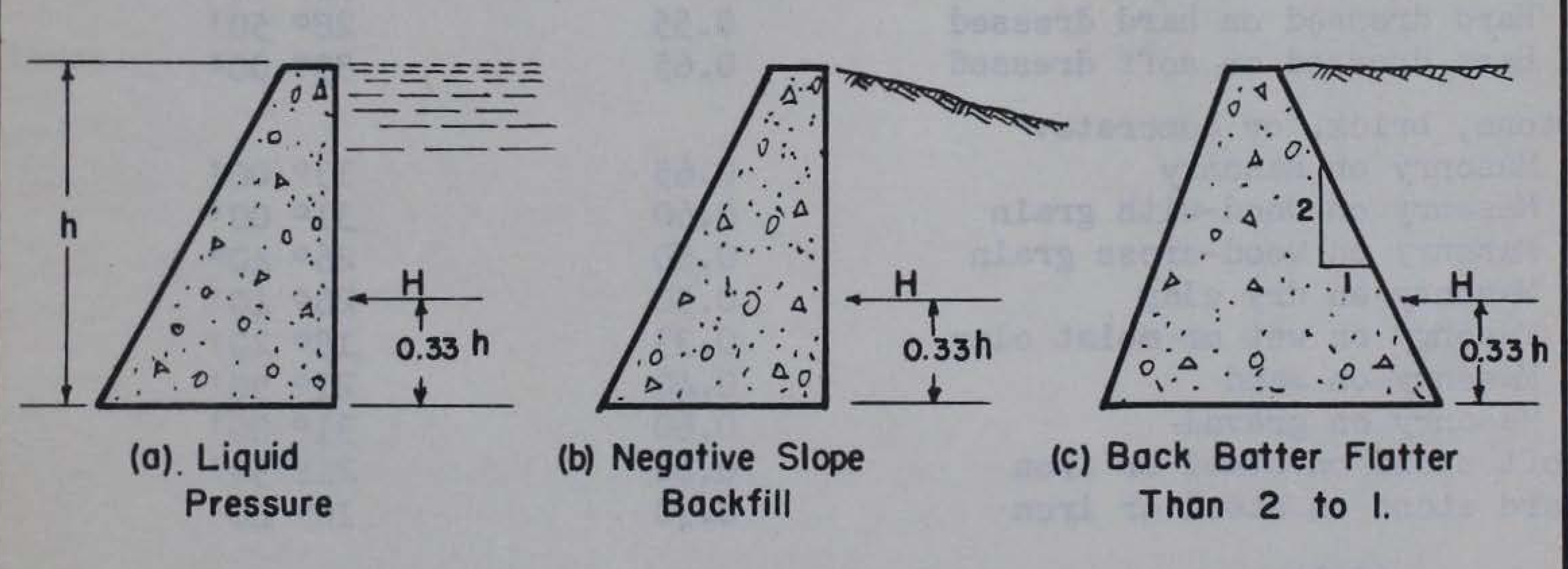
$$N = \sqrt{\frac{\sin(\phi + \phi_1)\sin(\phi - p)}{\sin(\phi_1 + \theta)\sin(\theta - p)}} \tag{60}$$

where P = total force in pounds
h = vertical height of fill in feet
w = unit weight of fill material
ϕ = internal friction angle of fill material
ϕ_1 = friction angle between backfill and face of wall
p = angle between surface of backfill and horizontal plane
θ = angle between back of retaining wall and a horizontal plane.

4.311 Unit Weights and Internal Friction Angles - The unit weight (w) of typical materials and their internal friction angle (ϕ) are given in Table 10. These are average or normal values of w and ϕ.



EARTH PRESSURE ON AN INCLINED WALL
FIGURE 88



APPLICATION OF RESULTANT OF EARTH PRESSURE
FIGURE 89

TABLE 10

UNIT WEIGHTS AND INTERNAL FRICTION ANGLES

Material	Weight Dry lbs./cu. ft.	Weight Natural Drained lbs./cu. ft.	Weight Submerged lbs./cu. ft.	Internal Friction Angle (Ø) degrees
Clay, soft	60 - 90	100 - 120	35 - 55	10
Clay, compact	90 - 115	120 - 135	55 - 70	20 - 25
Gravel	105 - 120	115 - 135	65 - 75	40 - 50
Silt, compact	85 - 105	115 - 130	55 - 65	25 - 40
Silt, loose	65 - 85	105 - 115	40 - 50	20 - 30
Sand	90 - 105	105 - 120	55 - 65	25 - 40
Sand, clay, compact	115 - 130	135 - 140	70 - 80	40 - 50

The resultant pressure is inclined from the normal to the back of the wall by the angle of wall friction $\emptyset_1$. Values for $\emptyset_1$ can be taken from Table 11, but should never exceed the internal friction angle of the back-fill material. The vertical component of the earth pressure (P) need not be considered in the stability analysis unless it has considerable effect on the structural design.

TABLE 11

COEFFICIENTS AND ANGLES OF FRICTION

Kind of Surface	Coefficient of Friction f	Angle of Friction $\emptyset_1$
Granite, Limestone, Marble:		
Soft dressed on soft dressed	0.70	35° 00'
Hard dressed on hard dressed	0.55	28° 50'
Hard dressed on soft dressed	0.65	33° 00'
Stone, brick, or concrete:		
Masonry on masonry	0.65	33° 00'
Masonry on wood-with grain	0.60	31° 00'
Masonry on wood-cross grain	0.50	26° 40'
Masonry on dry clay	0.50	26° 40'
Masonry on wet or moist clay	0.33	18° 20'
Masonry on sand	0.40	21° 50'
Masonry on gravel	0.60	31° 00'
Soft stone on steel or iron	0.40	21° 50'
Hard stone on steel or iron	0.30	16° 40'

Note:(86) Angles of friction should be reduced by about 5 degrees if the wall fill will support train or truck traffic. The coefficient of friction f would equal the tangent of the new angle $\emptyset_1$.

Earth pressure against structures of irregular section such as stepped stone blocks or those having two or more back batters may be computed by equation 59 by substituting an approximate average back batter to determine the angle (θ).

4.312 Application of the Resultant of Earth Force - For all structures with hydrostatic pressure or negative sloped backfills, the resultant of force should be considered to act at one-third of the height. For structures having batters on the back face equal to or greater than 2 on 1, the earth pressure should also be applied at 0.33 h. For structures with vertical back faces (or batters less than 2 on 1) and horizontal or positive sloped backfills the earth pressures should be applied at 0.375 h (see Figures 88 and 89).

4.32 PASSIVE FORCES - Though the passive resistance of an earth mass to movement is very much greater than the active pressure, to develop this passive resistance to movement, the wall must translate or rotate to a significant degree. Therefore, except for sheet pile structures, the passive resistance should not be considered in a stability analysis. For sheet pile structures the passive earth resistance may be computed as

$$R_p = wh^2 \tan^2 \frac{(90 + \emptyset)}{2} \qquad (61)$$

4.33 SURCHARGE LOADS - For a uniform surcharge the density of backfill material may be assumed to be increased in the following manner:

Let W = the surcharge load in pounds per square foot

w = the unit weight of backfill material either drained or saturated dependent on backfill condition

h = the vertical height of fill in feet

$h' = \frac{W}{w}$, the equivalent surcharge height

then w' the new value of unit backfill weight is given by

$$w' = w \frac{(h + 2h')}{h} \qquad (62)$$

When this procedure for dealing with surcharge loads is adopted, the point of application of the active thrust for vertical walls with horizontal or positive sloped backfill should be raised from 0.375 h, at the rate of 0.01h for each 10 percent increase in the artificial density, to a limit of 0.475h.

4.34 SUBMERGED MATERIALS - Pressures due to submerged fills may be calculated by substituting for w in the preceding equations the unit weight of the material reduced by buoyancy, and adding to the pressures so calculated, the full hydrostatic head of water. Note that for surcharge loads this buoyed unit weight of the material must be increased as shown in the preceding section.

4.35 UPLIFT - For design computations, uplift pressures should be considered as full hydrostatic pressure for walls whose bases are below sea level or for computations involving saturated backfill.

4.4 ICE FORCES(122)

The common forms of ice are usually classified by the use of terms which indicate the manner of formation or the effects produced. Usual classifications include; sheet ice, shale, slush, frazil ice, anchor ice, and agglomerate ice.

The amount of expansion of water in cooling from 39° F. to 32° F. is 1.32 hundredths of 1 percent, whereas in changing from water at 32° F. to ice at 32° F. the amount of expansion is about 9.05 percent or 685 times as great. It has been found that a change of structure to denser form takes place in the ice when, with a temperature lower than -8° F., it is subjected to pressures greater than about 30,000 pounds per square inch. Excessive pressure, with temperatures above -8°F., causes the ice to melt. With the temperature below -8°F., the change to a denser form at high pressure results in shrinkage which relieves pressure. Thus, the probable maximum pressure which can be produced by water freezing in an inclosed space is 30,000 pounds per square inch.

Designs for dams include allowances for ice pressures varying from no special allowance to as much as 45,000 to 50,000 pounds per linear foot. The crushing strength of ice has been found to be about 400 pounds per square inch and the thrust per linear foot for various thicknesses of ice as about 28,800 pounds for 6 inches, 57,600 pounds for 12 inches, etc. Structures subject to blows from floating ice should be capable of resisting from 10 to 12 tons per square foot 139 to 167 pounds per square inch) on the area exposed to the greatest thickness of floating ice.

Ice also expands when warmed from temperatures below freezing to a temperature of 32° F. without melting. Assuming a lake surface to be free from snow, with an average coefficient of expansion of ice between -20° F and 32°F equalling 0.0000284, the total expansion of a sheet of ice a mile long for a rise in temperature of 50° F. would be 3.75 feet. Normally, shore structures are subject to wave forces comparable in magnitude to the maximum probable pressure that might be developed by an ice sheet. As the maximum wave forces and ice thrust cannot occur at the same time, usually no special allowance for overturning stability to resist ice thrust is made. However, where heavy ice, either in the form of solid ice sheet or floating ice fields may occur, adequate precautions must be observed to insure that the structure is secure against sliding on its base. Ice breakers may be required in relatively sheltered water where wave action does not require a heavy type structure.

Floating ice fields may exert a major pressure on structures, when driven by a strong wind or current, by piling up in large ice packs against the obstructions. This condition must be given special attention in the design of small isolated structures. However, because of the flexibility of the ice field, the pressures exerted probably are not as great as would be caused by a solid ice sheet in a confined area.

Ice formations may at times cause considerable damage to shore lines in local areas, but their net effects are largely beneficial. Spray thrown up by wind and wave action during the winter may freeze on the banks and structures along the shore, covering them with a protective layer or ice. Ice piled on shore by wind and wave action does not, in general, cause serious damage to beaches, bulkheads, or protective riprap, and generally provides additional protection against damage from the severe winter storm waves. Ice often has a definite effect on impoundment of littoral drift. Updrift source material is less erodible in the frozen state and wind rowed ice acts as a barrier to shoreward moving wave energy, therefore, the quantity of material reaching an impounding structure is reduced. During the winter in 1951-52 it was estimated that ice caused a reduction in rate of impoundment of 40 to 50 percent at the Fort Sheridan, Illinois groin system. Some abrasion of timber or concrete structures may be caused and individual members may be broken or bent due to the weight of the ice mass. Piling has been slowly pulled by the repeated lifting effect of ice freezing to the piles or attached obstructions such as wales, and then being forced upward by a rise in water stage or wave action.

4.5 MATERIALS

The structural design of shore protective works must take into account the effects of the environmental conditions peculiar to the shore line area on the material used. General modifying criteria which should be applied to materials commonly used are discussed in the following paragraphs.

4.51 CONCRETE - Concrete exposed to sea water, freezing and thawing or other destructive agents or conditions should have an ultimate compressive strength of 3,000 pounds per square inch. A rich, dense, stiff mix is to be preferred where placement is to be done underwater. Care should be taken to cover reinforcing steel adequately, thereby minimizing possible spalling and exposure of the steel. Working stresses for those conditions may be found in the Corps of Engineers Engineering Manual for Civil Works Construction, Part CXXI, Chapter 1, August 1947.

4.52 STEEL - Where exposed to weathering, allowable working stresses must be reduced to take into account corrosive action, abrasion, or a combination thereof, which would result in loss of effective steel area. Working stresses for reinforcing and structural steel may be found in the preceding reference.

4.53 TIMBER - Allowable stresses for timber should be those for timbers more or less continuously damp or wet. These working stresses may be found in U. S. Department of Commerce publications dealing with American Lumber standards.

4.54 STONE - Usually the availability of stone sources determines the quality of stone used in waterfront structures. However, care should be taken to avoid use of stone which may decompose more or less rapidly under wave and water action. Where such stone has been used, the effective life of the structure was decreased considerably.

CHAPTER 5

STRUCTURAL ANALYSIS

5.1 SEAWALLS, BULKHEADS AND REVETMENTS

5.11 TYPES - Seawalls, bulkheads and revetments perform similar functions. Actually the distinction between these three kinds of shore structures is mainly one of nomenclature; overall design features are determined at the functional planning stage and the structure is named to suit its intended purpose. A seawall at one locality may be a bulkhead or revetment at another.

In general, however, seawalls tend to be the most massive of the three because consideration must be given to fill retention characteristics as well as to stability against wave forces. Bulkheads are ordinarily next in size as their primary function is to retain a fill. Revetments often are the lightest of the three because consideration in their design is given only to their effectiveness in reducing wave erosion.

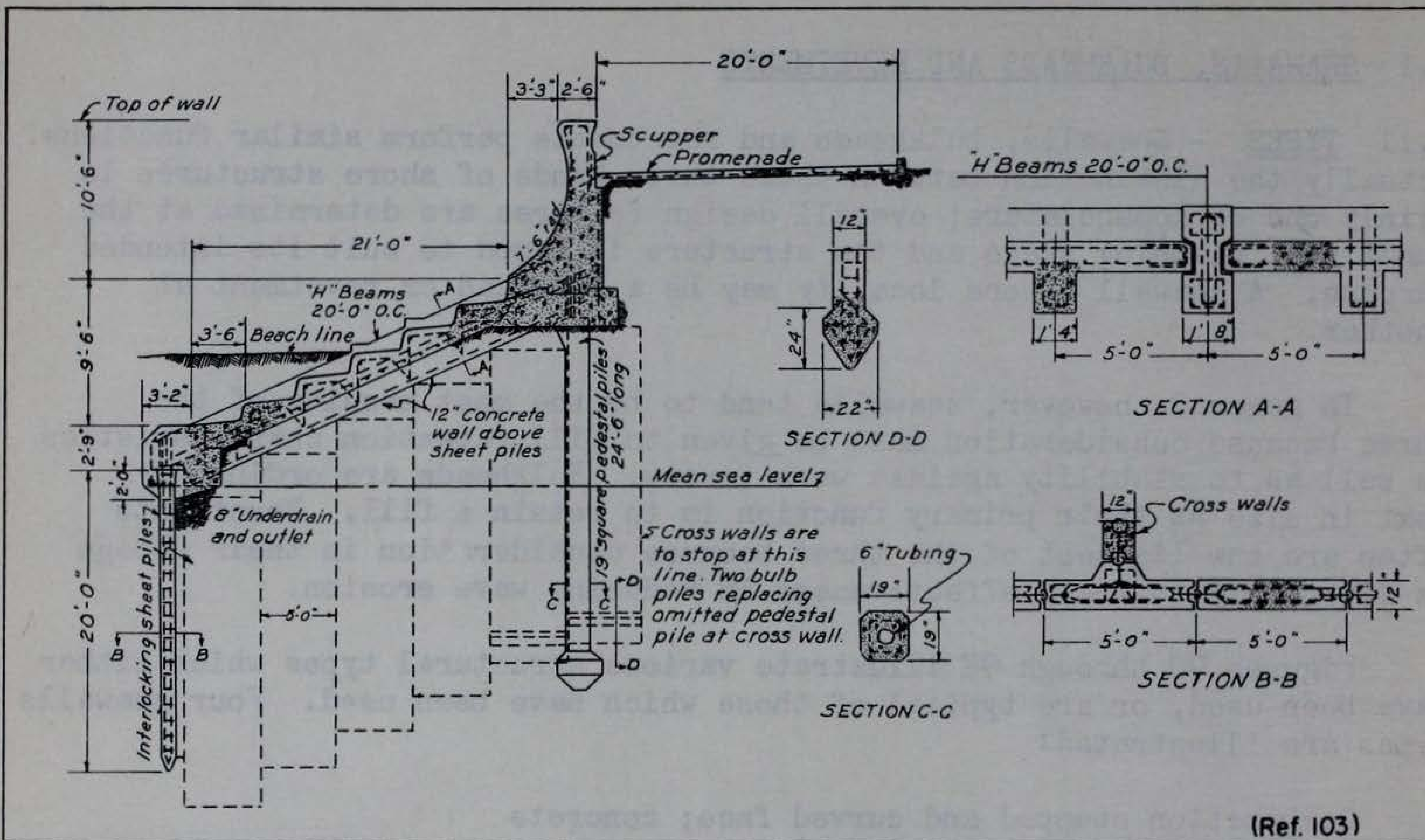
Figures 90 through 98 illustrate various structural types which either have been used, or are typical of those which have been used. Four seawalls types are illustrated:

Combination stepped and curved face; concrete (San Francisco, California)	Figure 90
Stepped face; concrete (Harrison County, Miss.)	Figure 91
Cellular sheet pile; steel (typical)	Figure 92
Stone; rubble-mound (typical)	Figure 93

The first of these, a combination stepped and curved face wall, is a relatively massive structure, built to resist high wave action and inhibit scour. The second, a stepped face wall, was designed for stability against more moderate wave conditions. The original design has been modified to provide a tongue the full height of the sheetpile. Formerly a groove, intended for grouting purposes, extended about two-thirds down the pile, but it was found to fill with sand making grouting impossible. Note in both of these, the presence of a sheet pile cut-off wall to prevent leaching of the backfill. The typical cellular sheet pile wall is one type which may be used on a rocky bottom where pile penetration is not adequate for inherent pile stability. A concrete, rubble-mound, or stone masonry (not illustrated) wall may also be used under these circumstances. A rubble-mound wall (Figure 93) is useful where bottom conditions may permit some settlement.

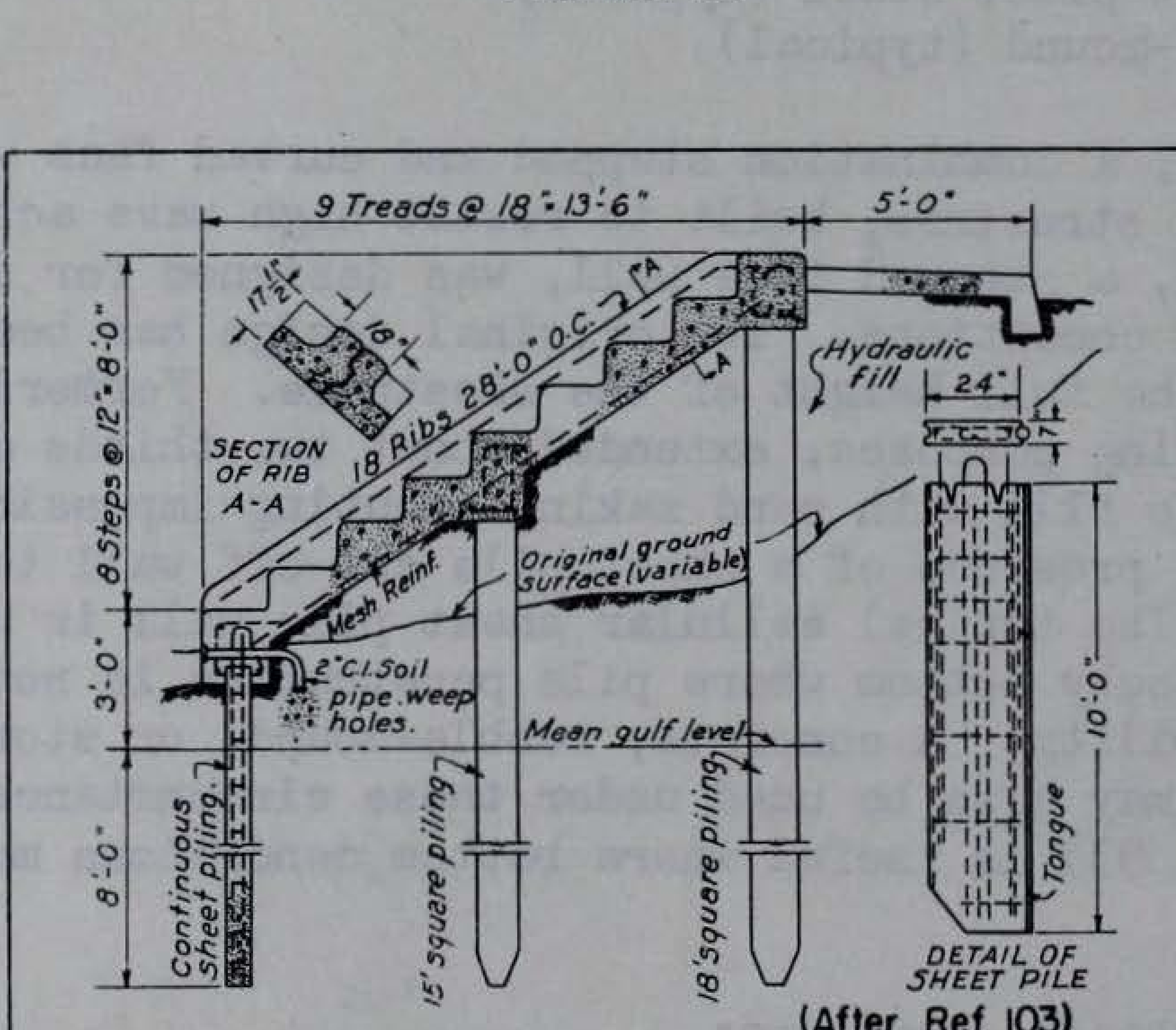
The bulkheads shown are:

Slab and king pile; concrete (Virginia Beach, Va.)	Figure 94
Sheet pile; steel (typical)	Figure 95
Sheet pile; timber (typical)	Figure 96



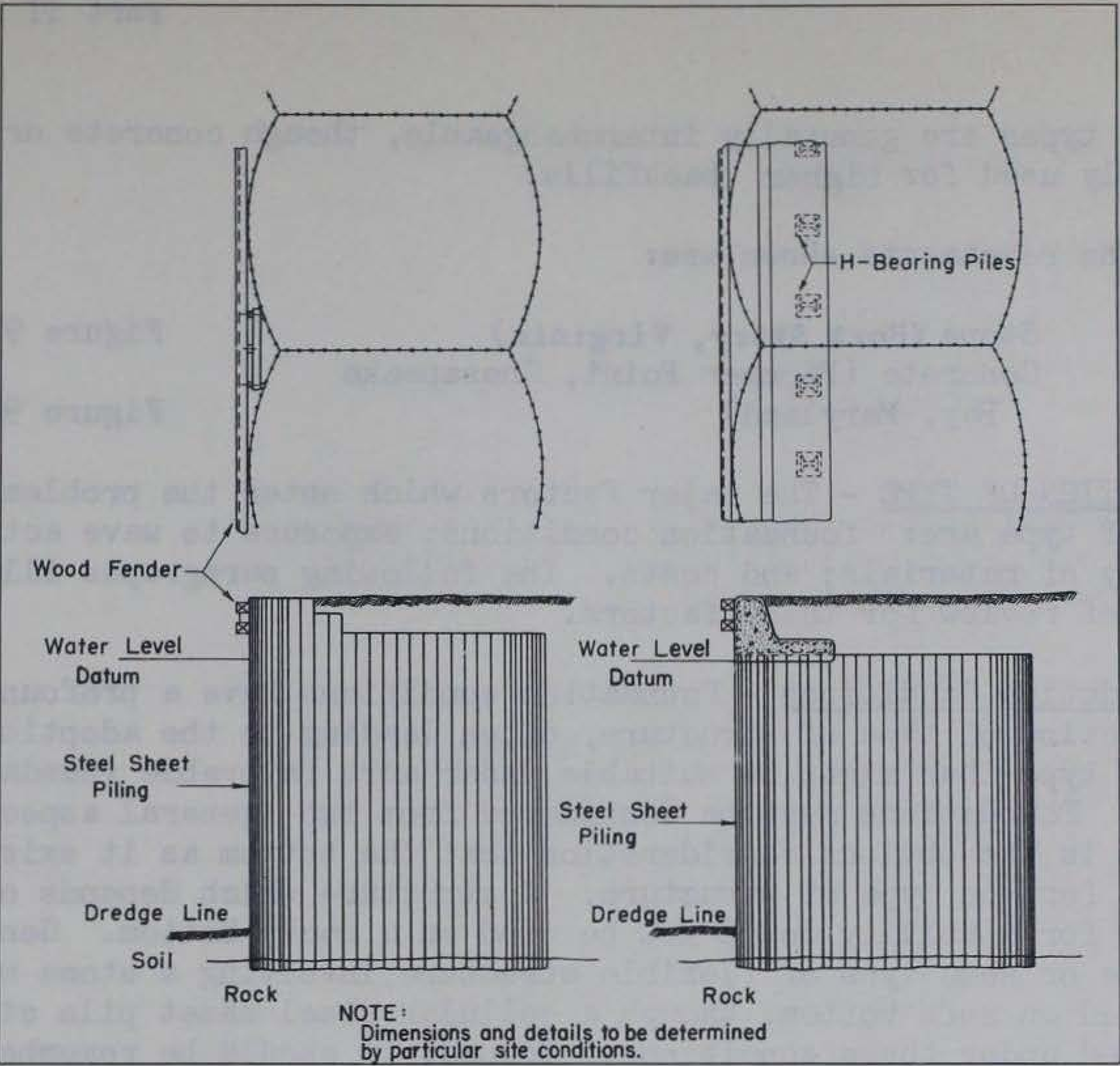
COMBINATION STEPPED AND CURVED FACE WALL; CONCRETE

FIGURE 90

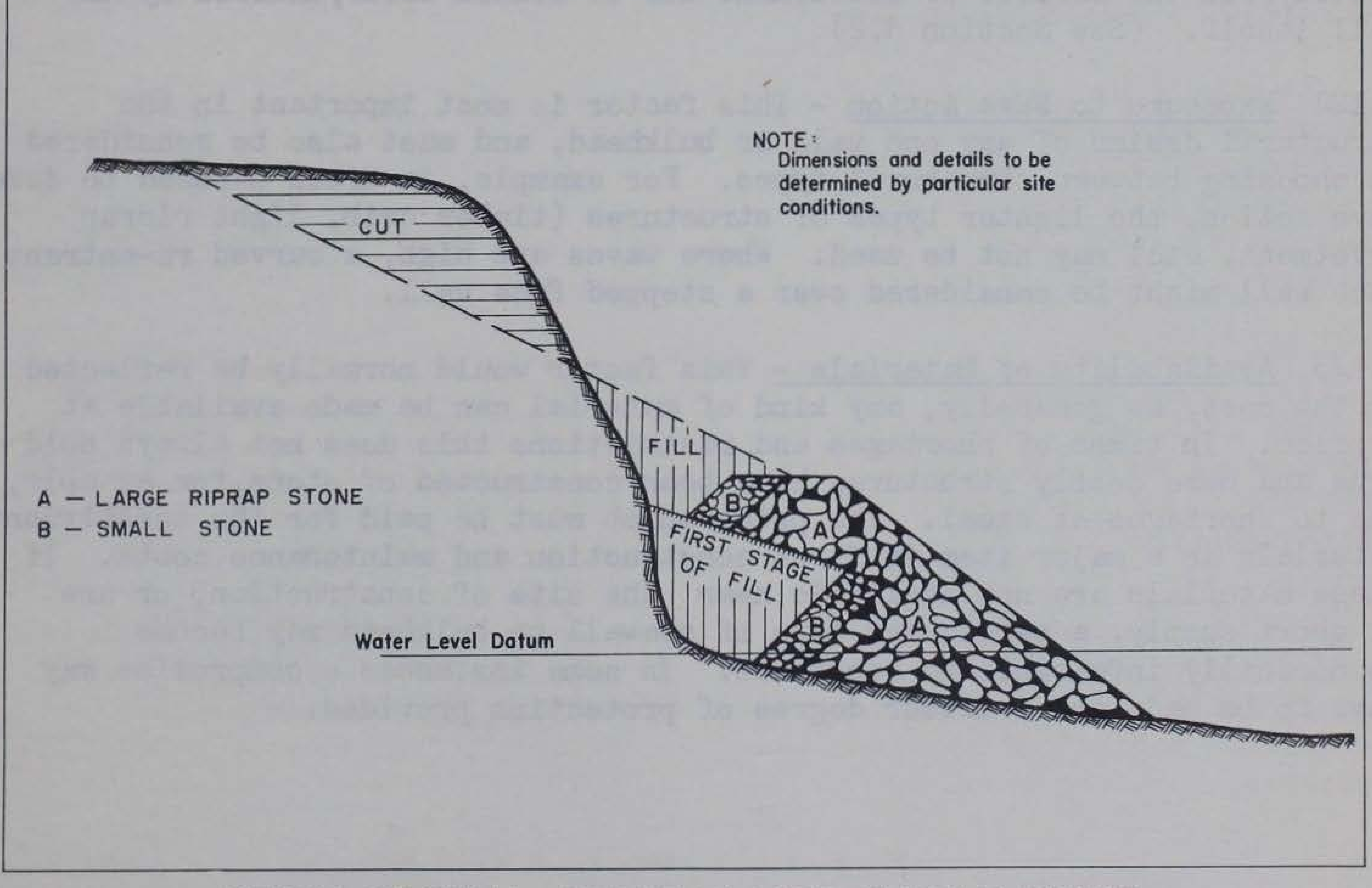


STEPPED FACE WALL; CONCRETE

FIGURE 91



CELLULAR SHEET PILE WALL; STEEL
FIGURE 92



STONE SEAWALL; STAGE PLACED RUBBLE MOUND
FIGURE 93

These three types are generally interchangeable, though concrete or steel are generally used for higher backfills.

The revetments shown are:

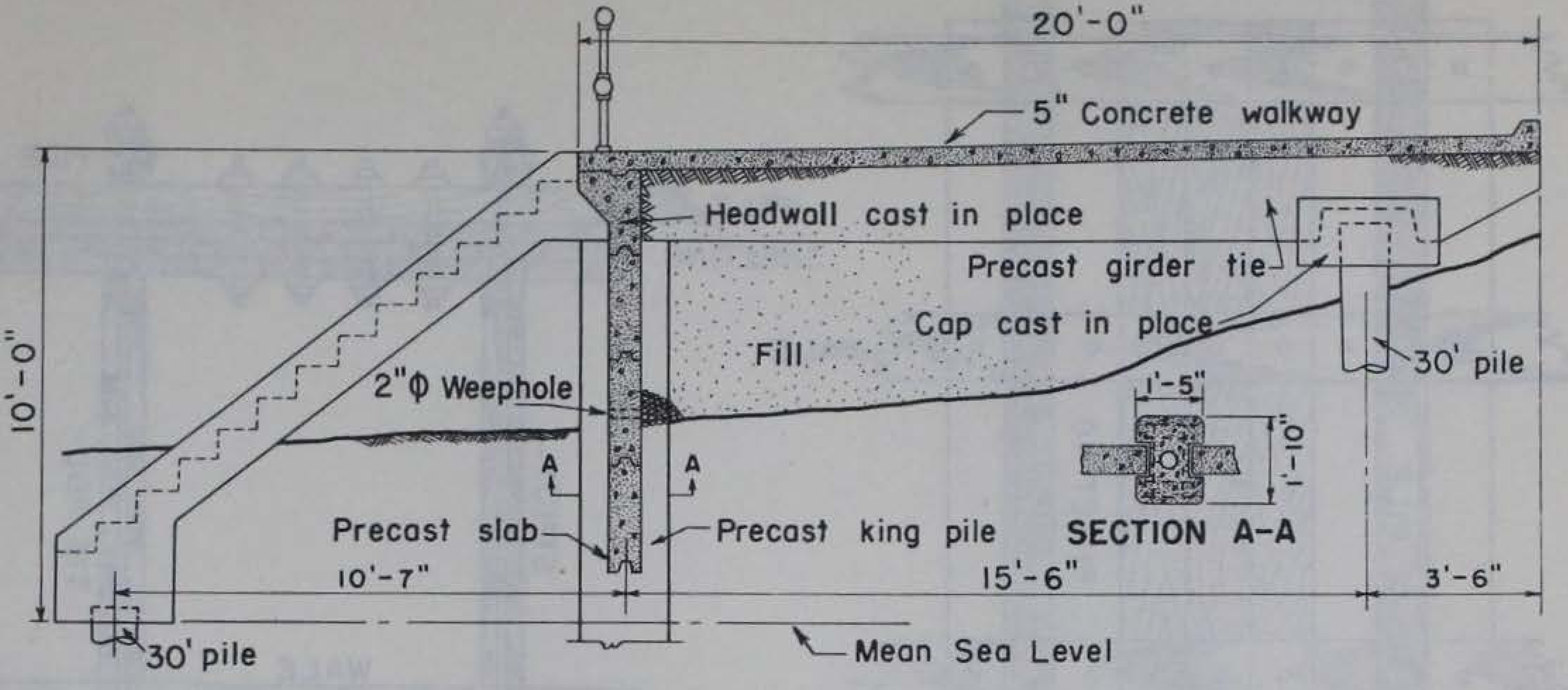
Stone **(Fort Story, Virginia)**	Figure 97
Concrete (Pioneer Point, Chesapeake Bay, Maryland)	Figure 98

5.12 <u>SELECTION OF TYPE</u> - The major factors which enter the problem of selection of type are: foundation conditions; exposure to wave action; availability of materials; and costs. The following paragraphs illustrate the manner of review for these factors.

5.121 <u>Foundation Conditions</u> - Foundation conditions have a profound effect on the selection of type of structure, often leading to the adoption of a **much more** costly type than might be suitable under more favorable foundation conditions. Foundations must be considered from two general aspects. First there is the obvious consideration that the bottom as it exists must be suitable for the type of structure. A structure which depends on bottom penetration for stability could not be used on a rocky bottom. Generally, random stone or some type of flexible structure involving a stone mat would be used on soft bottom, though a cellular steel sheet pile structure might be used under these conditions. Second, it should be remembered that the presence of a seawall or bulkhead may change the foundation conditions so that, unless precautions are taken, a structure might fail. Because of induced bottom scour, a foundation otherwise stable could become unstable. For example, a masonry wall or mass concrete wall must be protected from the effects of settlement due to bottom scour, induced by the wall itself. (See Section 3.2)

5.122 <u>Exposure to Wave Action</u> - This factor is most important in the structural design of any one wall or bulkhead, and must also be considered in choosing between structural types. For example, in areas exposed to severe wave action, the lighter types of structures (timber crib, light riprap revetment, etc) may not be used. Where waves are high, a curved re-entrant face wall might be considered over a stepped face wall.

5.123 <u>Availability of Materials</u> - This factor would normally be reflected in the cost, as generally, any kind of material can be made available at a price. In times of shortages and restrictions this does not always hold true and more costly structures have been constructed of stone, for example, due to shortages of steel. The price which must be paid for the constituent materials is a major item in first construction and maintenance costs. If these materials are not available near the site of construction, or are in short supply, a particular type of seawall or bulkhead may become economically infeasible to construct. In some instances a compromise may have to be made and a lesser degree of protection provided.



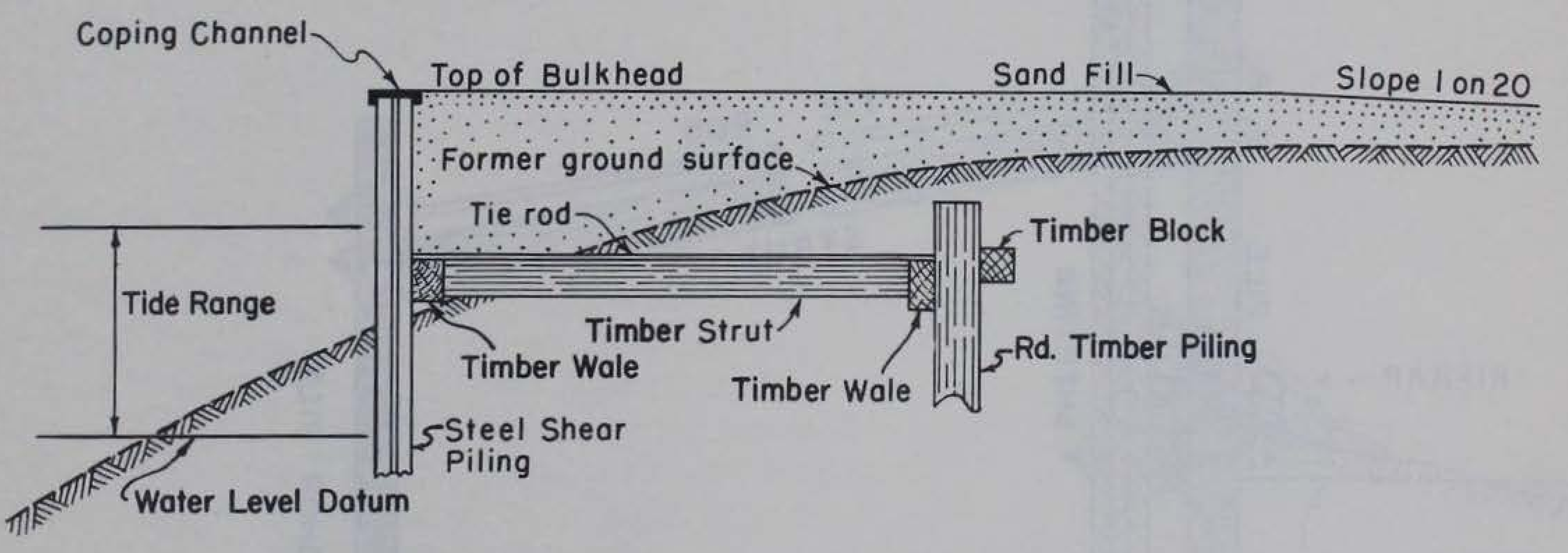
SLAB AND KING PILE BULKHEAD; CONCRETE

FIGURE 94

(Ref. 103)

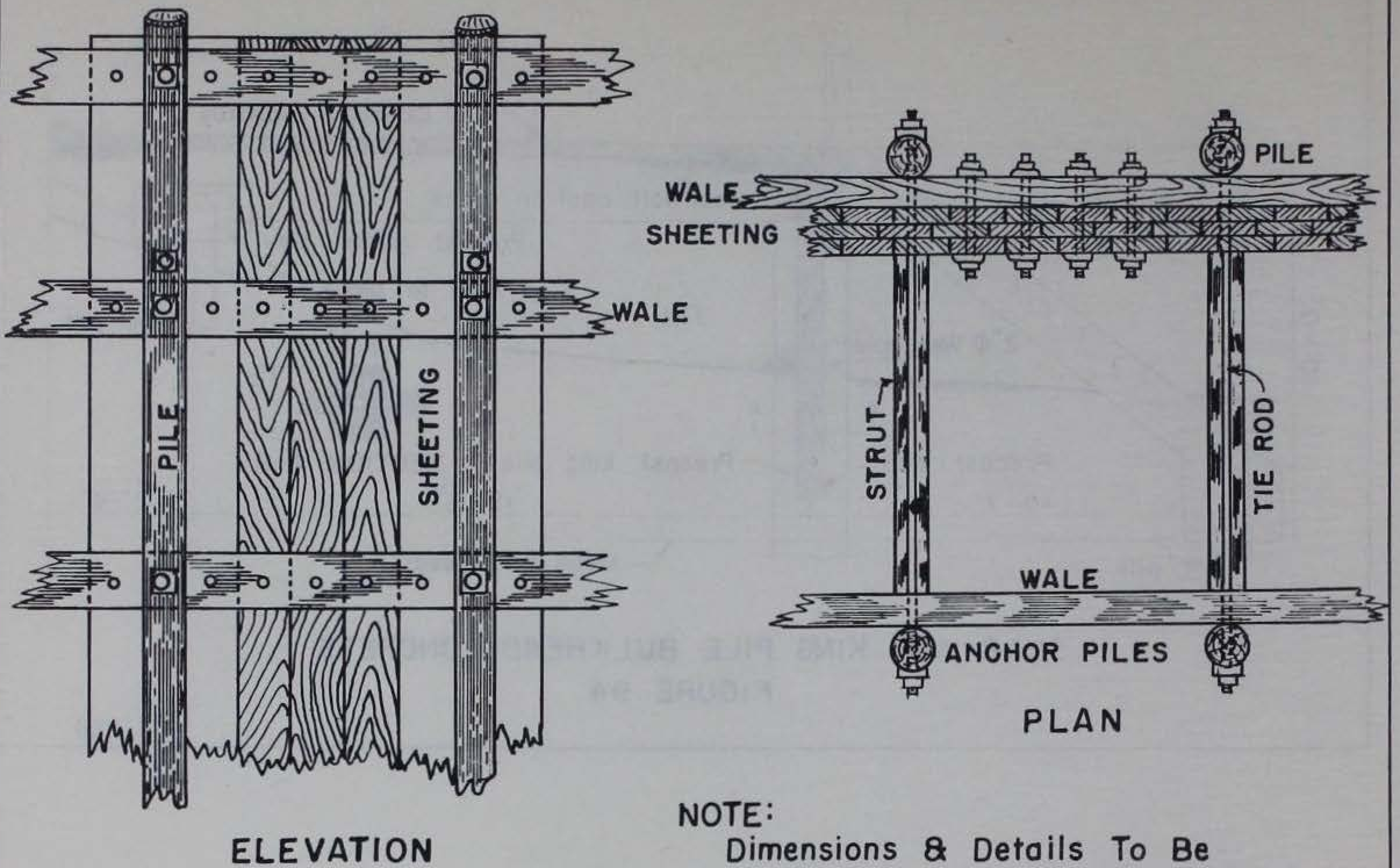
NOTE: A splash apron may be added next to coping channel to reduce damage due to overtoping.

NOTE: Dimensions and details to be determined by particular site conditions.

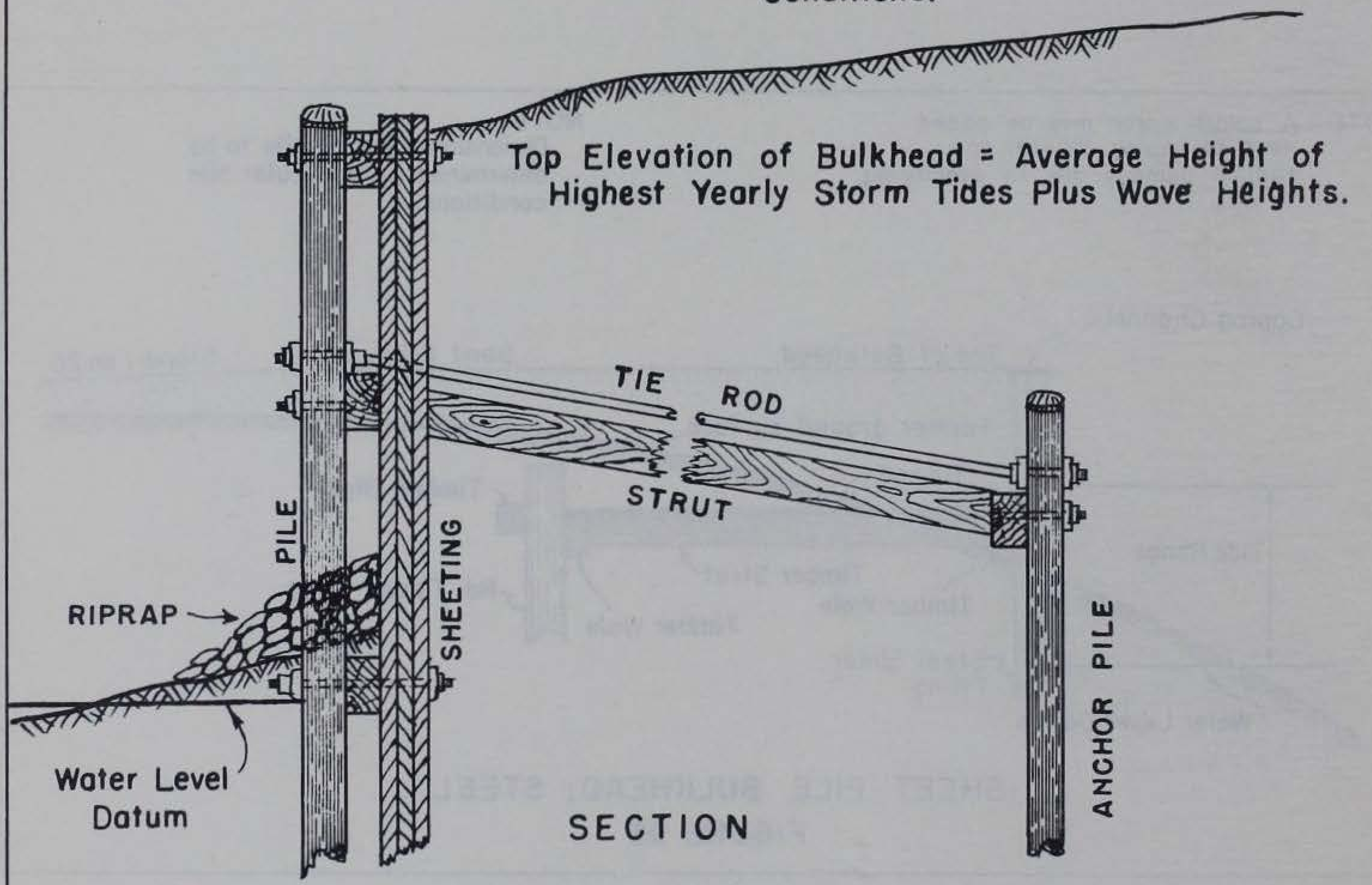


SHEET PILE BULKHEAD; STEEL

FIGURE 95

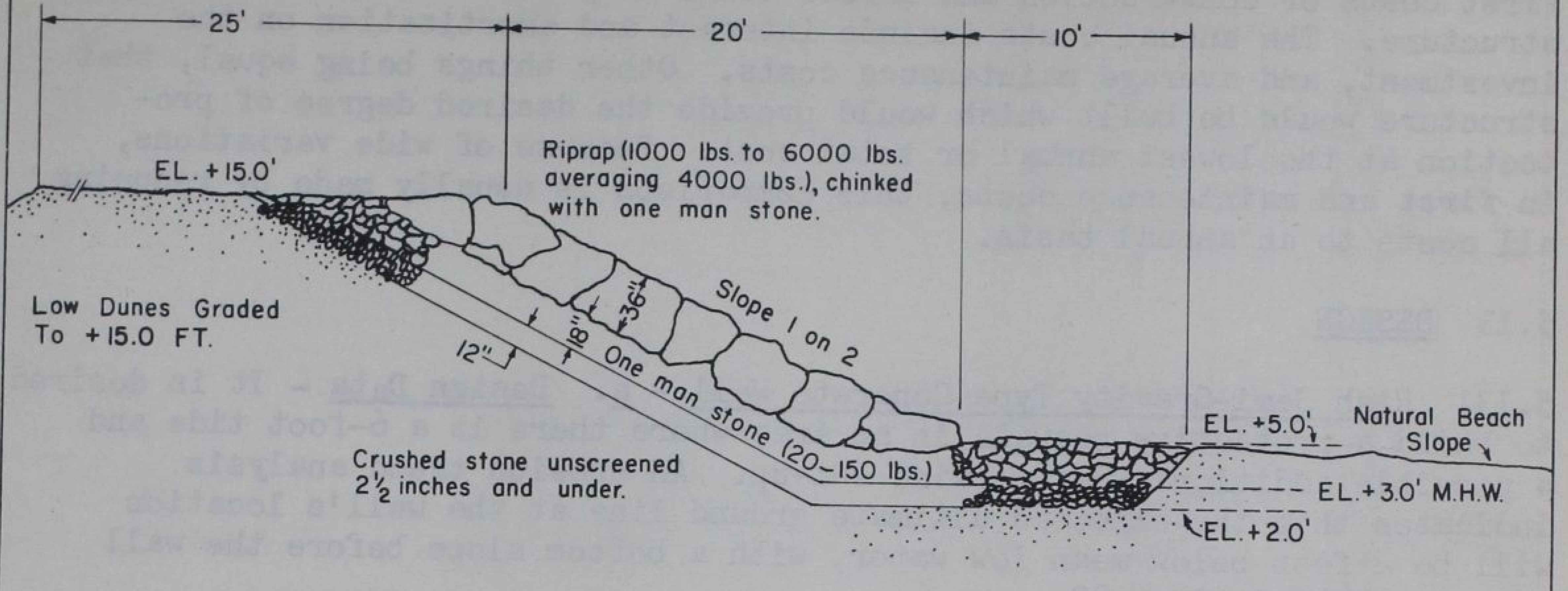


NOTE:
Dimensions & Details To Be Determined By Particular Site Conditions.



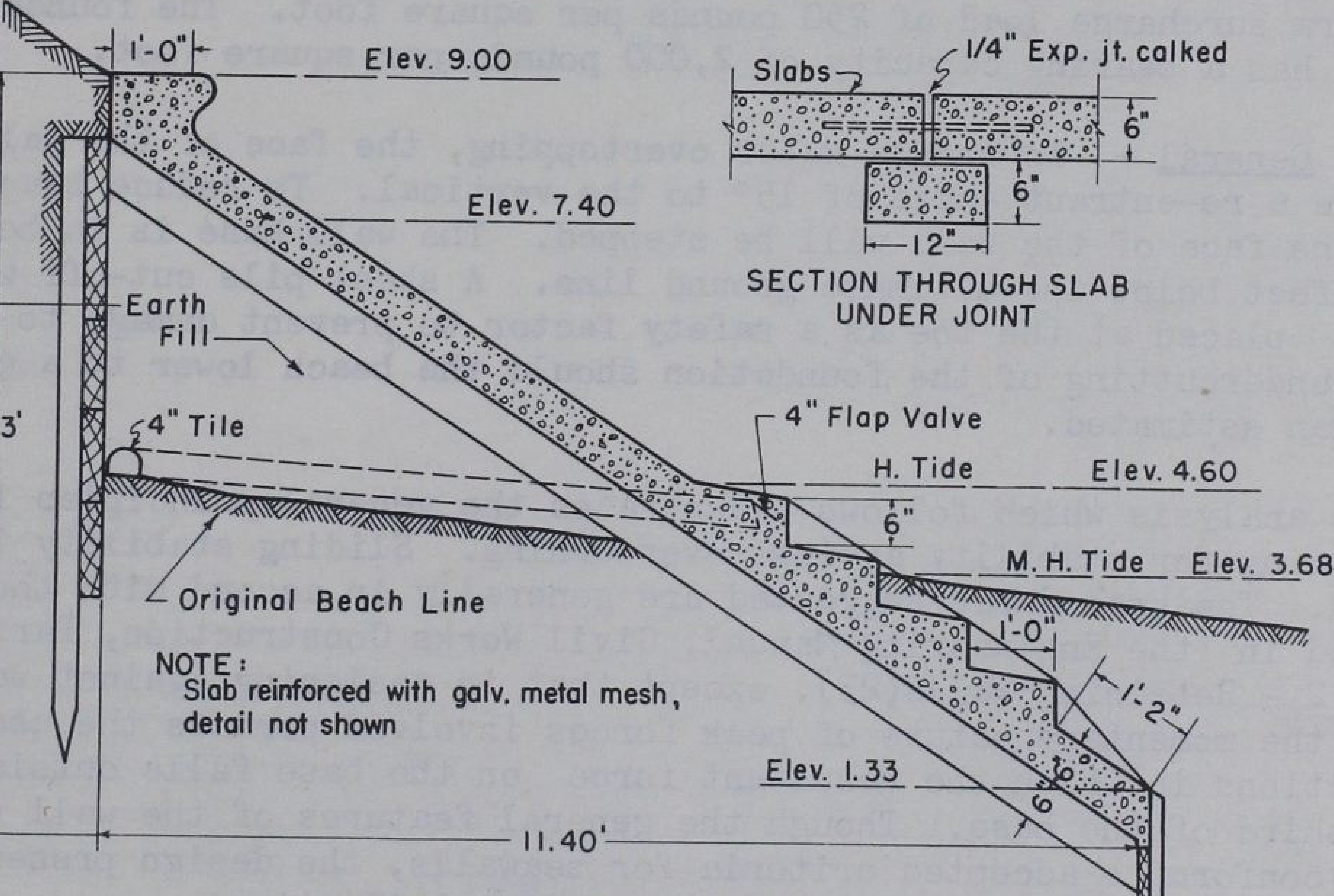
TIMBER SHEET PILE BULKHEAD

FIGURE 96



STONE REVETMENT
FORT STORY, VIRGINIA

FIGURE 97



CONCRETE REVETMENT
PIONEER POINT, CHESAPEAKE BAY, MARYLAND

FIGURE 98

5.124 Costs - The analysis of costs must include consideration of the first costs of construction and annual costs over the economic life of the structure. The annual costs include interest and amortization on the investment, and average maintenance costs. Other things being equal, that structure would be built which would provide the desired degree of protection at the lowest annual or total cost. Because of wide variations, in first and maintenance costs, this comparison is usually made by reducing all costs to an annual basis.

5.13 DESIGN

5.131 High Semi-Gravity Type Concrete Wall - a. Design Data.- It is desired to build a protective seawall in an area where there is a 6-foot tide and a possible additional 2-foot wind set-up. An erosion trend analysis indicates that the expected ultimate ground line at the wall's location will be 2 feet below mean low water, with a bottom slope before the wall of approximately 1 on 20.

Analysis of refraction diagrams indicates that waves from a certain direction may approach the area without decrease in height due to refraction while waves from other directions will be significantly decreased. Analysis of synoptic weather charts indicates that waves from this direction, under storm conditions, may have deep water heights up to 8 feet and periods from 7 to 11 seconds.

The backfill material has a unit weight of 120 pounds per cubic foot, and an angle of internal friction of 25° . The backfill will be subject to a uniform surcharge load of 250 pounds per square foot. The foundation material has a bearing capacity of 2,000 pounds per square foot.

b. General - To reduce water overtopping, the face of the wall will have a re-entrant angle of 15° to the vertical. To reduce beach scour, the face of the wall will be stepped. The wall base is to be at least 2 feet below the ultimate ground line. A sheet pile cut-off wall is to be placed at the toe as a safety factor to prevent damage to the wall by undercutting of the foundation should the beach lower to a greater depth than estimated.

The analysis which follows illustrates the general principles involved in designing for stability against overturning. Sliding stability is not analyzed. The procedures suggested are generally in accord with those presented in the Engineering Manual, Civil Works Construction, Part CXXV, Chapter 2 - Retaining Walls(23), except that in designing against wave forces, the momentary nature of peak forces involved permits the use of wall sections in which the resultant force on the base falls outside the middle third of the base. Though the general features of the wall section adopted conform to accepted criteria for seawalls, the design presented is not to be construed as a restriction on design initiative.

c. Wave Forces - Deep water wave lengths may be computed from the relationship $L_o = 5.12\ T^2$. With these the following table should be drawn up to determine whether waves may break on the structure.

BREAKER POSITION

H_o (feet) (1)	T (seconds) (2)	L_o (feet) (3)	H_o/L_o (4)	d_b/H'_o(curves) (5)	d'_b/H'_o(calculated) (6)
8	7	251	0.032	1.25	1.25
8	9	415	0.019	1.3	1.25
8	11	620	0.013	1.5	1.25

Column 1 - H_o = deep water design wave height from synoptic charts

Column 2 - T = wave period from synoptic charts

Column 3 - L_o = wave length from $L_o = 5.12\ T^2$

Column 4 - H_o/L_o = column 1 divided by column 3

Column 5 - d_b/H'_o (curve) = These values are tabulated from the curve of Figure 38

Column 6 - d'_b/H'_o (calculated) = The design depth at the structure divided by column 1 = 10/8 = 1.25

For the 7-second period, the fact that columns 5 and 6 are the same, indicates that a 7-second wave may break directly on the wall. The values for the 9-second period are so close that breaking waves may again be assumed. For these two periods, then, wave forces must be determined by Minikin's criteria. For the 11-second wave, column 5 is larger than column 6 indicating that waves would break seaward of the wall, and the force criteria for broken waves apply. If column 5 had been smaller than column 6 the Sainflou criteria would then have been used.

d. Breaking Waves - The curve IVERSEN(61) of Figure 37 may be used to determine the breaking wave height H_b.

BREAKING WAVE HEIGHTS

Period (T) (seconds)	H_o/L_o	H_b/H_o	H_b (feet)
7	0.032	1.1	9
9	0.019	1.25	10
11	0.013	1.4	11

H_b = the design wave height at the structure

The Minikin relationships for shock pressures, thrusts, and overturning moments due to wave action are:

$P_m = 101w(H_b/L_D) \times d(1 + d/D)$ for maximum pressure (27)

$R_m = P_m \times H_b/3$ for total thrust per foot of wall (33 first term)

$M_m = R_m \times (d + 2)$ for overturning moment (note that the moment arm is $d + 2$ since the wall base is 2 feet below ultimate ground level) (34 first term)

where L_D and D may be determined as follows for the 7 and 9-second wave periods with d = 10 feet. Values for d/L_d and D/L_D for the various values of d/L_o and D/L_o respectively are taken from Table D-1, Appendix D.

DETERMINATION OF D AND L_D

Period (T) (seconds) (1)	L_o (feet) (2)	d/L_o (3)	d/L_d (4)	L_d (feet) (5)	D (feet) (6)	D/L_o (7)	D/L_D (8)	L_D (9)
7	251	0.040	0.083	120	16	0.064	0.11	145
9	415	0.024	0.063	160	18	0.043	0.087	207

Accordingly, the shock pressures, thrust, and moments about the base due to the 7 and 9-second waves respectively are given in the following table.

SHOCK PRESSURES, THRUST AND MOMENTS

Period (T) (seconds)	Breaker Height (H_b) (feet)	Pressure (P_m) (lbs./sq. ft.)	Thrust (R_m) (lbs./sq. ft.)	Moment (M_m) (ft. lbs./ft.)
7	9	6540	19,620	236,000
9	10	4860	16,200	194,500

Since the wall is to be founded 2 feet below the ultimate ground line, the static forces on the face of the wall due to wave action may be computed as that force due to a head of water of height $(d + 2 + H_b/2)$. Therefore, the maximum static pressure will occur at the wall base (the depth d_b) and is given by

$$P_d = w(d + 2 + \frac{H_b}{2}) \quad (30)$$

the total thrust per linear foot of wall is given by

$$R_d = P_d \frac{(d + 2 + \frac{H_b}{2})}{2} \quad \text{(33-2nd term)}$$

and the overturning moment is given by

$$M_d = R_d\left(\frac{d + 2 + {}^{H_b}/_2}{3}\right) \qquad (34\text{-2nd term})$$

For the 7 and 9-second waves, these values are

STATIC PRESSURES, THRUSTS AND MOMENTS

Period (T) (seconds)	Breaker Height (H_b) (feet)	Pressure (P_d) (lb./ft.^{2})	Thrust (R_d) (lbs./ft.)	Moment (M_d) (ft.-lbs./ft.)
7	9	1,060	8,750	48,000
9	10	1,090	9,260	52,500

The manner in which these breaking wave pressures are applied is illustrated in Figure 99.

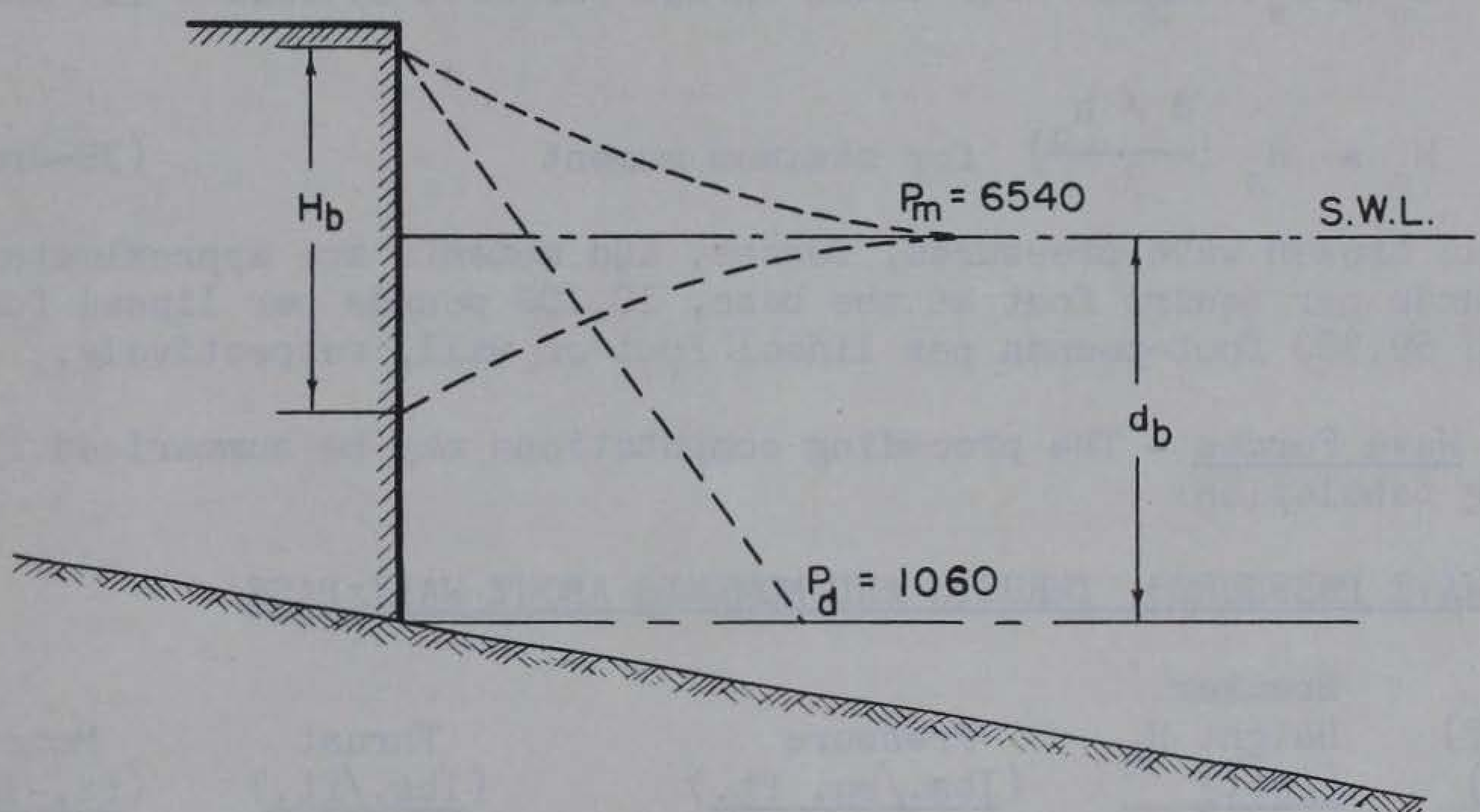


FIGURE 99 - BREAKING WAVE PRESSURE DIAGRAM

e. Broken Waves - To evaluate the dynamic forces due to the 11-second wave, the following relationship should be used:

$$P_m = \frac{wd_b}{2} \quad \text{for maximum pressure} \qquad (35)$$

$$R_m = P_m\,(h_c) \text{ for total thrust per foot of wall} \qquad (37\text{-1st term})$$

$$M_m = R_m\,(d + \frac{h_c}{2}) \quad \text{for maximum moment} \qquad (38\text{-1st term})$$

where d_D = the breaker depth for the 11-second wave and equals 8 x 1.5 = 12 feet (see tabulation on breaker position).

d = the depth of water at the structure = 10 feet

h_c = the elevation of the breaking wave crest over still water level = 0.7 x H_b = 7.7 feet

w = the unit weight of sea water = 64.2 pounds per cubic foot.

Therefore, for the 11-second wave, the dynamic pressure ≐ 385 pounds per square foot, the dynamic thrust = 3,000 pounds and the overturning moment ≐ 41,100 foot pounds per lineal foot of wall.

The equivalent expressions for static loadings are:

$$P_s = w(d + h_c) \text{ for maximum pressure at the wall base} \quad (36)$$

$$R_s = P_s\left(\frac{d + h_c}{2}\right) \text{ for total thrust per foot of wall} \quad (37\text{-2nd term})$$

$$M_s = R_s\left(\frac{d + h_c}{3}\right) \text{ for maximum moment} \quad (38\text{-2nd term})$$

The static broken wave pressures, forces, and moments are approximately 1,140 pounds per square foot at the base, 10,100 pounds per lineal foot of wall, and 59,300 foot-pounds per lineal foot of wall, respectively.

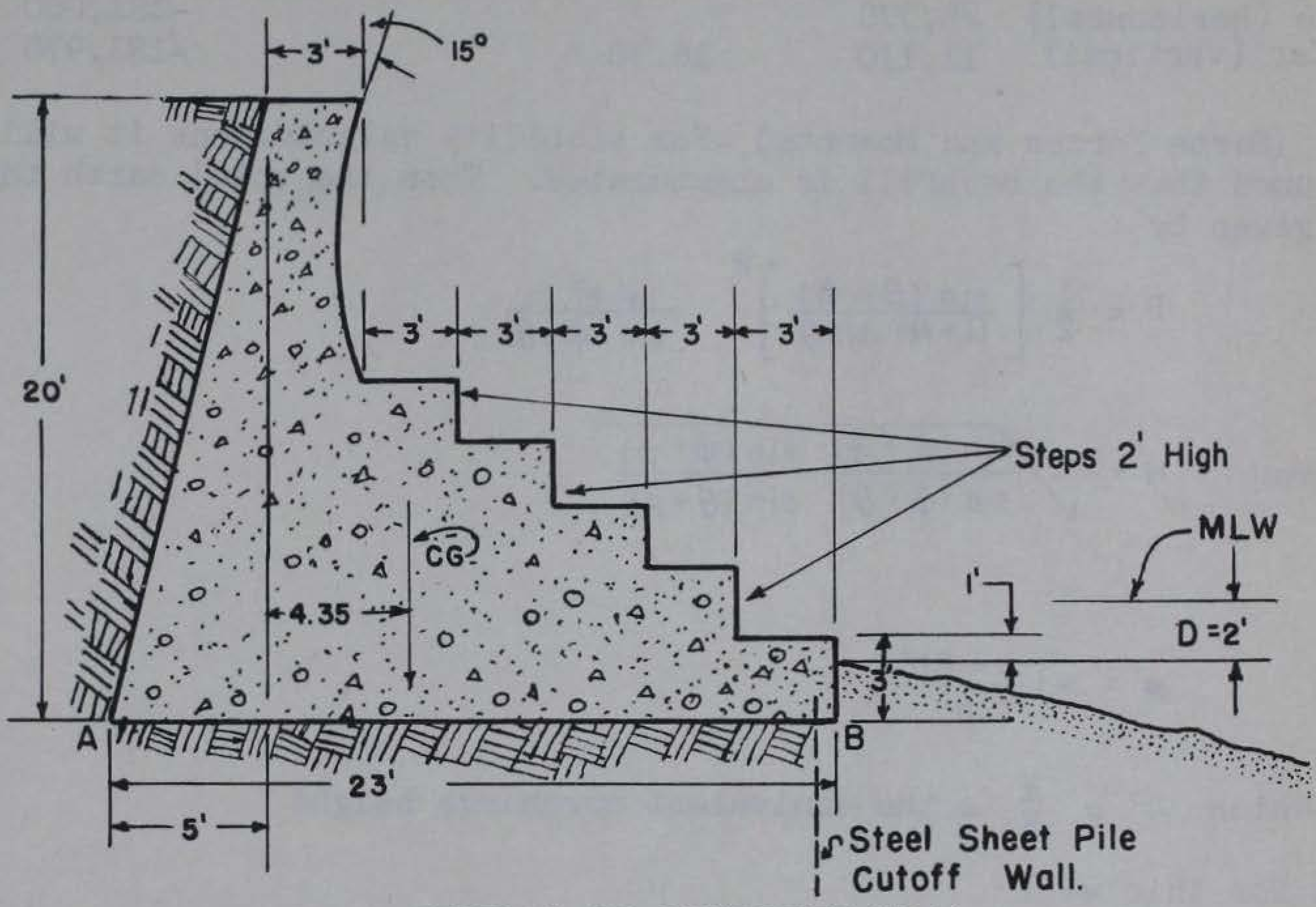
f. Wave Forces - The preceding computations may be summarized in the following tabulation:

WAVE PRESSURES, THRUSTS AND MOMENTS ABOUT WALL BASE

Period (T) (seconds)	Breaker Height H_b (feet)	Pressure (lbs./sq. ft.)		Thrust (lbs./ft.)	Moments (ft.-lbs./ft)
7	9	Dynamic	6,540	19,620	236,000
		Static	1,060	8,750	48,000
		TOTAL		28,370	284,000
		Dynamic	4,860	16,200	194,500
9	10	Static	1,090	9,260	52,500
		TOTAL		25,460	247,000
		Dynamic	385	3,000	41,100
11	11	Static	1,140	10,100	59,300
		TOTAL		13,100	100,400

Note that in this case the thrusts and moments are highest for the lowest breaking wave heights.

g. Wall Height - Though the forces due to the 11-second wave would be far less than those due to the 7 and 9-second waves, the wave crest height above sea level would be greater. To prevent significant overtopping, this height would be used for the design wall height. That is, the wall crest would be 8 feet above the still water level.



CROSS SECTION OF CONCRETE WALL
FIGURE 100

A wall section conforming to the previously noted general design criteria and to the above is shown in Figure 100. The weight of concrete in the wall, considering concrete to have a unit weight of 150 pounds per cubic foot, is 31,710 pounds per lineal foot. The wall center of gravity determined by taking moment areas about A is 9.35 feet seaward of A.

h. Stability Calculations -

(Wall Overturning Landward) - Maximum landward overturning moments would occur with maximum wave action on the face of the wall and with the backfill unsaturated. Wave forces and uplift would tend to cause the wall to overturn landward. (From the preceding table the design wave forces are those due to the 7-second period wave). The overturning moment would be resisted by the moments due to the weight of water over the toe of the seawall, the weight of the wall, active earth pressure on the back of the wall, and the weight of earth on the back of the wall.

(Wave Forces and Moments) - Total wave forces and moments about A are tabulated below, the vertical water value being computed assuming a maximum water level of $0.7 \times H_b = 6.3$ feet above storm sea level.

WAVE FORCES AND MOMENTS

Item	Force (lbs./ft.)	Arm (A) (feet)	Moment about A (ft.-lbs./ ft.)
Wave (horizontal)	28,370		-284,000
Water (vertical)	11,110	16.38	+181,970

(Earth Forces and Moments) -For stability calculations it will be assumed that the backfill is unsaturated. Then the total earth thrust is given by

$$P = \frac{1}{2}\left[\frac{\sin(\theta-\phi)}{(1+N)\sin\theta}\right]^2 \frac{w' h^2}{\sin(\phi_1+\theta)} \tag{59}$$

where $$N = \sqrt{\frac{\sin(\phi+\phi_1)\ \sin(\phi-\rho)}{\sin(\phi_1+\theta)\ \sin(\theta+\rho)}} \tag{60}$$

and $$w' = w\left(\frac{h+2h'}{h}\right) \tag{61}$$

in which $h' = \frac{W}{w}$ = the equivalent surcharge height

and for this wall

W = 250 pounds per square foot = the surcharge load

w = 120 pounds per cubic foot = the backfill unit weight

h = 20 ft. = the backfill height

w' = 145 pounds per cubic foot = backfill unit weight with surcharge load

$\theta = 90 + \tan^{-1}\left(\frac{5}{20}\right) = 104° 2'$ = the wall backface angle with the horizontal

$\phi = 25°$ = the backfill internal friction angle

$\phi_1 = 17°$ = the backfill wall friction angle

$\rho = 0°$ = the angle between surface of backfill and horizontal plane

This thrust will act at a distance $\left(0.375 + \frac{w' - w}{10w}\right)h$ above the base and at an angle $(\theta - 90 + \phi_1)$ with the horizontal. By resolving P into horizontal (P_H) and vertical (P_V) components we may find the thrust and moments about A. These are as follows:

EARTH FORCES AND MOMENTS

Item	Force (lbs./ft.)	Arm (from A) (feet)	Moment (ft. - lbs./ft.)
Earth (horizontal)	P_H = 11,850	7.92	93,830
Earth (vertical)	P_V = 7,130	1.98	14,110

(Uplift Forces and Moments)- Uplift forces may be computed as a straight line triangular variation, assuming a head of 18.3 feet at the toe and none at the heel.

UPLIFT FORCES AND MOMENTS

Item	Force (lbs./ft.)	Arm (from A) (feet)	Moments (ft. - lbs./ft.)
Uplift (vertical)	13,500	15.33	-207,120

(Stability Table) - Summarizing the preceding tables, the following table may be set up:

STABILITY TABLE, OVERTURNING LANDWARD

Item	Force (lbs./ft.)	Arm (from A) (feet)	Moments about A (ft. - lbs./ft.)	
Concrete (vertical)	31,710	9.35	+296,450	
Earth (horizontal)	11,850	7.92	+ 93,830	
Earth (vertical)	7,130	1.98	+ 14,110	
Water (vertical)	11,110	16.38	+181,970	
Wave (horizontal)	28,370			-284,000
Uplift (vertical)	13,500	15.33		-207,120
		TOTAL	+586,360	-491,120
		NET TOTAL	+95,240	

The moments which would cause overturning around point A are negative, therefore, the wall is stable under wave forces. It should be remembered that designing against wave forces in this manner is equivalent to assuming their uniform application over the entire length of wall at one time. That this is never so, plus the fact that the impact nature of the computed maximum force makes more effective the high inertia of the wall, are indications that a wall so designed is far more stable under wave forces than the above computations would indicate.

(Wall Overturning Seaward) - Maximum moment causing the wall to overturn seaward would occur with no water in front of the wall, and assuming a saturated backfill and a surcharge of 250 pounds per square foot. The earth forces and moments may be computed in the same manner as for the unsaturated case. All angles would be the same, but

$w = 145.7 = 120 + 0.40 \times 64.2$, assuming 40% voids in the material

$h' = 1.72$

so

$w' = 171$ pounds per cubic foot.

since w' for the unsaturated case was 145 pounds per cubic foot, the earth thrust will be in the proportion 171/145 to the force calculated for an unsaturated fill, and will be applied at a height of 7.84 feet above the base. On resolving the thrust into horizontal and vertical components, the forces and moments about B may be tabulated.

EARTH FORCES AND MOMENTS

Item	Force (lbs./ft.)	Arm (from B) (feet)	Moment about B (ft.-lbs./ft.)
Earth (horizontal)	$P_H = 13,520$	7.84	106,000
Earth (vertical)	$P_V = 8,390$	21.04	176,480

(Uplift Forces and Moments) - Uplift may be calculated for conservative conditions by assuming the water level at the toe to be at the base. Then the uplift force diagram is triangular and the thrust and moment about B are given in the following table.

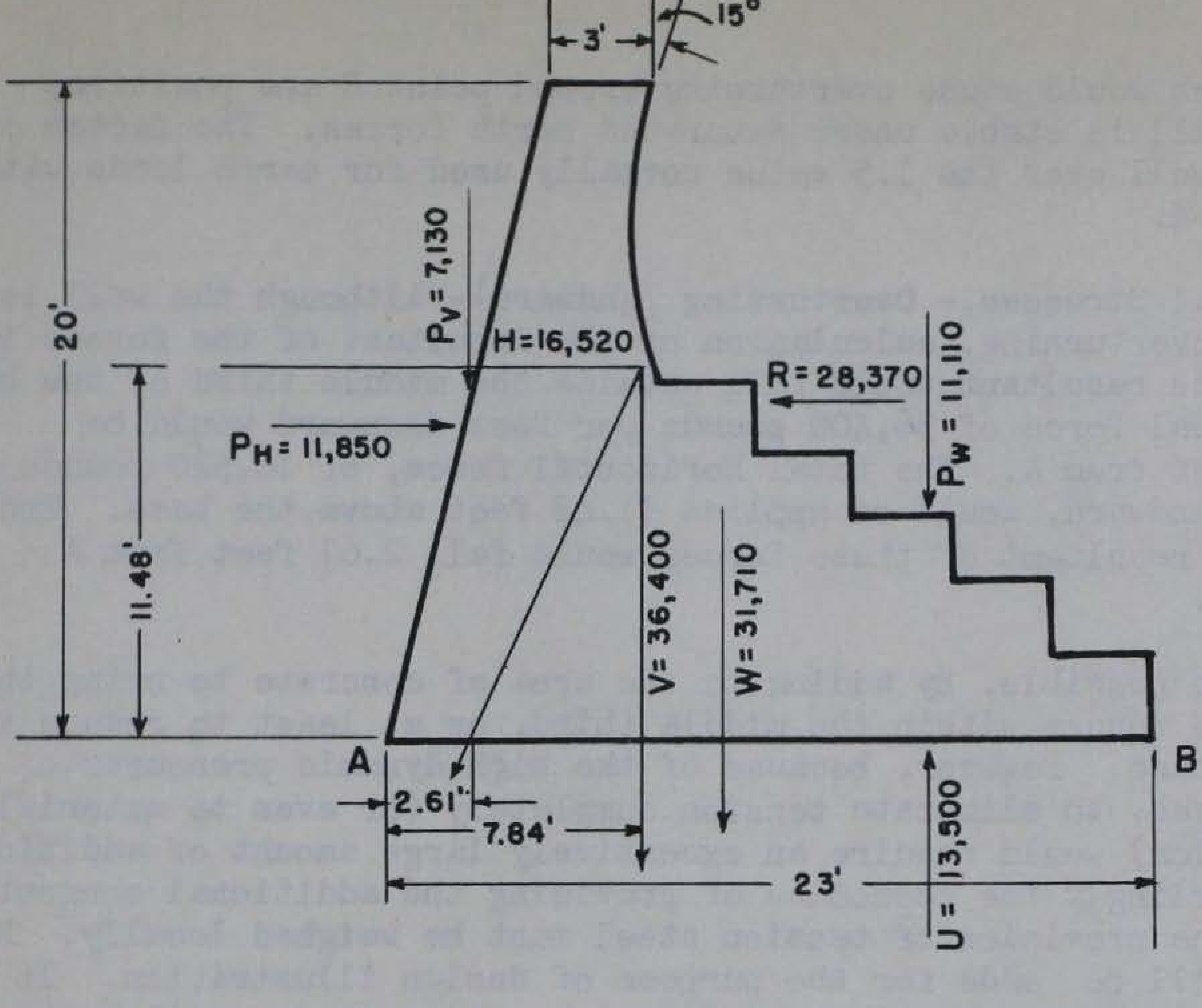
UPLIFT FORCES AND MOMENTS

Item	Force (lbs./ft.)	Arm (from B) (feet)	Moment about B (ft.-lbs./ft.)
Uplift	14,770	15.33	226,360

(Stability Table) - Summarizing the preceding the following table may be set up:

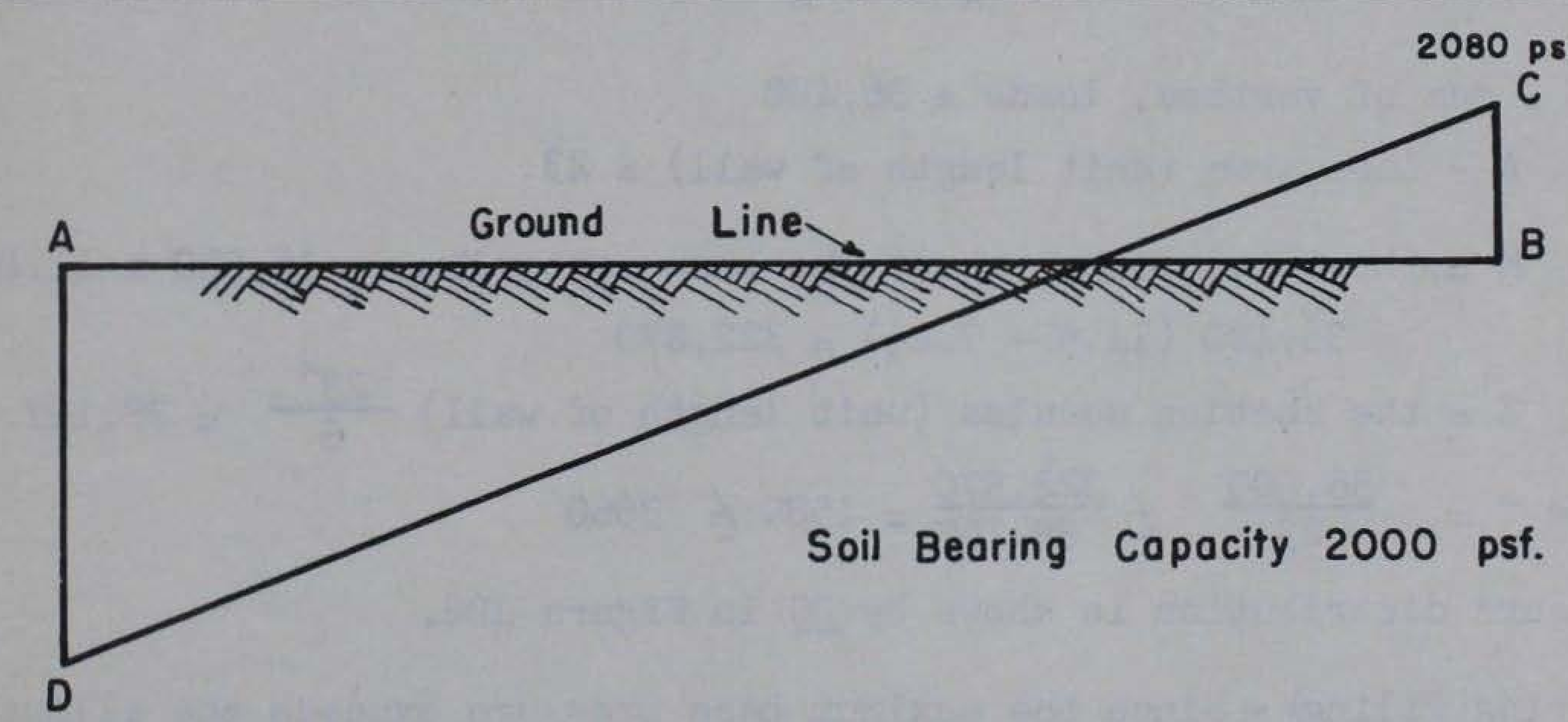
STABILITY, OVERTURNING SEAWARD

Item	Force (lbs./ft.)	Arm (from B) (feet)	Moment about B (ft.-lbs/ft.)	
Concrete (vertical)	31,710	13.65		-432,840
Earth (vertical)	8,390	21.04		-176,480
Earth (horizontal)	13,520	7.84	+106,000	
Uplift (vertical)	14,770	15.33	+226,360	
		TOTAL	+332,360	-609,320
		NET TOTAL		-276,960



FORCE RESULTANT, OVERTURNING LANDWARD

FIGURE 101



BEARING PRESSURES, OVERTURNING LANDWARD

FIGURE 102

The moments which would cause overturning around point B are positive, therefore the wall is stable under saturated earth forces. The factor of safety here is well over the 1.5 value normally used for earth loads with uplift considered.

(Internal Stresses.- Overturning Landward)- Although the wall is stable against overturning, calculation of the resultant of the forces indicates that this resultant would fall outside the middle third of the base. The total vertical force of 36,400 pounds per foot downward would be applied 7.84 feet from A. The total horizontal force, of 16,520 pounds per foot directed landward, would be applied 11.48 feet above the base. From Figure 101, the resultant of these forces would fall 2.61 feet from A inside the base.

It would be possible, by adding to the area of concrete to bring the resultant of the forces within the middle third, or at least to reduce the tension in the base. However, because of the high dynamic pressures of the breaking waves, to eliminate tension completely (or even to materially reduce the tension) would require an excessively large amount of additional concrete. Accordingly the economics of providing the additional concrete as opposed to the provision of tension steel must be weighed locally. No such analysis will be made for the purpose of design illustration. It should also be noted that tension steel for the given design will be required in the front face of the wall.

(Foundation Pressures. - Overturning Landward) - The bearing pressure of the loads may be calculated from

$$f = \frac{P_v}{A} \pm \frac{M}{Z} \tag{63}$$

where P_v = sum of vertical loads = 36,400

A - base area (unit length of wall) = 23

M = the total moment about the base centerline = $16,520 \times 11.48 + 36,400\ (11.5 - 7.84) = 322,870$

Z = the section modulus (unit length of wall) $\frac{23^2}{6} = 88.167$

Therefore $f = \frac{36,400}{23} \pm \frac{322,870}{88.167} = 1580 \pm 3660$

The pressure distribution is shown by DC in Figure 102.

(Bearing Piling) - Since the maximum base pressure exceeds the allowable bearing capacity of the foundation material, piles must be used to support the wall. Even if the maximum base pressure did not exceed the bearing capacity of the foundation material, for safety, bearing piling would be used. The piles should be designed for the total bearing pressure for the loads as shown on Figure 102. A network of piles under the entire base should be used to prevent unequal settlement. In a case such as this, where large horizontal thrusts are involved, it may be economical to use batter piles as well as vertical bearing piles.

Allowable pile loading varies with the material in which the pile is driven. If an individual pile rests on a hard stratum its bearing capacity is determined by the ultimate strength in compression of the pile material. Essentially the pile becomes a totally laterally supported column. However, when a pile's bearing load is supported primarily by skin friction with the material through which it is driven, its bearing capacity must be determined by empirical means.

The best practice in piling design is to rely upon statically loaded test piles to assess the safe loading for the piling. For most foundation materials on which seawalls will be built, the resistance to a static load is almost the same as that offered to the dynamic load of driving and for limited piling projects this relationship may be utilized to determine allowable loads. The allowable bearing load per pile can thus be calculated by relating the depth of penetration of a test pile per blow to the hammer weight. Relationships commonly in use for timber and concrete piling are as follows:

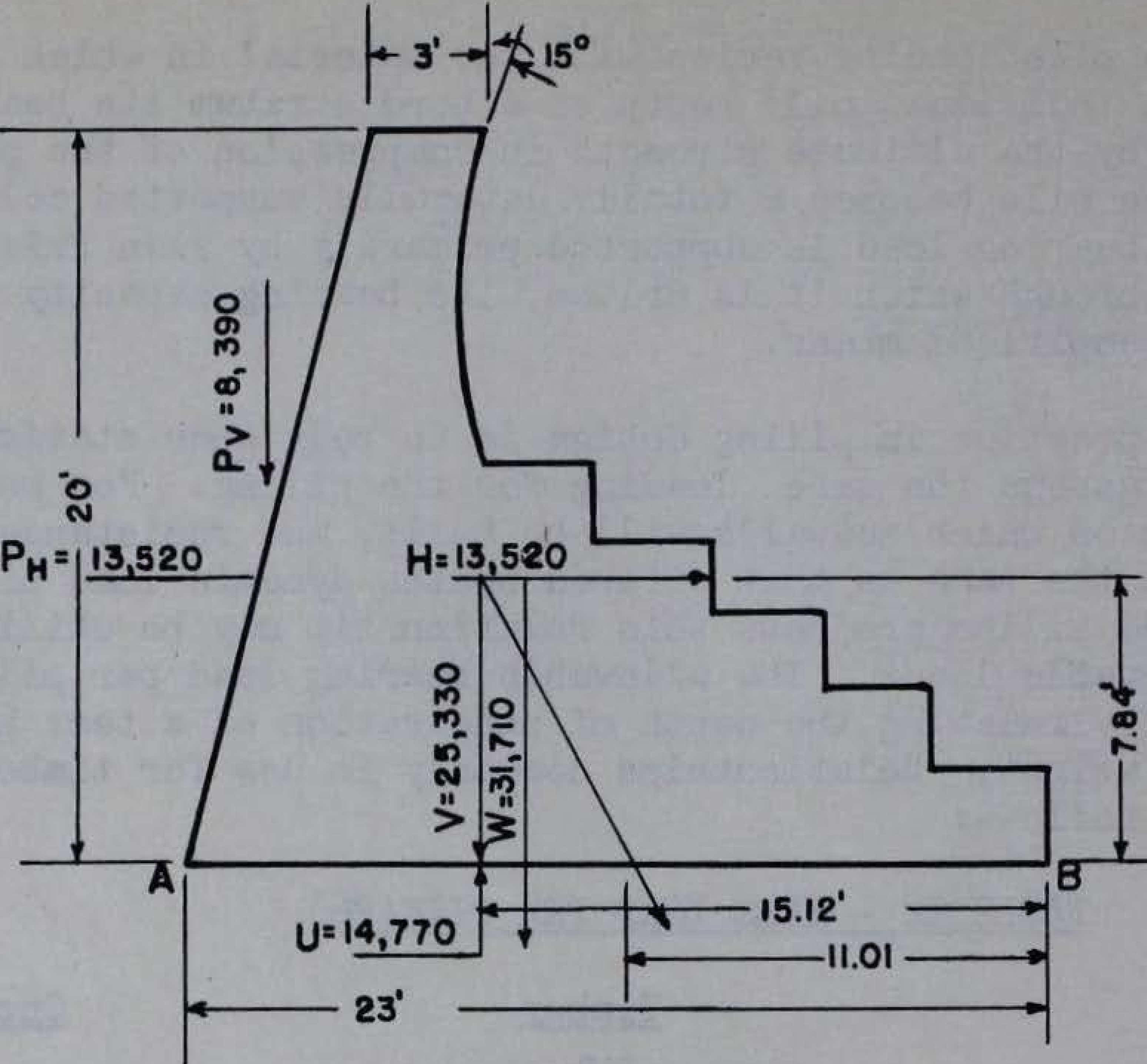
TABLE 12 - SAFE LOAD PER PILE(86)

Driver	Timber	Concrete
Drop Hammer	$P = \frac{2Wh}{S + 1}$	$P = \frac{2Wh}{s\ (1 + W_p/W)}$
Steam hammer, single acting	$P = \frac{2Wh}{s + 0.1}$	$P = \frac{2Wh}{s + 0.1\ W_p/W}$
Steam hammer, double acting	$P = \frac{2h\ (W + ap)}{s + 0.1}$	$P = \frac{2h\ (W + ap)}{s + 0.1\ W_p/W}$

P = safe bearing load, pounds
W = hammer weight, pounds
W_p = pile weight, pounds
h = free fall of hammer, feet
a = effective area of piston, inches^2
p = mean effective steam pressure, pounds per square inch
s = penetration of sinking, inches
(taken as average of last 5 to 10 blows for a drop hammer and as average of last 20 blows for a steam hammer.)

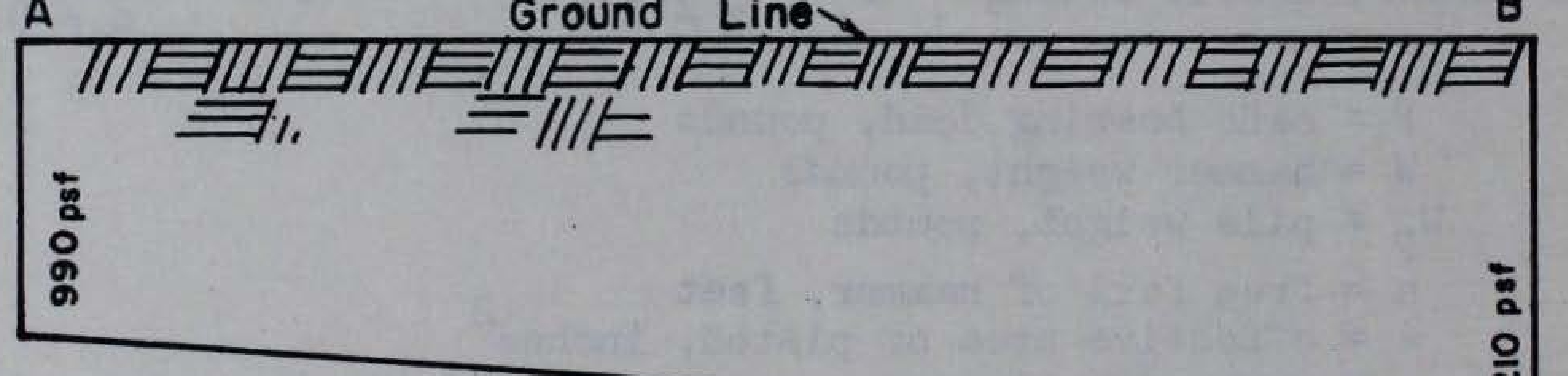
These formulas will give safe loadings of individual piles provided the individual piles of a group are not so closely spaced that skin friction is materially reduced.(125).

(Internal Stresses - Overturning Seaward)- On obtaining the resultant



FORCE RESULTANT, OVERTURNING SEAWARD

FIGURE 103



BEARING PRESSURES, OVERTURNING SEAWARD

FIGURE 104

of vertical and horizontal forces it is found that this resultant would fall within the middle third of the base. The total vertical force of 25,330 pounds per foot downward would be applied 15.12 feet from B. The total horizontal thrust of 13,520 pounds per foot directed seaward would be applied 7.84 feet above the base. From Figure 103 the resultant of these forces would fall 10.94 feet from B within the base, or well within the middle third. Therefore, there would be no tension in the base due to the force of the saturated fill.

(Foundation Pressures.- Overturning Seaward)-

Using $f = \frac{P_v}{A} \pm \frac{M}{Z}$ (see equation 63)

where $P_v = 25,330$

$A = 23$

$M = 13,520 \times 7.84 - 25,330\ (15.12 - 11.5) = 14,300$

$Z = \frac{23^2}{6} = 88.167$

$f = \frac{25,330}{23} \pm \frac{14,300}{88.167}$

$= 1,100 \pm 110$

The pressure distribution would be as shown in Figure 104 by the line CD.

5.132 Steel Sheet Pile Cellular Seawall - a. Design Data.- A wall is to be placed in fresh water at a low bluff area, to protect the bluff line from the action of storm waves. The water depth at the toe of the wall is to be 3 feet below low water datum. There is a maximum expected wind set-up of 2 feet. The slope of the bottom seaward of the site of the wall is approximately 1 on 40. The elevation of the top of the bluff is about 10 feet above low water datum. This would also be the elevation of the top of the backfill. Wave analysis indicates that the maximum deep water wave height expected is 10 feet, and that a wave this high will have a period of 12 seconds. The maximum refraction coefficient is 0.5, at a depth of 50 feet, for this 12-second wave. Refraction coefficients for other period waves are much smaller.

The bottom is composed of a 2-foot thickness of gravel and shale overlying bedrock. Because of poor penetration possibilities, a sheet pile, cellular type structure was chosen for design.

The backfill material has a natural drained unit weight of 130 pounds per cubic foot and an internal friction angle of 25°. The cell fill material (gravel) has a anatural drained unit weight of 120 pounds per cubic foot and an internal friction angle of 45°. The coefficient of friction between the cell fill and the bottom is 0.5.

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b. Wave Forces - The deep water wave length of a 12-second wave is $L_o = 5.12\ (12)^2 = 737$ feet. At a depth of 50 feet $d/L_o = 50/737 = 0.068$. From Table D-1 of Appendix D the corresponding $d/L = 0.112$ and $H/H'_o = 0.98$ Therefore, the wave height in this depth of water where the refraction coefficient k is 0.5 for a wave 10 feet high in deep water is

$$H = H_o \times k \times (H/H'_o) = 10 \times 0.5 \times 0.98 \doteq 5 \text{ feet} \qquad (13)$$

Where k is also equal to (H'_o/H_o) and H'_o is the deep water wave height which, in the absence of refraction, would give a wave height of H in a depth of 50 feet. This equivalent deep water wave height H'_o is 5 feet.

A wave with deep water length of 737 feet and deep water height of 5 feet will have a steepness of $H'_o/L_o = 5/737 = 0.0068$. From Figure 38, the ratio $d_b/H'_o = 2.0$, from which d_b, the breaking depth, is $d_b = 2.0 \times 5 = 10$ feet. The maximum anticipated depth at the wall including tidal rise is 5 feet, which indicates that waves will break before reaching the structure, and that the wave force criteria for broken waves must be used for this deep water design wave.

c. Forces Due to Broken Waves - Referring to Figure 37 H_b/H'_o for a wave whose deep water steepness $(H_o/L_o) = 0.0068$, moving up a 1:40 slope is $H_b/H'_o = 1.5$. Therefore, the breaking wave height is 7.5 feet.

The dynamic pressures, thrusts and moments are given by:

$$P_m = \frac{w\, d_b}{2} \quad \text{for maximum pressure} \qquad (35)$$

$$R_m = P_m(h_c) \quad \text{for total thrust per foot of wall} \qquad \text{(37-1st term)}$$

$$M_m = R_m\left(d + \frac{h_c}{2}\right) \quad \text{for overturning moment per foot of wall} \qquad \text{(38-1st term)}$$

where d_b = the breaker depth referred to the maximum anticipated water level = 10 feet

d = the depth of water at the structure = 5 feet

h_c = the elevation of the breaking wave crest above still water level = 0.7 x 7.5 = 5.25 feet

w = the unit weight of fresh water = 62.4 pounds per cubic foot.

The numerical values for these are approximately: P_m = 312 pounds per square foot; R_m = 1640 pounds per foot of wall; and M_m = 12,500 foot-pounds per foot of wall.

The equivalent expressions for static loadings are:

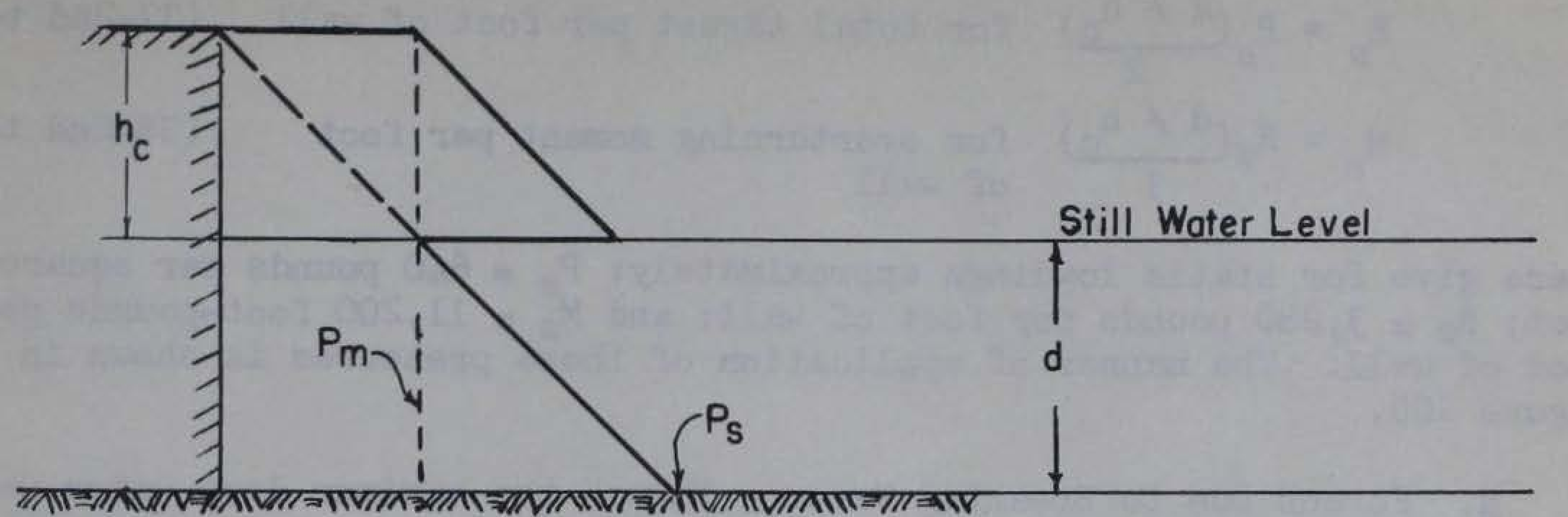


FIGURE 105 - DISTRIBUTION OF PRESSURES; BROKEN WAVES

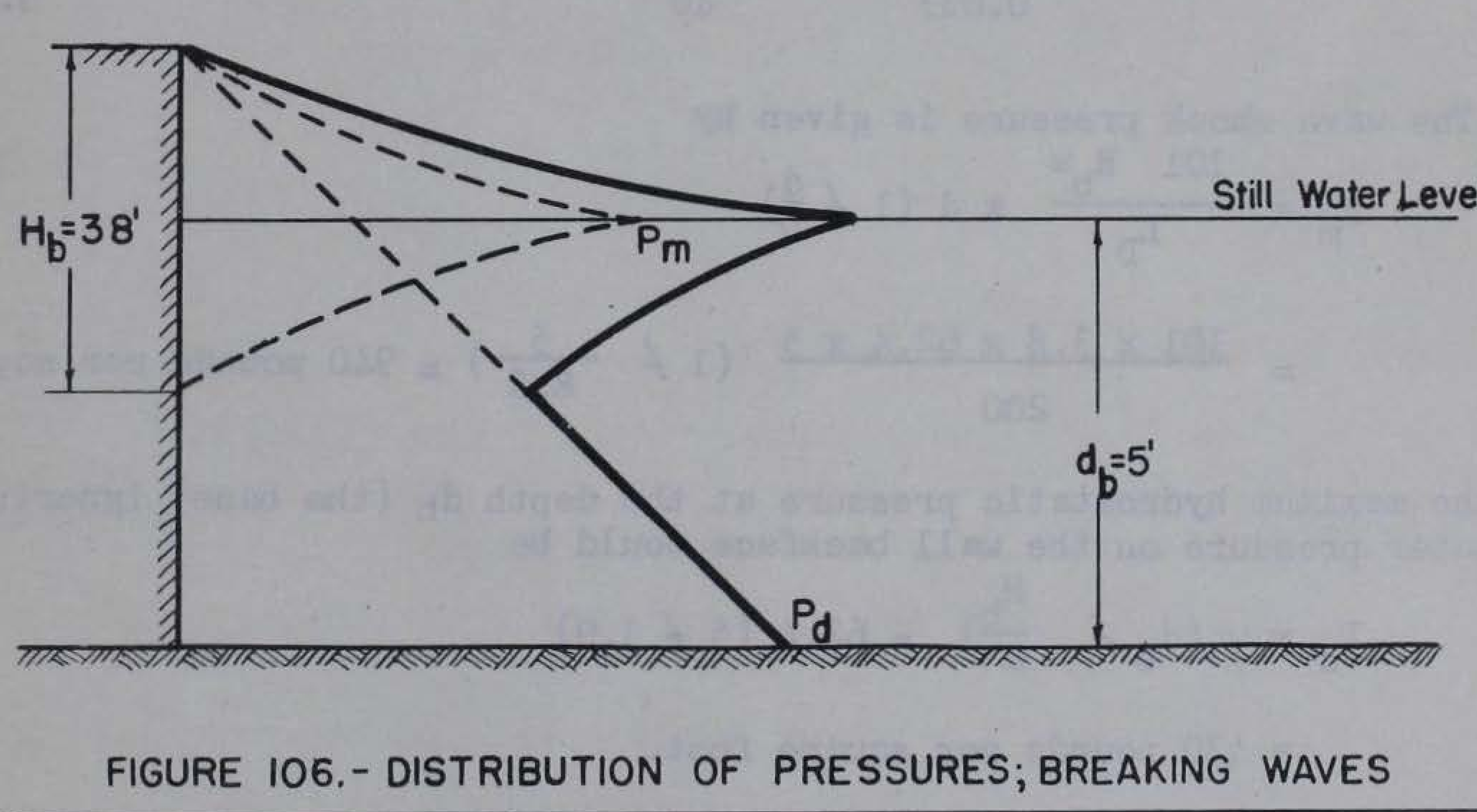


FIGURE 106. - DISTRIBUTION OF PRESSURES; BREAKING WAVES

$$P_s = w(d + h_c) \text{ for maximum pressure at wall base} \qquad (36)$$

$$R_s = P_s\left(\frac{d + h_c}{2}\right) \text{ for total thrust per foot of wall} \qquad (37\text{-2nd term})$$

$$M_s = R_s\left(\frac{d + h_c}{3}\right) \text{ for overturning moment per foot of wall} \qquad (38\text{-2nd term})$$

These give for static loadings approximately: P_s = 640 pounds per square foot; R_s = 3,280 pounds per foot of wall; and M_s = 11,200 foot-pounds per foot of wall. The manner of application of these pressures is shown in Figure 105.

d. Forces Due to Breaking Waves - Though the maximum deep water wave breaks before reaching the wall, some lesser height wave will break right at it. This wave height may be approximated by the relationship $d_b/H_b = 1.3$ where d_b is the maximum water depth at the wall. In this case then, $H_b = 5/1.3 = 3.8$ feet. To use the Minikin relationships for forces, it is necessary to determine D and L_D as indicated in Section 5.131 d. In tabular form, the computations are as follows:

DETERMINATION OF D AND L_D FOR d = 5 FEET

L_o (feet)	d/L_o	d/L_d	L_d (feet)	D (feet)	D/L_o	D/L_D	L_D (feet)
737	0.0068	0.033	$\frac{d}{0.033} = 152$	$\frac{152}{40} + 5 = 8.8$	0.0119	0.044	$\frac{D}{0.044} = 200$

The wave shock pressure is given by

$$P_m = \frac{101 \; H_b w}{L_D} \times d \left(1 + \frac{d}{D}\right) \qquad (27)$$

$$= \frac{101 \times 3.8 \times 62.4 \times 5}{200} \left(1 + \frac{5}{8.8}\right) = 940 \text{ pounds per sq. ft.}$$

and the maximum hydrostatic pressure at the depth d_b (the base) ignoring any water pressure on the wall backface would be

$$P_d = w\left(d_b + \frac{H_b}{2}\right) = 62.4\,(5 + 1.9) \qquad (30)$$

$$= 430 \text{ pounds per square foot.}$$

The manner of application of these pressures is shown in Figure 106.

From this figure, the total thrust is given by

$$R = R_m + R_d$$

$$= P_m \left(\frac{H_b}{3}\right) + \frac{P_d}{2} \left(d + \frac{H_b}{2}\right) \qquad (33)$$

$$= 940 \times \frac{3.8}{3} + 215\ (6.9)$$

$= 1190 + 1480 = 2670$ pounds per lineal foot of wall.

and the total overturning moment would be

$$M = R_m d + \frac{R_d}{3} \left(d + \frac{H_b}{2}\right) \qquad (34)$$

$$= 1190 \times 5 + \frac{1480}{3}\ (6.9)$$

$$= 5950 + 3400$$

$= 9350$ foot-pounds per lineal foot of wall.

e. Tabulation of Design Wave Forces and Moments - The forces and moments exerted on the wall by the deep water design wave breaking in 10 feet of water are greater than those exerted by some lesser height of wave breaking right on the structure. These larger forces will be used for design.

WAVE FORCES AND MOMENTS

Item		Force (pounds per foot)	Moment (foot-pounds per foot)
Dynamic		1,640	12,500
Static		3,280	11,200
	TOTAL	4,920	23,700

f. Earth Forces - The backfill thrust is given by

$$P = \frac{wh^2}{2} \tan^2 \left(\frac{90 - \phi}{2}\right) \qquad (57)$$

where

$h = 13$ feet = height of backfill above low water level

$\phi = 25°$ = internal friction angle of the backfill

$w = 130$ pounds per cubic foot = the unit weight of unsaturated fill or

$w = 130 + 0.4 \times 62.4 = 154.8$ pounds per cubic foot, the unit weight of saturated fill assuming the material has 40% voids.

The point of application of this thrust is at a distance 0.375h above the cell base.

Therefore, for an unsaturated backfill

$$P = \frac{130 \times 169}{2} (0.637)^2$$

$$= 4,460 \text{ pounds per lineal foot of wall}$$

and for a saturated backfill

$$P = \frac{154.8 \times 169}{2} (0.637)^2$$

$$= 5,300 \text{ pounds per lineal foot of wall}$$

each applied at a distance 4.88 feet above the base.

The thrust on the sheet pile due to the fill material is similarly calculated with

$$\emptyset = 45°$$

w = 120 pounds per cubic foot for unsaturated fill

= 120 + 0.4 x 62.4 = 144.8 pounds per cubic foot for saturated fill.

Therefore, for an unsaturated cell fill

$$P = \frac{120 \times 169}{2} (0.414)^2$$

$$= 1,730 \text{ pounds per lineal foot of wall}$$

and for a saturated cell fill

$$P = \frac{144.8 \times 169}{2} (0.414)^2$$

$$= 2100 \text{ pounds per lineal foot of wall.}$$

Note especially for the cell fill, that the maximum pressure (not thrust) is given by

$$p = wh \tan^2 \left(\frac{90 - \emptyset}{2}\right) \tag{64}$$

and occurs at the cell bottom. For the case being considered with a saturated cell fill, this pressure is

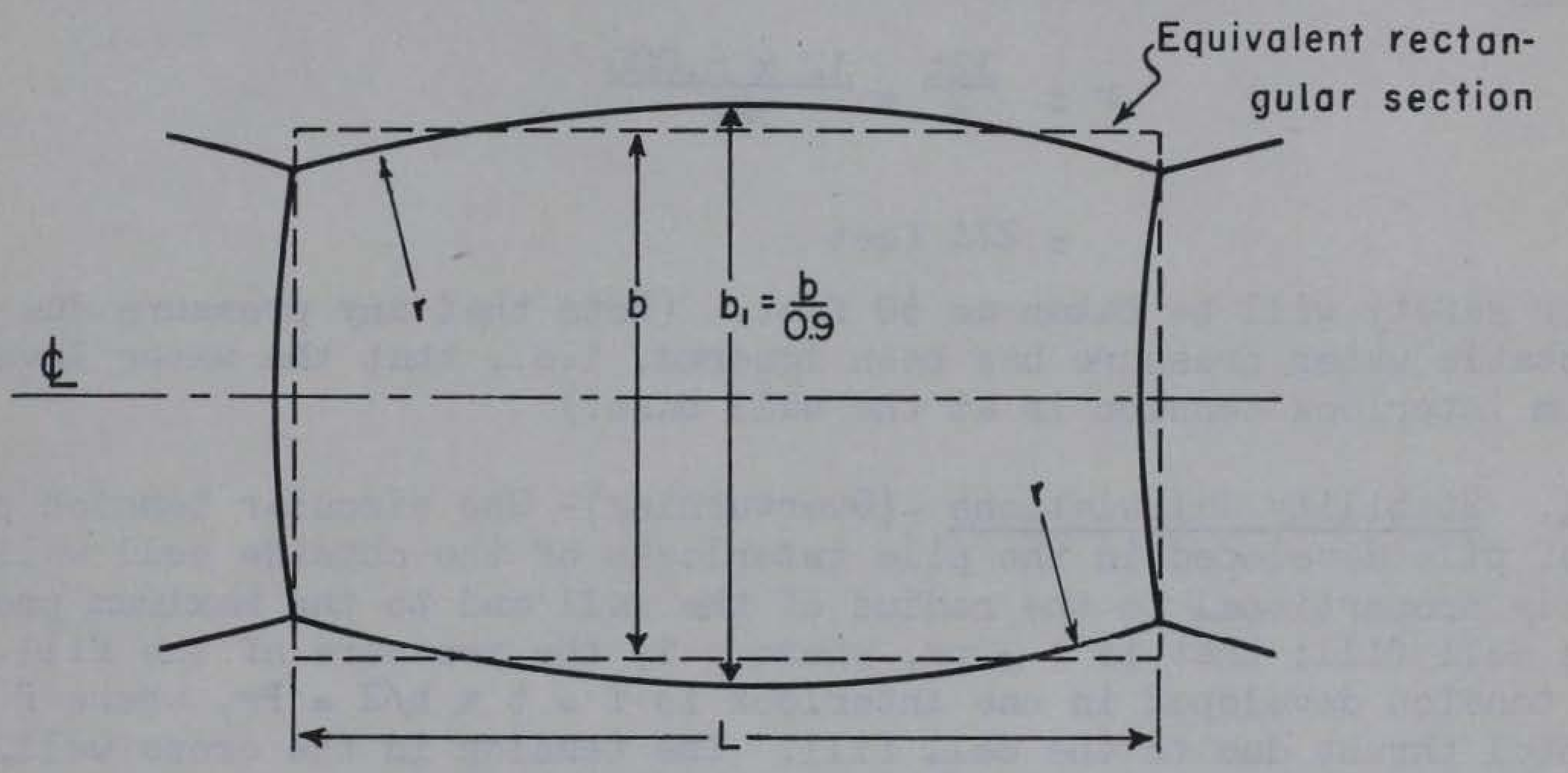
$$p = 144.8 \times 13\ (0.414)^2$$

$$= 322 \text{ pounds per square foot,}$$

and with an unsaturated cell fill

$$p = 120 \times 13\ (0.414)^2$$

$$= 267 \text{ pounds per square foot.}$$



DIAPHRAGM TYPE CELLULAR WALL
FIGURE 107

g. General Design - The type wall chosen is the so-called diaphragm type illustrated in Figure 107. The dashed lines indicate the dimensions of an equivalent rectangular cell used for stability calculations. The width (b) of the equivalent rectangle is the average width of the actual cell, the length (L) is the length of one cell section; and the distance (r) is the radius of the outside walls. Note that the crosswalls have a slight arc for stability against differential earth pressures due to partial filling of one cell before filling of the adjacent cell is started.

Failure criteria for such walls, if the piling has a significant amount of penetration, are difficult to determine. However, there will be an adequate factor of safety if they are considered to be open end boxes resting on the bottom. In the example chosen, the rock stratum is only 2 feet below the lake bottom and the open end box calculations will be fairly accurate.

h. Piling Calculations - Pile interlock tension resistance to cell rupture is directly proportional to the cell fill pressure and the radius of the cell wall. Expressing the fill pressure p in pounds per square foot and the radius in feet, the interlock tension in pounds per linear inch is given by

$$t = \frac{pr}{12} \tag{65}$$

Assuming an M112 section piling would be used, with an allowable interlock tension of 6,000 pounds per inch, the maximum allowable cell wall radius for this problem, with p being the pressure of the saturated cell fill would be

$$r = \frac{12t}{p} = \frac{12 \times 6{,}000}{322}$$

$$= 224 \text{ feet}$$

and for safety will be taken as 50 feet. (Note that any pressure due to hydrostatic water pressure has been ignored, i.e., that the water level for maximum interlock tension is at the wall base.)

i. Stability Calculations -(Overturning)- The circular tension per foot of pile developed in the pile interlocks of the outside cell wall is directly proportional to the radius of the wall and to the maximum pressure of the cell fill; that is $t = pr$, where p is the pressure of the fill. The total tension developed in one interlock is $T = t \times h/2 = Pr$, where P is the total thrust due to the cell fill. The tension in the cross wall, if two cells are joined by a standard Y-pile with 120° legs is the same as the tensions in the individual wall arcs, that is $T = Pr$ again.

For the cell to overturn, the cross wall piles must slip along the interlocks and the fill material must slip along the piling and shear through a vertical section. Therefore, if f is the coefficient of lock friction, and L is the length of one cell section, the resistance to overturning developed by the cross walls per unit length of structure is

$$S_p = \frac{Prf}{L} \tag{66}$$

In addition to this resistance, there is the shear resistance developed by the cell fill itself which is given by

$$S_f = P \tan \phi \tag{67}$$

that is, the lateral fill thrust times the coefficient of internal friction. Therefore, the total resistance to overturning per unit length of wall developed by the crosswall and cell fill is

$$S = S_p + S_f = P \left(\frac{rf}{L} + \tan \phi\right) \tag{68}$$

The exterior overturning moment (M) per unit length of wall about the center of the cell may be replaced by a couple about the neutral axis (also the center of the cell). If it is assumed that pressures along the wall bottom which may cause this couple are linearly distributed along the wall width, the "couple diagram" may be drawn as shown in Figure 108.

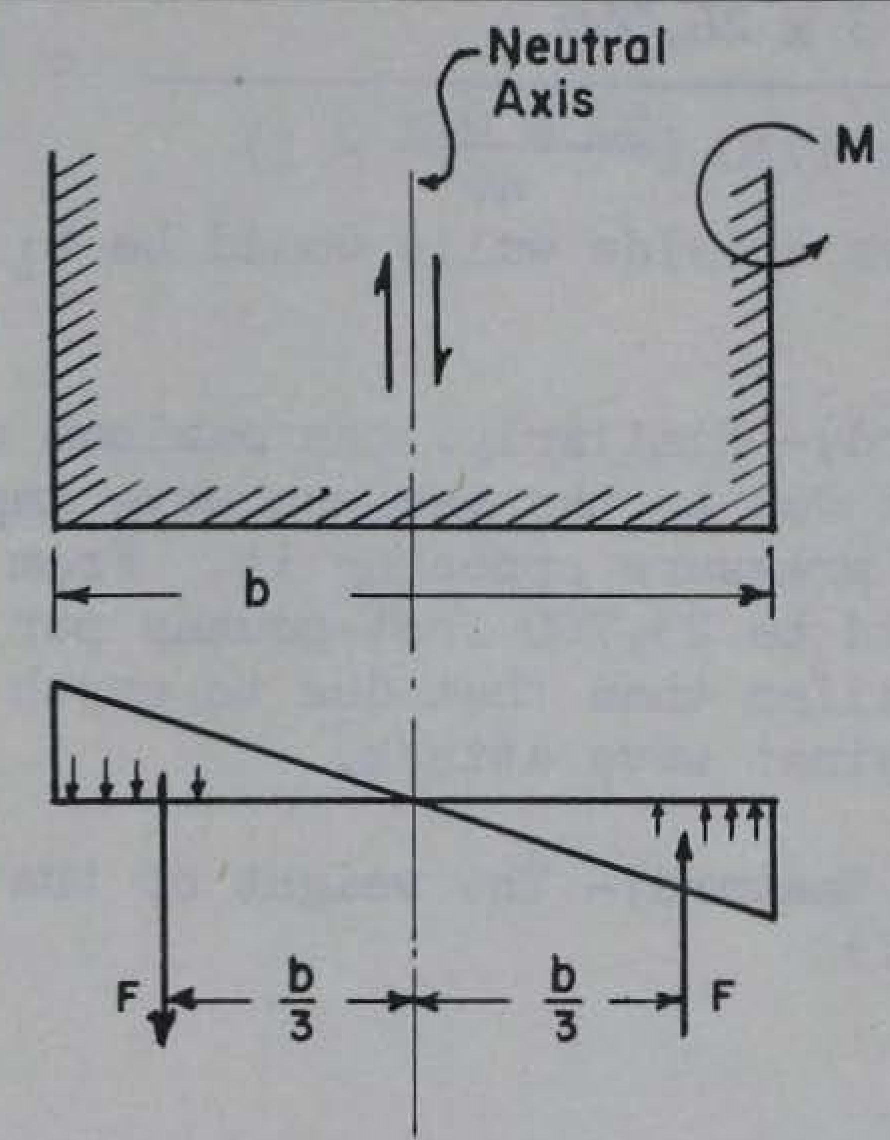


FIGURE 108-COUPLE REPLACING OVERTURNING MOMENT

The couple (F) due to the two pressure diagrams is applied at the centroids of the two triangles, and the moment due to this couple must equal the overturning moment about the neutral axis. Therefore, the overturning forces may be represented by

$$F = \frac{3\ M}{2\ b} \tag{69}$$

For stability, these forces should be least equal the resisting forces due to the crosswalls and fill, or F = S. Solving equations 68 and 69 for b, the minimum width of the equivalent rectangular cell section for stability is

$$b = \frac{3M}{2P\ (\frac{rf}{L} + \tan \phi)} \tag{70}$$

where M = the total overturning moment about the center line
P = the total force due to the cell fill
r = the radius of the outside cell wall
f = the coefficient of interlock friction
L = cell section length
$\tan \phi$ = the coefficient of internal friction of the cell fill.

(Overturning Seaward) - The maximum overturning moment due to the backfill will be caused by a saturated backfill with no water in front of the wall.

From the section on earth forces, this backfill force would be 5,340 pounds per lineal foot of wall applied at a distance 4.88 feet above the base, giving a moment of 26,060 foot-pounds per lineal foot of wall. Assuming an interlock friction coefficient of 0.3 , an unsaturated cell fill, and a cell length of 40 feet, the equation for minimum width would become

$$b = \frac{3 \times 26{,}060}{2 \times 1730 \left(\frac{50 \times 0.3}{40} + 1\right)} = 16.4 \text{ feet, say } 17 \text{ feet}$$

the maximum width between outside walls would be $b_1 = b/0.9 = 18.2$ feet, say 19 feet.

(Overturning Landward)- Similarly, the maximum overturning moment due to wave forces will be that due to the maximum wave impinging on the structure, with no active backfill pressure opposing it. From the section on wave forces, this moment would be 23,700 foot-pounds per lineal foot of wall. Since this moment is smaller than that due to earth pressures alone, the wall would be stable against wave attack.

(Stability; Sliding Seaward)- The weight of the filling material per foot of seawall length is

$$W = whb \tag{71}$$

where w = the unsaturated cell fill unit weight
h = the fill height
b = the average cell width

so

$$W = 120 \times 13 \times 17 = 26{,}520 \text{ pounds per foot.}$$

The known forces for seaward sliding are as shown in Figure 109.

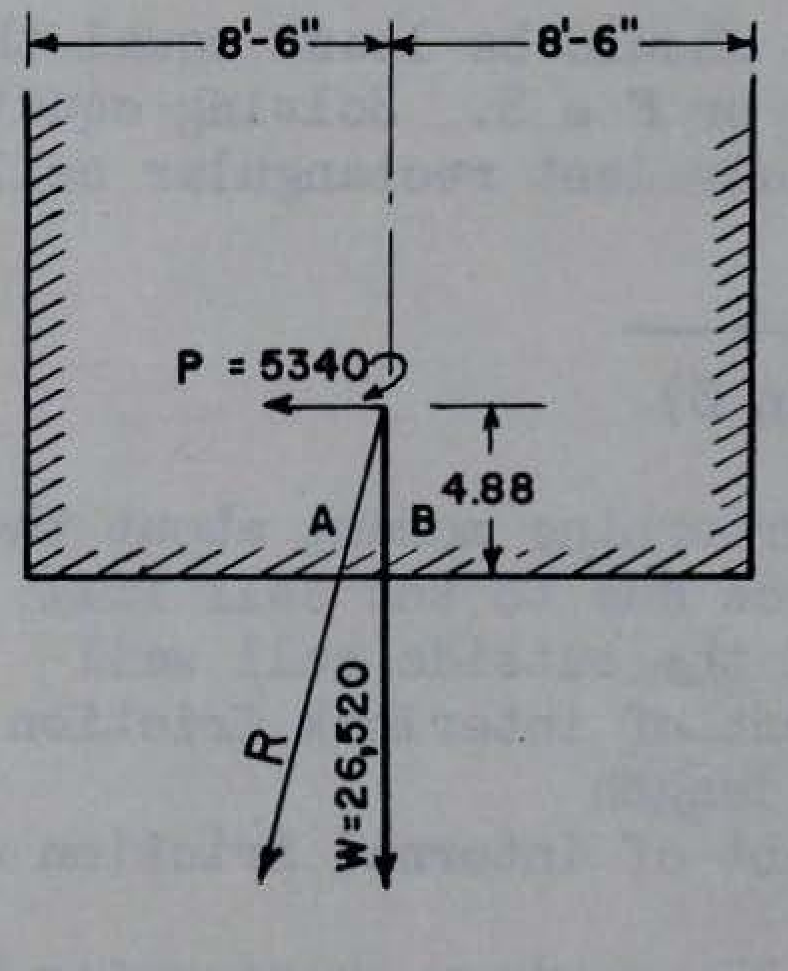


FIGURE 109-RESULTANT FORCE

For stability against sliding, the horizontal force divided by the total weight must be less than the coefficient of friction multiplied by the factor of safety. Because of the penetration of the piling, a factor of safety of 1 is satisfactory, or

$$\frac{P}{W} < 0.5$$

or $\frac{5340}{26,520} = 0.201$ which is less than that allowable, and the wall is stable against sliding seaward.

(Stability, Sliding Landward) - The numerator of the above equation is smaller when a wave force of 5,020 foot-pounds per foot is substituted for 5,340. Since the denominator remains the same, the wall is also stable against wave forces. A cell section designed for stability is shown on Figure 110.

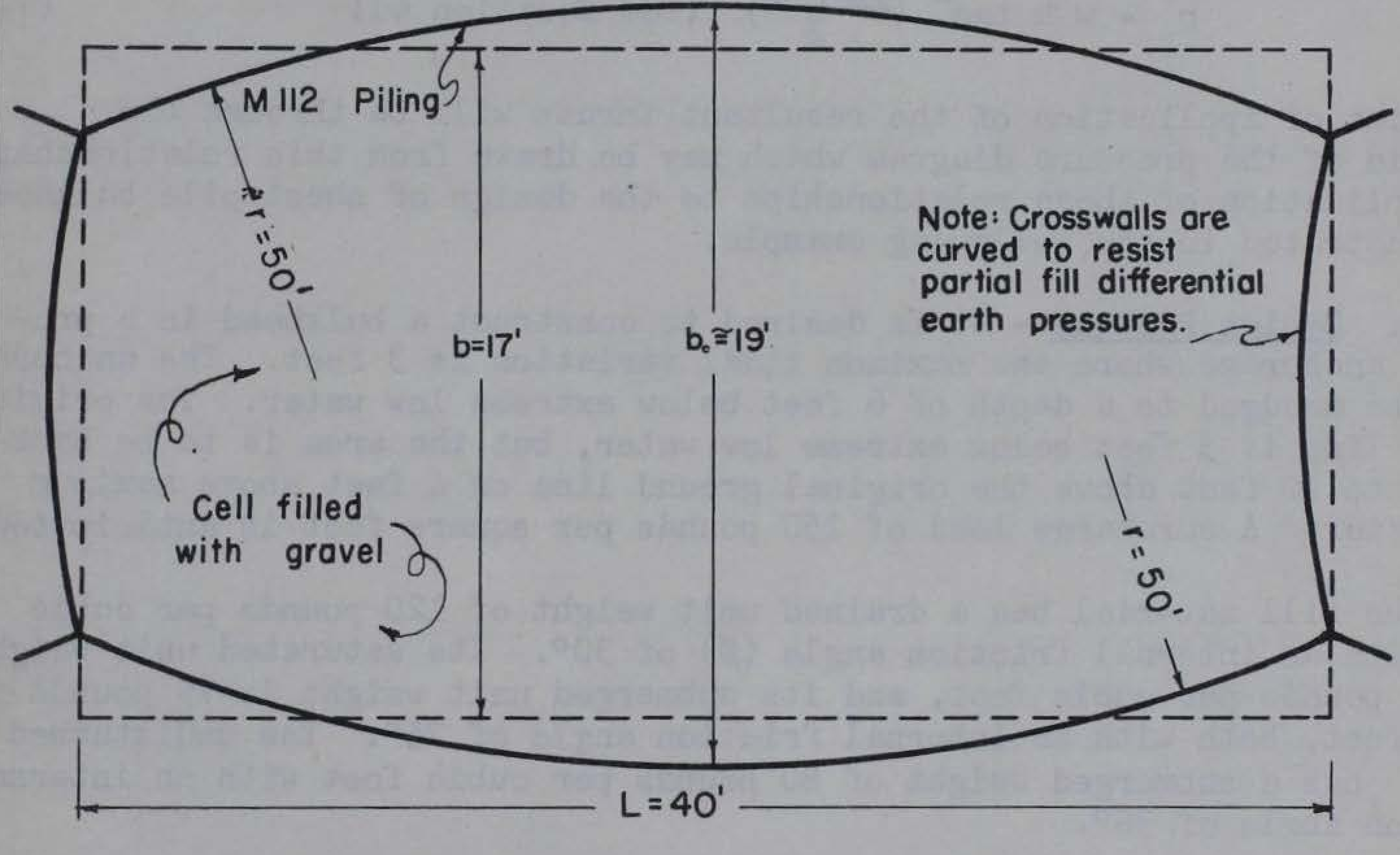


FIGURE 110 - DIAPHRAM TYPE CELL

5.133 Steel Sheet Pile Bulkhead(17)

a. Design Assumptions - Sheet pile bulkheads may be constructed where adequate penetration is obtainable. For design computations, the following assumptions for earth forces calculations may be made:

(1) A surcharge load on a fill may be considered to be replaced

by an equivalent height of fill which will produce the same pressure as the surcharge at its point of application;

(2) Active pressures may be computed by use of relationship:

$$p_a = w\ h\ \tan^2 \left(\frac{90 - \emptyset}{2}\right) \quad \text{(See Equation 58)} \qquad (64)$$

where
w = the unit weight of the material
h = the height (including the equivalent height due to surcharge if any) of the fill
$\emptyset$ = the internal friction angle of the fill material.

The point of application of the resultant thrust will be through the centroid of the pressure diagram drawn from this relationship;

(3) The maximum passive pressure may be computed by use of the relationship:

$$p_p = w\ h\ \tan^2 \left(\frac{90 + \emptyset}{2}\right) \quad \text{(see Equation 61)} \qquad (72)$$

The point of application of the resultant thrust will be through the centroid of the pressure diagram which may be drawn from this relationship. The application of these relationships to the design of sheet pile bulkheads is illustrated in the following example.

b. Design Problem - It is desired to construct a bulkhead in a protected anchorage where the maximum tidal variation is 3 feet. The anchorage is to be dredged to a depth of 6 feet below extreme low water. The original ground line is 3 feet below extreme low water, but the area is to be backfilled to 10 feet above the original ground line or 4 feet above maximum high water. A surcharge load of 150 pounds per square foot is anticipated.

The fill material has a drained unit weight of 120 pounds per cubic foot with an internal friction angle ($\emptyset$) of 30°. Its saturated unit weight is 140 pounds per cubic foot, and its submerged unit weight is 75 pounds per cubic foot, both with an internal friction angle of 25°. The undisturbed ground has a submerged weight of 80 pounds per cubic foot with an internal friction angle of 30°.

Figure 111 is a diagram of the desired placement for the proposed sheet pile bulkhead. For design purposes, it will be assumed that the backfill is saturated to the high water line. The heights h_1, h_2, and h_3, refer respectively to the backfill height above the high water line and the original ground line. The height h_4 refers to the height of the original ground above the dredged bottom. The corresponding unit weights and internal friction angles are w_1 and $\emptyset_1$, w_2 and $\emptyset_2$, w_3 and $\emptyset_3$, and w_4 and $\emptyset_4$.

c. Loading Diagram - A loading diagram may be constructed by drawing horizontal lines at various points of lengths proportional to the pressure intensities (as given by equations 64 and 72) at the points.

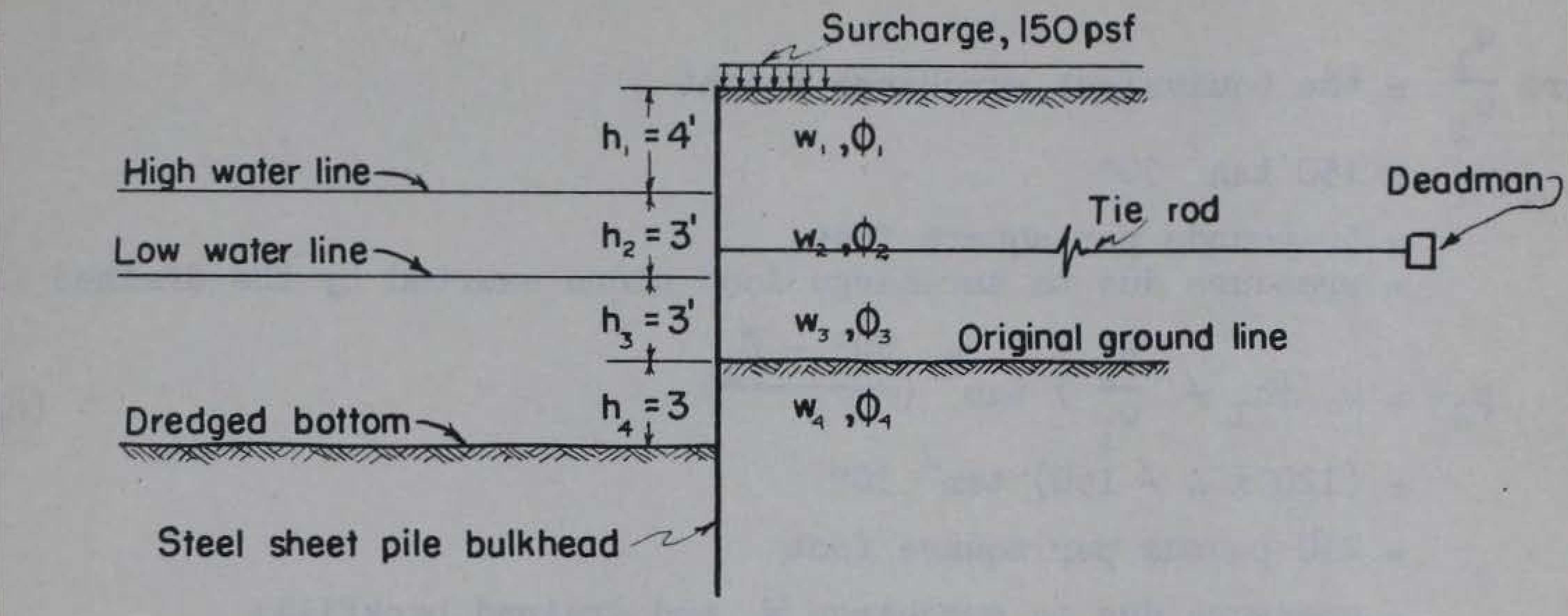


FIGURE 111 — BULKHEAD PLACEMENT

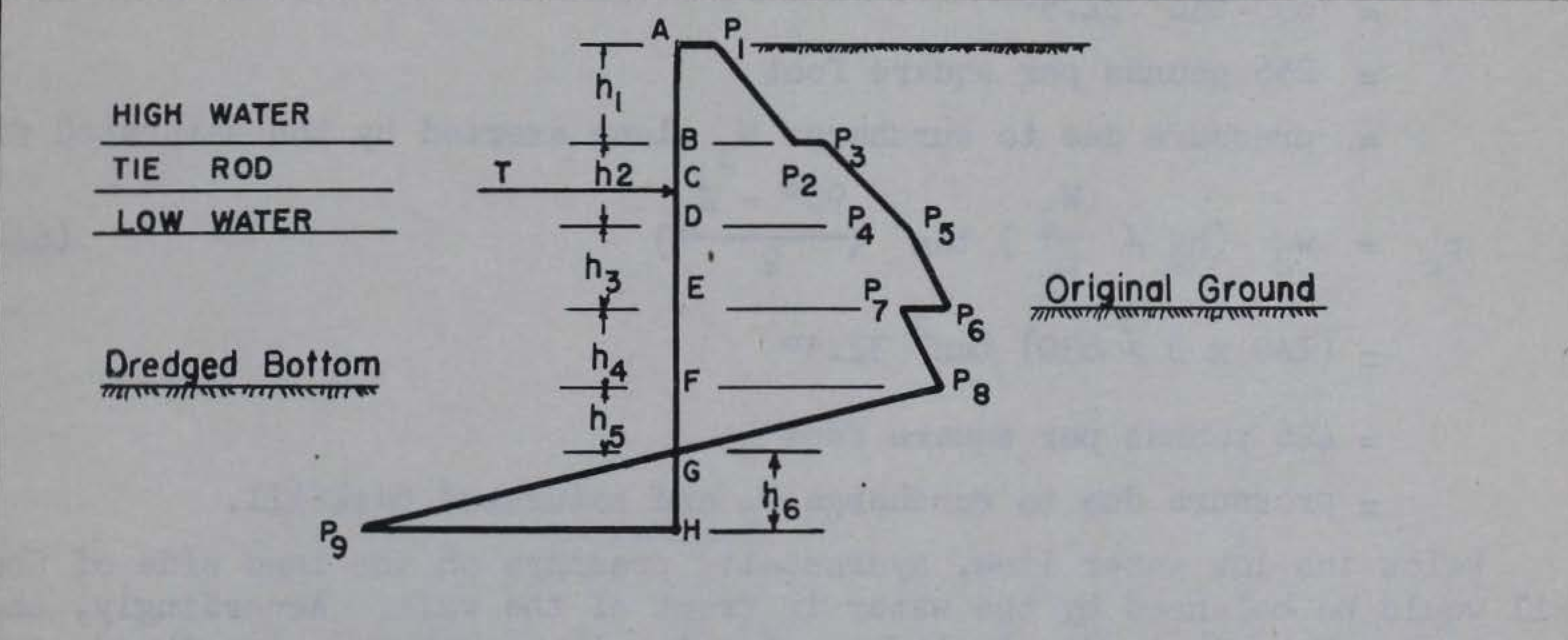


FIGURE 112 - LOADING DIAGRAM FOR MINIMUM PILE LENGTH

Thus in Figure 112

$$p_1 = w_1 \times \frac{W_1}{w_1} \tan^2 \left(\frac{90° - \phi_1}{2}\right) \qquad (64\text{-}p_1)$$

where $\frac{W_1}{w_1}$ = the equivalent surcharge height

$$\therefore p_1 = 150 \tan^2 30°$$

= 50 pounds per square foot

= pressure due to surcharge load alone exerted by the drained fill

$$p_2 = w_1 \left(h_1 + \frac{W_1}{w_1}\right) \tan^2 \left(\frac{90° - \phi_1}{2}\right) \qquad (64\text{-}p_2)$$

$$= (120 \times 4 + 150) \tan^2 30°$$

= 210 pounds per square foot

= pressure due to surcharge W_1 and drained backfill.

Considering the surcharge plus drained backfill to be a new surcharge applied to the saturated fill with a pressure of $W_2 = w_1 \left(h_1 + \frac{W_1}{w_1}\right) = 630$ pounds per square foot, then

$$p_3 = w_2 \left(\frac{W_2}{w_2}\right) \tan^2 \left(\frac{90° - \phi_2}{2}\right) \qquad (64\text{-}p_3)$$

$$= 630 \tan^2 32.5°$$

= 256 pounds per square foot

= pressure due to surcharge W_2 alone exerted by the saturated fill

$$p_4 = w_2 \left(h_2 + \frac{W_2}{w_2}\right) \tan^2 \left(\frac{90° - \phi_2}{2}\right) \qquad (64\text{-}p_4)$$

$$= (140 \times 3 + 630) \tan^2 32.5°$$

= 426 pounds per square foot

= pressure due to surcharge W_2 and saturated backfill.

Below the low water line, hydrostatic pressure on the land side of the wall would be balanced by the water in front of the wall. Accordingly, these pressures may be ignored. Again, considering the surcharge plus the drained backfill plus the saturated fill to be a new surcharge,

$$W_3 = w_2 \left(h_2 + \frac{W_2}{w_2}\right) = 1050 \text{ pounds per square foot}$$

then

$$p_5 = w_3 \frac{W_3}{w_3} \tan^2 \left(\frac{90° - \phi_3}{2}\right) \qquad (64\text{-}p_5)$$

$$p_5 = 1050 \tan^2 32.5°$$

= 426 pounds per square foot

= pressure due to surcharge W_3 exerted by submerged fill

and

$$p_6 = w_3 \left(h_3 \; \frac{W_3}{w_3}\right) \tan^2 \left(\frac{90° - \phi_3}{2}\right) \qquad (64\text{-}p_6)$$

$$= (75 \times 3 + 1050) \tan^2 32.5°$$

= 518 pounds per square foot

= pressure due to surcharge W_3 and submerged fill.

Using

$$W_4 = w_3 \left(h_3 + \frac{W_3}{w_3}\right) = 1{,}275 \text{ pounds per square foot}$$

$$p_7 = w_4 \times \frac{W_4}{w_4} \tan^2 \left(\frac{90° - \phi_4}{2}\right) \qquad (64\text{-}p_7)$$

$$= 1275 \tan^2 30°$$

= 425 pounds per square foot

= pressure due to surcharge W_4 exerted by original ground,

and

$$p_8 = w_4 \left(h_4 + \frac{W_4}{w_4}\right) \tan^2 \left(\frac{90 - \phi_4}{2}\right) \qquad (64\text{-}p_8)$$

$$= (80 \times 3 + 1275) \tan^2 30°$$

= 505 pounds per square foot

= pressure due to surcharge W_4 and original ground to the dredged bottom at 6 feet below the low water line.

At the point of application of p_8, both active pressure on the pile back and passive pressure on the pile face would be applied. The active pressure on the pile back would increase in the same manner as it increased from p_7 to p_8, because there would be no change of material at the level of p_8. From the expressions for p_7 and p_8, this rate of increase of active pressure with increase in depth would be $R_a = w_4 \tan^2 \left(\frac{90 - \phi_4}{2}\right)$ per unit increase in h_4. Similarly, the maximum passive pressure on the pile face would increase with depth at the rate $R_p = w_4 \tan^2 \left(\frac{90 + \phi_4}{2}\right)$ per unit increase in h_4, since at any point h below the level of the dredged bottom this passive pressure $p_8 = w_4 \, h \tan^2 (\frac{90 + \phi_4}{2})$. The rate of increase of passive pressure applied on the wall face would be greater than the increase of active pressure applied at the wall back, and the net rate of change of pressure is given by

$$R = R_p - R_a$$

$$= w_4 \left[\tan^2 \left(\frac{90 + \phi_4}{2}\right) - \tan^2 \left(\frac{90 - \phi_4}{2}\right)\right] \qquad (73)$$

$$= 80\ (3.00 - 0.33)$$

= 214 pounds per square foot per foot of depth

= rate of <u>decrease</u> of outward pressures

The point on the loading diagram at which the earth pressures would be zero is labelled G and its distance h_5 below the dredged bottom may be found by dividing p_8 by R.

$$h_5 = p_8/R = \frac{505}{214} \qquad (74)$$

= 2.36 feet

= depth of zero loading point below the dredged bottom.

The magnitude of the concentrated force (T) on the wall due to the deadman is dependent on the depth of penetration of the pile.

<u>d</u>. <u>Pile Length</u> - A pile which is just long enough to support the backfill will have a larger cross section than one, somewhat longer, which is designed for minimum bending moment. The choice between the two is a matter of economics.

<u>e</u>. <u>Pile Design</u> -(Minimum Pile Length) - For optimum pile design, the point of application of the deadman thrust should be such that the bending moment at this point equals the maximum bending moment in the pile below it. To attain this optimum design, the maximum moment below an assumed point of application must be found and compared with that at the assumed point. If these moments are not equal, the process must be repeated. For the purpose of this illustrative problem, only the methods of calculating the bending moments will be carried out.

Assuming the point of application (C) of the deadman tension (T) to be 1 foot above the low water line, the moment M_C of all the forces about this point between the surface and point G is given by

$$M_C = \left[p_8 h_5 \left(1 + h_3 + h_4 + \frac{h_5}{3}\right)\right] + \left[p_7 h_4 \left(1 + h_3 + \frac{h_4}{2}\right) + \frac{p_8\,p_7}{2} h_4 \left(1 + h_3 + \frac{2}{3} h_4\right)\right]$$

$$+ \left[p_5 h_3 \left(1 + \frac{h_3}{2}\right) + \left(\frac{p_6 - p_5}{2}\right) h_3 \left(1 + \frac{2h_3}{3}\right)\right] + \left[p_c \times 1 \times \frac{1}{2} + \frac{p_4 - p_c}{2} \times 1 \times \frac{2}{3}\right] \quad (75)$$

$$- \left[p_3 \times 2 \times \frac{1}{2} + \frac{p_c - p_3}{2} \times 2 \times \frac{2}{3}\right] - \left[(p_1 h_1)\left(2 + \frac{h_1}{2}\right) + \left(\frac{p_2 - p_1}{2}\right) h_1 \left(2 + \frac{h_1}{3}\right)\right]$$

where $p_C = p_4 - \left(\frac{p_4 - p_3}{h_2}\right) = 369$ pounds per square foot

$$M_C = \left[4620 + 7720 + 3600 + 209\right] - \left[331 + 1865\right]$$

$= 13,953$ foot-pounds per foot of wall

For stability, the sum of all moments about C must be zero. Then if H is the point of deepest pile penetration,

$$M_C = \left(\overline{CG} + \frac{2h_6}{3}\right) \left(h_6 \times \frac{h_6 \times R}{2}\right)$$

where $h_6 \times R = p_9$

Therefore $(h_6)^2 \left(\overline{CG} + \frac{2}{3} h_6\right) = 2 \frac{M_C}{R}$ (76)

or in this case

$$(h_6)^2 \left(9.36 + \frac{2}{3} h_6\right) = \frac{13,953 \times 2}{214} \cong 131$$

from which $h_6 \cong 3.4$ feet

Since point G is itself 2.36 feet below the dredged bottom, the pile must penetrate a distance $3.4 + 2.4 = 5.8$ feet below the dredged bottom, at which point the pressure p_9 would be

$$p_9 = R \times h_6 = 214 \times 3.4 = 728 \text{ pounds per square foot.} \quad (77)$$

The minimum length of pile for stability with the deadman tie-rod one foot above the low water line would be 18.8 feet.

(Maximum Bending Moment for Minimum Length Pile)- The maximum bending moment in the wall would occur at that point above the dredged bottom where the shear passes through zero. The shear S_F at the dredged bottom F is given by the sum of the forces below that point, which sum is the algebraic sum of the areas of the pressure diagrams below that point. Thus this shear

$$S_F = \frac{p_9 h_6}{2} - \frac{p_8 h_5}{2} \quad (78)$$

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$$= \frac{728 \times 3.4}{2} - \frac{505 \times 2.36}{2}$$

$= 1235 - 595 = 640$ pounds per foot of wall.

Above this point, the shear would decrease as pressure diagram area is added. The quantity $\frac{p_8 - p_7}{h_4} = 26.7$ pounds per square foot per foot of wall would be the rate of change of pressure over h_4, and calling this rate R_4 the equation for finding the point of zero shear when this point is within h_4 is

$$S_F = (p_8 - R_4 Z_1)Z_1 + \frac{R_4 Z_1^2}{2}$$

or

$$R_4 Z_1^2 - 2p_8 Z_1 + 2S_F = 0$$

where Z_1, is the distance of this zero shear point above the dredge bottom F. Solving for Z_1,

$$Z_1 = \frac{p_8 - \sqrt{p_8^2 - 2 R_4 S_F}}{R_4} \tag{79}$$

$$= 1.31 \text{ feet}$$

The pressure at Z would be $p_Z = p_8 - R_4 Z = 470$ pounds per square foot.

The bending moment M_F at the dredged bottom is the sum of the moments of the individual areas p_9 HG and p_8 FG

$$M_F = \frac{Rh_6^2}{2}\left(h_5 + \frac{2}{3} h_6\right) - \frac{h_5^2 p_8}{6} = \frac{Rh_6^2}{6}(2h_6 + 3h_5) - \frac{h_5^2 p_8}{6} \tag{80}$$

(Note: R is the rate of change of pressure between p_8 and p_9)

$$M_F = \frac{214 \times (3.4)^2}{2}(2.36 + 2.26) - \frac{(236)^2(505)}{6}$$

$M_F = 5230$ foot-pounds per foot of wall.

Figure 113 shows the loading, shear, and moment diagrams between F and Z_1. At any point x above F, the shear S is given by S_F less the area of the loading diagrams between F and x or

$$S = S_F - \left(p_8 x - \frac{R_4 x^2}{2}\right) \tag{81}$$

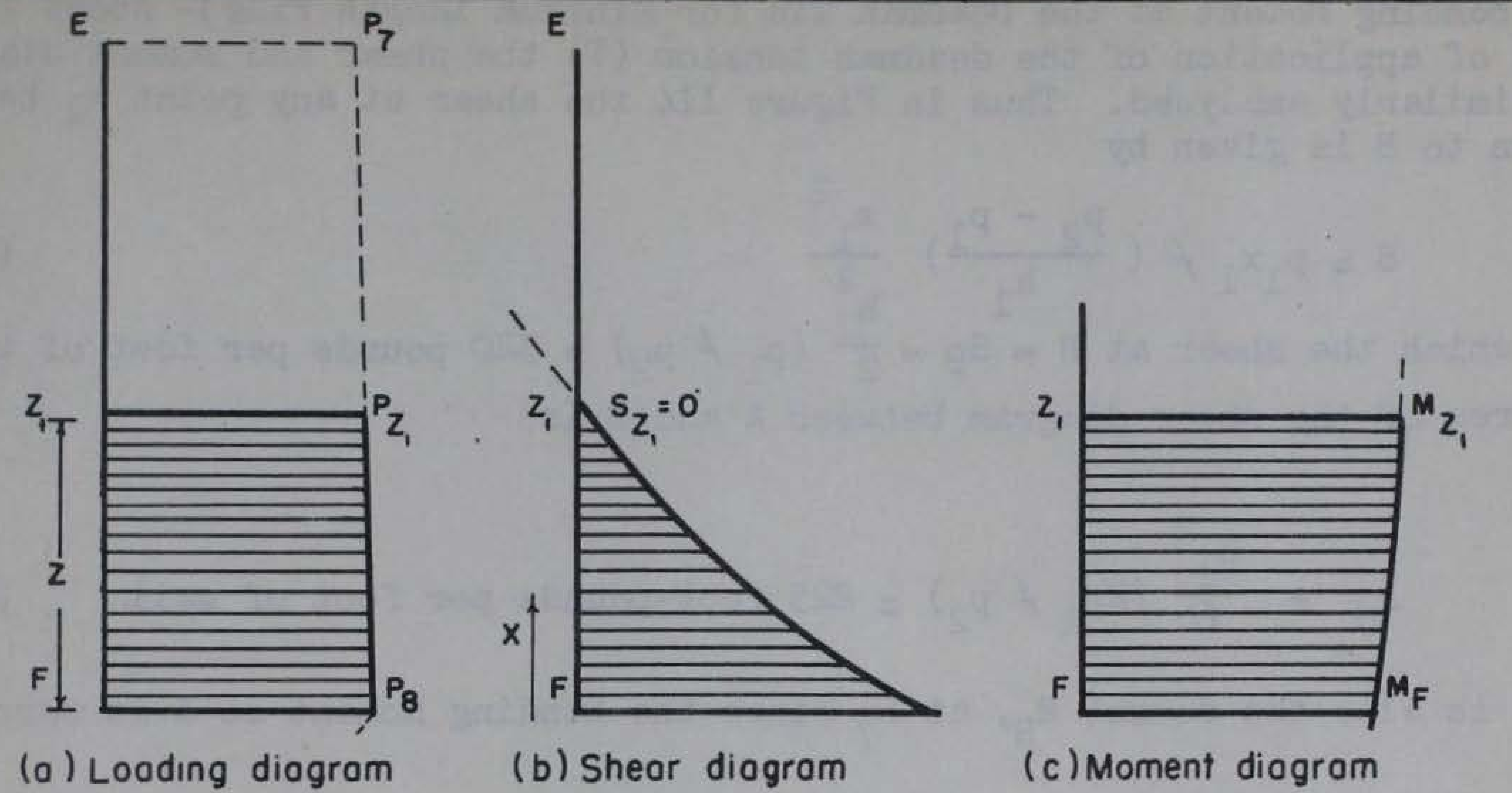

FIGURE 113 LOADING, SHEAR, AND BENDING MOMENT DIAGRAMS BETWEEN POINTS F AND Z_1

Integrating the area under this curve between the limits of 0 and Z_1, the shear diagram area A_s is found to be

$$A_s = \frac{Z_1}{2} \left(S_F - \frac{p_8 Z_1^2}{2} + \frac{R_4 Z_1^2}{6}\right)$$

or since

$$S_F = p_8 Z_1 - \frac{R_4 Z_1^2}{2}, \text{ and } p_{Z_1} = p_8 - R_4 Z_1$$

$$A_s = \frac{Z_1^2}{6} \left(p_8 + 2p_{Z_1}\right) \tag{82}$$

Since the area of the shear diagram between points F and Z_1 is the total increase of moment between these points, the bending moment at Z is

$$M_Z = M_F + \frac{Z_1^2}{6} \left(p_8 + 2p_{Z_1}\right) \tag{83}$$

$$= 5230 + \left(\frac{1.31^2}{6}\right) (505 + 940)$$

$$= 5230 + 415 = 5{,}645 \text{ foot-pounds per foot of wall}$$

= the maximum bending moment.

(Bending Moment at the Deadman Tie for Minimum Length Pile)- Above the point of application of the deadman tension (T) the shear and moment diagrams are similarly analyzed. Thus in Figure 114 the shear at any point x_1 below A down to B is given by

$$S = p_1 x_1 + \left(\frac{p_2 - p_1}{h_1}\right) \frac{x_1^2}{2} \tag{84}$$

from which the shear at B = $S_B = \frac{h_1}{2}(p_1 + p_2)$ = 520 pounds per foot of wall. The area of the shear diagram between A and B is

$$A_{S_1} = \frac{h_1^2}{6}(2p_1 + p_2) = 825 \text{ foot-pounds per foot of wall} \tag{85}$$

which is also the moment M_B, at B_7 since the bending moment at A is zero.

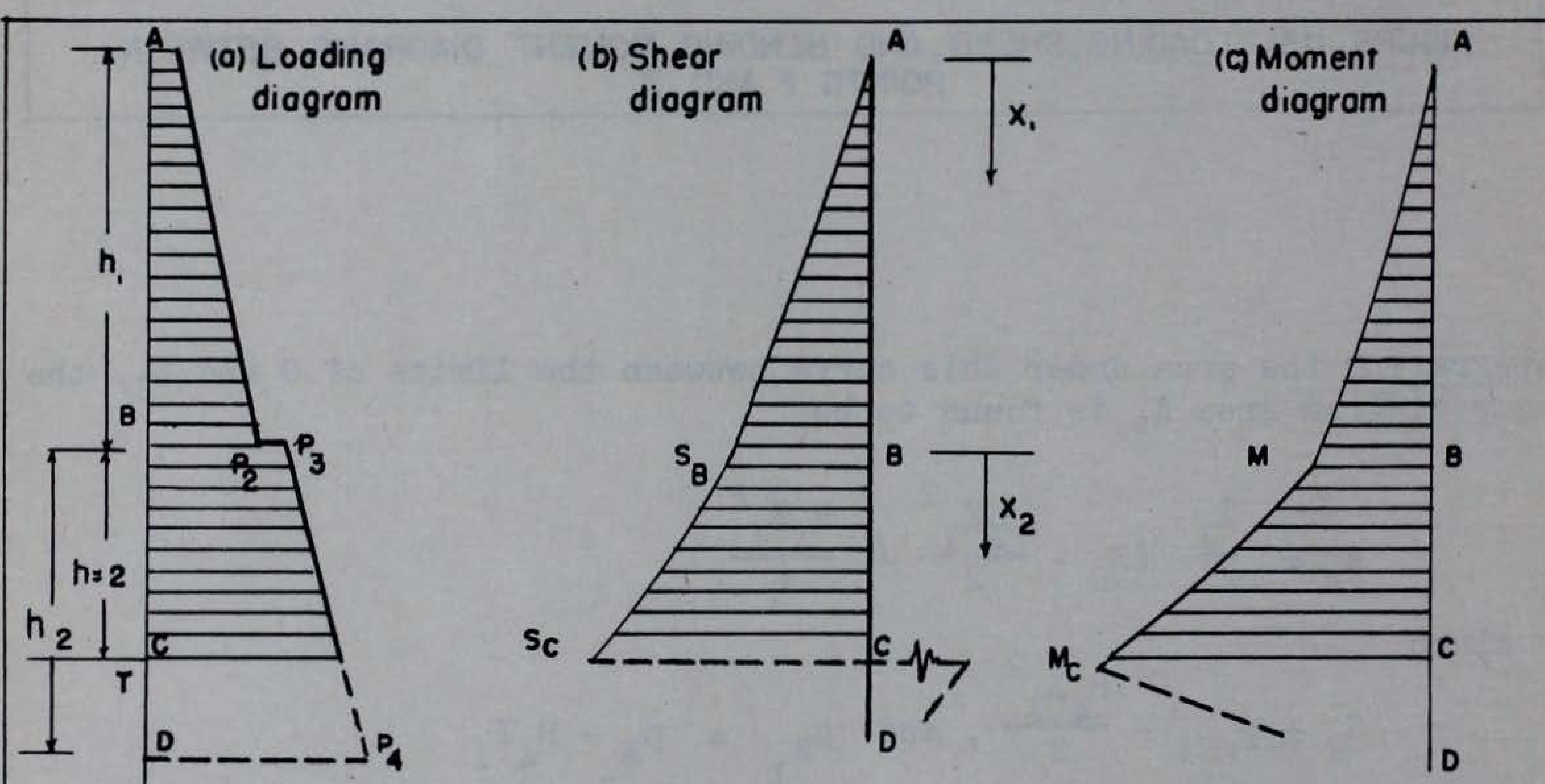

FIG. 114 LOADING, SHEAR, & BENDING MOMENT DIAGRAMS ABOVE DEADMAN TIE

Similarly the shear at any point x_2 below B down to C is given by

$$S = S_B + p_3 x_2 + \left(\frac{p_c - p_3}{h_2}\right) \frac{x_2^2}{2} \tag{86}$$

The area of the diagram between B and C is

$$A_{S_2} = S_B h + \frac{h^2}{6}(2p_3 + p_c) = 1{,}627 \text{ foot-pounds per foot of wall} \tag{87}$$

(Note: $p_c = p_4 - \frac{p_4 - p_3}{h_2}$ = 369 pounds per square foot)

The bending moment at C is

$$M_C = M_B + A_{S_2} \tag{88}$$

$$= 825 + 1627 = 2{,}452 \text{ foot-pounds per foot of wall.}$$

For this trial, it can be seen that the maximum positive bending moment in the pile is greater than that at the deadman tie. Lowering the point of application of the tie tension would bring these two values closer, but would put the tie under water where increase in construction costs would more than offset the saving in material costs.

The tension T in the tie rod is found by summing algebraically the forces due to the various earth pressure polygons.

$$T = \left[\left(\frac{p_1 + p_2}{2}\right) h_1 + \left(\frac{p_3 + p_4}{2}\right) h_2 + \left(\frac{p_5 + p_6}{2}\right) h_3 + \left(\frac{p_7 + p_8}{2}\right) h_4 + \frac{p_8}{2} \times h_5\right] - \left[\frac{p_9}{2} \times h_6\right] \tag{89}$$

$$= (520 + 1022 + 1418 + 1390 + 595) - 1240$$

$$= 4945 - 1240 = 3{,}705 \text{ pounds per foot of wall}$$

$$= F_a - F_p$$

F_a is the seaward (active) earth force, and F_p is the landward (passive) force. Note that the point of application of the force F_a is a distance ℓ below the tie rod given by

$$\ell = \frac{M_C}{F_a} = \frac{13{,}953}{4{,}945} = 2.82 \text{ feet} \tag{90}$$

where M_C is given by equation 75.

<u>f</u>. <u>Pile Design Minimum Bending Moment</u> - By driving the pile somewhat deeper, a cantilever would be formed near the pile bottom, which would develop passive pressures in the backfill. The loading diagram would be changed as shown in Figure 115.

For optimum pile design, that is for the least length of pile for which the maximum bending moment would be smallest, pressures p_9 and p_{10} must be as large as can be developed by the ground at these points. This may be proven for p_9 by finding the expression for the maximum negative bending moment. This maximum moment would occur below G, the point of zero pile load. Calling the moment at G, M_G, and the shear at the same point, S_G, the maximum moment would occur at the point of zero shear (say Z_2), and would equal M_G plus the area of the shear diagram between G and Z_2. Referring to Figure 116 the loading, shear, and bending moment diagrams for this section of pile, the shear at any point Z below G would be $S_z = S_G - RZ^2/2$ where R is the rate of change at pressure between p_8 and p_9. At $Z = Z_2$, this shear would be zero, therefore

$Z_2 = \sqrt{\frac{2S_G}{R}}$. The moment at Z_2 is

$$M_{Z_2} = M_G \neq A_s \tag{91}$$

$$= M_G \neq \frac{2}{3} \; S_G\sqrt{\frac{2S_G}{R}}$$

= the maximum negative moment.

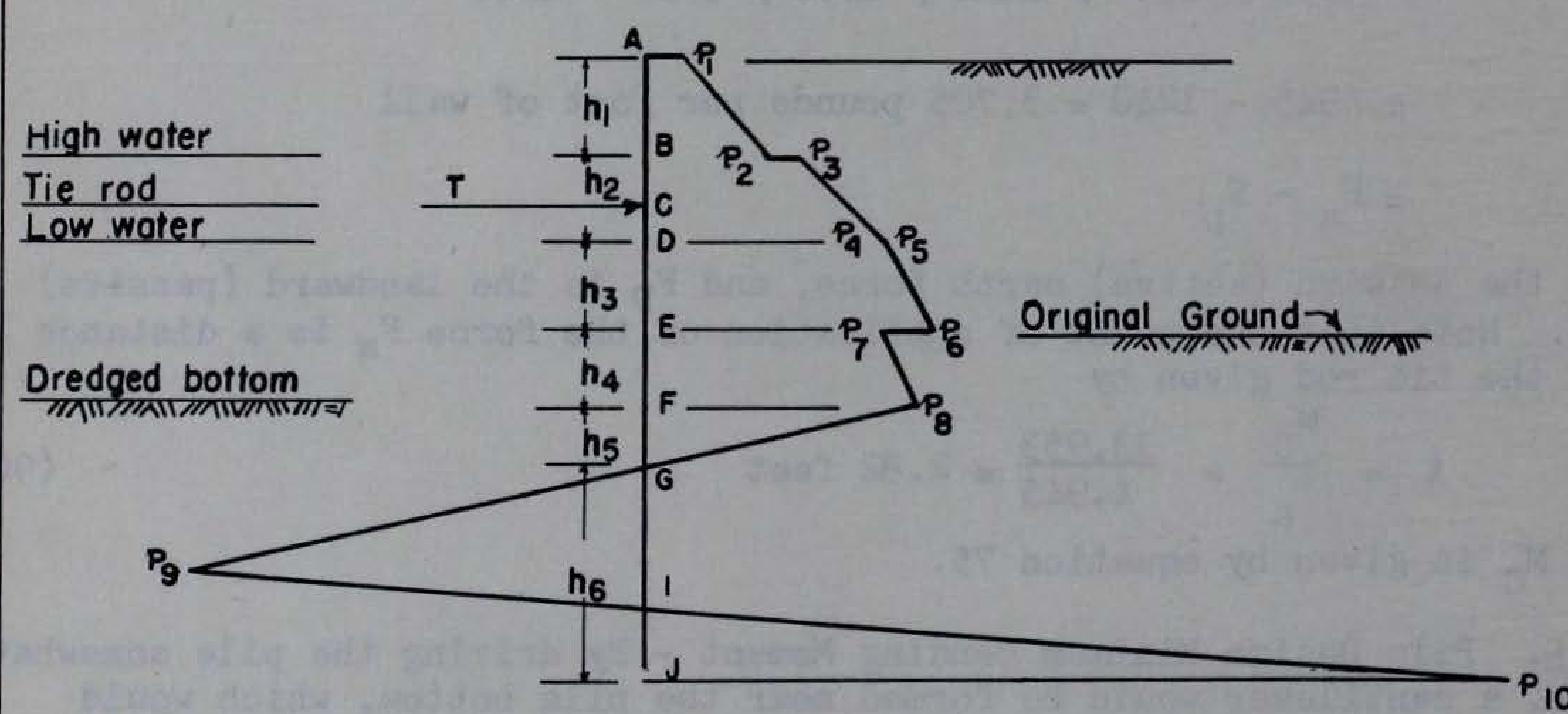

FIGURE 115 LOADING DIAGRAM FOR MINIMUM BENDING MOMENT

Now M_G and S_G are functions of the pressure area p_8 G F; as this area increases, both M_G and S_G will increase. These would be smallest when the rate of pressure change R is greatest (some lesser rate would appear as the dashed line in Figure 116 and would cause point G to be lowered). But the greatest possible rate of pressure change between p_8 and p_9 would make p_9 as large as the ground would permit at that point, which was to be shown.

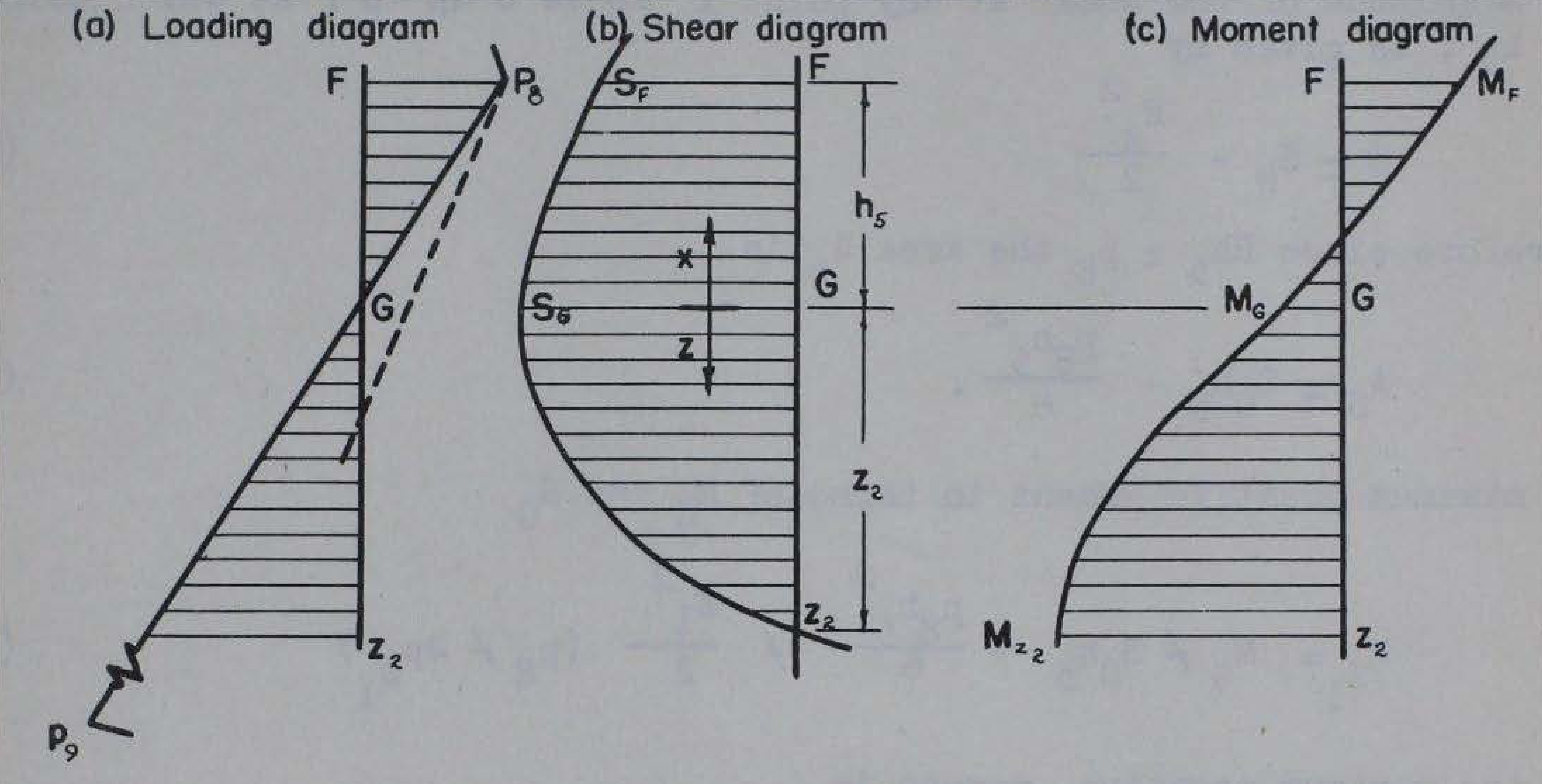

FIGURE 116 LOADING, SHEAR, & MOMENT DIAGRAMS BELOW POINT F

Referring again to Figure 115, one of the earth forces which cause the pressure p_9 to be developed is that due to the pressure triangle p_{10} I J. Whatever this force is, the least length of pile I J over which it is applied would be that length corresponding to the highest possible point of application of the pressure p_{10}. But the magnitude of p_{10} at this highest point can be no greater than the pressure which may be generated by the earth. Therefore for minimum pile length, p_{10} must be as large as this maximum permissible earth pressure.

The maximum rate of pressure change between p_8 and p_9 has already been determined as

$$R = w_4 \left[\tan^2 \left(\frac{90 + \phi_4}{2} \right) - \tan^2 \left(\frac{90 - \phi_4}{2} \right) \right] \qquad (73)$$

$$= 214 \text{ pounds per square foot per foot of depth}$$

and the distance between F and G with this rate was found to be $h_5 = 2.36$ (Equation 74). The maximum positive moment in terms of the moment at F, the dredged bottom, has been shown to be

$$M_{Z_1} = M_F + \frac{Z_1^{\,2}}{6} (p_8 + 2p_{Z_1}) \qquad (83)$$

It may be seen from Figure 115 that in going from F to G, the moments increase negatively. M_G is related to M_F by $M_G = M_F - |A_s|$ where A_s is the area of the shear diagram between F and G.

Therefore $M_{Z_1} = M_G + A_s + \frac{Z_1^2}{2} (p_8 + 2p_{Z_1})$

The magnitude of the shear at any point x above G up to F at which point $x = h_5$, is given by

$$S = S_G - \frac{Rx^2}{2} \tag{92}$$

therefore since $Rh_5 = p_8$ the area A_s is

$$A_S = S_G h_5 - \frac{p_8 h_5^2}{6}. \tag{93}$$

The maximum positive moment in terms of M_G and S_G

$$M_{Z_1} = M_G + S_G h_5 - \frac{p_8 h_5^2}{6} + \frac{Z_1^2}{2} (p_8 + 2p_{Z_1}) \tag{94}$$

and the maximum negative moment is

$$M_{Z_2} = M_G + \frac{2}{3} S_G \sqrt{2S_G/R} \tag{91}$$

In the above equations, S_G is given by the sum of all forces above G or

$$S_G = F_a - T \tag{95}$$

$$= 4945 - T \text{ (see equation 89)}$$

and the moment M_G is

$$M_G = \frac{M_C}{\ell} (\overline{CG} - \ell) - \overline{CG} \times T \tag{90}$$

$$= \frac{13,953}{2.82} (9.36 - 2.82) - 9.36 \text{ T}$$

$$= 32,300 - 9.36 \text{ T (see equations 75 and 90)}$$

In addition, $h_5 = 2.36$ feet (equation 74)

$p_8 = 505$ pounds per square foot

$R = 214$ pounds per square foot per foot of depth (equation 73)

$R_4 = 26.7$ pounds per square foot per foot of depth

and $Z_1 = \dfrac{p_8 - \sqrt{p_8^2 - 2R_4S_F}}{R_4}$ (equation 79)

where $S_F = S_G - \dfrac{Rh_5^2}{2} = (4945 - T) - 595 = 4350 - T$

therefore $Z_1 = \dfrac{505 - \sqrt{(505)^2 - 53.4\ (4350 - T)}}{26.7}$

$\approx 18.9 - \sqrt{32.3 + 0.075\ T}.$

For optimum design the maximum bending moments should be equal. The exact solution of the equation $M_{Z_1} = M_{Z_2}$ is difficult, and it is best solved by trial and error, assuming for the first trial $M_G = 0$. Then

$T = \dfrac{32,300}{9.36} = 3,450$ pounds per foot of wall

$S_G = 4945 - T, = 1495$ pounds per foot of wall

$Z_1 = 1.8$ feet.

Substituting these values in the expression for the maximum moments, the maximum positive moment is found to be

$M_{Z_1} = 5,355$ foot-pounds per foot of wall

and the maximum negative moment is found to be

$M_{Z_2} = 3,680$ foot-pounds per foot of wall.

By decreasing the value of T (that is, making M_G negative), these moments may be equated. Through trial and error the correct values of T, M_G, S_G, and Z_1 are found to be approximately

$T = 3,350$ pounds per foot of wall

$M_G \cong 1,000$ foot-pounds per foot of wall

$S_G = 1,595$ pounds per foot of wall

$Z_1 \cong 2.05$ feet

The bending moments $M_{Z_1} = M_{Z_2} \cong 5100$ foot-pounds per foot of wall.

For stability, the forces below G must equal S_G, and the moments below G must equal M_G. It has been shown that the pressure p_{10} for minimum pile length must be as great as can be generated by the earth at that point. Now the maximum passive pressure at the depth of p_{10} is that due to a "surcharge" consisting of all the earth above point G, applied to an undisturbed ground mass of height h_6. This surcharge load is given by

$$W_p = w_4 \ (h_4 + h_5) + \frac{W_4}{w_4} \tag{97}$$

where W_4 = 1,275 pounds per square foot = the surcharge load of the earth above h_4

$w_4 = w_5 = w_6 = 80$ pounds per cubic foot = the unit weight of the submerged ground

$h_4 = 3$ feet

$h_5 = 2.36$ feet

therefore, W_p = 1,705 pounds per square foot.

The maximum passive pressure p_p at the depth of p_{10} is applied at the wall back and is given by

$$p_p = w_6 \ (h_6 + \frac{W_p}{w_6}) \tan^2 (\frac{90 + \phi_6}{2}) \tag{72}$$

where $\phi_6 = 30°$.

The active pressure at the depth of p_{10} at the wall face is due to a surcharge composed of the ground above G applied to the earth mass of height h_6. The surcharge in this case is

$$W_a = w_5 \ h_5 \tag{98}$$

$$= 80 \times 2.36 = 186.4 \text{ pounds per square foot}$$

The maximum active pressure p_a at the depth of p_{10} is applied at the wall face and is given by

$$p_a = w_6 \ (h_6 + \frac{W_a}{w_6}) \tan^2 (\frac{90 - \phi}{2}) \tag{64}$$

The actual pressure p_{10} is given by

$$p_{10} = p_p - p_a \tag{99}$$

$$= w_6 h_6 \ \tan^2 (\frac{90 + \phi_6}{2}) - \tan^2 (\frac{90 - \phi_6}{2}) + w_p \tan^2 (\frac{90 + \phi_6}{2}) - W_a \tan^2 (\frac{90 - \phi_6}{2})$$

$$= Rh_6 + 1705 \ (3) - 186.4 \ (0.33)$$

$$= 214 \ h_6 + 5053 = 214 \ (h_6 + 23.6)$$

which may be written symbolically

$$p_{10} = R\ (h_6 + h_o)$$

The loading diagram for this section of piling is shown in Figure 117.

The sum of all forces below G must equal the shear S_G at G. Summing the forces, represented by the areas GH' H, H'H JJ', and H'J' p_{10} and equating them to S_G we have

$$S_G = R\ (\frac{h_6 - \ell}{2}^2) + R\ (h_6 - \ell\ )\ \ell \quad -R(h_6 - \ell) + R(h_6 + h_o)\ \frac{\ell}{2}$$

which, when solved for ℓ gives

$$\ell = \frac{Rh_6^{\ 2} - 2S_G}{R\ (h_o + 2h_6)} \tag{100}$$

Similarly the sum of all moments below G must equal the moment M_G at G. Using the same areas

$$M_G = R(\frac{h_6 - \ell}{2})^2\ \frac{2(h_6 - \ell)}{3} + R(H_6\ell - \ell^2)\ (h_6 - \frac{\ell}{2})$$

$$- R(2h_6 + h_o - \ell\ )\ \frac{\ell}{2}\ (h_6 - \frac{1}{3})$$

which may be reduced to

$$\frac{6M_G}{R} = 2h_6^{\ 3} - \ell\ (6h_6^{\ 2} + 3h_o h_6) + \ell^2\ (h_o + 2h_6)$$

Substituting the solution for ℓ from equation 100 the final expression for h_6 is

$$R^2\ h_6^{\ 3}\ (h_6 + h_o) - 2R\ 3M_G(2h_6 + h_o) + S_G h_6(4h_6 + 3h_o) - 4S_G^{\ 2} = 0 \tag{101}$$

Substituting the previously determined values for R, h_o, M_G, and S_G in this equation we have approximately

$$45.8\ h_6^{\ 4} + 1{,}080h_6^{\ 3} - 2{,}730h_6^{\ 2} - 50{,}970h_6 = 40{,}480$$

which when solved for h_6 gives

$$h_6 \cong 7.4 \text{ feet}$$

The length of piling to G is 15.36 feet, and the total length of pile of minimum moment is about 22.8 feet.

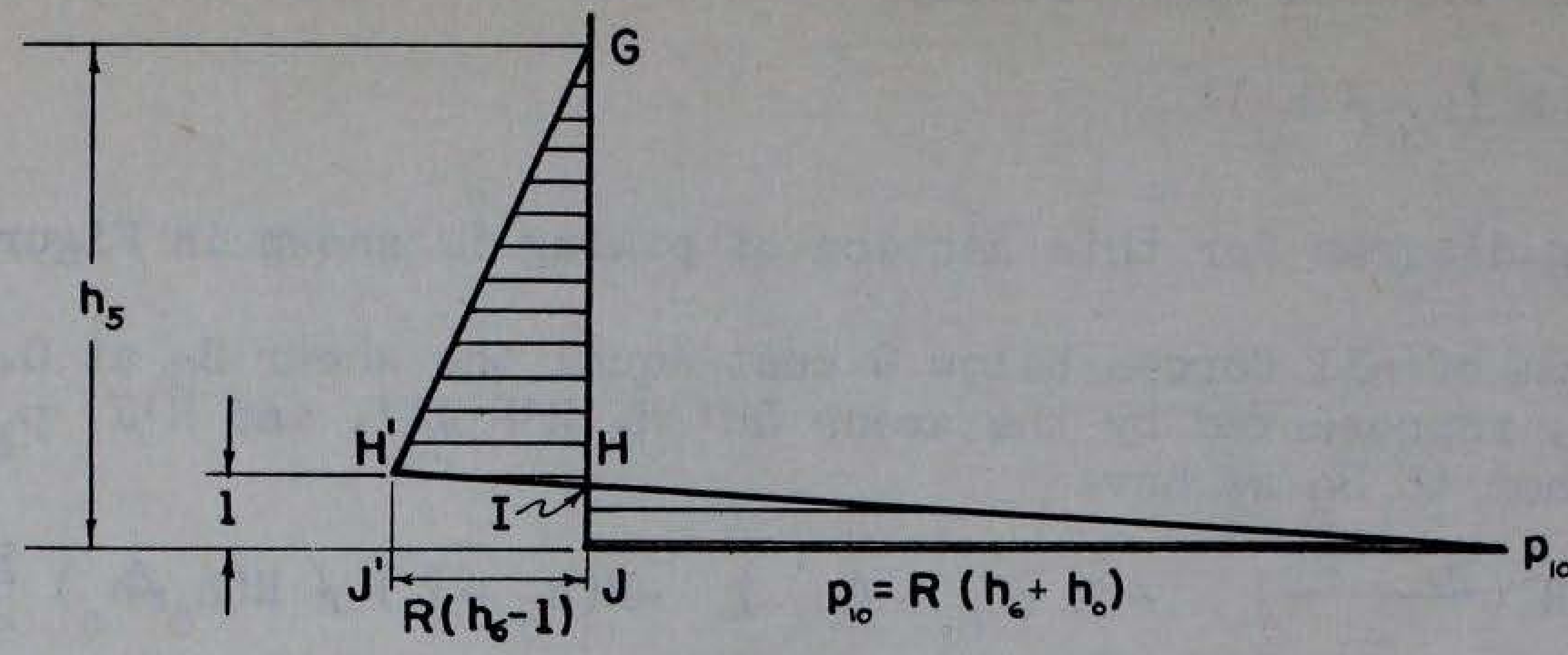

FIGURE 117 LOADING DIAGRAM BELOW POINT G

g. Final Pile Design - When designed for minimum length, the maximum bending moment in the pile was $M_Z = 5,645$ foot-pounds per foot of wall, and the length needed was 18.8 feet. When designed for minimum bending moment, M_Z was reduced to 5,100 foot-pounds per foot of wall, but the length needed for this reduced moment was 22.8 feet, an increase of 4 feet. The lightest pile section which can withstand a moment of 5,100 foot-pounds per foot of wall is an MP 115 which with a section modulus of 5.4 is good for a bending moment of 6,300 foot-pounds per foot. Since this is larger than the maximum moment which would occur in the pile when designed for minimum length, no saving is introduced by designing for minimum moment.

Accordingly, the design pile would be an MP 115 (or an MZ 22 which has the same weight per square foot of wall but greatly increased strength) and the total pile length would be 18.8 (19) feet.

h. Deadman Design - If the backfill is such that dependable passive resistance would be developed, either an intermittent or a continuous deadman could be used instead of one (say) of sheet pile which would probably be more expensive. The deadman may be of reinforced concrete, steel, timber, or any other material which would develop the required tie rod tensions over the life of the structure.

The loading diagram for the deadman is shown on Figure 118. The deadman would experience an active pressure on its back (the side farthest from the bulkhead wall) which would increase at the rate

$$R_a = w_1 \tan^2 \left(\frac{90 - \phi_1}{2}\right)$$

and a passive pressure on its front which would increase at the rate of

$$R_p = w_1 \tan^2 \left(\frac{90 + \phi_1}{2}\right)$$

assuming the entire deadman to be above the line of permanent saturation.

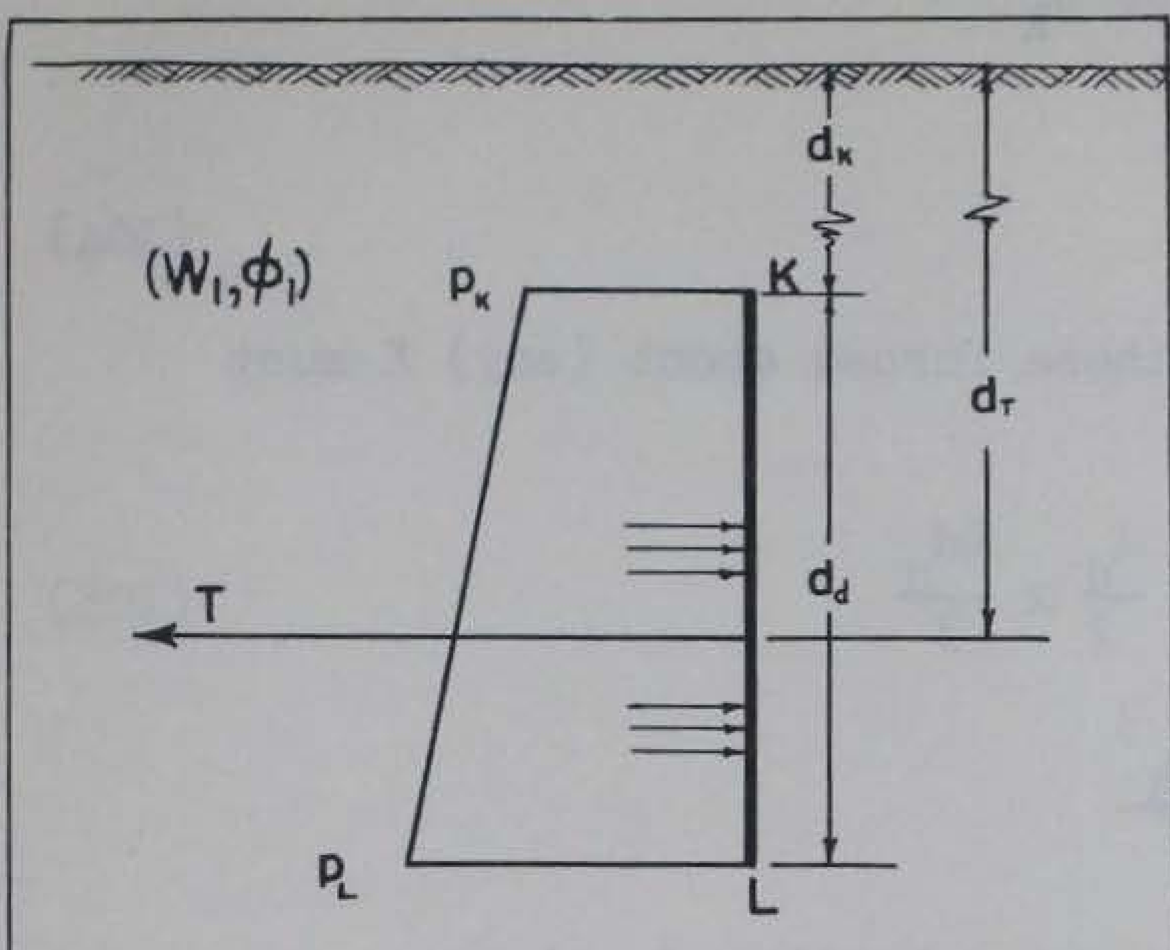

FIGURE 118 DEADMAN LOADING DIAGRAM

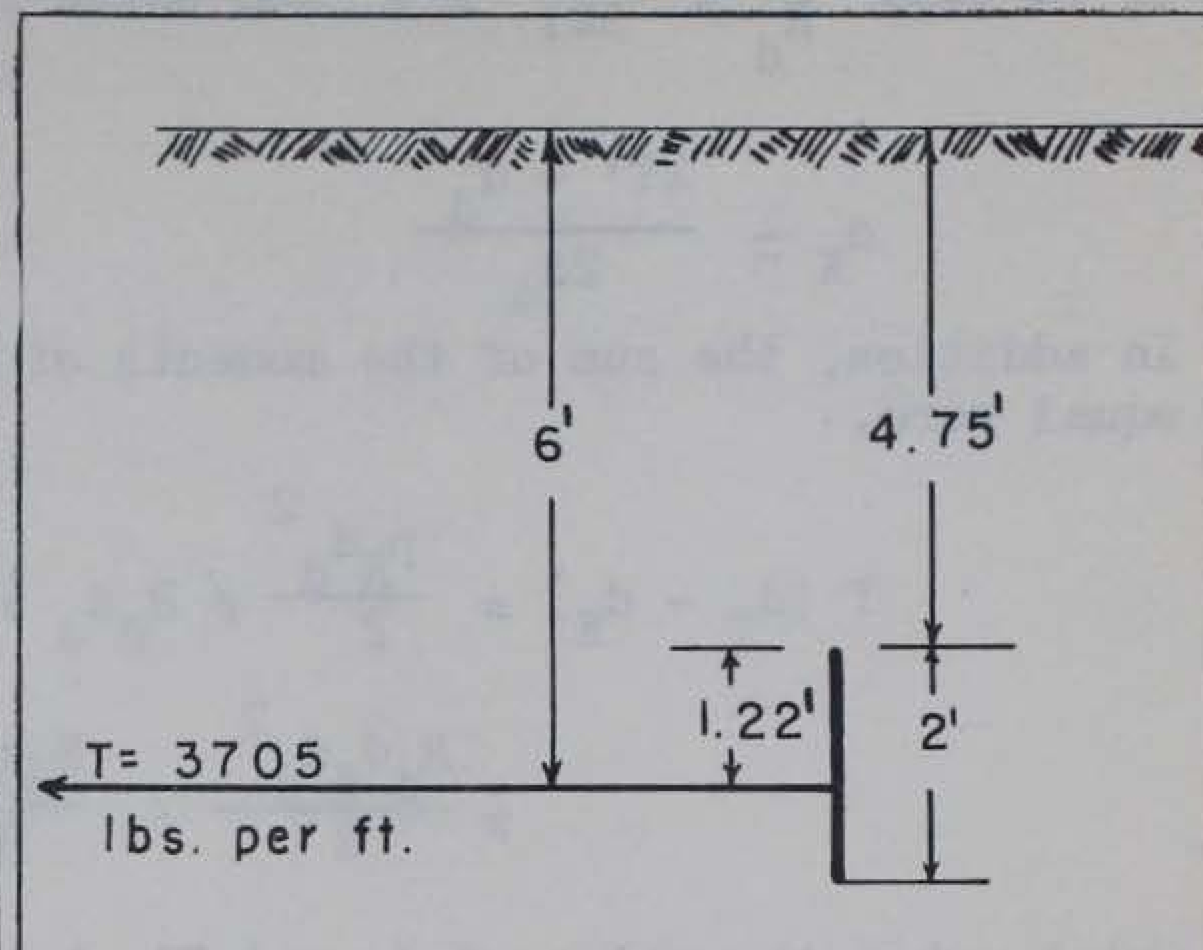

FIGURE 119 DEADMAN DIMENSIONS

The total rate of pressure increase then is

$$R_d = R_p - R_a \tag{102}$$

$$= w_1\left[\tan^2\left(\frac{90 + \phi_1}{2}\right) - \tan^2\left(\frac{90 - \phi_1}{2}\right)\right]$$

$$= 120\,(3.00 - 0.33)$$

$R_d = 321$ pounds per square foot per foot of depth.

The total earth pressure on the deadman, given by the area of the trapezoid $p_K KL\, p_L$ must equal the tie-rod tension per unit width of wall (T) times the length of wall supported by each deadman, divided by the deadman width. In the case of a continuous deadman this earth force must equal T. Assuming a continuous deadman,

$$T = \left(\frac{p_K + p_L}{2}\right) \times d_d \tag{103}$$

where $\quad p_K = R_d \times d_K$

and $\quad p_L = p_K + R_d \times d_d = R_d\,(d_K + d_d)$

Therefore

$$T = \frac{R_d\,(2d_K + d_d)}{2} \times d_d$$

if we call $\frac{T}{R_d} = \frac{T}{321} = T'$ and solve for d_K

$$d_K = \frac{2T' - d_d^{\,2}}{2d_d} \tag{104}$$

In addition, the sum of the moments of these forces about (say) K must equal zero.

$$T\ (d_T - d_K) = \frac{p_K d_d^{\,2}}{2} + R_d d_d \times \frac{d_d}{2} \times \frac{2d_d}{3} \tag{105}$$

$$= \frac{R_d d_K d_d^{\,2}}{2} + \frac{R_d d_d^{\,3}}{3}$$

which, when the value of d_K and T' above are substituted, gives

$$d_d^{\,4} - 12\ d_t\ T'\ d_d + 12\ (T')^2 = 0 \tag{106}$$

an equation in d_d which must be solved by trial and error.

This quartic equation may have as many as two positive roots but no more. One of these roots will be slightly smaller than $d_d = \sqrt{12 d_T T'}$ but there may be another root smaller than this. In the case of the pile designed for minimum length where $T = 3{,}705$ pounds per foot of wall, the quantity $\sqrt[3]{12 d_T T'} \cong 9.4$ feet and a positive root does exist between $d_d = 8$ and 9. If this value for d_d is substituted in equation 104, d_K is found to be negative, which is impossible.

However investigation shows that there is another root very close to $d_d = 2$. This root is the applicable solution.

Substituting in the equation for d_K

$$d_K = \frac{2T' - d_d^{\,2}}{2d_d} = \frac{23.05 - 4}{4}$$

$$= 4.76 \text{ feet.}$$

The deadman dimensions and placement are shown on Figure 119.

i. <u>Sheet Pile Deadman</u> - If the backfill is not dependable for passive resistance, a sheet pile deadman, or one which depends on active as well as passive pressures must be used. The analysis for a deadman of this type is similar to the analysis of the lower part of the pile wall designed for minimum moment. (See Figure 120).

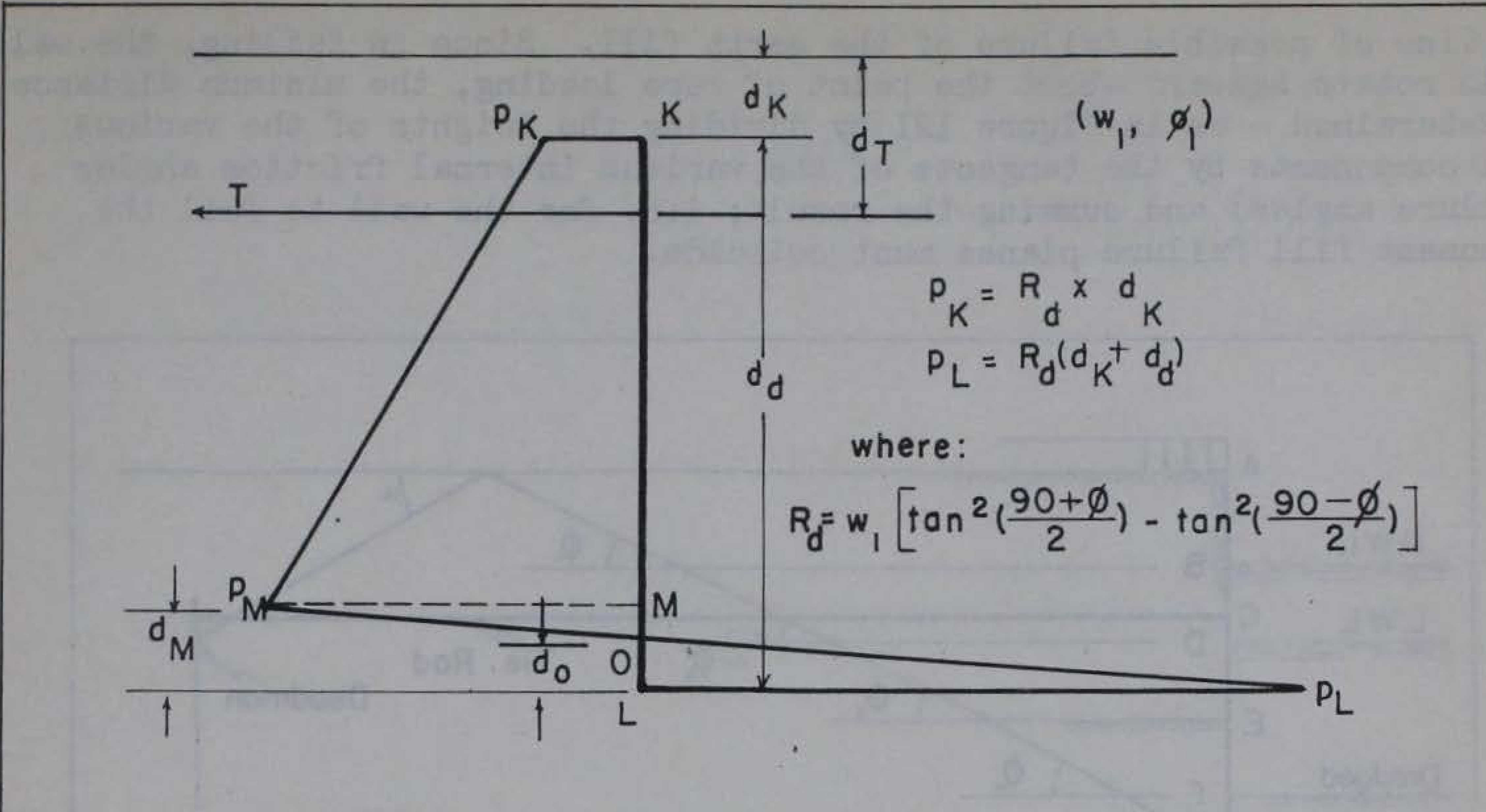

FIGURE 120 – SHEET PILE DEADMAN

By summing forces and moments about K as done previously, d_d is given by

$$\left[d_d\ (p_L + p_K) - 2T'\right]^2 = 2p_L \left[d_d^{\,2}\ (p_L + 2p_K) - 6T'\ (d_d - d_T + d_K)\right] \quad (107)$$

d_m and d_o are given by

$$d_m = \frac{d_d\ (p_K + p_L) - 2T'}{2p_L} \quad (108)$$

$$d_o = \frac{d_m p_L}{2p_L - R_d d_m} \quad (109)$$

The maximum negative and maximum positive moments in the pile are respectively

$$M_n = \left(\frac{d_T - d_K}{6}\right)^2 \left[3p_K + R_d\ (d_T - d_K)\right] \quad (110)$$

$$M_p = \frac{4p_L\, d_m^{\,3}}{6\ (p_L - R_d d_m)} \quad (111)$$

j. Distance of Deadman From Wall - The deadman should be located outside the possible failure planes of the various layers of earth fill and also outside the intersection of its own failure plane to the surface with the line of possible failure of the earth fill. Since in failing, the wall would rotate seaward about the point of zero loading, the minimum distance is determined - as in Figure 121 by dividing the heights of the various fill components by the tangents of the various internal friction angles (failure angles) and summing the result; i.e. for the wall to fail the component fill failure planes must coincide.

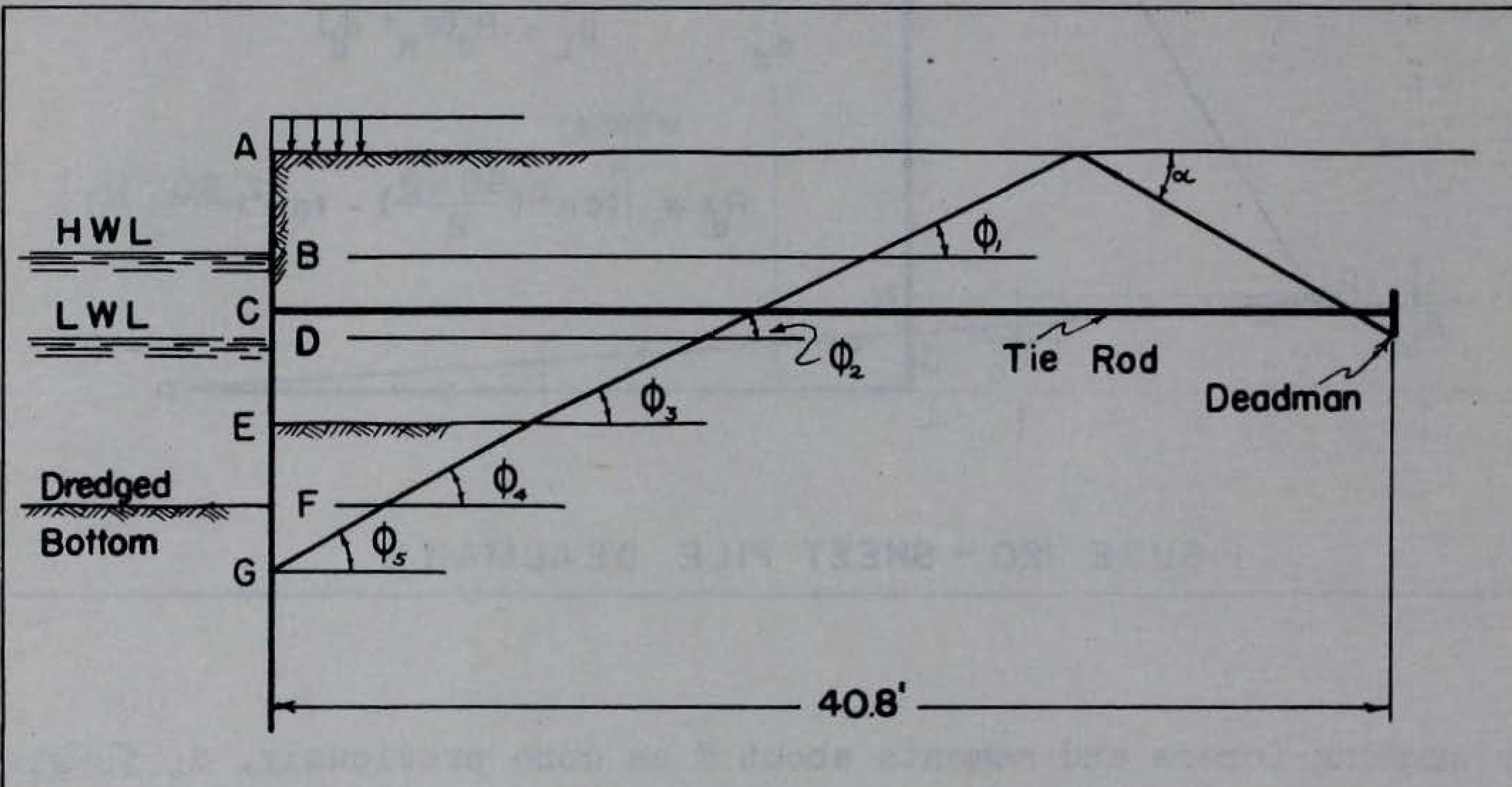

FIGURE 121 - DEADMAN DISTANCE FROM WALL

For the case being considered, the minimum distance of the deadman from the wall is given by

$$L = \frac{h_5}{\tan \phi_5} + \frac{h_4}{\tan \phi_4} + \frac{h_3}{\tan \phi_3} + \frac{h_2}{\tan \phi_2} + \frac{h_1}{\tan \phi_1} + \frac{(d_K + d_d)}{\tan \alpha} \qquad (112)$$

where $\tan \alpha = \sqrt{1 + \tan \phi_1^{\,2}} - \tan \phi_1 = 0.58$

therefore $L = \frac{2.36}{\tan 30°} + \frac{3}{\tan 30°} + \frac{3}{\tan 25°} + \frac{3}{\tan 25°} + \frac{4}{\tan 30°} + \frac{6.78}{0.58}$

$= 4.08 + 5.2 + 6.44 + 6.44 + 6.94 + 11.70$

$= 40.8$ feet from the wall.

5.134 Revetment - a. General - A reinforced concrete revetment as shown in Figure 98(103) will provide relatively inexpensive protection if the beach is subjected to comparatively moderate wave and current action. The wall is essentially a pavement slab cast in 12 1/2-foot lengths and reinforced with galvanized metal mesh with dowels across the joints. The lower portion of the slab is stepped to reduce beach scour and the toe is protected by timber sheet piling, driven into the foundation.

As pointed out in Section 4.27, the most sound engineering approach to the design of rubble-mound structures for a particular site is to study carefully all similar structures in the area, in order to evaluate their design in relation to the structure's present condition, and then consider these existing data along with presently available empirical formulas for rubble mound structures. In many cases no comparable structures exist in a study area, or those of comparable design have completely different shore line exposure and are of limited value for comparative purposes. The following example illustrates the design procedure and use of presently available formulas for the design of a stone type of revetment. The example is generalized since local conditions will necessitate modifications.

b Design Problem - It is desired to construct a stone revetment for the purpose of checking the recession of the foreshore. Let it be assumed that the area is subjected to a 3-foot spring tide and a possible additional 2-foot wind set-up. The erosion trend indicates that the expected position of the ground line at the toe of the structure will be 1 foot below MLW (see Section 3.28). Considering scour, spring tide and wind set-up, the depth of water at the toe of the structure would be 6 feet. Analysis of wave hindcasting data (section 1.2) and wave refraction diagrams indicates that a 4-foot, 5-second period, wave can reach the toe of the structure.

The design slope for the revetment is selected to be 1 on 2. The top of the revetment is to be made approximately 10 feet in width; this will provide a roadway during construction and provide sufficient material to allow flattening of the slope after construction without exposing the bank material. The top elevation of the structure should be constructed to 11 feet above mean low water (5 feet plus 150 percent of the height of a 4-foot wave which is capable of reaching the revetment.) The criteria for the design of a stone revetment is similar to a rubble-mound breakwater in that the face stones must be of sufficient weight to overcome any displacement due to wave action.

The Iribarren formula, as modified by Hudson, is considered the best guide presently available for determining the size of stone which will resist displacement due to wave forces. The size of the stone is determined in the following manner:

For seawater:

$$W = \frac{72\ (S_r)\ K'\ H^3}{(1.05 \cos\alpha - \sin\alpha)^3\ (S_r - 1.03)^3} \qquad (47a)$$

where:

S_r = 2.65, specific gravity of rock available to the area.

K' = 0.017, Table D-10 for a slope of 1 on 2

$(1.05 \cos\alpha - \sin\alpha)^3 = 0.1190$, Table D-11 for a slope ratio of 2

Substituting in equation 47a gives:

$$W = \frac{72(2.65)\ (0.017)\ (4)^3}{0.1190\ (2.65 - 1.03)^3} = 410 \text{ lbs.}$$

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The weight of the face rock would therefore be approximately 1/4 ton. The subsurface and filter blankets are dependent on local foreshore and bottom conditions and should be selected to retain the bank material from leaching by wave action.

5.2 OFFSHORE BREAKWATERS

5.21 GENERAL CONSIDERATIONS - The use of offshore breakwaters, solely for shore protection, has been limited because of the high costs involved. However, because they have use as a protective structure, because they may be required to provide a sand trap for by-passing material, and because they may be used to retain the toe of the beach fill, design data are included for typical examples of the most frequently constructed types. Other types may be designed by using the methods illustrated for the design of seawalls.

5.22 TYPES - Several conventional types of permanent offshore breakwaters are now in general use, varying as to material of which constructed and as to cross section. The principal construction materials are stone, cnncrete, steel and timber. In addition fascine mattresses, asphalt,(125) and bitumen have occasionally been used. The selection of a material or combination of materials for breakwater construction is based on cost and adaptability in relation to conditions imposed by a given site.

5.221 Rubble-Mound - The rubble-mound structure is adaptable to any depth of water and may be placed on practically any kind of foundation. This type of structure is used extensively throughout the United States and almost exclusively on the Pacific Coast(69). The chief advantages of the random stone structure are: damage is easily repaired, settlement of the structure results only in readjustment of component stones and increased stability rather than the incipient failure of the entire structure, and the reflected wave action is less severe than that from a solid wall structure. The chief disadvantage of rubble-mound construction is the large quantity of material required, which results in a high first cost if satisfactory material is not available within economic hauling distance.

The rubble-mound breakwater is a more or less heterogeneous assemblage of natural stones of different sizes and shapes either dumped at random or laid in courses. A typical cross section is shown in Figure 122. Side slopes and stone sizes are so designed that the structure will resist the expected wave action.

5.222 Composite Earth and Stone - The composite type of random stone structure is designed for relative economy and rapidity of construction. The principal difference from the rubble-mound breakwater lies in the substitution of sand and clay or incipient shale for the base and core material. Care must be used in selecting the clay or earth material with which the sand sub-base is covered. This type of construction was used for the 32,000-foot Los Angeles-Long Beach detached breakwater. As shown in Figure 123, this structure consisted of a sand inner core covered by a hard clay mound, all being armored by a graded rubble-stone covering. If satisfactory material can be secured from an approved or useful dredging area, savings may be effected.

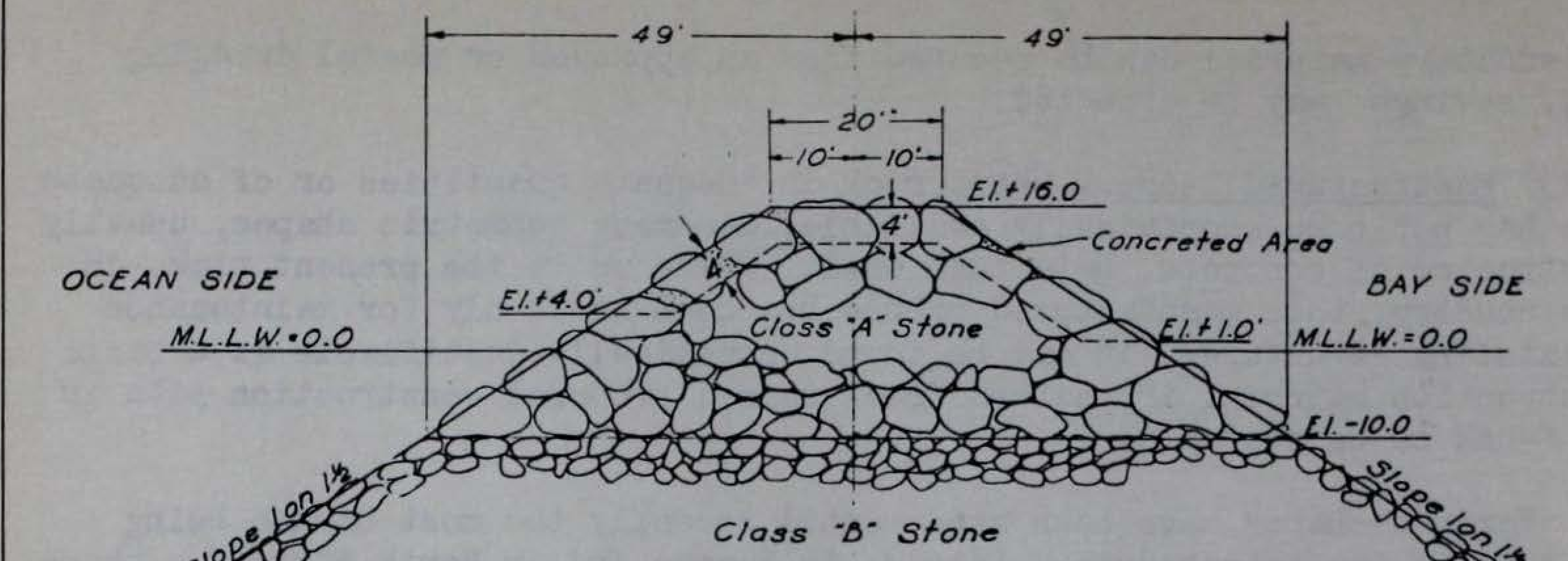
5.223 Manufactured Stone - Where rock in adequate quantities or of adequate size has not been economically available, man-made geometric shapes, usually constructed of concrete, have been used. While up to the present time, in this country, this manufactured rubble has been used only for maintenance of existing structures, it may be found economically justifiable as a basic construction material if natural rock near a proposed construction site is not found to be suitable.

Various shapes have been used, until recently the most common being the cube or the tetrahedron. Lately, in Europe and in North Africa, a shape known as a "Tetrapod" has been used for breakwater and jetty construction. A "Tetrapod" is essentially a dished faced tetrahedron, somewhat resembling a child's toy jack. Savings of materials have been claimed for tetrapods based on an apparent tendency for the individual members to interlock.

5.224 Stone and Concrete - The stone and concrete type structure is a combination of rubble-mound and concrete-wall types, ranging from a rubble mound, in which the voids in the upper part of the structure are filled with concrete, to massive concrete superstructures on rubble-mound substructures. The rubble mound usually is used either as a foundation for a vertical or nearly vertical concrete wall or as the main structure surmounted by a low concrete superstructure with a vertical, curved, stepped, or inclined face. The use of a composite concrete and stone structure reduces the quantity of material required and may be economical in great depths.

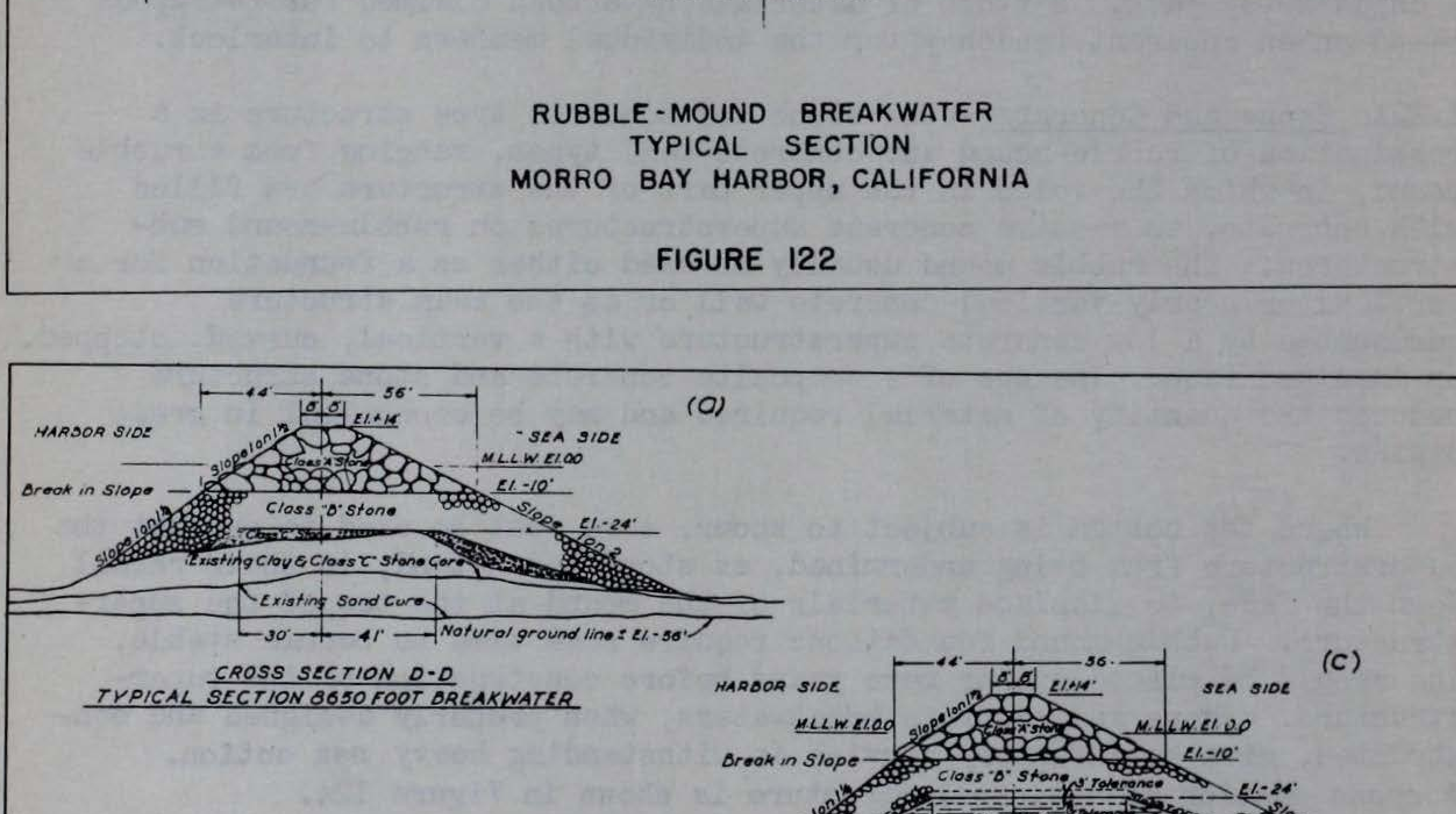
Where the bottom is subject to scour, care must be used to prevent the superstructure from being undermined, as storm waves tend, in their recoil down the face, to displace materials of the mound at the toe of the superstructure. Rubble-mound foundations require some time to become stable, and should be placed one or more years before construction of the superstructure. Stone and concrete breakwaters, when properly designed and constructed, give satisfactory service in withstanding heavy sea action. A cross section of a typical structure is shown in Figure 124.

5.225 Concrete Caissons - Breakwaters of this type are built of reinforced concrete shells, which are floated into position, settled upon a prepared foundation, filled with stone or sand to give stability, and then capped with concrete slabs or cap stones. These breakwaters may be constructed with or without parapet walls. In general, concrete caissons are of two types; one type having a bottom of reinforced concrete which is an integral part of the caisson, the other type not having a permanent bottom. The bottom opening of the latter type is closed with a temporary wooden bottom which is removed after the caisson is placed on the foundation. Stone, which is used to fill the compartments, combines with the foundation material to provide additional resistance against horizontal movement. A typical section is shown in Figure 125 of concrete caisson breakwater.

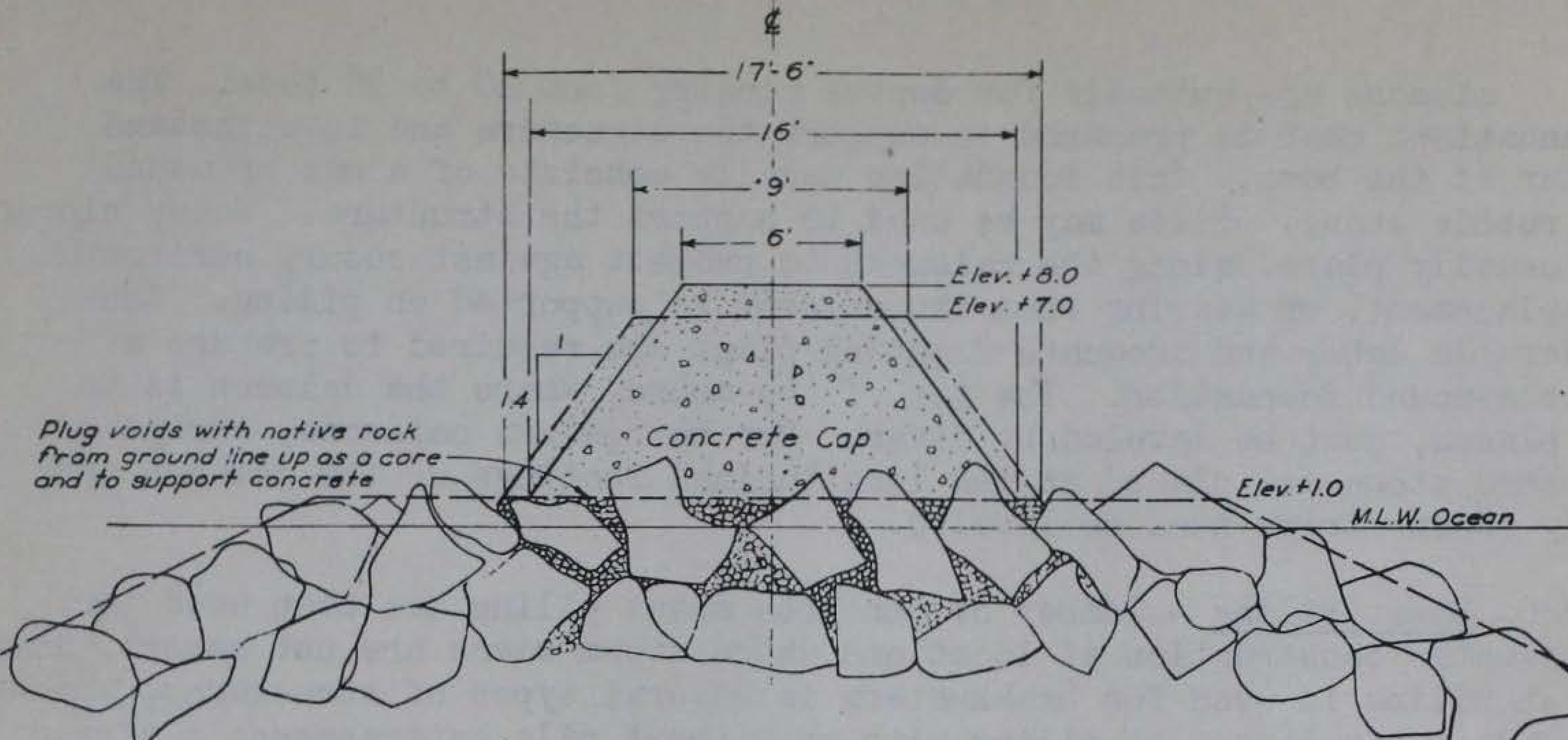


RUBBLE-MOUND BREAKWATER
TYPICAL SECTION
MORRO BAY HARBOR, CALIFORNIA

FIGURE 122



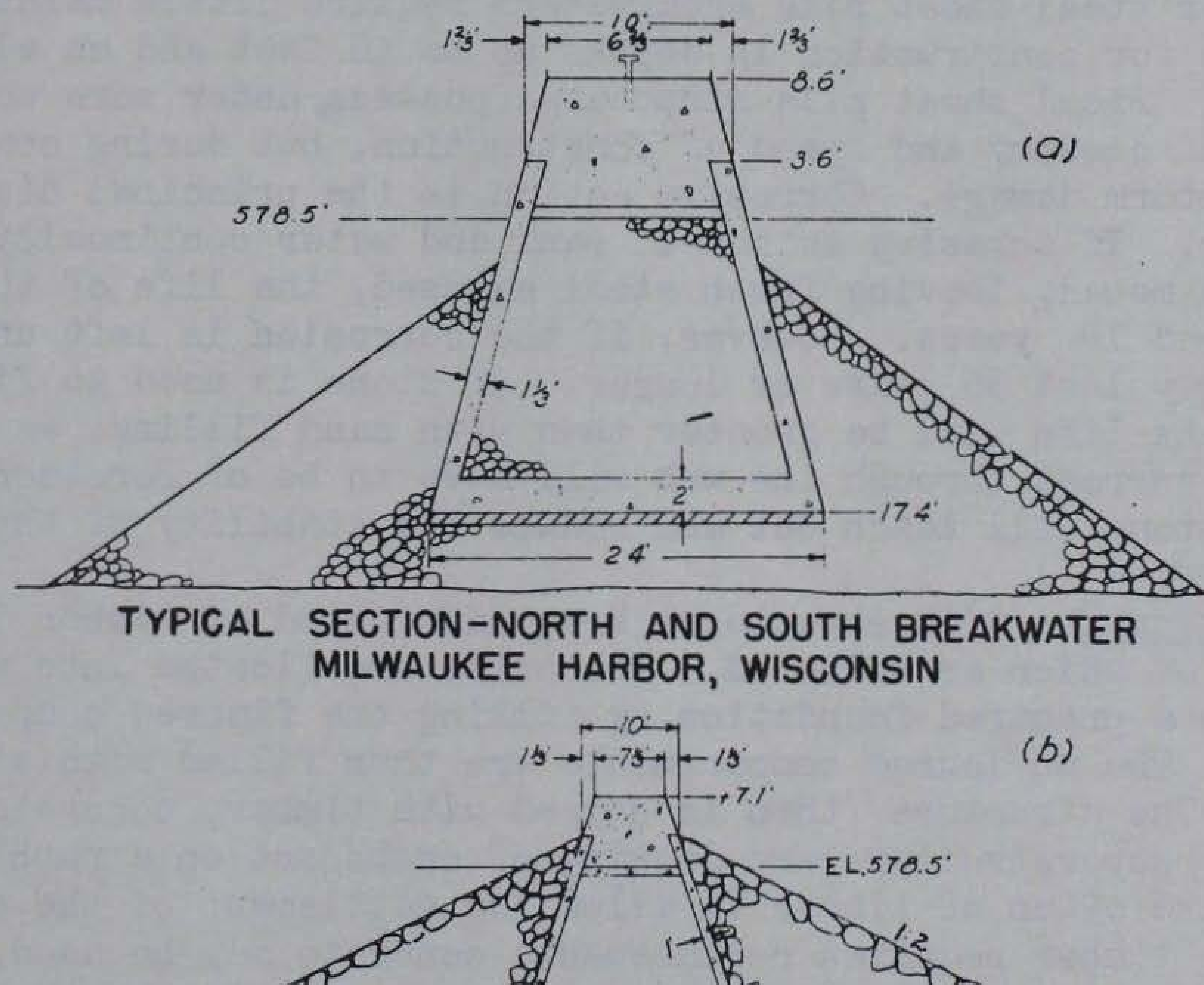
LOS ANGELES & LONG BEACH OUTER HARBOR
BREAKWATER
COMPOSITE EARTH & STONE
FIGURE 123



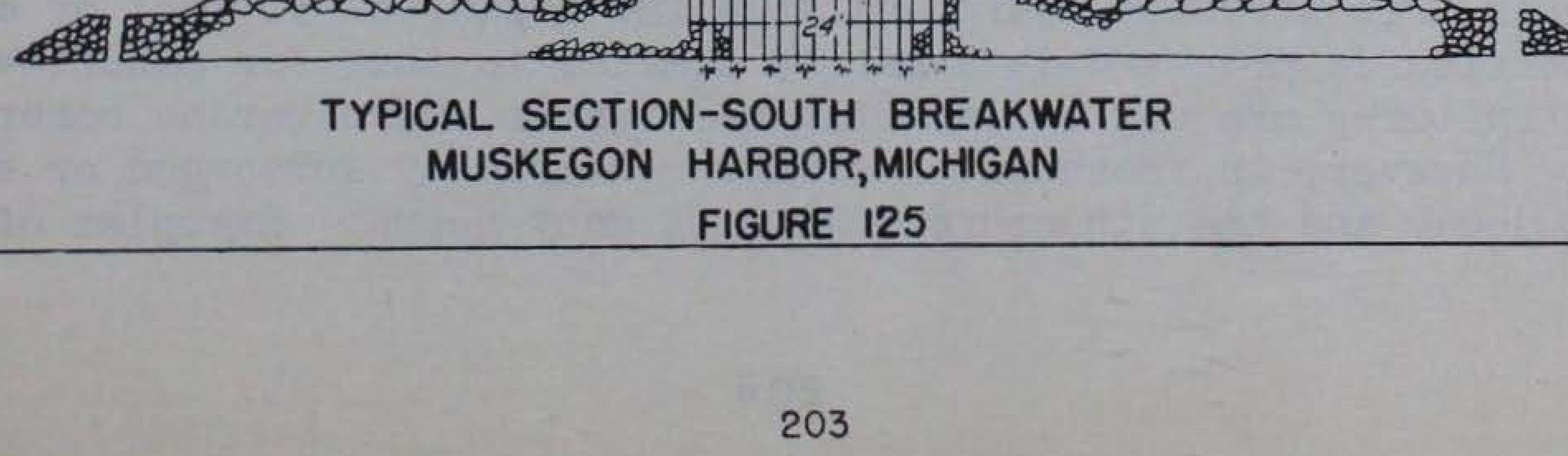
TYPICAL SECTION OF CONCRETE CAPPED JETTY

LAKE WORTH INLET, FLORIDA

FIGURE 124



TYPICAL SECTION-NORTH AND SOUTH BREAKWATER
MILWAUKEE HARBOR, WISCONSIN



TYPICAL SECTION-SOUTH BREAKWATER
MUSKEGON HARBOR, MICHIGAN

FIGURE 125

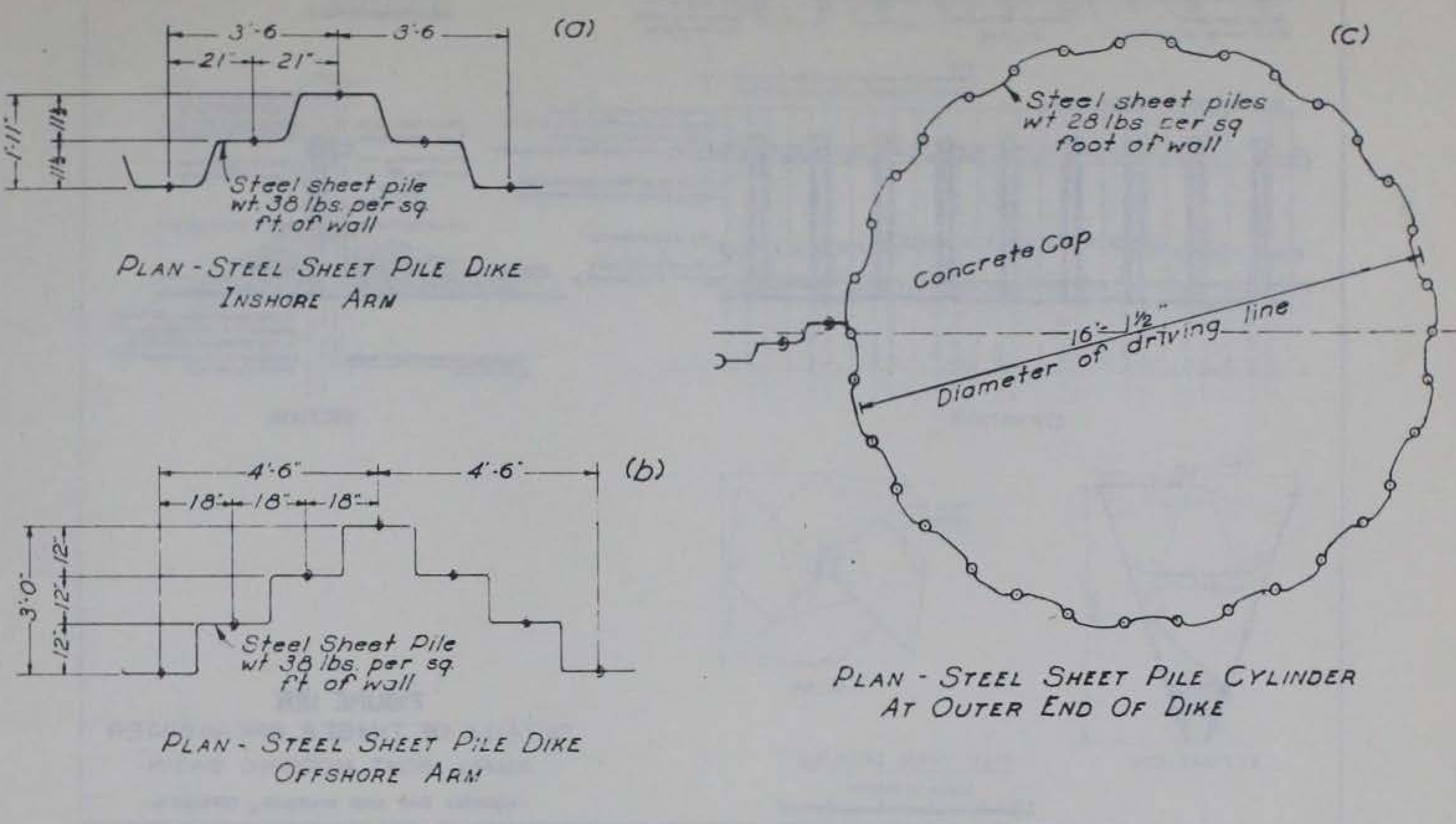
Caissons are suitable for depths ranging from 10 to 35 feet. The foundations must be prepared to support the structure and to withstand scour at the base. This foundation usually consists of a mat or mound of rubble stone. Piles may be used to support the structure. Heavy riprap is usually placed along the caissons to protect against scour, horizontal displacement, or weaving when the caisson is supported on piling. Considerable labor and adequate floating plant are required to prepare a rubble-mound foundation. The top of the mound, where the caisson is to be placed, must be leveled by diver. For bottomless caissons, strips of crushed stone are placed on the longitudinal footings of the caisson and only these strips need be leveled.

5.226 <u>Sheet Piling</u> - Timber or concrete sheet piling has been used for breakwater construction at locations where storm waves are not severe. Steel sheet piling is used for breakwaters in several types of structures, which include a single row of piling with or without pile buttressess; a single row of sheet piling arranged so that the row of piling acts as a buttressed wall, but requires no more piling than a straight wall; double walls of sheet piling held together with tie rods with the space between the walls filled with stone or sand (usually separated into compartments by cross walls if sand is used); and cellular steel sheet pile structures which are modifications of the double-wall type. Examples of pile breakwaters both typical and existing are shown in Figures 126 to 129 inclusive.

Cellular steel sheet pile breakwaters require little maintenance and are suitable for construction in depths up to 40 feet and on all kinds of foundations. Steel sheet pile structures possess, under some conditions, advantages of economy and speed of construction, but during construction are subject to storm damage. Corrosive action is the principal disadvantages in sea water. If abrasive action of sand and water continually wears away the corroded metal, leaving fresh steel exposed, the life of the piling may not exceed 10 years. However, if the corrosion is left undisturbed the piling may last 35 years or longer. If stone is used to fill the structure, its life will be greater than with sand filling, as the holes which first corrode through the web will have to be of considerable size before the stone will leach out and reduce the stability of the structure.

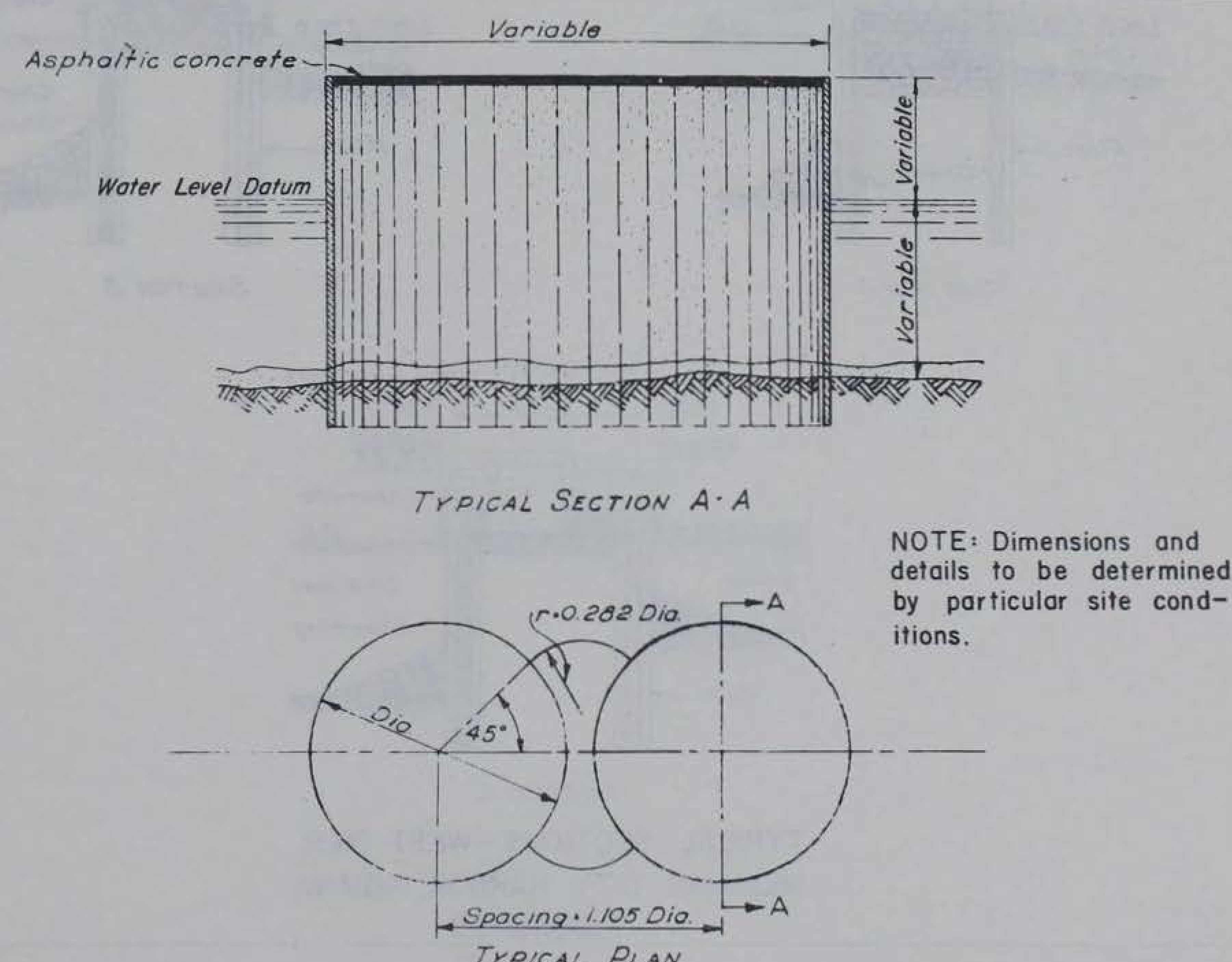
5.227 <u>Crib Types</u> - Crib structures are constructed of timber in compartments, some of which are floored. The cribs are floated into position and settled upon a prepared foundation by filling the floored compartments with stone. The unfloored compartments are then filled with stone for stability. The structure then is capped with timber, concrete, or capstones. The superstructure and decking of cribs set on a rubble-mound foundation are often of timber to allow for settlement of the crib. When decay of the timber requires replacement, concrete may be used, as the crib will probably have settled into permanent position by that time.

Timber crib structures are suitable for depths of 50 feet or more. The foundation is prepared in a manner similar to that for concrete caissons. Timber structures are not suitable in salt water where marine borers are present. However, in fresh water timber continually submerged or saturated does not decay and the structure will last many years. Examples of timber



PORT O'CONNOR DIKES—PORT O'CONNOR, TEXAS

FIGURE 126



CIRCULAR TYPE OF CELLULAR BREAKWATER

FIGURE 127

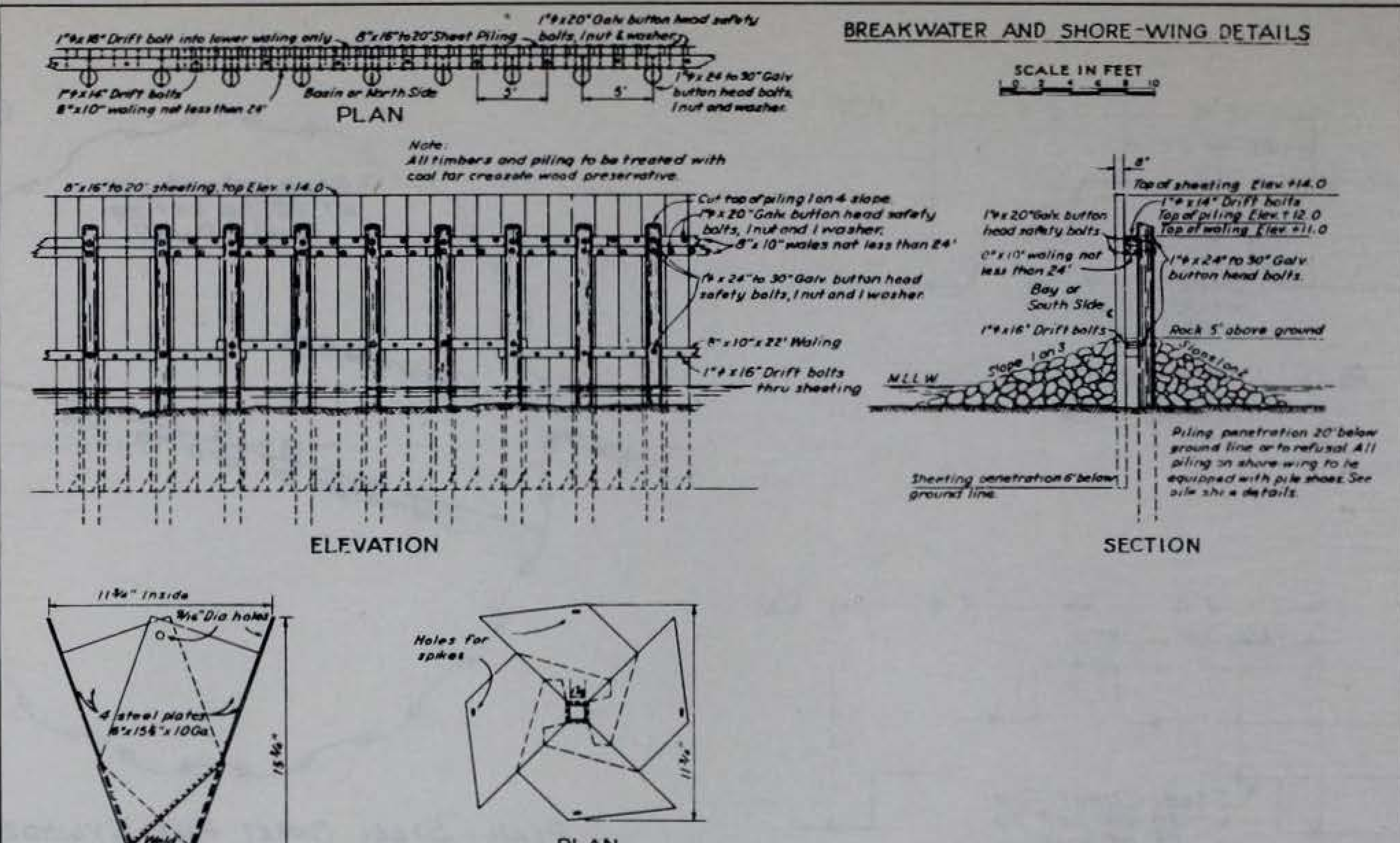
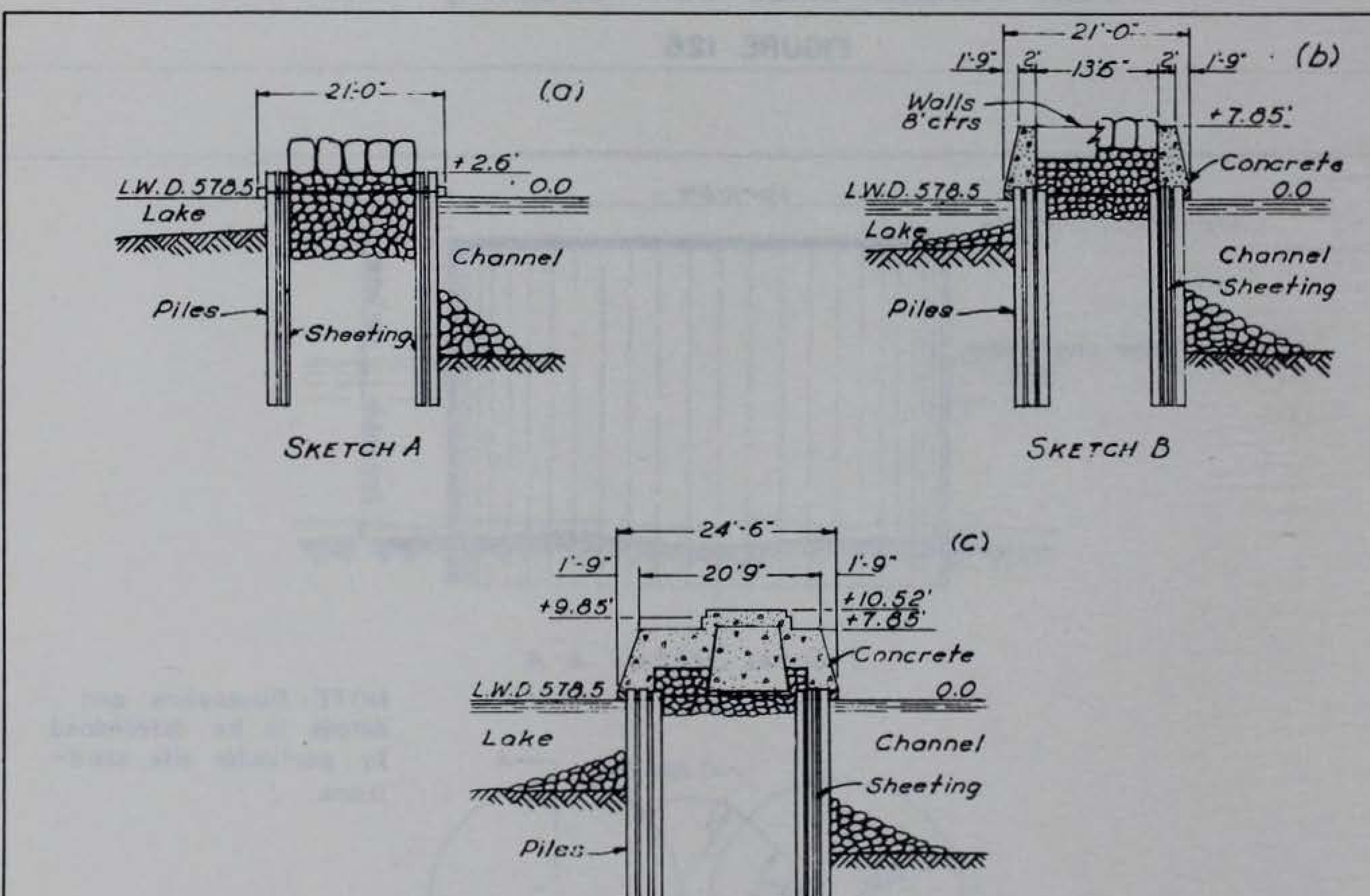


FIGURE 128
DETAILS OF TIMBER BREAKWATER
SMALL BOAT MOORING BASIN
YAQUINA BAY AND HARBOR, OREGON



TYPICAL SECTIONS – WEST PIER
MICHIGAN CITY HARBOR, INDIANA
FIGURE 129

crib breakwaters are shown in Figures 130 and 131.

5.228 Solid Fill. A solid fill structure is sometimes constructed where it is desired to prevent wave action passing through the structure. One common type of solid fill breakwater consists of hydraulically deposited sand fill between stone mounds, with an armor of heavy rubble-stone on the seaward side to protect the smaller material in the stone mounds against wave action. An example of solid fill breakwater is shown in Figure 132.

5.229 Asphaltic Materials. In some instances the core and capstone have been consolidated to various depths by forcing hot asphaltic concrete into the interstices between the stones. Typical sections of structures where asphaltic concrete has been used are shown on Figure 133. Although the asphaltic concrete appears to serve its purpose in the cases shown, adjacent sections indicate that the use of concrete would probably have been superior. Asphaltic concrete used at other locations such as the Columbia River and Los Angeles Harbor has proven ineffective.

5.23 SELECTION OF TYPE. The selection of the type of offshore breakwater is dependent on many factors. Principal factors which may affect the selection of type include natural forces to be resisted, foundations, availability of material, desired life, and cost. Probably all are reflected in the cost. The desired life is also a function of the use of the structure.

The principal natural force to be resisted by a detached breakwater is wave attack. This is of primary importance in determining the type of structure. Heavy wave action would require one of the more massive types of structure, whereas sheet piling might suffice where wave attack is light. In some few instances involving submerged breakwaters to retain a sand beach, the earth pressures also must be taken into consideration.

The type of foundation may be the governing factor in the selection of type. For instance a rock bottom would not permit use of one of the sheet pile types. Also, with a readily erodible bottom, some flexible type of structure such as a random stone mound would probably prove desirable. Timber crib structures are moderately flexible to uneven settlement. Obviously, undermining of the structure by scour of an erodible foundation would be most detrimental to a caisson or masonry wall type breakwater.

Availability of materials may also dictate the type of structure by its effect on cost. Lack of stone within economic hauling distance may make mandatory use of some other material. In many instances steel sheet pile cannot be obtained due to shortages, hence concrete or timber must be used.

The desired life of the structure also dictates its type. Obviously, an untreated timber structure could not be installed on a sea coast where a structure is desired to last 50 years. Conversely, a permanent stone structure would not be constructed if the need for protection was of a temporary nature.

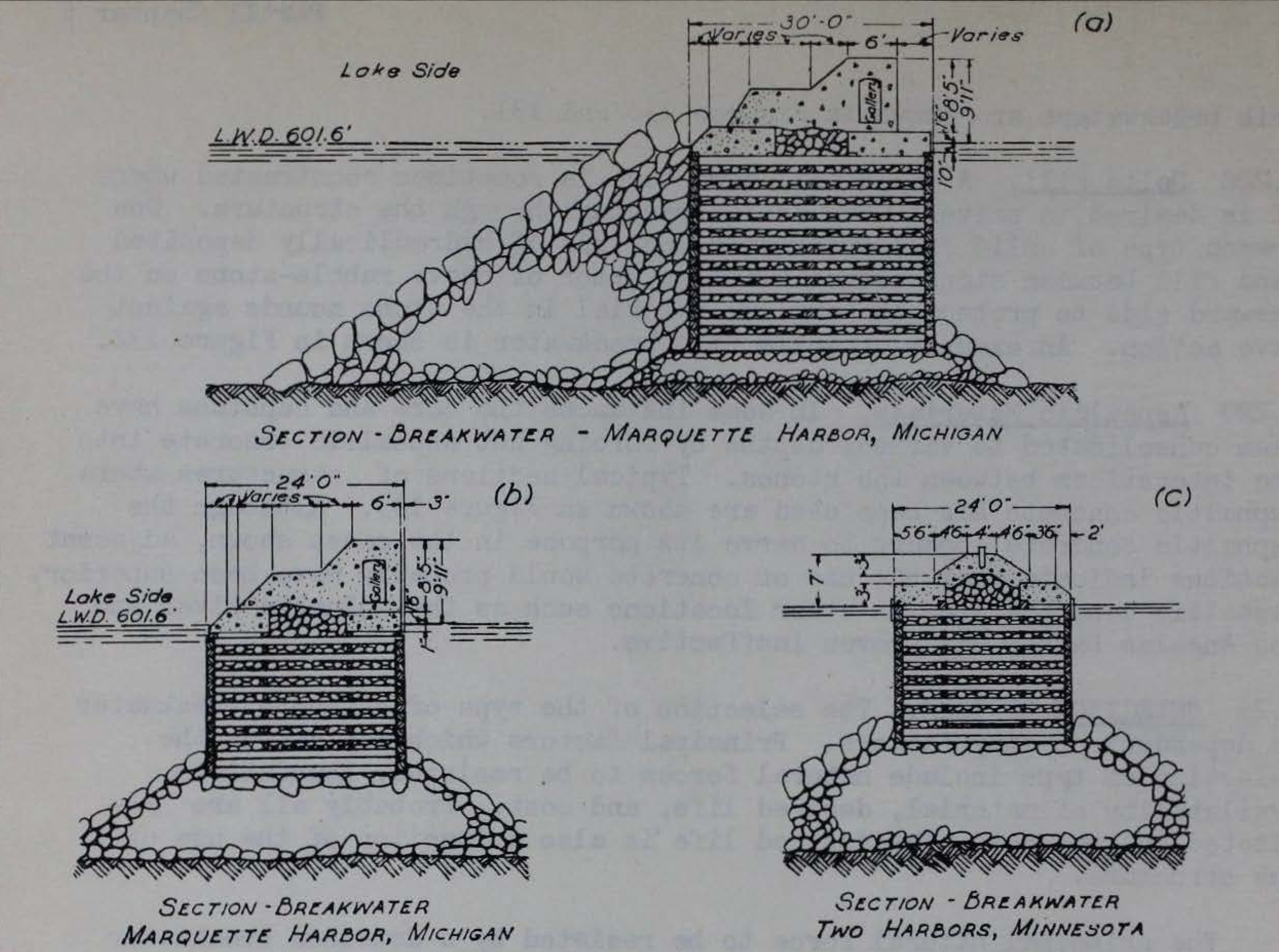
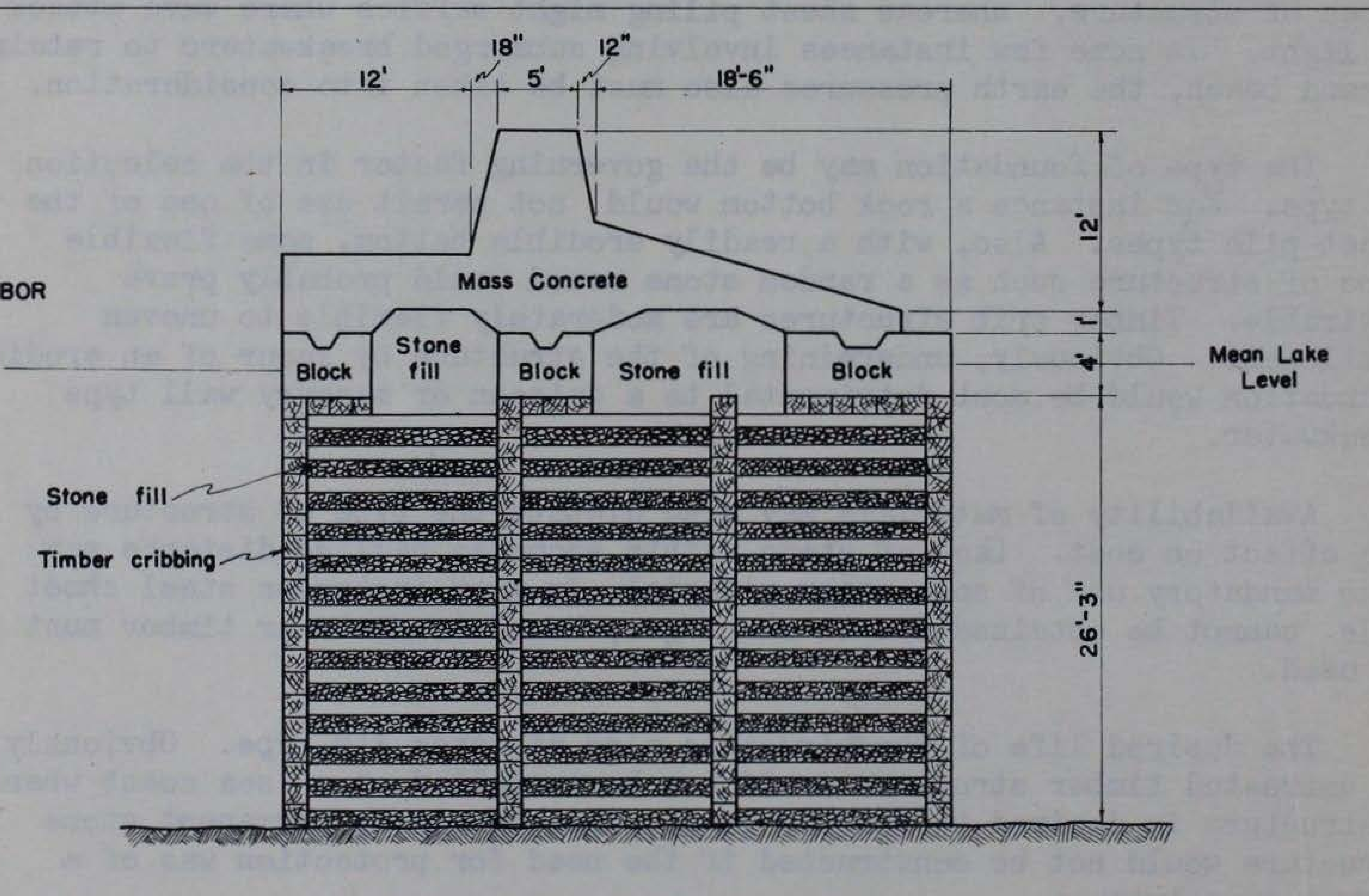


FIGURE 130 TIMBER CRIB TYPE BREAKWATERS



BREAKWATER AT HARBOR BEACH, MICH.

TIMBER CRIB TYPE

FIGURE 131

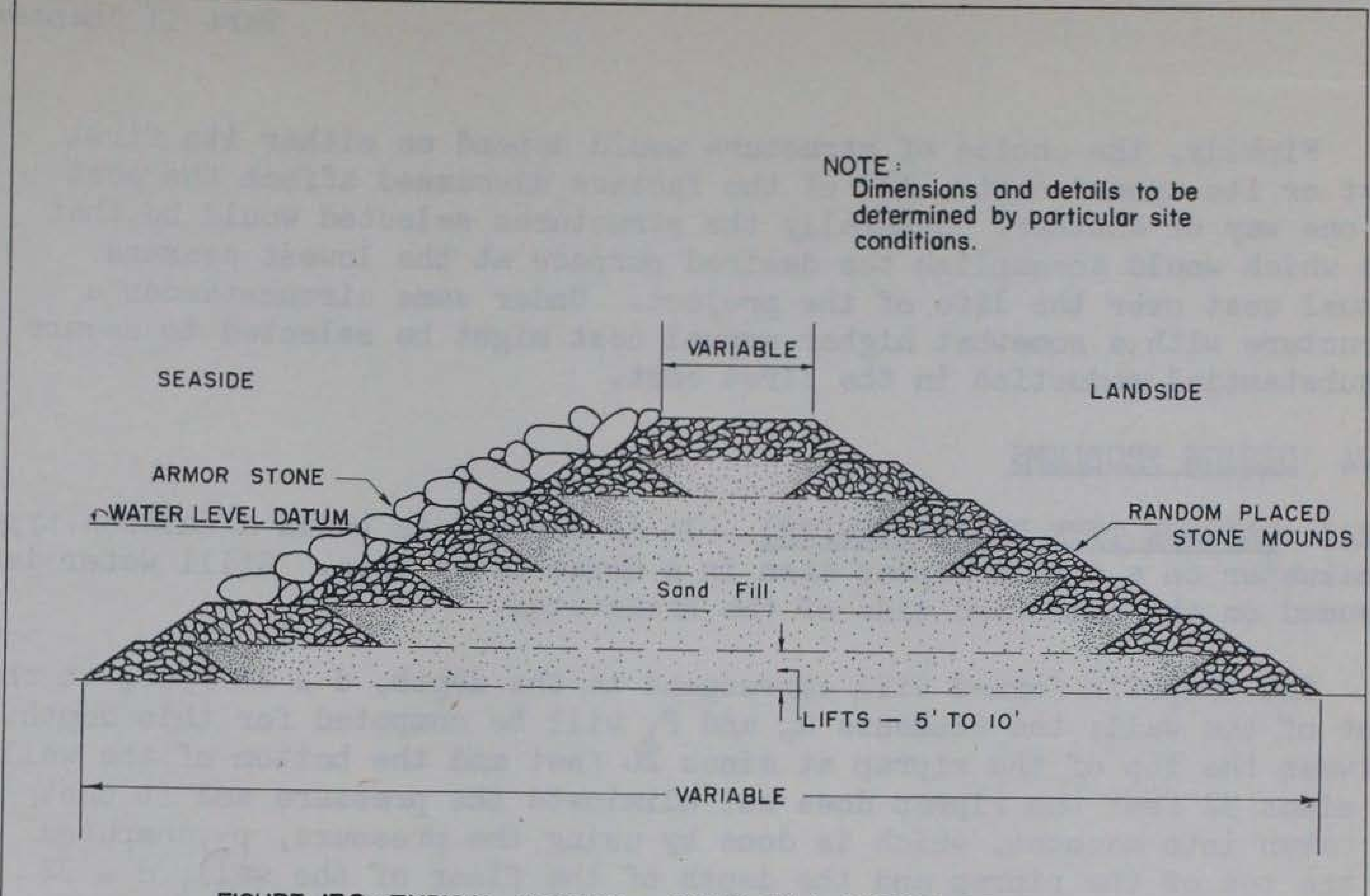
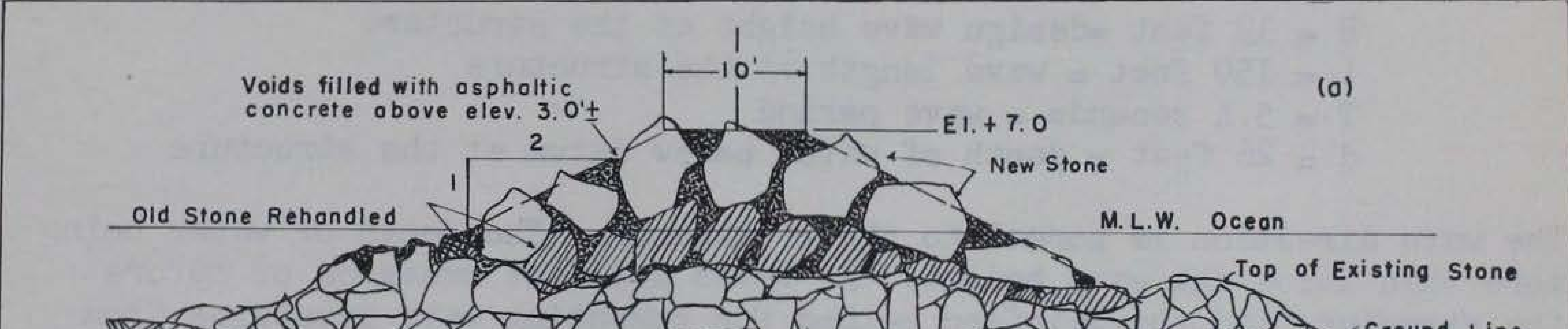
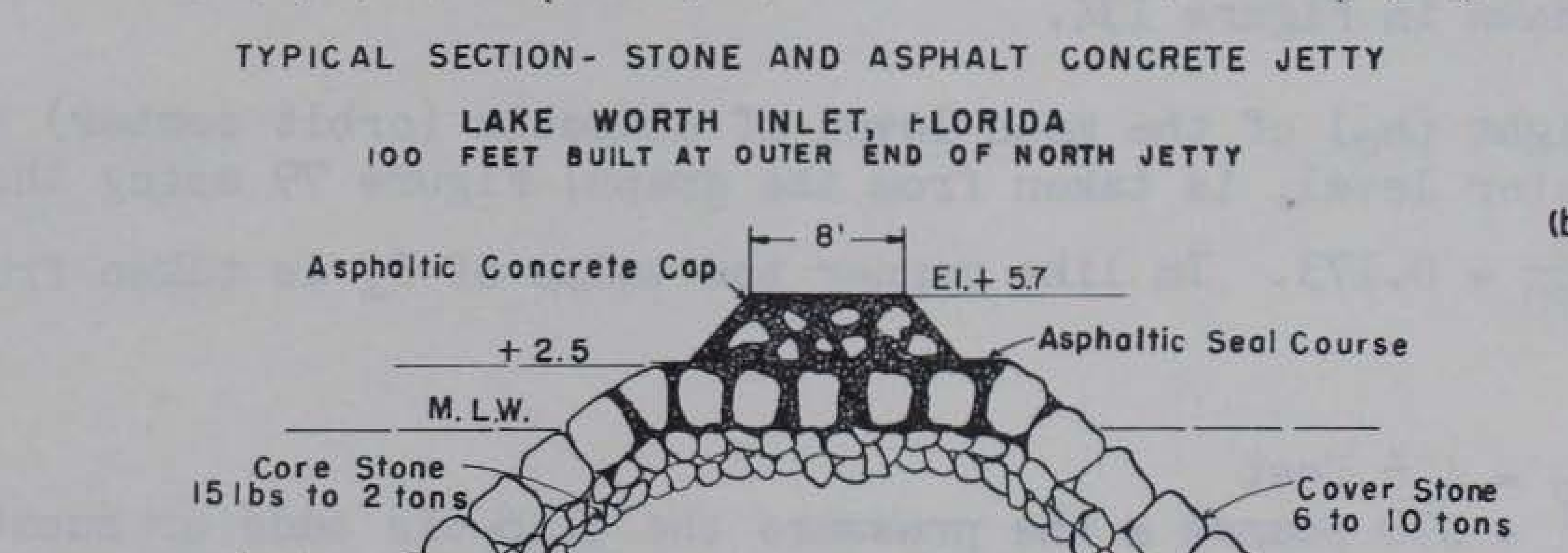


FIGURE 132 - TYPICAL SECTION — SOLID FILL BREAKWATER



TYPICAL SECTION - STONE AND ASPHALT CONCRETE JETTY

LAKE WORTH INLET, FLORIDA
100 FEET BUILT AT OUTER END OF NORTH JETTY



TYPICAL SECTION OF INNER PORTION OF SOUTH JETTY
GALVESTON HARBOR, TEXAS

FIGURE 133

Finally, the choice of structure would depend on either its first cost or its annual cost. All of the factors discussed affect the cost in one way or another. Generally the structures selected would be that one which would accomplish the desired purpose at the lowest average annual cost over the life of the project. Under some circumstances a structure with a somewhat higher annual cost might be selected to secure a substantial reduction in the first cost.

5.24 DESIGN PROBLEMS

5.241 CAISSON TYPE BREAKWATER(28) - It is desired to build a caisson type breakwater on a rubble riprap base in a Great Lakes area. Still water is assumed on the shoreward side of the breakwater.

The clapotis formed will correspond to the depth, $d = 26$ feet, at the foot of the wall; the elements h_o and P_1 will be computed for this depth. Between the top of the riprap at minus 26 feet and the bottom of the wall at minus 32 feet the riprap does not eliminate the pressure and it must be taken into account, which is done by using the pressure, p_1, computed at the top of the riprap and the depth of the floor of the wall, $d = 32$ feet in the formulas giving R_e and M_e. By application of methods for forecasting and determining the characteristics of the design wave, described in the section on wave action, the following data have been found:

$H = 13$ feet = design wave height at the structure
$L = 150$ feet = wave length at the structure
$T = 5.4$ seconds = wave period
$d = 26$ feet = depth of water below datum at the structure

The wave direction is normal to the breakwater. The depth of water being more than twice the wave height, the waves will not break on or before the structure and the Sainflou method for computing wave pressures from non-breaking waves can be used. The pressure diagram developed by this method is shown in Figure 134.

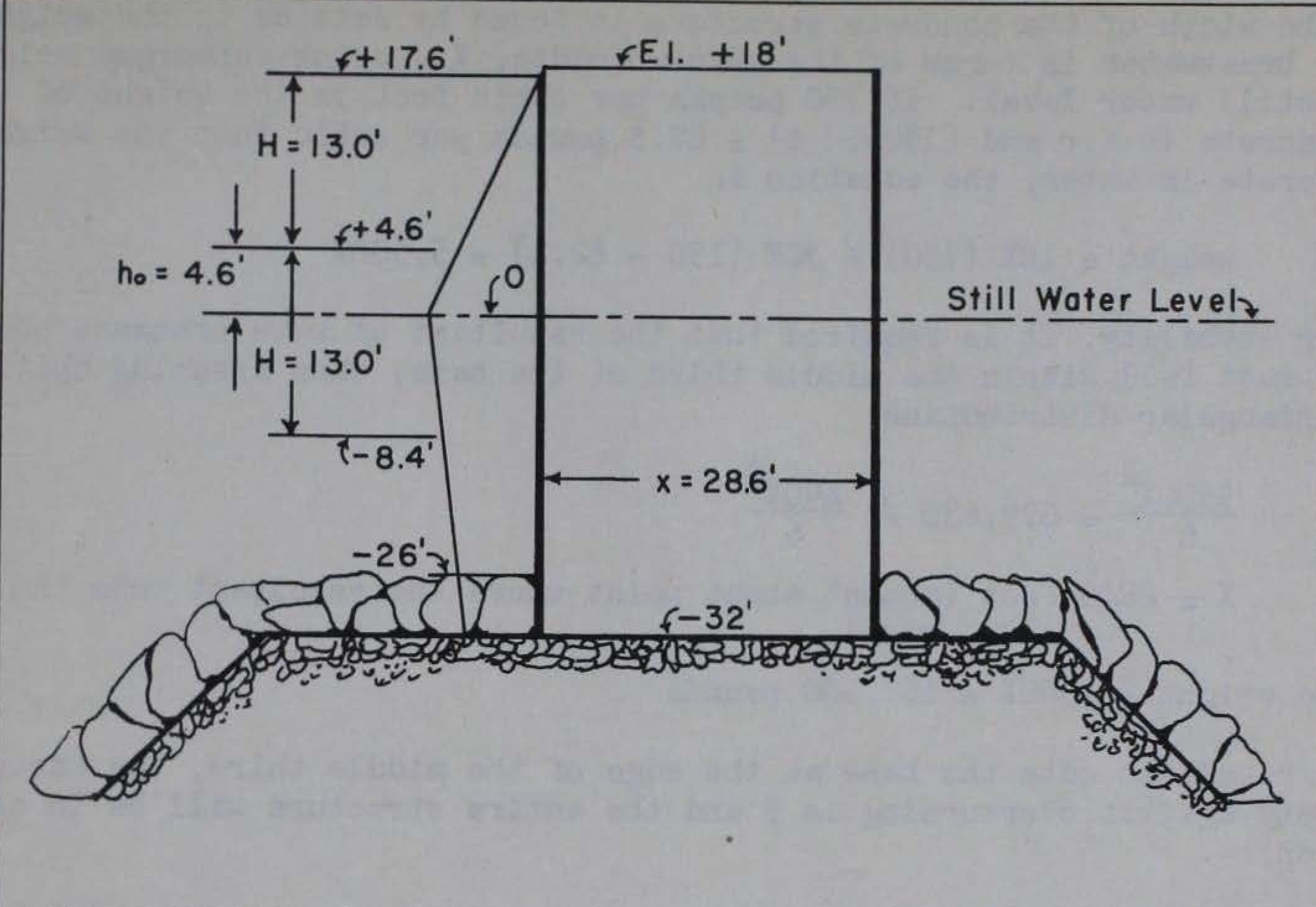
The height (h_o) of the mean level of clapotis (orbit center) above the still water level, is taken from the graph, Figure 79 using the value of $d/L = \frac{26}{150} = 0.173$. In like manner the value of P_1 is taken from Figure 80.

$h_o = 4.6$ feet
$P_1 = 490$ pounds = the pressure the clapotis adds or substracts from the still water pressure.

The upper and lower limits reached by the clapotis are

$h_o + H = 4.6 + 13 = 17.6$ feet above still water level

$h_o - H = 4.6 - 13 = 8.4$ feet below still water level



PRESSURE DIAGRAM FOR CAISSON TYPE BREAKWATER
FIGURE 134

Accordingly, to obstruct the oscillating wave completely, the breakwater should rise to not less than 17.6 feet above still water level.

With $d = 32$ feet, R_e and M_e may be found by

$$R_e = \frac{(d + H + h_o)\,(wd + P_1)}{2} - \frac{wd^2}{2} \qquad (23)$$

$$M_e = \frac{(d + h_o + H)^2\,(wd + P_1)}{6} - \frac{wd^3}{6} \qquad (24)$$

so

$$R_e = \frac{(32 + 17.6)\,[62.5\,(32) + 490]}{2} - \frac{62.5\,(32)^2}{2}$$

$$R_e = 29{,}750 \text{ pounds}$$

$$M_e = \frac{(49.6)^2\,[62.5\,(32) + 490]}{6} - \frac{62.5\,(32)^3}{6}$$

$$M_e = 679{,}630 \text{ foot-pounds}$$

The width of the concrete structure is found by setting up the weight of the breakwater in terms of the unknown width, X , using submerged weights below still water level. If 150 pounds per cubic foot is the weight of the concrete in air and (150-62.4) = 87.5 pounds per cubic foot the weight of concrete in water, the equation is

$$\text{weight} = 18X\ (150) + 32X\ (150 - 62.4) = 5{,}500X$$

If, for stability, it is required that the resultant of wave pressure and weight must fall within the middle third of the base, then assuming uplift in a triangular distribution,

$$\frac{5500X^2}{6} = 679{,}630 + \frac{490X^2}{6} \qquad (113)$$

$X = 28.6$ feet (moment about point where the resultant cuts the base)

and the weight $= 5500X = 157{,}300$ pounds

As the resultant cuts the base at the edge of the middle third, the factor of safety against overturning is 3 and the entire structure will be in compression.

To determine the factor of safety against sliding, multiply the effective downward force acting on the structure by a coefficient of static friction and divide by the horizontal thrust of the wave force. If the foundation is not dressed smooth, a friction coefficient of 0.5 or 0.6 is generally considered adequate, The factor of safety should not be less than 2.

$$\frac{\text{wt} \times 0.5}{R_e} = \frac{157{,}300 \times 0.5}{29{,}750} = 2.64$$

which exceeds two and is satisfactory.

The distance Z from the inner edge to the point where the resultant cuts the base, is $28.6/3 = 9.53$ feet since equation 113 was derived assuming that the total moment about this point is zero.

The maximum pressure against the foundation would be at the inside edge with maximum wave conditions. If Z is the distance between the inside edge of the base and the point where all the resultant forces intersect the base, V is the effective downward force per unit length of wall, G is the pressure on one square unit of foundation, and A is the width of the breakwater, then the ground pressure at the shoreward edge of the breakwater is:

$$G = \frac{2V}{A}\left(2 - \frac{3Z}{A}\right) \qquad (114)$$

$$G = \frac{2(157{,}300)}{28.6}\left(2 - \frac{3(9.53)}{28.6}\right) = 11{,}050 \text{ pounds}$$

This pressure would be large for most foundation conditions. Should this ground pressure be too great for the bearing power of the bed at the site, the base width of the structure must be increased until the pressure is within the allowable load.

5.242 Rubble-Stone Breakwaters(49)(58) - The design of a rubble-stone breakwater is best illustrated by numerical example. It is assumed that a breakwater is to be constructed in sea water in a depth of 30 feet. The stone available locally has a specific gravity of 2.65. The quarry is capable of producing adequate quantities of stone weighing as much as 10 tons.

Referring to the methods of Section 4.27, with the wave height at the structure's location, given by

$$H = H_o \left(\frac{H}{H_o'}\right) \sqrt{b_o/b} \tag{13}$$

assume $H_o = 17$, $L_o = 184$, $T = 6$, $\sqrt{b_o/b} = 0.84$ and $d = 30$, $d/L_o = 0.163$. From Table D-1, Appendix D, $d/L = 0.194$ and $H/H_o' = 0.913$. Therefore, from equation ~~12~~, 13 $H = 13$ feet. From Plates D-7a and Table D-10, stable slopes for various stone weights may be found in the following manner.

COMPUTATIONS FOR STABLE ABOVE SURFACE SLOPES

Slope	W/K' (Plate D-7a)	K' (Table D-10)	W (pounds)	W (tons)
1 on 3	3.4×10^5	0.036	12,200	6.1
1 on 2	8.8×10^5	0.017	15,000	7.5
1 on 1.5	3.1×10^6	0.008	24,800	12.4

Since 10-ton stone would be stable on a slope somewhere between 1 on 1.5 and 1 on 2, the slope of 1 on 1.75 will be chosen for this design. (Further refinements may be calculated by use of the equation

$$W = \frac{209\ K'\ H^3}{(1.05 \cos \alpha - \sin \alpha)^3\ (1.62)^3} \tag{47a}$$

and reference to Table D-11)

Sub-surface slopes may be determined by substituting, for H in equation 47a for any depth, a hypothetical wave height H' given by

$$H' = \frac{\pi\ H_s^{\ 2}}{L_o \left(\sinh \frac{2\pi d}{L}\right)^2} \tag{49}$$

in which H_s is determined by extending equation ~~12~~ 13 to points over the breakwater slope. Starting at depth 13 (one wave height below the surface), the computation for H' is illustrated in the following table.

COMPUTATION OF H_s AND H' FOR VARIOUS VALUES OF d

d	d/L_o	$\sinh \frac{2\pi d}{L}$	(H/H_o')	H_s*	H'*
13	0.0705	0.7823	0.9704	13.9	5.4
20	0.109	1.076	0.9263	13.2	2.6
30	0.163	1.546	0.9130	13.1	1.2

*H_s and H' are plotted on Figure 135.

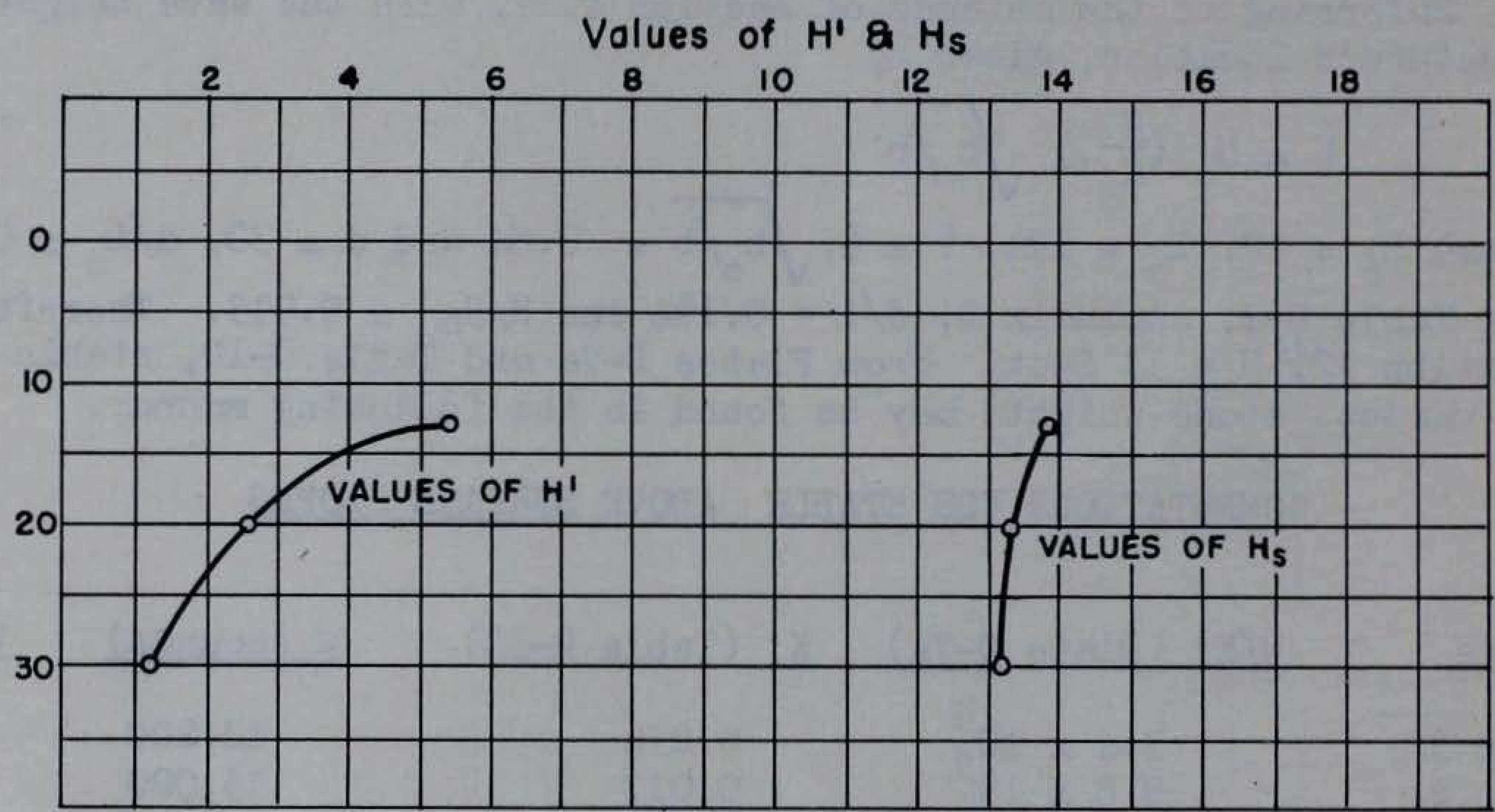


FIGURE 135 - VALUES OF H' AND H_s IN TERMS OF DEPTH (d)

Assuming that 6-ton stone is available and will be placed in the section from the elevation -13 to the bottom, and that quarry run stone averaging 1/4 ton is available, stable slopes may be found from Plate D-7a, Appendix D, with K' = 0.015. With the 6-ton stone, W/K' = 8 x 10^5 and with H' = 5.4 feet (d = 13 feet) the stable slope is found to be about 1 on 1.25. If the 1/4-ton stone had been placed below -20, W/K' = 3.3 x 10^4 and with H' = 2.6 feet (d = 20 feet) the stable slope would be about 1 on 1.4, however, in this case extending the secondary armor to the bottom does not materially increase the quantity needed.

In order to obstruct the waves effectively the breakwater crest should be at an elevation of about 19.5 feet (H x 1.50) above still water level. If only partial obstruction is necessary the crest elevation may be set somewhat lower. The choice between the two must be made on a use and economic basis. The crest width is related to the size of cap stone used. The width is selected so as to provide for a minimum of two interlocking cap stones of the design size. In the example of Figure 122, a crest width of 20 feet was used. Landward side slopes ordinarily should not be less than 1 on 1.5 to the depth of one wave height (13 feet), thence 1 on 1.25 to the bottom. With these criteria and the computed sea side slopes of 1 on 1.75 to elevation -13 and 1 on 1.25 to the bottom, a stable profile may be drawn as in Figure 136. For ease of construction, a shelf of some 5 feet more or less may be added to the lower seaward face stone. Note that end slopes should be designed in the same manner as sea side slopes.

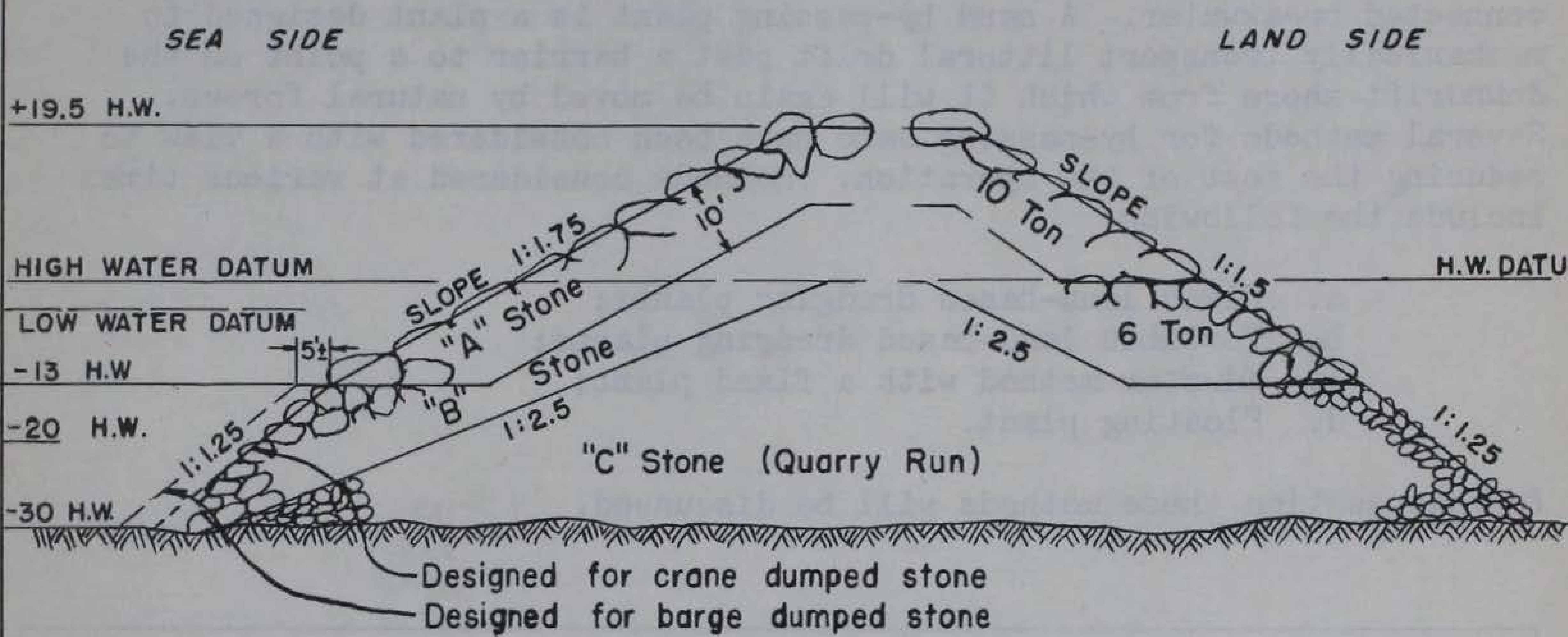


FIGURE 136 - PROFILE OF BREAKWATER SIDE SLOPES

5.243 Composite Type Breakwaters - A composite type breakwater is one comprised of two or more types and materials, or of two or more materials. There have been many types of composite breakwaters designed and constructed where it has appeared that a saving in time, material, or cost would be effected thereby. One type of composite breakwater consisted of a shale or earth core incased in a rubble-mound covering. Another type is comprised of a concrete caisson wall set on a rubble-mound base.

As wave action is affected by the profile of the substructure, the characteristics of the wave which act on the superstructure must be determined accordingly. Should it be desired to place the base of the concrete caisson breakwater described in a previous example on a rubble-mound substructure at elevation -10 feet, values for H and H' should be found for this depth by Iribarren's method. H' is used in computing the weight of the individual stone at elevation -10 feet, and H is the shallow water wave height at the superstructure. The wave height H should be used in the stability analysis of the concrete superstructure. Assuming the same conditions as in the previous examples, the wave characteristics are computed to be:

$$H' = 8.1' \quad H = 14.3' \quad d = 10' \quad d/L_o = 0.0543$$

$$d/L = 0.09859 \quad L = 101.4'$$

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Using these wave data, the concrete caisson would be designed according to the procedure described in Section 5.241.

5.3 SAND BY-PASSING

5.31 METHODS - It is often desired to nourish beaches downdrift from a natural or artificial barrier within the littoral zone, the trapped material acting as a source of supply. Such a littoral barrier may be a jettied entrance to a harbor, a natural inlet, an offshore breakwater, or a shore connected breakwater. A sand by-passing plant is a plant designed to mechanically transport littoral drift past a barrier to a point on the downdrift shore from which it will again be moved by natural forces. Several methods for by-passing sand have been considered with a view to reducing the cost of the operation. Methods considered at various times include the following:

a. Fixed land-based dredging plants;
b. Portable land-based dredging plants;
c. Eductor method with a fixed plant;
d. Floating plant.

In this section these methods will be discussed.

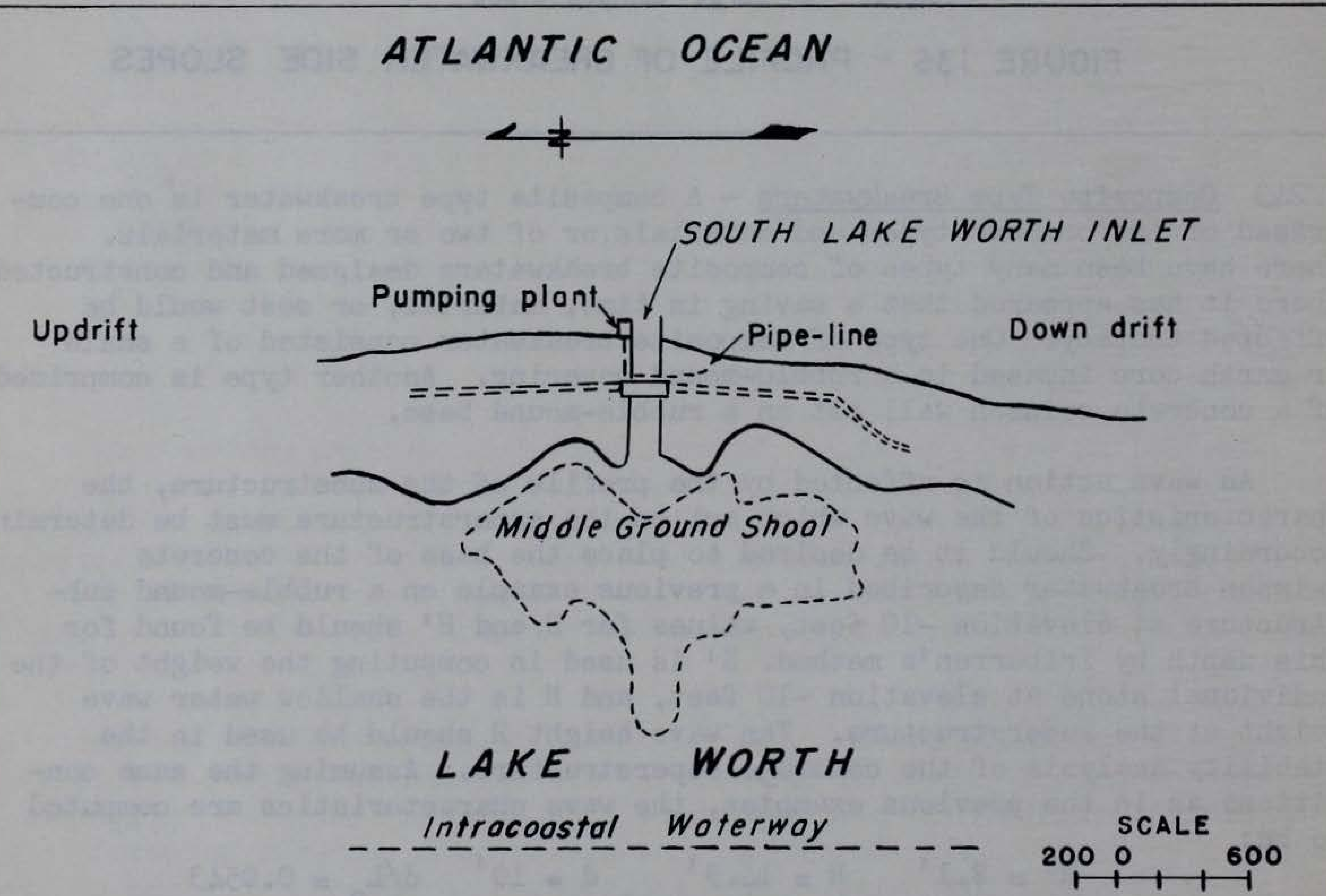


FIGURE 137 - SOUTH LAKE WORTH INLET BYPASSING PLANT

5.32 FIXED PLANT - A small fixed plant has been used for some time at South Lake Worth Inlet, Florida(136) (also see Figure 137). After construction of jetties at South Lake Worth Inlet, erosion occurred on the downcoast beach. A seawall and groin field failed to protect the shore line and a small by-passing plant was placed in operation in 1937. The basis of design of the pump and pumping plant as initially installed was not related to the rate of littoral transport along the shore. It was designed to transport the quantity of sand over a period of 2 years, estimated as required to fill the groins and give adequate protection to the seawall. Pumping was continued until 1942.

The pumping plant consisted essentially of an 8-inch suction line, a 6-inch 65 h.p. Diesel-driven centrifugal pump, and about 1,200 feet of 6-inch discharge line. The installation had a capacity of about 55 cubic yards an hour and averaged 48,000 cubic yards a year during 4 years of operation. The normal rate of littoral transport during that same period was on the order of 225,000 cubic yards a year. At the end of 5 years the beach was restored for a distance of over a mile downcoast.

During the 3-year period, 1942-1945, pumping was discontinued with the result that severe erosion of the beaches south of the inlet again occurred. In 1945, the plant was again placed in operation. In an attempt to reduce shoaling in the entrance channel, the size of the installation was increased to a 10-inch suction line, 8-inch pump with 275 h.p. Diesel motor, and 8-inch discharge line. Observations indicate that a larger pump is required for this purpose and the installation of a 12-inch pump and discharge line is under consideration.

A by-passing plant on a large scale has been constructed at Salina Cruz, Mexico(9). Salina Cruz is an artificial harbor in the Gulf of Tehuantepec on the Pacific Coast of Mexico. (See Figure 138) This plant essentially consists of six 18-inch suction pipes operating through two dredge pumps. Each dredge pump is motivated by a 450 h.p. Diesel engine. An 18-inch discharge line crosses the entrance to the inner harbor on a drawspan and discharges on the beach downcoast from the eastern breakwater.

The Salina Cruz installation immediately encountered operational difficulties. The suction pipes of the plant were not long enough to open a channel in the beach that would permit the free entrance of the sand to the dredge. An attempt has been made to open this channel by means of a dragline operating across the beach by a cable supported from two posts. As yet it is not known whether operational difficulties have been overcome.

In theory, the plant is designed with a capacity somewhat greater than the average rate of littoral transport. Its purpose is to pump initially a sufficient quantity to pull the shore line back to the future alignment as shown on Figure 138. Thereafter the plant would pump all material coming to it along the coast. This is expected to prevent shoaling of the harbor as well as prevent damage of downdrift areas. The plant has cost

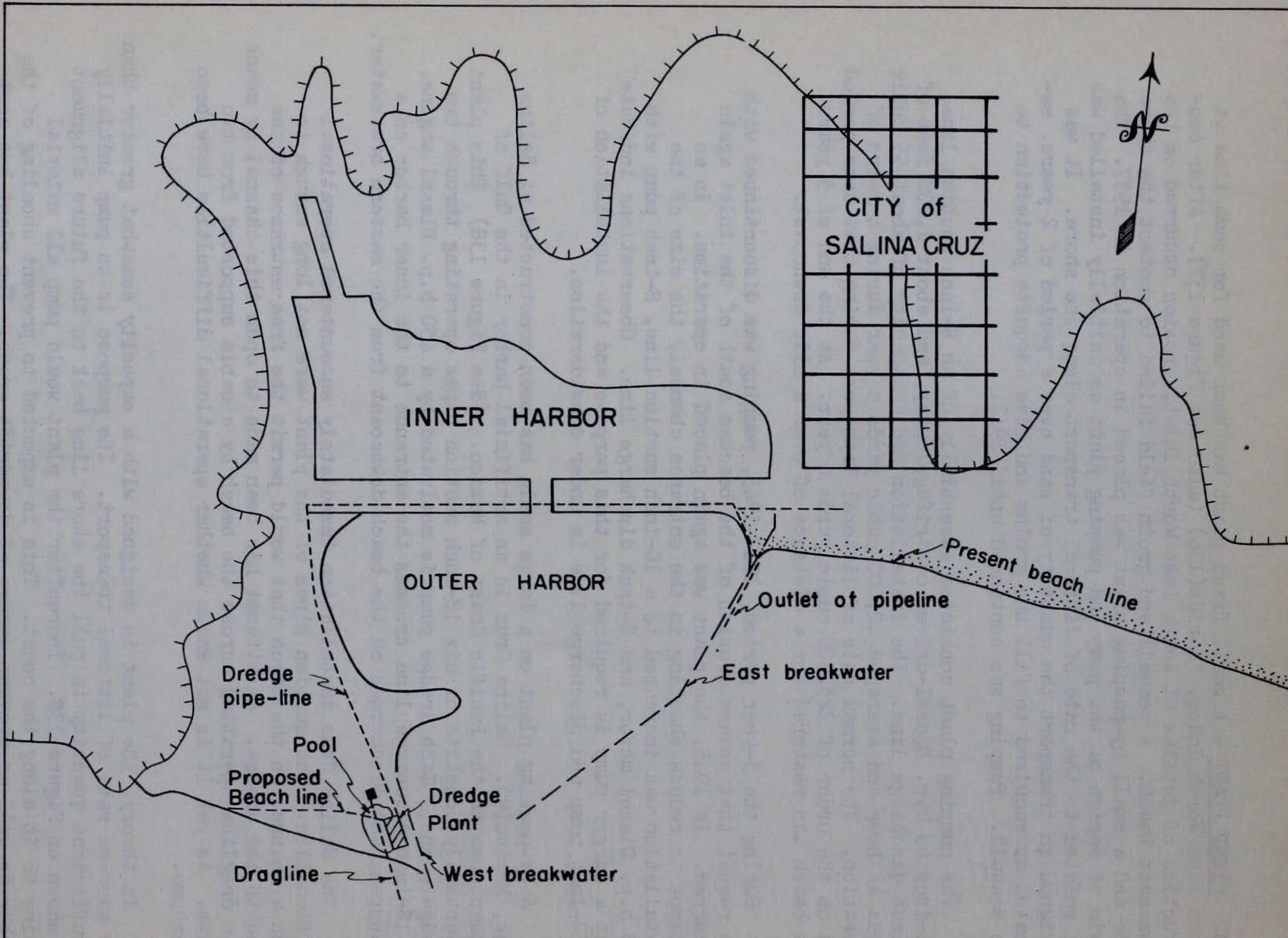


FIGURE 138-FIXED DREDGE, SALINA CRUZ, MEXICO

approximately $2 million to date. It remains to be seen if it can be made to function as designed.

5.33 PORTABLE LAND-BASED PLANT - Because of the wide range in the daily rate of littoral transport any by-passing plant designed to pump substantially all of the material moving along the coast, must necessarily have a capacity of several times the average daily rate computed from an annual basis. Thus, after the operating sequence has been established the plant would be required to operate only a fraction of the time, probably not over 4 to 5 hours a day. In an effort to reduce plant costs, consideration has been given to a portable land-based dredging plant. This plant would probably be operated from a fixed pier or other foundation. It would have to be highly portable so that it could be rapidly and easily assembled, disassembled, and transported between sites. By working full time, such a plant would be able to by-pass sand from several sites, provided adequate sand traps were constructed to impound littoral material between dredging. However, no known working plants of this nature are presently in operation.

5.34 EDUCTORS - The successful use of eductors in moving 14 million cubic yards of sand from dunes to the beach during the preparation of the Hyperion sewage treatment plant site on Santa Monica Bay, California, led to a consideration of the possibility of their use for by-passing sand. In principal the eductor is merely a venturi activated by a high pressure jet. It can be effectively used to move sand for distances up to 2,000 feet, depending on the size of the eductor and high pressure jet. The efficiency of the eductor is somewhat lower than that of an ordinary dredge pump with a suction pipe. Studies to date have failed to indicate sufficient savings in construction or operating labor costs as to offset the lower efficiency and show an overall saving in cost per cubic yard of material moved.

5.35 FLOATING PLANT - To date no satisfactory method has been found which offers any improvement on the proven methods of periodic movement of beach material by hydraulic pipe-line dredge. Sand has been successfully by-passed by this type of plant at Santa Barbara, California, for the last 15 years. (See Figure 139). The rate of littoral transport at Santa Barbara has been determined to be about 270,000 cubic yards a year. It moves downcoast. This drift was completely intercepted by a shore-connected breakwater, constructed in 1927. After its construction, the upcoast angle of the breakwater filled in and material passed along and around the seaward end of the structure, settling in the protected harbor area. The resultant shoal extended across the harbor entrance towards shore. Coincident with the accretion of material in and upcoast of the harbor, erosion began on the downcoast side of the harbor. Within a period of 10 years, the erosion zone extended a distance of 10 miles downcoast and the damage amounted to millions of dollars.

A project was inaugurated in 1935 for the maintenance of navigable depths in the harbor entrance and the restoration of the downcoast beaches by cooperation of the United States and the city and county of Santa Barbara. The first dredging in 1935 was done by hopper dredge. The material was deposited in 18 feet of water in a mound parallel to shore.

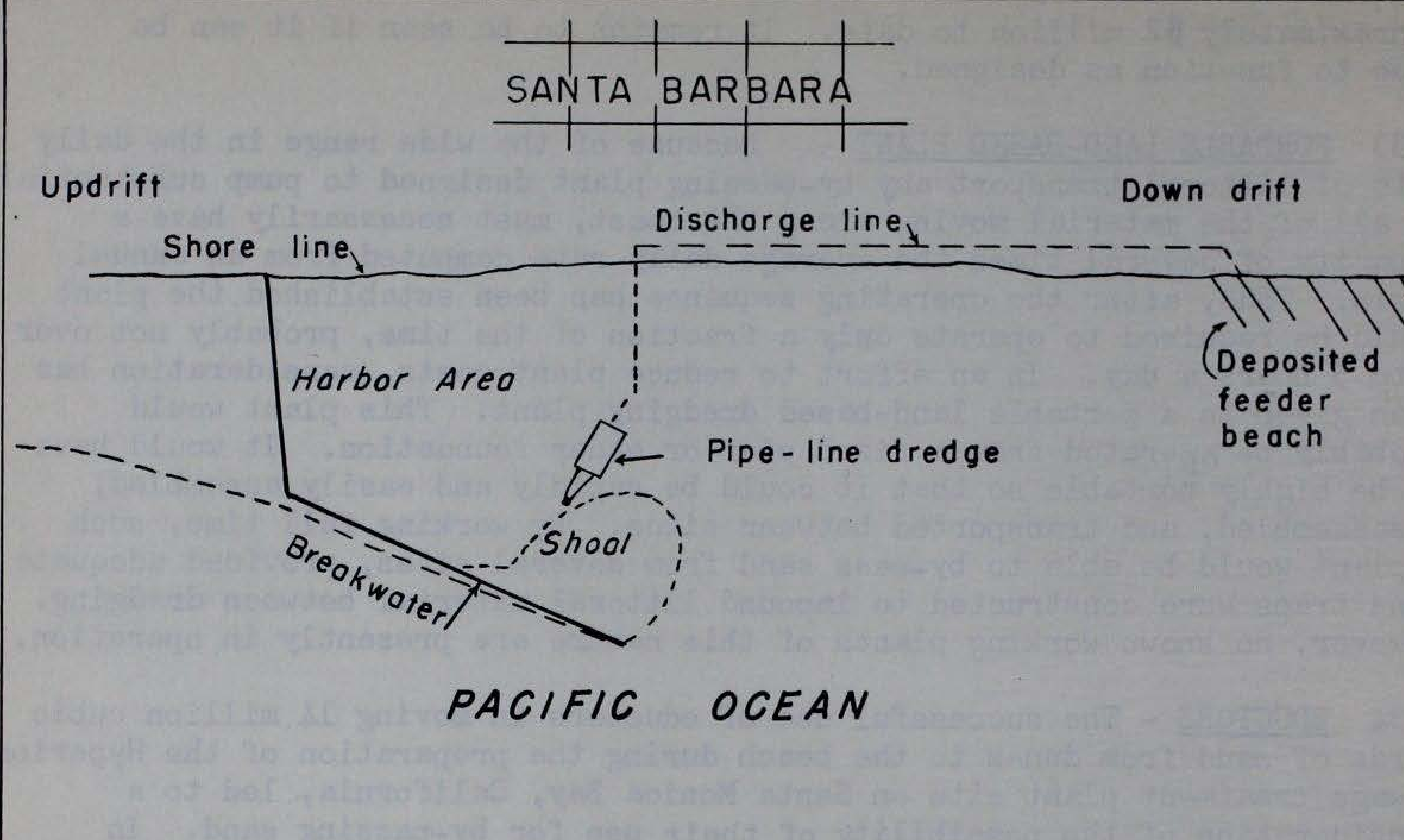


FIGURE 139 – SANTA BARBARA HARBOR-SAND BYPASSING

It was hoped that the material would be moved ashore by wave action, a hope that was never realized. Subsequent dredging has been accomplished at 2 to 3-year intervals by hydraulic pipe-line dredge. In these operations, amounts ranging from 500,000 to 800,000 cubic yards of material were dredged and deposited directly on the beach downcoast of the zone of influence of the breakwater. Within a second 10-year interval the beaches had been re-established and the shore line stabilized along its new alignment. The action at Santa Barbara has adequately demonstrated the efficacy of periodic by-passing of material to maintain normal littoral drift to the downdrift beaches.

Two things are required for the effective use of floating plant for periodic by-passing of sand: (1) a sand trap of sufficient size as to retain littoral material between dredging operations; and (2) a protected area within which the suction dredge can operate in safety. Both conditions were satisfied by the breakwater at Santa Barbara. The Santa Monica Breakwater (see Figure 73) has provided similar protection, as well as a sand trap, from which similar by-passing has been accomplished.

In a few instances, dredging with floating plant has been attempted on the open coast by confining operations to calm periods. Even so, much lost time has ensued and damage has been severe with the results that contractors either submit high prices or no bids at all on this kind of work. If conditions are favorable, consideration may well be given to the construction of a combination sand trap and protective structure.

5.4 SAND DUNES(71)(76)

The problem of dune control resolves itself into two fundamental objectives: (1) the stabilization and maintenance of sand dunes at locations where they exist naturally; and (2) the inducement of dune formation where they do not exist naturally, by the direct and permanent stoppage or impounding of sand before the location to be protected, and the stabilization of the dune formation after its creation.

Wind blown sand accumulates in several distinctive ways and characteristic forms. Any appreciable accrual is loosely termed a dune. Strictly, according to the most generally accepted definition, a true dune is one capable of moving freely as a unit and which can exist independently of any fixed surface structure. The loose definition is used herein. Sand accumulations caused directly by fixed obstructions in the path of the wind, unlike true dunes, are dependent for their continued existence on the presence of the obstacle which causes and fixes them so they cannot move away.

When an object is placed so as to interrupt the wind flow, the air path in front and behind the obstacle is divided into two parts by a somewhat ill-defined surface of discontinuity. Outside this surface the air stream flows smoothly by; but the volume within the wind shadow of the obstacle is filled with swirls and vortices of air whose average forward velocity is less than that of the air stream outside. Downwind from the obstacle, the forward velocity of the air inside the shadow gradually increases and the shadow fades away to merge eventually with the general flow of the wind. The sand grains which strike the obstacle rebound from it and come to rest in the relatively stagnant air in front. When the resulting heap has grown up so that its slopes stand at the limiting angle of repose (about 34°) all additional material slides down the slope to join the sand stream passing along the side of the obstacle.

5.41 DUNE BUILDING - Dunes can be caused to form by the use of sand fences or crude oil.

Sand fences can be constructed in movable sections or made of individual pickets driven into the sand. The width and length of the pickets may vary but the spacing of the pickets is important, with no more than 50 percent of the surface covered. For best results the space between the pickets should equal the width of the pickets. In order to widen the crest of the dune, and facilitate establishment of vegetation, two lines of fence about 30 feet apart should be used. The use of a single fence tends to make dunes with a sharp crest unfavorable to establishment of cover. As the dune builds up on the fence, the fence can be raised until the desired height is attained. The belt can be broadened by shifting the second fence windward as the dune grows, or by the addition of a third fence.

The sand fences should be constructed normal to the prevailing wind direction unless it is desired to cause the sand to move longitudinally along the fence to fill in low gaps. In this case the fences may be built slightly quartering to the general shifting direction of the sand movement or panelling may be resorted to.

Panelling, to divert or stop sand, consists of curved or flat barriers in a single slant or in V arrangement. As the line of the slant or V approaches the normal to the wind direction, there is more and more stoppage, and less and less diversion of sand. These barriers require frequent changing or cleaning. The dune development behind diversion fences is shown in Figure 140.

Oil coats of hot crude oil may be used to stabilize the face of the dune and cause the dune to widen. Penetration is deeper when the upper few inches of sand are dry. Oiling is expensive and special equipment is necessary for highest efficiency. Oiling is effective for only a few years and must be repeated frequently. Oiling increases the saltation coefficient of material flowing over the surface. The sand blows up and over the top, depositing in front of the slip face in successive advancing fill increments. This widening of the dune continues until a streamlined body is built downwind from the paved face. The entire mass is permanently stored for as long as the oiled surface remains intact. Figure 141 shows the successive steps of dune development behind an oiled surface.

5.42 DUNE STABILIZATION - After protective dunes have been formed they should be stabilized with vegetation. This is expensive in the beginning but minimizes future difficulty. The most satisfactory plants are long lived perennials, with extensive root systems, that spread rapidly either vegetatively or by seed or both, and maintain surface growth even though sand is accumulating around them to increasing depths. Such plants are not numerous but in practically every section a few satisfactory ones are obtainable at reasonable costs.

Usually grasses are available and can be transplanted from naturally established plantings. Transplanting is most satisfactory by using small clumps 1/2 to 1 inch in diameter, setting about 8 to 10 inches deep in staggering rows with plants in rows about 18 inches apart. Staggering the rows prevents direct wind action over long areas and offers more opportunity to hold sand than straight line planting. Dunes are low in fertility and contain practically no organic matter. Nitrogeneous fertilizers are stimulating and while not effective very long in the sand, they are of benefit in inducing vigor and enabling newly set plants to get firmly established the first year. The use of coarse fertilizers as manure is generally not practicable because its effect is so slow that little of its value would be realized.

Brush barriers may be used effectively to control the sand on newly filled areas while grasses, shrubs, or trees are gaining a foothold. These barriers are usually built in rows 4 feet apart, crosswise with the prevailing wind, about 2 feet high, 1 to 2 feet wide, and anchored firmly with

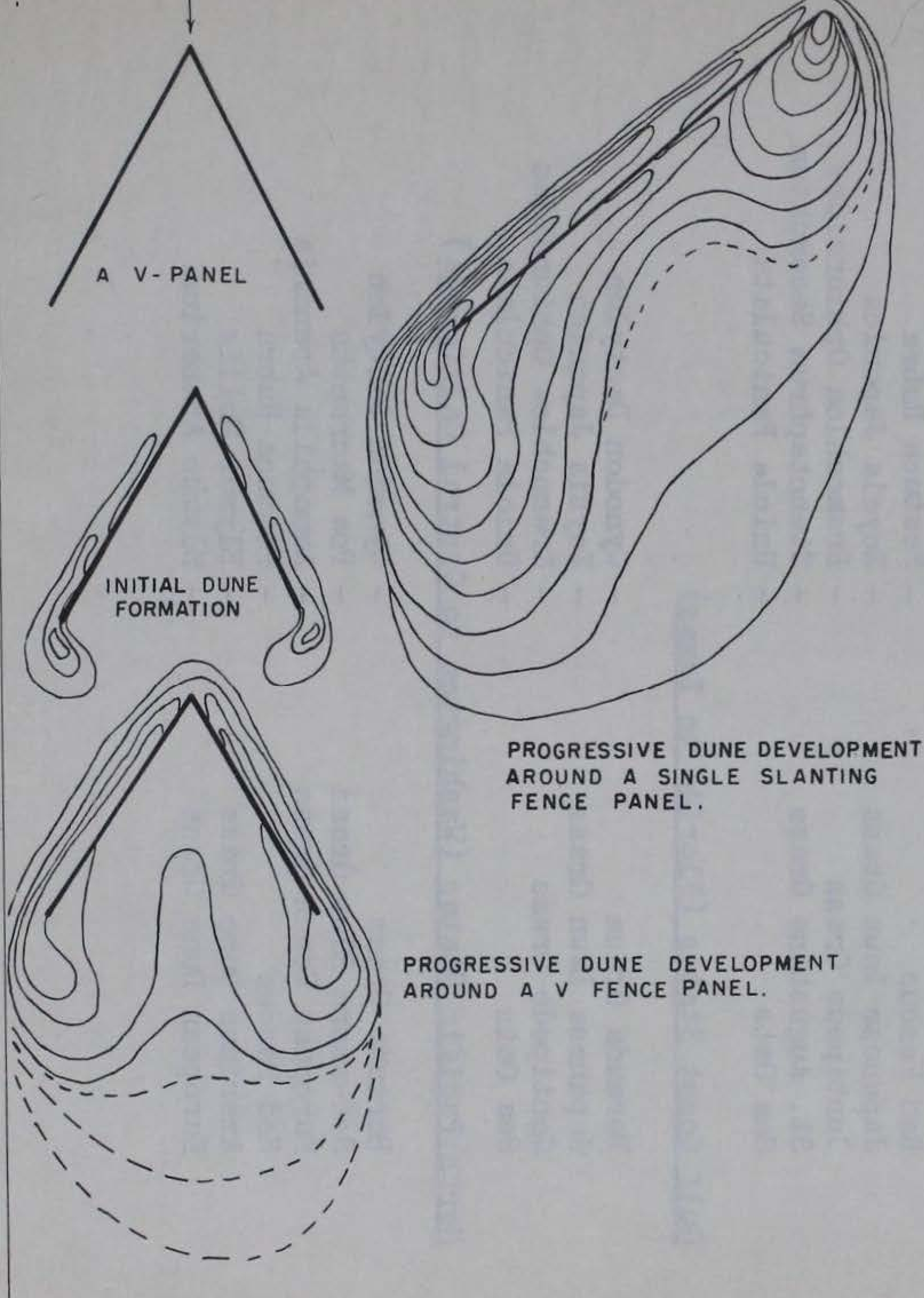


FIGURE 140-DUNE DEVELOPMENT AROUND FENCE PANELS.

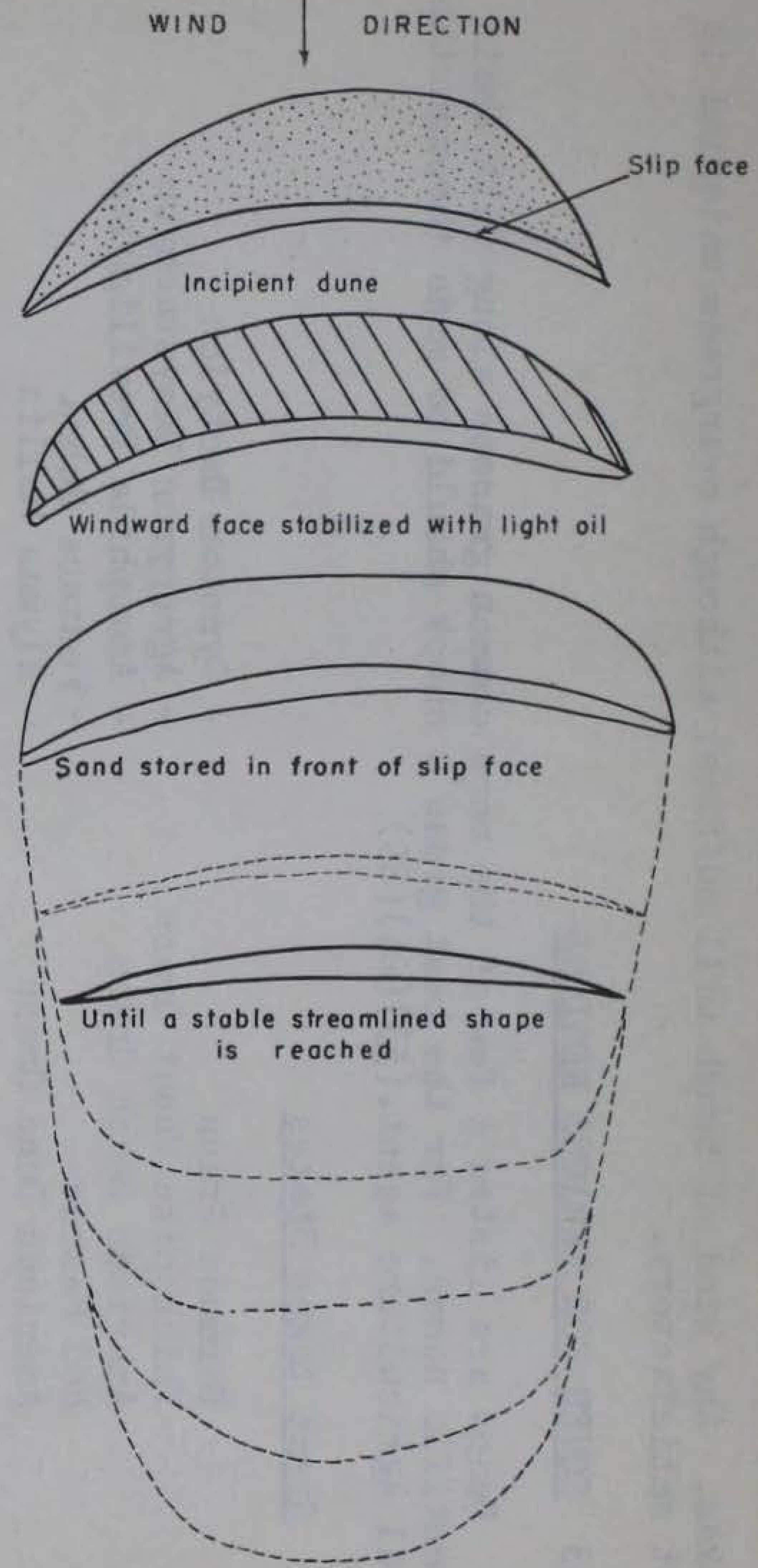


FIGURE 141-DEVELOPMENT OF AN OILED DUNE

stakes. Any kind of brush will suffice, although evergreen material is most satisfactory.

5.43 COVER FOR VARIOUS REGIONS

Below are listed a few of the more common grasses along with their scientific names. For the best grass a check should be made through the local Agriculture agent.(53)(84)(85)

Great Lakes States

Bermuda Grass	- Cynoton Dactylon
Thickspike Wheat Grass	- Agropyron Dasystachyum
American Beach Grass	- Ammophila Breviligulta
Red Fescue	- Festuca Rubra
American Dune Grass	- Elymus Mollis

North Atlantic (Maine to Maryland)

Bermuda Grass	- Cynoton Dactylon
American Beach Grass	- Ammophila Breviligulta
Red Fescue	- Festuca Rubra
American Dune Grass	- Elymus Mollis

South Atlantic (Virginia to Florida)

Bermuda Grass	- Cynodon Dactylon
Red Fescue	- Festuca Rubra
Japanese Lawn Grass	- Zoysia Japonica
Centipede Grass	- Eremochloa Ophiuroides
St. Augustine Grass	- Stenotaphrum Secondatum
Sea Oats	- Uniola Paniculata

Gulf Coast States (Florida to Texas)

Bermuda Grass	- Cynodon Dactylon
Japanese Lawn Grass	- Zoysia Japonica
Centipede Grass	- Eremochloa Ophiuroides
Sea Oats	- Uniola Paniculata

North Pacific States (Washington to Central California)

Bermuda Grass	- Cynodon Dactylon
Sea-shore Blue Grass	- Poa Macrantha
European Beach Grass	- Ammophila Arenaria
Red Fescue	- Festuca Rubra
American Dune Grass	- Elymus Mollis
European Dune Grass	- Elymus Arenarius

South Pacific State (Central to Southern California)

Bermuda Grass	- Cynodon Dactylon
Japanese Lawn Grass	- Zoysia Japonica
Red Fescue	- Festuca Rubra

After a stand of grass has been obtained, a well planned secondary growth program of scrubs and trees should be started to further stabilize the sand.

5.5 GROINS

5.51 GENERAL - In the past, groins generally have been built, not designed. As a result, many have been too light to withstand the wave forces and have failed. Others have been overbuilt to such an extent as to be uneconomical. Because of the many unknowns involved, this section on design is an attempt at standardization on a safe basis rather than any presentation of an exact design analysis.

As described in Section 3.45, based on functional considerations, groins are classified principally as to permeability and height. Groins built of common construction materials can be made permeable or impermeable, high or low. The materials commonly used are concrete, steel, stone and timber. Asphalt has found a limited application in groin construction.

5.52 CONCRETE GROINS

5.521 Concrete Permeable Groins - Of the various types of permeable concrete groins, that patented by Sydney M. Wood probably has been the most widely used. Accordingly, it has been taken as typical of this type of groin. This groin consists of precast reinforced concrete units shaped like flat dumbbells. (see Figure 142) These units are threaded on piles to form cribs. The piles contribute materially to the stability of the groin and may be used to support erecting equipment. A part of the theory of permeable groins is that the permeability may be varied, the longitudinal members may vary with respect to the number of vertical lugs. The lengths of the units range from 6 to 14 feet and the corresponding weights from 3/4 to 1-1/2-tons. The width of the groin is equal to the length of the units. The 14-foot lengths are used below water; varing the lengths from the surface of the water to the top of the groin makes a sloping side and a top width of 6 feet. The piles serve as guides to position the units and lock them together. Each group of four piles and connecting units comprise an independent crib unit, permitting a measure of unequal settlement without failure of the structure. These and other concrete groins often are constructed with concrete top slabs.

5.522 Concrete Impermeable Groins - Impermeable concrete groins may be either articulated or solid throughout their entire length. In general, the articulated type appears to be the most practicable as it will permit unequal settlement. Also, being handled in smaller units, it is simpler to construct as costly forming is avoided. Figures 143, 144, and 145 show three types of impermeable concrete groins. Figure 143 shows a combination

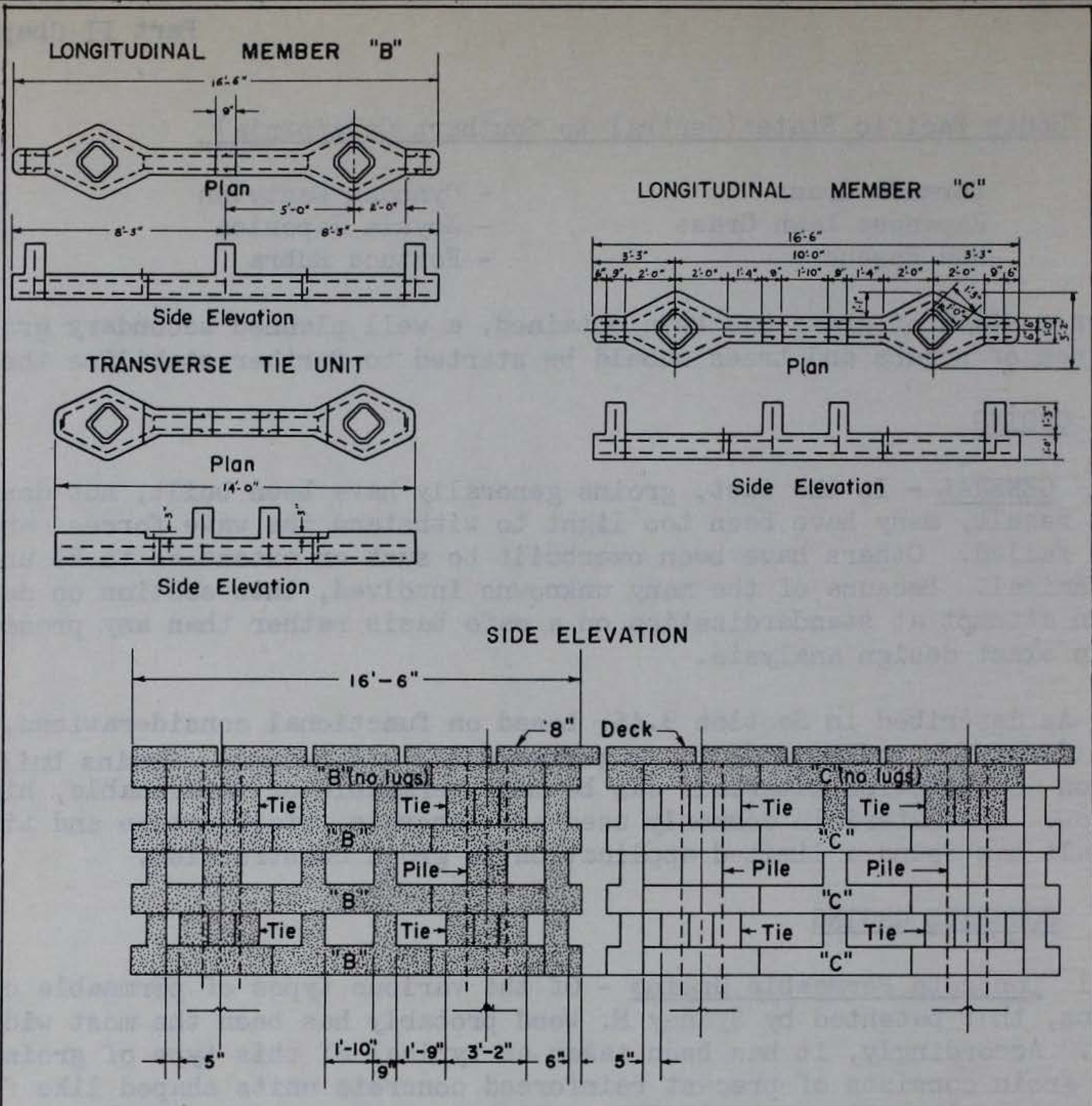


FIGURE 142 CONCRETE PERMEABLE GROIN (Sydney.M.Wood)

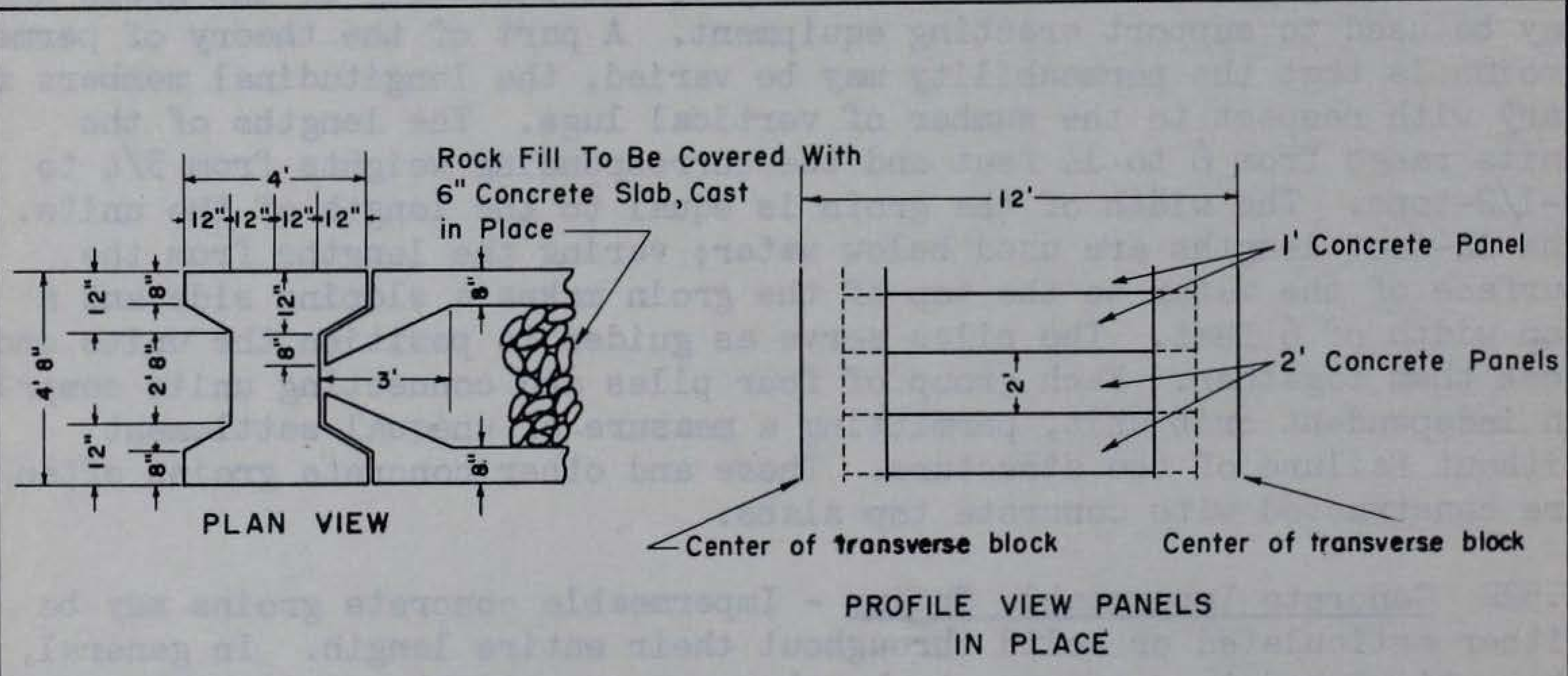


FIGURE 143-COMBINED PRECAST CONCRETE AND ROCK FILL GROIN

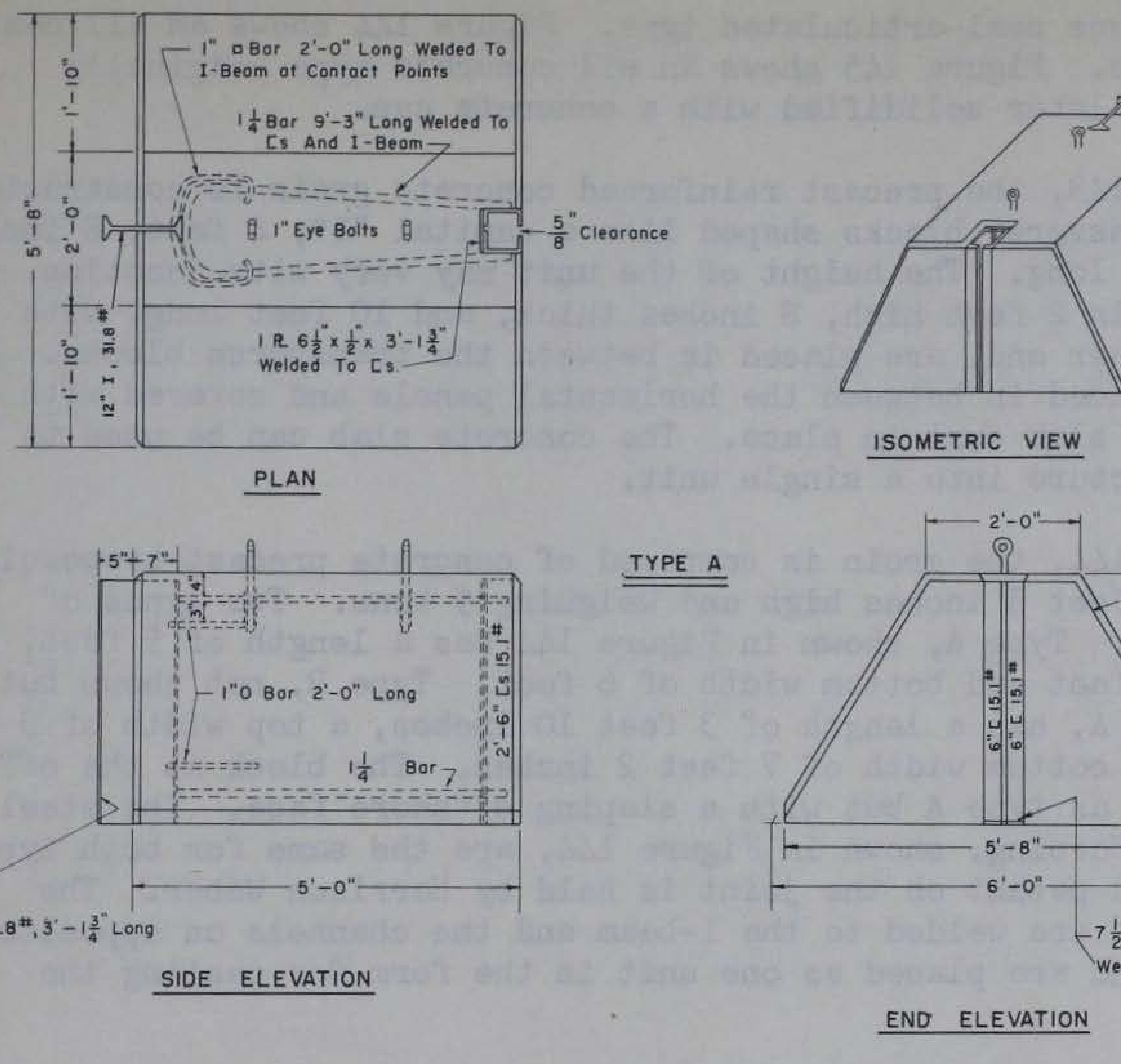


FIGURE 144 CONCRETE BLOCK GROIN (Harrison Weber)

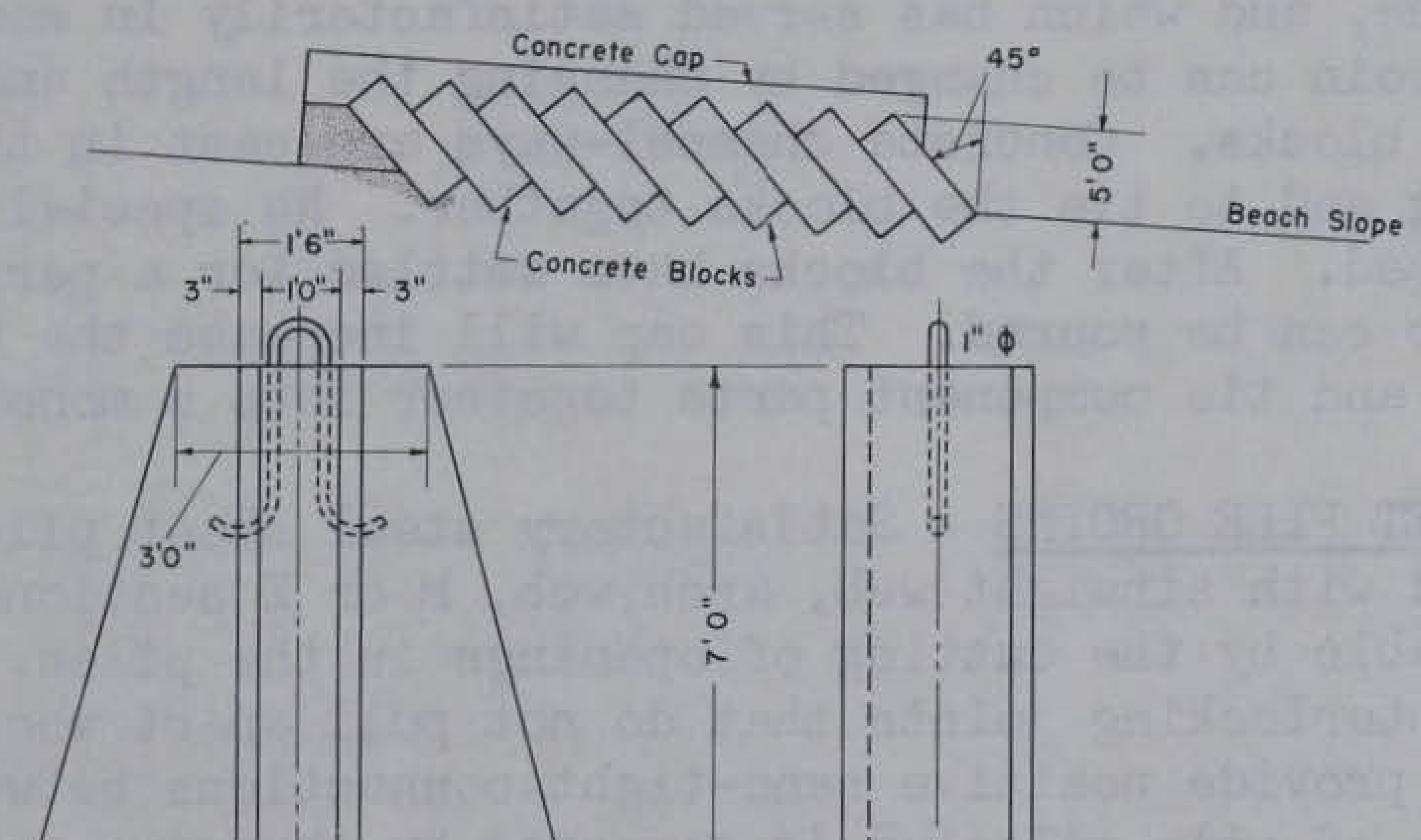


FIGURE 145 CONCRETE BLOCK GROIN (C. L. Leeds)

concrete and stone semi-articulated type. Figure 144 shows an all concrete articulated type. Figure 145 shows an all concrete type originally articulated and later solidified with a concrete cap.

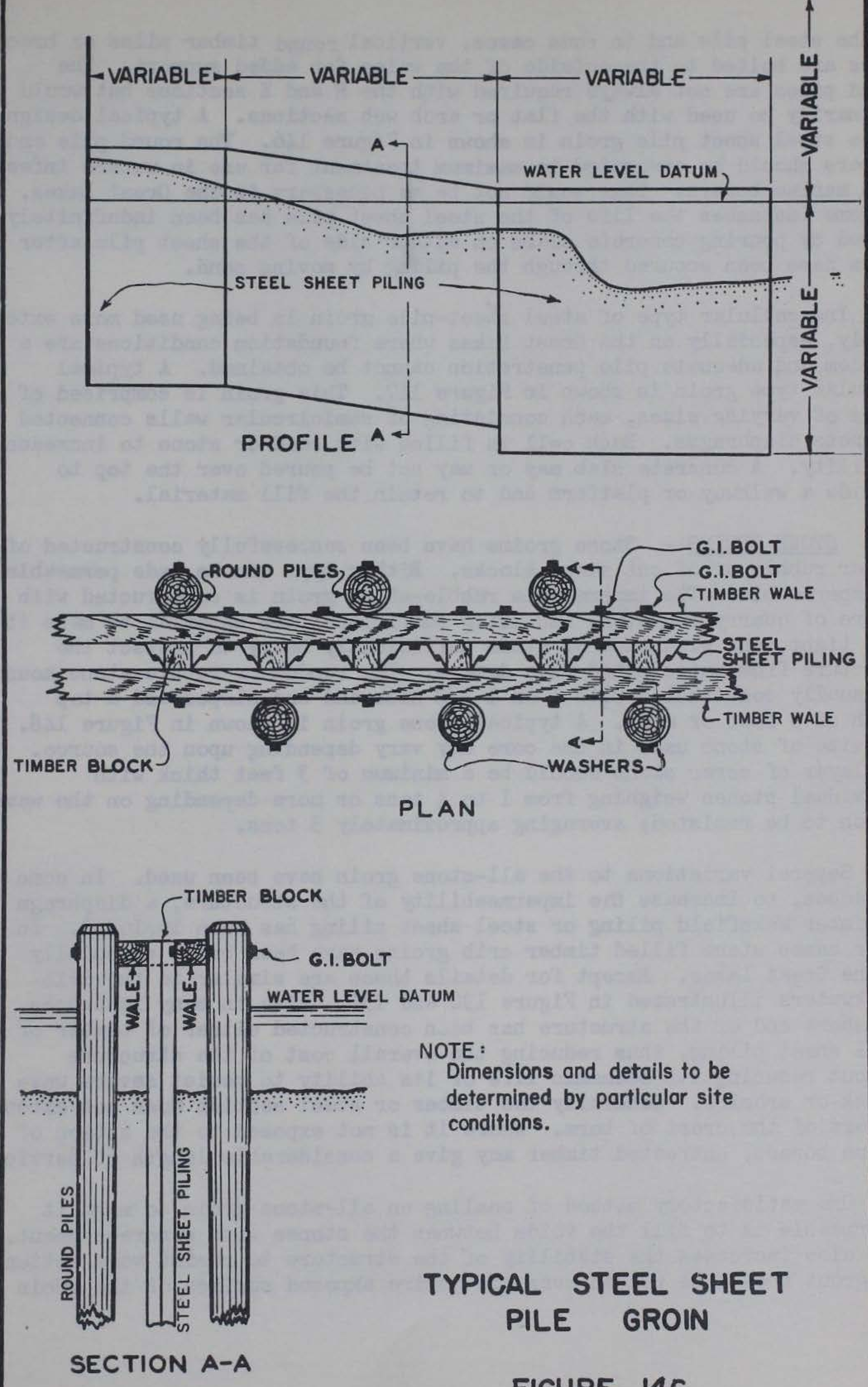
In Figure 143, the precast reinforced concrete groin is constructed of vertical transverse blocks shaped like a capital "I", 4 feet, 8 inches wide and 6 feet long. The height of the unit may vary with location. Horizontal panels 2 feet high, 8 inches thick, and 10 feet long, with notches on either end, are placed in between the transverse blocks. Rock fill is then placed in between the horizontal panels and covered with a 6-inch concrete slab cast in place. The concrete slab can be used to tie the entire structure into a single unit.

In Figure 144, the groin is composed of concrete precast trapezoidal blocks, each 3 feet 3 inches high and weighing 5 tons. Two types of blocks are used. Type A, shown in Figure 144 has a length of 5 feet, a top width of 2 feet and bottom width of 6 feet. Type B, not shown but similar to type A, has a length of 3 feet 10 inches, a top width of 3 feet 2 inches, and a bottom width of 7 feet 2 inches. The block at the offshore end is the same as type A but with a sloping offshore face. The steel joints and reinforcing, shown in Figure 144, are the same for both type A and type B. A patent on the joint is held by Harrison Weber. The reinforcing bars are welded to the I-beam and the channels on opposite ends of the block, and are placed as one unit in the form for casting the concrete block.

The blocks are placed on the beach surface, the flexible nature of the joints enabling the blocks to conform with the ground slope and to undergo a measure of settlement. The height of groin can be increased by maintaining the relationship between height and base width. As some settlement occurred where this type of groin was originally placed on a sandy beach a mat of crushed stone was used as a foundation in later installations.

Figure 145 shows another type of concrete block groin which is simple to cast and place, and which has served satisfactorily in some locations. The height of groin can be changed by changing the length and base width of the concrete blocks. Concrete channel-ways are cast in the block as a guide in placing and to tie the blocks together. No special type of end block is required. After the blocks have settled for a period of time, the concrete cap can be poured. This cap will increase the height of the groin somewhat, and tie component parts together into a monolithic groin.

5.53 STEEL SHEET PILE GROINS - Satisfactory steel sheet pile groins have been constructed with straight web, arch,web, M or Z sections. Some have been made permeable by the cutting of openings in the piles. All of these are made with interlocking joints that do not pull apart when subjected to wave forces and provide positive sand-tight connections between adjacent piles. The type of pile selected is governed by the wave action to be encountered. The section modulus increases as the depth of the arch increases in the arch web, M, and Z sections. The steel sheet pile groin is often constructed with horizontal timber or steel wales along the top



TYPICAL STEEL SHEET PILE GROIN

FIGURE 146

of the steel pile and in some cases, vertical round timber piles or brace piles are bolted to the outside of the wales for added support. The round piles are not always required with the M and Z sections but would ordinarily be used with the flat or arch web sections. A typical design for a steel sheet pile groin is shown in Figure 146. The round pile and timbers should be creosoted to maximum treatment for use in waters infested with marine borers. This would not be as necessary in the Great Lakes. In some instances the life of the steel sheet pile has been indefinitely prolonged by pouring concrete slabs on either side of the sheet pile after holes have been scoured through the piling by moving sand.

The cellular type of steel sheet-pile groin is being used more extensively, especially on the Great Lakes where foundation conditions are a problem and adequate pile penetration cannot be obtained. A typical cellular type groin is shown in Figure 147. This groin is comprised of cells of varying sizes, each consisting of semicircular walls connected by cross diaphragms. Each cell is filled with sand or stone to increase stability. A concrete slab may or may not be poured over the top to provide a walkway or platform and to retain the fill material.

5.54 <u>STONE GROINS</u> - Stone groins have been successfully constructed of either rubble or of cut stone blocks. Either type can be made permeable or impermeable. The impermeable rubble-stone groin is constructed with a core of quarry run stone including sufficient fine material to make it sand tight, and with caps of stone sufficiently heavy to protect the structure from anticipated wave damage. The random or rubble-stone mound is usually constructed with 1 on 1 1/2 side and end slopes and a top width of 5 feet or more. A typical stone groin is shown in Figure 148. The size of stone used in the core may vary depending upon the source. The layer of cover stone should be a minimum of 3 feet thick with individual stones weighing from 1 to 4 tons or more depending on the wave action to be resisted; averaging approximately 3 tons.

Several variations to the all-stone groin have been used. In some instances, to increase the impermeability of the structure, a diaphragm of timber Wakefield piling or steel sheet piling has been included. In other cases stone filled timber crib groins have been built especially on the Great Lakes. Except for details these are similar to the crib breakwaters illustrated in Figure 130 and 131. Also in many instances the shore end of the structure has been constructed either of timber or steel sheet piling, thus reducing the overall cost of the structure without reducing its economic life or its ability to resist severe wave attack or erosion. Generally the timber or steel section does not extend seaward of the crest of berm. Where it is not exposed to the action of marine borers, untreated timber may give a considerable length of service.

One satisfactory method of sealing an all-stone groin to make it impermeable is to fill the voids between the stones with concrete grout. This also increases the stability of the structure to resist wave action. The grout should be placed over the entire exposed surface of the groin

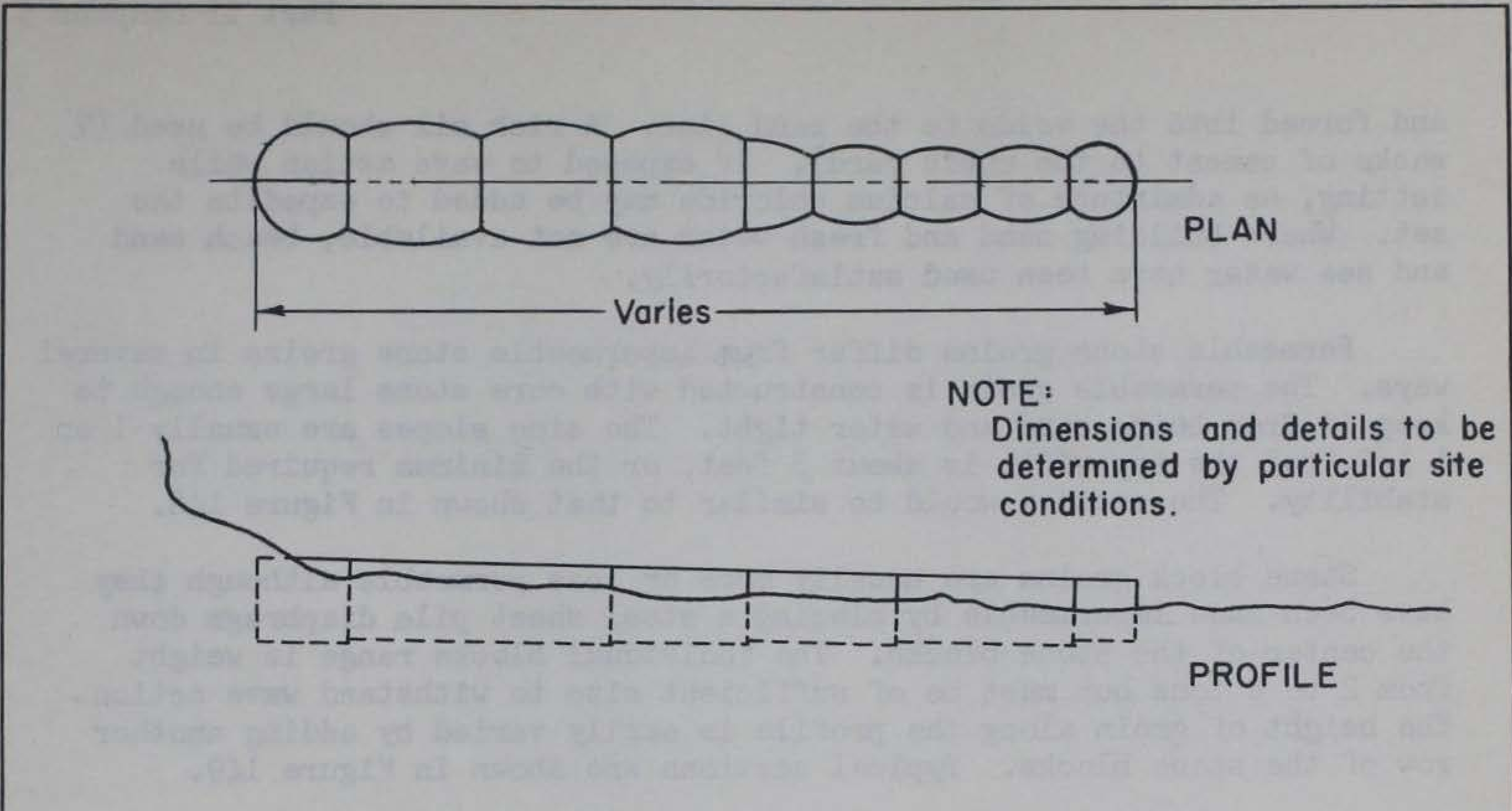
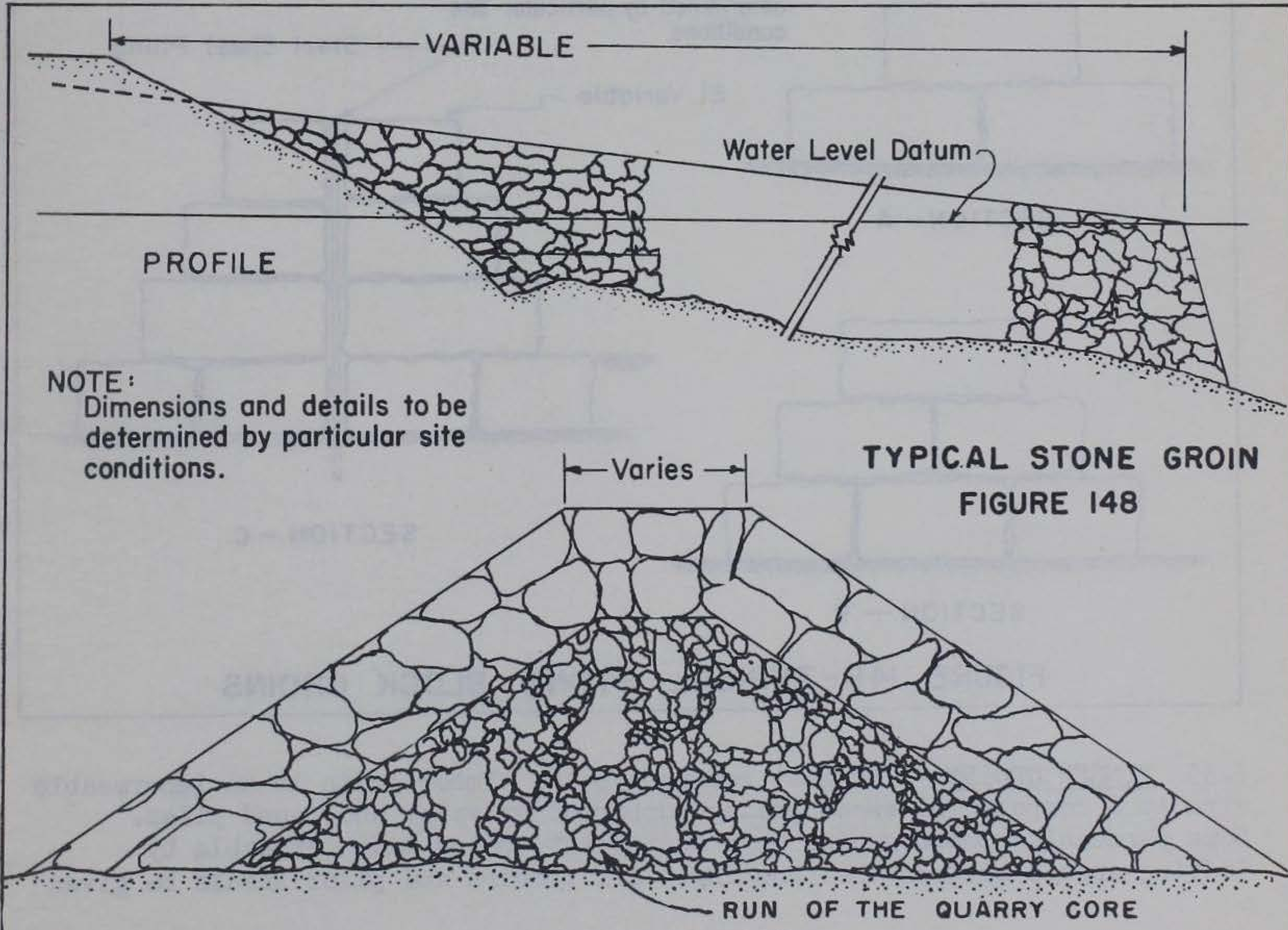


FIGURE 147 - TYPICAL CELLULAR STEEL GROIN



TYPICAL STONE GROIN
FIGURE 148

and forced into the voids to the sand line. A rich mix should be used (7 sacks of cement to the cubic yard). If exposed to wave action while setting, an admixture of calcium chloride may be added to expedite the set. Where building sand and fresh water are not available, beach sand and sea water have been used satisfactorily.

Permeable stone groins differ from impermeable stone groins in several ways. The permeable groin is constructed with core stone large enough to keep it from being sand and water tight. The side slopes are usually 1 on 1 1/2, but the top width is about 3 feet, or the minimum required for stability. The section would be similar to that shown in Figure 148.

Stone block groins are usually more or less permeable although they have been made impermeable by placing a steel sheet pile diaphragm down the center of the stone blocks. The individual blocks range in weight from 2 to 6 tons but must be of sufficient size to withstand wave action. The height of groin along the profile is easily varied by adding another row of the stone blocks. Typical sections are shown in Figure 149.

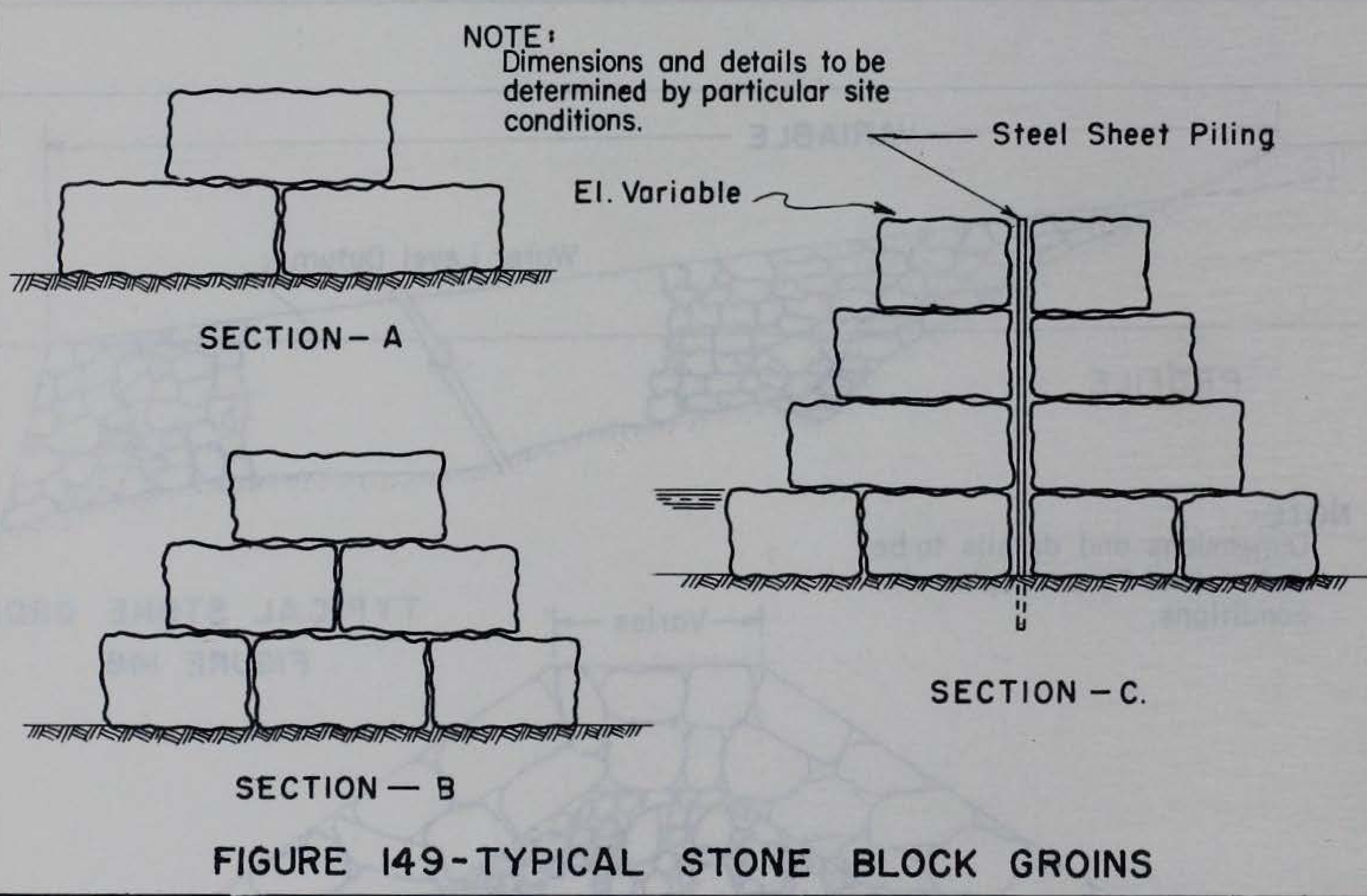
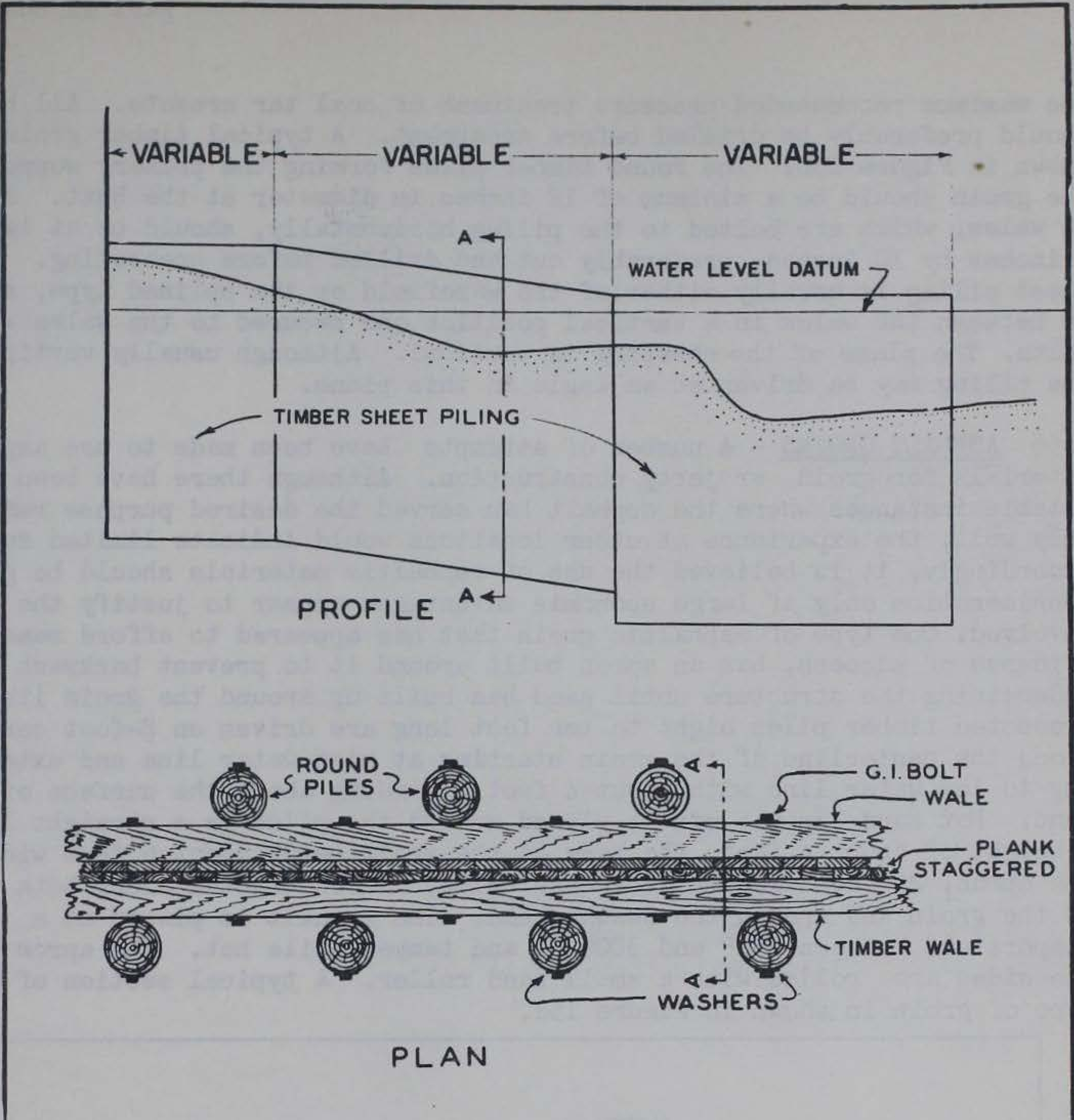


FIGURE 149 - TYPICAL STONE BLOCK GROINS

5.55 TIMBER GROINS - The most common type of timber groin is an impermeable structure composed of sheet piling supported by wales and round piles. Some permeable timber groins have been built, being made permeable by leaving spaces between the sheeting. All timbers and piles should be given



TYPICAL TIMBER GROIN
FIGURE 150

the maximum recommended pressure treatment of coal tar cresote. All holes should preferably be drilled before treatment. A typical timber groin is shown in Figure 150. The round timber piles forming the primary support of the groin should be a minimum of 12 inches in diameter at the butt. Stringers or wales, which are bolted to the piling horizontally, should be at least 8 inches by 10 inches, preferably cut and drilled before creosoting. The sheet piling is usually either of the Wakefield or the splined type, supported between the wales in a vertical position and secured to the wales with bolts. The plane of the sheeting is vertical. Although usually vertical the piling may be driven at an angle in this plane.

5.56 <u>ASPHALT GROINS</u> - A number of attempts have been made to use asphaltic materials for groin or jetty construction. Although there have been some notable instances where the asphalt has served the desired purpose reasonably well, the experience at other locations would indicate limited success. Accordingly, it is believed the use of asphaltic materials should be given consideration only if large economic advantages appear to justify the risk involved. One type of asphaltic groin that has appeared to afford some evidence of success, has an apron built around it to prevent backwash from undermining the structure until sand has built up around the groin itself. Creosoted timber piles eight to ten feet long are driven on 8-foot centers along the centerline of the groin starting at high water line and extending to low water line with about 4 feet extending above the surface of the sand. Hot sand-asphalt mix is placed around the piles in a straight line to a height of about 4 feet, the base of the groin being about 6 feet wide. The apron, 20 feet wide and 2 inches thick, is constructed along both sides of the groin and around the seaward end. The asphalt is placed at a temperature between 250° and 300° F. and tamped while hot. The apron on the sides are rolled with a small hand roller. A typical section of this type of groin in shown in Figure 151.

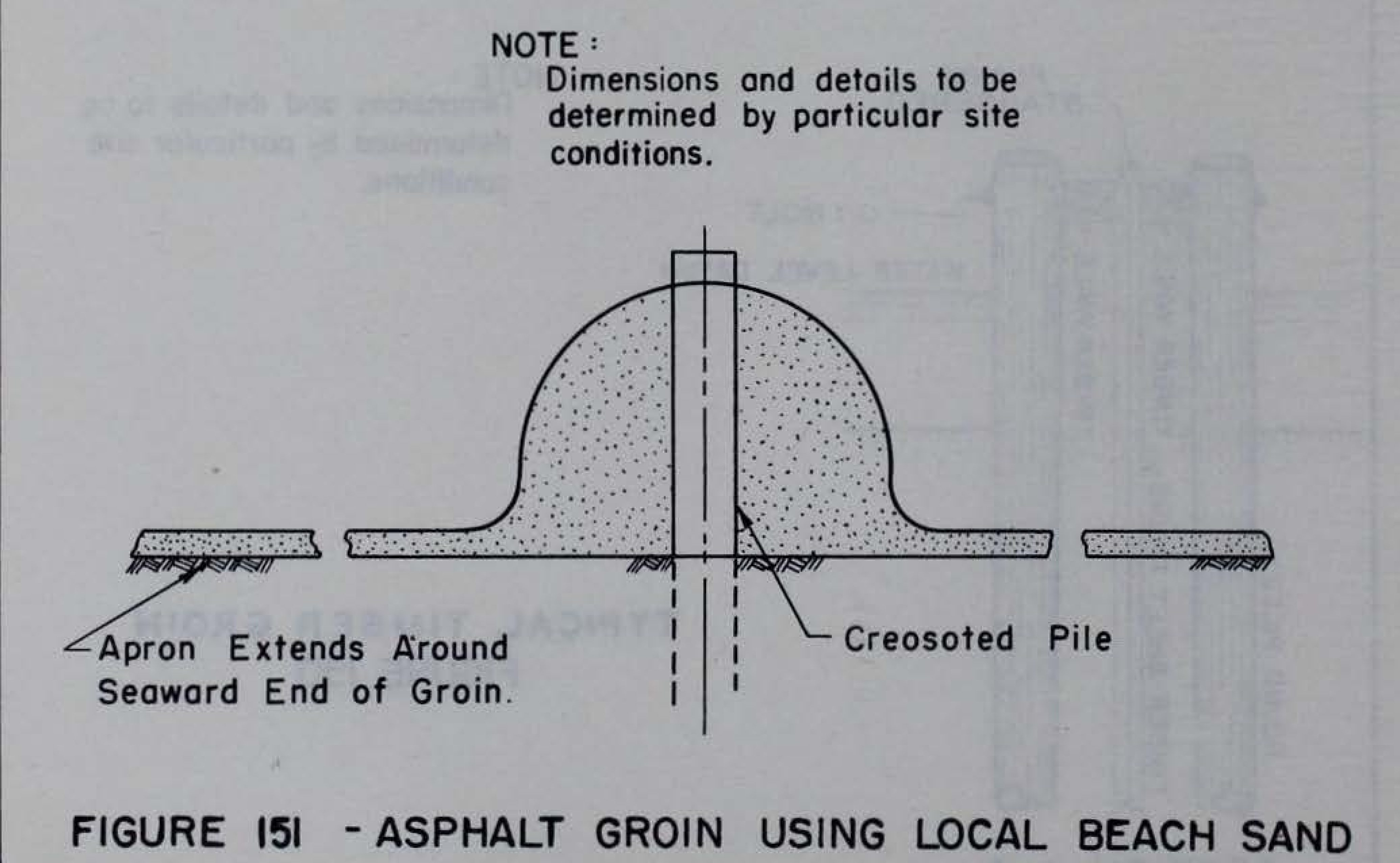


FIGURE 151 - ASPHALT GROIN USING LOCAL BEACH SAND

5.57 SELECTION OF TYPE - Every beach has its inherent physical characteristics. These include the range of tide, littoral transport, beach composition, wave characteristics, tidal currents, marine borers, and beach dimensions and slope. Other factors to be considered include beach and upland topography, the character of the foundation, the availability and cost of construction materials, maintenance costs, economic life, and the use made of the upland areas. After planning considerations have indicated that use of groins is practicable, the selection of groin type is affected in overlapping and varying degrees by the foregoing inter-related factors.

With consideration to beach and upland topography, a steep beach is an indication of heavy wave action and generally requires substantial structures, usually of concrete, stone, or steel, Under these conditions light timber groins will not be adequate seaward of the crest of berm. However, timber can be used for the inshore section above the crest of berm if the beach is backed by sandy areas such as sand dunes. On gently sloping beaches, lighter construction of timber, steel, or concrete generally can be used except in the vicinity of inlets. A thorough consideration of foundation materials is essential to the selection of groin type. Borings and probings should be taken to determine the subsurface conditions for penetration of piling. Where the foundations are poor or where little penetration is possible, some type of cellular or stone groin may be indicated despite its greater cost. Good penetration may indicate the economy of sheet piling, provided materials are available.

Availability of materials affects selection of type because of the economic aspects. It may happen that the material, which would normally be the most economic with full consideration given to life of the material and maintenance costs, is not available except at a cost that would make some other material or type of construction more economical. This involves the question of the economic life of the material together with the annual cost of maintenance to attain that economic life. The first costs of timber groins and of steel sheet pile groins, in that order, are often less than for other types of construction. The concrete groin is considerably more expensive, but often costs less than does the massive stone groin. However, concrete and stone groins require less maintenance and have much longer economic life than do the timber or steel sheet pile groins. These factors, the amount of funds available for initial construction, the annual charges, and the period during which protection will be required must all be studied before deciding on a particular type.

No universal plan of protection can be prescribed because of the wide variation in conditions at each location. However, the information required in a problem and the general factors to be considered to determine the groin type are generally the same and depend on the location and characteristics of the area for detail. The groins constructed at Coney Island, New York, are used as a typical example to demonstrate the selection of type. These groins were designed to retain the artificially placed protective beach. Profiles taken before fill or groin construction was started, showed a general slope of about 1 on 24, which would be considered steep and ordinarily require heavy stone structures. However, good penetration could be obtained throughout the area, and because the fill would be added as the groin construction progressed, providing protection from heavy waves to the landward ends of the groins, timber impermeable groins 160 feet long, heavily creosoted to resist attack by marine borers, were chosen for the inshore section. Seaward of this timber section, the groins were constructed of stone for a distance of 200 feet to withstand heavy waves and prevent scour or damage to the timber sections. The stone was transported to the site by barges and lighters. When the work was finished the beach between high and low water had a moderate slope and extended about 320 feet seaward of the original shore line. The experience with this beach and the groins was satisfactory over a period of 30 years, during which several hurricanes occurred.

5.58 DESIGN

5.581 Concrete Block Groins - a. Maximum Forces - The forces acting on the structure would be either earth pressures or wave forces or combination thereof. No combination of these forces would exceed the maximum that would occur by application of either singly. The maximum earth pressure would occur with the groin full on one side and unsupported on the other. The maximum wave pressure would occur at the seaward end of the groin where there would be no supporting fill on either side, and where the groin would be subject to attack by the largest wave. The end block would be subject to the full impact of the waves. Landward blocks would be subject to reduce wave forces by reason of the angle of wave approach.

b. Design Problem - A concrete block groin of the type shown on Figure 144 is to be built in an area in which the beach slope is 1 on 20, and the design wave period is 10 seconds. The sand composing the beach has a unit weight, w, (moist condition) of 110 pounds per cubic foot, an internal friction angle, ϕ, of 25°, and a friction angle with concrete ϕ_1, of 22°. The block to be used has, as the angle θ between its face and the horixontal plane, 110°, and a height of 5 feet.

c. Earth Pressures - Maximum earth pressures would occur with one side of the groin full and the other empty. For $\phi = 25°$, $\theta = 110°$, $\phi_1 = 22°$, h = 5 feet and the angle between the surface of the accretion and the horizontal plane being p = 0°, the total force in pounds P due to the updrift accretion is given by

$$P = 1/2 \left[\frac{\sin(\theta - \phi)}{(1 + N)\sin\theta} \right]^2 \frac{wh^2}{\sin(\phi_1 + \theta)} \qquad (59)$$

where

$$N = \sqrt{\frac{\sin(\phi + \phi_1)\sin(\phi - p)}{\sin(\phi_1 + \theta)\sin(\theta - p)}} \qquad (60)$$

by substitution

$$N = \sqrt{\frac{.731 \times .423}{.731 \times .934}} = 0.67$$

and

$$P = \frac{1}{2} \left[\frac{.998}{1.67\ (.936)}\right]^2 \quad \frac{110\ (5)^2}{.731} = 0.41\ (3760)$$

$$= 1,540 \text{ pounds}$$

The maximum moment of horizontal pressure would be:

$$M_h = \quad 1,540 \times 5/3 = 2,570 \text{ foot-pounds}$$

If the vertical component of earth pressure is neglected, which may be done unless it has a large effect on stability, the vertical load of the wall passes through the centerline of the base, and the above moment becomes the moment around the centerline of the base.

Assuming a block with a 7-foot base, a 3-foot top width, and 5 feet high, its weight per linear foot in air would be

$$5\,\frac{(7 + 3)}{2} \times 150 = 3,750 \text{ pounds}$$

With this block, the stress in the base would be

$$f = \frac{P}{A} + \frac{6M}{bd^2} = \frac{3750}{1 \times 7} + \frac{6(2570)}{(7)^2} = 536 \pm 315 \qquad (63)$$

where P = the sum of vertical loads
A = the base area (unit length of groin)
M = the total moment about the base centerline
$\frac{bd^2}{6}$ = the section modulus of the base (unit length of groin)

$$= 536 + 315 = 851 \text{ pounds per square foot}$$

$$= 536 - 315 = 221 \text{ pounds per square foot.}$$

These pressures are satisfactory both for concrete and for the foundations.

To check the possibility of sliding:

$$\frac{P}{W} = \frac{1540}{3750} = 0.41$$

This is about equal to the coefficient of friction for masonry on sand and is considered satisfactory.

d. Wave Pressures - With a 5-foot high block, the maximum wave conditions would occur when the water is about 3.7 feet deep at the groin, at which time a wave 3.7/1.3 = 2.8 feet high would be possible. Using a 10-second wave and a beach slope of 1 on 20

$$\frac{d}{L_o} = \frac{3.7}{512} = .0073 \quad \text{and} \quad \frac{d}{L} = .034$$

$$L = d/.034 = 3.7/.034 = 109 \text{ feet}$$

$$D = 3.7 + \frac{109}{20} = 9.2 \text{ feet}$$

$$\frac{D}{L} = \frac{9.2}{512} = .018 \quad \text{and} \quad \frac{D}{L_D} = .055$$

Therefore

$$L_D = 9.2/.055 = 167 \text{ feet.}$$

The dynamic pressure concentrated at still water level if the groin were normal to the direction of wave approach would be determined from equation 27 or

$$P_m = \frac{\pi g\ 2.8 \times 64.2}{167} \times \frac{3.7\ (9.2 + 3.7)}{9.2}$$

$$P_m = 109 \times 5.2 = 567 \text{ pounds per square foot,}$$

and the static pressure is (Equation 30)

$$P_s = \frac{wH}{2} = \frac{64.2 \times 2.8}{2} = 90 \text{ pounds per square foot.}$$

These pressures would be applied in a manner similar to that shown in Figure 152.

The resultant wave thrust if the groin were normal to wave approach, by equation 31, would be

$$R_m = (567 \times \frac{2.8}{3}) + 90\ (3.7 + \frac{2.8}{4})$$

$$= 530 + 396 = 926 \text{ pounds}$$

As the dynamic pressure acts at an angle (α) with the structure, the total force normal to the structure per unit length of structure would be, with α = 30 degrees. (see Figure 153)

$$R_n = 530 \sin^2\alpha + 396$$

$$= 133 + 396 = 529 \text{ pounds per foot of groin.}$$

The moments around the base of the block by equation 32 would be (with $\sin^2\alpha$ as a factor in the first term)

$$M = 133 \times 3.7 + \frac{90 \times 3.7^2}{2} + \frac{90 \times 2.8}{4}\ (3.7 + 0.47)$$

$$M = 492 + 616 + 263 = 1371 \text{ foot-pounds}$$

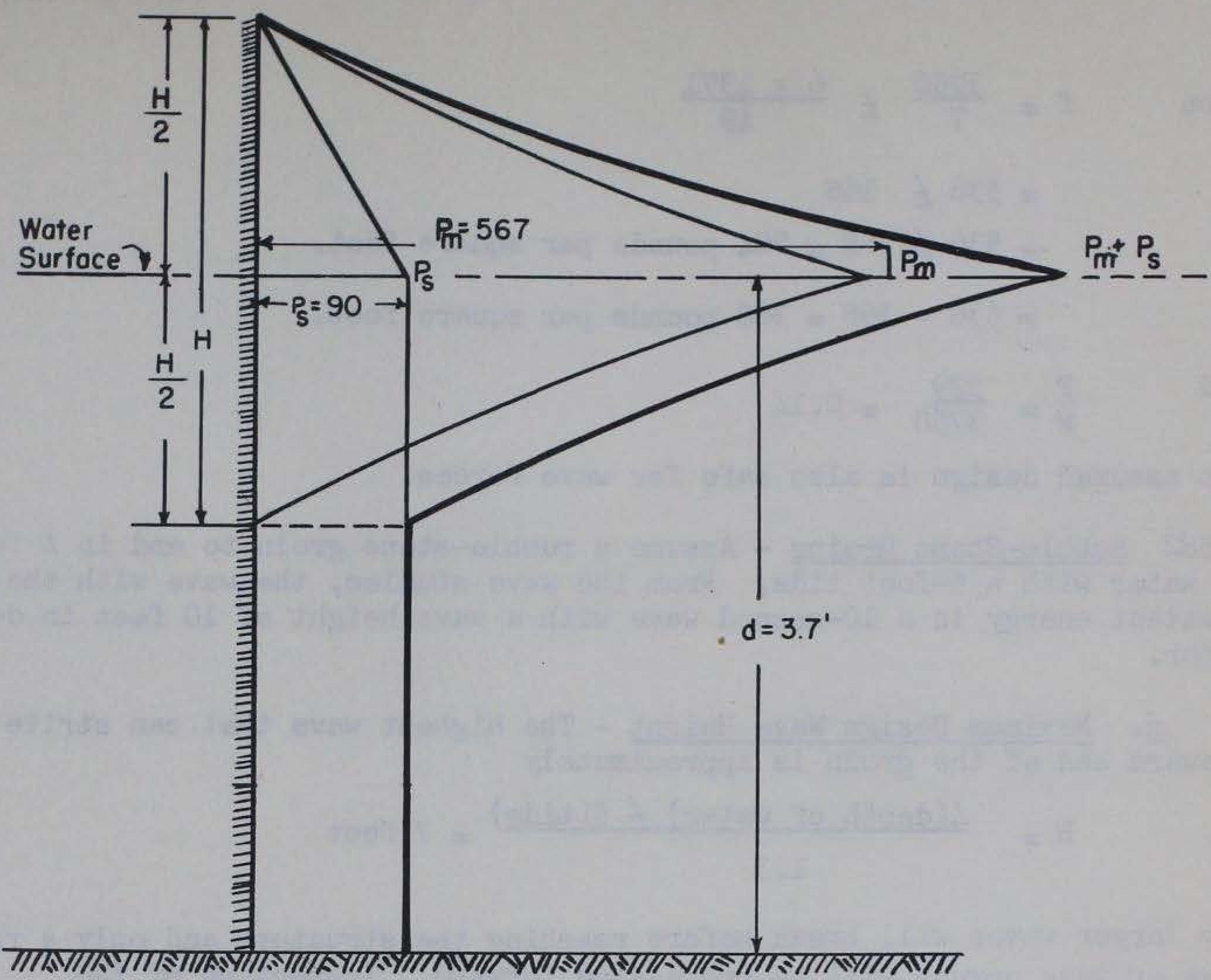


FIGURE 152 - WAVE PRESSURE DIAGRAM

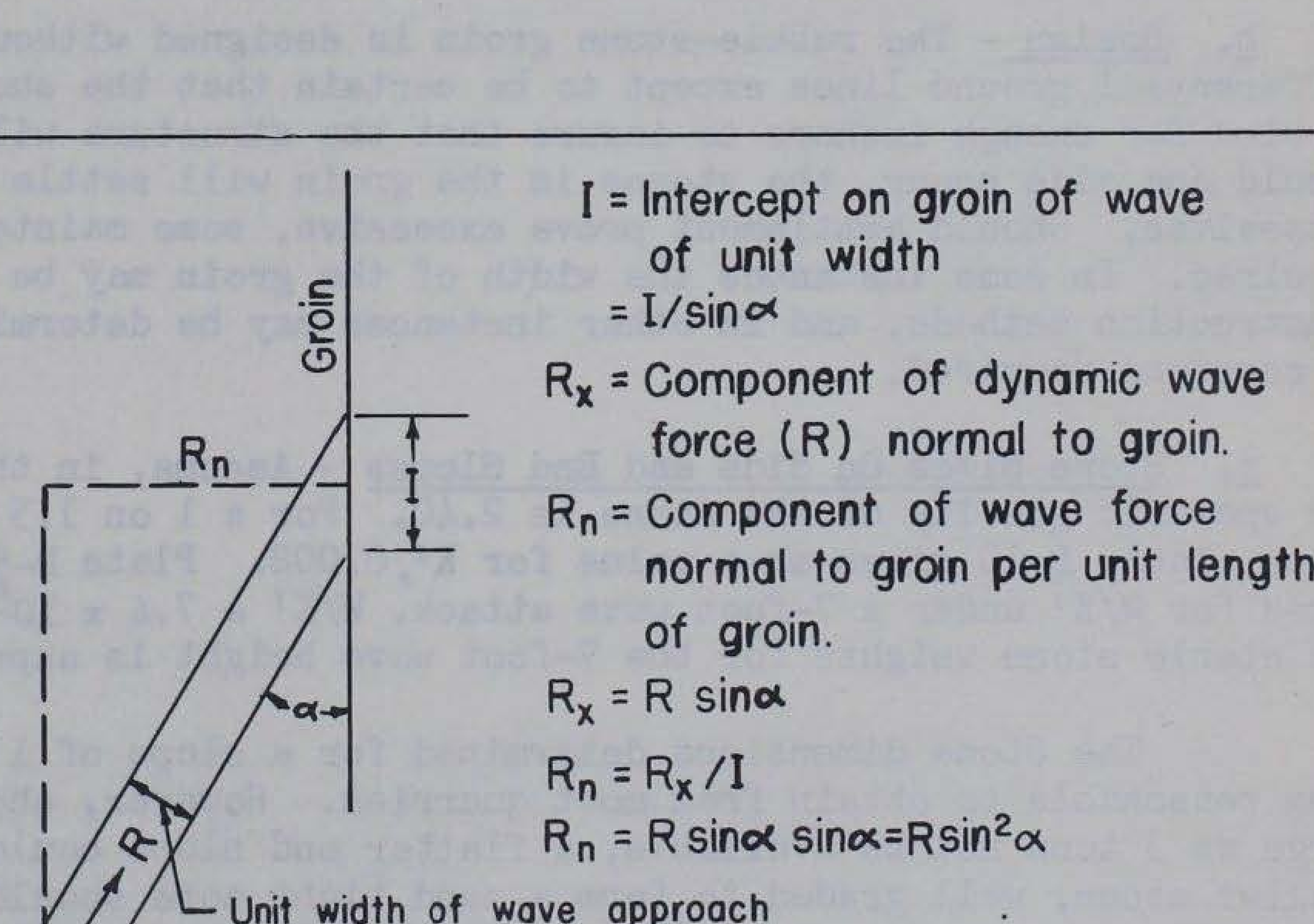


FIGURE 153 - ANGLE OF WAVE APPROACH

then $$f = \frac{3750}{7} \pm \frac{6 \times 1371}{49}$$

$$= 536 \pm 168$$

$$= 536 + 168 \doteq 704 \text{ pounds per square foot.}$$

$$= 536 - 168 = 368 \text{ pounds per square foot.}$$

and $$\frac{P}{W} = \frac{529}{3750} = 0.14$$

The assumed design is also safe for wave forces.

5.582 Rubble-Stone Groins - Assume a rubble-stone groin to end in 4 feet of water with a 5-foot tide. From the wave studies, the wave with the greatest energy is a 10-second wave with a wave height of 10 feet in deep water.

a. Maximum Design Wave Height - The highest wave that can strike the seaward end of the groin is approximately

$$H = \frac{4(\text{depth of water}) + 5(\text{tide})}{1.3} = 7 \text{ feet}$$

Any larger waves will break before reaching the structure and only a reformed wave or wave uprush will be propagated forward. Inspection of the orthogonal pattern indicates little divergence so that 7-foot waves at the structure can occur if the deep water design wave is 10 feet.

b. Design - The rubble-stone groin is designed without regard to differential ground lines except to be certain that the shore end is carried far enough inshore to insure that the structure will not be flanked. Should one side scour, the stones in the groin will settle and readjust themselves. Should settlement prove excessive, some maintenance may be required. In some instances the width of the groin may be determined by construction methods, and in other instances may be determined by the size of capstone required.

c. Stone Sizes On Side and End Slopes - Assume, in this case, that the specific gravity of the stone is 2.40. For a 1 on 1.5 groin face slope, Table D-10 gives as a value for K', 0.008. Plate D-5b, Appendix D, gives for W/K' under a 7-foot wave attack, $W/K' = 7.4 \times 10^5$. With $K' = 0.008$ the stable stone weights for the 7-foot wave height is approximately 3 tons.

The Stone dimensions determined for a slope of 1 on 1.5 would seem reasonable to obtain from most quarries. However, should stone as large as 3 tons not be available, a flatter end slope could be tried. Smaller stone, well graded to form a sand tight core should be used inside the armor or capstone. A typical groin of this type is shown in Figure 148.

5.583 Vertical Sheet Pile Groins - This type of groin may be constructed of timber, concrete, or steel, depending on life expectancy, cost, availability of materials, etc. The design of a groin of this type for wave forces is generally the same as that for a concrete block groin; for earth loading, the design follows that for a sheet pile bulkhead. Piling sections designed in this manner may be reduced somewhat if the customary practice is followed of using wales at the top of the sheet pile with secondary support furnished by round timber or other piling driven outside the wales.

5.6 MISCELLANEOUS DESIGN PRACTICES

The more important lessons learned from experience on the deterioration of concrete and steel and timber waterfront structures may be summarized as follows:

a - The elimination of bracing within the tidal zone to the maximum practicable extent is desirable, since maximum deterioration occurs in that zone;

b - Round members, because of their smaller area and better flow characteristics for wave action, generally have a longer life than other shapes;

c - It is imperative that all steel or concrete deck framing be located above normal spray level;

d - Untreated timber piles should never be used in waterfront structures unless located below the permanent wet line and protected from marine borer attack;

e - The most effective injected preservative appears to be creosote oil having a high phenolic content. For piles subject to marine borer attack a maximum penetration of creosote-coal tar solution is recommended;

f - Salt-treated timber gives satisfactory service when protected from the weather;

g - Boring and cutting of piles after treatment should be avoided, and where unavoidable, cut surfaces require field treatment;

h - Single timber caps have a longer life than pairs of cap timbers dapped into the piles;

i - Untreated timber piles when encased in a gunite armor and properly sealed at the top will give economical service;

j - Concrete to last in the tidal zone must have a high cement content; a minimum of 6-1/2 bags per cubic yard is recommended;

k - The lower the water-cement ratio, the more durable concrete will be in salt water;

l - Care must be exercised in the selection of coarse and fine aggregates both for density of grading and to avoid unfavorable chemical reaction with the cement.

m - Maintenance of specified clear cover over all reinforcing steel is of the greatest importance;

n - Smooth formwork and rounded corners improve the durability of concrete structures.

o - All steelwork in and above the tidal range will last longer if protected. A good method is to provide a concrete envelope.

APPENDIX A

GLOSSARY OF TERMS

This glossary has been compiled and reviewed by the staff of the Beach Erosion Board. Terms from the following publications were included:

(1) Waves, Tides, and Beaches: Glossary of Terms and List of Standard Symbols, R. L. Wiegel, University of California.

(2) The Hydrographic Manual, K. T. Adams, U. S. Coast and Geodetic Survey.

(3) Webster's Unabridged Dictionary, 2nd Edition.

(4) Tides and Currents Glossary, U. S. Coast and Geodetic Survey.

(5) U. S. Naval Photographic Interpretation Center Glossary.

APPENDIX A

GLOSSARY OF TERMS

ACCRETION - May be either NATURAL or ARTIFICIAL. Natural accretion is the gradual build-up of land over a long period of time solely by the action of the forces of nature, on a BEACH by deposition of water- or air-borne material. Artificial accretion is a similar build-up of land by reason of an act of man, such as the accretion formed by a groin, breakwater, or beach fill deposited by mechanical means. Also AGGRADATION.

ADVANCE (OF A BEACH) - (1) a continuing seaward movement of the shore line; (2) a net seaward movement of the shore line over a specified time. Also PROGRESSION.

AGE, WAVE - The ratio of wave velocity to wind velocity (in wave forecasting theory).

ALLUVIUM - Soil (sand, mud, or similar detrital material) deposited by flowing water, or the deposits formed thereby.

ALONGSHORE - Same as LONGSHORE.

AMPLITUDE, WAVE - (1) in hydrodynamics, one-half the wave height; (2) in engineering usage, loosely, the wave height from crest to trough.

ARTIFICIAL NOURISHMENT - The process of replenishing a beach by artificial means, e.g. by the deposition of dredged materials.

ATOLL - A ring-like "coral" island or islands encircling or nearly encircling a lagoon. It should be noted that the term "coral" island for most of these tropical islands is incorrect as calcareous algae (Lithothamnion) often forms much more than 50% of them.

ATOLL REEF - A ring-shaped, coral reef, often carrying low sand islands, enclosing a body of water.

AWASH - (1) (Nautical) Condition of an object which is nearly flush with the water level; (2) (Common usage) Condition of being tossed about or washed by waves or tide.

BACKBEACH - See BACKSHORE.

BACKRUSH - The seaward return of the water following the uprush of the waves. For any given tide stage the point of farthest return seaward of the backrush is known as the LIMIT of BACKRUSH or LIMIT of BACKWASH. (See Figure A-2)

BACKSHORE - That zone of the shore or beach lying between the foreshore and the coast line and acted upon by waves only during severe storms, especially when combined with exceptionally high water. Also BACKBEACH. (See Figure A-1)

BACKWASH - (1) See BACKRUSH; (2) Water or waves thrown back by an obstruction such as a ship, breakwater, cliff, etc.

BANK - (1) The rising ground bordering a lake, river, or sea, on a river designated as right or left as it would appear facing downstream; (2) An elevation of the sea floor of large area, surrounded by deeper water, but safe for surface navigation; a submerged plateau or shelf, a shoal, or shallow.

BAR - An offshore ridge or mound of sand, gravel, or other unconsolidated material submerged at least at high tide, especially at the mouth of a river or estuary, or lying a short distance from and usually parallel to, the beach. (See Figure A-2 and A-9)

BAR, BAYMOUTH - A bar extending partially or entirely across the mouth of a bay. (See Figure A-9)

BAR, CUSPATE - A crescent shaped bar uniting with shore at each end, It may be formed by a single spit growing from shore turning back to again meet the shore, or by two spits growing from shore uniting to form a bar of sharply cuspate form. (See Figure A-9)

BARRIER BEACH - A bar essentially parallel to the shore, the crest of which is above high water. Also OFFSHORE BARRIER. (See Figure A-9)

BARRIER REEF - A reef which roughly parallels land but is some distance offshore, with deeper water intervening.

BASIN, BOAT - A naturally or artificially enclosed or nearly enclosed body of water where small craft may lie.

BAY - A recess in the shore or an inlet of a sea or lake between two capes or headlands, not as large as a gulf but larger than a cove. See also BIGHT, EMBAYMENT. (See Figure A-9)

BAYMOUTH BAR - A bar extending partially or entirely across the mouth of a bay. (See Figure A-9)

BAYOU - A minor sluggish waterway or estuarial creek, tributary to, or connecting, other streams or bodies of water. Its course is usually through lowlands or swamps.

BEACH (n.) (1) The zone of unconsolidated material that extends landward from the low water line to the place where there is marked change in material or physiographic form...or to the line of permanent vegetation (usually the effective limit of storm waves). The seaward limit of the beach - unless otherwise specified - is the mean low water line. A beach includes FORESHORE and BACKSHORE; (2) Sometimes, the material which is in more or less active transport, alongshore or on-and-off shore, rather than the zone. (See Figure A-1)

BEACH ACCRETION - See ACCRETION.

BEACH, BARRIER - A bar essentially parallel to the shore, the crest of which is above high water level. Also OFFSHORE BARRIER.

BEACH BERM - A nearly horizontal portion of the beach or backshore formed by the deposit of material by wave action. Some beaches have no berms, others have one or several.

BEACH CUSP - One of a series of low mounds of beach material separated by crescent shaped troughs spaced at more or less regular intervals along the beach face. Also CUSP.

BEACH EROSION - The carrying away of beach materials by wave action, tidal currents, or littoral currents, or by wind.

BEACH FACE - The section of the beach normally exposed to the action of the wave uprush. The FORESHORE zone of a BEACH. (Not synonymous with SHOREFACE). (See Figure A-2)

BEACH, FEEDER - An artificially widened beach serving to nourish downdrift beaches by natural littoral currents or forces.

BEACH RIDGE - An essentially continuous mound of beach material behind the beach that has been heaped up by wave or other action. Ridges may occur singly or as a series of approximately parallel deposits. In England they are called FULLS.

BEACH SCARP - An almost vertical slope along the beach caused by erosion by wave action. It may vary in height from a few inches to several feet, depending on wave action and the nature and composition of the beach. (See Figure A-1)

BEACH WIDTH - The horizontal dimension of the beach as measured normal to the shore line.

BENCH - (1) A level or gently sloping erosion plane inclined seaward; (2) A nearly horizontal area at about the level of maximum high water on the sea side of a dike.

BENCH MARKS (B.M.) - A fized point used as a reference for elevations.

BERM, BEACH - A nearly horizontal portion of a beach formed by the deposit of material by wave action. Some beaches have no berrs, others have one or several. (See Figure A-1)

BERM CREST - The seaward limit of a berm. Also BERM EDGE. (See Figure A-1)

BIGHT - A slight indentation in the shore line of an open coast or of a bay, usually crescent shaped. (See Figure A-8)

BLIND ROLLERS - Long, high swells which have increased in height, almost to the breaking point, as they pass over shoals or run in shoaling water.

BLUFF - A high steep bank or cliff.

BOLD COAST - A prominent land mass that rises steeply from the sea.

BORE - A tidal flood with a high, abrupt front. (e.g. such as occurs in the Amazon in South America, the Hugli in India, and in the Bay of Fundy). Also EAGER.

BOTTOM - The ground or bed under any body of water; the bottom of the sea. (See Figure A-1)

BOTTOM (NATURE OF) - The composition or character of the bed of an ocean or other body of water; (e.g. clay, coral, gravel, mud, ooze, pebbles, rock, shell, shingle, hard, or soft).

BOULDER - A rounded rock more than 12 inches in diameter; larger than a cobble stone.

BREAKER - A wave breaking on the shore, over a reef, etc. Breakers may be (roughly) classified into three kinds although there is much overlapping
<u>Spilling</u> breakers break gradually over quite a distance;
<u>Plunging</u> breakers tend to curl over and break with a crash; and
<u>Surging</u> breakers peak up, but then instead of spilling or plunging they surge up the beach face. (See Figure A-4)

BREAKER DEPTH - The still water depth at the point where the wave breaks. Also BREAKING DEPTH. (See Figure A-2)

BREAKWATER - A structure protecting a shore area, harbor, anchorage, or basin from waves.

BULKHEAD - A structure separating land and water areas, primarily designed to resist earth pressures. See also SEAWALL.

BUOY - A float; especially a floating object moored to the bottom, to mark a channel, anchor, shoal rock, etc. Some common types:

A nun or nut buoy is conical in shape;

A can buoy is squat, and cylindrical or nearly cylindrical above water and conical belowwwater;

A spar buoy is a vertical, slender spar anchored at one end;

A bell buoy is one having a bell operated mechanically or by the action of waves, usually marking shoals or rocks;

A whistling buoy is similarly operated, marking shoals or channel entrances;

A dan buoy carries a pole with a flag or light on it.

BUOYANCY - The resultant of upward forces, exerted by the water on a submerged or floating body, equal to the weight of the water displaced by this body.

CANAL - An artificial watercourse cut through a land area for use in navigation, irrigation, etc.

CANYON - (1) (Oceanographical) A deep submarine depression of valley form with relatively steep sides; (2) (Geographical) A deep gorge or ravine with steep sides, often with a river flowing at the bottom of it.

CAPE - A relatively extensive land area jutting seaward from a continent or large island which prominently marks a change in, or interrupts notably, the coastal trend; a prominent feature. (See Figure A-8)

CAPILLARY WAVE - A wave whose velocity of propagation is controlled primarily by the surface tension of the liquid in which the wave is travelling. Water waves of length less than one inch are considered to be capillary waves.

CAUSEWAY - A raised road, across wet or marshy ground or across water.

CAUSTIC - In refraction of waves, the name given to the curve to which adjacent orthogonals of waves, refracted by a bottom whose contour lines are curved, are tangents. The occurrence of a caustic always marks a region of crossed orthogonals and high wave convergence.

CAY - See KEY.

CHANNEL - (1) A natural or artificial waterway of perceptible extent which either periodically or continuously contains moving water, or which forms a connecting link between two bodies of water; (2) The part of a body of water deep enough to be used for navigation through an area otherwise too shallow for navigation; (3) A large strait, as the English Channel; (4) The deepest portion of a stream, bay, or strait through which the main volume or current of water flows.

CHARACTERISTIC WAVE HEIGHT - See SIGNIFICANT WAVE HEIGHT.

CHART DATUM - The plane or level to which soundings on a chart are referred, usually taken to correspond to a low water stage of the tide. See also DATUM PLANE.

CHOP - The short-crested waves that may spring up quickly in a fairly moderate breeze, and break easily at the crest. Also WIND CHOP.

CLAPOTIS - (1) The French equivalent for a type of STANDING WAVE; (2) In American usage it is usually associated with the standing wave phenomenon caused by the reflection of a wave train from a breakwater, bulkhead, or steep beach.

CLAY - See SOIL CLASSIFICATION.

CLIFF - A high, steep face of rock; a precipice. See also SEA CLIFF.

COAST - A strip of land of indefinite width (may be several miles) that extends from the seashore inland to the first major change in terrain features. (See Figure A-1)

COASTAL AREA - The land and sea area bordering the shore line. (See Figure A-1)

COASTAL PLAIN - The plain composed of horizontal or gently sloping strata of clastic materials fronting the coast and generally representing a strip of recently emerged sea bottom.

COAST LINE - (1) Technically, the line that forms the boundary between the COAST and the SHORE; (2) Commonly, the line that forms the boundary between the land and the water.

COBBLES (COBBLESTONE) - See SOIL CLASSIFICATION.

COMBER - (1) A deep water wave whose crest is pushed forward by a strong wind, much larger than a whitecap; (2) A long-period spilling breaker.

CONTINENTAL SHELF - The zone bordering a continent extending from the line of permanent immersion to the depth (usually about 100 fathoms) where there is a marked or rather steep descent toward the great depths.

CONTOUR - (1) A line connecting the points, on a land or submarine surface, that have the same elevation; (2) In topographic or hydrographic work, a line connecting all points of equal elevation <u>above</u> or <u>below</u> a datum plane.

CONTROLLING DEPTH - The least depth of water in the navigable parts of a waterway, which limits the allowable draft of vessels.

CONVERGENCE - (1) In refraction phenomena, the decreasing of the distance between orthogonals in the direction of wave travel. This denotes an area of increasing wave height and energy concentration; (2) In wind set-up phenomena, the increase in set-up observed over that which would occur in an equivalent rectangular basin of uniform depth, caused by changes in planform or depth; also the decrease in basin width or depth causing such increase in set-up.

CORAL - The calcareous skeletons of various anthozoans and a few hydrozoans; also these skeletons when solidified into a stony mass. Many tropical islands, reefs, and atolls are formed of coral.

COVE - A small sheltered recess in a shore or coast, often inside a larger embayment. (See Figure A-8).

CREST LENGTH, WAVE - The length of a wave <u>along</u> its crest. Sometimes called CREST WIDTH.

CREST OF BERM - The seaward limit of a berm. Also BERM EDGE. (See Figure A-1)

CREST OF WAVE - (1) The highest part of a wave; (2) That part of the wave above still water level. (See Figure A-3)

CREST WIDTH, WAVE - See CREST LENGTH, WAVE.

CURRENT - A flow of water.

CURRENT, COASTAL - One of the offshore currents flowing generally parallel to the shore line with a relatively uniform velocity (as compared to the littoral currents). They are not related genetically to waves and resulting surf but may be composed of currents related to distribution of mass in ocean waters (or local eddies), wind-driven currents and/or tidal currents.

CURRENT, DRIFT - A broad, shallow, slow-moving ocean or lake current.

CURRENT, EBB - The movement of the tidal current away from shore or down a tidal stream.

CURRENT, EDDY - A circular movement of water of comparatively limited area formed on the side of a main current. Eddies may be created at points where the main stream passes projecting obstructions.

CURRENT, FEEDER - The current which flows parallel to shore before converging and forming the neck of a rip current. See also RIP.

CURRENT, FLOOD - The movement of the tidal current toward the shore or up a tidal stream.

CURRENT, INSHORE - An current inside the breaker zone.

CURRENT, LITTORAL - The nearshore currents primarily due to wave action, e.g. <u>Longshore</u> currents and <u>Rip</u> currents. See also CURRENT, NEARSHORE.

CURRENT, LONGSHORE - The inshore current moving essentially parallel to the shore, usually generated by waves breaking at an angle to the shore line.

CURRENT SYSTEM, NEARSHORE - The current system caused primarily by wave action in and near the breaker zone and which consists of four parts: the shoreward mass transport of water; longshore currents; seaward return flow, including rip currents; and the longshore movement of the expanding heads of rip currents.

CURRENT, OFFSHORE - (1) Any current in the offshore zone; (2) Any current flowing away from shore.

CURRENT, PERIODIC - A current, caused by the tide-producing forces of the moon and the sun, which is a part of the same general movement of the sea manifested in the vertical rise and fall of the tides. Also CURRENT, TIDAL.

CURRENT, PERMANENT - A current that runs continuously independent of the tides and temporary causes. Permanent currents include the fresh water discharge of a river and the currents that form the general circulatory systems of the oceans.

CURRENT, RIP - A narrow current of water flowing seaward through the breaker zone. A rip current consists of three parts: (1) The "feeder currents" flowing parallel to the shore inside the breakers; (2) the "neck" - where the feeder currents converge and flow through the breakers in a narrow band or "rip"; and (3) The "head" - where the current widens and slackens outside the breaker line. Also RIP SURF. (See Figure A-7)

CURRENT, STREAM - A narrow, deep, and fast-moving ocean current.

CURRENT, TIDAL - A current, caused by the tide-producing forces of the moon and the sun, which is a part of the same general movement of the sea manifested in the vertical rise and fall of the tides. Also CURRENT, PERIODIC. See also CURRENT, FLOOD AND CURRENT, EBB.

CUSP - One of a series of naturally formed low mounds of beach material separated by crescent-shaped troughs spaced at more or less regular intervals along the beach face. Also BEACH CUSP. (See Figure A-7)

CUSPATE BAR - A crescent-shaped bar uniting with shore at each end. It may be formed by a single spit growing from shore turning back to again meet the shore, or by two spits growing from shore uniting to form a bar of sharply cuspate form.

CYCLOIDAL WAVE - A very steep, symmetrical wave whose crest forms an angle of 120°. The wave form is that of a cycloid. A trochoidal wave of maximum steepness. See also WAVE, TROCHOIDAL.

DAILY RETARDATION (OF TIDES) - The amount of time by which corresponding tidal phases grow later day by day (averages approximately 50 minutes).

DATUM, CHART - See CHART DATUM.

DATUM PLANE - The horizontal plane to which soundings, ground elevations, or water surface elevations are referred. Also REFERENCE PLANE. The plane is called a TIDAL DATUM when defined by a certain phase of the tide. The following datums are ordinarily used on hydrographic charts:

MEAN LOW WATER - Atlantic Coast (U. S.), Argentina, Sweden and Norway;
MEAN LOWER LOW WATER - Pacific Coast (U. S.);
MEAN LOW WATER SPRINGS - Great Britain, Germany, Italy, Brazil, and Chile.
LOW WATER DATUM - Great Lakes (U. S. and Canada);
LOWEST LOW WATER SPRINGS - Portugal;
LOW WATER INDIAN SPRINGS - India and Japan (See INDIAN TIDE PLANE)
LOWEST LOW WATER - France, Spain, and Greece.

A common datum used on topographic maps is based upon MEAN SEA LEVEL. See also BENCH MARK.

DEBRIS LINE - A line near the limit of storm wave uprush marking the landward limit of debris deposits.

DECAY DISTANCE - The distance through which waves travel after leaving the generating area.

DECAY OF WAVES - The change that waves undergo after they leave a generating area (fetch) and pass through a calm, or region of lighter winds. In the process of decay, the significant wave height decreases and the significant wave length increases.

DEEP WATER - Water of depth such that surface waves are little affected by conditions on the ocean bottom. It is customary to consider water deeper than one-half the surface wave length as deep water.

DEFLATION - The removal of material from a beach or other land surface by wind action.

DELTA - An alluvial deposit, usually triangular, at the mouth of a river.

DEPTH - The vertical distance from the still water level (or datum as specified) to the bottom.

DEPTH OF BREAKING - The still water depth at the point where the wave breaks. Also BREAKER DEPTH.

DEPTH CONTOUR - See CONTOUR.

DEPTH, CONTROLLING - The least depth of water in the navigable parts of a waterway, which limits the allowable draft of vessels.

DEPTH FACTOR - See SHOALING COEFFICIENT.

DERRICK STONE - Stone of a sufficient size as to require handling in individual pieces by mechanical means, generally 1 ton up.

DIFFRACTION OF WATER WAVES - The phenomenon by which energy is transmitted laterally along a wave crest. When a portion of a train of waves is interrupted by a barrier such as a breakwater, the effect of diffraction is manifested by propagation of waves into the sheltered region within the barrier's geometric shadow.

DIKE (DYKE) - A wall or mound built around a low-lying area to prevent flooding.

DIURNAL - Daily, recurring once each day. (e.g. lunar day or solar day).

DIURNAL TIDE - A tide with one high water and one low water in a tidal day. (See Figure A-10)

DIVERGENCE - (1) In refraction phenomena, the spreading of orthogonals in the direction of wave travel. This denotes an area of decreasing wave height and energy concentration; (2) In wind set-up phenomena, the decrease in set-up observed under that which would occur in an equivalent rectangular basin of uniform depth, caused by changes in planform or depth. Also the increase in basin width or depth causing such decrease

DOWNCOAST - In United States usage, the coastal direction generally trending towards the south.

DOWNDRIFT - The direction of predominant movement of littoral materials.

DRIFT (noun) - (1) The speed at which a current runs; (2) Also, floating material deposited on a beach (driftwood); (3) A deposit of a continental ice sheet, as a ~~DRUNLIN~~; (4) Sometimes used as an abbreviation of LITTORAL DRIFT. DRUMLIN

DRIFT CURRENT - A broad, shallow, slow moving ocean or lake current.

DUKW -(Pronounced duck) Amphibian Truck, 2-1/2 ton, 6 x 6.

DUNES - Ridges or mounds of loose, wind-blown material, usually sand. (See Figure A-7).

DURATION - In wave forecasting, the length of time the wind blows in essentially the same direction over the FETCH (generating area).

DURATION, MINIMUM - The time necessary for steady state wave conditions to develop for a given wind velocity over a given fetch length.

EAGER - See BORE.

EBB CURRENT - The movement of the tidal current away from shore or down a tidal stream.

EBB TIDE - A non-technical term referring to that period of tide between a high water and the succeeding low water; falling tide. (See Figure A-10)

ECHO SOUNDER - A survey instrument that determines the depth of water by measuring the time required for a sound signal to travel to the bottom and return. It may be either "sonic" or "supersonic" depending on the frequency of sound wave, the "sonic" being generally within the audible ranges (under 15,000 cycles per second).

EDDY - A circular movement of water formed on the side of a main current. Eddies may be created at points where the main stream passes projecting obstructions.

EDDY CURRENT - See EDDY.

EELGRASS - A submerged marine plant with very long, narrow leaves, abundant along the North Atlantic Coast. See also KELP and SEAWEED.

EMBANKMENT - An artificial bank, mound, dike or the like, built to hold back water, carry a roadway, etc.

EMBAYED - Formed into a bay or bays, as an embayed shore.

EMBAYMENT - An indentation in a shore line forming an open bay.

ENERGY COEFFICIENT - The ratio of the energy in a wave per unit crest length transmitted forward with the wave at a point in shallow water to the energy in a wave per unit crest length transmitted forward with the wave in deep water. On refraction diagrams this is equal to the ratio of the distance between a pair of orthogonals at a selected point to the distance between the same pair of orthogonals in deep water. Also the square of the REFRACTION COEFFICIENT.

ENTRANCE - The avenue of access or opening to a navigable channel.

EROSION - The wearing away of land by the action of natural forces. (See also SCOUR). On a BEACH, by carrying away of beach material by wave action, tidal currents, or littoral currents or by the action of the wind (See DEFLATION).

ESCARPMENT - A more or less continuous line of cliffs or steep slopes facing in one general direction which are caused by erosion or faulting. Also SCARP. (See Figure A-1)

ESTUARY - (1) That portion of a stream influenced by the tide of the body of water into which it flows; (2) A bay, as the mouth of a river, where the tide meets the river current.

FAIRWAY - The parts of a waterway kept open and unobstructed for navigation.

FATHOM - A unit of measurement used for soundings. It is equal to 6 feet (1.83 meters).

FATHOMETER - The copyrighted trade name for a type of echo sounder.

FEEDER BEACH - An artificially widened beach serving to nourish downdrift beaches by natural littoral currents or forces.

FEEDER CURRENT - The current which flows parallel to shore before converging and forming the neck of a rip current. See also RIP.

FETCH - (1) In wave forecasting, the continuous area of water over which the wind blows in essentially a constant direction. Sometimes used synonymously with FETCH LENGTH. Also GENERATING AREA; (2) In wind set-up phenomena, for inclosed bodies of water, the distance between the points of maximum and minimum water surface elevations. This would usually coincide with the longest axis in the general wind direction.

FETCH LENGTH - In wave forecasting, the horizontal distance (in the direction of the wind) over which the wind blows.

FIRTH - A narrow arm of the sea; also the opening of a river into the sea.

FJORD (FIORD) - A long narrow arm of the sea between highlands.

FLOOD CURRENT - The movement of the tidal current toward the shore or up a tidal stream.

FLOOD TIDE - A non-technical term referring to that period of tide between low water and the succeeding high water; a rising tide. (See Figure A-10)

FOAM LINE - The front of a wave as it advances shoreward, after it has broken. (See Figure A-4)

FOLLOWING WIND - In wave forecasting, wind blowing in the same direction that waves are travelling.

FORESHORE - The part of the shore, lying between the crest of the seaward berm (or the upper limit of wave wash at high tide) and the ordinary low water mark, that is ordinarily traversed by the uprush and backrush of the waves as the tides rise and fall. (See Figure A-1)

FREEBOARD - The additional height of a structure above design high water level to prevent overflow. Also, at a given time the vertical distance between the water level and the top of the structure. On a ship, the distance from the water line to main deck or gunwale.

FRESHET - A rapidly rising flood in a stream resulting from snow melt or rainfall.

FRINGING REEF - A reef attached to an insular or continental shore.

FRONT OF THE FETCH - In wave forecasting it is that end of the generating area toward which the wind is blowing.

GENERATING AREA - In wave forecasting, the continuous area of water surface over which the wind blows in essentially a constant direction. Sometimes used synonymously with FETCH LENGTH. Also FETCH.

GENERATION OF WAVES - (1) The creation of waves by natural or mechanical means; (2) In wave forecasting, the creation and growth of waves caused by a wind blowing over a water surface for a certain period of time. The area involved is called the GENERATING AREA or FETCH.

GEOMETRIC MEAN DIAMETER - The diameter equivalent of the arithmetic mean of the logarithmic frequency distribution. In the analysis of beach sands it is taken as that grain diameter determined graphically by the intersection of a straight line through selected boundary sizes (generally points on the distribution curve where 16 and 84 percent of the sample by weight is coarser) and a vertical line through the median diameter of the sample.

GEOMETRIC SHADOW - In wave diffraction theory, the area outlined by drawing straight lines paralleling the direction of wave approach through the extremities of the protective structure. It differs from the actual protected area to the extent that the diffraction and refraction effects modify the wave pattern.

GEOMORPHOLOGY - That branch of both physiography and geology which deals with the form of the earth, the general configuration of its surface, and the changes that take place in the evolution of land forms.

GRADIENT (GRADE) - See SLOPE. With reference to winds or currents, the rate of increase or decrease in speed, usually in the vertical, or the curve which represents this rate.

GRAVEL - See SOIL CLASSIFICATION.

GRAVITY WAVE - A wave whose velocity of propagation is controlled primarily by gravity.. Water waves of a length greater than 2 inches are considered gravity waves.

GROIN (BRIT. GROYNE) - A shore protective structure (built usually perpendicular to the shore line) to trap littoral drift or retard erosion of the shore. It is narrow in width (measured parallel to the shore line), and its length may vary from less than one hundred to several hundred feet (extending from a point landward of the shore line out into the water). Groins may be classified as permeable or impermeable; impermeable groins having a solid or nearly solid structure, permeable groins having openings through them of sufficient size to permit passage of appreciable quantities of littoral drift.

GROUND SWELL - A long high ocean swell; also, this swell as it rises to prominent height in shallow water, however, usually so high or dangerous as BLIND ROLLERS.

GROUND WATER - Subsurface water occupying the zone of saturation. In a strict sense the term is applied only to water below the WATER TABLE.

GROUP VELOCITY - The velocity at which a wave group travels. In deep water, it is equal to one-half the velocity of the individual waves within the group.

GULF - A relatively large portion of sea, partly enclosed by land.

GUT - (1) A narrow passage such as a strait or inlet. (2) A channel in otherwise shallower water, generally formed by water in motion.

HARBOR (BRIT. HARBOUR) - A protected part of a sea, lake, or other body of water used by vessels as a place of safety and/or the transfer of passengers and cargo between water and land carriers. See also PORT.

HEAD (HEADLAND) - A point or portion of land jutting out into the sea, a lake, or other body of water; a cape or promontory; now, usually specifically, a promontory especially bold and cliff-like.

HEAD OF RIP - The section of a rip current that has widened out seaward of the breakers. See also CURRENT, RIP; CURRENT; FEEDER; and NECK (RIP).

HEIGHT OF WAVE - The vertical distance between a crest and the preceding trough. (See Figure A-3) See also SIGNIFICANT WAVE HEIGHT.

HIGH TIDE: HIGH WATER (HW) - The maximum height reached by each rising tide. See TIDE. (See Figure A-10)

HIGH WATER OF ORDINARY SPRING TIDES (HWOST) - A tidal datum appearing in some British publications, based on high water of ordinary spring tides.

HIGHER HIGH WATER (HHW) - The higher of the two high waters of any tidal day. The single high water occurring daily during periods when the tide is diurnal is considered to be a higher high water. (See Figure A-10).

HIGHER LOW WATER (HLW) - The higher of two low waters of any tidal day. (See Figure A-10)

HIGH WATER - See HIGH TIDE.

HIGH WATER LINE - In strictness, the intersection of the plane of mean high water with the shore. The shore line delineated on the nautical charts of the Coast and Geodetic Survey is an approximation of the mean high water line.

HINDCASTING, WAVE - The calculation from historic synoptic wind charts of the wave characteristics that probably occurred at some past time.

HINTERLAND - The region inland from the coast.

HOOK - A spit or narrow cape, turned landward at the outer end, resembling a hook in form.

HYDRAULIC JUMP - In fluid flow, a change in flow conditions accompanied by a stationary, abrupt turbulent rise in water level in the direction of flow. A type of STATIONARY WAVE.

HYDROGRAPHY - (1) A configuration of an underwater surface including its relief, bottom materials, coastal structures, etc. and (2) The description and study of sea, lakes, rivers, and other waters.

IMPERMEABLE GROIN - See under Groin.

INDIAN SPRING LOW WATER - The approximate level of the mean of lower low waters at spring tides, used principally in the Indian Ocean and along the east coast of Asia. Also INDIAN TIDE PLANE.

INDIAN TIDE PLANE - The datum of INDIAN SPRING LOW WATER.

INLET - A short, narrow waterway connecting a bay, lagoon, or similar body of water with a large parent body of water. An arm of the sea (or other body of water), that is long compared to its width, and that may extend a considerable distance inland. See also TIDAL INLET.

INSHORE (ZONE) - In beach terminology, the zone of variable width extending from the shore face through the breaker zone. (See Figure A-1)

INSHORE CURRENT - Any current in or landward of the breaker zone.

INSULAR SHELF - The zone surrounding an island extending from the line of permanent immersion to the depth (usually about 100 fathoms) where there is a marked or rather steep descent toward the great depths.

INTERNAL WAVES - Waves that occur within a fluid whose density changes with depth, either abruptly at a sharp surface of discontinuity (an interface) or gradually. Their amplitude is greatest at the density discontinuity or, in the case of a gradual density change, somewhere in the interior of the fluid and not at the free upper surface where the surface waves have their maximum amplitude.

ISTHMUS - A narrow strip of land, bordered on both sides by water, that connects two larger bodies of land.

JETTY - (1) (U. S. usage) On open seacoasts, a structure extending into a body of water, and designed to prevent shoaling of a channel by littoral materials, and to direct and confine the stream or tidal flow. Jetties are built at the mouth of a river or tidal inlet to help deepen and stabilize a channel. (2) (British usage) Jetty is synonymous with "wharf" or "pier".

KELP - The general name for several species of large seaweeds. A mass or growth of large seaweed or any of various large brown seaweeds.

KEY - A low insular bank of sand, coral, etc., as one of the islets off the southern coast of Florida, also CAY.

KINETIC ENERGY (OF WAVES) - In a progressive oscillatory wave, a summation of the energy of motion of the particles within the wave. This energy does not advance with the wave form.

KNOLL - (1) A submerged elevation of rounded shape rising from the ocean floor, but less prominent than a seamount. (2) A small rounded hill.

KNOT - (Abbreviation kt. or kts.) The unit of speed used in navigation. It is equal to 1 nautical mile (6,080.20 feet). per hour.

LAGGING - See DAILY RETARDATION (OF TIDES)

LAGOON - A shallow body of water, as a pond or lake, which usually has a shallow, restricted outlet to the sea. (See Figures A-8 and A-9).

LAND BREEZE - A light wind blowing from the land caused by unequal cooling of land and water masses.

LAND-SEA BREEZE - The combination of a land breeze and a sea breeze as a diurnal phenomenon.

LANDLOCKED - An area of water enclosed, or nearly enclosed, by land, as a bay, a harbor, etc. (thus, protected from the sea).

LANDMARK - A conspicuous object - natural or artificial - located near or on land which aids in fixing the position of an observer.

LEADLINE - A line, wire, or cord used in sounding. It is weighted at one end with a plummet (sounding lead). Also SOUNDING LINE.

LEE - (1) Shelter, or the part or side sheltered or turned away from the wind or waves. (2) (Chiefly nautical) The quarter or region toward which the wind blows.

LEEWARD - The direction toward which the wind is blowing; the direction toward which waves are travelling.

LENGTH OF WAVE - The horizontal distance between similar points on two successive waves measured perpendicularly to the crest. (See Figure A-3).

LEVEE - A dike or embankment for the protection of land from inundation.

LIMIT OF BACKRUSH)
LIMIT OF BACKWASH) See BACKWASH.

LITTORAL - Of or pertaining to a shore, especially of the sea. A coastal region.

LITTORAL CURRENT - See CURRENT, LITTORAL

LITTORAL DEPOSITS - Deposits of littoral drift.

LITTORAL DRIFT - The material moved in the littoral zone under the influence of waves and currents.

LITTORAL TRANSPORT - The movement of material along the shore in the littoral zone by waves and currents.

LONGSHORE CURRENT - A current in the surf zone moving essentially parallel to the shore, usually generated by waves breaking at an angle to the shore line.

LOWER HIGH WATER (LHW) - The lower of the two high waters of any tidal day. (See Figure A-10)

LOWER LOW WATER (LLW) - The lower of the two low waters of any tidal day. The single low water occurring daily during periods when the tide is diurnal is considered to be a lower low water. (See Figure A-10).

LOW TIDE (LOW WATER, LW) - The minimum height reached by each falling tide. See TIDE. (See Figure A-10)

LOW WATER DATUM - An approximation to the plane of mean low water that has been adopted as a standard reference plane. See also DATUM PLANE.

LOW WATER LINE - The intersection of any standard low tide datum plane with the shore.

LOW WATER OF ORDINARY SPRING TIDES (LWOST) -- A tidal datum appearing in some British publications, based on low water of ordinary spring tides.

MANGROVE - A particular kind of tropical tree or shrub with thickly matted roots, confined to low-lying brackish areas.

MARIGRAM - A graphic record of the rise and fall of the tide.

MARSH - A tract of soft, wet or periodically inundated land, generally treeless and usually characterized by grasses and other low growth.

MARSH, SALT - A marsh periodically flooded by salt water.

MASS TRANSPORT - The net transfer of water by wave action in the direction of wave travel. See under ORBIT.

MEAN DIAMETER, GEOMETRIC - The diameter equivalent of the arthmetic mean of the logarithmic frequency distribution. In the analysis of beach sands it is taken as that grain diameter determined graphically by the intersection of a straight line through selected boundary sizes (generally points on the distribution curve where 16 and 84 percent of the sample by weight is coarser) and a vertical line through the median diameter of the sample.

MEAN HIGHER HIGH WATER (MHHW) - The average height of the higher high waters over a 19-year period. For shorter periods of observation, corrections are applied to eliminate known variations and reduce the result to the equivalent of a mean 19 year value.

MEAN HIGH WATER (MHW) - The average height of the high waters over a 19-year period. For shorter periods of observations, corrections are applied to eliminate known variations and reduce the result to the equivalent of a mean 19-year value.
All high water heights are included in the average where the type of tide is either semidiurnal or mixed. Only the higher high water heights are included in the average where the type of tide is diurnal. So determined, mean high water in the latter case is the same as mean higher high water.

MEAN HIGH WATER SPRINGS - The average height of the high waters occurring at the time of spring tide. Frequently abbreviated to High Water Springs.

MEAN LOWER LOW WATER (MLLW) - Frequently abbreviated lower low water. The average height of the lower low waters over a 19-year period. For shorter periods of observations, corrections are applied to eliminate known variations and reduce the result to the equivalent of a mean 19-year value.

MEAN LOW WATER (MLW) - The average height of the low waters over a 19-year period. For shorter periods of observations, corrections are applied to eliminate known variations and reduce the result to the equivalent of a mean 19-year value.
All low water heights are included in the average where the type of tide is either semidiurnal or mixed. Only the lower low water heights are included in the average where the type of tide is diurnal. So determined, mean low water in the latter case is the same as mean lower low water.

MEAN LOW WATER SPRINGS - Frequently abbreviated low water springs. The average height of low waters occurring at the time of the spring tides. It is usually derived by taking a plane depressed below the half-tide level by an amount equal to one-half the spring range of tide, necessary corrections being applied to reduce the result to a mean value. This plane is used to a considerable extent for hydrographic work outside of the United States and is the plane of reference for the Pacific approaches to the Panama Canal.

MEAN SEA LEVEL - The average height of the surface of the sea for all stages of the tide over a 19-year period, usually determined from hourly height readings. See also SEA LEVEL DATUM.

MEAN TIDE LEVEL - Also called half-tide level. A plane midway between mean high water and mean low water.

MEDIAN DIAMETER - The diameter which marks the division of a given sample into two equal parts by weight, one part containing all grains larger than that diameter and the other part containing all grains smaller.

MINIMUM DURATION - The time necessary for steady state wave conditions to develop for a given wind velocity over a given fetch length.

MIXED TIDE - A type of tide in which the presence of a diurnal wave is conspicuous by a large inequality in either the high or low water heights with two high waters and two low waters usually occurring each tidal day. In strictness all tides are mixed but the name is usually applied without definite limits to the tides intermediate to those predominantly semidiurnal and those predominantly diurnal. (See Figure A-10)

MOLE - In coastal terminology, a massive solid-fill structure of earth, (generally revetted), masonry, or large stone. It may serve as a breakwater or pier.

MONOLITHIC - Like a single stone or block. Therefore in (say) breakwaters, the type of construction in which the structure's component parts are bound together to act as one.

MUD - A fluid-to-plastic mixture of finely divided particles of solid material and water.

NAUTICAL MILE - The length of a minute of arc, 1/21,600 of an average great circle of the earth. Generally one minute of latitude is considered equal to one nautical mile. The accepted United States value is 6,080.20 feet.... approximately 1.15 times as long as the statute mile of 5,280 feet. Also GEOGRAPHICAL MILE.

NEAP TIDE - A tide occurring near the time of quadrature of the moon. The neap tidal range is usually 10 to 30 percent less than the mean tidal range.

NEARSHORE (ZONE) - In beach terminology an indefinite zone extending seaward from the shore line somewhat beyond the breaker zone. It defines the area of NEARSHORE CURRENTS. The SHOREFACE. (See Figure A-1)

NEARSHORE CIRCULATION - The ocean circulation pattern composed of the NEARSHORE CURRENTS and COASTAL CURRENTS. See under CURRENT.

NEARSHORE CURRENT SYSTEM - The current system caused primarily by wave action in and near the breaker zone, and which consists of four parts: the shoreward mass transport of water; long-shore currents; seaward return flow, including rip currents; and the longshore movement of the expanding heads of rip currents.

NECK - The narrow band of water flowing seaward through the surf. Also RIP.

NIP - The cut made by waves in a shore line of emergence.

NODAL ZONE - An area at which the predominant direction of the littoral transport changes.

NOURISHMENT - The process of replenishing a beach. It may be brought about by natural means, e.g. littoral drift, or by artificial means, e.g. by the deposition of dredged materials.

OCEANOGRAPHY - That science treating of the oceans, their forms, physical features, and phenomena.

OFFSHORE (n. or adj.) - (1) In beach terminology, the comparatively flat zone of variable width, extending from the breaker zone to the seaward edge of the continental shelf. (2) A direction seaward from the shore (See Figure A-1)

OFFSHORE CURRENT - (1) Any current in the offshore zone. (2) Any current flowing away from shore.

OFFSHORE WIND - A wind blowing seaward from the land in the coastal area.

ONSHORE - A direction landward from the sea.

ONSHORE WIND - A wind blowing landward from the sea in the coastal area.

OPPOSING WIND - In wave forecasting, a wind blowing in the opposite direction to that in which the waves are travelling.

ORBIT - In water waves, the path of a water particle affected by the wave motion. In deep water waves the orbit is nearly circular and in shallow water waves the orbit is nearly elliptical. In general, the orbits are slightly open in the direction of wave motion giving rise to MASS TRANSPORT. (See Figure A-3)

ORBITAL CURRENT - The flow of water accompanying the orbital movement of the water particles in a wave. Not to be confused with wave-generated LITTORAL CURRENTS. (See Figure A-3)

ORTHOGONAL - On a refraction diagram, a line drawn perpendicular to the wave crests. (See Figure A-6)

OSCILLATION - A periodic motion to and fro, or up and down.

OSCILLATORY WAVE - A wave in which each individual particle oscillates about a point with little or no permanent change in position. The term is commonly applied to progressive oscillatory waves in which only the form advances, the individual particles moving in closed orbits. Distinguished from a WAVE of TRANSLATION. See Also ORBIT.

OUTFALL - (1) The vent of a river, drain, etc. (2) A structure extending into a body of water for the purpose of discharging sewage, storm runoff, or cooling water.

OVERWASH - That portion of the uprush that carries over the crest of a berm or of a structure.

PARAPET - A low wall built along the edge of a structure as on a seawall or quay.

PARTICLE VELOCITY - For waves, the velocity induced by wave motion with which a specific water particle moves.

PASS - In hydrographic usage a navigable channel, through a bar, reef, or shoal, or between closely adjacent islands.

PEBBLES - See SOIL CLASSIFICATION.

PENINSULA - An elongated portion of land nearly surrounded by water, and connected to a larger body of land.

PERIODIC CURRENT - A current caused by the tide-producing forces of the moon and the sun, a part of the same general movement of the sea that is manifested in the vertical rise and fall of the tides. See also CURRENT, FLOOD and CURRENT, EBB.

PERMAFROST - Permanently forzen subsoil.

PERMANENT CURRENTS - A current that runs continuously independent of the tides and temporary causes. Permanent currents include the fresh water discharge of a river and the currents that form the general circulatory systems of the oceans.

PERMEABLE GROIN - See under GROIN.

PETROGRAPHY - The description and systematic classification of rocks.

PIER - A structure, extending out into the water from the shore, to serve as a landing place, a recreational facility, etc., rather than to afford coastal protection.

PILE - A long, slender piece of wood, concrete, or metal to be driven or jetted into the earth or sea bed to serve as a support or protection.

PILING - A group of piles.

PILE, SHEET - A pile with a generally flat cross-section to be driven into the ground or sea bed and meshed or interlocked with like members to form a diaphragm, wall, or bulkhead.

PINNACLE - A tall, slender, pointed, rocky mass. See also REEF PINNACLE.

PLAIN - An extent of level or nearly level land.

PLAIN, COASTAL - A plain fronting the coast and generally representing a strip of recently emerged sea bottom.

PLANFORM - The outline or shape of a body of water as determined by the still water line.

PLATEAU - An elevated plain, table land, or flat-topped region of considerable extent.

PLUNGE POINT - (See Figure A-1) (1) For a plunging wave, the point at which the wave curls over and falls; (2) The final breaking point of the waves just before they rush up on the beach.

PLUNGING BREAKER - See under BREAKER.

POINT - The extreme end of a cape; or the outer end of any land area protruding into the water, usually less prominent than a cape.

PORT - A place where vessels may discharge or receive cargo; may be the entire harbor including its approaches and anchorages or may be the commercial part of a harbor, where the quays, wharves, facilities for transfer of cargo, docks, repair shops, etc. are situated.

POTENTIAL ENERGY OF WAVES - In a progressive oscillatory wave, the energy resulting from the elevation or depression of the water surface from the undisturbed level. This energy advances with the wave form.

PROFILE BEACH - The intersection of the ground surface with a vertical plane; may extend from the top of the dune line to the seaward limit of sand movement. (See Figure A-1)

PROGRESSION - See ADVANCE,

PROGRESSIVE WAVE - A wave which is manifested by the progressive movement of the wave form.

PROMONTORY - A high point of land projecting into a body of water; a headland.

PROPAGATION OF WAVES - The transmission of waves through water.

PROTOTYPE - In laboratory usage, the original structure, concept, or phenomenon used as a basis for constructing a scale model or copy.

QUAY - (pronounced KEY) - A stretch of paved bank, or a solid artificial landing place parallel to the navigable waterway, for use in loading and unloading vessels.

QUICKSAND - Loose, yielding, wet sand which offers no support to heavy objects. The upward flow of the water has a velocity that eliminates contact pressures between the sand grains, and causes the sand-water mass to behave like a fluid.

RECESSION (OF A BEACH) - (1) A continuing landward movement of the shore line. (2) A net landward movement of the shore line over a specified time. Also RETROGRESSION.

REEF - A chain or range of rock or coral, elevated above the surrounding bottom of the sea, generally submerged and dangerous to surface navigation.

REEF, ATOLL - A ring-shaped, coral reef, often carrying low sand islands, enclosing a body of water.

REEF, BARRIER - A reef which roughly parallels land but is some distance offshore, with deeper water intervening.

REEF, FRINGING - A reef attached to an insular or continental shore.

REEF, SAND - Synonymous with BAR.

REFERENCE PLANE - See DATUM PLANE.

REFERENCE POINT - A specified location (in plan and/or elevation) to which measurements are referred.

REFERENCE STATION - A station for which tidal constants have previously been determined and which is used as a standard for the comparison of simultaneous observations at a second station; also a station for which independent daily predictions are given in the tide or current tables from which corresponding predictions are obtained for other stations by means of differences or factors.

REFLECTED WAVE - The wave that is returned seaward when a wave impinges upon a very steep beach, barrier, or other reflecting surfaces.

REFRACTION OF WATER WAVES - (1) The process by which the direction of a wave moving in shallow water at an angle to the contours is changed. The part of the wave advancing in shallower water moves more slowly than that part still advancing in deeper water, causing the wave crest to bend toward alignment with the underwater contours. (2) The bending of wave crests by currents. (See Figure A-5)

REFRACTION COEFFICIENT - The square root of the ratio of the spacing between adjacent orthogonals in deep water and in shallow water at a selected point. When multiplied by the SHOALING FACTOR, this becomes the WAVE HEIGHT COEFFICIENT or the ratio of the refracted wave height at any point to the deep water wave height. Also the square root of the ENERGY COEFFICIENT.

REFRACTION DIAGRAM - A drawing showing positions of wave crests and/or orthogonals in a given area for a specific deep water wave period and direction. (See Figure A-6)

RETARDATION - The amount of time by which corresponding tidal phases grow later day by day (averages approximately 50 minutes).

RETROGRESSION OF A BEACH - (1) A continuing landward movement of the shore line; (2) A net landward movement of the shore line over a specified time. Also RECESSION.

REVETMENT - A facing of stone, concrete, etc., built to protect a scarp, embankment or shore structure against erosion by the wave action or currents.

RIA - A long narrow inlet, with depth gradually diminishing inward.

RIDE-UP - See RUN-UP.

RIDGE, BEACH - An essentially continuous mound of beach material that has been shaped up by wave or other action. Ridges may occur singly or as a series of approximately parallel deposits. (See Figure A-7) In England they are called FULLS.

RILL MARKS - Tiny drainage channels in a beach caused by the flow seaward of water left in the sands of the upper part of the beach after the retreat of the tide or after the dying down of storm waves.

RIP - A body of water made rough by waves meeting an opposing current, particularly a tidal current; often found where tidal currents are converging and sinking. A TIDE RIP.

RIPARIAN - Pertaining to the banks of a body of water.

RIPARIAN RIGHTS - The rights of a person owing land containing or bordering on a watercourse or other body of water in or to its banks, bed, or waters.

RIP CURRENTS - A strong surface current of short duration flowing seaward from the shore. It usually appars as a visible band of agitated water and is the return movement of water piled up on the shore by incoming waves and wind. With the seaward movement concentrated in a limited band its velocity is somewhat accentuated. A rip consists of three parts: the FEEDER CURRENT flowing parallel to the shore inside the breakers; the NECK, where the feeder currents converge and flow through the breakers in a narrow band or "rip"; and the HEAD, where the current widens and slackens outside the breaker line. A rip current is often miscalled a RIP TIDE. Also RIP SURF. (See Figure A-7).

RIP SURF - See under RIP CURRENT.

RIPPLE --(1) The ruffling of the surface of water, hence a little curling wave or undulation. (2) A wave controlled to a significant degree by both surface tension and gravity. See WAVE, CAPILLARY and WAVE, GRAVITY.

RIPPLE MARKS - Small, fairly regular ridges in the bed of a waterway or on a land surface caused by water currents or wind. As their form is approximately normal to the direction of current or wind, they indicate both the presence and the direction of currents or winds.

RIPRAP - A layer, facing, or protective mound of stones randomly placed to prevent erosion, scour, or sloughing of a structure or embankment; also the stone so used.

RISE, TIDAL - The height of tide as referred to the datum of a chart.

ROADSTEAD - (Nautical) A sheltered area of water near shore where vessels may anchor in relative safety. Also ROAD.

ROCK - (1) (Engineering) A natural aggregate of mineral particles connected by strong and permanent cohesive forces. In igneous and metamorphic rocks, it consists of interlocking crystals; in sedimentary rocks, of closely packed mineral grains, often bound together by a natural cement. Since the terms "strong" and "permanent" are subject to different interpretations, the boundary between rock and soil is necessarily an arbitrary one. (2) (Geological) The material that forms the essential part of the earth's solid crust, and includes loose incoherent masses, such as a bed of sand, gravel, clay or volcanic ash, as well as the very firm, hard, and solid masses of granite, sandstone, limestone, etc. Most rocks are aggregates of one or more minerals, but some are composed entirely of glassy matter, or of a mixture of glass and minerals.

ROLLER - An indefinite term, sometimes considered to be one of a series of long-crested, large waves which roll in upon a coast, as after a storm.

RUBBLE - (1) Losse angular water-worn stones along a beach. (2) Rough, irregular fragments of broken rock.

RUNNEL - A corrugation (trough) of the foreshore (or the bottom just offshore), formed by wave and/or tidal action. Larger than the trough between ripple marks.

RUN-UP - The rush of water up a structure on the breaking of a wave. Also UPRUSH. The amount of run-up is the vertical height above still water level that the rush of water reaches.

SALTATION - That method of sand movement in a fluid in which individual particles leave the bed by bounding nearly vertically and, because the motion of the fluid is not strong or turbulent enough to retain them in suspension, return to the bed at some distance downstream. The travel path of the particles is a series of hops and bounds.

SALT MARSH - A marsh periodically flooded by salt water.

SAND - See SOIL CLASSIFICATION.

SAND BAR - (1) See BAR. (2) In a river, a ridge of sand built up to or near the surface by river currents.

SAND REEF - Synonymous with BAR.

SCARP - A more or less continuous line of cliffs or steep slopes facing in one general direction which are caused by erosion or faulting. Also ESCARPMENT.

SCARP, BEACH - An almost vertical slope along the beach caused by erosion by wave action. It may vary in height from a few inches to several feet, depending on wave action and the nature and composition of the beach.

SCOUR - Erosion, especially by moving water. See also EROSION.

SEA - (1) An ocean, or alternatively a large body of (usually) salt water less than an ocean; (2) Waves caused by wind at the place and time of observation; (3) State of the ocean or lake surface in regard to waves.

SEA (STATE OF) - Description of the sea surface with regard to wave action.

SEA BREEZE - (1) A breeze blowing from the sea toward the land; (2) A light wind blowing toward the land caused by unequal heating of land and water masses.

SEA CLIFF - A cliff situated at the seaward edge of the coast.

SEA LEVEL - See MEAN SEA LEVEL.

SEA MOUNT - A submarine mountain rising more than 500 fathoms above the ocean floor.

SEA PUSS - A dangerous longshore current, a rip current, caused by return flow, loosely the submerged channel or inlet through a bar caused by those currents.

SEASHORE - The SHORE of a sea or ocean.

SEAWALL - A structure separating land and water areas primarily designed to prevent erosion and other damage due to wave action. See also BULKHEAD.

SEICHE - A periodic oscillation of a body of water whose period is determined by the resonant characteristics of the containing basin as controlled by its physical dimensions. These periods generally range from a few minutes to an hour or more. (Originally the term was applied only to lakes but now also to harbors, bays, oceans, etc.).

SEISMIC SEA WAVE - (TSUNAMI) - A generally long period wave caused by an underwater seismic disturbance or volcanic eruption. Commonly misnamed "tidal wave".

SEMIDIURNAL TIDES - A tide with two high and two low waters in a tidal day, with comparatively little diurnal inequality. (See Figure A-10)

SET OF CURRENT - The direction toward which a current flows.

SET-UP, WIND - (1) The vertical rise in the still water level on the leeward side of a body of water caused by wind stresses on the surface of the water; (2) The difference in still water level between the windward and the leeward sides of a body of water caused by wind stresses on the surface of the water; (3) Synonymous with WIND TIDE. WIND TIDE is usually reserved for use on the ocean and large bodies of water. WIND SET-UP is usually reserved for use on reservoirs and small bodies of water. (See Figure A-11)

SHALLOW WATER - (1) Commonly; water of such a depth that surface waves are noticeably affected by bottom topography. It is customary to consider water of depths less than half the surface wave length as shallow water. See TRANSITIONAL WATER; (2) More strictly; in hydrodynamics with regard to progressive gravity waves, water in which the depth is less than 1/25th the wave length. Also called VERY SHALLOW WATER.

SHEET PILE - See under PILE.

SHELF, CONTINENTAL - The zone bordering a continent extending from the line of permanent immersion to the depth (usually about 100 fathoms) where there is a marked or rather steep descent toward the great depths.

SHELF, INSULAR - The zone surrounding an island extending from the line of permanent immersion to the depth (usually about 100 fathoms) where there is a marked or rather steep descent toward the great depths.

SHINGLE - (1) Loosely and commonly; any beach material coarser than ordinary gravel, especially any having flat or flattish pebbles; (2) Strictly and accurately; beach material of smooth, well-rounded pebbles that are roughly the same size. The spaces between pebbles are not filled with finer materials. Shingle gives out a musical note when stepped on.

SHOAL (noun) - A detached elevation of the sea bottom comprised of any material except rock or coral, and which may endanger surface navigation.

SHOAL (verb) - (1) to become shallow gradually; (2) to cause to become shallow; (3) to proceed from a greater to a lesser depth of water.

SHOALING COEFFICIENT - The ratio of the height of a wave in water of any depth to its height in deep water with the effect of refraction eliminated. Sometimes SHOALING FACTOR or DEPTH FACTOR. See also ENERGY COEFFICIENT and REFRACTION COEFFICIENT.

SHORE - The strip of ground bordering any body of water. A shore of unconsolidated material is usually called a BEACH. (See Figure A-1)

SHORE FACE - The narrow zone seaward from the low tide SHORE LINE permanently covered by water, over which the beach sands and gravels actively oscillate with changing wave conditions.

SHORE LINE - The intersection of a specified plane of water with the shore or beach. (e.g. the high water shore line would be the intersection of the plane of mean high water with the shore or beach). The line delineating the shore line on U. S. Coast and Geodetic Survey nautical charts and surveys approximates the mean high water line.

SIGNIFICANT WAVE - A statistical term denoting waves with the average height and period of the one-third highest waves of a given wave group. The composition of the higher waves depends upon the extent to which the lower waves are considered. Experience so far indicates that a careful observer who attempts to establish the character of the higher waves will record values which approximately fit the definition. A wave of significant wave period and significant wave height.

SIGNIFICANT WAVE HEIGHT - The average height of the one-third highest waves of a given wave group. Note that the composition of the highest waves depends upon the extent to which the lower waves are considered. In wave record analysis, the average height of the highest 1/3 of a selected number of waves, this number being determined by dividing the time of record by the significant period. Also CHARACTERISTIC WAVE HEIGHT.

SIGNIFICANT WAVE PERIOD - An arbitrary period generally taken as the period of the 1/3 highest waves within a given group. Note that the composition of the highest waves depends upon the extent to which the lower waves are considered. In wave record analysis, this is determined as the average period of the most frequently recurring of the larger well-defined waves in the record under study.

SILT - See SOIL CLASSIFICATION.

SLACK TIDE (SLACK WATER) - The state of a tidal current when its velocity is near zero, especially the moment when a reversing current changes direction and its velocity is zero. Sometimes considered the intermediate period between ebb and flood currents during which the velocity of the currents is less than 0.1 knot. See TIDAL STAND.

SLIP - A space between two piers, wharves, etc. for the berthing of vessels.

SLOPE - The degree of inclination to the horizontal. Usually expressed as a ratio, such as 1:25 or 1 on 25, indicating 1 unit rise in 25 units of horizontal distance; or in a decimal fraction (0.04); degrees (2° 18'); or percent (4%). It is sometimes described by such adjectives as; steep, moderate, gentle, mild, or flat.

SLOUGH - (pronounced - sloo): - (1) A small muddy marshland or tidal waterway which usually connects other tidal areas; (2) A tide-land or bottom-land creek.

SOIL CLASSIFICATION (Size) - An arbitrary division of a continuous scale of sizes such that each scale unit or grade may serve as a convenient class interval for conducting the analysis or for expressing the results of an analysis. There are many classifications used; some of those most often used are presented below:

1. Wentworth's Size Classification

Grade Limits (Diameters)	Name
Above 256 mm.	Boulder
256 - 64 mm.	Cobble
64 - 4 mm.	Pebble
4 - 2 mm.	Granule
2 - 1 mm.	Very coarse sand
1 - 1/2 mm.	Coarse sand
1/2 - 1/4 mm.	Medium sand
1/4 - 1/8 mm.	Fine sand
1/8 - 1/16 mm.	Very fine sand
1/16 - 1/256 mm.	Silt
Below 1/256 mm.	Clay

2. U. S. Army Corps of Engineers' Classification

Grade Limits (Diameters)	U.S. Standard Sieve Size	Name
Above 76 mm.	Above 3"	Cobbles
76 mm. - 19 mm.	3" to 3/4"	Coarse gravel
19 mm. - 4.76 mm.	3/4" to No. 4	Fine gravel
4.76 - 2.00 mm.	No. 4 to No. 10	Coarse sand
2.00 - 0.42 mm.	No. 10 to No. 40	Medium sand
0.42 - 0.074 mm.	No. 40 to No. 200	Fine sand
Below 0.074 mm.	Below No. 200	Silt or clay

3. U. S. Bureau of Soils Classification

~~Grad~~ Grade Limits (Diameters)	Name
2 - 1 mm.	Fine gravel
1 - 1/2 mm.	Coarse sand
1/2 - 1/4 mm.	Medium sand
1/4 - 1/10 mm.	Fine sand
1/10 - 1/20 mm.	Very fine sand
1/20 - 1/200 mm.	Silt
Below 1/200 mm.	Clay

4. Atterberg's Size Classification

Grade Limits (Diameters)	Name
2,000 - 200 mm.	Blocks
200 - 20 mm.	Cobbles
20 - 2 mm.	Pebbles
2 - 0.2 mm.	Coarse sand
0.2 - 0.02 mm.	Fine sand
0.02 - 0.002 mm.	Silt
Below 0.002 mm.	Clay

SOLITARY WAVE - A wave consisting of a single elevation (above the water surface) of height not necessarily small compared to the depth, and neither followed nor preceded by another elevation or depression of the water surfaces.

SORTING COEFFICIENT - A coefficient used in describing the distribution of grain sizes in a sample of unconsolidated material. It is defined as $S_o = \sqrt{Q_1/Q_3}$, where Q_1 is the diameter which has 75% of the cumulative size-frequency (by wt.) distribution smaller than itself and 25% larger than itself, and Q_3 is that diameter having 25% smaller and 75% larger than itself.

SOUND (noun) - (1) A wide waterway between the mainland and an island, or a wide waterway connecting two sea areas. See also STRAIT. (2) A relatively long arm of the sea or ocean forming a channel between an island and a mainland or connecting two larger bodies , as a sea and the ocean, or two parts of the same body; usually wider and more extensive than a strait.

SOUND (verb) - To measure or ascertain the depth of water as with sounding lines.

SOUNDING - A measured depth of water. On hydrographic charts the soundings are adjusted to a specific plane of reference (SOUNDING DATUM).

SOUNDING DATUM - The plane to which soundings are referred. See also DATUM, CHART.

SOUNDING LINE - A line, wire, or cord used in sounding. It is weighted at one end with a plummet (sounding lead). Also LEADLINE.

SPILLING BREAKER - See under BREAKER.

SPIT - A small point of land or submerged ridge running into a body of water from the shore. (See Figure A-9).

SPRING TIDE - A tide that occurs at or near the time of new and full moon and which rises highest and falls lowest from the mean level.

STAND OF TIDE - An interval at high or low water when there is no sensible change in the height of the tide. The water level is stationary at high and low water for only an instant, but the change in level near these times is so slow that it is not usually perceptible. See TIDE, SLACK.

STANDING WAVE - A type of wave in which the surface of the water oscillates vertically between fixed points, called nodes, without progression. The points of maximum vertical rise and fall are called antinodes or loops. At the nodes, the underlying water particles exhibit no vertical motion but maximum horizontal motion. At the antinodes the underlying water particles have no horizontal motion and maximum vertical motion. They may be the result of two equal progressive wave trains travelling through each other in opposite directions. Sometimes called STATIONARY WAVE.

STATIONARY WAVE - A wave of essentially stable form which does not move with respect to a selected reference point; a fixed swelling. Sometimes called STANDING WAVE.

STILL WATER LEVEL - The elevation of the surface of the water if all wave action were to cease. (See Figure A-3)

STONE - (1) Rock or rocklike matter used as a material for building; (2) A small piece of rock or a specific piece of rock.

STONE, DERRICK - Stone of a sufficient size as to require handling in individual pieces by mechanical means, generally 1 ton up.

STORM TIDE - The rise of water accompanying a storm caused by wind stresses on the water surface. See also SET-UP, WIND.

STRAIT - A relatively narrow waterway between two larger bodies of water. See also SOUND (noun).

STREAM - (1) A course of water flowing along a bed in the earth; (2) A current in the sea formed by wind action, water density differences, etc. (Gulf Stream) See also CURRENT, STREAM.

SURF - The wave activity in the area between the shore line and the outermost limit of breakers.

SURF BEAT - Irregular oscillations of the nearshore water level, with periods of the order of several minutes.

SURF ZONE - The area between the outermost breaker and the limit of wave uprush (See Figures A-2 and A-5).

SURGE - (1) The name applied to wave motion with a period intermediate between that of the ordinary wind wave and that of the tide, say from 1/2 to 60 minutes. It is of low height; usually less than 0.3 foot. See also SEICHE. (2) In fluid flow, long interval variations in velocity and pressure, not necessarily periodic, perhaps even transient in nature.

SURGING BREAKER - See under Breaker.

SWAMP (noun) - A tract of wet spongy land, frequently inundated by fresh or salt water, and characteristically dominated by trees and shrubs.

SWAMP (verb) - To overset, sink, or fill up a craft with water.

SWASH - The rush of water up onto the beach following the breaking of a wave. Also UPRUSH, RUN-UP.

SWASH CHANNEL - (1) On the open shore, a channel cut by flowing water in its return to the parent body (e.g. a rip channel); (2) A secondary channel passing through or shoreward of an inlet or river bar. (See Figure A-9)

SWASH MARK - The thin wavy line of fine sand, mica scales, bits of seaweed, etc. left by the uprush when it recedes from its upward limit of movement on the beach face.

SWELL - Wind-generated waves that have advanced into regions of weaker winds or calm.

TERRACE - A horizontal or nearly horizontal natural or artificial topographic feature interrupting a steeper slope, sometimes occurring in a series.

TIDAL CURRENT - A current caused by the tide-producing forces of the moon and the sun, a part of the same general movement of the sea that is manifested in the vertical rise and fall of the tides. Also CURRENT, PERIODIC. See also CURRENT, FLOOD and CURRENT, EBB.

TIDAL DATUM - See DATUM, CHART, and DATUM PLANE.

TIDAL DAY - The time of the rotation of the earth with respect to the moon, or the interval between two successive upper transits of the moon over the meridian of a place, about 24.84 solar hours (24 hours and 50 minutes) in length or 1.035 times as great as the mean solar day. (See Figure A-10)

TIDAL FLATS - Marshy or muddy land areas which are covered and uncovered by the rise and fall of the tide.

TIDAL INLET - (1) A natural inlet maintained by tidal flow; (2) Loosely any inlet in which the tide ebbs and flows. Also TIDAL OUTLET.

TIDAL PERIOD - The interval of time between two consecutive like phases of the tide. (See Figure A-10)

TIDAL POOL - A pool of water remaining on a beach or reef after recession of the tide.

TIDAL PRISM - The total amount of water that flows into the harbor or out again with movement of the tide, excluding any fresh water flow.

TIDAL RANGE - The difference in height between consecutive high and low waters. (See Figure A-10)

TIDAL RISE - The height of tide as referred to the datum of a chart. (See Figure A-10)

TIDAL WAVE - See TSUNAMI.

TIDE - The periodic rising and falling of the water that results from gravitational attraction of the moon and sun acting upon the rotating earth. Although the accompanying horizontal movement of the water resulting from the same cause is also sometimes called the tide, it is preferable to designate the latter as TIDAL CURRENT, reserving the name tide for the vertical movement.

TIDE, DAILY RETARDATION OF - The amount of time by which corresponding tides grow later day by day.

TIDE, DIURNAL - A tide with one high water and one low water in a tidal day. (See Figure A-10)

TIDE, EBB - That period of tide between a high water and the succeeding low water; falling tide. (See Figure A-10)

TIDE, FLOOD - That period of tide between low water and the succeeding high water; a rising tide. (See Figure A-10)

TIDE, MIXED - A type of tide in which the presence of a diurnal wave is conspicuous by a large inequality in either the high or low water heights with two high waters and two low waters usually occurring each tidal day. In strictness all tides are mixed but the name is usually applied without definite limits to the tides intermediate to those predominantly semidiurnal and those predominantly diurnal. (See Figure A-10)

TIDE, NEAP - A tide occurring near the time of quadrature of the moon. The neap tidal range is usually 10 to 30 percent less than the mean tidal range.

TIDE, SEMIDIURNAL - A tide with two high and two low waters in a tidal day, with comparatively little diurnal inequality. (See Figure A-10)

TIDE, SLACK - The state of a tidal current when its velocity is near zero, especially the moment when a reversing current changes direction and its velocity is zero. Sometimes considered the intermediate period between ebb and flood currents during which the velocity of the currents is less than 0.1 knot. See TIDAL STAND. Also SLACK WATER.

TIDE, SPRING - A tide that occurs at or near the time of new and full moon and which rises highest and falls lowest from the mean level.

TIDE, STORM - The rise af water accompanying a storm caused by wind stresses on the water surface. See also SET-UP, WIND.

TOMBOLO - An area of unconsolidated material, deposited by wave action or currents, that connects a rock, or island, etc. to the mainland or to another island. (See Figure A-9)

TOPOGRAPHY - The configuration of a surface including its relief, the position of its streams, roads, buildings, etc.

TRAINING WALL - A wall or jetty to direct current flow.

TRANSITIONAL ZONE
TRANSITIONAL WATER - In regard to progressive gravity waves, water whose depth is less than 1/2 but more than 1/25 the wave length. Often called SHALLOW WATER.

TROCHOIDAL WAVE - A progressive oscillatory wave whose form is that of a prolate cycloid or trochoid. It is approximated by waves of small amplitude. See also WAVE, CYCLOIDAL.

TROUGH OF WAVE - The lowest part of a wave form between successive crests. Also that part of a wave below still water level. (See Figure A-3).

TSUNAMI - A generally long period wave caused by underwater seismic disturbance or volcanic eruption. Commonly misnamed "tidal wave".

UNDERTOW - A current, below water surface, flowing seaward; also the receding water below the surface from waves breaking on a shelving beach. Actually "undertow" is largely mythical. As the backwash of each wave flows down the beach, a current is formed which flows seaward, however, it is a periodic phenomenon. The most common phenomena expressed as "undertow" are actually the rip currents in the surf. Often uniform return flows seaward or lakeward are termed "undertow" though these flows will not be as strong as rip currents. See also CURRENT SYSTEM, NEARSHORE.

UNDERWATER GRADIENT - The slope of the sea bottom. See also SLOPE.

UNDULATION - A continuously propagated motion to and fro, in any fluid or elastic medium, with no permanent translation of the particles themselves.

UPCOAST - In United States usage; the coastal direction generally trending towards the north.

UPDRIFT - The direction opposite that of the predominant movement of littoral materials.

UPLIFT - The upward water pressure on the base of a structure or pavement.

UPRUSH - The rush of water up onto the beach following the breaking of a wave. Also SWASH, RUN-UP. (See Figure A-2)

VALLEY, SEA - A submarine depression of broad valley form without the steep side slopes which characterize a canyon.

VALLEY, SUBMARINE - A prolongation of a land valley into or across the continental or insular shelf, which generally gives evidence of having been formed by stream erosion.

VARIABILITY OF WAVES - (1) The variation of heights and periods between individual waves within a wave train. (Wave trains are not composed of waves of equal height and period, but rather of heights and periods which vary in a statistical manner). (2) The variation in direction of propagation of waves leaving the generating area. (3) The variation in height along the crest. This is usually called "variation along the wave".

VELOCITY OF WAVES - The speed with which an individual wave advances.

VISCOSITY - Internal friction due to molecular cohesion in fluids. The internal properties of a fluid which offer resistance to flow.

WATER LINE - A juncture of land and sea. This line migrates, changing with the tide or other fluctuation in the water level. Where waves are present on the beach, this line is also known as the limit of backrush. (Approximately the intersection of the land with the still water level).

WAVE - A ridge, deformation, or undulation of the surface of a liquid.

WAVE AGE - The ratio of wave velocity to wind velocity.

WAVE, CAPILLARY - A wave whose velocity of propagation is controlled primarily by the surface tension of the liquid in which the wave is travelling. Water waves of a length less than one inch are considered to be capillary waves.

WAVE CREST - The highest part of a wave. Also that part of the wave above still water level. (See Figure A-3)

WAVE CREST LENGTH - The length of a wave along its crest. Sometimes called CREST WIDTH.

WAVE, CYCLOIDAL - A very steep, symmetrical wave whose crest forms an angle of 120°. The wave form is that of a cycloid. A trochoidal wave of maximum steepness. See also WAVE, TROCHOIDAL.

WAVE DECAY - The change which waves undergo after they leave a generating area(fetch) and pass through a calm, or region of lighter or opposing winds. In the process of decay, the significant wave height decreases and the significant wave length increases.

WAVE DIRECTION - The direction from which a wave approaches.

WAVE FORECASTING - The theoretical determination of future wave characteristics, usually from observed or predicted meteorological phenomena.

WAVE GENERATION - (1) The creation of waves by natural or mechanical means. (2) In wave forecasting, the growth of waves caused by a wind blowing over a water surface for a certain period of time. The area involved is called the GENERATING AREA or FETCH.

WAVE, GRAVITY - A wave whose velocity of propagation is controlled primarily by gravity. Water waves of a length greater than 2 inches are considered gravity waves.

WAVE GROUP - A series of waves in which the wave direction, wave length, and wave height vary only slightly. See also GROUP VELOCITY.

WAVE HEIGHT - The vertical distance between a crest and the preceding trough. See also SIGNIFICANT WAVE HEIGHT.

WAVE HEIGHT COEFFICIENT - The ratio of the wave height at a selected point to the deep water wave height. The refraction coefficient multiplied by the shoaling factor.

WAVE HINDCASTING - The calculation from historic synoptic wind charts of the wave characteristics that probably occurred at some past time.

WAVE LENGTH,- The horizontal distance between similar points on two successive waves measured perpendicularly to the crest.

WAVE, OSCILLATORY - A Wave in which each individual particle oscillates about a point with little or no permanent change in position. The term is commonly applied to progressive oscillatory waves in which only the form advances, the individual particles moving in closed or nearly closed orbits. Distinguished from a WAVE of TRANSLATION. See also ORBIT.

WAVE PERIOD - The time for a wave crest to traverse a distance equal to one wave length. The time for two successive wave crests to pass a fixed point. See also SIGNIFICANT WAVE PERIOD.

WAVE, PROGRESSIVE - A wave which is manifested by the progressive movement of the wave form.

WAVE PROPAGATION - The transmission of waves through water.

WAVE RAY - See ORTHOGONAL.

WAVE, REFLECTED - The wave that is returned seaward when a wave impinges upon a very steep beach or barrier.

WAVE REFRACTION - (1) The process by which the direction of a train of waves moving in shallow water at an angle to the contours is changed. The part of the wave train advancing in shallower water moves more slowly than that part still advancing in deeper water, causing the wave crests to bend toward alignment with the underwater contours. (See Figures A-5 and A-6). (2) The bending of wave crests by currents.

WAVE, SEISMIC - A TSUNAMI - A generally long period wave caused by an underwater seismic disturbance or volcanic eruption. Commonly misnamed "tidal wave".

WAVE, SOLITARY - A wave consiting of a single elevation (above the water surface) of height not necessarily small compared to the depth and neither followed nor preceded by another elevation or depression of the water surfaces.

WAVE, STANDING - A type of wave in which there are nodes, or points of no vertical motion and maximum horizontal motion, between which the water oscillates vertically. The points of maximum vertical motion and least horizontal motion are called antinodes or loops. It is caused by the meeting of two similar wave groups travelling in opposing directions.

WAVE, STATIONARY - A wave of essentially stable form which does not move with respect to a selected reference point.

WAVE STEEPNESS - The ratio of a wave's height to its length.

WAVE TRAIN - A series of waves from the same direction.

WAVE OF TRANSLATION - A wave in which the water particles are permanently displaced to a significant degree in the direction of wave travel. Distinguished from an OSCILLATORY WAVE.

WAVE, TROCHOIDAL - A progressive oscillatory wave whose form is that of a prolate cycloid or trochoid. It is approximated by waves of small amplitude. See also WAVE, CYCLOIDAL.

WAVE TROUGH - The lowest part of a wave form between successive crests. Also that part of a wave below still water level.

WAVE VARIABILITY - (1) The variation of heights and periods between individual waves within a wave train. (Wave trains are not composed of waves of equal height and period, but rather of heights and periods which vary in a statistical manner). (2) The variation in direction of propagation of waves leaving the generating area. (3) The variation in height along the crest. This is usually called "variation along the wave".

WAVE VELOCITY - The speed with which an individual wave advances.

WAVE, WIND - A wave that has been formed and built up by the wind.

WAVES, INTERNAL - Waves that occur within a fluid whose density changes with depth, either abruptly at a sharp surface of discontinuity (an interface) or gradually. Their amplitude is greatest at the density discontinuity or, in the case of a gradual density change, somewhere in the interior of the fluid and not at the free upper surface where the surface waves have their maximum amplitude.

WHARF - A structure built on the shore of a harbor, river, canal, etc., so that vessels may lie alongside to receive and discharge cargo, passengers, etc.

WHITECAP - On the crest of a wave, the white froth caused by wind.

WIND - The horizontal natural movement of air; air naturally in motion with any degree of velocity.

WIND CHOP - The short-crested waves that may spring up quickly in a fairly moderate breeze, and break easily at the crest.

WIND, FOLLOWING - In wave forecasting, wind blowing in the same direction that waves are travelling.

WIND, OFFSHORE - A wind blowing seaward over the coastal area.

WIND, ONSHORE - A wind blowing landward over the coastal area.

WIND, OPPOSING - In wave forecasting, wind blowing in the opposite direction to that in which the waves are travelling.

WIND SET-UP - (1) The vertical rise in the still water level on the leeward side of a body of water caused by wind stresses on the surface of the water; (2) the difference in still water levels on the windward and the leeward sides of a body of water caused by wind stresses on the surface of the water; (3) Synonymous with WIND TIDE. WIND TIDE is usually reserved for use on the ocean and large bodies of water. WIND SET-UP is usually reserved for use on reservoirs and smaller bodies of water (See Figure A-11)

WIND TIDE - See WIND SET-UP.

WINDWARD - The direction from which the wind is blowing.

WIND WAVES (1) Waves being formed and built up by the wind, (2) Loosely, any wave generated by wind.

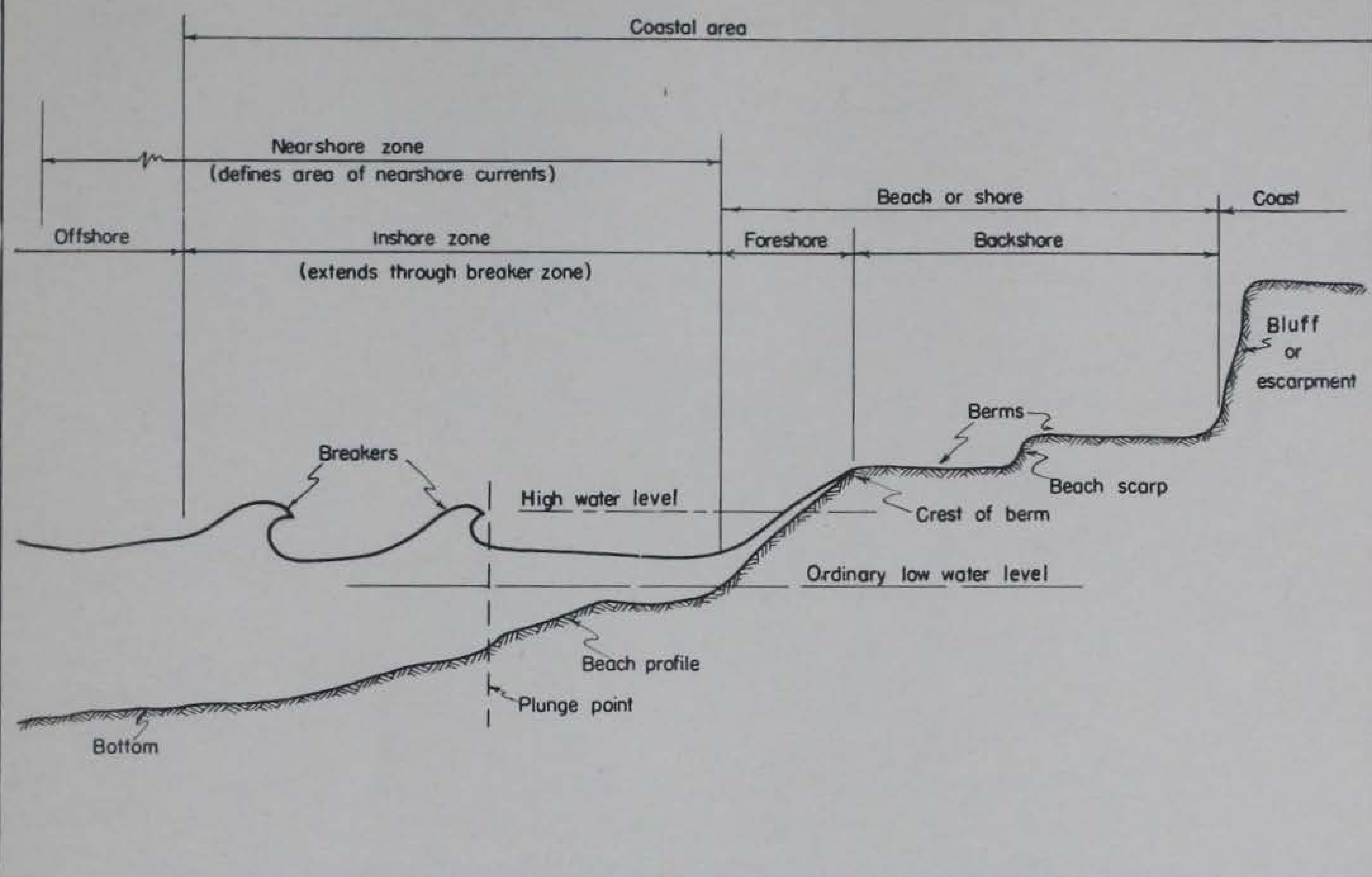


FIGURE A-1 — BEACH PROFILE – RELATED TERMS

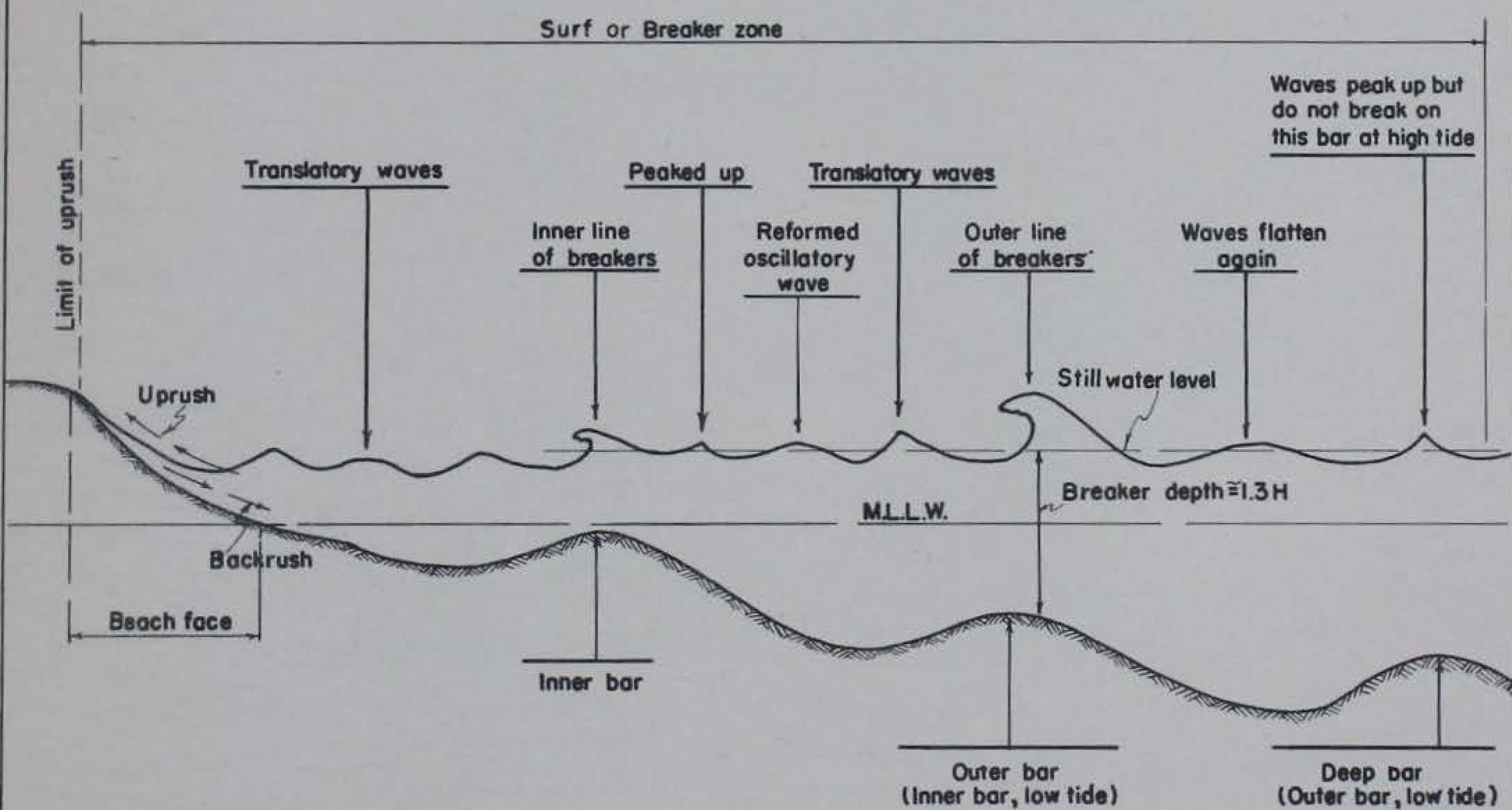
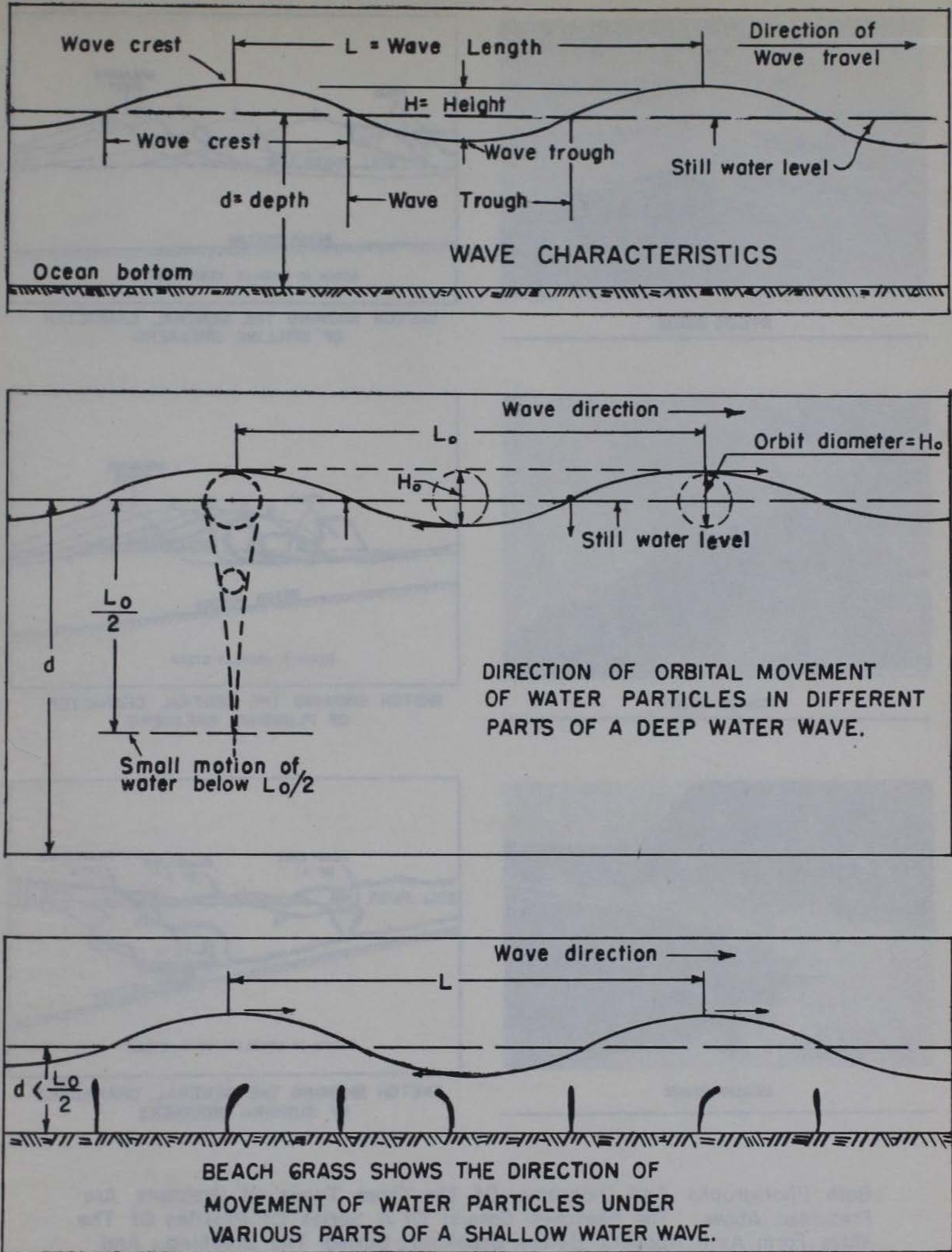


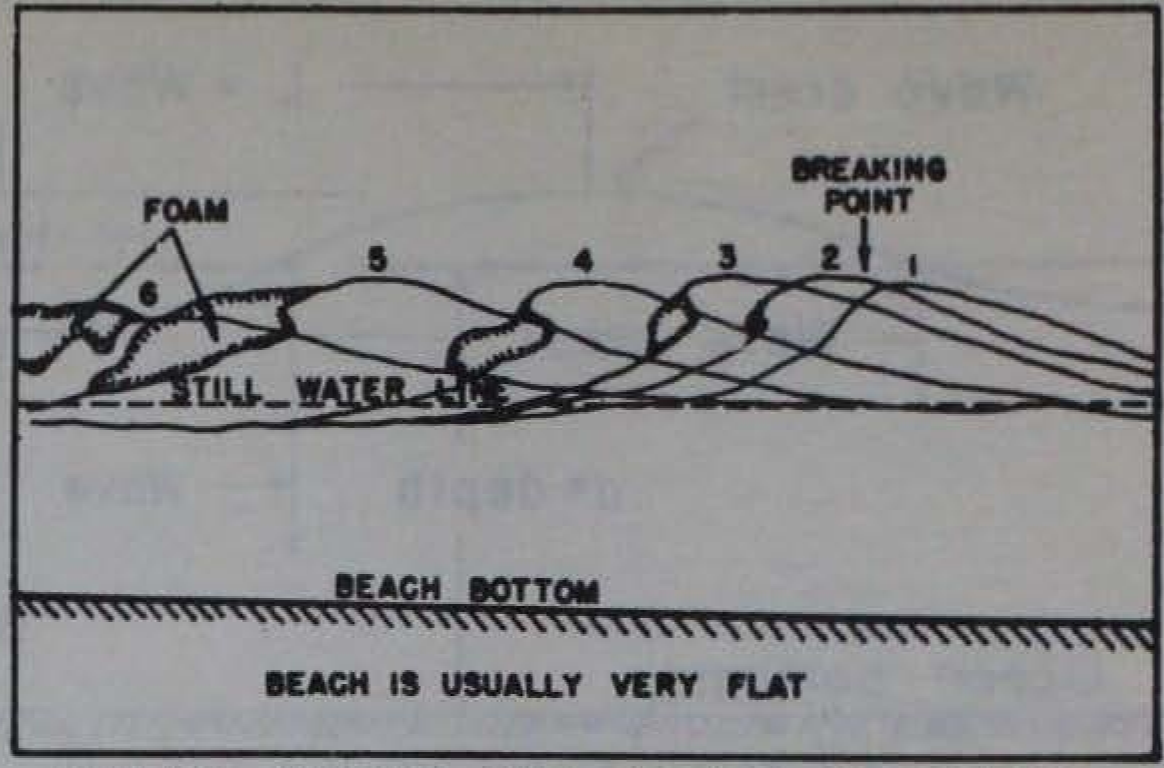
FIGURE A-2 SCHEMATIC DIAGRAM OF WAVES IN THE BREAKER ZONE



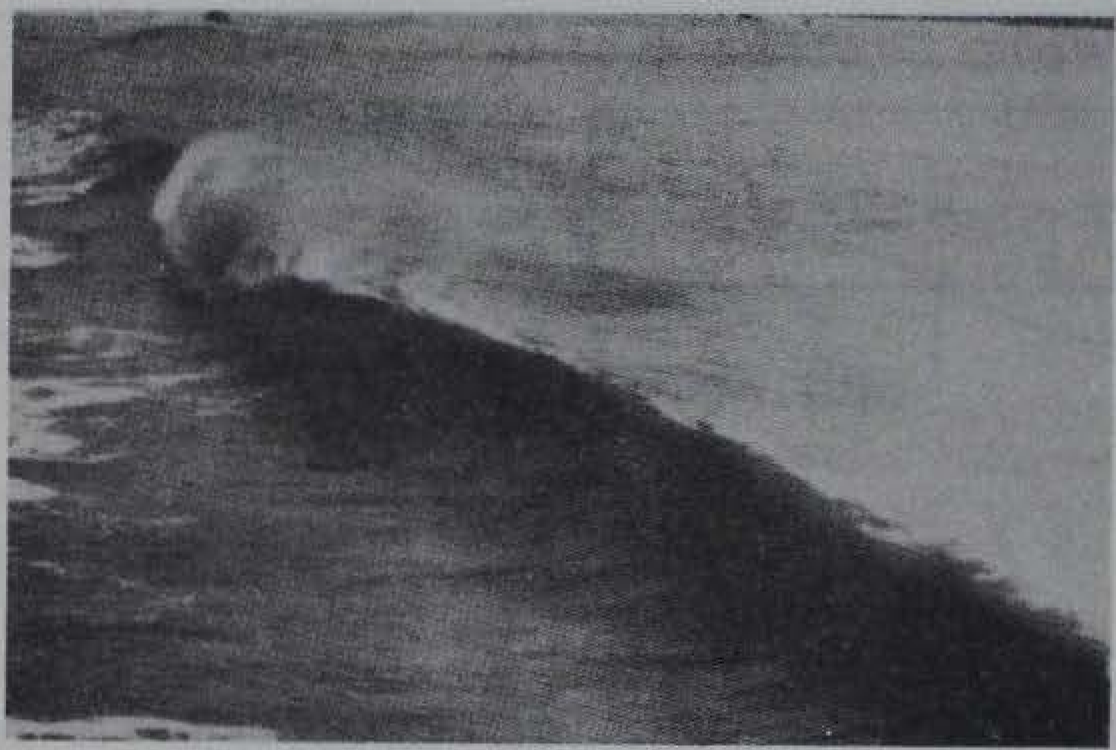
(Wiegel, 1953)

FIGURE A-3 WAVE CHARACTERISTICS AND DIRECTION OF WATER PARTICLE MOVEMENT

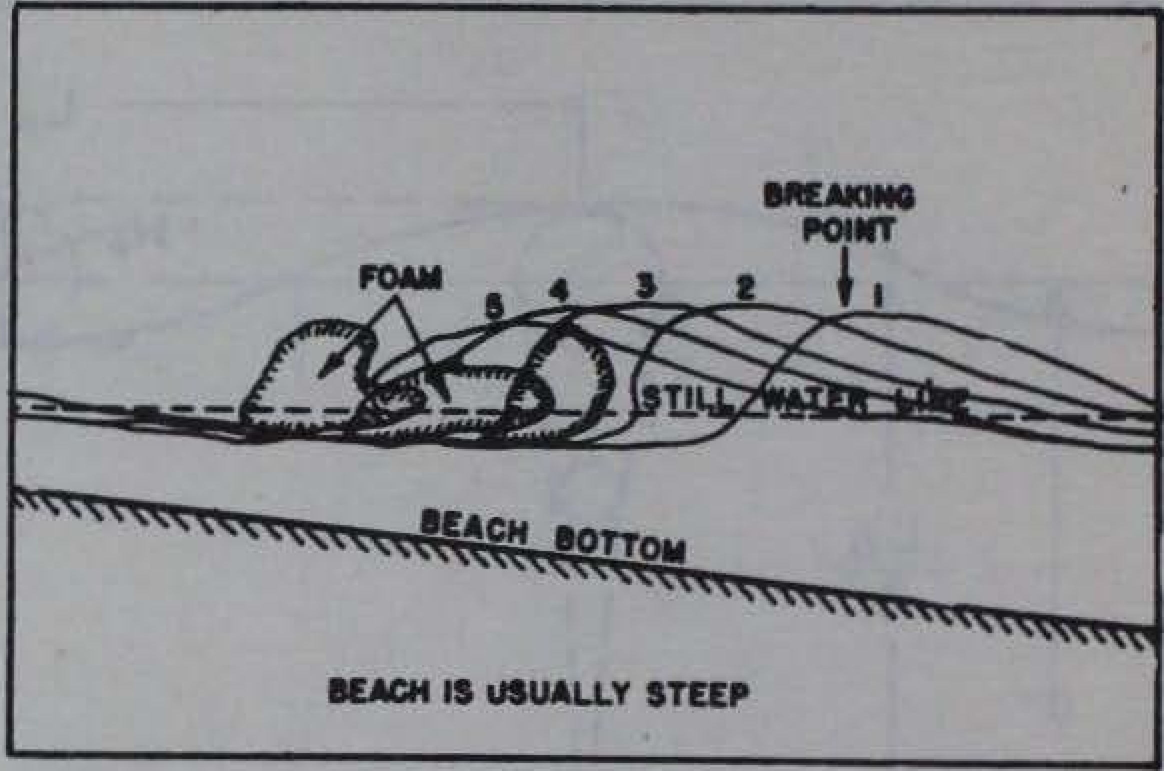
SPILLING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF SPILLING BREAKERS

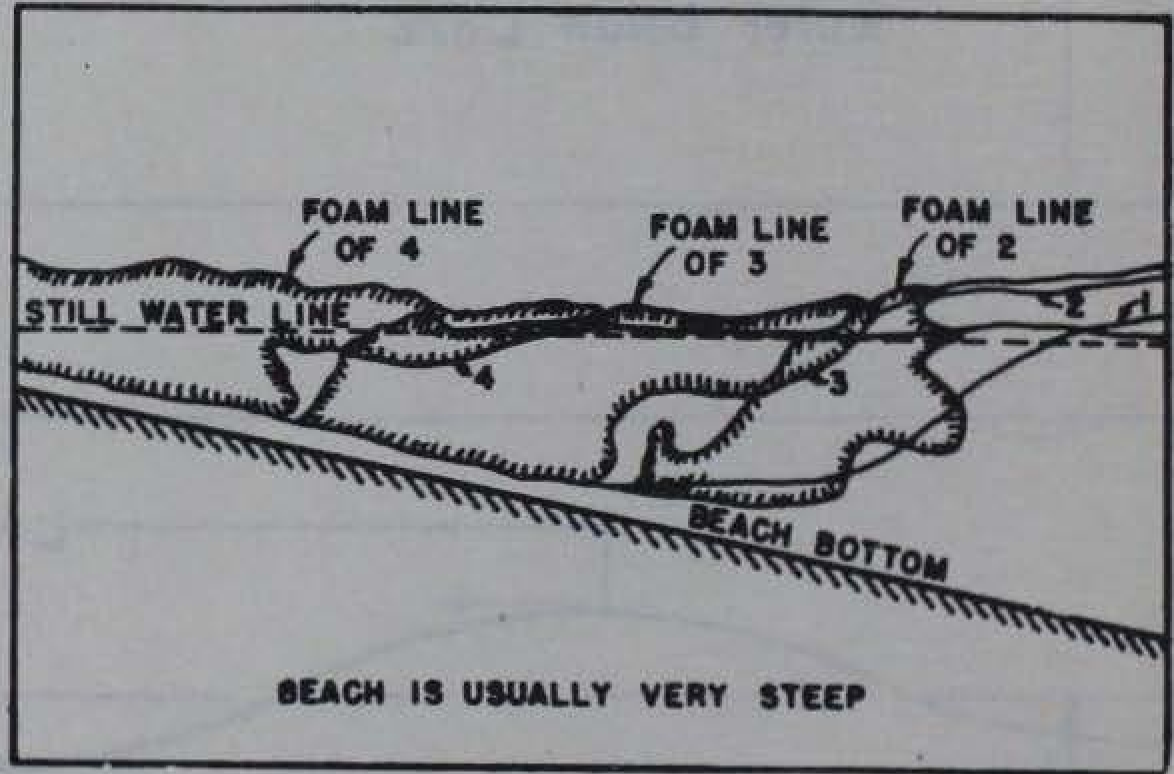


PLUNGING BREAKER



SKETCH SHOWING THE GENERAL CHARACTER OF PLUNGING BREAKERS

SURGING BREAKER

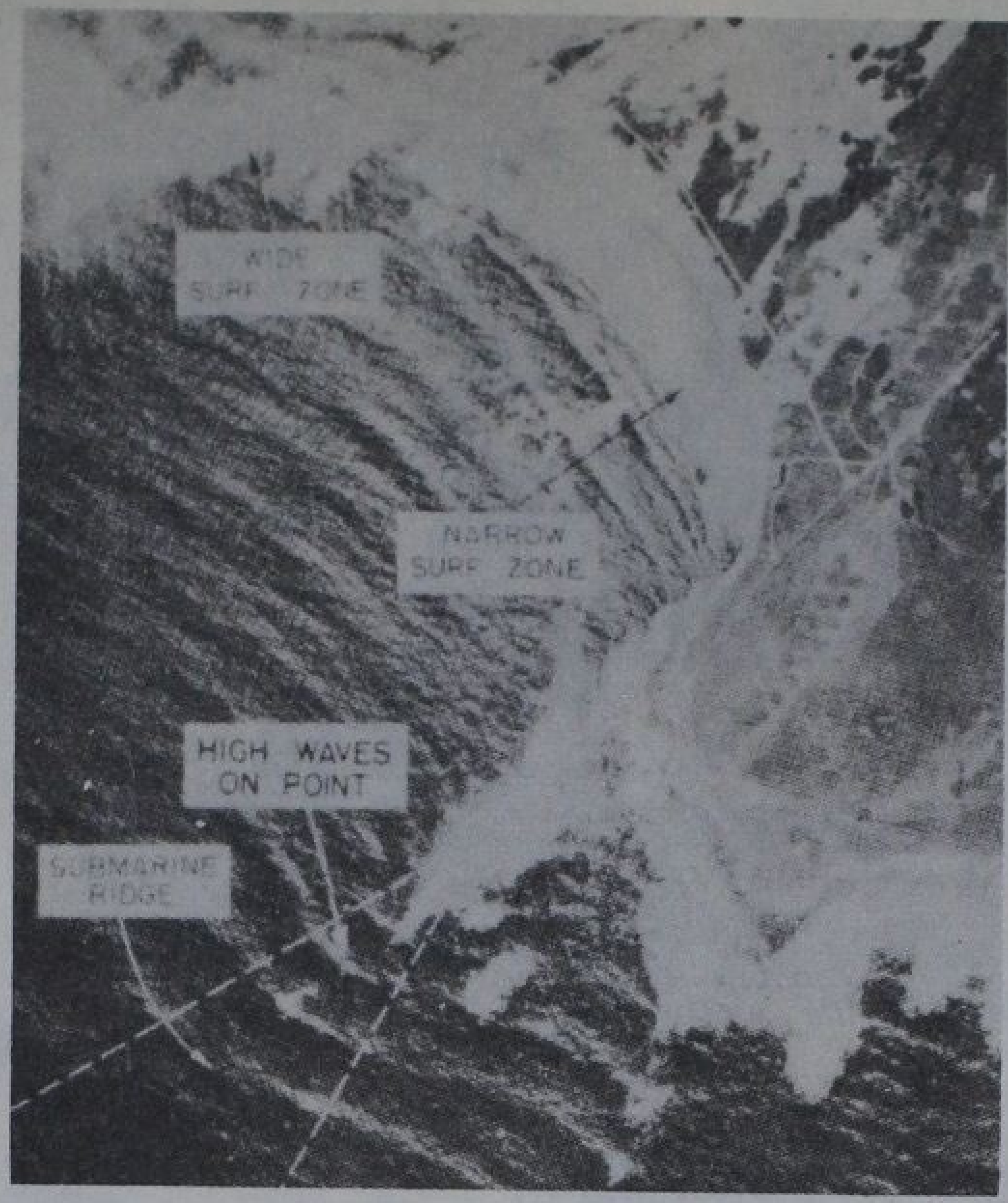


SKETCH SHOWING THE GENERAL CHARACTER OF SURGING BREAKERS

Both Photographs And Diagrams Of The Three Types Of Breakers Are Presented Above. The Sketches Consist Of A Series Of Profiles Of The Wave Form As It Appears Before Breaking, During The Breaking, And After Breaking. The Numbers Opposite The Profile Lines Indicate The Relative Times Of The Occurrences.

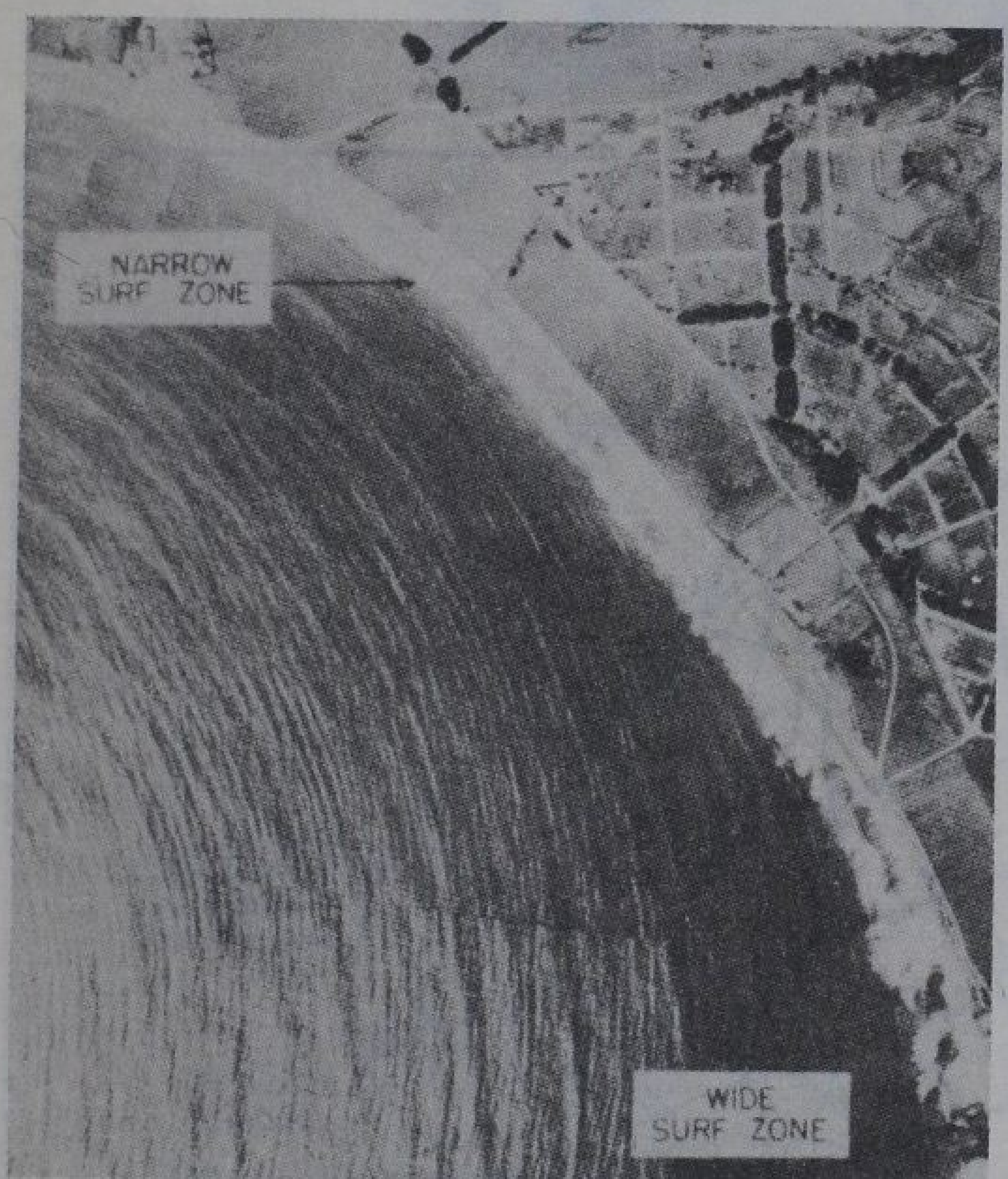
FIGURE A-4 BREAKER TYPES

(Wiegel, 1953)



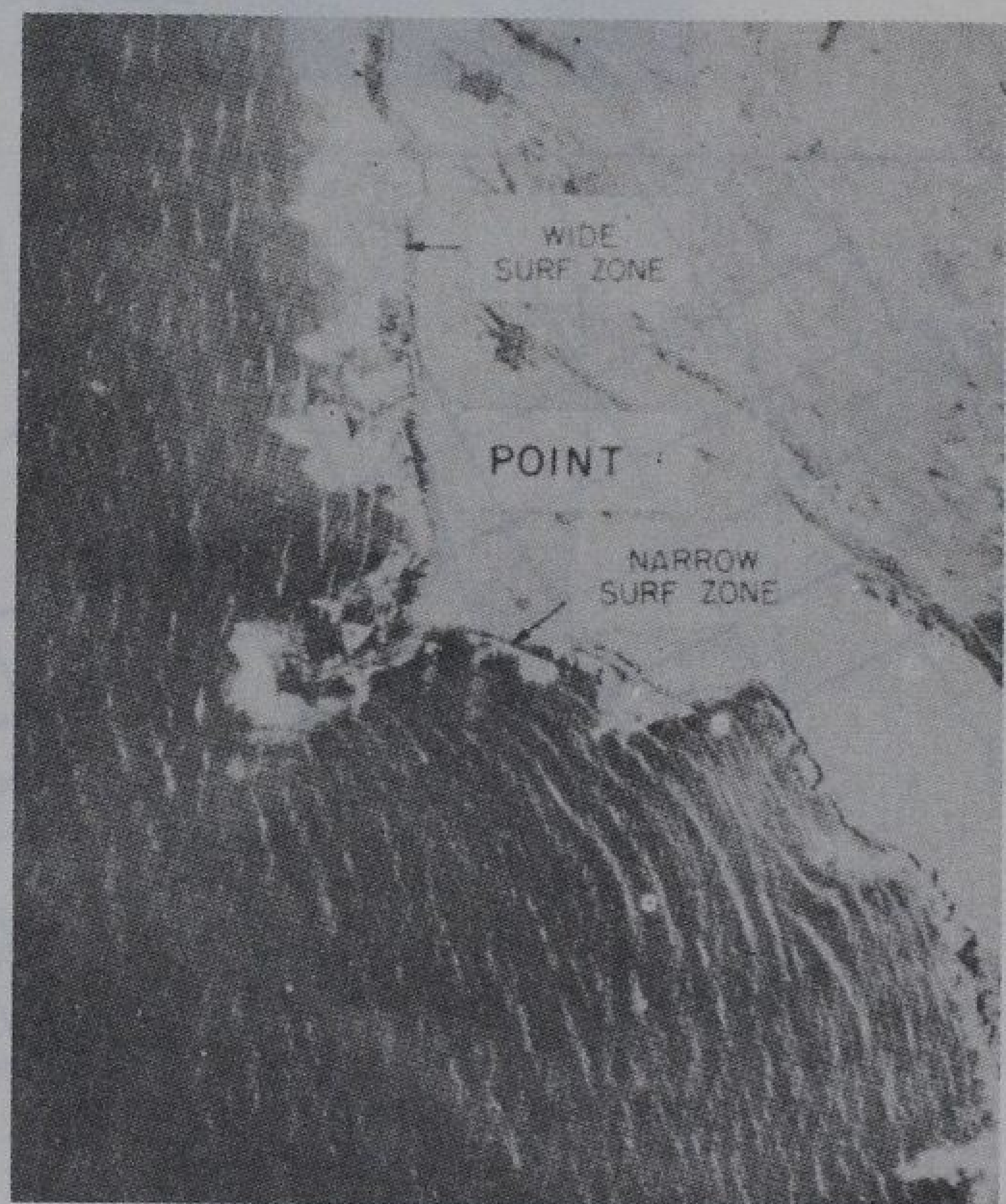
Pt. Pinos, California

Waves moving over a submarine ridge concentrate to give large wave heights on a point.



Halfmoon Bay, California

Note the increasing width of the surf zone with increasing degree of exposure to the south.



Purisima Pt., California

Refraction of waves around a headland produces low waves and a narrow surf zone where bending is greatest.

FIGURE A-5 REFRACTION OF WAVES

(Wiegel, 1953)

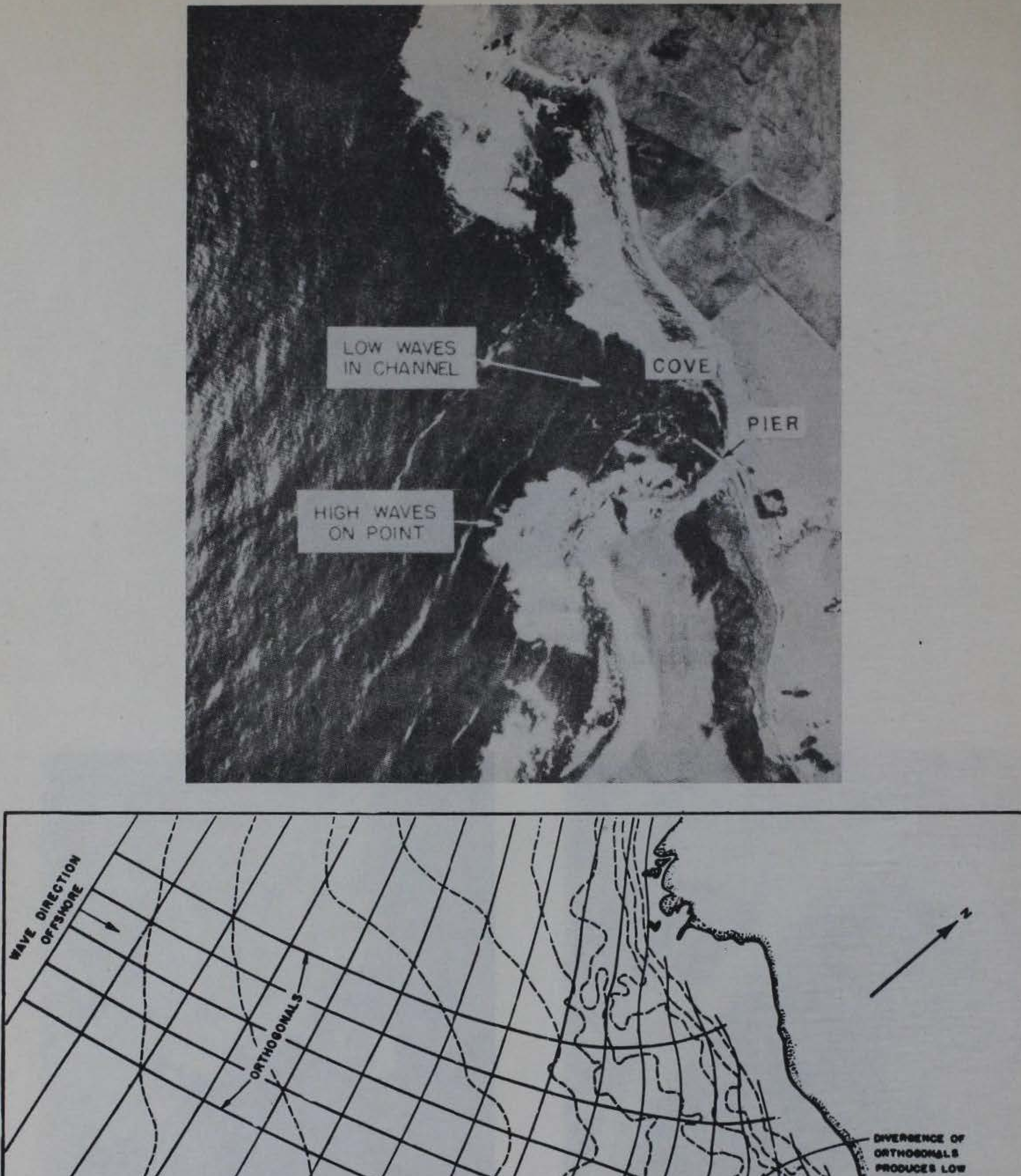


FIGURE A-6 REFRACTION DIAGRAM

(Wiegel, 1953)

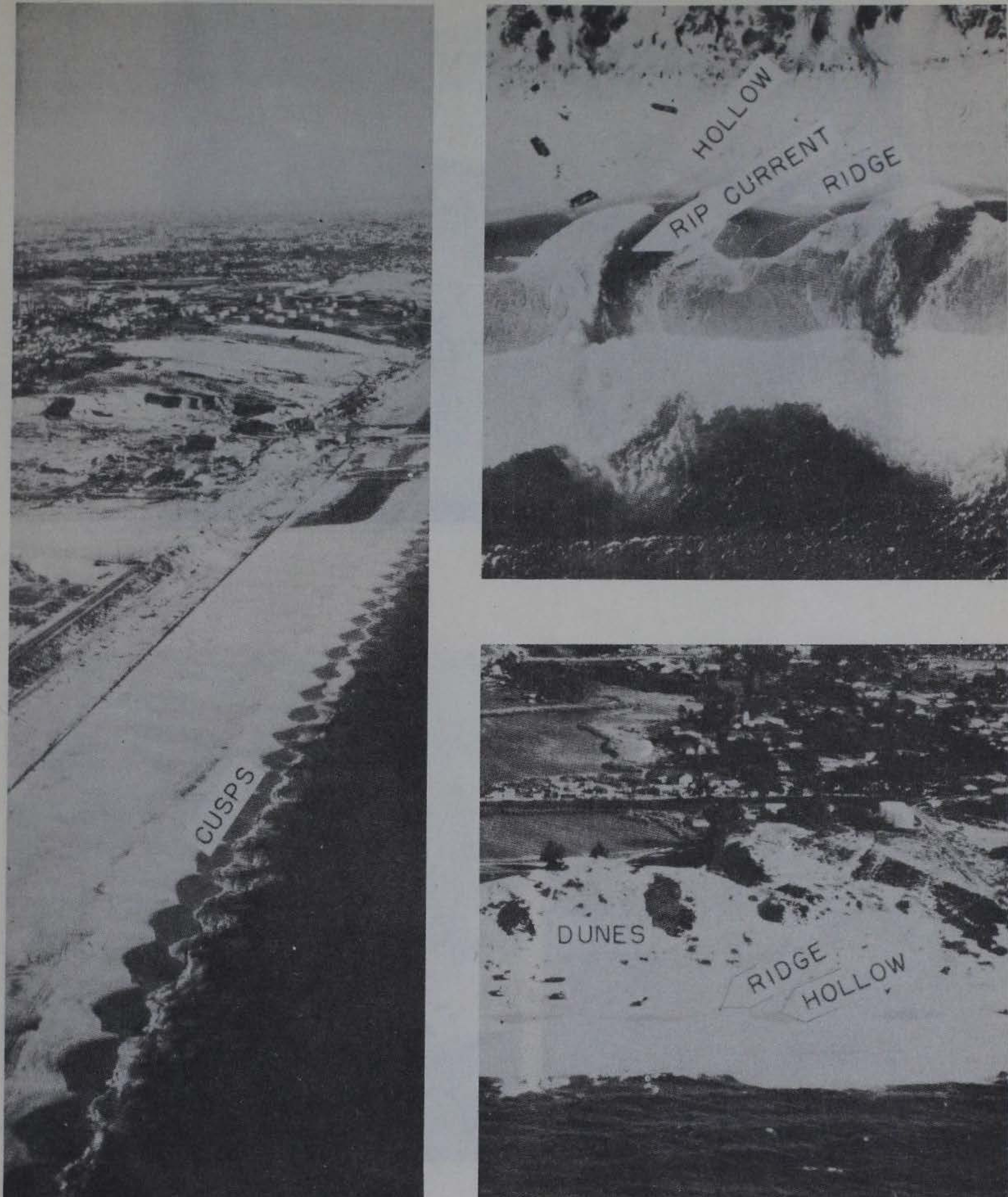


FIGURE A-7 BEACH FEATURES

(Wiegel, 1953)

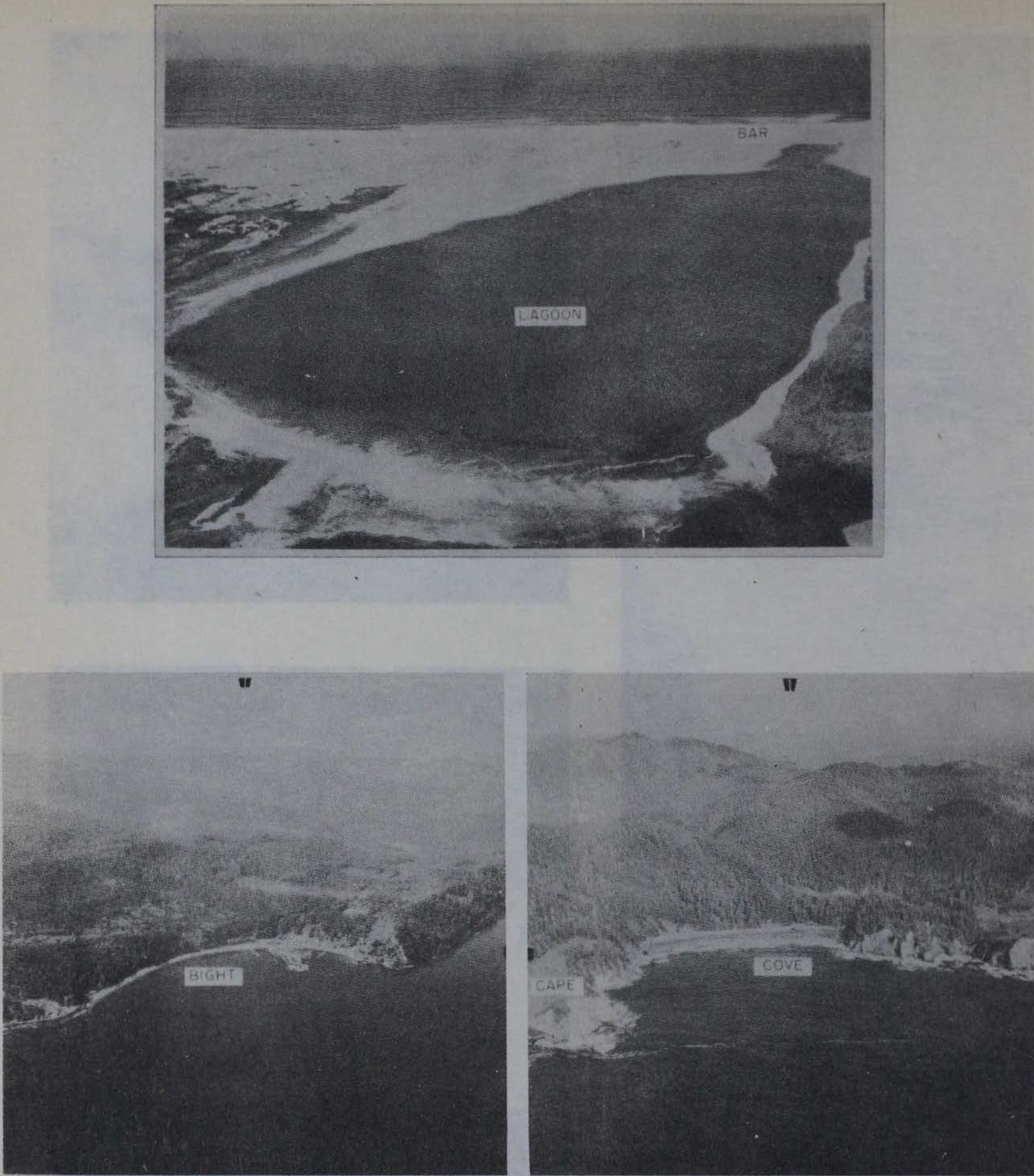


FIGURE A-8 SHORELINE FEATURES

(Wiegel, 1953)

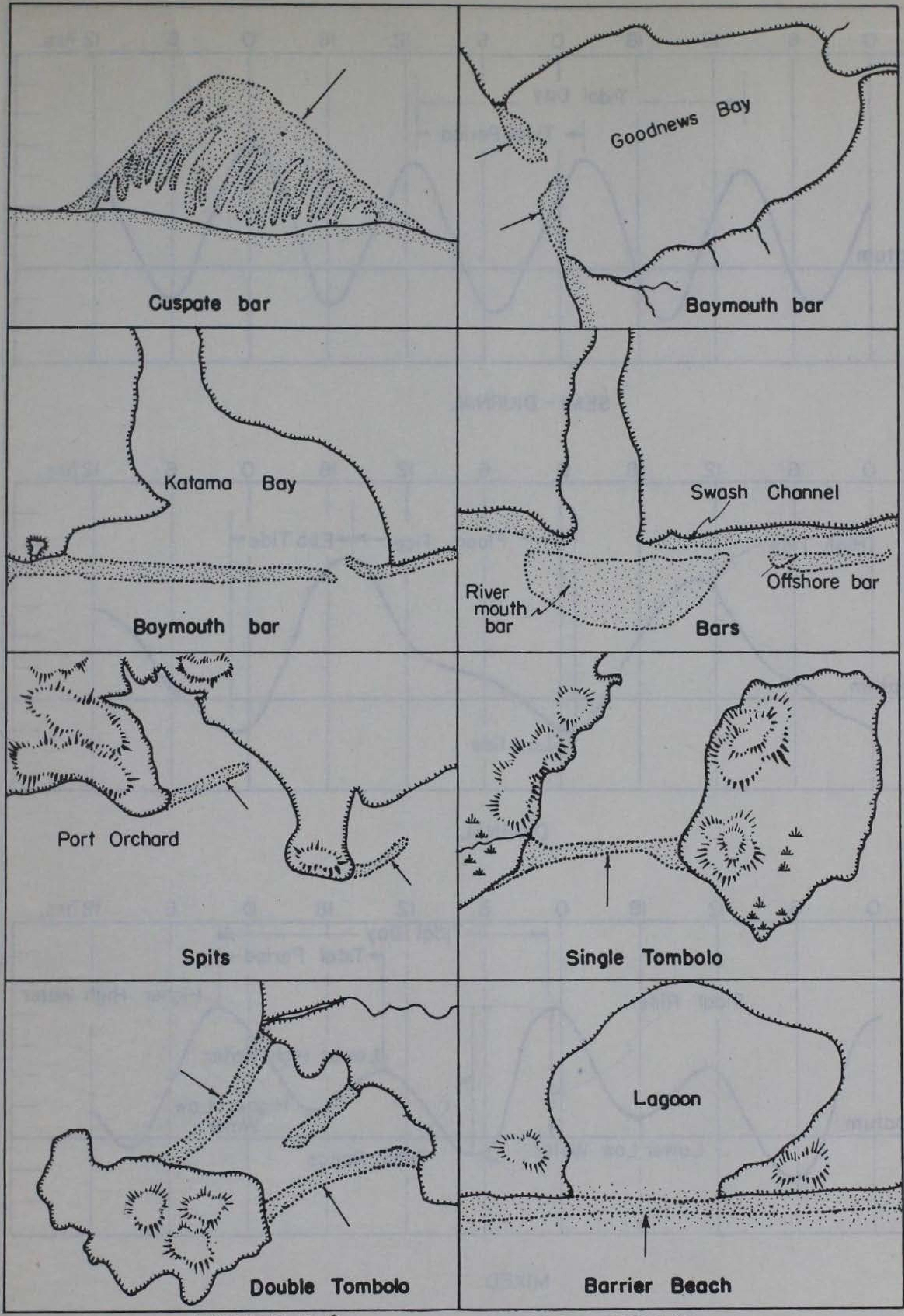


FIGURE A-9 BAR AND BEACH FORMS

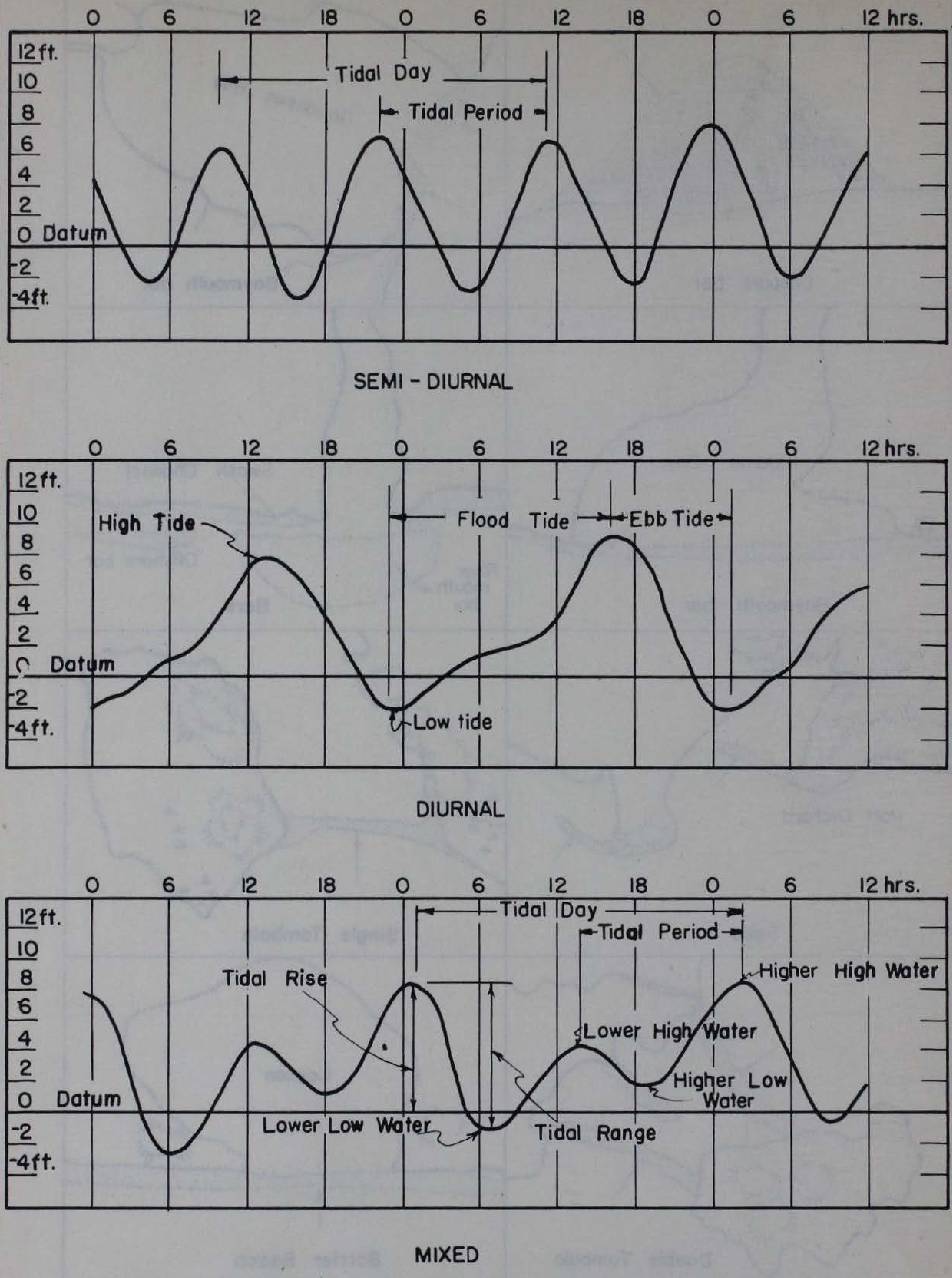
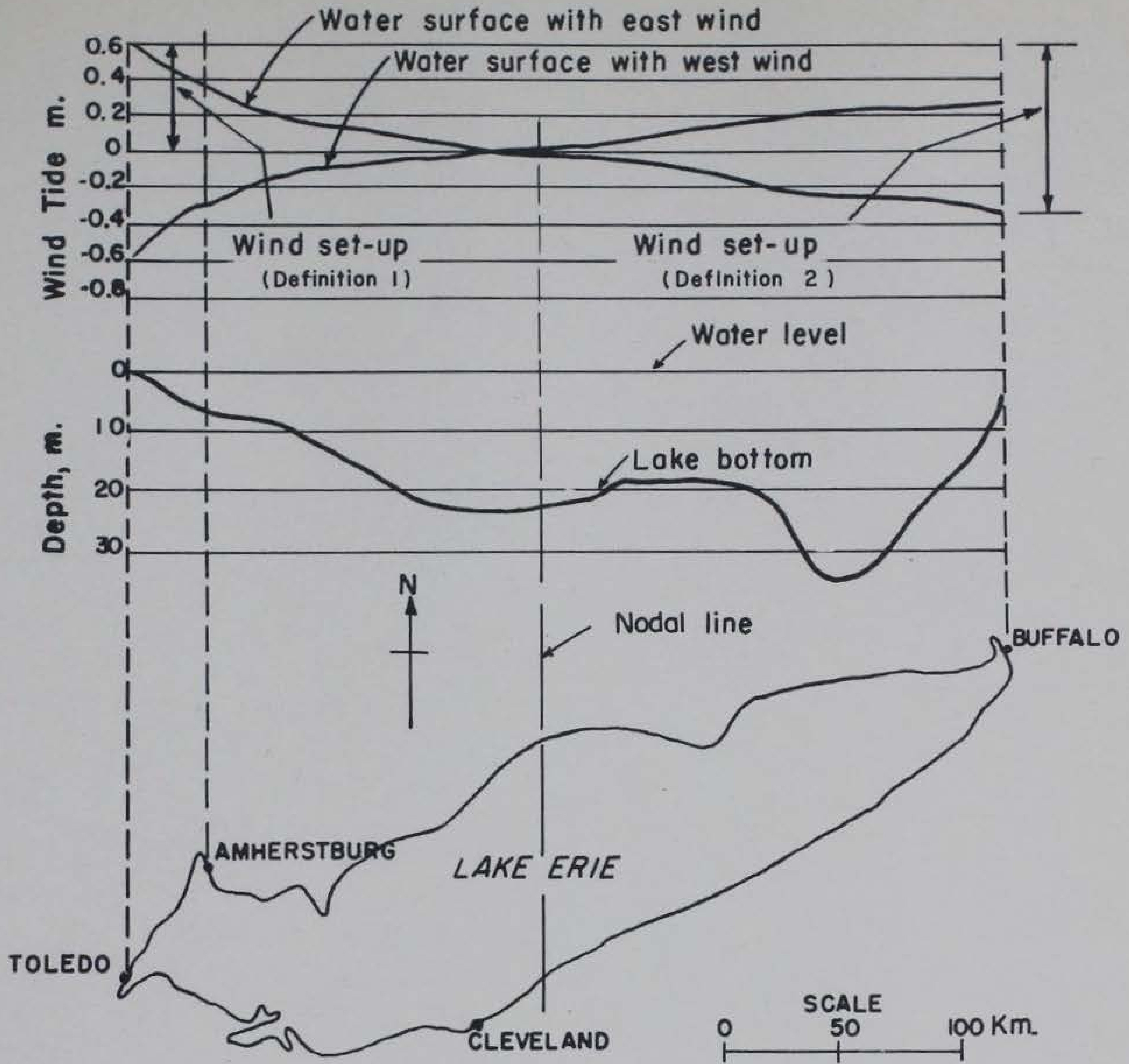


FIGURE A-10 TYPES OF TIDES (Wiegel, 1953)



SET-UP OR WIND TIDE

FIGURE A-11

(After Hellstrom, 1941)

APPENDIX B

LIST OF COMMON SYMBOLS

APPENDIX B

LIST OF COMMON SYMBOLS

Symbol	Definition	Basic Units (in F, L, T System)	Example in American Units
A	Area	L^2	$feet^2$
a	Acceleration, Also:	L/T^2	ft/sec^2
	Amplitude	L	feet
a_Z	Horizontal displacement of a water particle	L	feet
a', a_s	Length of semi-major axis of orbit of water particle	L	feet
$-_a$	Subscript "a" may refer to active earth pressures	--	--
$-_{av}$	Subscript "av" refers to average	--	--
B	Breakwater gap width	L	feet
B.F.	Beaufort wind force	--	--
b	Length of wave crest between orthogonals (measured perpendicular to the local direction of travel)	L	feet
b_Z	Vertical displacement of a water particle	L	feet
b', b_s	Length of semi-minor axis of orbit of water particle	L	feet
$-_b$	Subscript "b" refers to breaking wave conditions	--	--
C, c	Wave velocity	L/T	ft/sec, knots
C_d, C_D	Coefficient of drag	--	--
C_g, C_G	Group velocity	L/T	ft/sec, knots
C_H	Velocity of waves of finite height	L/T	ft/sec
C_M	Coefficient of mass	--	--
C_o	Deep water wave velocity	L/T	ft/sec (knots, as noted)

Symbol	Definition	Basic Units (in F,L, T System)	Example in American Units
D	Decay distance	L	miles or nautical miles
	Also: Diameter	L	feet
	Also: A depth	L	feet
D_d	Coefficient of shoaling	--	--
D_e	Effective decay distance	L	miles or nautical miles
d	Depth of water, measured from the still water level to the bottom	L	feet or fathoms
d_b	Depth of water at a breaker's position	L	feet or fathoms
E	Portion of total energy of one wave per unit length of crest transmitted forward with the wave form	LF/L	ft-lbs/ft of crest
E_k	Mean kinetic energy of one wave per unit length of crest	LF/L	ft-lbs/ft of crest
E_p	Mean potential energy of one wave per unit length of crest	LF/L	ft-lbs/ft of crest
E_t	Mean total energy of one wave per unit length of crest	LF/L	ft-lbs/ft of crest
F	Force, one of the basic units	F	pounds
	Also: a function of one or more variables, as F(x,y).	--	--
	Also: Fetch length	L	miles or nautical miles
F(H)	Percent of wave heights below the height H	--	--
F_H	Horizontal component of force	F	pounds
F_m, F_{min}	Minimum fetch length	L	miles or nautical miles
F_v	Vertical component of force	F	pounds

Symbol	Definition	Basic Units (in F, L, T System)	Example in American Units
$F_x(x)$	Prob. $(X \leq x)$ = cumulative distribution function of the random variable X.	--	--
f	Function of one or more variables, as $f(x,y)$ Also: Coriolis parameter $(f = 2\ \Omega\ \sin \phi)$	-- $1/T$	-- radian/sec
f =	Friction factor	---	---
f'_c	Compressive strength of concrete	F/L^2	psi
g	Acceleration of gravity Also: Function of one or more variables, as $g(x,y)$	L/T^2 --	ft/sec^2 --
H	Wave height Also: Horizontal component of force	L F	feet pounds
$H_1/10$	Average height of the highest one-tenth of the waves for a specified period of time	L	feet
$H_1/3$ also H_s	Average height of the highest one-third of the waves (significant wave height) for a specified period of time	L	feet
H_b	Wave height on breaking	L	feet
H_D	Significant wave height at end of decay distance	L	feet
H_F	Significant wave height at end of fetch	L	feet
H_{av}	Average of the wave heights for a specified period of time	L	feet
H_m	Highest wave for a specified period of time	L	feet
H	Horizontal component (force etc.)	--	--
h	A height	L	feet

Symbol	Definition	Basic Units (in F, L, T System)	Example in American Units
h_o	Height of the mean level of the clapotis (orbit center) above SWL	L	feet
J	Distance between two underwater contours as used in the orthogonal method of wave refraction coefficient determination	L	feet
K	Sub-surface pressure response coefficient	--	--
	Also: Any constant	--	--
	Also: Refraction coefficient	--	--
K_d	Refraction coefficient	--	--
K_b	Refraction coefficient for breakers	--	--
K'	Diffraction coefficient	--	--
	Also: Dimensionless coefficient (Iribarren)		
i	Beach slope, as specified	L/L	$\frac{\text{Ver. rise (ft)}}{\text{Hor. dist. (ft)}}$
	Also: $\sqrt{-1}$	--	--
k	$\sqrt{m^2 + n^2}$, where $m = 2\pi/L$ and $n = 2\pi/L'$ (short crested waves)	1/L	1/feet
	Also: $2\pi/L$ (long crested waves)	1/L	1/feet
L	Wave length (distance between two successive crests in the direction of propagation)	L	feet
	Also: Length, one of the basic units	L	feet
L_b	Wave length of breaking	L	feet
L_D	Wave length at end of decay	L	feet
L_D	Wave length at deeper water depth (Minikin formula)	L	feet
L_F	Wave length at end of fetch	L	feet
L'	Wave length in the crest direction (short-crested theory)	L	feet
ℓ	a length	L	feet

Symbol	Definition	Basic Units (in F, L, T System)	Example in American Units
ℓ'	Crest length (the linear distance measured along the wave crest between consecutive intersections of the crest and the still water level)	L	feet
M	Mass, a basic unit in the (M,L,T) system	$(\frac{FT^2}{L})$	slugs
	Also: Energy coefficient		
	Also: Moment	LF	foot-pounds
m	$2\pi/L$	1/L	1/feet
$-_m$	In wave forecasting subscript "m" refers to minimum conditions (eg. F_m)	--	--
N	A number	--	--
n	Ratio of group velocity to wave velocity	--	--
	Also: Crest interval in refraction drawings	--	--
	Also: $2\pi/L'$	1/L	1/feet
$-_o$	Subscript "o" refers to deep water conditions	--	--
P	Power transmitted by one wave per unit length of crest	$\frac{LF/T}{L}$	ft-lbs/sec per foot
	Also: Pressure	F/L^2	pounds/square foot
	Also: Force	F	pounds
p	Probability	--	--
	Also: Sub-surface pressure associated with wave motion	F/L^2	pounds/square foot
	Also: Atmospheric pressure	F/L^2	milibars
	Also: Any pressure	F/L^2	pounds/square foot
	Also: Angle of earth fill surface to the horizontal	--	degrees
Prob (A) or p (A)	The probability of a statement, A	--	--

Symbol	Definition	Basic Units (in F,L, T System)	Example in American Units
$-_p$	Subscript "p" refers to passive earth pressures	--	--
q	Water particle velocity in direction of interest	L/T	ft/sec
R	A resultant force	F	pounds
	Also: Distance between contours measured along an orthogonal	L	feet
R_e	Reynolds number	--	--
r	Radius	L	feet
	Also: Coefficient of energy partition	--	--
	Also: Distance from end of breakwater to point (x,y) in diffraction theory	L	feet
S	Distance from the ocean bottom to a water particle		
	Also: Sum of the real parts of diffraction function $\Sigma s + i \Sigma w$	--	--
	Also: Specific gravity	--	--
s	Sheltering coefficient	$1/L^3$	$1/ft^3$
	Also: Specific gravity	--	--
	Also: Real part of diffraction Function $f(-u) = s + iw$	--	---
	Also: Slope as specified	L/L	Ver. rise (ft) / Hor. dist. (ft)
$-_s$	Subscript "s" refers to surface terms	--	--
T	Wave period; seiche period	T	seconds
	Also: Temperature	θ	degrees F.
	Also: Time, a basic unit	T	seconds
T_C, T_s	Significant wave period	T	seconds
T_D	Significant wave period at end of decay	T	seconds
T_F	Significant wave period at end of fetch	T	seconds

Symbol	Definition	Basic Units (in F,L, T System)	Example in American Units
t	A time	T	seconds
t_D	Travel time of waves (from end of fetch to end of decay distance)	T	hours
t_d	Wind duration, interval of time wind blows at constant velocity in generating waves	T	hours
t_μ	Modulus of wave decay due to viscosity	T	hours
t_m, t_{min}	Miniumum duration of wind in fetch area	T	hours
U	Velocity of surface wind	L/T	knots
$\bar{U}$	Velocity of mass transport	L/T	ft/sec
U'	Horizontal velocity of motion left after wave motion has been destroyed (Gerstner Theory) Also: Approximate velocity of surface wind	L/T L/T	ft/sec knots
u	Water particle (horizontal component positive in the direction of wave advance) orbital velocity Also: Upper limit of integral term in solutions of diffraction problem	L/T --	ft/sec --
V	A velocity Also: A volume	L/T L^3	ft/sec $feet^3$
V_F	Velocity of storm or fetch front	L/T	knots
V_g	Geostrophic wind velocity	L/T	knots, m.p.h.
$\bar{v}$	Vertical component (force etc.)	--	--
v	Water particle (vertical component orbital velocity)	L/T	ft/sec
W	Weight Also: Work performed by one wave per unit length of crest	F LF/L	pounds ft-lbs/ft of crest

Symbol	Definition	Basic Units (in F,L, T System)	Example in American Units
w	Unit weight (Specific weight) of water, or backfill material	F/L^3	lbs/ft^3
	Also: Imaginary part of diffraction function $f(-u) = S \not+ iw$	--	--
X	Extraneous force in x direction	F	pounds
x	Coordinate, usually horizontal	L	feet
x_o	Equilibrium position of particle, horizontal coordinate	L	feet
$\bar{x}$	Horizontal displacement of particles from equilibrium position	L	feet
Y	Extraneous force in y direction	F	pounds
y	A coordinate, usually vertical	L	feet
y_o	Equilibrium position of particle vertical coordinate	L	feet
$\bar{y}$	Vertical displacement of particle from equilibrium position	L	feet
Z	Extraneous force in z direction	F	pounds
	Also: Section modulus per length of wall	L^3/L	ft^2
Z_t	Time between successive weather maps	T	hours
	Also: Greenwich mean time	T	hours
z	A coordinate, usually horizontal perpendicular to x and y	L	feet
α (alpha)	Angle of wave crest to bottom contours	--	degrees
	Also: Angle of wave approach, measured between the shore line and the line of wave advance	--	degrees
	Also: Angle between gradient and surface winds	--	degrees
	Also: Phase difference between axis of "f" and "g" terms in diffraction theory	--	radians

Symbol	Definition	Basic Units (in F, L, T System)	Example in American Units
α (cont'd)	Also: Angle of breakwater slope with horizontal	--	degrees
α_3	Skewness coefficient	--	--
β (beta)	Angular position of the water particle when the maximum horizontal particle velocity occurs.	--	degrees
	Also: Wave age, the ratio of wave velocity to the velocity of the generating wind	--	--
	Also: An angle	--	degrees
γ^2 (gamma)	Resistance coefficient applicable to wind	--	--
Δ (delta)	Change	--	as stated
δ (delta)	Wave steepness, H/L	--	--
η (eta)	Wave surface elevation	L	feet
θ (Theta)	Angle of wall face with horizontal (for earth pressures)	--	degrees
	Also: Angular displacement	--	Radians
	Also: Temperature, a basic unit	θ	degrees F.
	Also: Angle of variability in the direction of wave travel	--	degrees
	Also: Angle of the face of a structure with the horizontal		degrees
λ (lambda)	Wave length	L	feet
μ (mu)	Arithmetic mean	--	appropriate units
	Also: Absolute viscosity	FT/L^2	$lb\text{-}sec/ft^2$
	Also: Coefficient of friction	--	--
ν (nu)	Kinematic viscosity	L^2/T	ft^2/sec
π (pi)	3.1416 - Ratio of circumference of a circle to the diameter	--	--

Symbol	Definition	Basic Units (in F,L, T System)	Example in American Units
ρ (rho)	Correlation coefficient	--	--
	Also: Mass density, any substance	FT^2/L^4	
		(M/L^3)	$slugs/ft^3$
ρ_a	Mass density of air	FT^2/L^4	
		(M/L^3)	$slugs/ft^3$
ρ_w	Mass density of water	FT^2/L^4	
		(M/L^3)	$slugs/ft^3$
σ (sigma)	Standard deviation	--	appropriate units
	Also: 2 /T	1/T	1/sec
σ_w	Surface tension of water	LF/L^2	$ft\text{-}lbs/ft^2$
τ (tau)	Drag force per unit area	F/L^2	lbs/ft^2
ϕ (phi)	Velocity potential	L^2/T	ft^2/sec
	Also: Angle of internal friction of earth	--	degrees
	Also: Phase of diffracted wave	--	radians
ψ (psi)	Aximuth of direction of wave travel, in the direction of travel	--	degrees
	Also: Stream function	L^2/T	ft^2/sec
	Also: Direction from which the wave comes	--	Compass direction (true)
Ω (omega)	Angular velocity of the earth	1/T	radians/sec
ω (omega)	Angular velocity	1/t	radian/sec

Symbol	Definition	Basic Units (in F, L, T System)	Example in American Units
D_f	Correction factor	---	---
E_f	Related to wave energy	L^2	feet^2
f_i	Minimum wave frequency	1/T	waves/second
f_{max}	Maximum wave frequency	1/T	waves/second
f_L	Lower limit of wave frequency	1/T	waves/second
f_u	Upper limit of wave frequency	1/T	waves/second
T_{av}	Average wave period	T	seconds
T_i	Maximum wave period	T	seconds
T_L	Lower limit of significant period	T	seconds
T_u	Upper limit of significant period	T	seconds
T_{max}	Period of most energetic wave in the spectrum	T	seconds

APPENDIX C

BIBLIOGRAPHY

APPENDIX C

BIBLIOGRAPHY

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APPENDIX D

MISCELLANEOUS TABLES AND GRAPHS

APPENDIX D

MISCELLANEOUS TABLES AND GRAPHS

PLATES AND TABLES

February 1957

Table D-1 - Functions of d/L for Even Increments of d/L_o (139)

Table D-2 - Functions of d/L_o for Even Increments of d/L (139)

d/L_o = ratio of the depth of water at any specific location to the wave length in deep water.

d/L = ratio of the depth of water at any specific location to the wave length at that same location.

K = a pressure response factor used in connection with underwater pressure instruments, where

$$K = H'/H = P/P_o = \frac{\cosh 2\pi(d-Z)/L}{\cosh \frac{2\pi d}{L}}$$

where P is the pressure fluctuation at a depth Z below still water, P_o is the surface pressure fluctuation, d is the depth of water from still water level to the ocean bottom, L is the wave length in any particular depth of water, and H' is the corresponding variation of head at a depth Z. The values of K shown in the tables are for the instrument placed on the bottom where

$$K = \frac{1}{\cosh 2\pi d/L}$$

n = the fraction of wave energy that travels forward with the wave form: i.e., with the wave velocity C rather than the group velocity C_G.

$$n = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh 4\pi d/L}\right]$$

n is also the ratio of group velocity C_G to wave velocity C.

C_G/C_o = ratio of group velocity to deep water wave velocity where

$$\frac{C_G}{C_o} = \frac{C_G}{C} \times \frac{C}{C_o} = n\tanh 2\pi d/L$$

H/H_o = ratio of the wave height in shallow water to what its wave height would have been in deep water if unaffected by refraction.

$$\frac{H}{H'_o} = \sqrt{\frac{1}{2} \cdot \frac{1}{n} \cdot \frac{1}{C/C_o}}$$

M = an energy coefficient defined as $\dfrac{\pi^2}{2\tanh^2(2\pi d/L)}$

TABLE D-1

FUNCTIONS OF d/L FOR EVEN INCREMENTS OF d/L_o

from 0.0001 to 1.000

d/L_o	d/L	$2\pi d/L$	TANH* $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
0	0	0	0	0	1	1	0	0	1	1	0	∞	∞
.0001000	.003990	.02507	.02506	.02507	1.0003	.9997	.05014	.05016	1.001	.9998	.02506	4.467	7,855
.0002000	.005643	.03546	.03544	.03547	1.0006	.9994	.07091	.07097	1.003	.9996	.03543	3.757	3,928
.0003000	.006912	.04343	.04340	.04344	1.0009	.9991	.08686	.08697	1.004	.9994	.04336	3.395	2,620
.0004000	.007982	.05015	.05011	.05018	1.0013	.9987	.1003	.1005	1.005	.9992	.05007	3.160	1,965
.0005000	.008925	.05608	.05602	.05611	1.0016	.9984	.1122	.1124	1.006	.9990	.05596	2.989	1,572
.0006000	.009778	.06144	.06136	.06148	1.0019	.9981	.1229	.1232	1.008	.9988	.06128	2.856	1,311
.0007000	.01056	.06637	.06627	.06642	1.0022	.9978	.1327	.1331	1.009	.9985	.06617	2.749	1,124
.0008000	.01129	.07096	.07084	.07102	1.0025	.9975	.1419	.1424	1.010	.9983	.07072	2.659	983.5
.0009000	.01198	.07527	.07513	.07534	1.0028	.9972	.1505	.1511	1.011	.9981	.07499	2.582	874.3
.001000	.01263	.07935	.07918	.07943	1.0032	.9969	.1587	.1594	1.013	.9979	.07902	2.515	787.0
.001100	.01325	.08323	.08304	.08333	1.0035	.9966	.1665	.1672	1.014	.9977	.08285	2.456	715.6
.001200	.01384	.08694	.08672	.08705	1.0038	.9962	.1739	.1748	1.015	.9975	.08651	2.404	656.1
.001300	.01440	.09050	.09026	.09063	1.0041	.9959	.1810	.1820	1.016	.9973	.09001	2.357	605.8
.001400	.01495	.09393	.09365	.09407	1.0044	.9956	.1879	.1890	1.018	.9971	.09338	2.314	562.6
.001500	.01548	.09723	.09693	.09739	1.0047	.9953	.1945	.1957	1.019	.9969	.09663	2.275	525
.001600	.01598	.1004	.1001	.1006	1.0051	.9949	.2009	.2022	1.020	.9967	.09977	2.239	493
.001700	.01648	.1035	.1032	.1037	1.0054	.9946	.2071	.2086	1.022	.9965	.1028	2.205	463
.001800	.01696	.1066	.1062	.1068	1.0057	.9943	.2131	.2147	1.023	.9962	.1058	2.174	438
.001900	.01743	.1095	.1091	.1097	1.0060	.9940	.2190	.2207	1.024	.9960	.1087	2.145	415
.002000	.01788	.1123	.1119	.1125	1.0063	.9937	.2247	.2266	1.025	.9958	.1114	2.119	394
.002100	.01832	.1151	.1146	.1154	1.0066	.9934	.2303	.2323	1.027	.9956	.1141	2.094	376
.002200	.01876	.1178	.1173	.1181	1.0069	.9931	.2357	.2379	1.028	.9954	.1161	2.070	359
.002300	.01918	.1205	.1199	.1208	1.0073	.9928	.2410	.2433	1.029	.9952	.1193	2.047	343
.002400	.01959	.1231	.1225	.1234	1.0076	.9925	.2462	.2487	1.031	.9950	.1219	2.025	329
.002500	.02000	.1257	.1250	.1260	1.0079	.9922	.2513	.2540	1.032	.9948	.1243	2.005	316
.002600	.02040	.1282	.1275	.1285	1.0082	.9919	.2563	.2592	1.033	.9946	.1268	1.986	304
.002700	.02079	.1306	.1299	.1310	1.0085	.9916	.2612	.2642	1.034	.9944	.1292	1.967	292
.002800	.02117	.1330	.1323	.1334	1.0089	.9912	.2661	.2692	1.036	.9942	.1315	1.950	282
.002900	.02155	.1354	.1346	.1358	1.0092	.9909	.2708	.2741	1.037	.9939	.1338	1.933	272
.003000	.02192	.1377	.1369	.1382	1.0095	.9906	.2755	.2790	1.038	.9937	.1360	1.917	263
.003100	.02228	.1400	.1391	.1405	1.0098	.9903	.2800	.2837	1.040	.9935	.1382	1.902	255
.003200	.02264	.1423	.1413	.1427	1.0101	.9900	.2845	.2884	1.041	.9933	.1404	1.887	247
.003300	.02300	.1445	.1435	.1449	1.0104	.9897	.2890	.2930	1.042	.9931	.1425	1.873	240
.003400	.02335	.1467	.1456	.1472	1.0108	.9893	.2934	.2976	1.043	.9929	.1446	1.860	233
.003500	.02369	.1488	.1477	.1494	1.0111	.9890	.2977	.3021	1.045	.9927	.1466	1.847	226
.003600	.02403	.1510	.1498	.1515	1.0114	.9887	.3020	.3065	1.046	.9925	.1487	1.834	220
.003700	.02436	.1531	.1519	.1537	1.0117	.9884	.3061	.3109	1.047	.9923	.1507	1.822	214
.003800	.02469	.1551	.1539	.1558	1.0121	.9881	.3103	.3153	1.049	.9921	.1527	1.810	208
.003900	.02502	.1572	.1559	.1579	1.0124	.9878	.3144	.3196	1.050	.9919	.1546	1.799	203
.004000	.02534	.1592	.1579	.1599	1.0127	.9875	.3184	.3238	1.051	.9917	.1565	1.788	198
.004100	.02566	.1612	.1598	.1619	1.0130	.9872	.3224	.3280	1.052	.9915	.1584	1.777	193
.004200	.02597	.1632	.1617	.1639	1.0133	.9869	.3263	.3322	1.054	.9912	.1602	1.767	189
.004300	.02628	.1651	.1636	.1659	1.0137	.9865	.3302	.3362	1.055	.9910	.1621	1.756	184
.004400	.02659	.1671	.1655	.1678	1.0140	.9862	.3341	.3403	1.056	.9908	.1640	1.746	180
.004500	.02689	.1690	.1674	.1698	1.0143	.9859	.3380	.3444	1.058	.9906	.1658	1.737	176
.004600	.02719	.1708	.1692	.1717	1.0146	.9856	.3417	.3483	1.059	.9904	.1676	1.727	172
.004700	.02749	.1727	.1710	.1736	1.0149	.9853	.3454	.3523	1.060	.9902	.1693	1.718	169
.004800	.02778	.1745	.1728	.1754	1.0153	.9849	.3491	.3562	1.062	.9900	.1711	1.709	165
.004900	.02807	.1764	.1746	.1773	1.0156	.9846	.3527	.3601	1.063	.9898	.1728	1.701	162
.005000	.02836	.1782	.1764	.1791	1.0159	.9843	.3564	.3640	1.064	.9896	.1746	1.692	159
.005100	.02864	.1800	.1781	.1809	1.0162	.9840	.3599	.3678	1.066	.9894	.1762	1.684	156
.005200	.02893	.1818	.1798	.1827	1.0166	.9837	.3635	.3715	1.067	.9892	.1779	1.676	153
.005300	.02921	.1835	.1815	.1845	1.0169	.9834	.3670	.3753	1.068	.9889	.1795	1.669	150
.005400	.02948	.1852	.1832	.1863	1.0172	.9831	.3705	.3790	1.069	.9887	.1811	1.662	147
.005500	.02976	.1870	.1848	.1880	1.0175	.9828	.3739	.3827	1.071	.9885	.1827	1.654	145
.005600	.03003	.1887	.1865	.1898	1.0178	.9825	.3774	.3864	1.072	.9883	.1843	1.647	142
.005700	.03030	.1904	.1881	.1915	1.0182	.9822	.3808	.3900	1.073	.9881	.1859	1.640	140
.005800	.03057	.1921	.1897	.1932	1.0185	.9818	.3841	.3937	1.075	.9879	.1874	1.633	137
.005900	.03083	.1937	.1913	.1949	1.0188	.9815	.3875	.3972	1.076	.9877	.1890	1.626	135

*Also: b_s/a_s, C/C_o, L/L_o

Table D-1 Cont'd

f

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.006000	.03110	.1954	.1929	.1967	1.0192	.9812	.3908	.4008	1.077	.9875	.1905	1.620	133
.006100	.03136	.1970	.1945	.1983	1.0195	.9809	.3941	.4044	1.079	.9873	.1920	1.614	130
.006200	.03162	.1987	.1961	.2000	1.0198	.9806	.3973	.4079	1.080	.9871	.1935	1.607	128
.006300	.03188	.2003	.1976	.2016	1.0201	.9803	.4006	.4114	1.081	.9869	.1950	1.601	126
.006400	.03213	.2019	.1992	.2033	1.0205	.9799	.4038	.4148	1.083	.9867	.1965	1.595	124
.006500	.03238	.2035	.2007	.2049	1.0208	.9796	.4070	.4183	1.084	.9865	.1980	1.589	123
.006600	.03264	.2051	.2022	.2065	1.0211	.9793	.4101	.4217	1.085	.9863	.1994	1.583	121
.006700	.03289	.2066	.2037	.2081	1.0214	.9790	.4133	.4251	1.087	.9860	.2009	1.578	119
.006800	.03313	.2082	.2052	.2097	1.0217	.9787	.4164	.4285	1.088	.9858	.2023	1.572	117
.006900	.03338	.2097	.2067	.2113	1.0221	.9784	.4195	.4319	1.089	.9856	.2037	1.567	116
.007000	.03362	.2113	.2082	.2128	1.0224	.9781	.4225	.4352	1.091	.9854	.2051	1.561	114
.007100	.03387	.2128	.2096	.2144	1.0227	.9778	.4256	.4386	1.092	.9852	.2065	1.556	112
.007200	.03411	.2143	.2111	.2160	1.0231	.9774	.4286	.4419	1.093	.9850	.2079	1.551	111
.007300	.03435	.2158	.2125	.2175	1.0234	.9771	.4316	.4452	1.095	.9848	.2093	1.546	109
.007400	.03459	.2173	.2139	.2190	1.0237	.9768	.4346	.4484	1.096	.9846	.2106	1.541	108
.007500	.03482	.2188	.2154	.2205	1.0240	.9765	.4376	.4517	1.097	.9844	.2120	1.536	106
.007600	.03506	.2203	.2168	.2221	1.0244	.9762	.4406	.4549	1.099	.9842	.2134	1.531	105
.007700	.03529	.2218	.2182	.2236	1.0247	.9759	.4435	.4582	1.100	.9840	.2147	1.526	104
.007800	.03552	.2232	.2196	.2251	1.0250	.9756	.4464	.4614	1.101	.9838	.2160	1.521	102
.007900	.03576	.2247	.2209	.2265	1.0253	.9753	.4493	.4646	1.103	.9836	.2173	1.517	101
.008000	.03598	.2261	.2223	.2280	1.0257	.9750	.4522	.4678	1.104	.9834	.2186	1.512	100
.008100	.03621	.2275	.2237	.2295	1.0260	.9747	.4551	.4709	1.105	.9832	.2199	1.508	98.6
.008200	.03644	.2290	.2250	.2310	1.0263	.9744	.4579	.4741	1.107	.9830	.2212	1.503	97.5
.008300	.03666	.2304	.2264	.2324	1.0266	.9741	.4607	.4772	1.108	.9827	.2225	1.499	96.3
.008400	.03689	.2318	.2277	.2338	1.0270	.9737	.4636	.4803	1.109	.9825	.2237	1.495	95.2
.008500	.03711	.2332	.2290	.2353	1.0273	.9734	.4664	.4834	1.111	.9823	.2250	1.491	94.1
.008600	.03733	.2346	.2303	.2367	1.0276	.9731	.4691	.4865	1.112	.9821	.2262	1.487	93.0
.008700	.03755	.2360	.2317	.2381	1.0280	.9728	.4719	.4896	1.113	.9819	.2275	1.482	91.9
.008800	.03777	.2373	.2330	.2396	1.0283	.9725	.4747	.4927	1.115	.9817	.2287	1.478	90.9
.008900	.03799	.2387	.2343	.2410	1.0286	.9722	.4774	.4957	1.116	.9815	.2300	1.474	89.9
.009000	.03821	.2401	.2356	.2424	1.0290	.9718	.4801	.4988	1.118	.9813	.2312	1.471	88.9
.009100	.03842	.2414	.2368	.2438	1.0293	.9815	.4828	.5018	1.119	.9811	.2324	1.467	88.0
.009200	.03864	.2428	.2381	.2452	1.0296	.9712	.4855	.5049	1.120	.9809	.2336	1.463	87.1
.009300	.03885	.2441	.2394	.2465	1.0299	.9709	.4882	.5079	1.122	.9807	.2348	1.459	86.1
.009400	.03906	.2455	.2407	.2479	1.0303	.9706	.4909	.5109	1.123	.9805	.2360	1.456	85.2
.009500	.03928	.2468	.2419	.2493	1.0306	.9703	.4936	.5138	1.124	.9803	.2371	1.452	84.3
.009600	.03949	.2481	.2431	.2507	1.0309	.9700	.4962	.5168	1.126	.9801	.2383	1.448	83.5
.009700	.03970	.2494	.2443	.2520	1.0313	.9697	.4988	.5198	1.127	.9799	.2394	1.445	82.7
.009800	.03990	.2507	.2456	.2534	1.0316	.9694	.5014	.5227	1.128	.9797	.2406	1.442	81.8
.009900	.04011	.2520	.2468	.2547	1.0319	.9691	.5040	.5257	1.130	.9794	.2417	1.438	81.0
.01000	.04032	.2533	.2480	.2560	1.0322	.9688	.5066	.5286	1.131	.9792	.2429	1.435	80.2
.01100	.04233	.2660	.2598	.2691	1.0356	.9656	.5319	.5574	1.145	.9772	.2539	1.403	73.1
.01200	.04426	.2781	.2711	.2817	1.0389	.9625	.5562	.5853	1.159	.9751	.2643	1.375	67.1
.01300	.04612	.2898	.2820	.2938	1.0423	.9594	.5795	.6125	1.173	.9731	.2743	1.350	62.1
.01400	.04791	.3010	.2924	.3056	1.0456	.9564	.6020	.6391	1.187	.9710	.2838	1.327	57.8
.01500	.04964	.3119	.3022	.3170	1.0490	.9533	.6238	.6651	1.201	.9690	.2928	1.307	54.0
.01600	.05132	.3225	.3117	.3281	1.0524	.9502	.6450	.6906	1.215	.9670	.3014	1.288	50.8
.01700	.05296	.3328	.3209	.3389	1.0559	.9471	.6655	.7158	1.230	.9649	.3096	1.271	47.9
.01800	.05455	.3428	.3298	.3495	1.0593	.9440	.6856	.7405	1.244	.9629	.3176	1.255	45.3
.01900	.05611	.3525	.3386	.3599	1.0628	.9409	.7051	.7650	1.259	.9609	.3253	1.240	43.0
.02000	.05763	.3621	.3470	.3701	1.0663	.9378	.7242	.7891	1.274	.9588	.3327	1.226	41.0
.02100	.05912	.3714	.3552	.3800	1.0698	.9348	.7429	.8131	1.289	.9568	.3399	1.213	39.1
.02200	.06057	.3806	.3632	.3898	1.0733	.9317	.7612	.8368	1.304	.9548	.3468	1.201	37.4
.02300	.06200	.3896	.3710	.3995	1.0768	.9287	.7791	.8603	1.319	.9528	.3535	1.189	35.9
.02400	.06340	.3984	.3786	.4090	1.0804	.9256	.7967	.8837	1.335	.9508	.3600	1.178	34.4
.02500	.06478	.4070	.3860	.4184	1.0840	.9225	.8140	.9069	1.350	.9488	.3662	1.168	33.1
.02600	.06613	.4155	.3932	.4276	1.0876	.9195	.8310	.9310	1.366	.9468	.3722	1.159	31.9
.02700	.06747	.4239	.4002	.4367	1.0912	.9164	.8478	.9530	1.381	.9448	.3781	1.150	30.8
.02800	.06878	.4322	.4071	.4457	1.0949	.9133	.8643	.9760	1.397	.9428	.3838	1.141	29.8
.02900	.07007	.4403	.4138	.4546	1.0985	.9103	.8805	.9988	1.413	.9408	.3893	1.133	28.8

Table D-1 Cont'd

f

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
			4138									1,133	
.03000	.07135	.4483	.4205	.4634	1.1021	.9073	.8966	1.022	1.430	.9388	.3947	1.125	27.9
.03100	.07260	.4562	.4269	.4721	1.1059	.9042	.9124	1.044	1.446	.9369	.4000	1.118	27.1
.03200	.07385	.4640	.4333	.4808	1.1096	.9012	.9280	1.067	1.462	.9349	.4051	1.111	26.3
.03300	.07507	.4717	.4395	.4894	1.1133	.8982	.9434	1.090	1.479	.9329	.4100	1.104	25.6
.03400	.07630	.4794	.4457	.4980	1.1171	.8952	.9588	1.113	1.496	.9309	.4149	1.098	24.8
.03500	.07748	.4868	.4517	.5064	1.1209	.8921	.9737	1.135	1.513	.9289	.4196	1.092	24.19
.03600	.07867	.4943	.4577	.5147	1.1247	.8891	.9886	1.158	1.530	.9270	.4242	1.086	23.56
.03700	.07984	.5017	.4635	.5230	1.1285	.8861	1.0033	1.180	1.547	.9250	.4287	1.080	22.97
.03800	.08100	.5090	.4691	.5312	1.1324	.8831	1.018	1.203	1.564	.9230	.4330	1.075	22.42
.03900	.08215	.5162	.4747	.5394	1.1362	.8801	1.032	1.226	1.582	.9211	.4372	1.069	21.90
.04000	.08329	.5233	.4802	.5475	1.1401	.8771	1.047	1.248	1.600	.9192	.4414	1.064	21.40
.04100	.08442	.5304	.4857	.5556	1.1440	.8741	1.061	1.271	1.617	.9172	.4455	1.059	20.92
.04200	.08553	.5374	.4911	.5637	1.1479	.8711	1.075	1.294	1.636	.9153	.4495	1.055	20.46
.04300	.08664	.5444	.4964	.5717	1.1518	.8688	1.089	1.317	1.654	.9133	.4534	1.050	20.03
.04400	.08774	.5513	.5015	.5796	1.1558	.8652	1.103	1.340	1.672	.9114	.4571	1.046	19.62
.04500	.08883	.5581	.5066	.5876	1.1599	.8621	1.116	1.363	1.691	.9095	.4607	1.042	19.23
.04600	.08991	.5649	.5116	.5954	1.1639	.8592	1.130	1.386	1.709	.9076	.4643	1.038	18.85
.04700	.09098	.5717	.5166	.6033	1.1679	.8562	1.143	1.409	1.728	.9057	.4679	1.034	18.49
.04800	.09205	.5784	.5215	.6111	1.1720	.8532	1.157	1.433	1.747	.9037	.4713	1.030	18.15
.04900	.09311	.5850	.5263	.6189	1.1760	.8503	1.170	1.456	1.766	.9018	.4746	1.026	17.82
.05000	.09416	.5916	.5310	.6267	1.1802	.8473	1.183	1.479	1.786	.8999	.4779	1.023	17.50
.05100	.09520	.5981	.5357	.6344	1.1843	.8444	1.196	1.503	1.805	.8980	.4811	1.019	17.19
.05200	.09623	.6046	.5403	.6421	1.1884	.8415	1.209	1.526	1.825	.8961	.4842	1.016	16.90
.05300	.09726	.6111	.5449	.6499	1.1926	.8385	1.222	1.550	1.845	.8943	.4873	1.013	16.62
.05400	.09829	.6176	.5494	.6575	1.1968	.8356	1.235	1.574	1.865	.8924	.4903	1.010	16.35
.05500	.09930	.6239	.5538	.6652	1.2011	.8326	1.248	1.598	1.885	.8905	.4932	1.007	16.09
.05600	×.1003	×.6303	.5582	.6729	1.2053	.8297	1.261	1.622	1.906	.8886	.4960	1.004	15.84
.05700	.1013	.6366	.5626	.6805	1.2096	.8267	1.273	1.646	1.926	.8867	.4988	1.001	15.60
.05800	.1023	.6428	.5668	.6880	1.2138	.8239	1.286	1.670	1.947	.8849	.5015	.9985	15.36
.05900	.1033	.6491	.5711	.6956	1.2181	.8209	1.298	1.695	1.968	.8830	.5042	.9958	15.13
.06000	.1043	.6553	.5753	.7033	1.2225	.8180	1.311	1.719	1.989	.8811	.5068	.9932	14.91
.06100	.1053	.6616	.5794	.7110	1.2270	.8150	1.3231	1.744	2.011	.8792	.5094	.9907	14.70
.06200	.1063	.6678	.5834	.7187	1.2315	.8121	1.336	1.770	2.033	.8773	.5119	.9883	14.50
.06300	.1073	.6739	.5874	.7256	1.2355	.8093	1.348	1.795	2.055	.8755	.5143	.9860	14.30
.06400	.1082	.6799	.5914	.7335	1.2402	.8063	1.360	1.819	2.076	.8737	.5167	.9837	14.11
.06500	.1092	.6860	.5954	.7411	1.2447	.8035	1.372	1.845	2.098	.8719	.5191	.9815	13.92
.06600	.1101	.6920	.5993	.7486	1.2492	.8005	1.384	1.870	2.121	.8700	.5214	.9793	13.74
.06700	.1111	.6981	.6031	.7561	1.2537	.7977	1.396	1.896	2.144	.8682	.5236	.9772	13.57
.06800	.1120	.7037	.6069	.7633	1.2580	.7948	1.408	1.921	2.166	.8664	.5258	.9752	13.40
.06900	.1130	.7099	.6106	.7711	1.2628	.7919	1.420	1.948	2.189	.8646	.5279	.9732	13.24
.07000	.1139	.7157	.6144	.7783	1.2672	.7890	1.432	1.974	2.213	.8627	.5300	.9713	13.08
.07100	.1149	.7219	.6181	.7863	1.2721	.7861	1.444	2.000	2.236	.8609	.5321	.9694	12.92
.07200	.1158	.7277	.6217	.7937	1.2767	.7833	1.455	2.026	2.260	.8591	.5341	.9676	12.77
.07300	.1168	.7336	.6252	.8011	1.2813	.7804	1.467	2.053	2.284	.8572	.5360	.9658	12.62
.07400	.1177	.7395	.6289	.8088	1.2861	.7775	1.479	2.080	2.308	.8554	.5380	.9641	12.48
.07500	.1186	.7453	.6324	.8162	1.2908	.7747	1.490	2.107	2.332	.8537	.5399	.9624	12.34
.07600	.1195	.7511	.6359	.8237	1.2956	.7719	1.502	2.135	2.357	.8519	.5417	.9607	12.21
.07700	.1205	.7569	.6392	.8312	1.3004	.7690	1.514	2.162	2.382	.8501	.5435	.9591	12.08
.07800	.1214	.7625	.6427	.8386	1.3051	.7662	1.525	2.189	2.407	.8483	.5452	.9576	11.95
.07900	.1223	.7683	.6460	.8462	1.3100	.7634	1.537	2.217	2.432	.8465	.5469	.9562	11.83
.08000	.1232	.7741	.6493	.8538	1.3149	.7605	1.548	2.245	2.458	.8448	.5485	.9548	11.71
.08100	.1241	.7799	.6526	.8614	1.3198	.7577	1.560	2.274	2.484	.8430	.5501	.9534	11.59
.08200	.1251	.7854	.6558	.8687	1.3246	.7549	1.571	2.303	2.511	.8413	.5517	.9520	11.47
.08300	.1259	.7911	.6590	.8762	1.3295	.7522	1.583	2.331	2.537	.8395	.5533	.9506	11.36
.08400	.1268	.7967	.6622	.8837	1.3345	.7494	1.594	2.360	2.563	.8378	.5548	.9493	11.25
.08500	.1277	.8026	.6655	.8915	1.3397	.7464	1.605	2.389	2.590	.8360	.5563	.9481	11.14
.08600	.1286	.8080	.6685	.8989	1.3446	.7437	1.616	2.418	2.617	.8342	.5577	.9469	11.04
.08700	.1295	.8137	.6716	.9064	1.3497	.7409	1.628	2.448	2.644	.8325	.5591	.9457	10.94
.08800	.1304	.8193	.6747	.9141	1.3548	.7381	1.639	2.478	2.672	.8308	.5605	.9445	10.84
.08900	.1313	.8250	.6778	.9218	1.3600	.7353	1.650	2.508	2.700	.8290	.5619	.9433	10.74
	1322											9422	

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.09000	.1322	.8306	6778 .6808	.9295	1.3653	.7324	1.661	2.538	2.728	.8273	.5632	.9422	10.65
.09100	.1331	.8363	.6838	.9372	1.3706	.7296	1.672	2.568	2.756	.8255	.5645	.9411	10.55
.09200	.1340	.8420	.6868	.9450	1.3759	.7268	1.684	2.599	2.785	.8238	.5658	.9401	10.46
.09300	.1349	.8474	.6897	.9525	1.3810	.7241	1.695	2.630	2.814	.8221	.5670	.9391	10.37
.09400	.1357	.8528	.6925	.9600	1.3862	.7214	1.706	2.662	2.843	.8204	.5682	.9381	10.29
.09500	.1366	.8583	.6953	.9677	1.3917	.7186	1.717	2.693	2.873	.8187	.5693	.9371	10.21
.09600	.1375	.8639	.6982	.9755	1.3970	.7158	1.728	2.726	2.903	.8170	.5704	.9362	10.12
.09700	.1384	.8694	.7011	.9832	1.4023	.7131	1.739	2.757	2.933	.8153	.5716	.9353	10.04
.09800	.1392	.8749	.7039	.9908	1.4077	.7104	1.750	2.790	2.963	.8136	.5727	.9344	9.962
.09900	.1401	.8803	.7066	.9985	1.4131	.7076	1.761	2.822	2.994	.8120	.5737	.9335	9.884
.1000	.1410	.8858	.7093	1.006	1.4187	.7049	1.772	2.855	3.025	.8103	.5747	.9327	9.808
.1010	.1419	.8913	.7120	1.014	1.4242	.7022	1.783	2.888	3.057	.8086	.5757	.9319	9.734
.1020	.1427	.8967	.7147	1.022	1.4297	.6994	1.793	2.922	3.088	.8069	.5766	.9311	9.661
.1030	.1436	.9023	.7173	1.030	1.4354	.6967	1.805	2.956	3.121	.8052	.5776	.9304	9.590
.1040	.1445	.9076	.7200	1.037	1.4410	.6940	1.815	2.990	3.153	.8036	.5785	.9297	9.519
.1050	.1453	.9130	.7226	1.045	1.4465	.6913	1.826	3.024	3.185	.8019	.5794	.9290	9.451
.1060	.1462	.9184	.7252	1.053	1.4523	.6886	1.837	3.059	3.218	.8003	.5803	.9282	9.384
.1070	.1470	.9239	.7277	1.061	1.4580	.6859	1.848	3.094	3.251	.7986	.5812	.9276	9.318
.1080	.1479	.9293	.7303	1.069	1.4638	.6833	1.858	3.128	3.284	.7970	.5820	.9269	9.254
.1090	.1488	.9343	.7327	1.076	1.4692	.6806	1.869	3.164	3.319	.7954	.5828	.9263	9.191
.1100	.1496	.9400	.7352	1.085	1.4752	.6779	1.880	3.201	3.353	.7937	.5836	.9257	9.129
.1110	.1505	.9456	.7377	1.093	1.4814	.6752	1.891	3.237	3.388	.7920	.5843	.9251	9.068
.1120	.1513	.9508	.7402	1.101	1.4871	.6725	1.902	3.274	3.423	.7904	.5850	.9245	9.009
.1130	.1522	.9563	.7426	1.109	1.4932	.6697	1.913	3.312	3.459	.7888	.5857	.9239	8.950
.1140	.1530	.9616	.7450	1.117	1.4990	.6671	1.923	3.348	3.494	.7872	.5864	.9234	8.891
.1150	.1539	.9670	.7474	1.125	1.5051	.6645	1.934	3.385	3.530	.7856	.5871	.9228	8.835
.1160	.1547	.9720	.7497	1.133	1.5108	.6619	1.944	3.423	3.566	.7840	.5878	.9223	8.780
.1170	.1556	.9775	.7520	1.141	1.5171	.6592	1.955	3.462	3.603	.7824	.5884	.9218	8.726
.1180	.1564	.9827	.7543	1.149	1.5230	.6566	1.966	3.501	3.641	.7808	.5890	.9214	8.673
.1190	.1573	.9882	.7566	1.157	1.5293	.6539	1.977	3.540	3.678	.7792	.5896	.9209	8.621
.1200	.1581	.9936	.7589	1.165	1.5356	.6512	1.987	3.579	3.716	.7776	.5902	.9204	8.569
.1210	.1590	.9989	.7612	1.174	1.5418	.6486	1.998	3.620	3.755	.7760	.5907	.9200	8.518
.1220	.1598	1.004	.7634	1.182	1.5479	.6460	2.008	3.659	3.793	.7745	.5913	.9196	8.468
.1230	.1607	1.010	.7656	1.190	1.5546	.6433	2.019	3.699	3.832	.7729	.5918	.9192	8.419
.1240	.1615	1.015	.7678	1.198	1.5605	.6407	2.030	3.740	3.871	.7713	.5922	.9189	8.371
.1250	.1624	1.020	.7700	1.207	1.5674	.6381	2.041	3.782	3.912	.7698	.5926	.9186	8.324
.1260	.1632	1.025	.7721	1.215	1.5734	.6356	2.051	3.824	3.952	.7682	.5931	.9182	8.278
.1270	.1640	1.030	.7742	1.223	1.5795	.6331	2.061	3.865	3.992	.7667	.5936	.9178	8.233
.1280	.1649	1.036	.7763	1.231	1.5862	.6305	2.072	3.907	4.033	.7652	.5940	.9175	8.189
.1290	.1657	1.041	.7783	1.240	1.5927	.6279	2.082	3.950	4.074	.7637	.5944	.9172	8.146
.1300	.1665	1.046	.7804	1.248	1.5990	.6254	2.093	3.992	4.115	.7621	.5948	.9169	8.103
.1310	.1674	1.052	.7824	1.257	1.6060	.6228	2.104	4.036	4.158	.7606	.5951	.9166	8.061
.1320	.1682	1.057	.7844	1.265	1.6124	.6202	2.114	4.080	4.201	.7591	.5954	.9164	8.020
.1330	.1691	1.062	.7865	1.273	1.6191	.6176	2.125	4.125	4.245	.7575	.5958	.9161	7.978
.1340	.1699	1.068	.7885	1.282	1.6260	.6150	2.135	4.169	4.288	.7560	.5961	.9158	7.937
.1350	.1708	1.073	.7905	1.291	1.633	.6123	2.146	4.217	4.334	.7545	.5964	.9156	7.897
.1360	.1716	1.078	.7925	1.300	1.640	.6098	2.156	4.262	4.378	.7530	.5967	.9154	7.857
.1370	.1724	1.084	.7945	1.308	1.647	.6073	2.167	4.309	4.423	.7515	.5969	.9152	7.819
.1380	.1733	1.089	.7964	1.317	1.654	.6047	2.177	4.355	4.468	.7500	.5972	.9150	7.781
.1390	.1741	1.094	.7983	1.326	1.660	.6022	2.188	4.402	4.514	.7485	.5975	.9148	7.744
.1400	.1749	1.099	.8002	1.334	1.667	.5998	2.198	4.450	4.561	.7471	.5978	.9146	7.707
.1410	.1758	1.105	.8021	1.343	1.675	.5972	2.209	4.498	4.607	.7456	.5980	.9144	7.671
.1420	.1766	1.110	.8039	1.352	1.681	.5947	2.219	4.546	4.654	.7441	.5982	.9142	7.636
.1430	.1774	1.115	.8057	1.360	1.688	.5923	2.230	4.595	4.663	.7426	.5984	.9141	7.602
.1440	.1783	1.120	.8076	1.369	1.696	.5898	2.240	4.644	4.751	.7412	.5986	.9140	7.567
.1450	.1791	1.125	.8094	1.378	1.703	.5873	2.251	4.695	4.800	.7397	.5987	.9139	7.533
.1460	.1800	1.131	.8112	1.388	1.710	.5847	2.261	4.746	4.850	.7382	.5989	.9137	7.499
.1470	.1808	1.136	.8131	1.397	1.718	.5822	2.272	4.798	4.901	.7368	.5990	.9136	7.465
.1480	.1816	1.141	.8149	1.405	1.725	.5798	2.282	4.847	4.951	.7354	.5992	.9135	7.432
.1490	.1825	1.146	.8166	1.415	1.732	.5773	2.293	4.901	5.001	.7339	.5993	.9134	7.400

Table D-1 Cont'd

d/L_o	d/L	$2\pi\ d/L$	TANH $2\pi\ d/L$ .9166	SINH $2\pi\ d/L$	COSH $2\pi\ d/L$	K	$4\pi\ d/L$	SINH $4\pi\ d/L$	COSH $4\pi\ d/L$	n	C_G/C_o	H/H'_o	M
.1500	.1833	1.152	.8183	1.424	1.740	.5748	2.303	4.954	5.054	.7325	.5994	.9133	7.369
.1510	.1841	1.157	.8200	1.433	1.747	.5723	2.314	5.007	5.106	.7311	.5994	.9133	7.339
.1520	.1850	1.162	.8217	1.442	1.755	.5699	2.324	5.061	5.159	.7296	.5995	.9132	7.309
.1530	.1858	1.167	.8234	1.451	1.762	.5675	2.335	5.115	5.212	.7282	.5996	.9132	7.279
.1540	.1866	1.173	.8250	1.460	1.770	.5651	2.345	5.169	5.265	.7268	.5996	.9132	7.250
.1550	.1875	1.178	.8267	1.469	1.777	.5627	2.356	5.225	5.320	.7254	.5997	.9131	7.221
.1560	.1883	1.183	.8284	1.479	1.785	.5602	2.366	5.283	5.376	.7240	.5998	.9130	7.191
.1570	.1891	1.188	.8301	1.488	1.793	.5577	2.377	5.339	5.432	.7226	.5999	.9129	7.162
.1580	.1900	1.194	.8317	1.498	1.801	.5552	2.387	5.398	5.490	.7212	.5998	.9130	7.134
.1590	.1908	1.199	.8333	1.507	1.809	.5528	2.398	5.454	5.544	.7198	.5998	.9130	7.107
.1600	.1917	1.204	.8349	1.517	1.817	.5504	2.408	5.513	5.603	.7184	.5998	.9130	7.079
.1610	.1925	1.209	.8365	1.527	1.825	.5480	2.419	5.571	5.660	.7171	.5998	.9130	7.052
.1620	.1933	1.215	.8381	1.536	1.833	.5456	2.429	5.630	5.718	.7157	.5998	.9130	7.026
.1630	.1941	1.220	.8396	1.546	1.841	.5432	2.440	5.690	5.777	.7144	.5998	.9130	7.000
.1640	.1950	1.225	.8411	1.555	1.849	.5409	2.450	5.751	5.837	.7130	.5998	.9130	6.975
.1650	.1958	1.230	.8427	1.565	1.857	.5385	2.461	5.813	5.898	.7117	.5997	.9131	6.949
.1660	.1966	1.235	.8442	1.574	1.865	.5362	2.471	5.874	5.959	.7103	.5996	.9132	6.924
.1670	.1975	1.240	.8457	1.584	1.873	.5339	2.482	5.938	6.021	.7090	.5996	.9132	6.900
.1680	.1983	1.246	.8472	1.594	1.882	.5315	2.492	6.003	6.085	.7076	.5995	.9133	6.876
.1690	.1992	1.251	.8486	1.604	1.890	.5291	2.503	6.066	6.148	.7063	.5994	.9133	6.853
.1700	.2000	1.257	.8501	1.614	1.899	.5267	2.513	6.130	6.212	.7050	.5993	.9134	6.830
.1710	.2008	1.262	.8515	1.624	1.907	.5243	2.523	6.197	6.275	.7036	.5992	.9135	6.807
.1720	.2017	1.267	.8529	1.634	1.915	.5220	2.534	6.262	6.342	.7023	.5991	.9136	6.784
.1730	.2025	1.272	.8544	1.644	1.924	.5197	2.544	6.329	6.407	.7010	.5989	.9137	6.761
.1740	.2033	1.277	.8558	1.654	1.933	.5174	2.555	6.395	6.473	.6997	.5988	.9138	6.738
.1750	.2042	1.282	.8572	1.664	1.941	.5151	2.565	6.465	6.541	.6984	.5987	.9139	6.716
.1760	.2050	1.288	.8586	1.675	1.951	.5127	2.576	6.534	6.610	.6971	.5985	.9140	6.694
.1770	.2058	1.293	.8600	1.685	1.959	.5104	2.586	6.603	6.679	.6958	.5984	.9141	6.672
.1780	.2066	1.298	.8614	1.695	1.968	.5081	2.597	6.672	6.747	.6946	.5982	.9142	6.651
.1790	.2075	1.304	.8627	1.706	1.977	.5058	2.607	6.744	6.818	.6933	.5980	.9144	6.631
.1800	.2083	1.309	.8640	1.716	1.986	.5036	2.618	6.818	6.891	.6920	.5979	.9145	6.611
.1810	.2092	1.314	.8653	1.727	1.995	.5013	2.629	6.890	6.963	.6907	.5977	.9146	6.591
.1820	.2100	1.320	.8666	1.737	2.004	.4990	2.639	6.963	7.035	.6895	.5975	.9148	6.571
.1830	.2108	1.325	.8680	1.748	2.013	.4967	2.650	7.038	7.109	.6882	.5974	.9149	6.550
.1840	.2117	1.330	.8693	1.758	2.022	.4945	2.660	7.113	7.183	.6870	.5972	.9150	6.530
.1850	.2125	1.335	.8706	1.769	2.032	.4922	2.671	7.191	7.260	.6857	.5969	.9152	6.511
.1860	.2134	1.341	.8718	1.780	2.041	.4899	2.681	7.267	7.336	.6845	.5967	.9154	6.492
.1870	.2142	1.346	.8731	1.791	2.051	.4876	2.692	7.345	7.412	.6832	.5965	.9155	6.474
.1880	.2150	1.351	.8743	1.801	2.060	.4854	2.702	7.421	7.488	.6820	.5963	.9157	6.456
.1890	.2159	1.356	.8755	1.812	2.070	.4832	2.712	7.500	7.566	.6808	.5961	.9159	6.438
.1900	.2167	1.362	.8767	1.823	2.079	.4809	2.723	7.581	7.647	.6796	.5958	.9161	6.421
.1910	.2176	1.367	.8779	1.834	2.089	.4787	2.734	7.663	7.728	.6784	.5955	.9163	6.403
.1920	.2184	1.372	.8791	1.845	2.099	.4765	2.744	7.746	7.810	.6772	.5952	.9165	6.385
.1930	.2192	1.377	.8803	1.856	2.108	.4743	2.755	7.827	7.891	.6760	.5950	.9167	6.368
.1940	.2201	1.383	.8815	1.867	2.118	.4721	2.765	7.911	7.974	.6748	.5948	.9169	6.351
.1950	.2209	1.388	.8827	1.879	2.128	.4699	2.776	7.996	8.059	.6736	.5946	.9170	6.334
.1960	.2218	1.393	.8839	1.890	2.138	.4677	2.787	8.083	8.145	.6724	.5944	.9172	6.317
.1970	.2226	1.399	.8850	1.901	2.148	.4655	2.797	8.167	8.228	.6712	.5941	.9174	6.300
.1980	.2234	1.404	.8862	1.913	2.158	.4633	2.808	8.256	8.316	.6700	.5938	.9176	6.284
.1990	.2243	1.409	.8873	1.924	2.169	.4611	2.819	8.346	8.406	.6689	.5935	.9179	6.268
.2000	.2251	1.414	.8884	1.935	2.178	.4590	2.829	8.436	8.495	.6677	.5932	.9181	6.253
.2010	.2260	1.420	.8895	1.947	2.189	.4569	2.840	8.524	8.583	.6666	.5929	.9183	6.237
.2020	.2268	1.425	.8906	1.959	2.199	.4547	2.850	8.616	8.674	.6654	.5926	.9186	6.222
.2030	.2277	1.430	.8917	1.970	2.210	.4526	2.861	8.708	8.766	.6642	.5923	.9188	6.206
.2040	.2285	1.436	.8928	1.982	2.220	.4504	2.872	8.803	8.860	.6631	.5920	.9190	6.191
.2050	.2293	1.441	.8939	1.994	2.231	.4483	2.882	8.897	8.953	.6620	.5917	.9193	6.176
.2060	.2302	1.446	.8950	2.006	2.242	.4462	2.893	8.994	9.050	.6608	.5914	.9195	6.161
.2070	.2310	1.451	.8960	2.017	2.252	.4441	2.903	9.090	9.144	.6597	.5911	.9197	6.147
.2080	.2319	1.457	.8971	2.030	2.263	.4419	2.914	9.187	9.240	.6586	.5908	.9200	6.133
.2090	.2328	1.462	.8981	2.042	2.274	.4398	2.925	9.288	9.342	.6574	.5905	.9202	6.119

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
2090			.8981										
.2100	.2336	1.468	.8991	2.055	2.285	.4377	2.936	9.389	9.442	.6563	.5901	.9205	6.105
.2110	.2344	1.473	.9001	2.066	2.295	.4357	2.946	9.490	9.542	.6552	.5898	.9207	6.091
.2120	.2353	1.479	.9011	2.079	2.307	.4336	2.957	9.590	9.642	.6541	.5894	.9210	6.077
.2130	.2361	1.484	.9021	2.091	2.318	.4315	2.967	9.693	9.744	.6531	.5891	.9213	6.064
.2140	.2370	1.489	.9031	2.103	2.329	.4294	2.978	9.796	9.847	.6520	.5888	.9215	6.051
.2150	.2378	1.494	.9041	2.115	2.340	.4274	2.989	9.902	9.952	.6509	.5884	.9218	6.037
.2160	.2387	1.500	.9051	2.128	2.351	.4253	2.999	10.01	10.06	.6498	.5881	.9221	6.024
.2170	.2395	1.506	.9061	2.142	2.364	.4232	3.010	10.12	10.17	.6488	.5878	.9223	6.011
.2180	.2404	1.511	.9070	2.154	2.375	.4211	3.021	10.23	10.28	.6477	.5874	.9226	5.999
.2190	.2412	1.516	.9079	2.166	2.386	.4191	3.031	10.34	10.38	.6467	.5871	.9228	5.987
.2200	.2421	1.521	.9088	2.178	2.397	.4171	3.042	10.45	10.50	.6456	.5868	.9231	5.975
.2210	.2429	1.526	.9097	2.192	2.409	.4151	3.052	10.56	10.61	.6446	.5864	.9234	5.963
.2220	.2438	1.532	.9107	2.204	2.421	.4131	3.063	10.68	10.72	.6436	.5861	.9236	5.951
.2230	.2446	1.537	.9116	2.218	2.433	.4111	3.074	10.79	10.84	.6425	.5857	.9239	5.939
.2240	.2455	1.542	.9125	2.230	2.444	.4091	3.085	10.91	10.95	.6414	.5854	.9242	5.927
.2250	.2463	1.548	.9134	2.244	2.457	.4071	3.095	11.02	11.07	.6404	.5850	.9245	5.915
.2260	.2472	1.553	.9143	2.257	2.469	.4051	3.106	11.15	11.19	.6394	.5846	.9248	5.903
.2270	.2481	1.559	.9152	2.271	2.481	.4031	3.117	11.27	11.31	.6383	.5842	.9251	5.891
.2280	.2489	1.564	.9161	2.284	2.493	.4011	3.128	11.39	11.44	.6373	.5838	.9254	5.880
.2290	.2498	1.569	.9170	2.297	2.506	.3991	3.138	11.51	11.56	.6363	.5834	.9258	5.869
.2300	.2506	1.575	.9178	2.311	2.518	.3971	3.149	11.64	11.68	.6353	.5830	.9261	5.858
.2310	.2515	1.580	.9186	2.325	2.531	.3952	3.160	11.77	11.81	.6343	.5826	.9264	5.848
.2320	.2523	1.585	.9194	2.338	2.543	.3932	3.171	11.90	11.93	.6333	.5823	.9267	5.838
.2330	.2532	1.591	.9203	2.352	2.556	.3912	3.182	12.03	12.07	.6323	.5819	.9270	5.827
.2340	.2540	1.596	.9211	2.366	2.569	.3893	3.192	12.15	12.19	.6313	.5815	.9273	5.816
.2350	.2549	1.602	.9219	2.380	2.581	.3874	3.203	12.29	12.33	.6304	.5811	.9276	5.806
.2360	.2558	1.607	.9227	2.393	2.594	.3855	3.214	12.43	12.47	.6294	.5807	.9279	5.796
.2370	.2566	1.612	.9235	2.408	2.607	.3836	3.225	12.55	12.59	.6284	.5804	.9282	5.786
.2380	.2575	1.618	.9243	2.422	2.620	.3816	3.236	12.69	12.73	.6275	.5800	.9285	5.776
.2390	.2584	1.623	.9251	2.436	2.634	.3797	3.247	12.83	12.87	.6265	.5796	.9288	5.766
.2400	.2592	1.629	.9259	2.450	2.647	.3779	3.257	12.97	13.01	.6256	.5792	.9291	5.756
.2410	.2601	1.634	.9267	2.464	2.660	.3760	3.268	13.11	13.15	.6246	.5788	.9294	5.746
.2420	.2610	1.640	.9275	2.480	2.674	.3741	3.279	13.26	13.30	.6237	.5784	.9298	5.736
.2430	.2618	1.645	.9282	2.494	2.687	.3722	3.290	13.40	13.44	.6228	.5780	.9301	5.727
.2440	.2627	1.650	.9289	2.508	2.700	.3704	3.301	13.55	13.59	.6218	.5776	.9304	5.718
.2450	.2635	1.656	.9296	2.523	2.714	.3685	3.312	13.70	13.73	.6209	.5272	.9307	5.710
.2460	.2644	1.661	.9304	2.538	2.728	.3666	3.323	13.85	13.88	.6200	.5768	.9310	5.701
.2470	.2653	1.667	.9311	2.553	2.742	.3648	3.334	14.00	14.04	.6191	.5764	.9314	5.692
.2480	.2661	1.672	.9318	2.568	2.755	.3629	3.344	14.15	14.19	.6182	.5760	.9317	5.684
.2490	.2670	1.678	.9325	2.583	2.770	.3610	3.355	14.31	14.35	.6173	.5756	.9320	5.675
.2500	.2679	1.683	.9332	2.599	2.784	.3592	3.367	14.47	14.51	.6164	.5752	.9323	5.667
.2510	.2687	1.689	.9339	2.614	2.798	.3574	3.377	14.62	14.66	.6155	.5748	.9327	5.658
.2520	.2696	1.694	.9346	2.629	2.813	.3556	3.388	14.79	14.82	.6146	.5744	.9330	5.650
.2530	.2705	1.700	.9353	2.645	2.828	.3537	3.399	14.95	14.99	.6137	.5740	.9333	5.641
.2540	.2714	1.705	.9360	2.660	2.842	.3519	3.410	15.12	15.15	.6128	.5736	.9336	5.633
.2550	.2722	1.711	.9367	2.676	2.856	.3501	3.421	15.29	15.32	.6120	.5732	.9340	5.624
.2560	.2731	1.716	.9374	2.691	2.871	.3483	3.432	15.45	15.49	.6111	.5728	.9343	5.616
.2570	.2740	1.722	.9381	2.707	2.886	.3465	3.443	15.63	15.66	.6102	.5724	.9346	5.608
.2580	.2749	1.727	.9388	2.723	2.901	.3447	3.454	15.80	15.83	.6093	.5720	.9349	5.600
.2590	.2757	1.732	.9394	2.739	2.916	.3430	3.465	15.97	16.00	.6085	.5716	.9353	5.592
.2600	.2766	1.738	.9400	2.755	2.931	.3412	3.476	16.15	16.18	.6076	.5712	.9356	5.585
.2610	.2775	1.744	.9406	2.772	2.946	.3394	3.487	16.33	16.36	.6068	.5707	.9360	5.578
.2620	.2784	1.749	.9412	2.788	2.962	.3376	3.498	16.51	16.54	.6060	.5703	.9363	5.571
.2630	.2792	1.755	.9418	2.804	2.977	.3359	3.509	16.69	16.73	.6052	.5699	.9367	5.563
.2640	.2801	1.760	.9425	2.820	2.992	.3342	3.520	16.88	16.91	.6043	.5695	.9370	5.556
.2650	.2810	1.766	.9431	2.837	3.008	.3325	3.531	17.07	17.10	.6035	.5691	.9373	5.548
.2660	.2819	1.771	.9437	2.853	3.023	.3308	3.542	17.26	17.28	.6027	.5687	.9377	5.541
.2670	.2827	1.776	.9443	2.870	3.039	.3291	3.553	17.45	17.45	.6018	.5683	.9380	5.534
.2680	.2836	1.782	.9449	2.886	3.055	.3274	3.564	17.64	17.67	.6010	.5679	.9383	5.527
.2690	.2845	1.788	.9455	2.904	3.071	.3256	3.575	17.84	17.87	.6002	.5675	.9386	5.520

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.2700	.2854	1.793	.9461	2.921	3.088	.3239	3.587	18.04	18.07	.5994	.5671	.9390	5.513
.2710	.2863	1.799	.9467	2.938	3.104	.3222	3.598	18.24	18.27	.5986	.5667	.9393	5.506
.2720	.2872	1.804	.9473	2.956	3.120	.3205	3.610	18.46	18.49	.5978	.5663	.9396	5.499
.2730	.2880	1.810	.9478	2.973	3.136	.3189	3.620	18.65	18.67	.5971	.5659	.9400	5.493
.2740	.2889	1.815	.9484	2.990	3.153	.3172	3.631	18.86	18.89	.5963	.5655	.9403	5.486
.2750	.2898	1.821	.9490	3.008	3.170	.3155	3.642	19.07	19.10	.5955	.5651	.9406	5.480
.2760	.2907	1.826	.9495	3.025	3.186	.3139	3.653	19.28	19.30	.5947	.5647	.9410	5.474
.2770	.2916	1.832	.9500	3.043	3.203	.3122	3.664	19.49	19.51	.5940	.5643	.9413	5.468
.2780	.2924	1.837	.9505	3.061	3.220	.3106	3.675	19.71	19.74	.5932	.5639	.9416	5.462
.2790	.2933	1.843	.9511	3.079	3.237	.3089	3.686	19.93	19.96	.5925	.5635	.9420	5.456
.2800	.2942	1.849	.9516	3.097	3.254	.3073	3.697	20.16	20.18	.5917	.5631	.9423	5.450
.2810	.2951	1.854	.9521	3.115	3.272	.3057	3.709	20.39	20.41	.5910	.5627	.9426	5.444
.2820	.2960	1.860	.9526	3.133	3.289	.3040	3.720	20.62	20.64	.5902	.5623	.9430	5.438
.2830	.2969	1.866	.9532	3.152	3.307	.3024	3.731	20.85	20.87	.5895	.5619	.9433	5.432
.2840	.2978	1.871	.9537	3.171	3.325	.3008	3.742	21.09	21.11	.5887	.5615	.9436	5.426
.2850	.2987	1.877	.9542	3.190	3.343	.2992	3.754	21.33	21.35	.5880	.5611	.9440	5.420
.2860	.2996	1.882	.9547	3.209	3.361	.2976	3.765	21.57	21.59	.5873	.5607	.9443	5.414
.2870	.3005	1.888	.9552	3.228	3.379	.2959	3.776	21.82	21.84	.5866	.5603	.9446	5.409
.2880	.3014	1.893	.9557	3.246	3.396	.2944	3.787	22.05	22.07	.5859	.5600	.9449	5.403
.2890	.3022	1.899	.9562	3.264	3.414	.2929	3.798	22.30	22.32	.5852	.5596	.9452	5.397
.2900	.3031	1.905	.9567	3.284	3.433	.2913	3.809	22.54	22.57	.5845	.5592	.9456	5.392
.2910	.3040	1.910	.9572	3.303	3.451	.2898	3.821	22.81	22.83	.5838	.5588	.9459	5.386
.2920	.3049	1.916	.9577	3.323	3.471	.2882	3.832	23.07	23.09	.5831	.5584	.9463	5.380
.2930	.3058	1.922	.9581	3.343	3.490	.2866	3.843	23.33	23.35	.5824	.5580	.9466	5.375
.2940	.3067	1.927	.9585	3.362	3.508	.2851	3.855	23.60	23.62	.5817	.5576	.9469	5.371
.2950	.3076	1.933	.9590	3.382	3.527	.2835	3.866	23.86	23.88	.5810	.5572	.9473	5.366
.2960	.3085	1.938	.9594	3.402	3.546	.2820	3.877	24.12	24.15	.5804	.5568	.9476	5.361
.2970	.3094	1.944	.9599	3.422	3.565	.2805	3.888	24.40	24.42	.5797	.5564	.9480	5.356
.2980	.3103	1.950	.9603	3.442	3.585	.2790	3.900	24.68	24.70	.5790	.5560	.9483	5.351
.2990	.3112	1.955	.9607	3.462	3.604	.2775	3.911	24.96	24.98	.5784	.5556	.9486	5.347
.3000	.3121	1.961	.9611	3.483	3.624	.2760	3.922	25.24	25.26	.5777	.5552	.9490	5.342
.3010	.3130	1.967	.9616	3.503	3.643	.2745	3.933	25.53	25.55	.5771	.5549	.9493	5.337
.3020	.3139	1.972	.9620	3.524	3.663	.2730	3.945	25.82	25.83	.5764	.5545	.9496	5.332
.3030	.3148	1.978	.9624	3.545	3.683	.2715	3.956	26.12	26.14	.5758	.5541	.9499	5.328
.3040	.3157	1.984	.9629	3.566	3.703	.2700	3.968	26.42	26.44	.5751	.5538	.9502	5.323
.3050	.3166	1.989	.9633	3.587	3.724	.2685	3.979	26.72	26.74	.5745	.5534	.9505	5.318
.3060	.3175	1.995	.9637	3.609	3.745	.2670	3.990	27.02	27.04	.5739	.5530	.9509	5.314
.3070	.3184	2.001	.9641	3.630	3.765	.2656	4.002	27.33	27.35	.5732	.5527	.9512	5.309
.3080	.3193	2.007	.9645	3.651	3.786	.2641	4.013	27.65	27.66	.5726	.5523	.9515	5.305
.3090	.3202	2.012	.9649	3.673	3.806	.2627	4.024	27.96	27.98	.5720	.5519	.9518	5.300
.3100	.3211	2.018	.9653	3.694	3.827	.2613	4.036	28.28	28.30	.5714	.5515	.9522	5.296
.3110	.3220	2.023	.9656	3.716	3.848	.2599	4.047	28.60	28.62	.5708	.5511	.9525	5.292
.3120	.3230	2.029	.9660	3.738	3.870	.2584	4.058	28.93	28.95	.5701	.5507	.9528	5.288
.3130	.3239	2.035	.9664	3.760	3.891	.2570	4.070	29.27	29.28	.5695	.5504	.9531	5.284
.3140	.3248	2.041	.9668	3.782	3.912	.2556	4.081	29.60	29.62	.5689	.5500	.9535	5.280
.3150	.3257	2.046	.9672	3.805	3.934	.2542	4.093	29.94	29.96	.5683	.5497	.9538	5.276
.3160	.3266	2.052	.9676	3.828	3.956	.2528	4.104	30.29	30.31	.5678	.5494	.9541	5.272
.3170	.3275	2.058	.9679	3.851	3.978	.2514	4.116	30.64	30.65	.5672	.5490	.9544	5.268
.3180	.3284	2.063	.9682	3.873	4.000	.2500	4.127	30.99	31.00	.5666	.5486	.9547	5.264
.3190	.3294	2.069	.9686	3.896	4.022	.2486	4.139	31.35	31.37	.5660	.5483	.9550	5.260
.3200	.3302	2.075	.9690	3.919	4.045	.2472	4.150	31.71	31.72	.5655	.5479	.9553	5.256
.3210	.3311	2.081	.9693	3.943	4.068	.2459	4.161	32.07	32.08	.5649	.5476	.9556	5.252
.3220	.3321	2.086	.9696	3.966	4.090	.2445	4.173	32.44	32.46	.5643	.5472	.9559	5.249
.3230	.3330	2.092	.9700	3.990	4.114	.2431	4.185	32.83	32.84	.5637	.5468	.9562	5.245
.3240	.3339	2.098	.9703	4.014	4.136	.2418	4.196	33.20	33.22	.5632	.5465	.9565	5.241
.3250	.3349	2.104	.9707	4.038	4.160	.2404	4.208	33.60	33.61	.5627	.5462	.9568	5.237
.3260	.3357	2.110	.9710	4.061	4.183	.2391	4.219	33.97	33.99	.5621	.5458	.9571	5.234
.3270	.3367	2.115	.9713	4.085	4.206	.2378	4.231	34.37	34.38	.5616	.5455	.9574	5.231
.3280	.3376	2.121	.9717	4.110	4.230	.2364	4.242	34.77	34.79	.5610	.5451	.9577	5.227
.3290	.3385	2.127	.9720	4.135	4.254	.2351	4.254	35.18	35.19	.5605	.5448	.9580	5.223

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
3290			9720										
.3300	.3394	2.133	.9723	4.159	4.277	.2338	4.265	35.58	35.59	.5599	.5444	.9583	5.220
.3310	.3403	2.138	.9726	4.184	4.301	.2325	4.277	35.99	36.00	.5594	.5441	.9586	5.217
.3320	.3413	2.144	.9729	4.209	4.326	.2312	4.288	36.42	36.43	.5589	.5438	.9589	5.214
.3330	.3422	2.150	.9732	4.234	4.350	.2299	4.300	36.84	36.85	.5584	.5434	.9592	5.210
.3340	.3431	2.156	.9735	4.259	4.375	.2286	4.311	37.25	37.27	.5578	.5431	.9595	5.207
.3350	.3440	2.161	.9738	4.284	4.399	.2273	4.323	37.70	37.72	.5573	.5427	.9598	5.204
.3360	.3449	2.167	.9741	4.310	4.424	.2260	4.335	38.14	38.15	.5568	.5424	.9601	5.201
.3370	.3459	2.173	.9744	4.336	4.450	.2247	4.346	38.59	38.60	.5563	.5421	.9604	5.198
.3380	.3468	2.179	.9747	4.361	4.474	.2235	4.358	39.02	39.04	.5558	.5417	.9607	5.194
.3390	.3477	2.185	.9750	4.388	4.500	.2222	4.369	39.48	39.49	.5553	.5414	.9610	5.191
.3400	.3468	2.190	.9753	4.413	4.525	.2210	4.381	39.95	39.96	.5548	.5411	.9613	5.188
.3410	.3495	2.196	.9756	4.439	4.550	.2198	4.392	40.40	40.41	.5544	.5408	.9615	5.185
.3420	.3504	2.202	.9758	4.466	4.576	.2185	4.404	40.87	40.89	.5539	.5405	.9618	5.182
.3430	.3514	2.208	.9761	4.492	4.602	.2173	4.416	41.36	41.37	.5534	.5402	.9621	5.179
.3440	.3523	2.214	.9764	4.521	4.630	.2160	4.427	41.85	41.84	.5529	.5399	.9623	5.176
.3450	.3532	2.220	.9767	4.547	4.656	.2148	4.439	42.33	42.34	.5524	.5396	.9626	5.173
.3460	.3542	2.225	.9769	4.575	4.682	.2136	4.451	42.83	42.84	.5519	.5392	.9629	5.171
.3470	.3551	2.231	.9772	4.602	4.709	.2124	4.462	43.34	43.35	.5515	.5389	.9632	5.168
.3480	.3560	2.237	.9775	4.629	4.736	.2111	4.474	43.85	43.86	.5510	.5386	.9635	5.165
.3490	.3570	2.243	.9777	4.657	4.763	.2099	4.486	44.37	44.40	.5505	.5383	.9638	5.162
.3500	.3579	2.249	.9780	4.685	4.791	.2087	4.498	44.89	44.80	.5501	.5380	.9640	5.159
.3510	.3588	2.255	.9782	4.713	4.818	.2076	4.509	45.42	45.43	.5496	.5377	.9643	5.157
.3520	.3598	2.260	.9785	4.741	4.845	.2064	4.521	45.95	45.96	.5492	.5374	.9646	5.154
.3530	.3607	2.266	.9787	4.770	4.873	.2052	4.533	46.50	46.51	.5487	.5371	.9648	5.152
.3540	.3616	2.272	.9790	4.798	4.901	.2040	4.544	47.03	47.04	.5483	.5368	.9651	5.149
.3550	.3625	2.278	.9792	4.827	4.929	.2029	4.556	47.59	47.60	.5479	.5365	.9654	5.147
.3560	.3635	2.284	.9795	4.856	4.957	.2017	4.568	48.15	48.16	.5474	.5362	.9657	5.144
.3570	.3644	2.290	.9797	4.885	4.987	.2005	4.579	48.72	48.73	.5470	.5359	.9659	5.141
.3580	.3653	2.296	.9799	4.914	5.015	.1994	4.591	49.29	49.30	.5466	.5356	.9662	5.139
.3590	.3663	2.301	.9801	4.944	5.044	.1983	4.603	49.88	49.89	.5461	.5353	.9665	5.137
.3600	.3672	2.307	.9804	4.974	5.072	.1972	4.615	50.47	50.48	.5457	.5350	.9667	5.134
.3610	.3682	2.313	.9806	5.004	5.103	.1960	4.627	51.08	51.09	.5453	.5347	.9670	5.132
.3620	.3691	2.319	.9808	5.034	5.132	.1949	4.638	51.67	51.67	.5449	.5344	.9673	5.130
.3630	.3700	2.325	.9811	5.063	5.161	.1938	4.650	52.27	52.28	.5445	.5342	.9675	5.127
.3640	.3709	2.331	.9813	5.094	5.191	.1926	4.661	52.89	52.90	.5441	.5339	.9677	5.125
.3650	.3719	2.337	.9815	5.124	5.221	.1915	4.673	53.52	53.53	.5437	.5336	.9680	5.123
.3660	.3728	2.342	.9817	5.155	5.251	.1904	4.685	54.15	54.16	.5433	.5333	.9683	5.121
.3670	.3737	2.348	.9819	5.186	5.281	.1894	4.697	54.78	54.79	.5429	.5330	.9686	5.118
.3680	.3747	2.354	.9821	5.217	5.312	.1883	4.708	55.42	55.43	.5425	.5327	.9688	5.116
.3690	.3756	2.360	.9823	5.248	5.343	.1872	4.720	56.09	56.10	.5421	.5325	.9690	5.114
.3700	.3766	2.366	.9825	5.280	5.374	.1861	4.732	56.76	56.77	.5417	.5322	.9693	5.112
.3710	.3775	2.372	.9827	5.312	5.406	.1850	4.744	57.43	57.44	.5413	.5319	.9696	5.110
.3720	.3785	2.378	.9830	5.345	5.438	.1839	4.756	58.13	58.14	.5409	.5317	.9698	5.107
.3730	.3794	2.384	.9832	5.377	5.469	.1828	4.768	58.82	58.83	.5405	.5314	.9700	5.105
.3740	.3804	2.390	.9834	5.410	5.502	.1818	4.780	59.52	59.53	.5402	.5312	.9702	5.103
.3750	.3813	2.396	.9835	5.443	5.534	.1807	4.792	60.24	60.25	.5398	.5309	.9705	5.101
.3760	.3822	2.402	.9837	5.475	5.566	.1797	4.803	60.95	60.95	.5394	.5306	.9707	5.099
.3770	.3832	2.408	.9839	5.508	5.598	.1786	4.815	61.68	61.68	.5390	.5304	.9709	5.097
.3780	.3841	2.413	.9841	5.541	5.631	.1776	4.827	62.41	62.42	.5387	.5301	.9712	5.095
.3790	.3850	2.419	.9843	5.572	5.661	.1766	4.838	63.13	63.14	.5383	.5299	.9714	5.093
.3800	.3860	2.425	.9845	5.609	5.697	.1756	4.851	63.91	63.91	.5380	.5296	.9717	5.091
.3810	.3869	2.431	.9847	5.643	5.731	.1745	4.862	64.67	64.67	.5376	.5294	.9719	5.090
.3820	.3879	2.437	.9848	5.677	5.765	.1735	4.875	65.45	65.46	.5372	.5291	.9721	5.088
.3830	.3888	2.443	.9850	5.712	5.798	.1725	4.885	66.16	66.17	.5369	.5288	.9724	5.086
.3840	.3898	2.449	.9852	5.746	5.833	.1715	4.898	67.02	67.03	.5365	.5286	.9726	5.084
.3850	.3907	2.455	.9854	5.780	5.866	.1705	4.910	67.80	67.81	.5362	.5284	.9728	5.082
.3860	.3917	2.461	.9855	5.814	5.900	.1695	4.922	68.61	68.62	.5359	.5281	.9730	5.081
.3870	.3926	2.467	.9857	5.850	5.935	.1685	4.934	69.45	69.46	.5355	.5279	.9732	5.079
.3880	.3936	2.473	.9859	5.886	5.970	.1675	4.946	70.28	70.29	.5352	.5276	.9735	5.077
.3890	.3945	2.479	.9860	5.921	6.005	.1665	4.958	71.12	71.13	.5349	.5274	.9737	5.076

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.3900	.3955	2.485	.9862	5.957	6.040	.1656	4.970	71.97	71.98	.5345	.5271	.9739	5.074
.3910	.3964	2.491	.9864	5.993	6.076	.1646	4.982	72.85	72.86	.5342	.5269	.9741	5.072
.3920	.3974	2.497	.9865	6.029	6.112	.1636	4.993	73.72	73.72	.5339	.5267	.9743	5.071
.3930	.3983	2.503	.9867	6.066	6.148	.1627	5.005	74.58	74.59	.5336	.5265	.9745	5.069
.3940	.3993	2.509	.9869	6.103	6.185	.1617	5.017	75.48	75.49	.5332	.5262	.9748	5.067
.3950	.4002	2.515	.9870	6.140	6.221	.1608	5.029	76.40	76.40	.5329	.5260	.9750	5.066
.3960	.4012	2.521	.9872	6.177	6.258	.1598	5.041	77.31	77.32	.5326	.5258	.9752	5.064
.3970	.4021	2.527	.9873	6.215	6.295	.1589	5.053	78.24	78.24	.5323	.5255	.9754	5.063
.3980	.4031	2.532	.9874	6.252	6.332	.1579	5.065	79.19	79.19	.5320	.5253	.9756	5.062
.3990	.4040	2.538	.9876	6.290	6.369	.1570	5.077	80.13	80.13	.5317	.5251	.9758	5.060
.4000	.4050	2.544	.9877	6.329	6.407	.1561	5.089	81.12	81.12	.5314	.5248	.9761	5.058
.4010	.4059	2.550	.9879	6.367	6.445	.1552	5.101	82.07	82.08	.5311	.5246	.9763	5.056
.4020	.4069	2.556	.9880	6.406	6.483	.1542	5.113	83.06	83.06	.5308	.5244	.9765	5.055
.4030	.4078	2.562	.9882	6.444	6.521	.1533	5.125	84.07	84.07	.5305	.5242	.9766	5.053
.4040	.4088	2.568	.9883	6.484	6.561	.1524	5.137	85.11	85.12	.5302	.5240	.9768	5.052
.4050	.4098	2.575	.9885	6.525	6.601	.1515	5.149	86.14	86.14	.5299	.5238	.9770	5.050
.4060	.4107	2.581	.9886	6.564	6.640	.1506	5.161	87.17	87.17	.5296	.5236	.9772	5.049
.4070	.4116	2.586	.9887	6.603	6.679	.1497	5.173	88.19	88.20	.5293	.5234	.9774	5.048
.4080	.4126	2.592	.9889	6.644	6.718	.1488	5.185	89.28	89.28	.5290	.5232	.9776	5.046
.4090	.4136	2.598	.9890	6.684	6.758	.1480	5.197	90.38	90.39	.5287	.5229	.9778	5.045
.4100	.4145	2.604	.9891	6.725	6.799	.1471	5.209	91.44	91.44	.5285	.5227	.9780	5.044
.4110	.4155	2.610	.9892	6.766	6.839	.1462	5.221	92.54	92.55	.5282	.5225	.9782	5.043
.4120	.4164	2.616	.9894	6.806	6.879	.1454	5.233	93.67	93.67	.5279	.5223	.9784	5.041
.4130	.4174	2.623	.9895	6.849	6.921	.1445	5.245	94.83	94.83	.5277	.5221	.9786	5.040
.4140	.4183	2.629	.9896	6.890	6.963	.1436	5.257	95.95	95.96	.5274	.5219	.9788	5.039
.4150	.4193	2.635	.9898	6.932	7.004	.1428	5.269	97.13	97.13	.5271	.5217	.9790	5.037
.4160	.4203	2.641	.9899	6.974	7.046	.1419	5.281	98.29	98.30	.5269	.5215	.9792	5.036
.4170	.4212	2.647	.9900	7.018	7.088	.1411	5.294	99.52	99.52	.5266	.5213	.9794	5.035
.4180	.4222	2.653	.9901	7.060	7.130	.1403	5.305	100.7	100.7	.5263	.5211	.9795	5.034
.4190	.4231	2.659	.9902	7.102	7.173	.1394	5.317	101.9	101.9	.5261	.5209	.9797	5.033
.4200	.4241	2.665	.9904	7.146	7.215	.1386	5.329	103.1	103.1	.5258	.5208	.9798	5.031
.4210	.4251	2.671	.9905	7.190	7.259	.1378	5.341	104.4	104.4	.5256	.5206	.9800	5.030
.4220	.4260	2.677	.9906	7.234	7.303	.1369	5.353	105.7	105.7	.5253	.5204	.9802	5.029
.4230	.4270	2.683	.9907	7.279	7.349	.1361	5.366	107.0	107.0	.5251	.5202	.9804	5.028
.4240	.4280	2.689	.9908	7.325	7.392	.1353	5.378	108.3	108.3	.5248	.5200	.9806	5.027
.4250	.4289	2.695	.9909	7.371	7.438	.1345	5.390	109.[illegible]	109.7	.5246	.5198	.9808	5.026
.4260	.4298	2.701	.9910	7.412	7.479	.1337	5.402	110.9	110.9	.5244	.5196	.9810	5.025
.4270	.4308	2.707	.9911	7.457	7.524	.1329	5.414	112.2	112.2	.5241	.5195	.9811	5.024
.4280	.4318	2.713	.9912	7.503	7.570	.1321	5.426	113.6	113.6	.5239	.5193	.9812	5.023
.4290	.4328	2.719	.9913	7.550	7.616	.1313	5.438	115.0	115.0	.5237	.5191	.9814	5.022
.4300	.4337	2.725	.9914	7.595	7.661	.1305	5.450	116.4	116.4	.5234	.5189	.9816	5.021
.4310	.4347	2.731	.9915	7.642	7.707	.1298	5.462	117.8	117.8	.5232	.5187	.9818	5.020
.4320	.4356	2.737	.9916	7.688	7.753	.1290	5.474	119.2	119.3	.5230	.5186	.9819	5.019
.4330	.4366	2.743	.9917	7.735	7.800	.1282	5.486	120.7	120.7	.5227	.5184	.9821	5.018
.4340	.4376	2.749	.9918	7.783	7.847	.1274	5.499	122.2	122.2	.5225	.5182	.9823	5.017
.4350	.4385	2.755	.9919	7.831	7.895	.1267	5.511	123.7	123.7	.5223	.5181	.9824	5.016
.4360	.4395	2.762	.9920	7.880	7.943	.1259	5.523	125.2	125.2	.5221	.5179	.9826	5.015
.4370	.4405	2.768	.9921	7.922	7.991	.1251	5.535	126.7	126.7	.5218	.5177	.9828	5.014
.4380	.4414	2.774	.9922	7.975	8.035	.1244	5.547	128.3	128.3	.5216	.5176	.9829	5.013
.4390	.4424	2.780	.9923	8.026	8.088	.1236	5.560	129.9	129.9	.5214	.5174	.9830	5.012
.4400	.4434	2.786	.9924	8.075	8.136	.1229	5.572	131.4	131.4	.5212	.5172	.9832	5.011
.4410	.4443	2.792	.9925	8.124	8.185	.1222	5.584	133.0	133.0	.5210	.5171	.9833	5.010
.4420	.4453	2.798	.9926	8.175	8.236	.1214	5.596	134.7	134.7	.5208	.5169	.9835	5.009
.4430	.4463	2.804	.9927	8.228	8.285	.1207	5.608	136.3	136.3	.5206	.5168	.9836	5.008
.4440	.4472	2.810	.9928	8.274	8.334	.1200	5.620	137.9	137.9	.5204	.5166	.9838	5.007
.4450	.4482	2.816	.9929	8.326	8.387	.1192	5.632	139.6	139.7	.5202	.5165	.9839	5.006
.4460	.4492	2.822	.9930	8.379	8.438	.1185	5.644	141.4	141.4	.5200	.5163	.9841	5.005
.4470	.4501	2.828	.9930	8.427	8.486	.1178	5.657	143.1	143.1	.5198	.5161	.9843	5.005
.4480	.4511	2.834	.9931	8.481	8.540	.1171	5.669	144.8	144.8	.5196	.5160	.9844	5.004
.4490	.4521	2.840	.9932	8.532	8.590	.1164	5.681	146.6	146.6	.5194	.5158	.9846	5.003

Table D-1 Cont'd

d/L_o	d/L	$2\pi\ d/L$	TANH $2\pi\ d/L$	SINH $2\pi\ d/L$	COSH $2\ d/L$	K	$4\pi\ d/L$	SINH $4\pi\ d/L$	COSH $4\pi\ d/L$	n	C_G/C_o	H/H'_o	M
.4500	.4531	2.847	.9933	8.585	8.643	.1157	5.693	148.4	148.4	.5192	.5157	.9847	5.002
.4510	.4540	2.853	.9934	8.638	8.695	.1150	5.705	150.2	150.2	.5190	.5156	.9848	5.001
.4520	.4550	2.859	.9935	8.693	8.750	.1143	5.717	152.1	152.1	.5188	.5154	.9849	5.000
.4530	.4560	2.865	.9935	8.747	8.804	.1136	5.730	154.0	154.0	.5186	.5152	.9851	5.000
.4540	.4569	2.871	.9936	8.797	8.854	.1129	5.742	155.9	155.9	.5184	.5151	.9852	4.999
.4550	.4579	2.877	.9937	8.853	8.910	.1122	5.754	157.7	157.7	.5182	.5150	.9853	4.998
.4560	.4589	2.883	.9938	8.910	8.965	.1115	5.766	159.7	159.7	.5181	.5148	.9855	4.997
.4570	.4599	2.890	.9938	8.965	9.021	.1109	5.779	161.7	161.7	.5179	.5146	.9857	4.997
.4580	.4608	2.896	.9939	9.016	9.072	.1102	5.791	163.6	163.6	.5177	.5145	.9858	4.996
.4590	.4618	2.902	.9940	9.074	9.129	.1095	5.803	165.6	165.6	.5175	.5144	.9859	4.995
.4600	.4628	2.908	.9941	9.132	9.186	.1089	5.815	167.7	167.7	.5173	.5143	.9860	4.994
.4610	.4637	2.914	.9941	9.183	9.238	.1083	5.827	169.7	169.7	.5172	.5141	.9862	4.994
.4620	.4647	2.920	.9942	9.242	9.296	.1076	5.840	171.8	171.8	.5170	.5140	.9863	4.993
.4630	.4657	2.926	.9943	9.301	9.354	.1069	5.852	173.9	173.9	.5168	.5139	.9864	4.992
.4640	.4666	2.932	.9944	9.353	9.406	.1063	5.864	176.0	176.0	.5167	.5138	.9865	4.991
.4650	.4676	2.938	.9944	9.413	9.466	.1056	5.876	178.2	178.2	.5165	.5136	.9867	4.991
.4660	.4686	2.944	.9945	9.472	9.525	.1050	5.888	180.4	180.4	.5163	.5135	.9868	4.990
.4670	.4695	2.951	.9946	9.533	9.585	.1043	5.900	182.6	182.6	.5162	.5134	.9869	4.989
.4680	.4705	2.957	.9946	9.586	9.638	.1037	5.912	184.8	184.8	.5160	.5132	.9871	4.989
.4690	.4715	2.963	.9947	9.647	9.699	.1031	5.925	187.2	187.2	.5158	.5131	.9872	4.988
.4700	.4725	2.969	.9947	9.709	9.760	.1025	5.937	189.5	189.5	.5157	.5129	.9873	4.988
.4710	.4735	2.975	.9948	9.770	9.821	.1018	5.949	191.8	191.8	.5155	.5128	.9874	4.987
.4720	.4744	2.981	.9949	9.826	9.877	.1012	5.962	194.2	194.2	.5154	.5127	.9875	4.986
.4730	.4754	2.987	.9949	9.888	9.938	.1006	5.974	196.5	196.5	.5152	.5126	.9876	4.986
.4740	.4764	2.993	.9950	9.951	10.00	.1000	5.986	199.0	199.0	.5150	.5125	.9877	4.985
.4750	.4774	2.999	.9951	10.01	10.07	.09942	5.999	201.4	201.4	.5149	.5124	.9878	4.984
.4760	.4783	3.005	.9951	10.07	10.12	.09882	6.011	203.9	203.9	.5147	.5122	.9880	4.984
.4770	.4793	3.012	.9952	10.13	10.18	.09820	6.023	206.5	206.5	.5146	.5121	.9881	4.983
.4780	.4803	3.018	.9952	10.20	10.25	.09759	6.036	209.0	209.0	.5144	.5120	.9882	4.983
.4790	.4813	3.024	.9953	10.26	10.31	.09698	6.048	211.7	211.7	.5143	.5119	.9883	4.982
.4800	.4822	3.030	.9953	10.32	10.37	.09641	6.060	214.2	214.2	.5142	.5117	.9885	4.982
.4810	.4832	3.036	.9954	10.39	10.43	.09583	6.072	216.8	216.8	.5140	.5116	.9886	4.981
.4820	.4842	3.042	.9955	10.45	10.50	.09523	6.085	219.5	219.5	.5139	.5115	.9887	4.980
.4830	.4852	3.049	.9955	10.52	10.57	.09464	6.097	222.2	222.2	.5137	.5114	.9888	4.980
.4840	.4862	3.055	.9956	10.59	10.63	.09405	6.109	225.0	225.0	.5136	.5113	.9889	4.979
.4850	.4871	3.061	.9956	10.65	10.69	.09352	6.121	228.3	228.3	.5134	.5112	.9890	4.979
.4860	.4881	3.067	.9957	10.71	10.76	.09294	6.134	230.6	230.6	.5133	.5111	.9891	4.978
.4870	.4891	3.073	.9957	10.78	10.83	.09236	6.146	233.5	233.5	.5132	.5110	.9892	4.978
.4880	.4901	3.079	.9958	10.85	10.90	.09178	6.159	236.4	236.4	.5130	.5109	.9893	4.977
.4890	.4911	3.086	.9958	10.92	10.96	.09121	6.171	239.6	239.6	.5129	.5107	.9895	4.977
.4900	.4920	3.092	.9959	10.99	11.03	.09064	6.183	242.3	242.3	.5128	.5106	.9896	4.976
.4910	.4930	3.098	.9959	11.05	11.09	.09010	6.195	245.2	245.2	.5126	.5105	.9897	4.976
.4920	.4940	3.104	.9960	11.12	11.16	.08956	6.208	248.3	248.3	.5125	.5104	.9898	4.975
.4930	.4950	3.110	.9960	11.19	11.24	.08901	6.220	251.3	251.3	.5124	.5103	.9899	4.975
.4940	.4960	3.117	.9961	11.26	11.31	.08845	6.232	254.5	254.5	.5122	.5102	.9899	4.974
.4950	.4969	3.122	.9961	11.32	11.37	.08793	6.245	257.6	257.6	.5121	.5101	.9900	4.974
.4960	.4979	3.128	.9962	11.40	11.44	.08741	6.257	260.8	260.8	.5120	.5100	.9901	4.973
.4970	.4989	3.135	.9962	11.47	11.51	.08691	6.269	264.0	264.0	.5119	.5099	.9902	4.973
.4980	.4999	3.141	.9963	11.54	11.59	.08637	6.282	267.3	267.3	.5118	.5098	.9903	4.972
.4990	.5009	3.147	.9963	11.61	11.65	.08584	6.294	270.6	270.6	.5116	.5097	.9904	4.972
.5000	.5018	3.153	.9964	11.68	11.72	.08530	6.306	274.0	274.0	.5115	.5096	.9905	4.971
.5010	.5028	3.159	.9964	11.75	11.80	.08477	6.319	277.5	277.5	.5114	.5095	.9906	4.971
.5020	.5038	3.166	.9964	11.83	11.87	.08424	6.331	280.8	280.8	.5113	.5094	.9907	4.971
.5030	.5048	3.172	.9965	11.91	11.95	.08371	6.343	284.3	284.3	.5112	.5093	.9908	4.970
.5040	.5058	3.178	.9965	11.98	12.02	.08320	6.356	287.9	287.9	.5110	.5092	.9909	4.970
.5050	.5067	3.184	.9966	12.05	12.09	.08270	6.368	291.4	291.4	.5109	.5092	.9909	4.969
.5060	.5077	3.190	.9966	12.12	12.16	.08220	6.380	295.0	295.0	.5108	.5091	.9910	4.969
.5070	.5087	3.196	.9967	12.20	12.24	.08169	6.393	298.7	298.7	.5107	.5090	.9911	4.968
.5080	.5097	3.203	.9967	12.28	12.32	.08119	6.405	302.4	302.4	.5106	.5089	.9912	4.968
.5090	.5107	3.209	.9968	12.35	12.39	.08068	6.417	306.2	306.2	.5105	.5088	.9913	4.967

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.5100	.5117	3.215	.9968	12.43	12.47	.08022	6.430	310.0	310.0	.5104	.5087	.9914	4.967
.5110	.5126	3.221	.9968	12.50	12.54	.07972	6.442	313.8	313.8	.5103	.5086	.9915	4.967
.5120	.5136	3.227	.9969	12.58	12.62	.07922	6.454	317.7	317.7	.5102	.5086	.9915	4.966
.5130	.5146	3.233	.9969	12.66	12.70	.07873	6.467	321.7	321.7	.5101	.5085	.9916	4.966
.5140	.5156	3.240	.9970	12.74	12.78	.07824	6.479	325.7	325.7	.5100	.5084	.9917	4.965
.5150	.5166	3.246	.9970	12.82	12.86	.07776	6.491	329.7	329.7	.5098	.5083	.9918	4.965
.5160	.5176	3.252	.9970	12.90	12.94	.07729	6.504	333.8	333.8	.5097	.5082	.9919	4.965
.5170	.5185	3.258	.9971	12.98	13.02	.07682	6.516	337.9	337.9	.5096	.5082	.9919	4.964
.5180	.5195	3.264	.9971	13.06	13.10	.07634	6.529	342.2	342.2	.5095	.5081	.9920	4.964
.5190	.5205	3.270	.9971	13.14	13.18	.07587	6.541	346.4	346.4	.5094	.5080	.9921	4.964
.5200	.5215	3.277	.9972	13.22	13.26	.07540	6.553	350.7	350.7	.5093	.5079	.9922	4.963
.5210	.5225	3.283	.9972	13.31	13.35	.07494	6.566	355.1	355.1	.5092	.5078	.9923	4.963
.5220	.5235	3.289	.9972	13.39	13.43	.07449	6.578	359.6	359.6	.5092	.5077	.9924	4.963
.5230	.5244	3.295	.9973	13.47	13.51	.07404	6.590	364.0	364.0	.5091	.5077	.9924	4.962
.5240	.5254	3.301	.9973	13.55	13.59	.07358	6.603	368.5	368.5	.5090	.5076	.9925	4.962
.5250	.5264	3.308	.9973	13.64	13.68	.07312	6.615	373.1	373.1	.5089	.5075	.9926	4.962
.5260	.5274	3.314	.9974	13.73	13.76	.07266	6.628	377.8	377.8	.5088	.5074	.9927	4.961
.5270	.5284	3.320	.9974	13.81	13.85	.07221	6.640	382.5	382.5	.5087	.5074	.9927	4.961
.5280	.5294	3.326	.9974	13.90	13.94	.07177	6.652	387.3	387.3	.5086	.5073	.9928	4.961
.5290	.5304	3.333	.9975	13.99	14.02	.07134	6.665	392.2	392.2	.5085	.5072	.9929	4.960
.5300	.5314	3.339	.9975	14.07	14.10	.07091	6.677	397.0	397.0	.5084	.5071	.9930	4.960
.5310	.5323	3.345	.9975	14.16	14.19	.07047	6.690	402.0	402.0	.5083	.5070	.9931	4.960
.5320	.5333	3.351	.9976	14.25	14.28	.07003	6.702	406.9	406.9	.5082	.5070	.9931	4.959
.5330	.5343	3.357	.9976	14.34	14.37	.06959	6.714	412.0	412.0	.5082	.5069	.9932	4.959
.5340	.5353	3.363	.9976	14.43	14.46	.06915	6.727	417.2	417.2	.5081	.5068	.9933	4.959
.5350	.5363	3.370	.9976	14.52	14.55	.06872	6.739	422.4	422.4	.5080	.5068	.9933	4.959
.5360	.5373	3.376	.9977	14.61	14.64	.06829	6.752	427.7	427.7	.5079	.5067	.9934	4.958
.5370	.5383	3.382	.9977	14.70	14.73	.06787	6.764	433.1	433.1	.5078	.5066	.9935	4.958
.5380	.5393	3.388	.9977	14.79	14.82	.06746	6.776	438.5	438.5	.5077	.5066	.9935	4.958
.5390	.5402	3.394	.9977	14.88	14.91	.06705	6.789	444.0	444.0	.5077	.5065	.9936	4.958
.5400	.5412	3.401	.9978	14.97	15.01	.06664	6.801	449.5	449.5	.5076	.5065	.9936	4.957
.5410	.5422	3.407	.9978	15.07	15.10	.06623	6.814	455.1	455.1	.5075	.5064	.9937	4.957
.5420	.5432	3.413	.9978	15.16	15.19	.06582	6.826	460.7	460.7	.5074	.5063	.9938	4.957
.5430	.5442	3.419	.9979	15.25	15.29	.06542	6.838	466.4	466.4	.5073	.5063	.9938	4.956
.5440	.5452	3.426	.9979	15.35	15.38	.06501	6.851	472.2	472.2	.5073	.5062	.9939	4.956
.5450	.5461	3.432	.9979	15.45	15.48	.06461	6.863	478.1	478.1	.5072	.5061	.9940	4.956
.5460	.5471	3.438	.9979	15.54	15.58	.06420	6.876	484.3	484.3	.5071	.5060	.9941	4.956
.5470	.5481	3.444	.9980	15.64	15.67	.06380	6.888	490.3	490.3	.5070	.5060	.9941	4.955
.5480	.5491	3.450	.9980	15.74	15.77	.06341	6.901	496.4	496.4	.5070	.5059	.9942	4.955
.5490	.5501	3.456	.9980	15.84	15.87	.06302	6.913	502.5	502.5	.5069	.5059	.9942	4.955
.5500	.5511	3.463	.9980	15.94	15.97	.06263	6.925	508.7	508.7	.5068	.5058	.9942	4.955
.5510	.5521	3.469	.9981	16.04	16.07	.06224	6.937	515.0	515.0	.5067	.5058	.9942	4.954
.5520	.5531	3.475	.9981	16.14	16.17	.06186	6.950	521.6	521.6	.5067	.5057	.9943	4.954
.5530	.5541	3.481	.9981	16.24	16.27	.06148	6.962	528.1	528.1	.5066	.5056	.9944	4.954
.5540	.5551	3.488	.9981	16.34	16.37	.06110	6.975	534.8	534.8	.5065	.5056	.9944	4.954
.5550	.5560	3.494	.9982	16.44	16.47	.06073	6.987	541.4	541.4	.5065	.5056	.9945	4.953
.5560	.5570	3.500	.9982	16.54	16.57	.06035	7.000	548.1	548.1	.5064	.5055	.9945	4.953
.5570	.5580	3.506	.9982	16.65	16.68	.05997	7.012	554.9	554.9	.5063	.5054	.9946	4.953
.5580	.5590	3.512	.9982	16.75	16.78	.05960	7.025	562.0	562.0	.5063	.5053	.9947	4.953
.5590	.5600	3.519	.9982	16.85	16.88	.05923	7.037	569.1	569.1	.5062	.5053	.9947	4.953
.5600	.5610	3.525	.9983	16.96	16.99	.05887	7.050	576.1	576.1	.5061	.5053	.9947	4.952
.5610	.5620	3.531	.9983	17.06	17.09	.05850	7.062	583.3	583.3	.5061	.5052	.9948	4.952
.5620	.5630	3.537	.9983	17.17	17.20	.05814	7.074	590.7	590.7	.5060	.5051	.9949	4.952
.5630	.5640	3.543	.9983	17.28	17.31	.05778	7.087	598.0	598.0	.5059	.5051	.9949	4.952
.5640	.5649	3.550	.9984	17.38	17.41	.05743	7.099	605.0	605.0	.5059	.5050	.9950	4.951
.5650	.5659	3.556	.9984	17.49	17.52	.05707	7.112	613.2	613.2	.5058	.5050	.9950	4.951
.5660	.5669	3.562	.9984	17.60	17.63	.05672	7.124	620.8	620.8	.5057	.5049	.9951	4.951
.5670	.5679	3.568	.9984	17.71	17.74	.05637	7.136	628.5	628.5	.5057	.5049	.9951	4.951
.5680	.5689	3.575	.9984	17.82	17.85	.05602	7.149	636.4	636.4	.5056	.5048	.9952	4.951
.5690	.5699	3.581	.9985	17.94	17.97	.05567	7.161	644.3	644.3	.5056	.5048	.9952	4.950

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.5700	.5709	3.587	.9985	18.05	18.08	.05532	7.174	652.4	652.4	.5055	.5047	.9953	4.950
.5710	.5719	3.593	.9985	18.16	18.19	.05497	7.186	660.5	660.5	.5054	.5047	.9953	4.950
.5720	.5729	3.600	.9985	18.28	18.31	.05463	7.199	668.8	668.8	.5054	.5046	.9954	4.950
.5730	.5738	3.606	.9985	18.39	18.42	.05430	7.211	677.2	677.2	.5053	.5046	.9954	4.950
.5740	.5748	3.612	.9985	18.50	18.53	.05396	7.224	685.6	685.6	.5053	.5045	.9955	4.950
.5750	.5758	3.618	.9986	18.62	18.64	.05363	7.236	694.3	694.3	.5052	.5045	.9955	4.949
.5760	.5768	3.624	.9986	18.73	18.76	.05330	7.249	703.2	703.2	.5052	.5044	.9956	4.949
.5770	.5778	3.630	.9986	18.85	18.88	.05297	7.261	711.9	711.9	.5051	.5044	.9956	4.949
.5780	.5788	3.637	.9986	18.97	19.00	.05264	7.274	720.8	720.8	.5051	.5043	.9957	4.949
.5790	.5798	3.643	.9986	19.09	19.12	.05231	7.286	729.9	729.9	.5050	.5043	.9957	4.949
.5800	.5808	3.649	.9987	19.21	19.24	.05198	7.298	739.0	739.0	.5049	.5043	.9957	4.948
.5810	.5818	3.656	.9987	19.33	19.36	.05166	7.311	748.1	748.1	.5049	.5042	.9958	4.948
.5820	.5828	3.662	.9987	19.45	19.48	.05134	7.323	757.5	757.5	.5048	.5042	.9958	4.948
.5830	.5838	3.668	.9987	19.58	19.60	.05102	7.336	767.0	767.0	.5048	.5041	.9959	4.948
.5840	.5848	3.674	.9987	19.70	19.73	.05070	7.348	776.7	776.7	.5047	.5041	.9959	4.948
.5850	.5858	3.680	.9987	19.81	19.84	.05040	7.361	786.5	786.5	.5047	.5040	.9960	4.948
.5860	.5867	3.686	.9987	19.94	19.96	.05009	7.373	796.4	796.4	.5046	.5040	.9960	4.948
.5870	.5877	3.693	.9988	20.06	20.09	.04978	7.386	806.5	806.5	.5046	.5040	.9960	4.947
.5880	.5887	3.699	.9988	20.19	20.21	.04947	7.398	816.5	816.5	.5045	.5039	.9961	4.947
.5890	.5897	3.705	.9988	20.32	20.34	.04916	7.411	826.7	826.7	.5045	.5039	.9961	4.947
.5900	.5907	3.712	.9988	20.45	20.47	.04885	7.423	837.1	837.1	.5044	.5038	.9962	4.947
.5910	.5917	3.718	.9988	20.57	20.60	.04855	7.436	847.6	847.6	.5044	.5038	.9962	4.947
.5920	.5927	3.724	.9988	20.70	20.73	.04824	7.448	858.2	858.2	.5043	.5037	.9963	4.947
.5930	.5937	3.730	.9989	20.83	20.86	.04794	7.460	868.9	868.9	.5043	.5037	.9963	4.946
.5940	.5947	3.737	.9989	20.97	20.99	.04764	7.473	879.8	879.8	.5043	.5037	.9963	4.946
.5950	.5957	3.743	.9989	21.10	21.12	.04735	7.485	890.8	890.8	.5042	.5036	.9964	4.946
.5960	.5967	3.749	.9989	21.23	21.25	.04706	7.498	901.9	901.9	.5042	.5036	.9964	4.946
.5970	.5977	3.755	.9989	21.35	21.37	.04677	7.510	913.4	913.4	.5041	.5036	.9964	4.946
.5980	.5987	3.761	.9989	21.49	21.51	.04648	7.523	925.0	925.0	.5041	.5035	.9965	4.946
.5990	.5996	3.767	.9989	21.62	21.64	.04619	7.535	936.5	936.5	.5040	.5035	.9965	4.946
.6000	.6006	3.774	.9990	21.76	21.78	.04591	7.548	948.1	948.1	.5040	.5035	.9965	4.945
.6100	.6106	3.836	.9991	23.17	23.19	.04313	7.673	1,074	1,074	.5036	.5031	.9969	4.944
.6200	.6205	3.899	.9992	24.66	24.68	.04052	7.798	1,217	1,217	.5032	.5028	.9972	4.943
.6300	.6305	3.961	.9993	26.25	26.27	.03806	7.923	1,379	1,379	.5029	.5025	.9975	4.942
.6400	.6404	4.024	.9994	27.95	27.97	.03576	8.048	1,527	1,527	.5026	.5023	.9977	4.941
.6500	.6504	4.086	.9994	29.75	29.77	.03359	8.173	1,771	1,771	.5023	.5020	.9980	4.940
.6600	.6603	4.149	.9995	31.68	31.69	.03155	8.298	2,008	2,008	.5021	.5018	.9982	4.940
.6700	.6703	4.212	.9996	33.73	33.74	.02964	8.423	2,275	2,275	.5019	.5017	.9983	4.939
.6800	.6803	4.274	.9996	35.90	35.92	.02784	8.548	2,579	2,579	.5017	.5015	.9985	4.939
.6900	.6902	4.337	.9997	38.23	38.24	.02615	8.674	2,923	2,923	.5015	.5013	.9987	4.938
.7000	.7002	4.400	.9997	40.71	40.72	.02456	8.799	3,314	3,314	.5013	.5012	.9988	4.938
.7100	.7102	4.462	.9997	43.34	43.35	.02307	8.925	3,757	3,757	.5012	.5011	.9989	4.937
.7200	.7202	4.525	.9998	46.14	46.15	.02167	9.050	4,258	4,258	.5011	.5010	.9990	4.937
.7300	.7302	4.588	.9998	49.13	49.14	.02035	9.175	4,828	4.828	.5010	.5009	.9991	4.937
.7400	.7401	4.650	.9998	52.31	52.32	.01911	9.301	5,473	5,473	.5009	.5008	.9992	4.937
,7500	.7501	4.713	.9998	55.70	55.71	.01795	9.426	6,204	6,204	.5008	.5007	.9993	4.936
.7600	.7601	4.776	.9999	59.31	59.31	.01686	9.552	7,034	7,034	.5007	.5006	.9994	4.936
.7700	.7701	4.839	.9999	63.15	63.16	.01583	9.677	7,976	7,976	.5006	.5005	.9995	4.936
.7800	.7801	4.902	.9999	67.24	67.25	.01487	9.803	9,042	9,042	.5005	.5004	.9996	4.936
.7900	.7901	4.964	.9999	71.60	71.60	.01397	9.929	10,250	10,250	.5005	.5004	.9996	4.936
.8000	.8001	5.027	.9999	76.24	76.24	.01312	10.05	11,620	11,620	.5004	.5004	.9996	4.936
.8100	.8101	5.090	.9999	81.18	81.19	.01232	10.18	13,180	13,180	.5004	.5004	.9996	4.936
.8200	.8201	5.153	.9999	86.44	86.44	.01157	10.31	14,940	14,940	.5003	.5003	.9997	4.935
.8300	.8301	5.215	.9999	92.04	92.05	.01086	10.43	17,340	17,340	.5003	.5003	.9997	4.935
.8400	.8400	5.278	1.000	98.00	98.01	.01020	10.56	19,210	19,210	.5003	.5003	.9997	4.935
.8500	.8500	5.341	1.000	104.4	104.4	.009582	10.68	21,780	21,780	.5002	.5002	.9998	4.935
.8600	.8600	5.404	1.000	111.1	111.1	.009000	10.81	24,690	24,690	.5002	.5002	.9998	4.935
.8700	.8700	5.467	1.000	118.3	118.3	.008451	10.93	28,000	28,000	.5002	.5002	.9998	4.935
.8800	.8800	5.529	1.000	126.0	126.0	.007934	11.06	31,750	31,750	.5002	.5002	.9998	4.935
.8900	.8900	5.592	1.000	134.2	134.2	.007454	11.18	36,000	36,000	.5002	.5002	.9998	4.935

Table D-1 Cont'd

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.9000	.9000	5.655	1.000	142.9	142.9	.007000	11.31	40,810	40,810	.5001	.5001	.9999	4.935
.9100	.9100	5.718	1.000	152.1	152.1	.006574	11.44	46,280	46,280	.5001	.5001	.9999	4.935
.9200	.9200	5.781	1.000	162.0	162.0	.006173	11.56	52,470	52,470	.5001	.5001	.9999	4.935
.9300	.9300	5.844	1.000	172.5	172.5	.005797	11.69	59,500	59,500	.5001	.5001	.9999	4.935
.9400	.9400	5.906	1.000	183.7	183.7	.005445	11.81	67,470	67,470	.5001	.5001	.9999	4.935
.9500	.9500	5.969	1.000	195.6	195.6	.005113	11.94	76,490	76,490	.5001	.5001	.9999	4.935
.9600	.9600	6.032	1.000	203.5	203.5	.004914	12.06	86,740	86,740	.5001	.5001	.9999	4.935
.9700	.9700	6.095	1.000	222.8	222.8	.004489	12.19	98,350	98,350	.5001	.5001	.9999	4.935
.9800	.9800	6.158	1.000	236.1	236.1	.004235	12.32	111,500	111,500	.5001	.5001	.9999	4.935
.9900	.9900	6.220	1.000	251.4	251.4	.003977	12.44	126,500	126,500	.5000	.5000	1.000	4.935
1.000	1.000	6.283	1.000	267.7	267.7	.003735	12.57	143,400	143,400	.5000	.5000	1.000	4.935

TABLE D-2

FUNCTIONS OF d/L FOR EVEN INCREMENTS OF d/L

from 0.0001 to 0.2890

(This covers the region where interpolation of d/L in Table I is inconvenient. Values of d/L of 0.2890 to 1.0000 can be obtained from Table I by interpolation)

d/L	d/Lo	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
0	0	0	0	0	1.0000	1.000	0	0	1.000	1.000	0	∞	∞
.0001000	.0000000 6283	.0006283	.0006283	.0006283	1.0000	1.000	.001257	.001257	1.000	1.000	.0006283	28.21	12,500,000
.0002000	.000000 2514	.001257	.001257	.001257	1.0000	1.000	.002513	.002513	1.000	1.000	.001257	19.95	3,125,000
.0003000	.000000 5655	.001885	.001885	.001885	1.0000	1.000	.003770	.003770	1.000	1.000	.001885	16.29	1,389.000
.0004000	.00000 1005	.002513	.002513	.002513	1.0000	1.000	.005027	.005027	1.000	1.000	.002513	14.10	781,300
.0005000	.00000 1571	.003142	.003142	.003142	1.0000	1.000	.006283	.006283	1.000	1.000	.003142	12.62	500,000
.0006000	.00000 2262	.003770	.003770	.003770	1.0000	1.000	.007540	.007540	1.000	1.000	.003770	11.52	347,200
.0007000	.00000 3079	.004398	.004398	.004398	1.0000	1.000	.008796	.008797	1.000	1.000	.004398	10.66	255,100
.0008000	.00000 4022	.005027	.005027	.005027	1.0000	1.000	.01005	.01005	1.000	1.000	.005026	9.974	195,300
.0009000	.00000 5090	.005655	.005655	.005655	1.0000	1.000	.01131	.01131	1.000	1.000	.005655	9.403	154,300
.001000	.00000 6283	.006283	.006283	.006283	1.0000	1.000	.01257	.01257	1.000	1.000	.006283	8.921	125,000
.001100	.00000 7603	.006912	.006911	.006912	1.0000	1.000	.01382	.01382	1.000	1.000	.006911	8.506	103,300
.001200	.00000 9048	.007540	.007540	.007540	1.0000	1.000	.01508	.01508	1.000	1.000	.007540	8.144	86,810
.001300	.00001062	.008168	.008168	.008168	1.0000	1.000	.01634	.01634	1.000	1.000	.008168	7.824	73,970
.001400	.00001231	.008796	.008796	.008797	1.0000	1.000	.01759	.01759	1.000	1.000	.008796	7.539	63,780
.001500	.00001414	.009425	.009425	.009425	1.0000	1.000	.01885	.01885	1.000	1.000	.009424	7.284	55,560
.001600	.00001608	.01005	.01005	.01005	1.0001	.9999	.02011	.02011	1.000	1.000	.01005	7.052	48,830
.001700	.00001816	.01068	.01068	.01068	1.0001	.9999	.02136	.02136	1.000	1.000	.01068	6.842	43,260
.001800	.00002036	.01131	.01131	.01131	1.0001	.9999	.02262	.02262	1.000	1.000	.01131	6.649	38,580
.001900	.00002269	.01194	.01194	.01194	1.0001	.9999	.02388	.02388	1.000	1.000	.01194	6.472	34,630

Table D-2 Cont'd

d/L	d/Lo	2π d/L	TANH 2π d/L	SINH 2π d/L	COSH 2π d/L	K	4π d/L	SINH 4π d/L	COSH 4π d/L	n	C_G/C_o	H/H_o'	M
.002000	.00002514	.01257	.01257	.01257	1.0001	.9999	.02513	.02514	1.000	.9999	.01257	6.308	31,250
.002100	.00002772	.01319	.01319	.01320	1.0001	.9999	.02639	.02639	1.000	.9999	.01319	6.156	28,350
.002200	.00003040	.01382	.01382	.01382	1.0001	.9999	.02765	.02765	1.000	.9999	.01382	6.015	25,830
.002300	.00003324	.01445	.01445	.01445	1.0001	.9999	.02890	.02891	1,000	.9999	.01445	5.882	23,630
.002400	.00003619	.01508	.01508	.01508	1.0001	.9999	.03016	.03016	1.000	.9999	.01508	5.759	21,700
.002500	.00003928	.01571	.01571	.01571	1.0001	.9999	.03142	.03142	1.000	.9999	.01571	5.642	20,000
.002600	.00004248	.01634	.01633	.01634	1.0001	.9999	.03267	.03268	1.001	.9999	.01633	5.533	18,490
.002700	.00004579	.01696	.01696	.01697	1.0001	.9999	.03393	.03394	1.001	.9999	.01696	5.429	17,150
.002800	.00004925	.01759	.01759	.01759	1.0002	.9998	.03519	.03519	1.001	.9999	.01759	5.332	15,950
.002900	.00005284	.01822	.01822	.01822	1.0002	.9998	.03644	.03645	1.001	.9999	.01822	5.239	14,870
.003000	.00005652	.01885	.01885	.01885	1.0002	.9998	.03770	.03771	1.001	.9999	.01885	5.151	13,890
.003100	.00006039	.01948	.01948	.01948	1.0002	.9998	.03896	.03897	1.001	.9999	.01947	5.067	13,010
.003200	.00006435	.02011	.02010	.02011	1.0002	.9998	.04021	.04022	1.001	.9999	.02010	4.987	12,210
.003300	.00006841	.02073	.02073	.02073	1.0002	.9998	.04147	.04148	1.001	.9999	.02073	4.911	11,480
.003400	.00007262	.02136	.02136	.02136	1.0002	.9998	.04273	.04274	1.001	.9998	.02136	4.838	10,820
.003500	.00007697	.02199	.02199	.02199	1.0002	.9998	.04398	.04399	1.001	.9998	.02199	4.769	10,210
.003600	.00008140	.02262	.02262	.02262	1.0003	.9997	.04524	.04525	1.001	.9998	.02261	4.702	9,648
.003700	.00008599	.02325	.02324	.02325	1.0003	.9997	.04650	.04652	1.001	.9998	.02324	4.638	9,134
.003800	.00009071	.02388	.02387	.02388	1.0003	.9997	.04775	.04777	1.001	.9998	.02387	4.577	8,660
.033900	.00009551	.02450	.02450	.02451	1.0003	.9997	.04901	.04903	1.001	.9998	.02449	4.518	8,221
.004000	.0001005	.02513	.02513	.02513	1.0003	.9997	.05027	.05029	1.001	.9998	.02511	4.462	7,815
.004100	.0001056	.02576	.02576	.02576	1.0003	.9997	.05152	.05154	1.001	.9998	.02574	4.407	7,439
.004200	.0001108	.02639	.02638	.02639	1.0003	.9997	.05278	.05280	1.001	.9998	.02637	4.354	7,090
.004300	.0001161	.02702	.02701	.02702	1.0004	.9996	.05404	.05406	1.001	.9998	.02700	4.303	6,764
.004400	.0001216	.02765	.02764	.02765	1.0004	.9996	.05529	.05531	1.002	.9997	.02763	4.254	6,460
.004500	.0001272	.02827	.02827	.02828	1.0004	.9996	.05655	.05658	1.002	.9997	.02825	4.207	6,176
.004600	.0001329	.02890	.02889	.02890	1.0004	.9996	.05781	.05784	1.002	.9997	.02888	4.161	5,911
.004700	.0001387	.02953	.02952	.02953	1.0004	.9996	.05906	.05909	1.002	.9997	.02951	4.116	5,662
.004800	.0001447	.03016	.03015	.03016	1.0005	.9995	.06032	.06035	1.002	.9997	.03014	4.073	5,429
.004900	.0001508	.03079	.03078	.03079	1.0005	.9995	.06158	.06161	1.002	.9997	.03076	4.032	5,209
.005000	.0001570	.03142	.03141	.03143	1.0005	.9995	.06283	.06287	1.002	.9997	.03139	3.991	5,003
.005100	.0001634	.03204	.03203	.03205	1.0005	.9995	.06409	.06413	1.002	.9997	.03202	3.951	4,809
.005200	.0001698	.03267	.03266	.03268	1.0005	.9995	.06535	.06539	1.002	.9996	.03265	3.913	4,626
.005300	.0001764	.03330	.03329	.03331	1.0005	.9995	.06660	.06665	1.002	.9996	.03328	3.876	4,453
.005400	.0001832	.03393	.03392	.03394	1.0006	.9994	.06786	.06791	1.002	.9996	.03391	3.840	4,290
.005500	.0001900	.03456	.03455	.03457	1.0006	.9994	.06911	.06916	1.002	.9996	.03454	3.805	4,135
.005600	.0001970	.03519	.03517	.03520	1.0006	.9994	.07037	.07042	1.002	.9996	.03517	3.771	3,989
.005700	.0002041	.03581	.03580	.03582	1.0006	.9994	.07163	.07169	1.003	.9996	.03579	3.738	3,851
.005800	.0002112	.03644	.03642	.03645	1.0007	.9993	.07288	.07294	1.003	.9996	.03641	3.706	3,719
.005900	.0002186	.03707	.03705	.03708	1.0007	.9993	.07414	.07420	1.003	.9995	.03703	3.675	3,594
.006000	.0002261	.03770	.03768	.03771	1.0007	.9993	.07540	.07547	1.003	.9995	.03766	3.644	3,475
.006100	.0002337	.03833	.03831	.03834	1.0007	.9993	.07665	.07672	1.003	.9995	.03829	3.614	3,363
.006200	.0002414	.03896	.03894	.03897	1.0008	.9992	.07791	.07798	1.003	.9995	.03892	3.584	3,255
.006300	.0002492	.03958	.03956	.03959	1.0008	.9992	.07917	.07925	1.003	.9995	.03954	3.556	3,153
.006400	.0002570	.04021	.04019	.04022	1.0008	.9992	.08042	.08050	1.003	.9995	.04017	3.528	3,055
.006500	.0002653	.04084	.04082	.04085	1.0008	.9992	.08168	.08177	1.003	.9994	.04080	3.501	2,962
.006600	.0002735	.04147	.04144	.04148	1.0009	.9991	.08294	.08303	1.003	.9994	.04142	3.475	2,873
.006700	.0002819	.04210	.04207	.04211	1.0009	.9991	.08419	.08428	1.004	.9994	.04204	3.449	2,788
.006800	.0002904	.04273	.04270	.04274	1.0009	.9991	.08545	.08555	1.004	.9994	.04267	3.423	2,707
.006900	.0002990	.04335	.04333	.04336	1.0009	.9991	.08671	.08681	1.004	.9994	.04330	3.398	2,629
.007000	.0003077	.04398	.04395	.04399	1.0010	.9990	.08796	.08807	1.004	.9994	.04392	3.374	2,554
.007100	.0003165	.04461	.04458	.04462	1.0010	.9990	.08922	.08933	1.004	.9993	.04455	3.350	2,483
.007200	.0003254	.04524	.04521	.04525	1.0010	.9989	.09048	.09060	1.004	.9993	.04518	3.327	2,415
.007300	.0003346	.04587	.04584	.04589	1.0011	.9989	.09173	.09185	1.004	.9993	.04581	3.304	2,349
.007400	.0003439	.04650	.04646	.04652	1.0011	.9989	.09299	.09312	1.004	.9993	.04644	3.281	2,286
.007500	.0003532	.04712	.04709	.04714	1.0011	.9989	.09425	.09438	1.004	.9993	.04706	3.260	2,226
.007600	.0003627	.04775	.04772	.04777	1.0011	.9989	.09550	.09565	1.005	.9992	.04768	3.238	2,167
.007700	.0003722	.04838	.04834	.04840	1.0012	.9988	.09676	.09681	1.005	.9992	.04830	3.217	2,112
.007800	.0003820	.04901	.04897	.04903	1.0012	.9988	.09802	.09817	1.005	.9992	.04893	3.197	2,058
.007900	.0003918	.04964	.04960	.04966	1.0012	.9988	.09927	.09943	1.005	.9992	.04956	3.176	2,006

Table D-2 Cont'd

d/L	d/Lo	2π d/L	TANH 2π d/L	SINH 2π d/L	COSH 2π d/L	K	4π d/L	SINH 4π d/L	COSH 4π d/L	n	C_G/C_o	H/H'_o	M
.008000	.0004018	.05027	.05022	.05029	1.0013	.9987	.1005	.1007	1.005	.9992	.05018	3.157	1,956
.008100	.0004118	.05089	.05085	.05091	1.0013	.9987	.1018	.1020	1.005	.9991	.05080	3.137	1,909
.008200	.0004221	.05152	.05147	.05154	1.0013	.9987	.1030	.1032	1.005	.9991	.05142	3.118	1,862
.008300	.0004324	.05215	.05210	.05217	1.0014	.9986	.1043	.1045	1.005	.9991	.05205	3.099	1,818
.008400	.0004429	.05278	.05273	.05280	1.0014	.9986	.1056	.1058	1.006	.9991	.05268	3.081	1,775
.008500	.0004536	.05341	.05336	.05343	1.0014	.9986	.1068	.1070	1.006	.9991	.05331	3.062	1,733
.008600	.0004644	.05404	.05398	.05406	1.0015	.9985	.1081	.1083	1.006	.9990	.05394	3.044	1,693
.008700	.0004751	.05466	.05461	.05469	1.0015	.9985	.1093	.1095	1.006	.9990	.05456	3.027	1,655
.008800	.0004860	.05529	.05524	.05533	1.0015	.9985	.1106	.1108	1.006	.9990	.05518	3.010	1,617
.008900	.0004972	.05592	.05586	.05595	1.0016	.9984	.1118	.1121	1.006	.9990	.05580	2.993	1,581
.009000	.0005084	.05655	.05649	.05658	1.0016	.9984	.1131	.1133	1.006	.9989	.05643	2.977	1,546
.009100	.0005198	.05718	.05712	.05721	1.0016	.9984	.1144	.1146	1.006	.9989	.05706	2.960	1,513
.009200	.0005312	.05781	.05774	.05784	1.0017	.9983	.1156	.1158	1.007	.9989	.05768	2.944	1,480
.009300	.0005427	.05843	.05836	.05846	1.0017	.9983	.1169	.1171	1.007	.9989	.05830	2.929	1,449
.009400	.0005545	.05906	.05899	.05909	1.0017	.9983	.1181	.1184	1.007	.9988	.05892	2.913	1,418
.009500	.0005664	.05969	.05962	.05973	1.0018	.9982	.1194	.1196	1.007	.9988	.05955	2.898	1,388
.009600	.0005784	.06032	.06025	.06036	1.0018	.9982	.1206	.1209	1.007	.9988	.06018	2.882	1,360
.009700	.0005905	.06095	.06087	.06099	1.0019	.9981	.1219	.1222	1.007	.9988	.06080	2.867	1,332
.009800	.0006027	.06158	.06150	.06162	1.0019	.9981	.1232	.1235	1.008	.9987	.06142	2.853	1,305
.009900	.0006150	.06220	.06212	.06224	1.0019	.9981	.1244	.1247	1.008	.9987	.06204	2.839	1,279
.01000	.0006275	.06283	.06275	.06287	1.0020	.9980	.1257	.1260	1.0079	.9987	.06267	2.825	1,253
.01100	.0007591	.06912	.06901	.06917	1.0024	.9976	.1382	.1387	1.0096	.9984	.06890	2.694	1,036
.01200	.0009031	.07540	.07526	.07547	1.0028	.9972	.1508	.1513	1.0114	.9981	.07511	2.580	871.0
.01300	.001060	.08168	.08150	.08177	1.0033	.9967	.1634	.1641	1.0134	.9978	.08131	2.480	742.9
.01400	.001228	.08795	.08774	.08808	1.0039	.9961	.1759	.1768	1.0155	.9974	.08751	2.389	641.1
.01500	.001410	.09425	.09397	.09439	1.0044	.9956	.1885	.1896	1.0178	.9970	.09369	2.310	558.9
.01600	.001603	.1005	.1002	.1007	1.0051	.9949	.2011	.2024	1.0203	.9966	.09986	2.238	491.6
.01700	.001809	.1068	.1064	.1070	1.0057	.9943	.2136	.2153	1.0229	.9962	.1060	2.172	435.8
.01800	.002027	.1131	.1126	.1133	1.0064	.9936	.2262	.2281	1.0257	.9958	.1121	2.112	389.1
.01900	.002258	.1194	.1188	.1197	1.0071	.9929	.2388	.2410	1.0286	.9953	.1183	2.056	349.5
.02000	.002500	.1257	.1250	.1260	1.008	.9922	.2513	.2540	1.032	.9947	.1244	2.005	315.8
.02100	.002755	.1320	.1312	.1323	1.009	.9914	.2639	.2669	1.035	.9942	.1305	1.958	286.8
.02200	.003022	.1382	.1374	.1387	1.010	.9905	.2765	.2800	1.038	.9937	.1365	1.915	261.5
.02300	.003301	.1445	.1435	.1450	1.011	.9896	.2890	.2931	1.042	.9931	.1425	1.873	239.6
.02400	.003592	.1508	.1497	.1514	1.011	.9887	.3016	.3062	1.046	.9925	.1485	1.834	220.3
.02500	.003895	.1571	.1558	.1577	1.012	.9878	.3142	.3194	1.050	.9919	.1545	1.799	203.3
.02600	.004210	.1634	.1619	.1641	1.013	.9868	.3267	.3326	1.054	.9912	.1605	1.765	188.2
.02700	.004537	.1697	.1680	.1705	1.014	.9858	.3393	.3458	1.058	.9905	.1665	1.733	174.8
.02800	.004876	.1759	.1741	.1768	1.016	.9847	.3519	.3592	1.063	.9898	.1724	1.703	162.7
.02900	.005226	.1822	.1802	.1832	1.017	.9836	.3644	.3725	1.067	.9891	.1783	1.675	151.9
.03000	.005589	.1885	.1863	.1896	1.018	.9825	.3770	.3860	1.072	.9884	.1841	1.648	142.2
.03100	.005963	.1948	.1924	.1960	1.019	.9813	.3896	.3995	1.077	.9876	.1900	1.622	133.4
.03200	.006347	.2011	.1984	.2024	1.020	.9801	.4021	.4131	1.082	.9868	.1958	1.598	125.4
.03300	.006746	.2073	.2044	.2088	1.022	.9789	.4147	.4267	1.087	.9860	.2016	1.575	118.1
.03400	.007155	.2136	.2104	.2153	1.023	.9776	.4273	.4404	1.093	.9851	.2073	1.553	111.4
.03500	.007575	.2199	.2164	.2217	1.024	.9763	.4398	.4541	1.098	.9843	.2130	1.532	105.3
.03600	.008007	.2262	.2224	.2281	1.026	.9749	.4524	.4680	1.104	.9834	.2187	1.512	99.75
.03700	.008450	.2325	.2284	.2346	1.027	.9736	.4650	.4819	1.110	.9824	.2244	1.493	94.61
.03800	.008905	.2388	.2343	.2410	1.029	.9722	.4775	.4959	1.116	.9815	.2300	1.475	89.88
.03900	.009370	.2450	.2403	.2527	1.030	.9708	.4901	.5099	1.123	.9805	.2356	1.457	85.50
.04000	.009847	.2513	.2462	.2540	1.032	.9693	.5027	.5241	1.129	.9795	.2411	1.440	81.43
.04100	.01033	.2576	.2521	.2605	1.033	.9677	.5152	.5383	1.136	.9785	.2467	1.424	77.67
.04200	.01083	.2639	.2579	.2670	1.035	.9662	.5278	.5526	1.143	.9775	.2521	1.408	74.17
.04300	.01134	.2702	.2638	.2735	1.037	.9646	.5404	.5670	1.150	.9765	.2576	1.393	70.91
.04400	.01186	.2765	.2696	.2800	1.039	.9630	.5529	.5815	1.157	.9754	.2630	1.379	67.88
.04500	.01239	.2827	.2754	.2865	1.040	.9613	.5655	.5961	1.164	.9743	.2684	1.365	65.05
.04600	.01294	.2890	.2812	.2931	1.042	.9596	.5781	.6108	1.172	.9732	.2737	1.352	62.39
.04700	.01349	.2953	.2870	.2996	1.044	.9579	.5906	.6256	1.180	.9721	.2790	1.339	59.91
.04800	.01405	.3016	.2928	.3062	1.046	.9562	.6032	.6404	1.188	.9709	.2843	1.326	57.57
.04900	.01463	.3079	.2985	.3128	1.048	.9544	.6158	.6554	1.196	.9697	.2895	1.314	55.38

Table D-2 Cont'd

d/L	d/Lo	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.05000	.01521	.3142	.3042	.3194	1.050	.9526	.6283	.6705	1.204	.9685	.2947	1.303	53.32
.05100	.01580	.3204	.3099	.3260	1.052	.9508	.6409	.6857	1.213	.9673	.2998	1.291	51.38
.05200	.01641	.3267	.3156	.3326	1.054	.9489	.6535	.7010	1.221	.9661	.3049	1.281	49.55
.05300	.01702	.3330	.3212	.3392	1.056	.9470	.6660	.7164	1.230	.9649	.3099	1.270	47.82
.05400	.01765	.3393	.3269	.3458	1.058	.9451	.6786	.7319	1.239	.9636	.3149	1.260	46.19
.05500	.01829	.3456	.3325	.3525	1.060	.9431	.6912	.7475	1.249	.9623	.3199	1.250	44.65
.05600	.01893	.3519	.3380	.3592	1.063	.9411	.7037	.7633	1.258	.9610	.3248	1.241	43.19
.05700	.01958	.3581	.3436	.3658	1.065	.9391	.7163	.7791	1.268	.9597	.3297	1.231	41.80
.05800	.02025	.3644	.3491	.3726	1.067	.9371	.7289	.7951	1.278	.9583	.3346	1.222	40.49
.05900	.02092	.3707	.3546	.3793	1.070	.9350	.7414	.8112	1.288	.9570	.3394	1.214	39.24
.06000	.02161	.3770	.3601	.3860	1.072	.9329	.7540	.8275	1.298	.9556	.3441	1.205	38.06
.06100	.02230	.3833	.3656	.3927	1.074	.9308	.7666	.8439	1.308	.9542	.3488	1.197	36.93
.06200	.02300	.3896	.3710	.3995	1.077	.9286	.7791	.8604	1.319	.9528	.3534	1.189	35.86
.06300	.02371	.3958	.3764	.4062	1.079	.9265	.7917	.8770	1.330	.9514	.3581	1.182	34.83
.06400	.02444	.4021	.3818	.4130	1.082	.9243	.8043	.8938	1.341	.9499	.3626	1.174	33.86
.06500	.02516	.4084	.3871	.4199	1.085	.9220	.8168	.9107	1.353	.9484	.3672	1.167	32.93
.06600	.02590	.4147	.3925	.4267	1.087	.9198	.8294	.9278	1.364	.9470	.3716	1.160	32.04
.06700	.02665	.4210	.3978	.4335	1.090	.9175	.8419	.9450	1.376	.9455	.3761	1.153	31.19
.06800	.02739	.4273	.4030	.4404	1.093	.9152	.8545	.9624	1.388	.9440	.3804	1.147	30.38
.06900	.02817	.4335	.4083	.4473	1.095	.9128	.8671	.9799	1.400	.9424	.3848	1.140	29.61
.07000	.02895	.4398	.4135	.4541	1.098	.9105	.8796	.9976	1.412	.9409	.3891	1.134	28.86
.07100	.02973	.4461	.4187	.4611	1.101	.9081	.8922	1.015	1.425	.9393	.3933	1.128	28.15
.07200	.03052	.4524	.4239	.4680	1.104	.9057	.9048	1.033	1.438	.9378	.3975	1.122	27.47
.07300	.03132	.4587	.4290	.4749	1.107	.9033	.9173	1.052	1.451	.9362	.4016	1.116	26.81
.07400	.03213	.4650	.4341	.4819	1.110	.9008	.9299	1.070	1.464	.9346	.4057	1.110	26.18
.07500	.03294	.4712	.4392	.4889	1.113	.8984	.9425	1.088	1.478	.9330	.4098	1.105	25.58
.07600	.03377	.4775	.4443	.4958	1.116	.8959	.9551	1.107	1.492	.9314	.4138	1.099	25.00
.07700	.03460	.4838	.4493	.5029	1.119	.8934	.9676	1.126	1.506	.9298	.4177	1.094	24.45
.07800	.03543	.4901	.4542	.5100	1.123	.8909	.9802	1.145	1.520	.9281	.4216	1.089	23.92
.07900	.03628	.4964	.4593	.5170	1.126	.8883	.9927	1.164	1.534	.9264	.4255	1.084	23.40
.08000	.03714	.5027	.4642	.5241	1.129	.8857	1.005	1.183	1.549	.9248	.4293	1.079	22.90
.08100	.03799	.5089	.4691	.5312	1.132	.8831	1.018	1.203	1.564	.9231	.4330	1.075	22.42
.08200	.03887	.5152	.4740	.5383	1.136	.8805	1.030	1.223	1.580	.9214	.4367	1.070	21.96
.08300	.03975	.5215	.4789	.5455	1.139	.8779	1.043	1.243	1.595	.9197	.4404	1.066	21.52
.08400	.04063	.5278	.4837	.5526	1.143	.8752	1.056	1.263	1.611	.9179	.4440	1.061	21.09
.08500	.04152	.5341	.4885	.5598	1.146	.8726	1.068	1.283	1.627	.9162	.4476	1.057	20.68
.08600	.04242	.5404	.4933	.5670	1.150	.8699	1.081	1.304	1.643	.9145	.4511	1.053	20.28
.08700	.04333	.5466	.4980	.5743	1.153	.8672	1.093	1.324	1.660	.9127	.4545	1.049	19.90
.08800	.04424	.5529	.5027	.5815	1.157	.8645	1.106	1.346	1.676	.9109	.4579	1.045	19.53
.08900	.04516	.5592	.5074	.5888	1.160	.8617	1.118	1.367	1.693	.9092	.4613	1.041	19.17
.09000	.04608	.5655	.5120	.5961	1.164	.8590	1.131	1.388	1.711	.9074	.4646	1.037	18.82
.09100	.04702	.5718	.5167	.6034	1.168	.8562	1.144	1.410	1.728	.9056	.4679	1.034	18.49
.09200	.04796	.5781	.5213	.6108	1.172	.8534	1.156	1.431	1.746	.9038	.4711	1.030	18.16
.09300	.04890	.5843	.5258	.6182	1.176	.8506	1.169	1.453	1.764	.9020	.4743	1.027	17.85
.09400	.04985	.5906	.5303	.6256	1.180	.8478	1.181	1.476	1.783	.9002	.4774	1.023	17.55
.09500	.05081	.5969	.5348	.6330	1.184	.8450	1.194	1.498	1.801	.8984	.4805	1.020	17.26
.09600	.05177	.6032	.5393	.6404	1.188	.8421	1.206	1.521	1.820	.8966	.4835	1.017	16.97
.09700	.05275	.6095	.5438	.6479	1.192	.8392	1.219	1.544	1.840	.8947	.4865	1.014	16.69
.09800	.05372	.6158	.5482	.6554	1.196	.8364	1.232	1.567	1.859	.8929	.4894	1.011	16.42
.09900	.05470	.6220	.5526	.6629	1.200	.8335	1.244	1.591	1.879	.8910	.4923	1.008	16.16
.1000	.05569	.6283	.5569	.6705	1.204	.8306	1.257	1.615	1.899	.8892	.4952	1.005	15.91
.1010	.05668	.6346	.5612	.6781	1.208	.8277	1.269	1.638	1.920	.8873	.4980	1.002	15.67
.1020	.05768	.6409	.5655	.6857	1.213	.8247	1.282	1.663	1.940	.8854	.5007	.9993	15.43
.1030	.05869	.6472	.5698	.6933	1.217	.8218	1.294	1.687	1.961	.8836	.5034	.9966	15.20
.1040	.05970	.6535	.5740	.7010	1.221	.8189	1.307	1.712	1.983	.8817	.5061	.9940	14.98
.1050	.06071	.6597	.5782	.7087	1.226	.8159	1.319	1.737	2.004	.8798	.5087	.9914	14.76
.1060	.06173	.6660	.5824	.7164	1.230	.8129	1.332	1.762	2.026	.8779	.5113	.9891	14.55
.1070	.06276	.6723	.5865	.7241	1.235	.8100	1.345	1.788	2.049	.8760	.5138	.9865	14.35
.1080	.06378	.6786	.5906	.7319	1.239	.8070	1.357	1.814	2.071	.8741	.5163	.9841	14.15
.1090	.06482	.6849	.5947	.7397	1.244	.8040	1.370	1.840	2.094	.8722	.5187	.9818	13.95

Table D-2 Cont'd

d/L	d/Lo	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H'_o	M
.1100	.06586	.6912	.5987	.7475	1.249	.8010	1.382	1.867	2.118	.8703	.5211	.9797	13.77
.1110	.06690	.6974	.6027	.7554	1.253	.7980	1.395	1.893	2.141	.8684	.5234	.9775	13.58
.1120	.06795	.7037	.6067	.7633	1.258	.7949	1.407	1.920	2.165	.8665	.5257	.9753	13.41
.1130	.06901	.7100	.6107	.7712	1.263	.7919	1.420	1.948	2.189	.8645	.5279	.9731	13.23
.1140	.07006	.7163	.6146	.7791	1.268	.7888	1.433	1.975	2.214	.8626	.5301	.9711	13.06
.1150	.07113	.7226	.6185	.7871	1.273	.7858	1.445	2.003	2.239	.8607	.5323	.9691	12.90
.1160	.07220	.7289	.6224	.7951	1.278	.7827	1.458	2.032	2.264	.8587	.5344	.9672	12.74
.1170	.07327	.7351	.6262	.8032	1.283	.7797	1.470	2.060	2.290	.8568	.5365	.9654	12.59
.1180	.07434	.7414	.6300	.8112	1.288	.7766	1.483	2.089	2.316	.8549	.5386	.9635	12.43
.1190	.07542	.7477	.6338	.8193	1.293	.7735	1.495	2.118	2.343	.8529	.5406	.9617	12.29
.1200	.07650	.7540	.6375	.8275	1.298	.7704	1.508	2.148	2.369	.8510	.5425	.9600	12.14
.1210	.07759	.7603	.6412	.8357	1.303	.7673	1.521	2.178	2.397	.8491	.5444	.9583	12.00
.1220	.07868	.7666	.6449	.8439	1.309	.7642	1.533	2.208	2.424	.8471	.5463	.9567	11.87
.1230	.07978	.7728	.6486	.8521	1.314	.7612	1.546	2.239	2.452	.8452	.5482	.9551	11.73
.1240	.08085	.7791	.6520	.8604	1.319	.7581	1.558	2.270	2.480	.8432	.5500	.9535	11.61
.1250	.08198	.7854	.6558	.8687	1.325	.7549	1.571	2.301	2.509	.8413	.5517	.9520	11.48
.1260	.08308	.7917	.6594	.8770	1.330	.7518	1.583	2.333	2.538	.8393	.5534	.9505	11.35
.1270	.08419	.7980	.6629	.8854	1.336	.7487	1.596	2.365	2.568	.8374	.5551	.9490	11.23
.1280	.08530	.8043	.6664	.8938	1.341	.7456	1.609	2.398	2.598	.8354	.5568	.9476	11.11
.1290	.08642	.8105	.6699	.9022	1.347	.7424	1.621	2.430	2.628	.8335	.5584	.9463	11.00
.1300	.08753	.8168	.6733	.9107	1.353	.7393	1.634	2.464	2.659	.8316	.5599	.9450	10.89
.1310	.08866	.8231	.6768	.9192	1.358	.7362	1.646	2.497	2.690	.8296	.5614	.9437	10.78
.1320	.08978	.8294	.6801	.9278	1.364	.7331	1.659	2.531	2.722	.8277	.5629	.9424	10.67
.1330	.09091	.8357	.6835	.9364	1.370	.7299	1.671	2.566	2.754	.8257	.5644	.9412	10.56
.1340	.09204	.8420	.6868	.9450	1.376	.7268	1.684	2.600	2.786	.8238	.5658	.9401	10.46
.1350	.09317	.8482	.6902	.9537	1.382	.7237	1.696	2.636	2.819	.8218	.5672	.9389	10.36
.1360	.09431	.8545	.6934	.9624	1.388	.7205	1.709	2.671	2.852	.8199	.5685	.9378	10.26
.1370	.09544	.8608	.6967	.9711	1.394	.7174	1.722	2.707	2.886	.8179	.5698	.9367	10.17
.1380	.09659	.8671	.6999	.9799	1.400	.7142	1.734	2.744	2.920	.8160	.5711	.9357	10.07
.1390	.09773	.8734	.7031	.9887	1.406	.7111	1.747	2.781	2.955	.8141	.5724	.9347	9.983
.1400	.09888	.8797	.7063	.9976	1.412	.7080	1.759	2.818	2.990	.8121	.5736	.9337	9.894
.1410	.1000	.8859	.7094	1.006	1.419	.7048	1.772	2.856	3.026	.8102	.5748	.9327	9.806
.1420	.1012	.8922	.7125	1.015	1.425	.7017	1.784	2.894	3.062	.8083	.5759	.9318	9.721
.1430	.1023	.8985	.7156	1.024	1.432	.6985	1.797	2.933	3.099	.8064	.5770	.9309	9.638
.1440	.1035	.9048	.7186	1.033	1.438	.6954	1.810	2.972	3.136	.8044	.5781	.9300	9.556
.1450	.1046	.9111	.7216	1.042	1.445	.6923	1.822	3.012	3.173	.8025	.5791	.9292	9.476
.1460	.1058	.9174	.7247	1.052	1.451	.6891	1.835	3.052	3.211	.8006	.5801	.9284	9.398
.1470	.1070	.9236	.7276	1.061	1.458	.6860	1.847	3.092	3.250	.7987	.5811	.9276	9.321
.1480	.1081	.9299	.7306	1.070	1.464	.6829	1.860	3.133	3.289	.7968	.5821	.9268	9.246
.1490	.1093	.9362	.7335	1.079	1.471	.6797	1.872	3.175	3.329	.7949	.5830	.9261	9.173
.1500	.1105	.9425	.7364	1.088	1.478	.6766	1.885	3.217	3.369	.7930	.5839	.9254	9.101
.1510	.1116	.9488	.7392	1.098	1.485	.6734	1.898	3.260	3.410	.7911	.5848	.9247	9.031
.1520	.1128	.9551	.7421	1.107	1.492	.6703	1.910	3.303	3.451	.7892	.5856	.9240	8.962
.1530	.1140	.9613	.7449	1.116	1.499	.6672	1.923	3.346	3.493	.7873	.5864	.9234	8.894
.1540	.1151	.9676	.7477	1.126	1.506	.6641	1.935	3.391	3.535	.7854	.5872	.9228	8.828
.1550	.1163	.9739	.7504	1.135	1.513	.6610	1.948	3.435	3.578	.7835	.5880	.9222	8.763
.1560	.1175	.9802	.7531	1.145	1.520	.6579	1.960	3.481	3.621	.7816	.5887	.9216	8.700
.1570	.1187	.9865	.7558	1.154	1.527	.6547	1.973	3.526	3.665	.7797	.5893	.9211	8.638
.1580	.1199	.9928	.7585	1.164	1.535	.6516	1.985	3.573	3.710	.7779	.5900	.9205	8.577
.1590	.1210	.9990	.7612	1.174	1.542	.6485	1.998	3.620	3.755	.7760	.5907	.9200	8.517
.1600	.1222	1.005	.7638	1.183	1.549	.6454	2.011	3.667	3.801	.7741	.5913	.9196	8.459
.1610	.1234	1.012	.7664	1.193	1.557	.6423	2.023	3.715	3.847	.7723	.5919	.9191	8.401
.1620	.1246	1.018	.7690	1.203	1.564	.6392	2.036	3.764	3.894	.7704	.5925	.9186	8.345
.1630	.1258	1.024	.7716	1.213	1.572	.6361	2.048	3.813	3.942	.7686	.5930	.9182	8.290
.1640	.1270	1.030	.7741	1.223	1.580	.6331	2.061	3.863	3.990	.7667	.5935	.9179	8.236
.1650	.1281	1.037	.7766	1.233	1.587	.6300	2.073	3.913	4.039	.7649	.5940	.9175	8.183
.1660	.1293	1.043	.7791	1.243	1.595	.6269	2.086	3.964	4.088	.7631	.5945	.9171	8.131
.1670	.1305	1.049	.7815	1.253	1.603	.6239	2.099	4.016	4.138	.7613	.5950	.9167	8.079
.1680	.1317	1.056	.7840	1.263	1.611	.6208	2.111	4.068	4.189	.7595	.5954	.9164	8.029
.1690	.1329	1.062	.7864	1.273	1.619	.6177	2.124	4.121	4.241	.7576	.5958	.9161	7.980

Table D-2 Cont'd

d/L	d/Lo	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.1700	.1341	1.068	.7887	1.283	1.627	.6147	2.136	4.175	4.293	.7558	.5962	.9158	7.932
.1710	.1353	1.074	.7911	1.293	1.635	.6117	2.149	4.229	4.346	.7540	.5965	.9155	7.885
.1720	.1365	1.081	.7935	1.304	1.643	.6086	2.161	4.284	4.399	.7523	.5969	.9153	7.838
.1730	.1377	1.087	.7958	1.314	1.651	.6056	2.174	4.340	4.454	.7505	.5972	.9150	7.793
.1740	.1389	1.093	.7981	1.325	1.660	.6026	2.187	4.396	4.508	.7487	.5975	.9148	7.748
.1750	.1401	1.100	.8004	1.335	1.668	.5995	2.199	4.453	4.564	.7469	.5978	.9146	7.704
.1760	.1413	1.106	.8026	1.345	1.676	.5965	2.212	4.511	4.620	.7451	.5980	.9144	7.661
.1770	.1425	1.112	.8048	1.356	1.685	.5935	2.224	4.569	4.677	.7434	.5983	.9142	7.619
.1780	.1437	1.118	.8070	1.367	1.693	.5905	2.237	4.628	4.735	.7416	.5985	.9140	7.577
.1790	.1449	1.125	.8092	1.377	1.702	.5875	2.249	4.688	4.793	.7399	.5987	.9138	7.536
.1800	.1460	1.131	.8114	1.388	1.711	.5845	2.262	4.749	4.853	.7382	.5989	.9137	7.496
.1810	.1472	1.137	.8135	1.399	1.720	.5816	2.275	4.810	4.918	.7364	.5991	.9136	7.457
.1820	.1484	1.144	.8156	1.410	1.728	.5786	2.287	4.872	4.974	.7347	.5992	.9135	7.419
.1830	.1496	1.150	.8177	1.420	1.737	.5757	2.300	4.935	5.035	.7330	.5993	.9134	7.381
.1840	.1508	1.156	.8198	1.431	1.746	.5727	2.312	4.999	5.098	.7313	.5995	.9133	7.343
.1850	.1520	1.162	.8218	1.442	1.755	.5697	2.325	5.063	5.161	.7296	.5996	.9132	7.307
.1860	.1532	1.169	.8239	1.454	1.764	.5668	2.337	5.129	5.225	.7279	.5997	.9131	7.271
.1870	.1544	1.175	.8259	1.465	1.773	.5639	2.350	5.195	5.290	.7262	.5997	.9131	7.235
.1880	.1556	1.181	.8278	1.476	1.783	.5610	2.362	5.262	5.356	.7245	.5998	.9131	7.201
.1890	.1568	1.188	.8298	1.487	1.792	.5581	2.375	5.329	5.422	.7228	.5998	.9130	7.167
.1900	.1580	1.194	.8318	1.498	1.801	.5551	2.388	5.398	5.490	.7212	.5998	.9130	7.133
.1910	.1592	1.200	.8337	1.510	1.811	.5522	2.400	5.467	5.558	.7195	.5998	.9130	7.100
.1920	.1604	1.206	.8356	1.521	1.820	.5493	2.413	5.538	5.625	.7179	.5998	.9130	7.068
.1930	.1616	1.213	.8375	1.533	1.830	.5465	2.425	5.609	5.697	.7162	.5998	.9130	7.036
.1940	.1628	1.219	.8393	1.544	1.840	.5436	2.438	5.681	5.768	.7146	.5998	.9131	7.005
.1950	.1640	1.225	.8412	1.556	1.849	.5408	2.450	5.754	5.840	.7129	.5997	.9131	6.974
.1960	.1652	1.232	.8430	1.567	1.859	.5379	2.463	5.827	5.913	.7113	.5997	.9131	6.944
.1970	.1664	1.238	.8448	1.579	1.869	.5350	2.476	5.902	5.988	.7097	.5996	.9132	6.914
.1980	.1676	1.244	.8466	1.591	1.879	.5322	2.488	5.978	6.061	.7081	.5995	.9133	6.885
.1990	.1688	1.250	.8484	1.603	1.889	.5294	2.501	6.055	6.137	.7065	.5994	.9133	6.856
.2000	.1700	1.257	.8501	1.614	1.899	.5266	2.513	6.132	6.213	.7049	.5993	.9134	6.828
.2010	.1712	1.263	.8519	1.626	1.909	.5238	2.526	6.211	6.291	.7033	.5992	.9135	6.801
.2020	.1724	1.269	.8535	1.638	1.920	.5210	2.538	6.290	6.369	.7018	.5990	.9137	6.774
.2030	.1736	1.276	.8552	1.651	1.930	.5182	2.551	6.371	6.449	.7002	.5988	.9138	6.747
.2040	.1748	1.282	.8570	1.663	1.940	.5154	2.564	6.452	6.529	.6987	.5987	.9139	6.720
.2050	.1760	1.288	.8586	1.675	1.951	.5127	2.576	6.535	6.611	.6971	.5986	.9140	6.694
.2060	.1772	1.294	.8602	1.687	1.961	.5099	2.589	6.619	6.694	.6956	.5984	.9141	6.669
.2070	.1784	1.301	.8619	1.700	1.972	.5071	2.601	6.703	6.777	.6941	.5982	.9142	6.644
.2080	.1796	1.307	.8635	1.712	1.983	.5044	2.614	6.789	6.862	.6925	.5980	.9144	6.619
.2090	.1808	1.313	.8651	1.725	1.994	.5016	2.626	6.876	6.948	.6910	.5978	.9146	6.594
.2100	.1820	1.320	.8667	1.737	2.004	.4989	2.639	6.963	7.035	.6895	.5976	.9147	6.570
.2110	.1832	1.326	.8682	1.750	2.015	.4962	2.652	7.052	7.123	.6880	.5973	.9149	6.547
.2120	.7844	1.332	.8697	1.762	2.026	.4935	2.664	7.143	7.219	.6865	.5971	.9151	6.524
.2130	.1856	1.338	.8713	1.775	2.037	.4908	2.677	7.234	7.302	.6850	.5969	.9153	6.501
.2140	.1868	1.345	.8728	1.788	2.049	.4881	2.689	7.326	7.394	.6835	.5966	.9155	6.479
.2150	.1880	1.351	.8743	1.801	2.060	.4854	2.702	7.420	7.487	.6821	.5963	.9157	6.457
.2160	.1892	1.357	.8757	1.814	2.071	.4828	2.714	7.514	7.580	.6806	.5960	.9159	6.435
.2170	.1904	1.364	.8772	1.827	2.083	.4801	2.727	7.610	7.675	.6792	.5958	.9161	6.413
.2180	.1915	1.370	.8786	1.840	2.094	.4775	2.739	7.707	7.772	.6777	.5955	.9164	6.393
.2190	.1927	1.376	.8801	1.853	2.106	.4749	2.752	7.805	7.869	.6763	.5952	.9166	6.372
.2200	.1939	1.382	.8815	1.867	2.118	.4722	2.765	7.905	7.968	.6749	.5949	.9168	6.351
.2210	.1951	1.389	.8829	1.880	2.129	.4696	2.777	8.006	8.068	.6735	.5946	.9170	6.331
.2220	.1963	1.395	.8842	1.893	2.141	.4670	2.790	8.108	8.169	.6720	.5943	.9173	6.312
.2230	.1975	1.401	.8856	1.907	2.153	.4644	2.802	8.211	8.272	.6706	.5939	.9175	6.292
.2240	.1987	1.407	.8869	1.920	2.165	.4619	2.815	8.316	8.375	.6692	.5936	.9178	6.273
.2250	.1999	1.414	.8883	1.934	2.177	.4593	2.827	8.422	8.481	.6679	.5933	.9181	6.254
.2260	.2011	1.420	.8896	1.948	2.189	.4567	2.840	8.529	8.587	.6665	.5929	.9183	6.236
.2270	.2022	1.426	.8909	1.962	2.202	.4542	2.853	8.637	8.695	.6651	.5925	.9186	6.218
.2280	.2034	1.433	.8922	1.975	2.214	.4516	2.865	8.756	8.800	.6637	.5921	.9189	6.200
.2290	.2046	1.439	.8935	1.989	2.227	.4491	2.878	8.859	8.915	.6624	.5918	.9191	6.182

Table D-2 Cont'd

d/L	d/Lo	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	H/H_o'	M
.2300	.2058	1.445	.8947	2.003	2.239	.4466	2.890	8.971	9.027	.6611	.5915	.9194	6.165
.2310	.2070	1.451	.8960	2.017	2.252	.4441	2.903	9.085	9.140	.6597	.5911	.9197	6.148
.2320	.2082	1.458	.8972	2.032	2.264	.4416	2.915	9.201	9.255	.6584	.5907	.9200	6.131
.2330	.2093	1.464	.8984	2.046	2.277	.4391	2.928	9.318	9.372	.6571	.5904	.9203	6.114
.2340	.2105	1.470	.8996	2.060	2.290	.4366	2.941	9.437	9.489	.6558	.5900	.9206	6.097
.2350	.2117	1.477	.9008	2.075	2.303	.4342	2.953	9.557	9.609	.6545	.5896	.9209	6.081
.2360	.2129	1.483	.9020	2.089	2.316	.4318	2.966	9.678	9.730	.6532	.5892	.9212	6.066
.2370	.2141	1.489	.9032	2.104	2.329	.4293	2.978	9.801	9.852	.6519	.5888	.9215	6.050
.2380	.2152	1.495	.9043	2.118	2.343	.4269	2.991	9.926	9.976	.6507	.5884	.9218	6.034
.2390	.2164	1.502	.9055	2.133	2.356	.4244	3.003	10.05	10.10	.6494	.5880	.9221	6.019
.2400	.2176	1.508	.9066	2.148	2.370	.4220	3.016	10.18	10.23	.6481	.5876	.9225	6.004
.2410	.2188	1.514	.9077	2.163	2.383	.4196	3.029	10.31	10.36	.6469	.5872	.9228	5.990
.2420	.2199	1.521	.9088	2.178	2.397	.4172	3.041	10.44	10.49	.6456	.5868	.9231	5.976
.2430	.2211	1.527	.9099	2.193	2.410	.4149	3.054	10.57	10.62	.6444	.5863	.9234	5.961
.2440	.2223	1.533	.9110	2.208	2.424	.4125	3.066	10.71	10.75	.6432	.5859	.9238	5.947
.2450	.2234	1.539	.9120	2.224	2.438	.4101	3.079	10.84	10.89	.6420	.5855	.9241	5.933
.2460	.2246	1.546	.9131	2.239	2.452	.4078	3.091	10.98	11.03	.6408	.5851	.9244	5.919
.2470	.2258	1.552	.9141	2.255	2.466	.4055	3.104	11.12	11.17	.6396	.5846	.9248	5.906
.2480	.2270	1.558	.9151	2.270	2.480	.4032	3.116	11.26	11.31	.6384	.5842	.9251	5.893
.2490	.2281	1.565	.9162	2.286	2.495	.4008	3.129	11.40	11.45	.6372	.5838	.9255	5.880
.2500	.2293	1.571	.9172	2.301	2.509	.3985	3.142	11.55	11.59	.6360	.5833	.9258	5.867
.2510	.2305	1.577	.9182	2.317	2.524	.3962	3.154	11.70	11.74	.6348	.5829	.9262	5.854
.2520	.2316	1.583	.9191	2.333	2.538	.3940	3.167	11.84	11.89	.6337	.5824	.9265	5.841
.2530	.2328	1.590	.9201	2.349	2.553	.3917	3.179	11.99	12.04	.6325	.5820	.9269	5.829
.2540	.2339	1.596	.9210	2.365	2.568	.3894	3.192	12.15	12.19	.6314	.5815	.9273	5.817
.2550	.2351	1.602	.9220	2.381	2.583	.3872	3.204	12.30	12.34	.6303	.5811	.9276	5.805
.2560	.2363	1.609	.9229	2.398	2.598	.3849	3.217	12.46	12.50	.6291	.5807	.9280	5.793
.2570	.2374	1.615	.9239	2.414	2.613	.3827	3.230	12.61	12.65	.6280	.5802	.9283	5.782
.2580	.2386	1.621	.9248	2.430	2.628	.3805	3.242	12.77	12.81	.6269	.5797	.9287	5.770
.2590	.2398	1.627	.9257	2.447	2.643	.3783	3.255	12.94	12.98	.6258	.5793	.9291	5.759
.2600	.2409	1.634	.9266	2.464	2.659	.3761	3.267	13.10	13.14	.6247	.5788	.9294	5.748
.2610	.2421	1.640	.9275	2.480	2.674	.3739	3.280	13.27	13.31	.6236	.5784	.9298	5.737
.2620	.2432	1.646	.9283	2.497	2.690	.3717	3.292	13.44	13.47	.6225	.5779	.9301	5.726
.2630	.2444	1.653	.9292	2.514	2.706	.3696	3.305	13.61	13.64	.6215	.5775	.9305	5.716
.2640	.2455	1.659	.9301	2.531	2.722	.3674	3.318	13.78	13.81	.6204	.5770	.9309	5.705
.2650	.2467	1.665	.9309	2.548	2.737	.3653	3.330	13.95	13.99	.6193	.5765	.9313	5.695
.2660	.2478	1.671	.9317	2.566	2.754	.3632	3.343	14.13	14.17	.6183	.5761	.9316	5.685
.2670	.2490	1.678	.9326	2.583	2.770	.3610	3.355	14.31	14.34	.6172	.5756	.9320	5.675
.2680	.2501	1.684	.9334	2.600	2.786	.3589	3.368	14.49	14.53	.6162	.5752	.9324	5.665
.2690	.2513	1.690	.9342	2.618	2.803	.3568	3.380	14.67	14.71	.6152	.5747	.9328	5.655
.2700	.2524	1.697	.9350	2.636	2.819	.3547	3.393	14.86	14.89	.6142	.5742	.9331	5.645
.2710	.2536	1.703	.9357	2.653	2.835	.3527	3.405	15.05	15.08	.6132	.5737	.9335	5.636
.2720	.2547	1.709	.9365	2.671	2.852	.3506	3.418	15.24	15.27	.6122	.5733	.9339	5.627
.2730	.2559	1.715	.9373	2.689	2.869	.3485	3.431	15.43	15.46	.6112	.5728	.9343	5.617
.2740	.2570	1.722	.9381	2.707	2.886	.3465	3.443	15.63	15.66	.6102	.5724	.9346	5.608
.2750	.2582	1.728	.9388	2.726	2.903	.3444	3.456	15.83	15.86	.6092	.5719	.9350	5.599
.2760	.2593	1.734	.9396	2.744	2.920	.3424	3.468	16.03	16.06	.6082	.5714	.9354	5.590
.2770	.2605	1.740	.9403	2.762	2.938	.3404	3.481	16.23	16.26	.6072	.5710	.9358	5.582
.2780	.2616	1.747	.9410	2.781	2.955	.3384	3.493	16.43	16.47	.6063	.5705	.9362	5.573
.2790	.2627	1.753	.9417	2.799	2.973	.3364	3.506	16.64	16.67	.6053	.5701	.9366	5.565
.2800	.2639	1.759	.9424	2.818	2.990	.3344	3.519	16.85	16.88	.6044	.5696	.9369	5.556
.2810	.2650	1.766	.9431	2.837	3.008	.3324	3.531	17.07	17.10	.6035	.5691	.9373	5.548
.2820	.2662	1.772	.9438	2.856	3.026	.3305	3.544	17.28	17.31	.6025	.5687	.9377	5.540
.2830	.2673	1.778	.9445	2.875	3.044	.3285	3.556	17.50	17.53	.6016	.5682	.9381	5.532
.2840	.2684	1.784	.9452	2.894	3.062	.3266	3.569	17.72	17.75	.6007	.5677	.9384	5.524
.2850	.2696	1.791	.9458	2.913	3.080	.3247	3.581	17.95	17.98	.5998	.5673	.9388	5.516
.2860	.2707	1.797	.9465	2.933	3.099	.3227	3.594	18.18	18.20	.5989	.5668	.9392	5.509
.2870	.2718	1.803	.9472	2.952	3.117	.3208	3.607	18.40	18.43	.5980	.5664	.9396	5.501
.2880	.2730	1.810	.9478	2.972	3.136	.3189	3.619	18.64	18.67	.5971	.5659	.9400	5.493
.2890	.2741	1.816	.9484	2.992	3.154	.3170	3.632	18.88	18.90	.5962	.5654	.9404	5.486

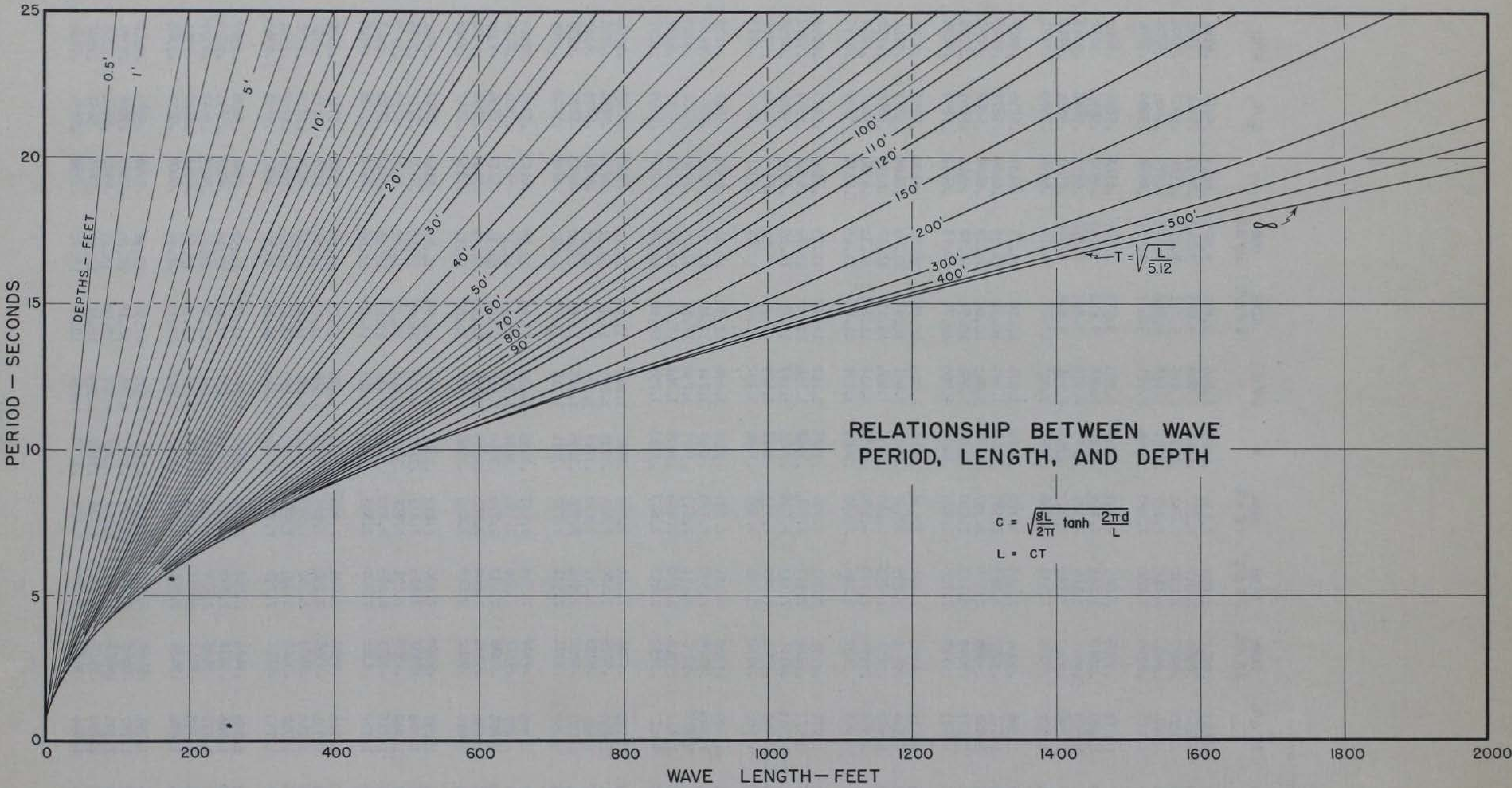

RELATIONSHIP BETWEEN WAVE
PERIOD, LENGTH, AND DEPTH
$C = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}}$
$L = CT$
$T = \sqrt{\frac{L}{5.12}}$
PERIOD — SECONDS
WAVE LENGTH — FEET
DEPTHS - FEET
0.5'
1'
5'
10'
20'
30'
40'
50'
60'
70'
80'
90'
100'
110'
120'
150'
200'
300'
400'
500'
∞
0
5
10
15
20
25
0
200
400
600
800
1000
1200
1400
1600
1800
2000

PLATE D-1

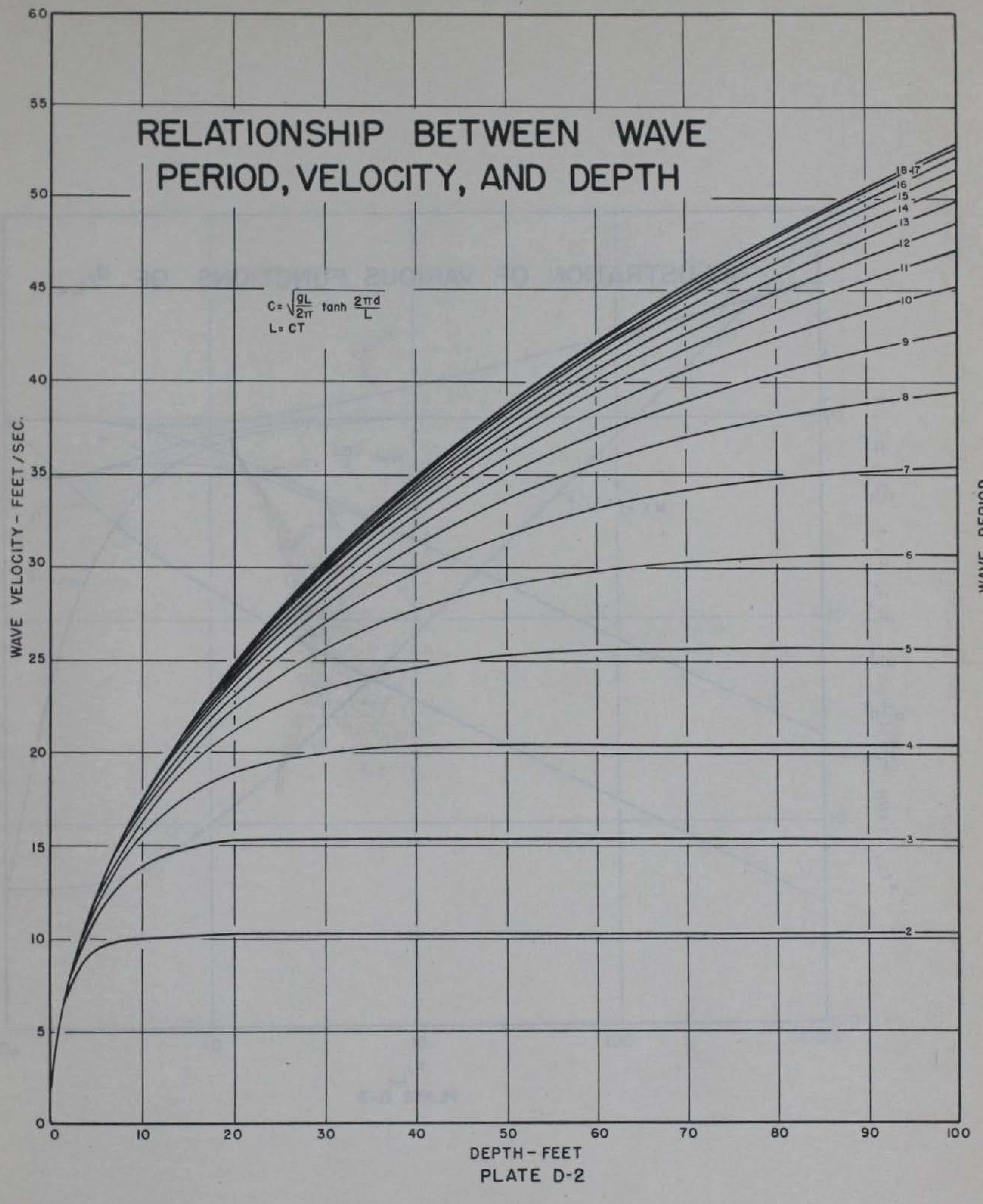


PLATE D-2

312838 O - 54 - 23

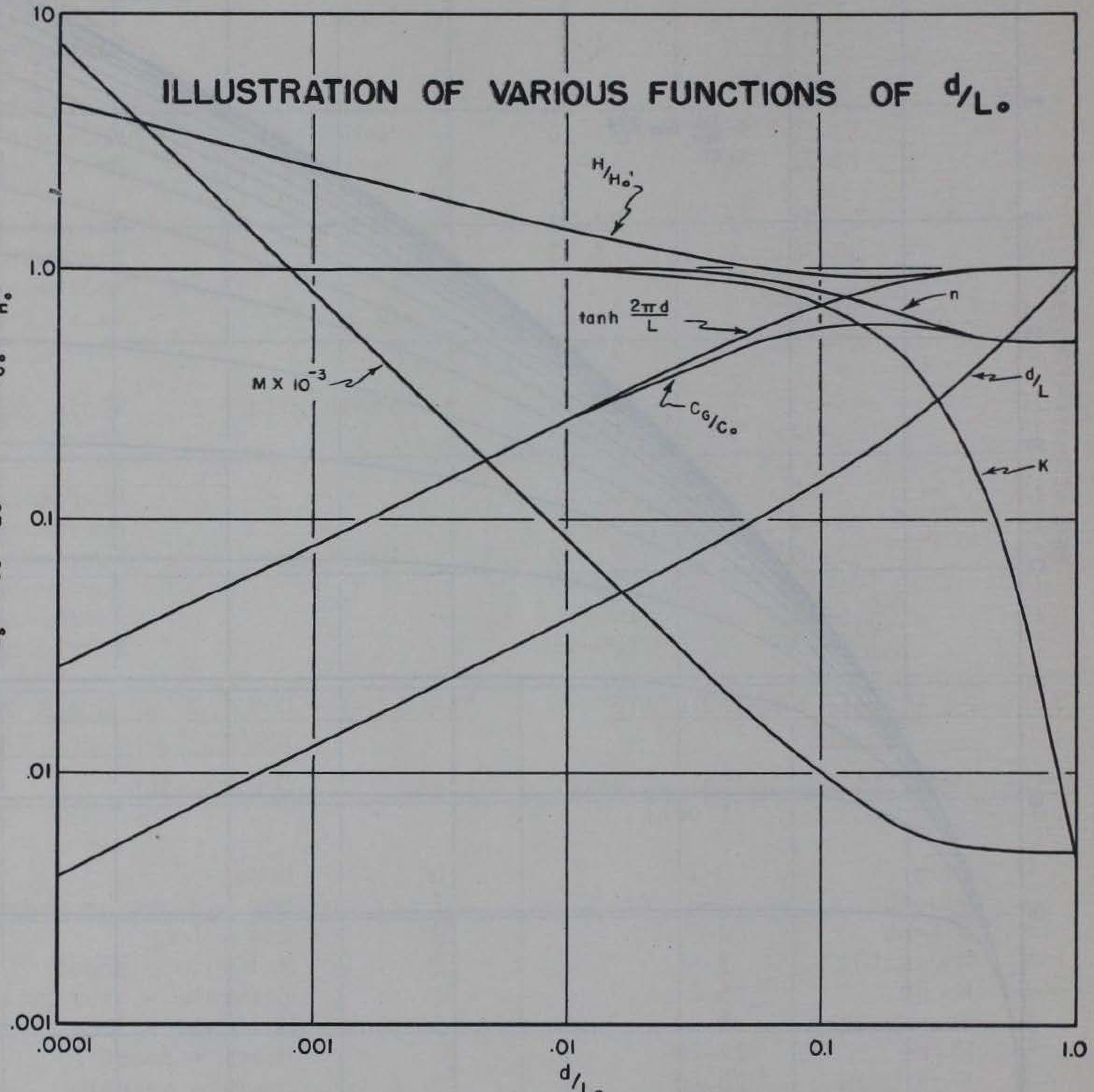


ILLUSTRATION OF VARIOUS FUNCTIONS OF d/Lo
H/Ho'
tanh 2πd/L
n
M X 10^-3
CG/Co
d/L
K
d/L, TANH 2πd/L, (bs/as), C/Co, L/Lo), K, n, CG/Co, H/Ho', M
10
1.0
0.1
.01
.001
.0001
.001
.01
0.1
1.0
d/Lo

PLATE D-3

TABLE D-3

CONVERSION TABLE FOR WIND FORCE, BEAUFORT SCALE

Beaufort Number	Statute Miles per Hour	Knots	Terms used in Forecasts	Explanatory Terms
0	Less than 1	Less than 1		Calm
1	1-3	1-3	Light	Light air
2	4-7	4-6		Light breeze
3	8-12	7-10	Gentle	Gentle breeze
4	13-18	11-16	Moderate	Moderate breeze
5	19-24	17-21	Fresh	Fresh breeze
6	25-31	22-27	Strong	Strong breeze
7	32-38	28-33		High wind (moderate gale)
8	39-46	34-40	Gale	Gale (fresh gale)
9	47-54	41-47		Strong gale
10	55-63	48-55	Whole gale	Whole gale
11	64-72	56-63		Storm
12	73-82	64-71		Hurricane
13	83-92	72-80		Hurricane
14	93-103	81-89	Hurricane	Hurricane
15	104-114	90-99		Hurricane
16	115-125	100-108		Hurricane
17	126-136	109-118		Hurricane

TABLE D-4

WIND WAVE (SEA) HEIGHT CODE

(U.S. Navy H.O. Pub. #607)

Code Figure	Approximate height of sea	Seaman's description
0	0	Calm - Sea like mirror.
1	Less than 1 foot	Smooth - Small wavelets or ripples with the appearance of scale but without crests.
2	1 to 3 feet	Slight - The waves or small rollers are short and more pronounced, when capping the foam is not white but more of a glassy appearance.
3	3 to 5 feet	Moderate - The waves or large rollers become longer and begin to show white caps occasionally. The sea produces short rustling sounds.
4	5 to 8 feet	Rough - Medium waves that take a more pronounced long form with extensive whitecapping and white foam crests. The noise of the sea is like a dull murmur.
5	8 to 12 feet	Very rough - The medium waves become larger and begin to heap up, the whitecapping is continuous, and the seas break occasionally; the foam from the capping and breaking waves begins to be blown along in the direction of the wind. The breaking and capping seas produce a perpetual murmur.
6	12 to 20 feet	High - Heavy, whitecapped waves that show a visible increase in height and are breaking extensively. The foam is blown in dense streaks along in the direction of the wind. The sea begins to roll and the noise of the breaking seas is like a dull roar, audible at greater distance.
7	20 to 40 feet	Very high - High, heavy waves developed with long overhanging crests that are breaking continuously, with a perpetual roaring noise. The whole surface of the sea takes on a white appearance from the great amount of foam being blown along with the wind. The rolling of the sea becomes heavy and shocklike.
8	40 feet and over	Mountainous - The heavy waves become so high that ships within close distances drop so low in the wave troughs that for a time they are lost from view. The rolling of the sea becomes tumultuous. The wind beats the breaking edge of the seas into a froth, and the whole sea is covered with dense streaks of foam being carried along with the wind. Owing to the violence of the wind the air is so filled with foam and spray that relatively close objects are no longer visible.
9		Note - Qualifying condition applicable to the previous conditions, e.g. (5-9). A very rough, confused sea.

TABLE D-5

SWELL CONDITION CODE (U.S. Navy H.O. Pub. #607)

Code	Height in feet	Description	Approximate length in feet
0	0 - no swell		0
1	1 to 6 low swell	Short or average	0 to 600
2		Long	Above 600
3	6 to 12 moderate	Short	0 to 300
4		Average	300 to 600
5		Long	Above 600
6	Greater than 12 high	Short	0 to 300
7		Average	300 to 600
8		Long	Above 600
9	Confused		

TABLE D-6

International Code 69 for Wave Period(138)

SYMBOL "P_w" PERIOD OF THE WAVES

Code Figure	
2	5 seconds or less
3	5-7 seconds
4	7-9 seconds
5	9-11 seconds
6	11-13 seconds
7	13-15 seconds
8	15-17 seconds
9	17-19 seconds
0	19-21 seconds
1	Over 21 seconds
X	Calm or period not determined

TABLE D-7

International Code 42 for Wave Height(138)

SYMBOL "H_w"

MEAN MAXIMUM HEIGHT OF THE WAVES

Code Figure	Height	Height*
0	Less than 1 foot (1/4 m.)	16 ft. (5 m.)
1	$1\frac{1}{2}$ ft. (1/2 m.)	$17\frac{1}{2}$ ft. ($5\frac{1}{2}$ m.)
2	3 ft. (1 m.)	19 ft. (6 m.)
3	5 ft. ($1\frac{1}{2}$ m.)	21 ft. ($6\frac{1}{2}$ m.)
4	$6\frac{1}{2}$ ft. (2 m.)	$22\frac{1}{2}$ ft. (7 m.)
5	8 ft. ($2\frac{1}{2}$ m.)	24 ft. ($7\frac{1}{2}$ m.)
6	$9\frac{1}{2}$ ft. (3 m.)	$25\frac{1}{2}$ ft. (8 m.)
7	11 ft. ($3\frac{1}{2}$ m.)	27 ft. ($8\frac{1}{2}$ m.)
8	13 ft. (4 m.)	29 ft. (9 m.)
9	14 ft ($4\frac{1}{2}$ m.)	$30\frac{1}{2}$ ft. ($9\frac{1}{2}$ m.)
X	Height not determined	

* If 50 is added to $d_w d_w$ or if code figure for the true direction from which the waves come (Symbol $d_w d_w$) is between 50 and 99 inclusive.

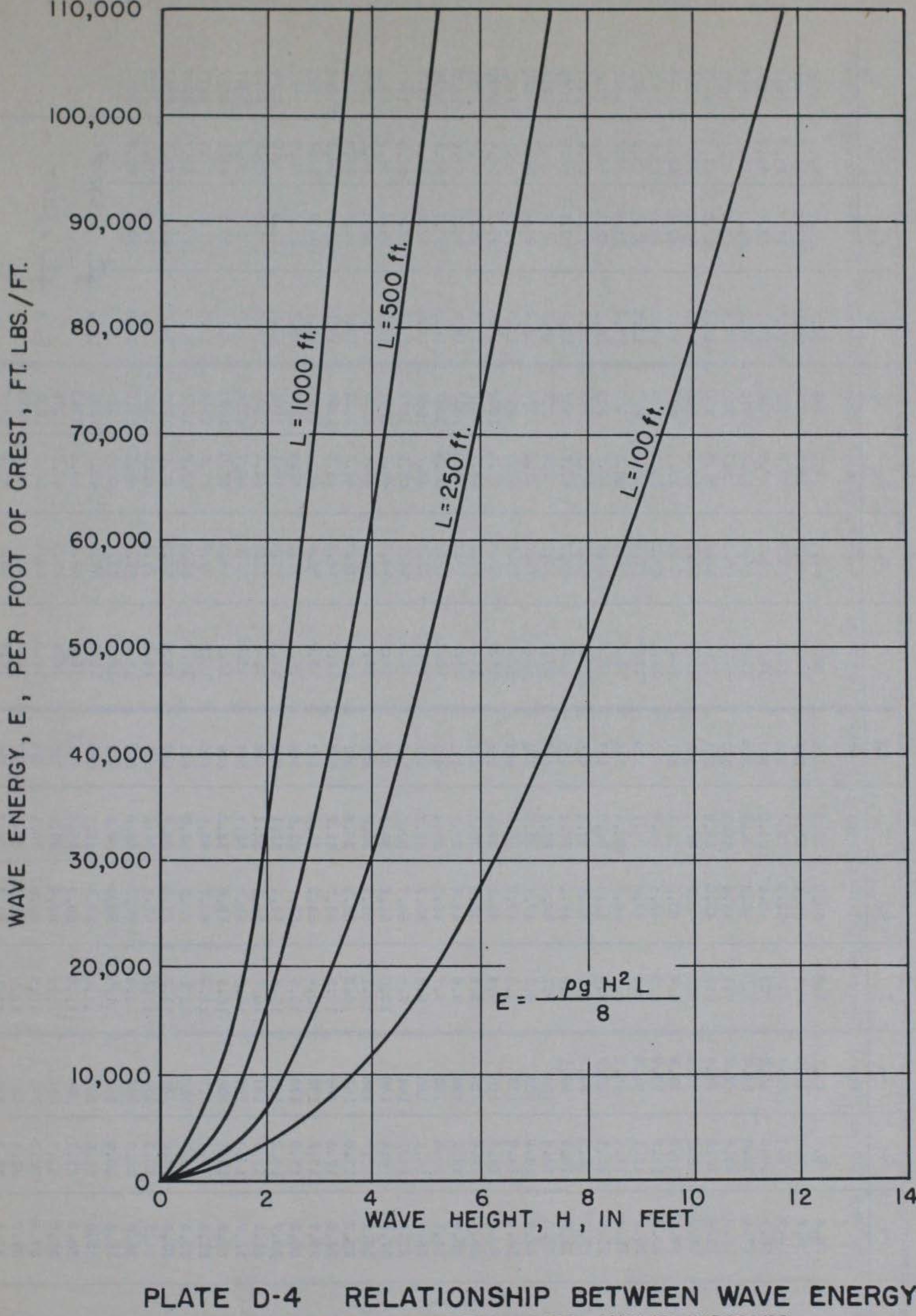


PLATE D-4 RELATIONSHIP BETWEEN WAVE ENERGY, WAVE LENGTH, AND WAVE HEIGHT

Table D-8 Deep water wave length (L_o) and velocity (C_o) as a function of wave period (T).

T	C_o	C_o	L_o	T	C_o	C_o	L_o	T	C_o	C_o	L_o	T	C_o	C_o	L_o
(Seconds)	(Ft./Sec)	(Knots)	(Feet)	(Seconds)	(Ft./Sec)	(Knots)	(Feet)	(Seconds)	(Ft./Sec)	(Knots)	(Feet)	(Seconds)	(Ft./Sec)	(Knots)	(Feet)
3.0	15.4	9.1	46.1	7.6	38.9	23.9	296	12.2	62.4	37.0	762	16.8	86.0	50.9	1444
3.1	15.9	9.4	49.2	7.7	39.4	23.3	304	12.3	63.0	37.3	775	16.9	86.5	51.2	1461
3.2	16.4	9.7	52.4	7.8	39.9	23.6	312	12.4	63.5	37.6	787	17.0	87.0	51.5	1479
3.3	16.9	10.0	55.8	7.9	40.4	23.9	320	12.5	64.0	37.9	800	17.1	87.5	51.8	1496
3.4	17.4	10.3	59.2	8.0	40.9	24.2	328	12.6	64.5	38.2	813	17.2	88.0	52.1	1514
3.5	17.9	10.6	62.7	8.1	41.4	24.5	336	12.7	65.0	38.5	826	17.3	88.5	52.4	1531
3.6	18.4	10.9	66.4	8.2	42.0	24.8	344	12.8	65.5	38.8	839	17.4	89.0	52.7	1549
3.7	18.9	11.2	70.1	8.3	42.5	25.1	353	12.9	66.0	39.1	852	17.5	89.6	53.0	1567
3.8	19.4	11.5	73.9	8.4	43.0	25.4	361	13.0	66.5	39.4	865	17.6	90.1	53.3	1585
3.9	20.0	11.8	77.9	8.5	43.5	25.7	370	13.1	67.0	39.7	879	17.7	90.6	53.6	1603
4.0	20.5	12.1	81.9	8.6	44.0	26.1	379	13.2	67.6	40.0	892	17.8	91.1	53.9	1621
4.1	21.0	12.4	86.1	8.7	44.5	26.4	388	13.3	68.1	40.3	906	17.9	91.6	54.2	1639
4.2	21.5	12.7	90.3	8.8	45.0	26.7	397	13.4	68.6	40.6	919	18.0	92.1	54.5	1658
4.3	22.0	13.0	94.7	8.9	45.6	27.0	406	13.5	69.1	40.9	933	18.1	92.6	54.8	1677
4.4	22.5	13.3	99.1	9.0	46.1	27.3	415	13.6	69.6	41.2	947	18.2	93.1	55.1	1695
4.5	23.0	13.6	104	9.1	46.6	27.6	424	13.7	70.1	41.5	961	18.3	93.6	55.4	1714
4.6	23.5	13.9	108	9.2	47.1	27.9	433	13.8	70.6	41.8	975	18.4	94.2	55.8	1732
4.7	24.0	14.2	113	9.3	47.6	28.2	442	13.9	71.1	42.1	989	18.5	94.7	56.1	1751
4.8	24.6	14.5	118	9.4	48.1	29.5	452	14.0	71.6	42.4	1004	18.6	95.2	56.4	1770
4.9	25.1	14.8	123	9.5	48.6	28.8	461	14.1	72.2	42.7	1018	18.7	95.7	56.7	1789
5.0	25.6	15.2	128	9.6	49.1	29.1	471	14.2	72.7	43.0	1032	18.8	96.2	57.0	1809
5.1	26.1	15.5	133	9.7	49.6	29.4	491	14.3	73.2	43.3	1047	18.9	96.7	57.3	1828
5.2	26.6	15.8	138	9.8	50.2	29.7	491	14.4	73.7	43.6	1062	19.0	97.2	57.6	1847
5.3	27.1	16.1	144	9.9	50.7	30.0	502	14.5	74.2	43.9	1076	19.1	97.8	57.9	1867
5.4	27.6	16.4	149	10.0	51.2	30.3	512	14.6	74.7	44.2	1091	19.2	98.3	58.2	1886
5.5	28.1	16.7	155	10.1	51.7	30.6	522	14.7	75.2	44.5	1106	19.3	98.8	58.5	1906
5.6	28.7	17.0	161	10.2	52.2	30.9	533	14.8	75.7	44.8	1121	19.4	99.3	58.8	1926
5.7	29.2	17.3	166	10.3	52.7	31.2	543	14.9	76.2	45.1	1137	19.5	99.8	59.1	1946
5.8	29.7	17.6	172	10.4	53.2	31.5	554	15.0	76.8	45.4	1152	19.6	100.3	59.4	1966
5.9	30.2	17.9	178	10.5	53.7	31.8	564	15.1	77.3	45.8	1167	19.7	100.8	59.7	1986
6.0	30.7	18.2	184	10.6	54.2	32.1	575	15.2	77.8	46.1	1183	19.8	101.3	60.0	2006
6.1	31.2	18.5	191	10.7	54.8	32.4	586	15.3	78.3	46.4	1199	19.9	101.8	60.3	2027
6.2	31.7	18.8	197	10.8	55.3	32.7	597	15.4	78.8	46.7	1214	20.0	102.4	60.6	2047
6.3	32.2	19.1	203	10.9	55.8	33.0	608	15.5	79.3	47.0	1230	21.0	107.5	63.6	2257
6.4	32.8	19.4	210	11.0	56.3	33.3	620	15.6	79.8	47.3	1246	22.0	112.6	66.7	2477
6.5	33.3	19.7	216	11.1	56.8	33.6	631	15.7	80.4	47.6	1263	23.0	117.7	69.7	2707
6.6	33.8	20.0	223	11.2	57.3	53.9	642	15.8	80.9	47.9	1277	24.0	122.8	72.7	2948
6.7	34.3	20.3	230	11.3	57.8	34.2	654	15.9	81.4	48.2	1293	25.0	128.0	75.7	3199
6.8	34.8	20.6	237	11.4	58.3	34.5	665	16.0	81.9	48.5	1310	26.0	133.1	78.8	3460
6.9	35.3	20.9	244	11.5	58.9	34.8	677	16.1	82.4	48.8	1326				
7.0	35.8	21.2	251	11.6	59.4	35.1	689	16.2	82.9	49.1	1343				
7.1	36.3	21.5	258	11.7	59.9	35.4	701	16.3	83.4	49.4	1359				
7.2	36.8	21.8	265	11.8	60.4	35.8	713	16.4	83.9	49.7	1376				
7.3	37.4	22.1	273	11.9	60.9	36.1	725	16.5	84.4	50.0	1393				
7.4	37.9	22.4	280	12.0	61.4	36.4	737	16.6	85.0	50.3	1410				
7.5	38.4	22.7	288	12.1	61.9	36.7	750	16.7	85.5	50.6	1427				

$$L_o = \frac{gT^2}{2} = 5.12\ T^2$$

$$C_o = \frac{gT}{2} = 5.12\ T$$

TABLE D-9 - Values used for plotting orthogonals

Wave period (T) = 5 seconds
Deep water wave length (L_o) = 128 feet

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
10			
	0.003	1.004	0.996
9			
	0.007	1.007	0.993
8			
	0.011	1.012	0.988
7			
	0.019	1.020	0.980
6			
	0.032	1.034	0.968
5			
	0.052	1.055	0.948
4			
	0.085	1.090	0.917
3			
	0.145	1.158	0.864
2			
	0.290	1.341	0.746
1			

Wave period (T) = 6 seconds
Deep water wave length = (L_o) = 184.2 ft.

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
15			
	0.005	1.005	0.995
13			
	0.011	1.011	0.989
11			
	0.009	1.009	0.991
10			
	0.014	1.013	0.987
9			
	0.019	1.019	0.981
8			
	0.026	1.026	0.974
7			
	0.037	1.037	0.964
6			
	0.050	1.052	0.950
5			
	0.074	1.075	0.930
4			
	0.093	1.111	0.900
3			
	0.154	1.180	0.847
2			
	0.330	1.363	0.734
1			

Wave period (T) = 7 seconds
Deep water wave length (L_o) = 251 feet

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
21			
	0.006	1.006	0.994
17.5			
	0.010	1.010	0.990
15			
	0.014	1.015	0.985
13			
	0.025	1.025	0.976
11			
	0.018	1.018	0.982
10			
	0.023	1.023	0.977
9			
	0.029	1.030	0.971
8			
	0.037	1.039	0.964
7			
	0.047	1.049	0.953
6			
	0.064	1.065	0.939
5			
	0.082	1.086	0.921
4			
	0.117	1.123	0.890
3			
	0.175	1.192	0.839
2			
	0.310	1.378	0.726
1			

Wave period (T) = 8 seconds
Deep water wave length (L_o) = 327 feet

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
27			
	0.006	1.015	0.986
20			
	0.013	1.013	0.987
17.5			
	0.021	1.021	0.979
15			
	0.013	1.012	0.988
14			
	0.014	1.014	0.986
13			
	0.016	1.017	0.983
12			
	0.020	1.021	0.980
11			
	0.025	1.025	0.976
10			
	0.031	1.031	0.970
9			
	0.038	1.037	0.964
8			
	0.046	1.046	0.956
7			
	0.056	1.059	0.946
6			
	0.069	1.069	0.933
5			
	0.090	1.095	0.914
4			
	0.124	1.131	0.844
3			
	0.183	1.199	0.834
2			
	0.30	1.389	0.720
1			

* Note:- C_d is the wave velocity in deeper water; C_s is the wave velocity in shallower water. When an orthogonal is being drawn from deep to shallow water, $C_d/C_s = C_1/C_2$. When an orthogonal is being drawn from shallow to deep water, $C_s/C_d = C_1/C_2$.

42.73
256

364

Table D-9 Cont'd

Wave period (T) = 9 seconds. Deep water wave length (L_o) = 415 feet			
Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
35			
	0.011	1.005	0.995
30			
	0.012	1.011	0.989
25			
	0.025	1.026	0.975
20			
	0.022	1.022	0.978
17.5			
	0.033	1.032	0.969
15			
	0.015	1.017	0.983
14			
	0.019	1.019	0.981
13			
	0.022	1.022	0.978
12			
	0.025	1.026	0.975
11			
	0.029	1.031	0.970
10			
	0.035	1.036	0.966
9			
	0.043	1.042	0.959
8			
	0.048	1.051	0.952
7			
	0.064	1.062	0.941
6			
	0.071	1.077	0.929
5			
	0.098	1.100	0.909
4			
	0.136	1.136	0.881
3			
	0.196	1.206	0.829
2			
	0.350	1.390	0.720
1			

Wave period (T) = 10 seconds. Deep water wave length (L_o) = 512 feet			
Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
43			
	0.010	1.007	0.993
35			
	0.010	1.011	0.989
30			
	0.020	1.021	0.979
25			
	0.040	1.040	0.962
20			
	0.029	1.031	0.970
17.5			
	0.041	1.042	0.960
15			
	0.021	1.020	0.980
14			
	0.023	1.023	0.978
13			
	0.026	1.026	0.975
12			
	0.029	1.030	0.971
11			
	0.034	1.034	0.967
10			
	0.039	1.040	0.962
9			
	0.045	1.046	0.956
8			
	0.054	1.055	0.948
7			
	0.063	1.066	0.938
6			
	0.078	1.081	0.925
5			
	0.100	1.103	0.906
4			
	0.132	1.140	0.878
3			
	0.189	1.211	0.826
2			
	0.326	1.397	0.716
1			

Wave period (T) = 11 seconds. Deep water wave length (L_o) = 620 feet			
Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
50			
	0.01	1.010	0.990
40			
	0.01	1.012	0.989
35			
	0.02	1.019	0.982
30			
	0.03	1.032	0.969
25			
	0.05	1.053	0.950
20			
	0.04	1.037	0.964
17.5			
	0.05	1.049	0.953
15			
	0.020	1.023	0.978
14			
	0.027	1.026	0.975
13			
	0.029	1.029	0.972
12			
	0.032	1.033	0.968
11			
	0.038	1.037	0.964
10			
	0.044	1.043	0.959
9			
	0.050	1.049	0.954
8			
	0.058	1.058	0.947
7			
	0.070	1.068	0.936
6			
	0.025	1.084	0.923
5			
	0.105	0.105	0.905
4			
	0.137	1.143	0.875
3			
	0.203	1.208	0.825
2			
	0.325	1.400	0.714
1			

Wave period (T) = 12 seconds. Deep water wave length (L_o) = 737 feet			
Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
61			
	0.007	1.007	0.994
50			
	0.010	1.007	0.993
45			
	0.008	1.011	0.989
40			
	0.012	1.018	0.983
35			
	0.032	1.027	0.974
30			
	0.040	1.042	0.960
25			
	0.065	1.062	0.941
20			
	0.044	1.043	0.959
17.5			
	0.051	1.054	0.949
15			
	0.024	1.025	0.976
14			
	0.027	1.028	0.973
13			
	0.031	1.031	0.970
12			
	0.035	1.034	0.967
11			
	0.039	1.039	0.962
10			
	0.044	1.044	0.958
9			
	0.049	1.051	0.951
8			
	0.057	1.059	0.944
7			
	0.068	1.070	0.935
6			
	0.083	1.086	0.921
5			
	0.105	1.108	0.904
4			
	0.135	1.148	0.871
3			
	0.20	1.210	0.829
2			
	0.33	1.406	0.709
1			

Table D-9 Cont'd

Wave period (T) = 13 seconds
Deep water wave length (L_o) = 865 feet

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
72			
	0.006	1.006	0.994
60			
	0.013	1.013	0.987
50			
	0.012	1.011	0.989
45			
	0.016	1.017	0.984
40			
	0.024	1.024	0.977
35			
	0.033	1.034	0.967
30			
	0.048	1.049	0.953
25			
	.0.070	1.072	0.933
20			
	0.047	1.047	0.955
17.5			
	0.055	1.058	0.945
15			
	0.027	1.027	0.974
14			
	0.029	1.029	0.972
13			
	0.032	1.033	0.968
12			

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
12			
	0.036	1.036	0.966
11			
	0.040	1.041	0.961
10			
	0.045	1.046	0.956
9			
	0.051	1.052	0.950
8			
	0.059	1.061	0.942
7			
	0.070	1.071	0.933
6			
	0.085	1.088	0.919
5			
	0.103	1.109	0.902
4			
	0.135	1.147	0.872
3			
	0.200	1.213	0.824
2			
	0.341	1.410	0.709
1			

Wave period (T) = 14 seconds
Deep water wave length (L_o) = 1,003.5 feet

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
85			
	0.006	1.007	0.993
70			
	0.030	1.030	0.970
50			
	0.038	1.038	0.963
40			
	0.029	1.030	0.971
35			
	0.040	1.041	0.961
30			
	0.055	1.056	0.947
25			
	0.075	1.078	0.928
20			
	0.049	1.050	0.952
17.5			
	0.059	1.061	0.942
15			
	0.028	1.029	0.972
14			
	0.030	1.031	0.970
13			

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
13			
	0.033	1.033	0.967
12			
	0.037	1.037	0.964
11			
	0.041	1.042	0.960
10			
	0.046	1.047	0.955
9			
	0.052	1.054	0.949
8			
	0.058	1.062	0.942
7			
	0.070	1.073	0.932
6			
	0.085	1.089	0.918
5			
	0.105	1.109	0.902
4			
	0.144	1.148	0.871
3			
	0.188	1.217	0.822
2			
	0.338	1.405	0.712
1			

Table D-9 Cont'd

Wave period (T) = 15 seconds
Deep water wave length (L_o) = 1,152 feet

Depth (fathoms)	L/L_{av}	C_d/C_s*	C_s/C_d*
96			
	0.060	1.060	0.944
50			
	0.047	1.048	0.954
40			
	0.080	1.083	0.923
30			
	0.059	1.061	0.942
25			
	0.080	1.084	0.923
20			
	0.050	1.053	0.949
17.5			
	0.063	1.064	0.940
15			
	0.029	1.029	0.972
14			
	0.031	1.032	0.969
13			
	0.034	1.035	0.966
12			

Depth (fathoms)	L/L_{av}	C_d/C_s*	C_s/C_d*
12			
	0.038	1.038	0.963
11			
	0.043	1.043	0.959
10			
	0.047	1.048	0.954
9			
	0.053	1.054	0.948
8			
	0.061	1.063	0.941
7			
	0.073	1.076	0.930
6			
	0.080	1.089	0.918
5			
	0.106	1.112	0.899
4			
	0.138	1.148	0.871
3			
	0.197	1.218	0.821
2			
	0.336	1.405	0.712
1			

Wave period (T) = 16 seconds
Deep water wave length (L_o) = 1,310 feet

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
109			
	0.028	1.028	0.973
70			
	0.055	1.057	0.946
50			
	0.055	1.056	0.947
40			
	0.038	1.039	0.962
35			
	0.049	1.050	0.952
30			
	0.065	1.066	0.938
25			
	0.084	1.088	0.919
20			
	0.051	1.055	0.948
17.5			
	0.067	1.066	0.938
15			
	0.030	1.030	0.971
14			
	0.033	1.032	0.969
13			
	0.035	1.035	0.966
12			

Depth (fathoms)	$\Delta L/L_{av}$	C_d/C_s*	C_s/C_d*
12			
	0.038	1.039	0.962
11			
	0.042	1.044	0.958
10			
	0.046	1.049	0.953
9			
	0.053	1.056	0.947
8			
	0.063	1.063	0.941
7			
	0.074	1.075	0.930
6			
	0.088	1.090	0.917
5			
	0.108	1.114	0.898
4			
	0.133	1.149	0.870
3			
	0.200	1.215	0.823
2			
	0.344	1.407	0.711
1			

Table D-9 Cont'd

Wave period (T) = 17 seconds Deep water wave length (L_o) = 1,480 feet				Wave period (T) = 18 seconds Deep water wave length (L_o) = 1,659 ft.			
Depth	L/L_{av}	C_d/C_s*	C_s/C_d*	Depth	L/L_{av}	C_d/C_s*	C_s/C_d*
(fathoms)				(fathoms)			
123				138			
	0.007	1.008	0.992		0.015	1.016	0.984
100				100			
	0.036	1.037	0.965		0.047	1.048	0.954
70				70			
	0.067	1.070	0.935		0.079	1.081	0.925
50				50			
	0.060	1.063	0.941		0.067	1.069	0.935
40				40			
	0.044	1.043	0.959		0.099	1.106	0.904
35				30			
	0.052	1.054	0.949		0.069	1.073	0.932
30				25			
	0.065	1.069	0.935		0.095	1.095	0.913
25				20			
	0.091	1.092	0.916		0.125	1.132	0.884
20				15			
	0.055	1.057	0.946		0.181	1.199	0.834
17.5				10			
	0.065	1.068	0.937		0.106	1.111	0.900
15				8			
	0.077	1.083	0.923		0.126	1.145	0.873
12.5				6			
	0.102	1.106	0.904		0.117	1.091	0.917
10				5			
	0.135	1.142	0.876		0.10	1.112	0.899
7.5				4			
	0.195	1.211	0.826		0.14	1.155	0.866
5				3			
	0.112	1.115	0.897		0.20	1.219	0.820
4				2			
	0.140	1.147	0.872		0.73	1.409	0.710
3				1			
	0.189	1.222	0.819				
2							
	0.339	1.400	0.714				
1							

Plates D-5 to D-9 incl. - Graphic determination of the weight of cap stone in terms of wave height, side slope and slope coefficient K' for sea water.

These curves are graphic solutions of the Iribarren formula as modified by Hudson for the determination of the side and end slopes and the weight of cap stone at those slopes required to withstand various wave heights.

$$W = \frac{K' \; w \; S_f^3 \; S_r \; \mu^3 \; H^3}{(\mu \cos \alpha - \sin \alpha)^3 \; (S_r - S_f)^3} \quad \text{(Modified Formula)}$$

where W = weight of stone in pounds

K' = slope coefficient - see Plate D-10

w = specific weight of fresh water

S_f = specific gravity of the water (in sea water $S_f = 1.03$)

S_r = specific gravity of the stone

μ = effective coefficient - stone on stone = 1.05

H = wave height at the structure

α = the angle the sea side slope makes with the horizontal

In sea water the formula reduces to

$$W = \frac{78.8K' \; S_r \; H^3}{(\mu \cos \alpha - \sin \alpha)^3 \; (S_r - 1.03)^3}$$

In fresh water it reduces to

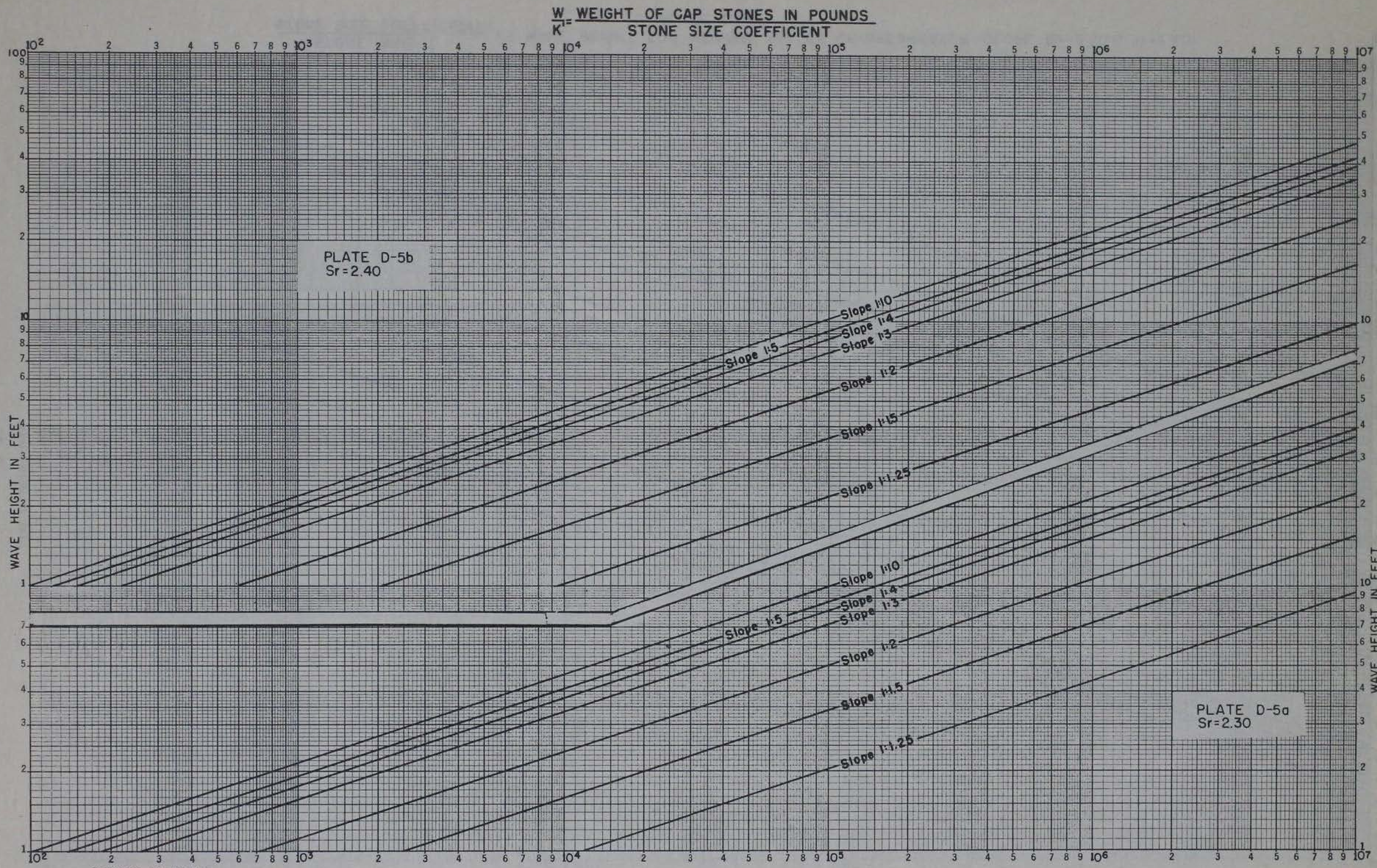
$$W = \frac{72.0 \; K' \; S_r \; H^3}{(\mu \cos \alpha - \sin \alpha)^3 \; (S_r - 1.0)^3}$$

The curves of Plate D-10 show the variation of K' in the basic equation with d/L and α.

An example of the use of these curves follows:

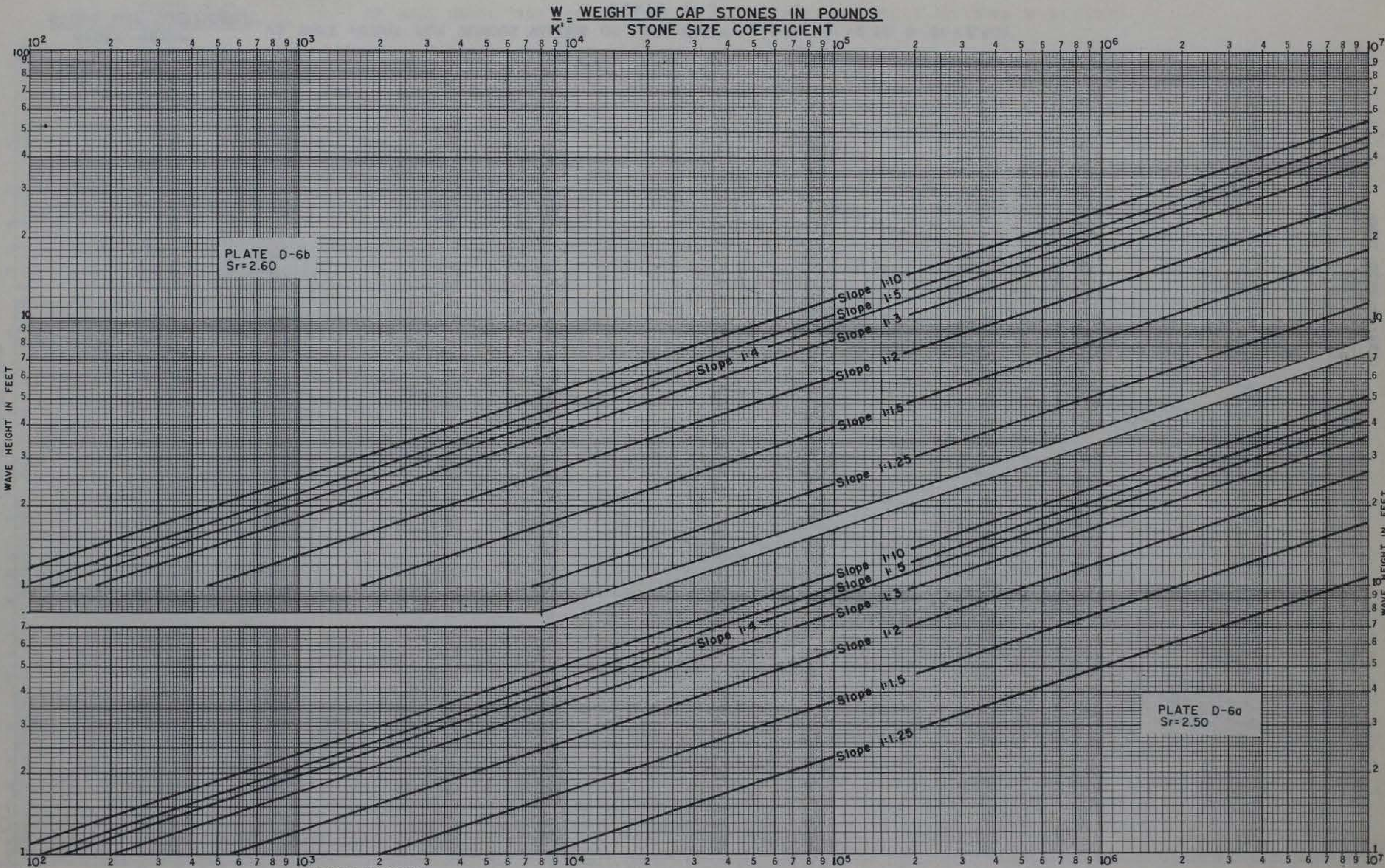
Given a breakwater composed of quarry stone having a specific gravity of 2.65, with side slopes of 1 on 2, founded in 30 feet of sea water and exposed to 10-foot waves, the weight of the cap stone is determined by using Plate D-7a and Table D-10. From Plate D-7a, W/K' for a side slope of 1 on 2 is 4.0×10^5 and from Table D-10, K' = 0.0175. Therefore the weight of the cap stone is determined to be ($4.0 \times 10^5 \times 0.0175$ = 7,000 pounds) or about 3.5 tons.

Plate D-11 - All curves on Plate D-11 (Plate III, Breakers and Surf)(131) deal with waves that approach the shore line directly, so that there are no changes due to refraction. When waves approach a shore line at an angle, the refraction correction must first be applied. This plate should only be used for quick calculations.



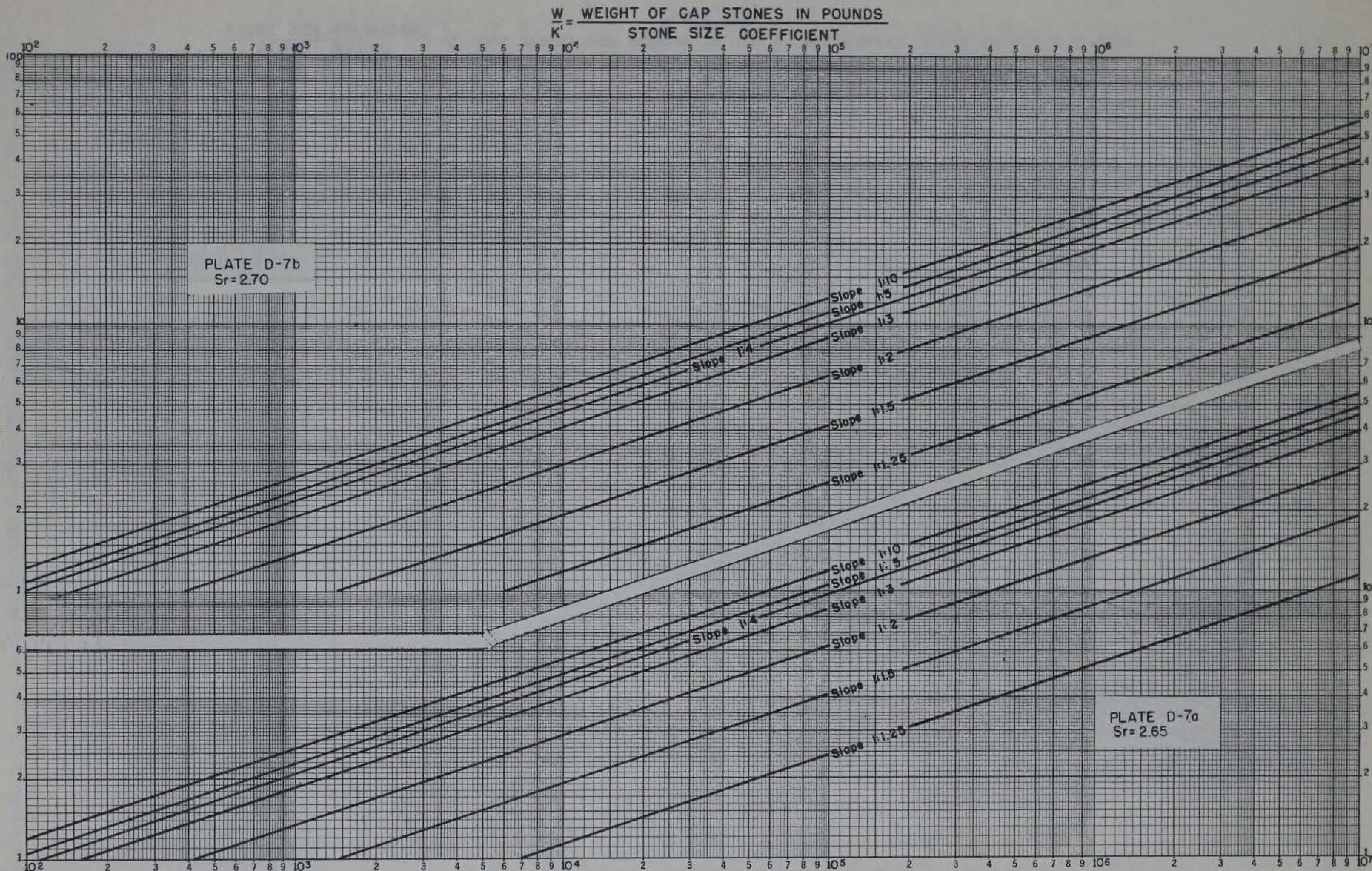
STONE WEIGHT / STONE SIZE COEFFICIENT VS WAVE HEIGHT FOR VARIOUS VALUES OF BREAKWATER SLOPE (Sr = 2.30 & Sr = 2.40)

PLATE D-5



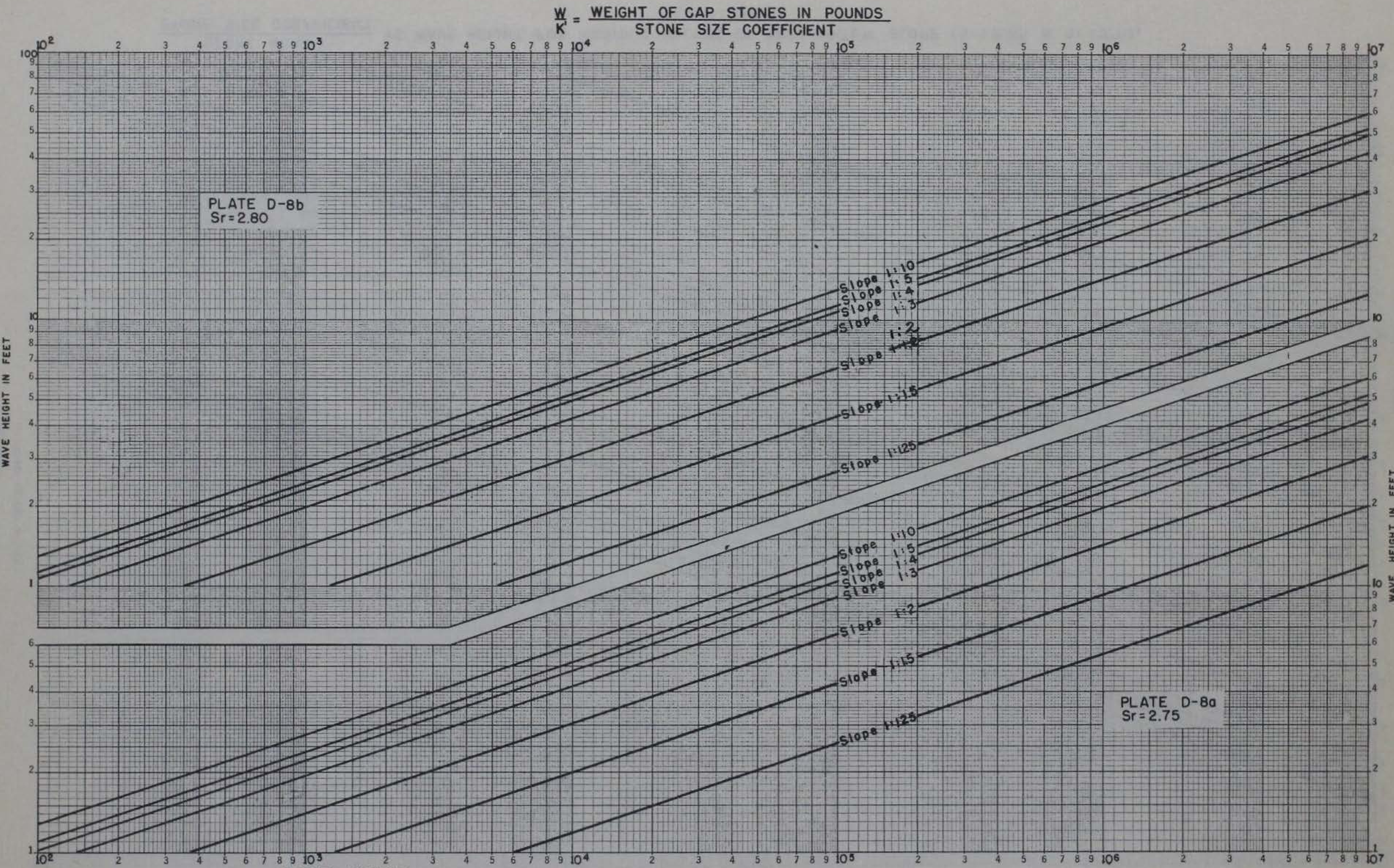
$\frac{\text{STONE WEIGHT}}{\text{STONE SIZE COEFFICIENT}}$ VS WAVE HEIGHT FOR VARIOUS VALUES OF BREAKWATER SLOPE (Sr=2.50 & Sr=2.60)

PLATE D-6



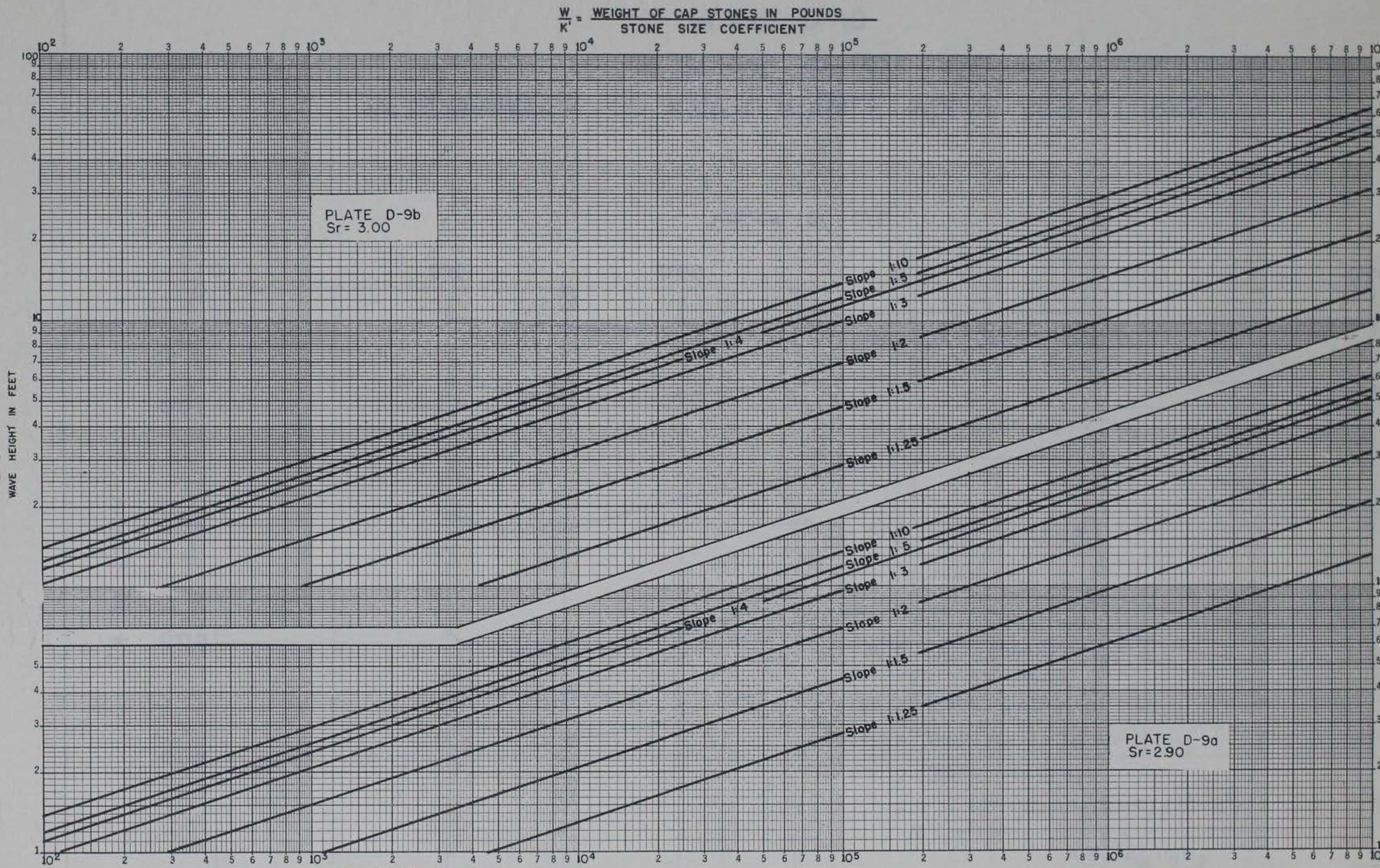
STONE WEIGHT / STONE SIZE COEFFICIENT VS WAVE HEIGHT FOR VARIOUS VALUES OF BREAKWATER SLOPE (Sr = 2.65 & Sr = 2.70)

PLATE D-7



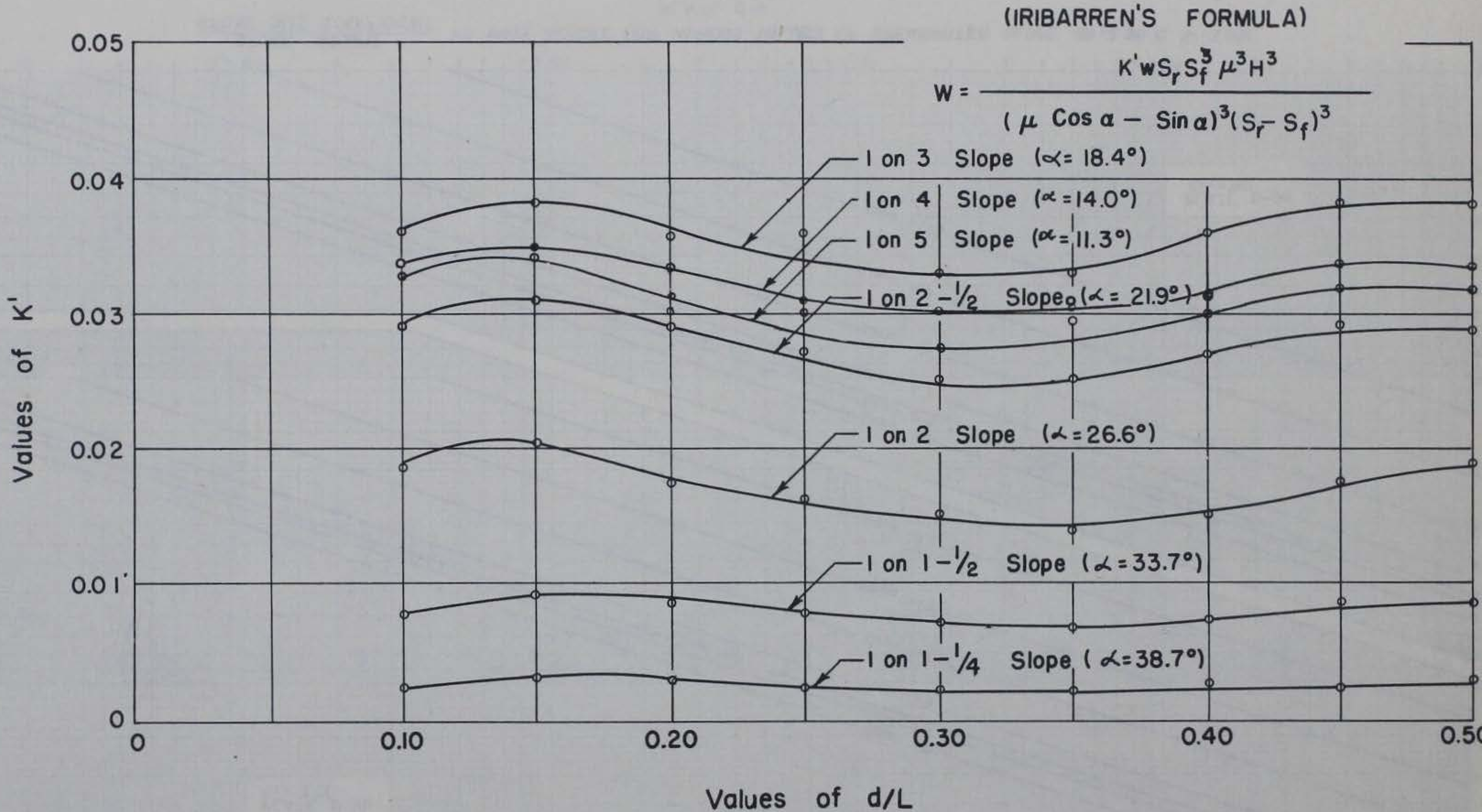
STONE WEIGHT / STONE SIZE COEFFICIENT VS WAVE HEIGHT FOR VARIOUS VALUES OF BREAKWATER SLOPE (Sr=2.75 & Sr=2.80)

PLATE D-8



$\frac{\text{STONE WEIGHT}}{\text{STONE SIZE COEFFICIENT}}$ VS WAVE HEIGHT FOR VARIOUS VALUES OF BREAKWATER SLOPE (Sr=2.90 & Sr=3.00)

PLATE D-9



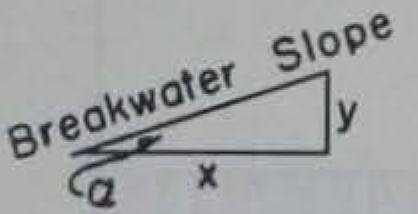
DETERMINATION OF K'(FOR RUBBLE MOUND STRUCTURES)
IN TERMS OF d/L AND α

PLATE D-10

TABLE D-10 - VARIATION OF K' WITH BREAKWATER SIDE SLOPE

Side Slope	K'	Side Slope	K'
1 on 1.25	0.0035	1 on 3	0.0365
1 on 1.5	0.0085	1 on 4	0.0325
1 on 2	0.0175	1 on 5	0.0300
1 on 2.5	0.0285		

TABLE D-11 - VALUES OF $(1.05 \cos \alpha - \sin \alpha)^3$ FOR VARIOUS BREAKWATER SLOPES



$$\text{Slope} = \frac{y}{x} = \frac{1}{\cot \alpha}$$

Slope ratio = $\cot \alpha$

Slope Angle α	Slope Ratio (cot α)	Slope and Friction Function $(1.05 \cos \alpha - \sin \alpha)^3$
45°	1	4.42×10^{-5}
42° 16'	1.1	1.14×10^{-3}
39° 48'	1.2	4.62×10^{-3}
38° 40'	1.25	7.41×10^{-3}
37° 34'	1.3	0.0110
35° 32'	1.4	0.0204
33° 41'	1.5	0.0325
32° 00'	1.6	0.0469
29° 45'	1.75	0.0715
26° 34'	2.0	0.1190
23° 58'	2.25	0.1693
21° 48'	2.5	0.2198
19° 59'	2.75	0.2684
18° 26'	3.0	0.3143
17° 06'	3.25	0.3572
15° 57'	3.5	0.3967
14° 56'	3.75	0.4335
14° 02'	4.0	0.4676
13° 14'	4.25	0.4991
12° 32'	4.5	0.5274
11° 53'	4.75	0.5546
11° 19'	5.0	0.5788
10° 18'	5.5	0.6234
9° 28'	6.0	0.6613
8° 49'	6.5	0.6947
8° 08'	7.0	0.7240
7° 36'	7.5	0.7499
7° 08'	8.0	0.7728
6° 43'	8.5	0.7936
6° 20'	9.0	0.8129
6° 01'	9.5	0.8290
5° 43'	10.0	0.8444

TABLE D-12 - VALUES OF $\tan^2 (90 - \phi)/2$

ϕ	$\tan \phi$	$\tan^2 \left(\frac{90 - \phi}{2}\right)$	ϕ	$\tan \phi$	$\tan^2 \left(\frac{90 - \phi}{2}\right)$
0	0	1.00	45	1.000	0.17
10	0.176	0.70	50	1.192	0.13
20	0.364	0.49	60	1.732	0.07
25	0.466	0.41	70	2.748	0.03
30	0.577	0.33	80	5.671	0.01
35	0.700	0.27	90	∞	0
40	0.839	0.22			

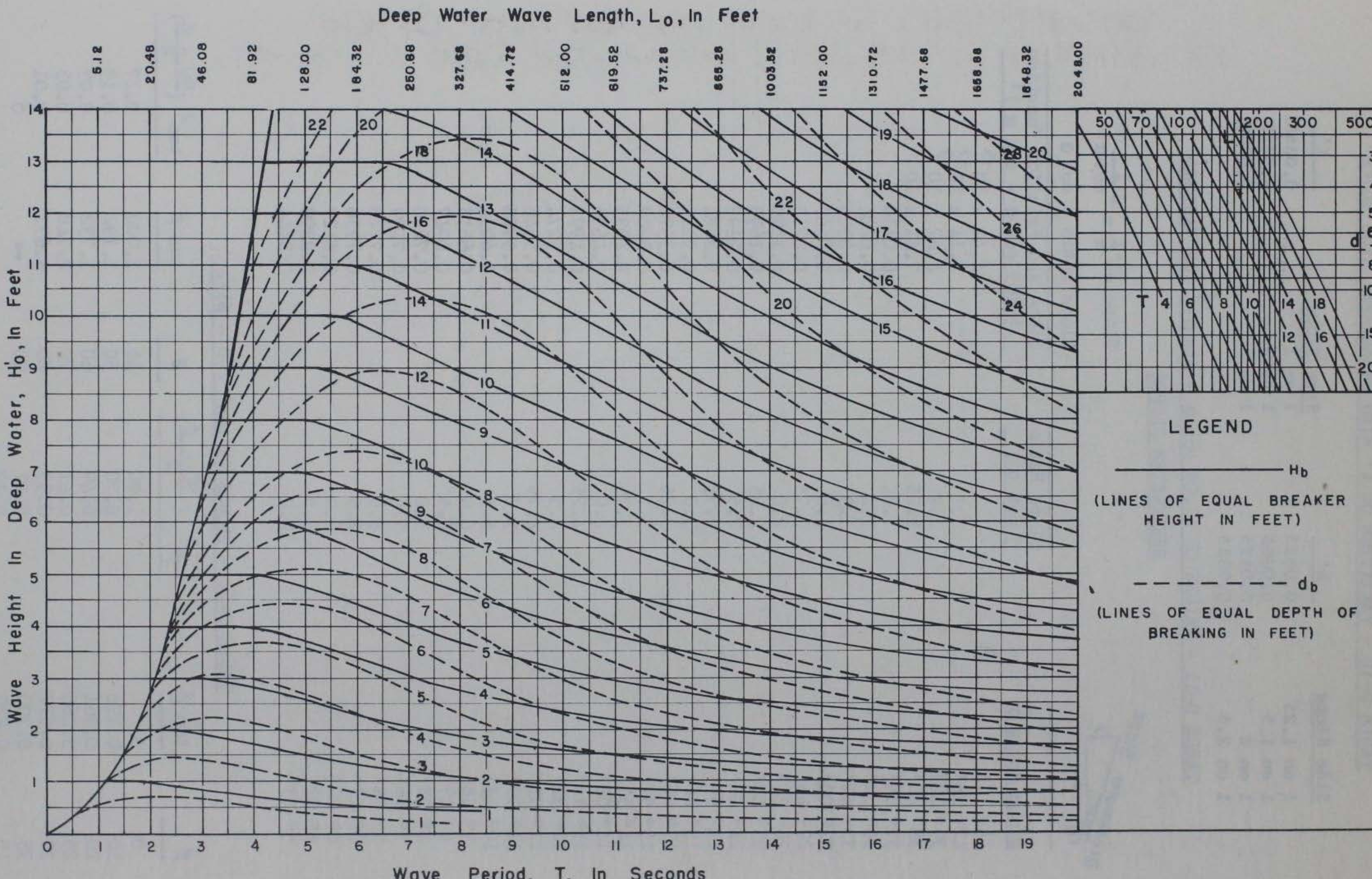
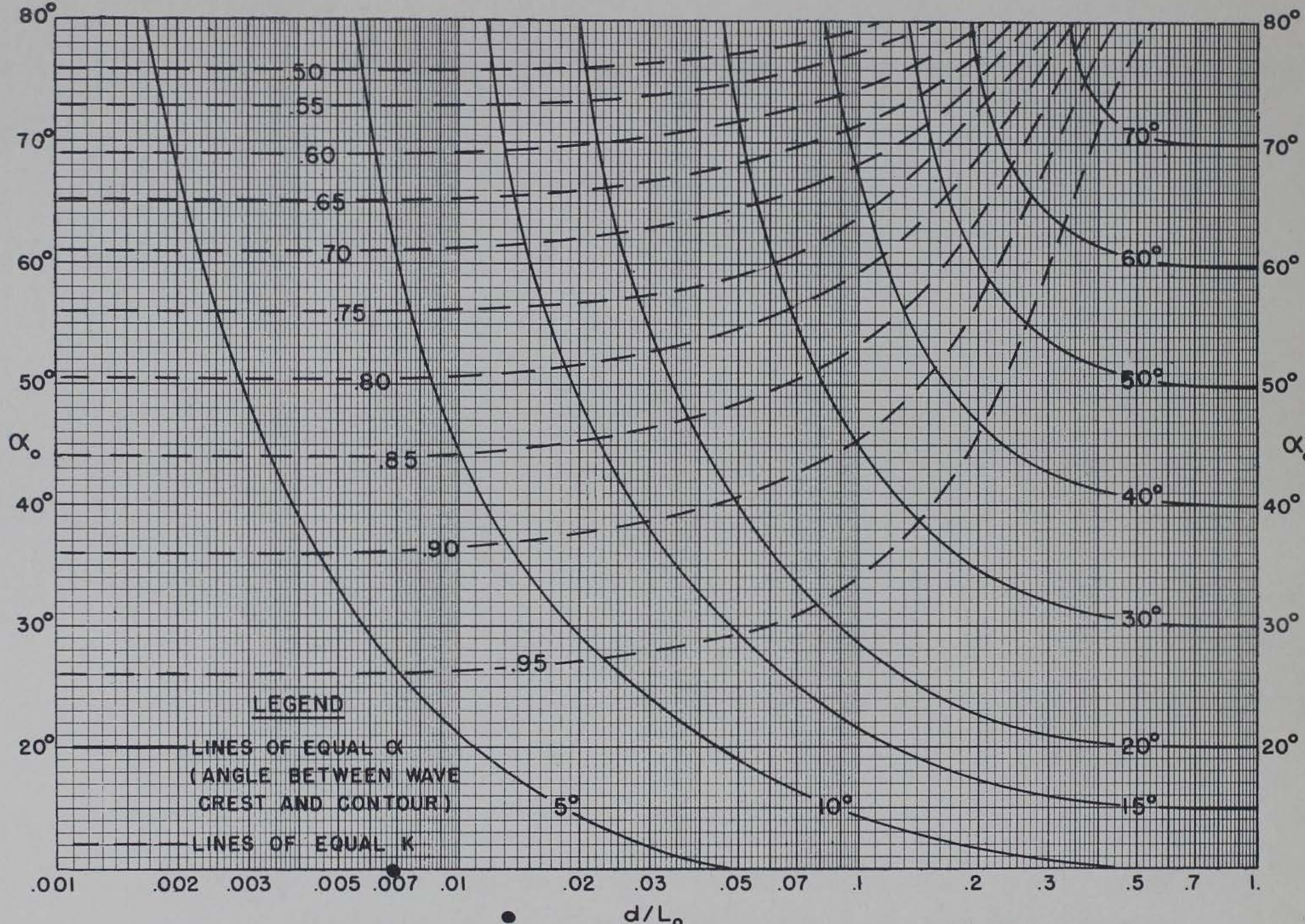


PLATE D-11 DETERMINATION OF WAVE HEIGHT AND DEPTH OF WATER AT POINT OF BREAKING.



CHANGE IN WAVE DIRECTION AND HEIGHT DUE TO REFRACTION ON BEACHES WITH STRAIGHT, PARALLEL DEPTH CONTOURS

PLATE D-12

APPENDIX E

MISCELLANEOUS DERIVATIONS

APPENDIX E

MISCELLANEOUS DERIVATIONS

1. Refraction Diagrams; derivation of method and design of template.

a. Derivation. The parametric equations for an orthogonal given by Arthur, Munk and Isaacs(3) are:

$$\frac{dx}{dt} = C \cos \beta ; \frac{dy}{dt} = C \sin \beta ; \frac{d\beta}{dt} = \sin \beta \frac{\partial C}{\partial x} - \cos \beta \frac{\partial C}{\partial y} \tag{E-1}$$

These may be solved by simple separation of variables for a velocity field which is a function of y alone. (Here $\beta = 90° - \alpha$ where α = the angle between a tangent to a contour and a normal to an orthogonal, see Figure E-1). In particular, for a field which varies linearly with y, $C = C_o (1 - ay)$, the solutions for x and y are

$$x = \frac{1}{a \sin \alpha_0} (\cos \alpha - \cos \alpha_0)$$

$$y = \frac{1}{a \sin \alpha_0} (\cos \alpha_0 - \cos \alpha) \tag{E-2}$$

which are the parametric equations of a circle of radius

$$r = \sqrt{\frac{\sin^2 (2\alpha_0) \neq 4}{a \sin (2\alpha_0)}} \text{ and center at} \tag{E-3}$$

$$x = - \frac{2}{a \sin (2\alpha_0)} ; y = \frac{1}{a} \tag{E-4}$$

The solution for y may be put in the form

$$\sin \alpha = \sin (\alpha_0 - \Delta \alpha) = (1 - \frac{\Delta C}{C_o}) \sin \alpha_0 \tag{E-5}$$

$$= \frac{C}{C_o} \sin \alpha_0, \text{ which is Snell's law.}$$

From these, exact values of $\Delta \alpha$ and x at any point in the field may be found.

b. Template Design. Referring to Figure E-1, if from the point of intersection P of the mid-contour with an incoming orthogonal, a perpendicular is dropped to an arbitrary point R, then the line RQ perpendicular to the tangent to the mid-contour = sin α, x PR (angle RPQ = α_1). If another line from R equal in length to C_1/C_2 x PR (where C_1 and C_2 are the velocities at contours 1 and 2 respectively) is drawn to intersect the tangent to the mid-contour, the following relationships hold:

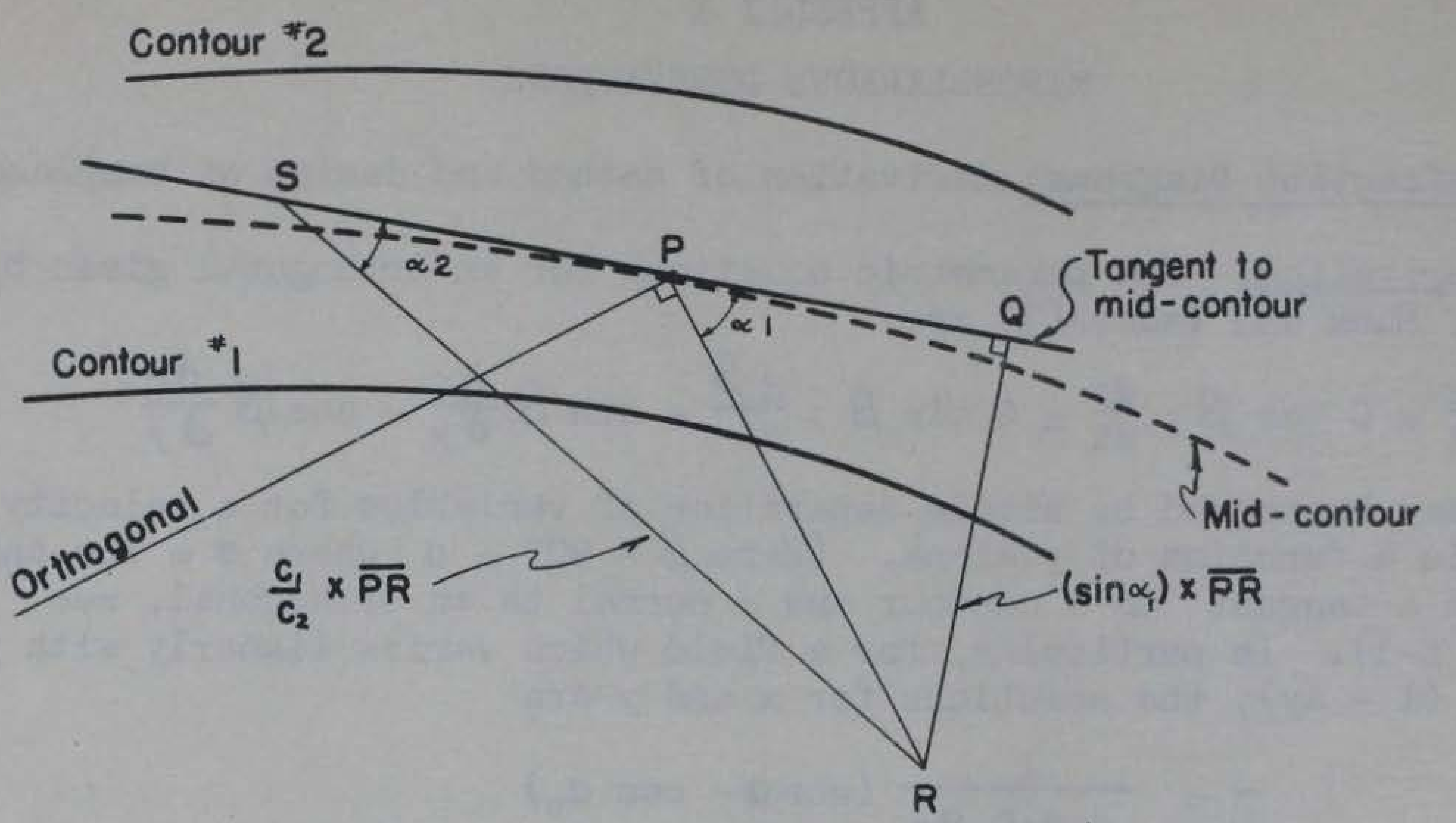


FIGURE E-1 APPLICATION OF SNELL'S LAW TO TEMPLATE CONSTRUCTION

$$\sin\,(R\ S\ P) = \frac{\sin\alpha_1 \times \underline{PR}}{C_1/C_2 \times \underline{PR}} = C_2/C_1 \sin\alpha_1 \qquad \text{(E-6)}$$

but $\quad C_2/C_1 \sin{}_1 = \sin\alpha_2$;

by Snell's law: $(R\ S\ P) = \alpha_2$ (E-7)

c. Template Construction. To construct a template on these principles, a line, the length of which represents $\underline{RP}$ in Figure E-1, is laid out. This length is entirely arbitrary; any length found convenient for use may be utilized. At one end of this line, a perpendicular is constructed. This perpendicular represents the orthogonal in Figure E-1, its intersection with the first line corresponds to point P, and the other end of the first line drawn corresponds to point R. The distance between points R and P is then divided into tenths, and when convenient, into even smaller intervals. These divisions are labelled, starting with zero at point R to 1.0 at point P, and continuing the designations on $\underline{RP}$ extended.

Figure 17 of the text is one such template. The two perpendicular lines, one marked "orthogonal," and the other, divided into tenths and hundreds marked "C_1/C_2", are the basic template. For convenience in using the R/J method (See Figure 14), a protractor centered on R ("Turning Point") with radius $\underline{RP}$, and a graph showing $\Delta\,\alpha$ as a function of R/J and C_2/C_1 (Note: not C_1/C_2) have been plotted.

2. Diffraction

a. Waves Passing a Single Breakwater - The general equation for progressive irrotational waves of small amplitude may be written;

$$\eta = \frac{AikC}{g} \cosh (kd) \cdot e^{ikCt} \cdot F(x,y) \qquad \text{(E-8)}$$

where η = the surface elevation at time t, given by the real part of equation E-8

$k = 2\pi/L$

L = the wave length

$i = \sqrt{-1}$

C = the wave velocity

$\frac{AkC}{g} \times e^{ikCt} \times \cosh (kd)$ = the maximum amplitude of wave motion.

For waves travelling in the direction of the positive y axis with no barrier present,

$$F(x,y) = e^{-iky} \qquad \text{(E-9)}$$

With a single rigid barrier present, Putnam and Arthur(105) give as a solution

$$F(x,y) = e^{-iky} \cdot f(u_1) + e^{iky} \cdot g(u_2) \qquad \text{(E-10)}$$

where $f(u_1)$, and $g(u_2)$ are given by

$$f(u_1) = \frac{1}{\sqrt{2}} \; e^{i\,\pi/4} \int_{-\infty}^{u_1} e^{-i\pi v^2/2} \, dv \qquad \text{(E-11)}$$

$$g(u_2) = \frac{1}{\sqrt{2}} \; e^{i\,\pi/4} \int_{-\infty}^{u_2} e^{-i\pi v^2/2} \, dv \qquad \text{(E-12)}$$

u_1 and u_2 being given by,

$$u_1 = \sqrt{\frac{4(r-y)}{L}} \qquad \text{(E-13)}$$

$$u_2 = \sqrt{\frac{4(r+y)}{L}} \qquad \text{(E-14)}$$

In Figure E-2 for both x and y positive (protected region) the signs of the upper limit of the integrals in equations E-11 and E-12 are negative. For x positive and y negative both limits are positive. For x negative and y positive the upper limit of the integral in E-11 is positive and that in E-12 is negative. As a simplified solution, Putnam and Arthur give

$$F(x,y) = e^{-iky} f(u) \qquad (u = u_1) \qquad \text{(E-15)}$$

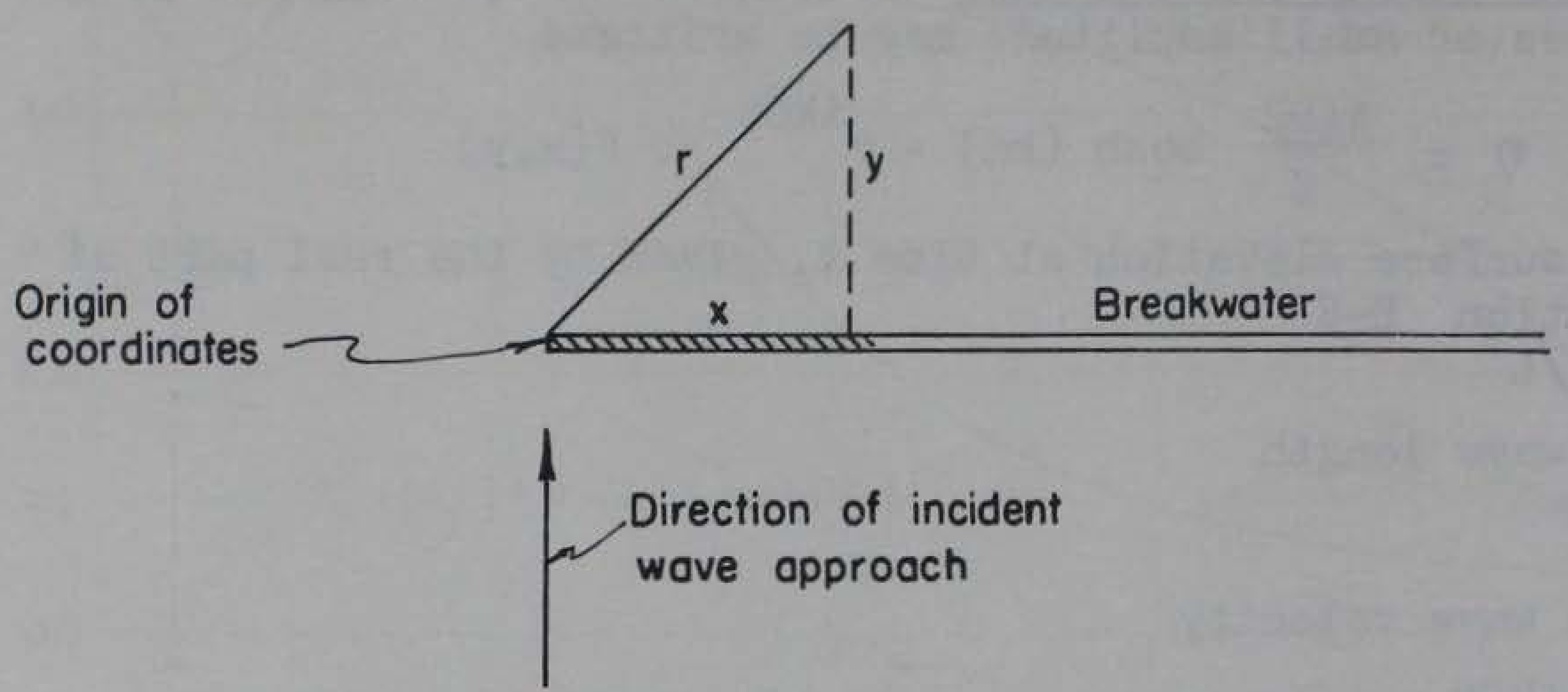


FIGURE E-2 DIFFRACTION COORDINATES FOR A SINGLE BREAKWATER

Using the simplified solution and comparing equations E-9 and E-15 it can be seen that the modulus of $f(u)$ (written $|f(u)|$) determines the relative height of waves with a barrier present to those without a barrier. That is, the ratio

$$\frac{\text{diffracted wave height}}{\text{incident wave height}} = K' = |f(u)| \tag{E-16}$$

Also from equation E-9 and E-15 diffracted waves differ in phase from undiffracted waves by the argument of $f(u)$ (written $\arg f(u)$). i.e. equation E-15 may be written

$$F(x,y) = e^{-ik\left(y - \frac{\arg f(u)}{k}\right)} \circ |f(u)| \tag{E-17}$$

Both the modulus and argument of $f(u)$ may be determined from tabulated values of the Fresnel integrals $C(u)$ and $S(u)$ since

$$C(u) - iS(u) = \int_0^u e^{-i\pi v^2/2}\, dv \quad \text{for } u > 0 \tag{E-18}$$

and it can be shown that for $u < 0$, $f(-u) = 1 - f(u)$ (E-19)

The modulus and argument of $f(u)$ are plotted on Figure E-3 as a function of u. Equation E-13 may be solved for X/L.

$$X/L = (\mp)\ 0.707\, u \sqrt{Y/L + 0.125\, u^2} \tag{E-20}$$

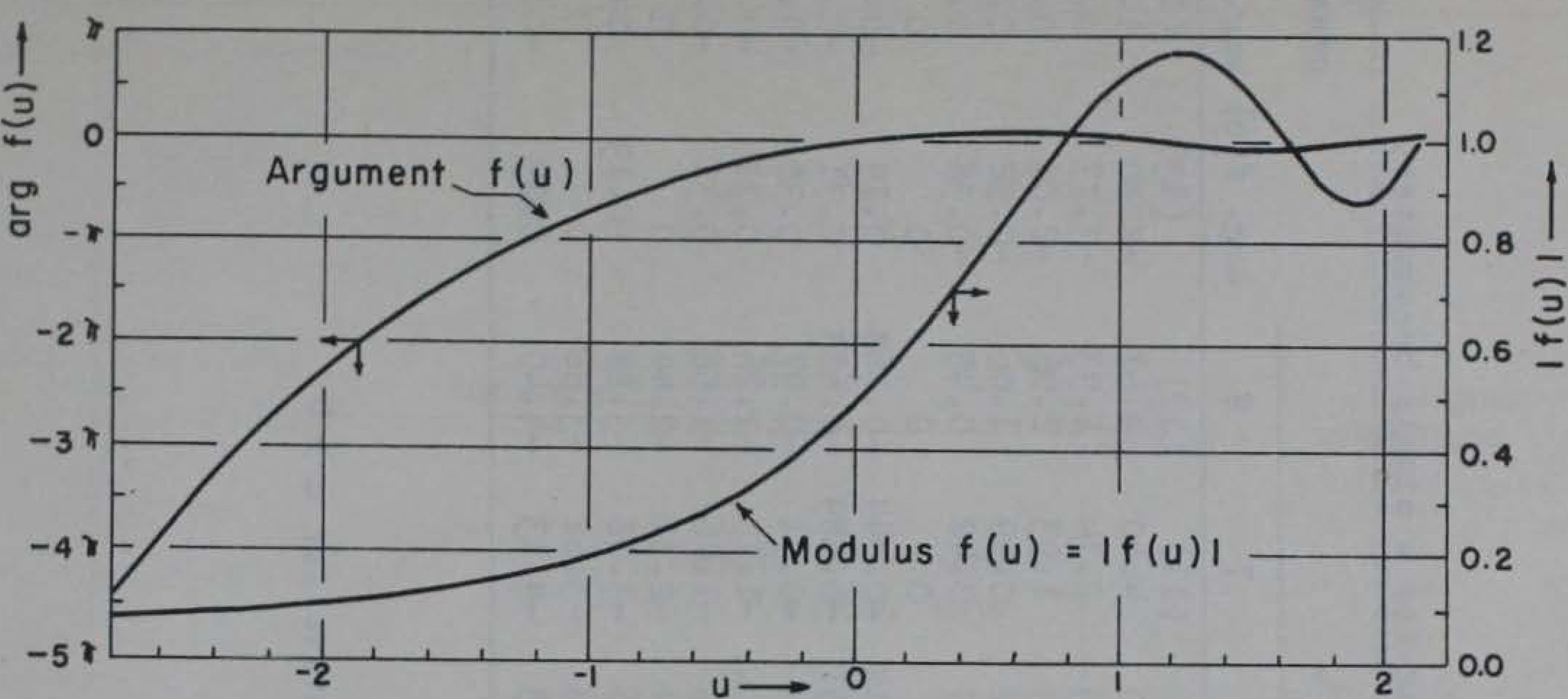


FIGURE E-3 FUNCTIONS OF u: ARGUMENT f(u) AND MODULUS f(u)

A diffraction diagram consisting of lines of equal wave height reduction (K') and wave crest advance positions may be drawn from computations similar to those shown in Table E-1. Values of K' are chosen (including the maximum and minimum at the points of reversal of the curve $|f(u)| = K'$ vs. u), and are entered in Column 1. From Figure E-3 corresponding values of u are found and entered in Column 2. Columns 3 and 4 are computed as indicated. With columns 3 and 4, for every value of Y/L in the heading of columns 5 through 12 (these values represent distances in wave lengths leeward of the end of the breakwater) corresponding values of X/L are computed with equation E-20 and are entered in the table. Curves of constant K' may be drawn from these X/L and Y/L values of columns 5 through 12. (See Figure 21 of text).

Along these curves, since u is constant, arg f(u) will also be constant; which means that the lines of constant K' may be considered to be lines of constant phase lag. The amount of crest lag in percent of wave length along any of these lines is given by

$$\text{Crest lag} = \frac{\arg f(u)}{2\,\pi} \qquad \text{(E-21)}$$

in which u is taken to correspond to each value of K', and arg f(u) is taken from Figure E-3, and entered in column 13. With arg f(u), the crest lag is computed from equation E-21 and entered in column 14.

Since the wave crest lag is constant along any one line of K', crest positions along these lines after diffraction may be plotted as on Figure 21 by marking points on them, from and normal to the undiffracted position of any wave crest, a distance equal to the calculated values of crest lag. Positive values of crest lag represent lag and negative values represent lead of the new position of wave crest. All linear dimensions on the graph, Figure 21, are divided by L the incident wave length. Note that Figure 21,

Table E-1-- Calculation of diffraction coefficients and crest lag for a single breakwater.

				(a) Values of x/L for y/L =									(b) crest lag =
K'	u	0.707 u	0.125 u^2	1	2	3	4	5	6	7	8	arg. f(u)	arg. f(u)/2π
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
0.1	-2.25	-1.59	0.635	2.03	2.58	3.03	3.42	3.78	4.10	4.40	4.67	-8.80	1.4
0.15	-1.44	-1.018	0.259	1.14	1.53	1.84	2.10	2.34	2.55	2.74	2.93	-3.77	0.60
0.2	-1.02	-0.721	0.130	0.77	1.05	1.28	1.46	1.64	1.79	1.93	2.06	-2.05	0.33
0.3	-0.528	-0.373	0.035	0.38	0.53	0.65	0.75	0.84	0.92	0.99	1.06	-0.82	0.13
0.4	-0.225	-0.159	0.006	0.16	0.23	0.28	0.32	0.36	0.39	0.42	0.45	-0.38	0.06
0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
0.6	0.184	0.130	0.0042	-0.130	-0.184	-0.227	-0.260	-0.29	-0.318	-0.344	-0.368	0.19	-0.03
0.7	0.341	0.241	0.0145	-0.243	-0.349	-0.420	-0.485	-0.54	-0.591	-0.689	-0.683	0.31	-0.05
0.8	0.486	0.343	0.0295	-0.348	-0.488	-0.597	-0.69	-0.77	-0.84	-0.91	-0.97	0.38	-0.06
0.9	0.631	0.446	0.0498	-0.456	-0.637	-0.776	-0.895	-1.00	-1.10	-1.18	-1.26	0.31	-0.05
1.0	0.719	0.551	0.076	-0.576	-0.804	-0.976	-1.12	-1.24	-1.36	-1.46	-1.56	0.31	-0.05
1.17	1.218	0.861	0.186	-0.94	-1.26	-1.525	-1.75	-1.96	-2.14	-2.31	-2.44	0	0
1.0	1.610	1.138	0.324	-1.30	-1.72	-2.06	-2.35	-2.66	-2.86	-3.08	-3.28	-0.19	0.03
0.88	1.878	1.328	0.441	-1.56	-2.03	-2.42	-2.75	-3.10	-3.38	-3.62	-3.86	0	0
1.0	2.124	1.502	0.564	-1.73	-2.24	-2.64	-3.00	-3.54	-3.85	-4.13	-4.40	0.09	-0.02

(a) $x/L = (\mp)\ 0.707\ u - \sqrt{y/L + 0.125\ u^2}$; x/L positive for $u < 0$; x/L negative for $u > 0$

(b) negative values of crest lag indicate crest <u>lead</u>.

being dimensionless, may be used for all wave conditions and for all size drawings by increasing or decreasing the scale of the diagram to correspond to the scale of the drawing.

Equation E-15 may be written in the following forms by use of the relationship E-19. When the upper limit in equation E-11 is positive:

$$F(x,y) = e^{-iky} - f \quad \text{for } x \leq 0 \quad \text{and} \begin{bmatrix} x \geq 0 \\ y \leq 0 \end{bmatrix} \tag{E-22}$$

When the upper limit in equation E-11 is negative;

$$F(x,y) = f \quad \text{for } x \geq 0, \; y \geq 0 \tag{E-23}$$

where $f = e^{-iky} f(-u)$

The simplified solution for a non-reflecting type barrier is the same. Graphically these solutions are shown on Figure E-4. Now, x does not appear directly in the solution in the form E-24 . As can be seen from E-24 only the regions of effect of the barrier are delineated without reference to any coordinate system. (See Figure E-4 for equation E-24)

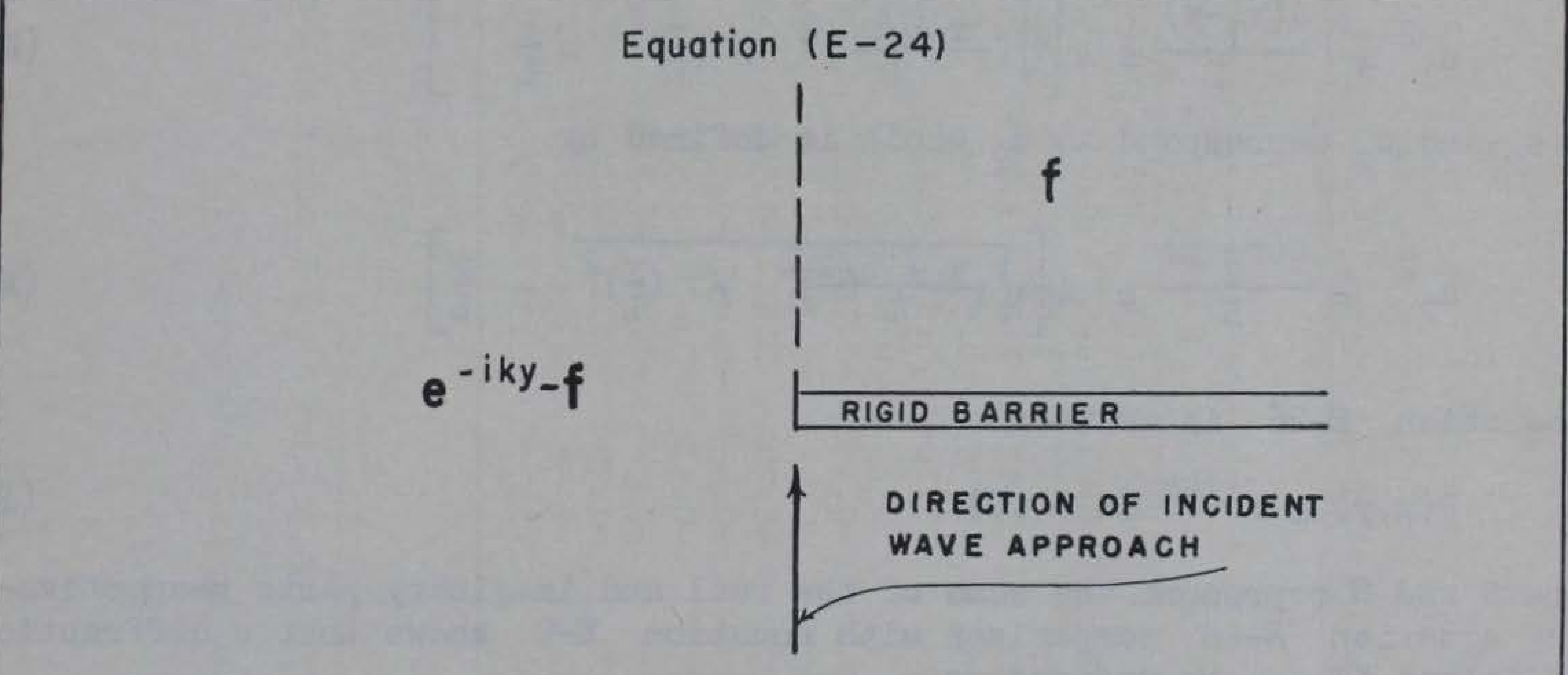


FIGURE E-4 GRAPHIC REPRESENTATION OF THE AREAS OF APPLICATION OF EQUATIONS (E-15) & (E-16)

b. Waves Passing a Gap - Gap Width Less Than 5 Wave Lengths - Blue has shown that a simplified solution of the gap diffraction problem may be found by adding together the separate solutions due to each arm of the breakwater, and subtracting e^{-iky}. In this manner we may graph the gap solution as on Figure E-5, the factors in brackets indicating the effects of the individual arms. This is the simplified solution. The complete solution contains additional terms computed from relationships similar to those in equation E-12.

The computations for the gap problem are somewhat more complex than those for a single breakwater. If we establish the following coordinate

System, (Figure E-6) equation E-25 (Figure E-5) may be written

$$F(x,y) = e^{-iky} - f_1 - f_2 \text{ for } 0 \leq x \leq b/2$$

and

$$F(x,y) = f_1 - f_2 \qquad \text{for } x \geq b/2 \tag{E-26}$$

(the solution for $x<0$ will be the mirror image of that for $x>0$).

Computations may be made by use of equation E-23 and Figure E-7, which show real and imaginary parts of $f(-u)$. Writing $f(-u)$ as

$$f(-u) = s + iw \tag{E-27}$$

equation E-26 may be written as

$$F(x,y) = e^{-iky}\left[(1-s_1-s_2) + i(-w_1-w_2)\right] \text{ for } 0 \leq x \leq b/2 \tag{E-28}$$

$$F(x,y) = e^{-iky}\left[(s_1-s_2) + i(w_1-w_2)\right] \text{ for } x \geq b/2$$

s_1 and w_1 correspond to u_1 which is defined by

$$u_1^2 = \frac{4(r_1-y)}{L} = 4\left[\sqrt{\left(\frac{x-b/2}{L}\right)^2 + \left(\frac{y}{L}\right)^2} - \frac{y}{L}\right] \tag{E-29}$$

and s_2 and w_2 correspond to u_2 whcih is defined by

$$u_2^2 = \frac{4(r_2-y)}{L} = 4\left[\sqrt{\left(\frac{x+b/2}{L}\right)^2 + \left(\frac{y}{L}\right)^2} - \frac{y}{L}\right] \tag{E-30}$$

If equation E-26 is written

$$F(x,y) = e^{-iky}\,(S + iW) \tag{E-31}$$

Where S and W represent the sums of the real and imaginary parts respectively of equation E-28 comparison with equation E-9 shows that a diffraction coefficient K' may be defined as

$$K' = \frac{|F(x,y)| \text{ for diffracted wave}}{|F(x,y)| \text{ for incident wave}} = |F(x,y)| \text{ for diffracted wave} \tag{E-32}$$

which is equal to

$$K' = \sqrt{S^2 + W^2} \tag{E-33}$$

The amount of crest lag in percent of wave length at any point is given by

$$\text{crest lag} = \frac{\arg\,(S + iW)}{2\pi} = \tan^{-1}\left(\frac{W}{S}\right) \tag{E-34}$$

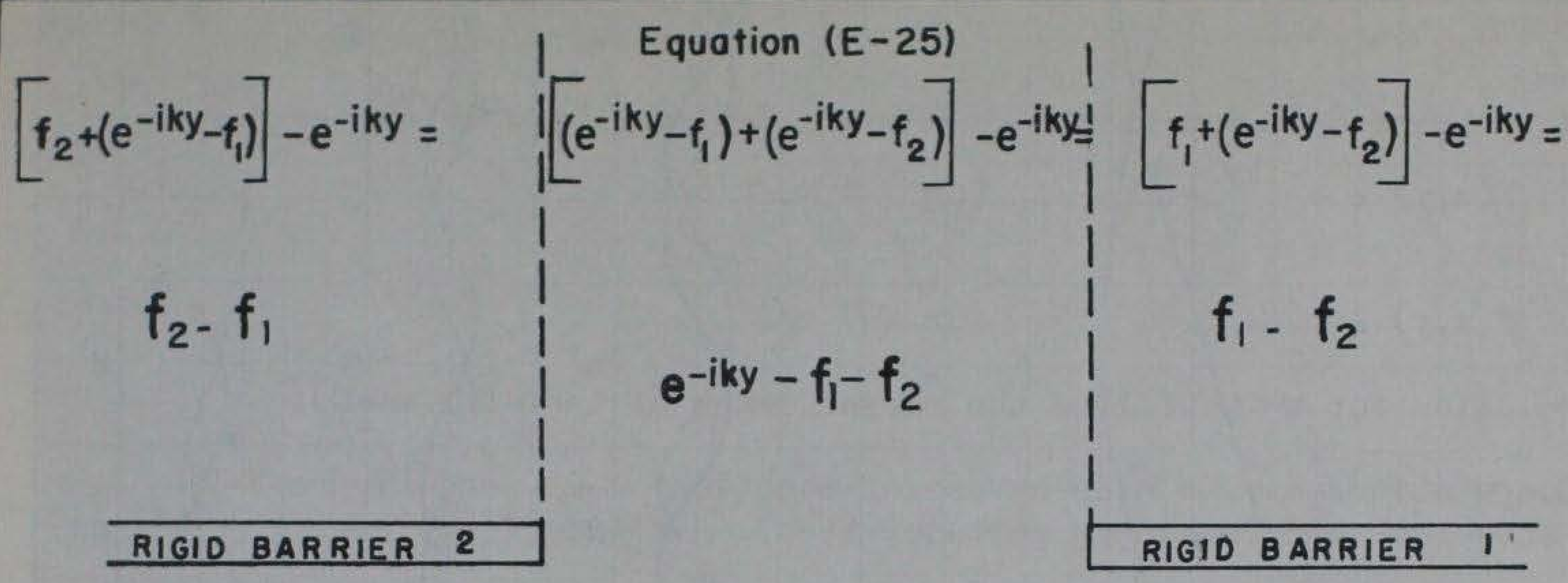


FIGURE E-5 GRAPHIC REPRESENTATION OF THE AREAS OF APPLICATION OF THE SOLUTIONS TO THE DIFFRACTION PROBLEM AT A BREAKWATER GAP.

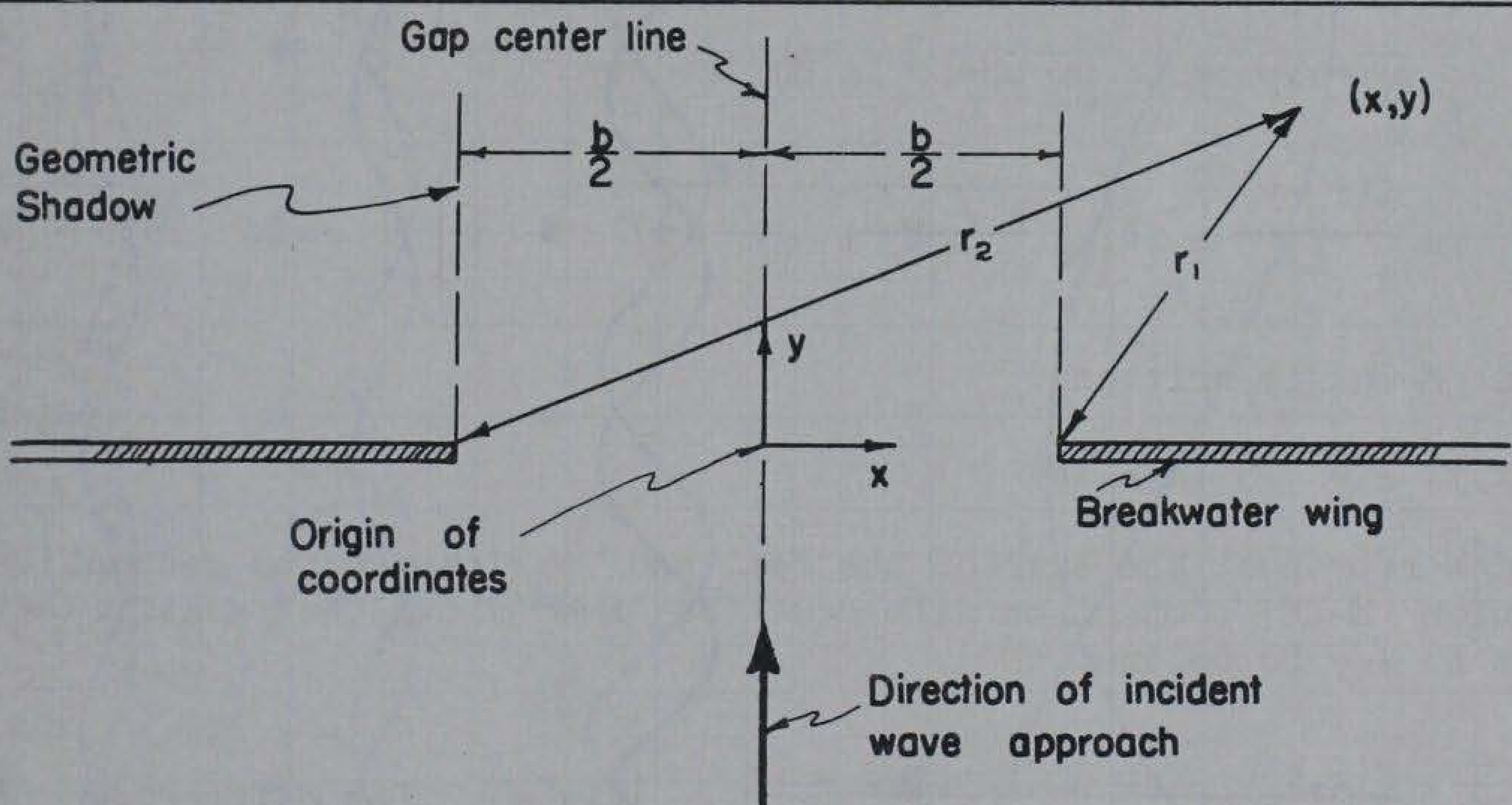


FIGURE E-6 DIFFRACTION COORDINATES AT A BREAKWATER GAP

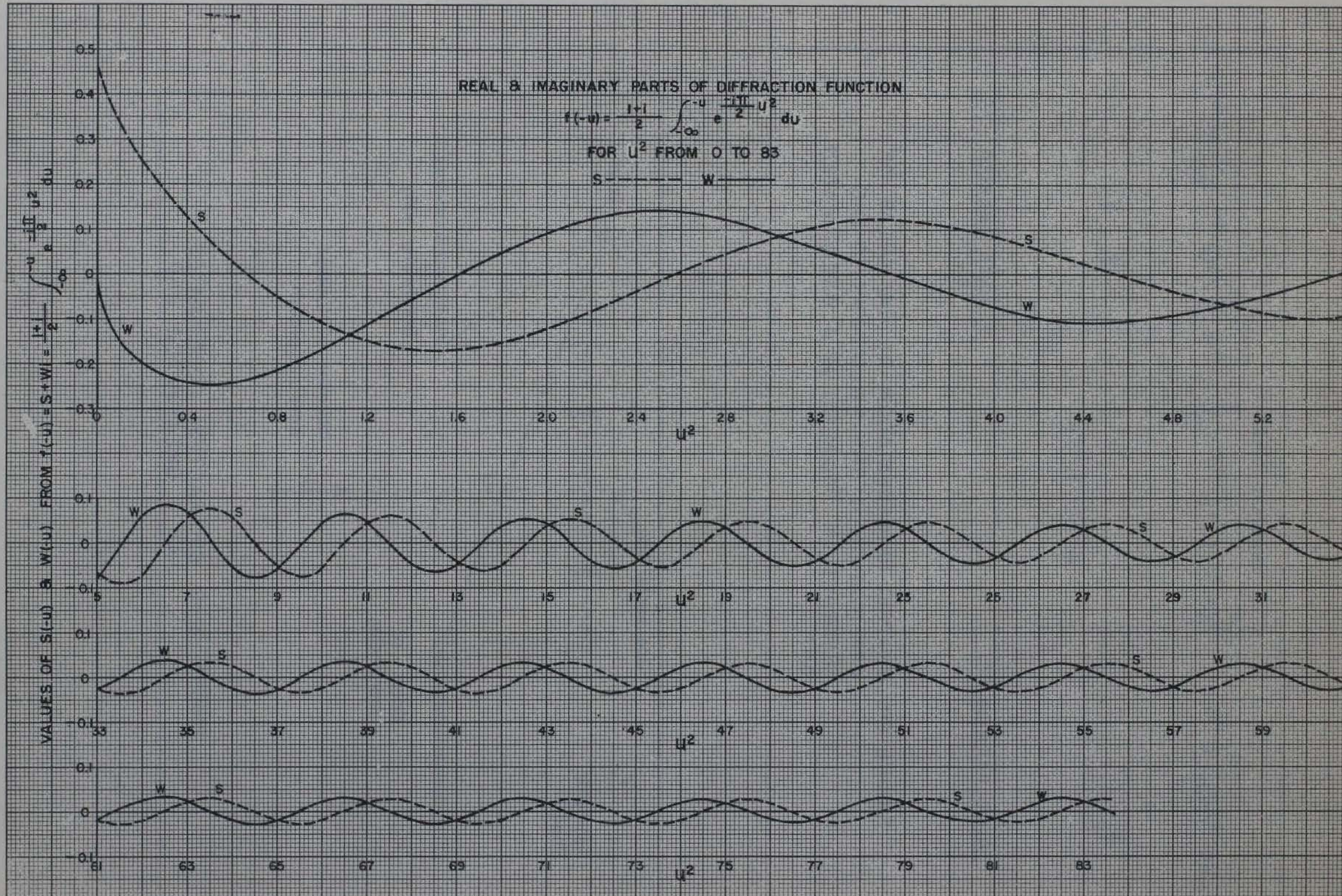

REAL & IMAGINARY PARTS OF DIFFRACTION FUNCTION
$f(-u) = \frac{1+i}{2} \int_{-\infty}^{-u} e^{\frac{-i\pi}{2}u^2} du$
FOR u^2 FROM 0 TO 83
S ------ W ——
VALUES OF S(-u) & W(-u) FROM $f(-u) = S + Wi = \frac{1+i}{2} \int_{0}^{-u} e^{\frac{-i\pi}{2}u^2} du$
u^2

FIGURE E-7

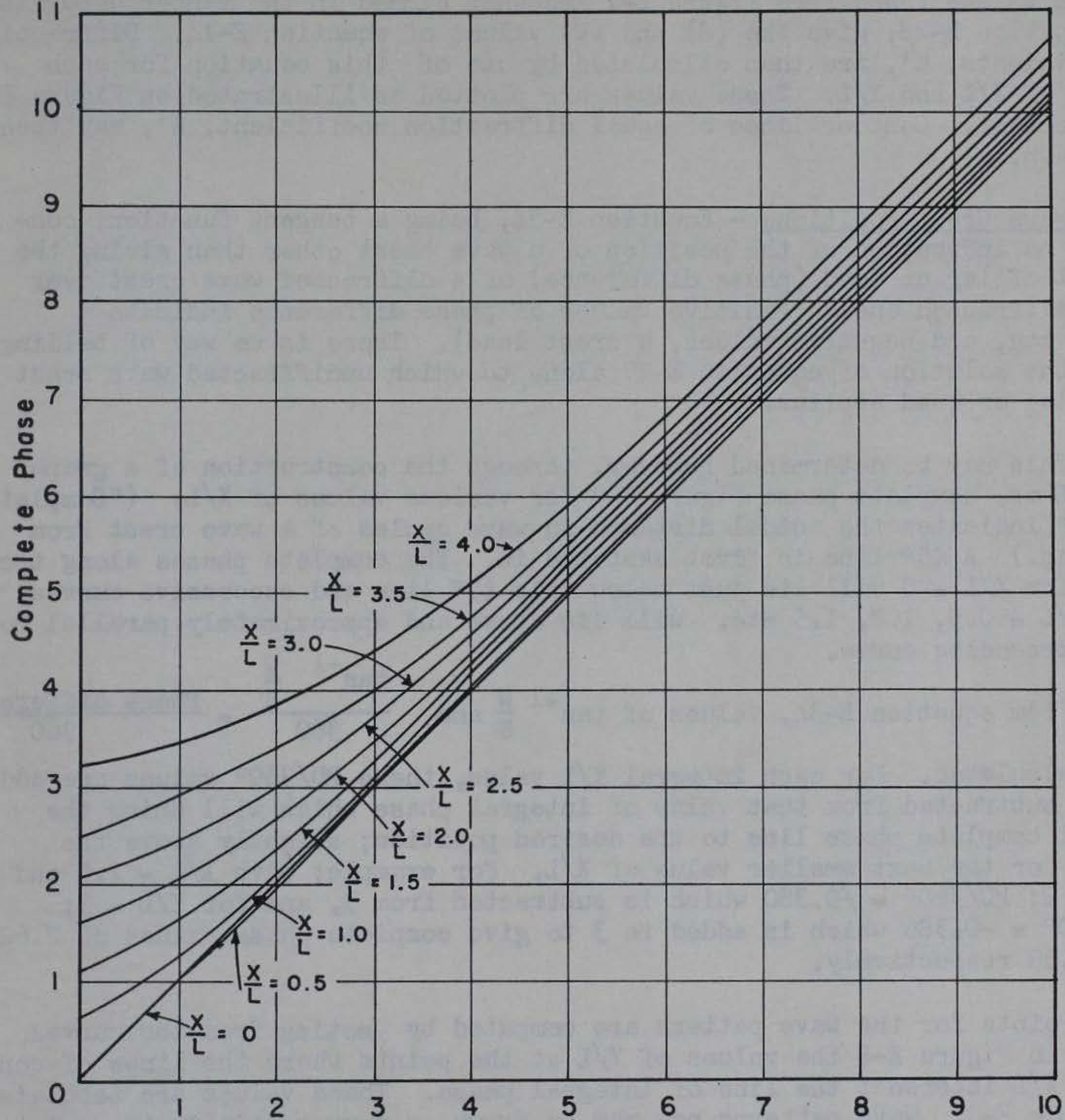


FIGURE E-8 BREAKWATER DIFFRACTION FOR GAP WIDTH OF TWO WAVE LENGTHS

Diffraction Coefficient - The manner in which lines of equal diffraction coefficients are found is illustrated in Table E-2. These computations are for diffraction effects where the gap width in twice the wave length. For a particular value of X/L and various values of Y/L, u_1^2 and u_2^2 are calculated by use of equations E-29 and E-30. With these values s_1, w_1, s_2 and w_2 are found from Figure E-7 and when summed in the manner determined by equation E-28, give the (S) and (W) values of equation E-14. Diffraction coefficients, K', are then calculated by use of this equation for each value of X/L and Y/L. These values are plotted as illustrated on Figure 23 of the text. Contour lines of equal diffraction coefficient, K', may then be drawn.

Wave Crest Positions - Equation E-34, being a tangent function, contains no indication of the position of a wave crest other than giving the amount of lag or lead (phase difference) of a diffracted wave crest over an undiffracted one. (Positive values of phase difference indicate a crest lag, and negative values, a crest lead). There is no way of telling from the solution of equation E-27 alone to which undiffracted wave crest this lag or lead applies.

This may be determined however, through the construction of a graph of Y/L vs. complete phase Figure E-8 for various values of X/L. ("Complete phase" indicates the actual distance in wave cycles of a wave crest from the gap.) A 45° line is first sketched in. The complete phases along the line for X/L = 0 will lie just below this 45° line and successive curves for X/L = 0.5, 1.0, 1.5 etc. will lie above and approximately parallel to each preceding curve.

From equation E-34, values of $\tan^{-1} \frac{W}{S}$ and

$$\frac{\tan^{-1} \frac{W}{S}}{360} = \frac{\text{Phase difference(PD)}}{360}$$

are calculated. For each integral Y/L value, these PD/360° values are added to or subtracted from that value of integral phase which will bring the actual complete phase line to the desired position; slightly above the curve for the next smaller value of X/L. For example; with X/L = 2.5 and Y/L = 2; PD/360° = +0.380 which is subtracted from 3, and for Y/L = 3; PD/360° = -0.386 which is added to 3 to give complete phase values of 2.62 and 3.39 respectively.

Points for the wave pattern are computed by noting from the curves shown in Figure E-8 the values of Y/L at the points where the lines of constant X/L intersect the line of integral phase. These values are tabulated in Table E-3. Wave patterns now may be drawn as curves joining the points having the same integral phases. The patterns for the gap width of 2L are shown in Figure 23 of the text together with the contours of equal diffraction coefficients.

Table E-2. - Calculation of diffraction coefficients and phases for 2 breakwaters; gap width 2 wave lengths

y/L	u_1^2	s_1	w_1	u_2^2	s_2	w_2	S $1-(s_1+s_2)$	W $-(w_1+w_2)$	S^2	W^2	S^2+W^2	K′	W/S	$\tan^{-1}W/S$ (degrees)	$\frac{\tan^{-1}W/S}{360}$	Complete phase
								x/L = 0								
0	4.0	+0.085	-0.07	4.0	+0.085	-0.07	+0.83	+0.14	0.689	0.020	0.709	0.84	+0.169	+ 9.6	+0.0267	
1	1.66	-0.16	+0.02	1.66	-0.16	+0.02	+1.32	-0.04	1.742	0.002	1.744	1.32	-0.030	- 1.7	-0.0047	1.01
2	0.944	-0.09	-0.18	0.944	-0.09	-0.18	+1.18	+0.36	1.392	0.030	1.522	1.23	+0.305	+17.0	+0.047	1.95
3	0.65	0.00	-0.24	0.65	+0.00	-0.24	+1.00	+0.48	1.0	0.230	1.230	1.11	+0.48	+25.6	+0.071	2.93
4	0.493	+0.08	-0.245	0.493	+0.08	-0.245	+0.84	+0.49	0.706	0.240	0.946	0.97	+0.583	+30.2	+0.084	3.92
5	0.396	+0.13	-0.24	0.396	+0.13	-0.24	+0.74	+0.48	0.548	0.230	0.778	0.88	+0.649	+33.0	+0.092	4.91
6	0.328	+0.17	-0.23	0.328	+0.17	-0.23	+0.66	+0.46	0.436	0.212	0.648	0.81	+0.698	+34.9	+0.097	5.90
7	0.284	+0.19	-0.22	0.284	+0.19	-0.22	+0.62	+0.44	0.384	0.194	0.578	0.76	+0.710	+35.4	+0.098	6.90
8	0.248	+0.22	-0.215	0.248	+0.22	-0.215	+0.56	+0.43	0.314	0.185	0.499	0.71	+0.768	+37.5	+0.104	7.90
9	0.220	+0.24	-0.205	0.220	+0.24	-0.205	+0.52	+0.41	0.270	0.168	0.48	0.66	+0.788	+38.3	+0.106	8.89
10	0.199	+0.25	-0.20	0.199	+0.25	-0.20	+0.50	+0.40	0.250	0.160	0.410	0.64	+0.80	+38.7	+0.108	9.89
								x/L = 0.5								
0	2.0	-0.12	+0.10	6.0	-0.07	+0.065	+1.19	-0.165	1.416	0.027	1.443	1.20	-0.139	- 7.9	-0.022	0.02
1	0.472	+0.09	-0.245	3.21	+0.11	+0.06	+0.80	+0.185	0.640	0.034	0.674	0.82	+0.232	+13.1	+0.0364	0.96
2	0.248	+0.22	-0.22	2.0	-0.12	+0.10	+0.90	+0.12	0.810	0.014	0.824	0.91	+0.134	+ 7.7	+0.0214	1.98
3	0.164	+0.28	-0.18	1.42	-0.17	-0.05	+0.89	+0.23	0.792	0.053	0.845	0.92	+0.259	+14.5	+0.0403	2.96
4	0.124	+0.31	-0.17	1.09	-0.13	-0.145	+0.82	+0.31	0.672	0.096	0.772	0.88	+0.378	+20.7	+0.0584	3.94
5	0.100	+0.33	-0.15	0.88	-0.07	-0.20	+0.74	+0.35	0.548	0.123	0.671	0.82	+0.473	+25.3	+0.0704	4.93
6	0.084	+0.35	-0.14	0.75	-0.03	-0.22	+0.72	+0.36	0.518	0.130	0.648	0.80	+0.500	+26.6	+0.0739	5.93
7	0.072	+0.37	-0.13	0.636	+0.01	-0.235	+0.62	+0.365	0.384	0.133	0.517	0.72	+0.588	+30.5	+0.0847	6.92
8	0.064	+0.38	-0.12	0.551	+0.06	-0.24	+0.56	+0.36	0.314	0.130	0.444	0.67	+0.643	+32.8	+0.0910	7.91
9	0.056	+0.39	-0.11	0.504	+0.08	-0.245	+0.53	+0.355	0.281	0.126	0.407	0.64	+0.670	+33.8	+0.0940	8.91
10	0.052	+0.40	-0.10	0.448	+0.10	-0.245	+0.50	+0.345	0.250	0.119	0.369	0.61	+0.690	+34.6	+0.0961	9.90
								x/L = 1.								
0	0	0.5	0	8.0	+0.06	-0.05	+0.44	+0.05	0.194	0.003	0.197	0.44	+0.144	+ 6.5	+0.018	
1	0	0.5	0	4.95	-0.06	-0.08	+0.56	+0.08	0.314	0.006	0.320	0.57	+0.143	+ 8.3	+0.023	0.98
2	0	0.5	0	3.31	+0.11	+0.05	+0.39	-0.05	0.152	0.003	0.155	0.39	-0.128	- 7.3	-0.020	2.02
3	0	0.5	0	2.42	-0.03	+0.14	+0.53	-0.14	0.281	0.020	0.301	0.55	-0.264	-14.8	-0.041	3.04
4	0	0.5	0	1.89	-0.14	+0.07	+0.64	-0.07	0.410	0.005	0.415	0.64	-0.109	- 6.2	-0.017	4.02
5	0	0.5	0	1.54	-0.17	-0.02	+0.67	+0.02	0.449	0.0004	0.449	0.67	+0.030	+ 1.7	+0.0047	5.00
6	0	0.5	0	1.30	-0.16	-0.09	+0.66	+0.09	0.436	0.008	0.444	0.67	+0.136	+ 7.8	+0.021	5.98
7	0	0.5	0	1.12	-0.13	-0.13	+0.63	+0.13	0.397	0.017	0.414	0.64	+0.206	+11.6	+0.032	6.97
8	0	0.5	0	0.985	-0.10	-0.17	+0.60	+0.17	0.360	0.029	0.389	0.62	+0.283	+15.8	+0.044	7.96
9	0	0.5	0	0.875	-0.07	-0.20	+0.57	+0.20	0.325	0.040	0.365	0.60	+0.351	+19.3	+0.054	8.95
10	0	0.5	0	0.792	-0.04	-0.21	+0.54	+0.21	0.292	0.044	0.336	0.58	+0.389	+21.3	+0.059	9.94

Table E-2 (cont'd)

y/L	U_1^2	s_1	w_1	u_2^2	s_2	w_2	S (s_1-s_2)	W (w_1-w_2)	S^2	W^2	S^2+W^2	K'	W/S	$\tan^{-1}W/S$ (degrees)	$\tan^{-1}W/S$ / 360	Complete phase
								x/L = 1.5								
0	2.0	-0.12	+0.10	10.0	-0.055	+0.05	-0.065	+0.05	0.0042	0.0025	0.0067	0.09	-0.782	+142.0	+0.395	0.60
1	0.472	+0.090	-0.245	6.78	+0.04	+0.08	+0.05	-0.325	0.0025	0.1056	0.1081	0.33	-6.50	-81.3	-0.226	1.23
2	0.248	+0.22	-0.20	4.81	+0.04	+0.09	+0.26	-0.13	0.0676	0.0169	0.0845	0.29	-0.50	-26.6	-0.074	2.07
3	0.164	+0.028	-0.18	3.62	+0.12	-0.01	+0.16	-0.17	0.0256	0.0289	0.0545	0.23	-1.06	-46.7	-0.129	3.13
4	0.124	+0.32	-0.17	2.87	+0.06	+0.12	+0.26	-0.29	0.0676	0.0841	0.1517	0.39	-1.11	-48.0	-0.133	4.13
5	0.100	+0.33	-0.15	2.36	-0.05	+0.14	+0.38	-0.29	0.1444	0.0841	0.2285	0.48	-0.673	-37.3	-0.104	5.10
6	0.084	+0.35	-0.14	2.00	-0.12	+0.10	+0.47	-0.24	0.2289	0.0576	0.2785	0.53	-0.511	-27.1	-0.080	6.08
7	0.072	+0.37	-0.13	1.73	-0.16	+0.04	+0.53	-0.17	0.2809	0.0289	0.3098	0.56	-0.321	-17.8	-0.049	7.05
8	0.064	+0.38	-0.12	1.53	-0.17	-0.02	+0.55	-0.10	0.3025	0.0100	0.3125	0.56	-0.182	-10.3	-0.029	8.03
9	0.056	+0.39	-0.11	1.36	-0.16	-0.07	+0.55	-0.04	0.3025	0.0016	0.3041	0.55	-0.073	- 4.2	-0.017	9.02
10	0.052	+0.40	-0.10	1.23	-0.15	-0.11	+0.55	+0.01	0.3025	0.0001	0.3026	0.55	+0.018	+ 1.0		10.00
								x/L = 2								
0	4.0	+0.085	-0.07	12.0	+0.04	-0.04	+0.45	-0.03	0.002	0.001	0.003	0.055	-0.666	-33.7	-0.094	1.09
1	1.66	-0.16	+0.02	8.65	-0.01	-0.07	-0.15	+0.09	0.023	0.008	0.031	0.176	-0.600	+149.0	+0.414	1.59
2	0.944	-0.09	-0.18	6.42	-0.01	+0.09	-0.08	-0.27	0.006	0.073	0.079	0.281	+3.38	-106.5	-0.296	2.30
3	0.64	+0.01	-0.24	4.97	-0.06	-0.08	+0.07	-0.16	0.005	0.026	0.031	0.176	-2.29	-66.4	-0.184	3.18
4	0.493	+0.08	-0.245	4.0	+0.085	-0.07	-0.01	-0.175	0.000	0.031	0.031	0.176	+17.5	-93.3	-0.258	4.26
5	0.396	+0.13	-0.24	3.32	+0.12	+0.04	+0.01	-0.28	0.000	0.078	0.078	0.279	-28.0	-88.0	-0.244	5.24
6	0.328	+0.17	-0.23	2.83	+0.05	+0.12	+0.12	-0.35	0.014	0.123	0.137	0.370	-2.92	-71.1	-0.199	6.20
7	0.284	+0.19	-0.22	2.46	-0.02	+0.14	+0.21	-0.36	0.44	0.130	0.174	0.417	-1.71	-59.7	-0.166	7.17
8	0.248	+0.22	-0.215	2.17	-0.09	+0.12	+0.31	-0.335	0.096	0.112	0.208	0.456	-1.08	-47.2	-0.131	8.13
9	0.220	+0.24	-0.205	1.95	-0.13	+0.09	+0.37	-0.295	0.137	0.087	0.224	0.473	-0.798	-38.6	-0.107	9.11
10	0.199	+0.25	-0.20	1.76	-0.15	+0.04	+0.40	-0.24	0.160	0.058	0.218	0.467	-0.60	-31.0	-0.086	10.09
								x/L = 2.5								
0	6.0	-0.07	+0.065	14.0	-0.04	+0.04	-0.03	+0.025	0.0009	0.0063	0.0072	0.085	-0.83	+140.2	+0.39	1.61
1	3.21	+0.11	+0.06	10.58	+0.01	+0.07	+0.10	-0.01	0.01	0.0001	0.0101	0.1005	-0.10	- 5.7	-0.058	2.06
2	2.0	-0.12	+0.10	8.12	+0.05	-0.06	-0.17	+0.16	0.0289	0.0256	0.0545	0.233	-0.94	+136.8	+0.380	2.62
3	1.42	-0.17	-0.05	6.44	-0.01	+0.09	-0.16	-0.14	0.0256	0.196	0.0452	0.213	+0.87	-139.0	-0.386	3.39
4	1.09	-0.13	-0.145	5.25	-0.09	-0.04	-0.04	-0.105	0.0016	0.0110	0.0126	0.112	+2.63	-110.8	-0.308	4.31
5	0.88	-0.07	-0.20	4.42	+0.02	-0.10	-0.09	-0.10	0.0081	0.01	0.0181	0.135	+1.11	-132.0	-0.367	5.37
6	0.75	-0.03	-0.22	3.78	+0.11	-0.04	-0.14	-0.18	0.0196	0.0324	0.0520	0.226	+1.29	-127.8	-0.355	6.36
7	0.636	+0.01	-0.235	3.30	+0.12	+0.05	-0.11	-0.285	0.0121	0.0812	0.0933	0.305	+2.59	-111.3	-0.309	7.31
8	0.551	+0.06	-0.24	2.93	+0.07	+0.11	-0.01	-0.35	0.0001	0.1225	0.1225	0.351	+35.0	- 91.6	-0.255	8.26
9	0.504	+0.08	-0.245	2.62	+0.01	+0.14	+ 0.07	-0.385	0.0049	0.1482	0.1531	0.392	-5.50	- 79.7	-0.222	9.22
10	0.448	+0.10	-0.245	2.38	-0.04	+0.14	0.14	-0.385	0.0196	0.1482	0.1678	0.41	-2.75	- 70.0	-0.195	10.20

Table F-2 (cont'd)

y/L	u_1^2	s_1	w_1	u_2^2	s_2	w_2	S (s_1-s_2)	W (w_1-w_2)	S^2	W^2	S^2+W^2	K′	W/S	$\tan^{-1}W/S$ (degrees)	$\frac{\tan^{-1}W/S}{360}$	Complete phase
								x/L =3								
0	8.0	+0.06	-0.05	16.0	+0.04	-0.04	+0.02	-0.01	0.000	0.000	0.01	0.10	-0.5	- 26.6	-0.074	2.07
1	4.95	-0.06	-0.08	12.5	+0.0	-0.06	-0.06	-0.02	0.004	0.000	0.004	0.063	+0.333	-161.6	-0.449	2.45
2	3.31	+0.11	+0.05	9.9	-0.06	+0.04	+0.17	+0.01	0.029	0.000	0.029	0.17	+0.059	+ 3.4	+0.009	2.99
3	2.42	-0.03	+0.14	8.0	+0.06	-0.05	-0.09	+0.19	0.008	0.036	0.044	0.21	-2.11	+115.4	+0.321	3.68
4	1.89	-0.14	+0.07	6.62	+0.02	+0.09	-0.16	-0.02	0.26	0.000	0.026	0.16	+0.125	-172.6	-0.480	4.48
5	1.54	-0.17	-0.02	5.61	-0.09	+0.01	-0.08	-0.03	0.006	0.001	0.007	0.084	+0.38	-159.2	-0.444	5.44
6	1.30	-0.16	-0.29	4.85	-0.04	-0.09	-0.12	+0.0	0.014	0.000	0.014	0.12	0	+180.0	+0.50	6.50
7	1.12	-0.13	-0.13	4.25	+0.05	-0.10	-0.18	-0.03	0.032	0.001	0.033	0.18	+0.17	-170.4	-0.474	7.41
8	0.985	-0.10	-0.17	3.78	+0.11	-0.04	-0.21	-0.13	0.044	0.017	0.061	0.25	+0.618	-148.3	-0.413	8.41
9	0.875	-0.07	-0.20	3.39	+0.12	+0.03	-0.19	-0.23	0.036	0.053	0.089	0.30	+1.21	-129.5	-0.359	9.36
10	0.792	-0.04	-0.21	3.08	+0.09	+0.08	-0.13	-0.29	0.017	0.084	0.101	0.32	+2.23	-114.2	-0.317	10.32
								x/L = 3.5								
0	10.0	-0.555	+0.05	18.0	-0.035	+0.035	-0.02	-0.015	0.0004	0.0002	0.0006	0.024	+0.75	-143.1	-0.449	2.45
1	6.78	+0.04	+0.08	14.4	-0.01	+0.06	+0.05	+0.02	0.0025	0.0004	0.0029	0.054	+0.40	+ 26.6	+0.061	2.94
2	4.81	-0.04	-0.09	11.70	+0.06	-0.02	-0.10	-0.07	0.010	0.0049	0.0149	0.039	+0.07	-145.0	-0.404	3.40
3	3.62	+0.12	-0.01	9.63	-0.07	+0.01	+0.19	-0.02	0.0361	0.0004	0.0365	0.191	-0.105	+ 6.0	-0.017	4.02
4	2.87	+0.06	+0.12	8.08	+0.05	-0.06	+0.01	+0.18	0.0001	0.0324	0.0325	0.080	+18.0	+ 86.7	+0.241	4.76
5	2.38	-0.05	+0.14	6.90	+0.05	+0.07	-0.10	+0.07	0.01	0.0049	0.0149	0.122	-0.70	+142.1	+0.404	5.60
6	2.00	-0.12	-0.10	6.00	-0.07	+0.06	-0.05	+0.04	0.0025	0.0016	0.0041	0.064	-0.80	+141.3	+0.393	6.61
7	1.73	-0.16	-0.04	5.29	-0.09	-0.04	-0.07	+0.08	0.0049	0.0064	0.0113	0.106	-1.14	+131.3	+0.365	7.64
8	1.53	-0.17	-0.02	4.72	-0.03	-0.10	-0.14	+0.08	0.0256	0.0064	0.0320	0.179	-0.572	+150.2	+0.417	8.58
9	1.36	-0.16	-0.07	4.25	+0.05	-0.10	-0.21	+0.03	0.0441	0.0009	0.0450	0.212	-0.143	+171.8	+0.478	9.52
10	1.23	-0.15	-0.11	3.86	+0.10	-0.05	-0.25	-0.06	0.0625	0.0036	0.0661	0.257	+0.240	-166.5	-0.463	10.46
								x/L = 4.0								
0	12.0	+0.04	-0.04	20.0	+0.04	-0.035	0	-0.005	0	0.000025	0.000025	0.005		+ 90.0	+0.25	3.25
1	8.65	-0.01	-0.07	16.35	+0.02	-0.05	-0.03	-0.02	0.0009	0.0004	0.0013	0.036	+0.666	-146.3	-0.407	3.41
2	6.42	-0.01	+0.09	13.54	-0.06	0	+0.05	+0.09	0.0025	0.0081	0.0106	0.103	+1.8	+ 61.0	+0.169	3.82
3	4.97	-0.06	-0.08	11.30	+0.06	+0.02	-0.12	-0.10	0.0144	0.01	0.0244	0.156	+0.832	-140.1	-0.390	4.39
4	4.0	+0.087	-0.07	9.60	-0.07	-0.01	+0.155	-0.08	0.0240	0.0064	0.0304	0.174	-0.516	- 12.7	-0.076	5.08
5	3.32	+0.12	+0.04	8.28	+0.03	-0.07	+0.09	+0.11	0.0081	0.0121	0.0202	0.142	+1.22	+ 53.2	+0.141	5.86
6	2.83	+0.05	+0.12	7.24	+0.07	+0.04	-0.02	+0.08	0.0004	0.0064	0.0068	0.083	-4.0	+104.0	+0.289	6.71
7	2.46	-0.02	+0.14	6.41	-0.01	+0.09	-0.01	+0.05	0.0001	0.0025	0.0026	0.051	-5.1	+101.3	+0.280	7.72
8	2.17	-0.09	+0.12	5.73	-0.09	+0.03	0	+0.09	0	0.0081	0.0081	0.09		+ 90.0	+0.25	8.75
9	1.95	-0.13	+0.09	5.18	-0.09	-0.05	-0.04	+0.14	0.0016	0.0196	0.0202	0.142	-3.5	+108.2	+0.301	9.76
10	1.76	-0.15	+0.04	4.72	-0.03	-0.10	-0.12	+0.14	0.0144	0.0196	0.0340	0.185	-1.16	+130.8	+0.364	10.64

TABLE E -3. - PLOTTING VALUES OF y/L and x/L FOR DIFFRACTION DIAGRAM FOR TWO BREAKWATERS; GAP WIDTH = 2 WAVE LENGTHS.

Comp. phase	$\frac{x}{L}$ 0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
	(Values of y/L)								
0	0	0	0						
1	1.03	1.	0.95	0.68					
2	2.06	2.02	2.00	1.92	1.62	0.95			
3	3.08	3.04	2.96	2.88	2.72	2.50	2.00	1.18	
4	4.08	4.05	3.98	3.90	3.74	3.62	3.40	2.96	2.32
5	5.10	5.07	5.00	4.90	4.80	4.65	4.50	4.30	3.92
6	6.10	6.07	6.01	5.92	5.80	5.65	5.52	5.38	5.15
7	7.10	7.08	7.03	6.95	6.82	6.68	6.52	6.36	6.20
8	8.10	8.08	8.05	7.95	7.85	7.72	7.57	7.41	7.22
9	9.10	9.09	9.05	8.97	8.85	8.76	8.62	8.45	8.25
10				10.00	9.90	9.80	9.69	9.45	9.26

APPENDIX F

EXAMPLE BEACH EROSION CONTROL STUDY

APPENDIX F

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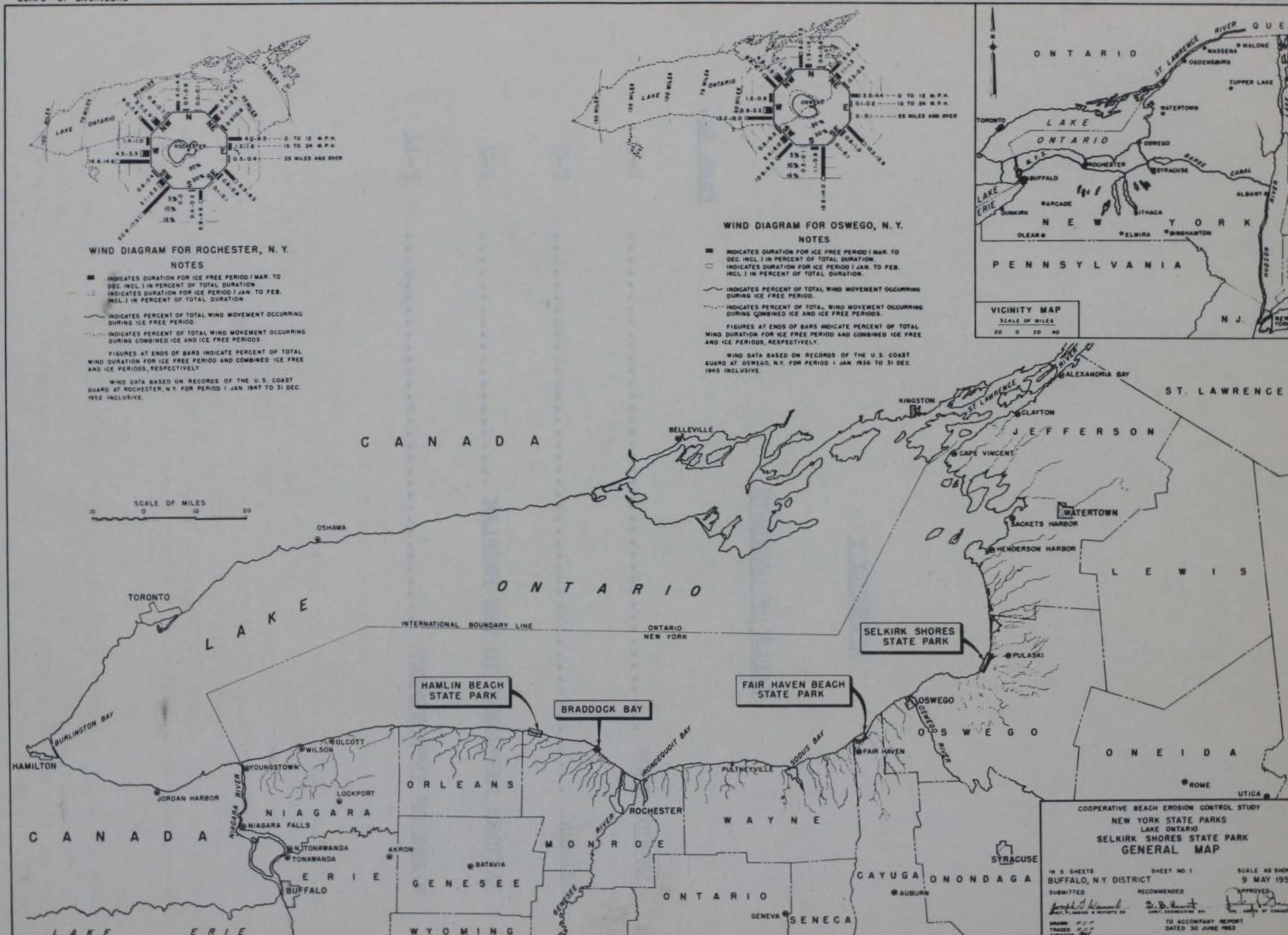
WIND DIAGRAM FOR ROCHESTER, N. Y.
NOTES
INDICATES DURATION FOR ICE FREE PERIOD (MAR. TO DEC. INCL.) IN PERCENT OF TOTAL DURATION.
INDICATES DURATION FOR ICE PERIOD (JAN. TO FEB. INCL.) IN PERCENT OF TOTAL DURATION.
INDICATES PERCENT OF TOTAL WIND MOVEMENT OCCURRING DURING ICE FREE PERIOD.
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FIGURES AT ENDS OF BARS INDICATE PERCENT OF TOTAL WIND DURATION FOR ICE FREE PERIOD AND COMBINED ICE FREE AND ICE PERIODS, RESPECTIVELY.
WIND DATA BASED ON RECORDS OF THE U. S. COAST GUARD AT ROCHESTER, N.Y. FOR PERIOD 1 JAN. 1947 TO 31 DEC. 1952 INCLUSIVE.
0 TO 12 M.P.H.
13 TO 24 M.P.H.
25 MILES AND OVER
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NOTES
INDICATES DURATION FOR ICE FREE PERIOD (MAR. TO DEC. INCL.) IN PERCENT OF TOTAL DURATION.
INDICATES DURATION FOR ICE PERIOD (JAN. TO FEB. INCL.) IN PERCENT OF TOTAL DURATION.
INDICATES PERCENT OF TOTAL WIND MOVEMENT OCCURRING DURING ICE FREE PERIOD.
INDICATES PERCENT OF TOTAL WIND MOVEMENT OCCURRING DURING COMBINED ICE AND ICE FREE PERIODS.
FIGURES AT ENDS OF BARS INDICATE PERCENT OF TOTAL WIND DURATION FOR ICE FREE PERIOD AND COMBINED ICE FREE AND ICE PERIODS, RESPECTIVELY.
WIND DATA BASED ON RECORDS OF THE U. S. COAST GUARD AT OSWEGO, N.Y. FOR PERIOD 1 JAN. 1936 TO 31 DEC. 1945 INCLUSIVE.
VICINITY MAP
SCALE OF MILES
20 0 20 40
ONTARIO
QUE.
ST. LAWRENCE RIVER
MALONE
MASSENA
OGDENSBURG
TUPPER LAKE
LAKE CHAMPLAIN
WATERTOWN
TORONTO
LAKE ONTARIO
OSWEGO
ROCHESTER
SYRACUSE
BARGE CANAL
BUFFALO
LAKE ERIE
DUNKIRK
ARCADE
ITHACA
ALBANY
NEW YORK
OLEAN
ELMIRA
BINGHAMTON
HUDSON RIVER
PENNSYLVANIA
N. J.
NEW YORK
SCALE OF MILES
10 0 10 20
CANADA
KINGSTON
ALEXANDRIA BAY
ST. LAWRENCE
ST. LAWRENCE RIVER
CLAYTON
BELLEVILLE
JEFFERSON
CAPE VINCENT
WATERTOWN
SACKETS HARBOR
HENDERSON HARBOR
OSHAWA
LEWIS
LAKE ONTARIO
TORONTO
INTERNATIONAL BOUNDARY LINE
ONTARIO
NEW YORK
SELKIRK SHORES STATE PARK
PULASKI
HAMLIN BEACH STATE PARK
BRADDOCK BAY
FAIR HAVEN BEACH STATE PARK
OSWEGO
OSWEGO RIVER
BURLINGTON BAY
HAMILTON
OLCOTT
WILSON
YOUNGSTOWN
IRONDEQUOIT BAY
SODUS BAY
FAIR HAVEN
PULTNEYVILLE
OSWEGO
ONEIDA
ROME
UTICA
JORDAN HARBOR
NIAGARA RIVER
LOCKPORT
ORLEANS
NIAGARA
NIAGARA FALLS
ROCHESTER
GENESEE RIVER
MONROE
WAYNE
CANADA
N. TONAWANDA
TONAWANDA
AKRON
SYRACUSE
ERIE
BATAVIA
GENESEE
CAYUGA
ONONDAGA
BUFFALO
AUBURN
ONTARIO
GENEVA
SENECA
LAKE ERIE
WYOMING
COOPERATIVE BEACH EROSION CONTROL STUDY
NEW YORK STATE PARKS
LAKE ONTARIO
SELKIRK SHORES STATE PARK
GENERAL MAP
IN 5 SHEETS
SHEET NO. 1
SCALE AS SHOWN
BUFFALO, N.Y. DISTRICT
9 MAY 1953
SUBMITTED
RECOMMENDED
APPROVED
TO ACCOMPANY REPORT DATED 30 JUNE 1953
DRAWN
TRACED
CHECKED

PLATE F-1

APPENDIX F

EXAMPLE BEACH EROSION CONTROL STUDY

INTRODUCTION

1. The purpose of this Appendix is to present an example beach erosion problem to illustrate the relationship of the physical factors to functional planning, and structural analysis and design. The present example is a brief summary of a problem and its analysis at Selkirk Shores State Park, New York, prepared by Buffalo District, Corps of Engineers, U. S. Army, June 1953.

2. Although the complete structural design is not carried through in the example problem, the overall analysis is similar to that presented in Chapter 5 of the preceding text.

GENERAL

3. Purpose. The purpose of this beach erosion control study was to develop the most suitable plans for the protection of the shore of Selkirk Shores State Park. Consideration was to be given to restoring a beach for the dual purpose of shore protection and recreational use and protecting the remainder of the shore by the most economical means.

4. Description. Lake Ontario, the smallest and most easterly of the Great Lakes chain, has an area of 7,540 square miles. It is 190 miles long and 50 miles wide with its axis in an east-west direction. It is comparatively deep, the maximum recorded depth being 778 feet. Low water datum, to which elevations are referred, is 244.0 feet above mean tide at New York City. The flow from Lake Erie through the Niagara River enters the lake on the south shore approximately 35 miles east of its western extremity, while the outlet at the head of the St. Lawrence River is located at the northeastern extremity of the lake (see Plate F-1).

5. Between the west end of the lake and Sodus Bay, a distance of about 140 miles, the south shore of Lake Ontario extends generally in an east-west direction with intermediate sections having flat reverse curvatures. In the general vicinity of Sodus Bay the shore line turns to a northeasterly direction and follows that alignment to a point which is about 10 miles almost due west of Selkirk Shores State Park. From there to about the south limit of the park the shore line runs in a easterly direction. Across the park frontage the shore line runs in a northerly direction and continues in this general direction with a smooth alignment for a distance of about 30 miles, where it becomes very irregular and is characterized by prominent headlands, bays and islands near the head of the St. Lawrence River.

6. Selkirk Shores State Park is located in the town of Richland, Oswego County, New York, at the southeast corner of Lake Ontario, approximately 15 miles northeast of Oswego Harbor. It contains about 631 acres with a shore frontage of approximately 6,000 feet, lying between Salmon River

and Grindstone Creek. Park land bordering these streams is low and swampy. The lower half mile of Grinstone Creek flows through a swamp area about 1,500 feet wide. The central portion of the park is wooded rolling land rising to a maximum elevation of about 50 feet above the lake.

FACTORS PERTINENT TO THE PROBLEM

7. General Geology. The south shore of Lake Ontario is located in the Eastern Lake Section of the physiographic area known as the Central Lowlands, and is a post-glacially elevated region of moderate relief consisting of a series of terraces ascending southward to the Allegheny Plateau, separated by escarpments facing northward. The terraces, consisting of the Ontario, Huron, and Erie Plains, are elevated marine terraces modified by lake changes and the Wisconsin Ice Sheet, which moved across the area in a general southwesterly direction. The shore line is in a submature stage of submergence, but has risen from a previous lower elevation.

8. Local Geology. An outstanding feature of the surficial geology is the prevalence of drumlins in the vicinity of the park. The drumlins were formed under the comparatively thin edge of the continental ice sheet when it had reached the limit of its transporting capacity, and had become incompetent to carry further its load of rock, sand, gravel and clay. Their molded forms attest to the overriding effect of the glacier, which resulted in their main axes lying in the southwesterly direction of movement of the ice sheet.

9. Rivers. There are three important rivers entering the lake along its south shore: the Niagara, Genesee, and Oswego. The Niagara enters the lake about 35 miles east of the western extremity of the lake. Due to its relatively high velocity and large volume of flow, the main current carries most of the sediment far into the lake before depositing it on the Niagara Bar. The outer edge of the bar is some 5 miles from shore. There is a weak littoral current carrying little or no sediment, which follows closely along the shore easterly from the river mouth.

10. The Genessee River enters the lake at Rochester Harbor, approximately 80 miles east of the Niagara River. Although it has a drainage area of 2,479 square miles, and carries considerable sediment, it discharges little beach building material into the lake. About 250,000 cubic yards of material, much of which is sand, have been removed annually in the past 10 years by hopper dredges and deposited in deep water in the lake. In 1949 material was placed immediately east of the harbor in depths of about 20 feet, which was as close to shore as the dredge could safely navigate. Surveys over a 2-year period following placement of this material showed little or no movement of the material and further attempts at beach nourishment by this method were therefore abandoned.

11. The Oswego River, having a drainage area of about 5,122 square miles, enters the lake at Oswego Harbor about 15 miles southwest of Selkirk Shores

State Park. It is the closest major stream to the study area. Under a Federal navigation project the lower portion of the river has been improved and a relatively large outer harbor is enclosed by breakwaters. Little, if any, beach building material from the river becomes available to the south shore of Lake Ontario.

12. Local Streams. All streams in the vicinity of the study area discharge into swamps, ponds, or bays, where they deposit most of their sediment in comparatively still water before passing through barrier beaches to the lake. Thus little or no material is furnished the shore from such streams. Grindstone Creek, with a drainage area of approximately 50 square miles, is the only stream entering the lake within the park area.

13. Shore Characteristics. The shore at the park and to the south presents the undulating profile of eroding faces of drumlins, alternating with low-lying barrier beaches fronting swamps, ponds, or bays. The drumlins consist, in general, of layers of boulder clay covered by a stratum of fine sand and silt. The boulder clay varies from compact, blue boulder clay to loosely consolidated brown clay. All the deposits contain rock flour, pebbles, boulders and occasional, large sharp-edged blocks of limestone or sandstone up to 6 tons in weight. They contain, however, little or no medium or coarse sand. From a point about 2-1/2 miles south of the park to the southerly limit of the park, there are two drumlins, their lakeward ends eroded to almost vertical bluffs. The southerly one rises to a maximum height of 60 feet, the second to a height of only 40 feet. Overlying the base of boulder clay is a 10-foot layer of sand and silt. Between these bluffs, a narrow barrier beach of gravel and boulders separates low swamp land from the lake. During high lake stages, the swamp and most of the barrier beach are inundated. For 1,200 feet on each side of the park's southerly boundary there is a gravel barrier beach between the lake and the swamp area through which Grindstone Creek passes. The outlet of Grindstone Creek passes through this barrier beach about 900 feet north of the southerly park boundary. Bluffs reach a maximum height of about 44 feet along the shore between 1,200 feet and 5,000 feet north of the park's southerly boundary, which includes the shore frontage at the bathhouse, the clubhouse, and some of the most popular camp sites in the vicinity of the community building. In the southerly 400 feet of this frontage in the vicinity of the clubhouse, the bluff is stratified and consists of layers of brown and gray sand interspersed with layers of gravel. Northerly from this vicinity the sand gravel content diminishes rapidly and the bluffs are composed of fine sand, silt, and boulder clay. The remaining northerly 1,000 feet of park frontage consists of a gravel and cobble beach in front of low-lying land. Immediately beyond the shore end of the northerly limit of the park, a sand dune begins which reaches a maximum height of 60 feet about midway to the Salmon River, then diminishes in height to only a few feet above lake level at the river. Beyond the river, a high sand dune continues northward.

14. Beaches. The beaches to the south of and at the park are steep and narrow and consist generally of gravel and boulders derived from the

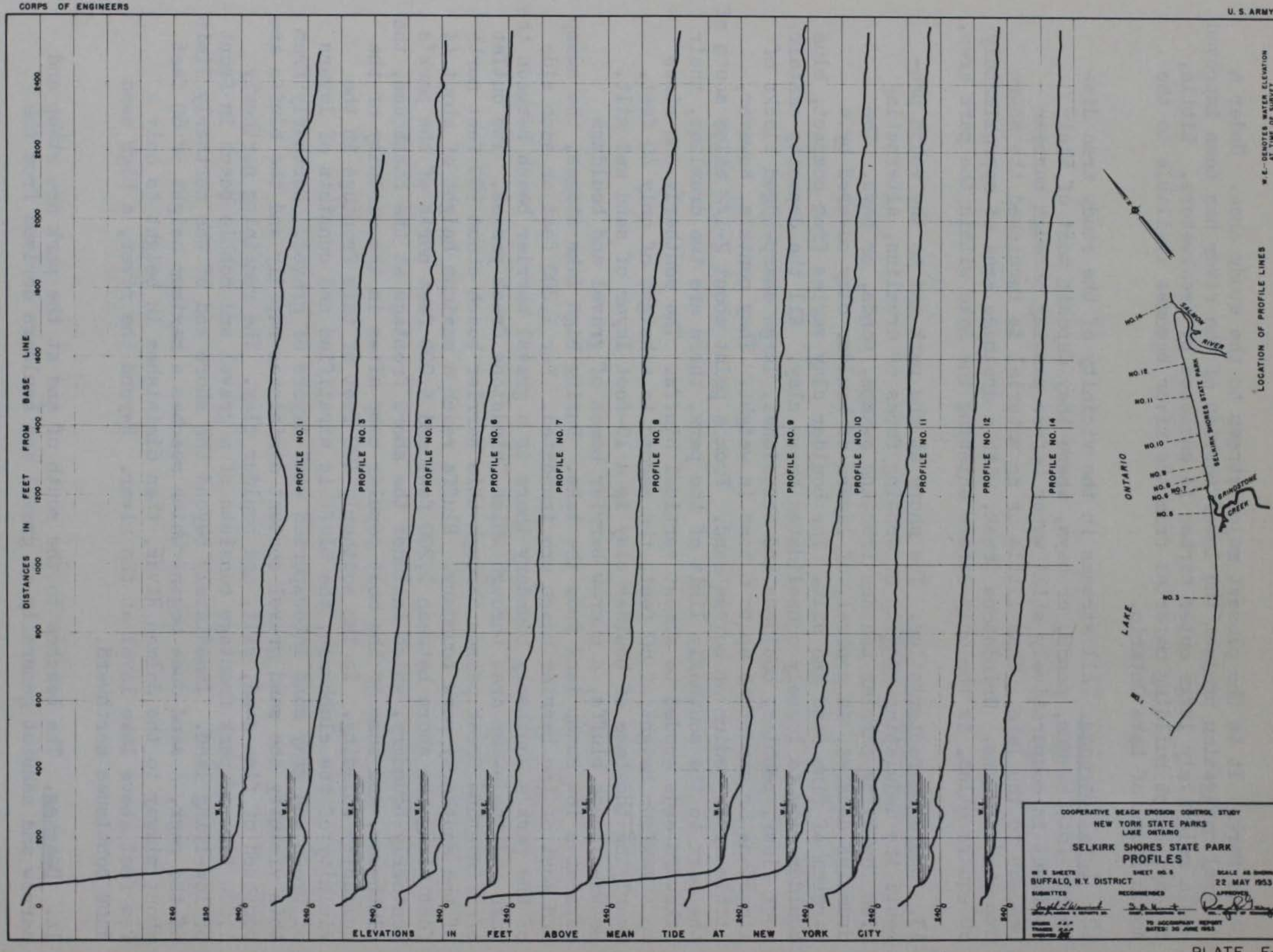

CORPS OF ENGINEERS
U. S. ARMY
DISTANCES IN FEET FROM BASE LINE
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
ELEVATIONS IN FEET ABOVE MEAN TIDE AT NEW YORK CITY
PROFILE NO. 1
PROFILE NO. 3
PROFILE NO. 5
PROFILE NO. 6
PROFILE NO. 7
PROFILE NO. 8
PROFILE NO. 9
PROFILE NO. 10
PROFILE NO. 11
PROFILE NO. 12
PROFILE NO. 14
W.E.
W.E.—DENOTES WATER ELEVATION AT TIME OF SURVEY
LOCATION OF PROFILE LINES
LAKE ONTARIO
SALMON RIVER
GRINDSTONE CREEK
SELKIRK SHORES STATE PARK
NO. 14
NO. 12
NO. 11
NO. 10
NO. 9
NO. 8
NO. 7
NO. 6
NO. 5
NO. 3
NO. 1
COOPERATIVE BEACH EROSION CONTROL STUDY
NEW YORK STATE PARKS
LAKE ONTARIO
SELKIRK SHORES STATE PARK
PROFILES
IN 5 SHEETS
SHEET NO. 5
SCALE AS SHOWN
BUFFALO, N.Y. DISTRICT
22 MAY 1953
SUBMITTED
RECOMMENDED
APPROVED
TO ACCOMPANY REPORT
DATED: 30 JUNE 1953

PLATE F-2

TABLE F-1

SAMPLE ANALYSES

Location	Cumulative percentages retained on sieve							Total %
	Gravel	Sand						Silt
	Fine	Coarse		Medium		Fine		and
(U. S. Mesh No.)	4	10	20,	40	60	100	200	Clay
	Bluff Samples							
Profile 1, upper	10	15	21	24	28	32	42	58
Profile 1, lower	25	34	40	45	47	50	57	43
Profile 4, upper	24	38	42	44	46	50	58	42
Profile 4, lower	15	24	32	37	40	44	54	46
200' N Profile 7	0	0	1	6	25	61	87	13
Profile 8, upper 40'	0	31	44	51	56	61	68	32
Profile 8, lower	0	0	1	11	58	90	99	1
200' S Profile 9 entire	0	20	25	29	32	36	46	54
200' N Profile 9 upper	36	60	75	89	95	98	100	0
600' N Profile 9 lower 4'	0	31	44	47	49	53	59	41
600' N Profile 9 upper 40'	0	1	1	2	2	4	13	87
200' N Profile 10 lower 19'	0	18	25	31	35	41	51	49
200' N Profile 10 upper 4'	0	20	23	27	32	55	83	17
500' N Profile 10 lower 10'	0	22	28	32	34	38	47	53
500' N Profile 10 upper 4'	0	47	56	72	87	96	98	2
600' S of Profile 11 mid 6'	0	9	11	13	16	22	32	68
	Beach Samples, El. 248 feet							
Profile 1	Gravel and cobbles 1 to 12" diameter							
Profile 2	Gravel and cobbles 1 to 4" diameter							
Profile 3	0	0	0	1	35	84	99	1
Profile 5	90	99	99	99	99	99	100	0
Profile 6	45	69	78	84	98	100	100	0
Profile 8	Gravel and cobbles 2 to 4" diameter							
Profile 9	Gravel and cobbles 2 to 8" diameter							
Profile 10	Gravel and cobbles 2 to 8" diameter							
Profile 11	0	30	53	96	100	100	100	0
Profile 12	Gravel 1" to 2" diameter							
Profile 13	66	78	84	91	97	99	100	0
	Bottom Samples, Elev. 244 feet. (low water datum)							
Profile 1	0	0	0	1	15	77	100	0
Profile 2	0	0	0	2	15	80	99	1
Profile 3	0	0	0	2	40	91	100	0
Profile 4	0	1	2	4	36	86	100	0
Profile 5	0	0	0	1	41	93	100	0
Profile 6	Boulders							
Profile 9	0	0	0	2	33	89	99	1
Profile 11	0	0	1	5	38	84	99	1
Profile 13	0	0	0	2	50	93	100	0
Average	0	0	0	2	33	87	99	

TABLE F-1 Continued

Location	(U. S. Mesh No.)	Cumulative percentages retained on sieve: Gravel Fine 4	Sand Coarse 10	20	Sand Medium 40	60	Sand Fine 100	200	Total % Silt and Clay
	Bottom Samples, 5-foot depth (low water datum)								
Profile 1		0	0	0	0	3	64	97	3
Profile 2		Boulders							
Profile 3		0	0	0	1	3	51	100	0
Profile 4		0	0	0	1	6	62	99	1
Profile 5		0	0	1	2	14	66	100	0
Profile 6		0	0	0	1	11	69	99	1
Profile 9		Boulders							
Profile 11		0	0	0	0	8	70	99	1
Profile 13		0	0	0	0	4	55	100	0
	Average	0	0	0	1	7	62	99	1
	Bottom Samples, 10-foot depth (low water datum)								
Profile 1		No Sample							
Profile 2		Boulders							
Profile 3		0	1	2	3	5	27	83	17
Profile 4		0	1	1	2	3	18	83	17
Profile 5		0	1	2	3	6	26	95	5
Profile 6		0	0	0	0	2	44	99	1
Profile 9		Boulders							
Profile 11		No Samples							
Profile 13		0	0	0	0	1	7	93	7
	Average	0	1	1	2	3	22	91	9
	Bottom Samples, 15-foot depth (low water datum)								
Profile 1		Boulders							
Profile 2		Boulders							
Profile 3		0	1	2	3	5	26	90	11
Profile 4		0	0	1	1	3	14	83	17
Profile 5		Boulders							
Profile 6		0	0	0	0	1	11	83	17
Profile 9		Boulders							
Profile 11		0	6	7	8	9	16	88	12
Profile 13		0	0	0	0	1	7	93	7
	Average	0	1	2	2	4	15	87	13
	Bottom Samples, 20-foot depth (low water datum)								
Profile 1		Boulders							
Profile 2		Boulders							
Profile 3		0	0	0	0	2	14	86	14
Profile 4		0	4	5	6	8	22	92	8
Profile 5		0	0	0	0	0	2	5	95
Profile 6		0	0	0	0	1	6	38	62
Profile 9		Boulders							
Profile 11		0	1	2	3	5	11	89	11
Profile 13		0	0	1	3	8	16	71	29
	Average	0	1	1	2	4	12	64	36

boulder clay bluffs. Beginning at the dune area, immediately north of the park, sand begins to appear in appreciable quantities mixed with the gravel. North of the mouth of Salmon Creek the beach material is fine sand. During former periods of average lake levels, the beaches at the park contained more sand, particularly in the vicinity of the existing groin. During the field survey 16 representative samples of material found in the eroding bluffs within the study area were obtained. Attempts were made to obtain samples from the beach at elevation 248 feet, and from the bottom at low water datum, elevation 244 feet, and at the 5, 10, 15 and 20-foot depths on 9 profiles located as shown on Plate F-2. Because of the inability of the sampler to obtain samples in cobbles and boulders only 30 bottom samples were obtained for analysis. Detailed analyses of these samples are shown in Table F-1.

15. Lake Levels. Levels of the Great Lakes fluctuate irregularly from year to year and from month to month, depending upon several factors. The levels change with variation in supply of water from precipitation on the lake surfaces and from runoff of land area of the drainage basins, and with variation in extraction of water by outflow and evaporation from lake surfaces. Over the period of record since 1860, the range of fluctuations for Lake Ontario between the lowest monthly mean level and the highest monthly mean level has been 6.61 feet. The pattern of seasonal variation is low levels in winter and high levels in summer, with extremes usually in January and June.

16. Lake Ontario reached its lowest monthly mean level of record in 1934. Since that time there has been a continuing trend of rising levels through 1952, except in 1949 and the spring of 1950 when a slight recession occurred.

17. In addition to the cyclic and seasonal variations of level, Lake Ontario is subject to fluctuations of irregular duration and amount. Variations in barometric pressure produce changes in level ranging from a few inches to several feet. These temporary fluctuations may extend over a few minutes or several days. At times the lake level is affected by winds of sufficient velocity to drive the surface water forward in greater volume than it can be carried by return currents, thus raising the lake elevation on the lee shore and lowering it on the weather shore. The magnitude of these short period fluctuations depends to a great extent upon local conditions. Fluctuations of short duration, of 1.2 feet have been recorded at Oswego Harbor about once a year and fluctuations of 2.0 feet have occurred about once in 8 or 9 years. Pertinent data on fluctuations of Lake Ontario as recorded at Oswego Harbor since 1859 are given in Table F-2.

18. Wind. Wind records of the U. S. Coast Guard Station at Rochester, New York, for the period 1 January 1947 to 31 December 1952, and for Oswego, New York, for the period 1 January 1936 to 31 December 1945, were compiled and converted to yearly averages. The resulting data are shown diagramatically on Plate F-1. The wind duration for each direction is classified in three groups of velocities: 0-12, 13-24, and 25 miles per hour

TABLE F-2

WATER LEVEL FLUCTUATIONS - LAKE ONTARIO

Item	High stage feet(1)	Date	Low stage feet(1)	Date	Variation, feet	Elevation, feet(2)
Extreme stage, monthly mean (1860-1952)	+5.29	June 1952	-1.32	Nov. 1934	6.61	
Extreme stage, instantaneous (1900-1952)	+6.19	June 1952	-1.83	Dec. 1934	8.02	
Maximum seasonal variation in calendar year	+4.46	June 1867	+0.88	Dec.1867	3.58	
Minimum seasonal variation in calendar year	+1.63	July 1920	+0.94	Feb.1920	0.69	
Mean Lake Level	From Jan. 1860 to Dec.1952 incl.					246.02
	From Jan. 1900 to Dec.1952 incl.					245.85
Average Lake Level for months March to Dec., incl.	From Jan. 1900 to Dec.1952 incl.					245.97

(1) Referred to low water datum, 244.0 feet above mean tide at New York City. All stages computed from monthly means, except instantaneous high and low.

(2) Referred to mean tide at New York City.

and over. The data have also been broken down into an ice period, January and February, and an ice-free period, March to December, inclusive. The length of the solid portion of each bar represents the percent of the total duration for winds of that direction and velocity group for the ice-free period; the outlined portion represents the ice period. The curves in the center of the diagram show the percent of the total wind movement occurring during the ice-free period and the combined ice and ice-free periods. A summary of wind data is given in Table F-3.

19. The shore line of Selkirk Shores State Park is exposed to the effects of winds from the southwest, west, northwest, and north, with fetches of 35, 170, 45, and 20 miles, respectively. Although southwest winds cannot act directly on the area, waves generated by these winds with the above fetch are refracted and do affect the shore. The effective winds during the ice-free period from southwest, west, northwest, and north account for 11.9, 16.1, 11.6, and 7.6 percent of the total wind duration and 12.4, 17.8, 17.7, and 7.8 percent of the total wind movement, respectively.

TABLE F-3

YEARLY AVERAGE WINDS AT ROCHESTER AND OSWEGO, N. Y.

Direction	Wind duration, percent of time				Wind movement, percent of total			
	Rochester		Oswego		Rochester		Oswego	
	Total	Ice-free	Total	Ice-free	Total	Ice-free	Total	Ice-free
North	5.39	4.75	9.19	7.61	4.69	4.02	9.46	7.78
Northeast	9.49	8.08	8.05	6.74	11.95	9.65	8.04	6.88
East	8.47	7.64	4.68	3.68	8.43	7.34	2.67	2.09
Southeast	9.35	7.41	13.71	11.12	6.97	5.54	9.07	7.26
South	6.34	4.81	17.54	15.03	3.87	2.82	11.24	9.38
Southwest	24.80	20.52	13.77	11.94	21.95	16.59	14.87	12.41
West	22.60	19.10	18.27	16.09	25.20	19.97	21.64	17.80
Northwest	13.56	11.41	14.79	11.55	16.95	13.85	23.01	17.75
Total	100.00	83.72	100.00	83.76	100.00	79.78	100.00	81.35

20. Waves. No observations of wave heights and periods were made during the field survey. However, from a study of wind data and synoptic weather charts(113), it was determined that waves of 12 feet in height with periods of 8 to 9 seconds, can be expected about once a year in deep water on Lake Ontario. Waves up to 25 feet in height have been reported, and theoretically waves of this height with a period of about 12 seconds can be developed. However, the combination of wind velocity, duration and fetch required for their generation is of rare occurrence. As deep water waves approach shore, there characteristics are changed by the effects of refraction and shoaling.

21. Littoral Currents and Drift. In general, the predominant direction of littoral currents and transport along the south shore of Lake Ontario from Burlington Bay, Ontario, to Selkirk Shores State Park is from west to east, as indicated by the accumulation of material to the west of harbor structures at Rochester Harbor, Little Sodus Bay Harbor and Oswego Harbor. There are local reversals due to the configuration of the shore line and short term reversals caused by changes in the direction of waves. The predominant direction of the littoral currents and transport within the study area, is from south to north as indicated by the following conditions. The high inner end of the existing groin is retaining a narrow beach to the south, but the beach berm does not prograde lakeward of the high section of the groin since the material passes over the top of the lower portion of the groin. The mouth of Grindstone Creek becomes blocked by drift and is gradually forced northerly. As a result the creek has in the past run along and behind the beach berm gradually approaching the existing sea wall. Before it is dangerously close, the State opens the original channel by dredging through the barrier and the process is repeated. The meandering of the creek outlet interferes with the natural development of a beach south of the existing groin. The Salmon River mouth, to the north of the park, has moved continuously northerly until halted by existing structures along the right bank. After a storm in November 1952, it was observed that the outlet of the river

was reduced to a narrow opening only 2 feet wide. Forces causing littoral movement are relatively strong as evidenced by the movement of gravel and cobbles along the shore, but the rate of transport on which there is little available information, has not been computed. The fact that erosion has occurred along the park frontage indicates that rate of loss is greater than the rate of supply.

22. Sources of Beach Building Material. The eroding bluffs of the two drumlins to the south of the park are a minor source of supply of beach material. However, after excluding cobbles and boulders only about one-half of the remainder is suitable beach material because of a high percentage of very fine material which is carried offshore. Since these bluffs have a combined length along shore of only 2,000 feet, the amount of material derived from their erosion is incapable of maintaining the intervening 2 miles of shore line and the park frontage to the north. The streams in the area are small and show little evidence of supplying any beach building material, and sand in the offshore areas is too fine to remain on the beach. It is evident, therefore, that the only feasible way a sand bathing beach can be constructed is by adding sand from an inland source.

23. Ice Action. Under normal conditions, during late December spray freezes and forms a coating of ice on the shore to heights up to 10 feet above lake level, thereby temporarily armoring the beach and bluffs against further wave action. During the latter part of December, ice usually begins to form in the bays and in shallow water near shore. During severe winters, the ice cover extends a mile or more offshore and may attain a thickness of up to 3 feet near shore. Storms often break up the ice cover and force it into windrows which become grounded just offshore and effectively protect the shore from storm wave action during the winter. Normally the ice fields break up early in March. The windrowed ice melts slowly and generally protects shore structures from the battering they would otherwise receive from floating ice fields. Aside from increased loads on shore structures due to its weight, ice generally has a net beneficial effect by preventing wave action from reaching the shore and bluffs during the ice period.

24. History of Structures and Effect of Storms. About 1929, erosion at the toe of the bluff induced the State to commence construction of a timber retaining wall built of logs. By 1933, 1,500 feet of wall extending northerly from approximately the location of the existing groin had been completed. (See Plate F-3)

25. In 1930, the existing 276-foot groin was constructed. It consists of parallel rows of steel sheet piling set 10 feet apart inclosing a gravel fill. The top is capped by a 2-foot thick concrete deck 15 feet wide at elevation 250.0 feet for the outer 200 feet. The inner 76 feet sloped upward to an elevation of about 252.2 feet.

26. The timber retaining wall did not provide sufficient protection and in 1934 construction was commenced on a closed-face concrete crib wall, founded on a concrete footing. The cribs were backfilled with gravel and boulders.

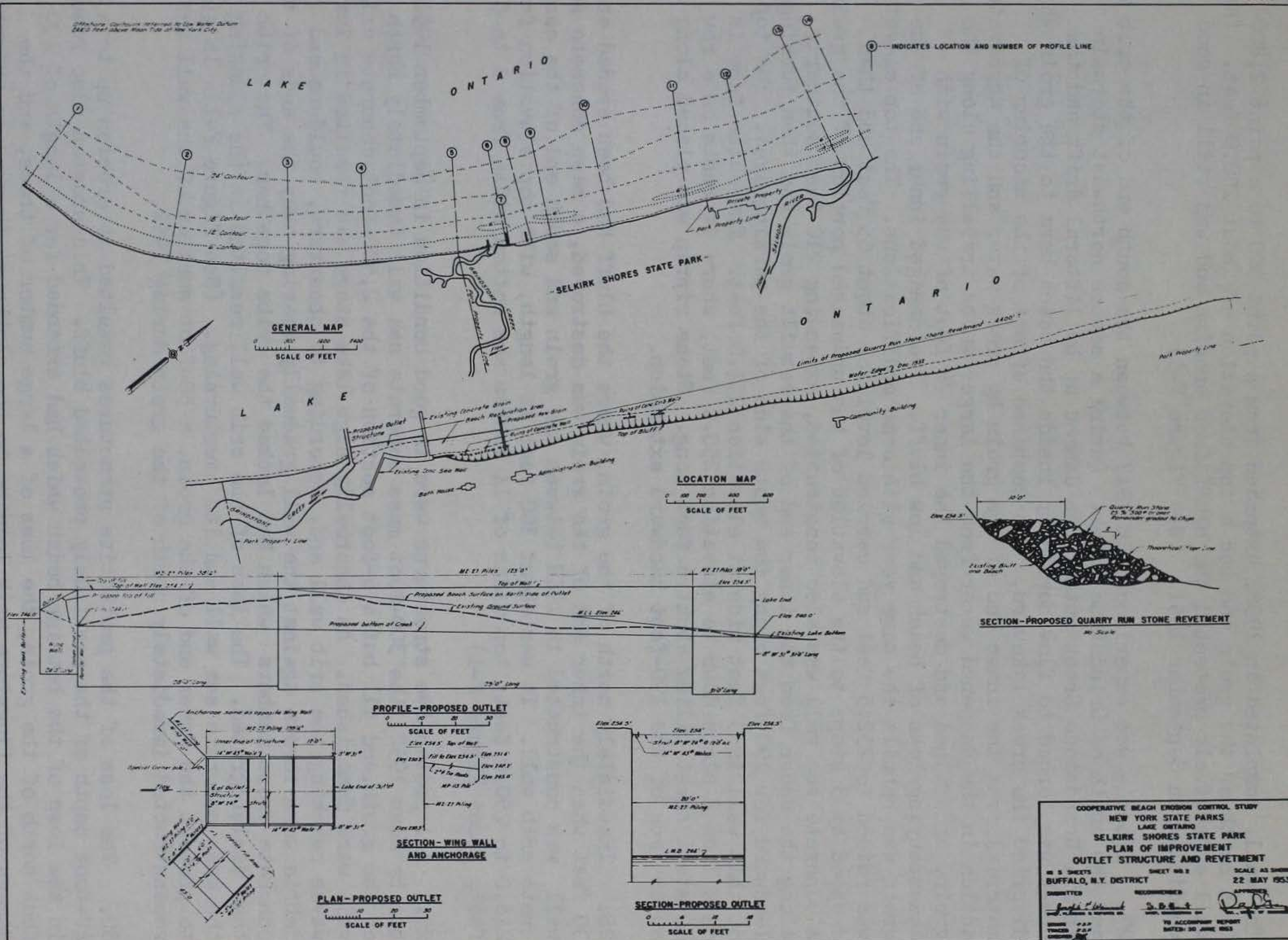

LAKE ONTARIO
INDICATES LOCATION AND NUMBER OF PROFILE LINE
24' Contour
18' Contour
12' Contour
6' Contour
SELKIRK SHORES STATE PARK
GRINDSTONE CREEK
SALMON RIVER
Private Property
Park Property Line
GENERAL MAP
SCALE OF FEET
LAKE ONTARIO
Limits of Proposed Quarry Run Stone Shore Revetment
Water Edge
Park Property Line
Community Building
Proposed Outlet Structure
Existing Concrete Groin
Beach Restoration Area
Proposed New Groin
Top of Bluff
Existing Conc Sea Wall
Administration Building
Bath House
GRINDSTONE CREEK
Park Property Line
LOCATION MAP
SCALE OF FEET
SECTION-PROPOSED QUARRY RUN STONE REVETMENT
No Scale
Existing Bluff and Beach
Theoretical Toe Line
Proposed Beach Surface on North side of Outlet
Existing Ground Surface
Proposed bottom of Creek
Existing Lake Bottom
Lake End
PROFILE-PROPOSED OUTLET
SCALE OF FEET
SECTION-WING WALL AND ANCHORAGE
PLAN-PROPOSED OUTLET
SCALE OF FEET
SECTION-PROPOSED OUTLET
SCALE OF FEET
COOPERATIVE BEACH EROSION CONTROL STUDY
NEW YORK STATE PARKS
LAKE ONTARIO
SELKIRK SHORES STATE PARK
PLAN OF IMPROVEMENT
OUTLET STRUCTURE AND REVETMENT
BUFFALO, N.Y. DISTRICT
22 MAY 1953

PLATE F-3

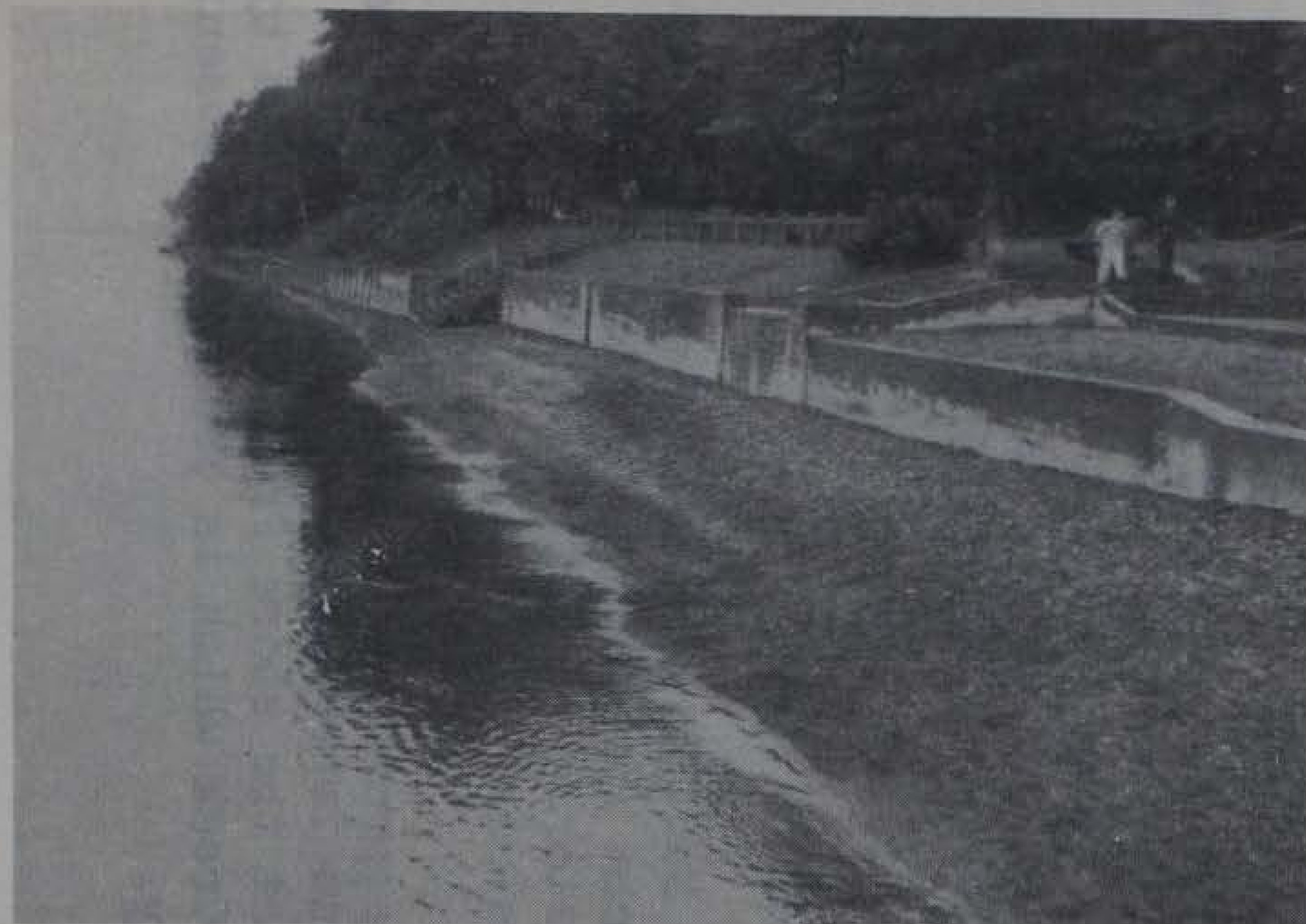
The wall, completed in 1936, extended from a point 300 to a point 2,800 feet north of the groin and had a top elevation of about 250.0 feet. The wall effectively protected the bluff from erosion and was still in good condition in September 1951. (See Figure F-1)

27. Trouble was experienced in 1947 between the south end of the crib wall and the mouth of Grindstone Creek. During a severe northwest storm in April, Grindstone Creek outlet was dammed up by littoral drift and the creek was forced to flow northerly inside the beach berm to the groin which diverted the creek lakeward. The combined effect of the scouring of material from the inner end of the groin by creek water and the aggravated action in the deepened water from the larger waves travelling along the groin, undermined and destroyed the inner 76 feet of the groin with accompanying loss of beach and low bluff. The damaged inner end of the groin was rebuilt the same year with some modifications. The top elevation was raised to 252.5 feet and carried level for about 65 feet and then dropped by 3 steps to the elevation of the undamaged portion of the groin. A concrete sea wall was also constructed, extending 210 feet southerly along the shore from the inner end of the rebuilt groin and then turning landward for 190 feet along the south side of the parking area. The top of this wall is 3 feet wide at elevation 254.0 feet. The outer face is on a 1 on 2 slope down to elevation 250.0 feet, where it rests on a row of steel sheet piling about 6 feet long. Stone riprap was placed along the outer face of the 190-foot landward extension.

28. Immediately north of the groin, where the bluff had been eroded about 30 feet when the inner end of the groin was destroyed, a mass concrete sea wall was constructed in 1948 between the groin and south end of the concrete crib wall. It was about 300 feet in length, with top elevation from 246.0 to 250.0 feet, top width of 14 inches and bottom width from 3 to 5 feet. (See Figure F-1)

29. All protective structures were in good condition in September 1951, but by June 1952, the 300-foot mass concrete sea wall immediately north of the groin,and all but a 6-foot section of the 2,500-foot concrete crib wall were demolished. An abnormally high lake stage had resulted in larger waves reaching the crib wall and, according to observers, boulders and debris were thrown against the wall gradually wearing away the ends of the concrete cross members which had locked the cribs together. The cribs then disintegrated. The loss of the crib wall resulted in the flanking of the mass concrete sea wall and its overturning. (See Figure F-1) In order to protect the inner end of the groin, a 50-foot section of the wall was reconstructed immediately north of the groin during 1952.

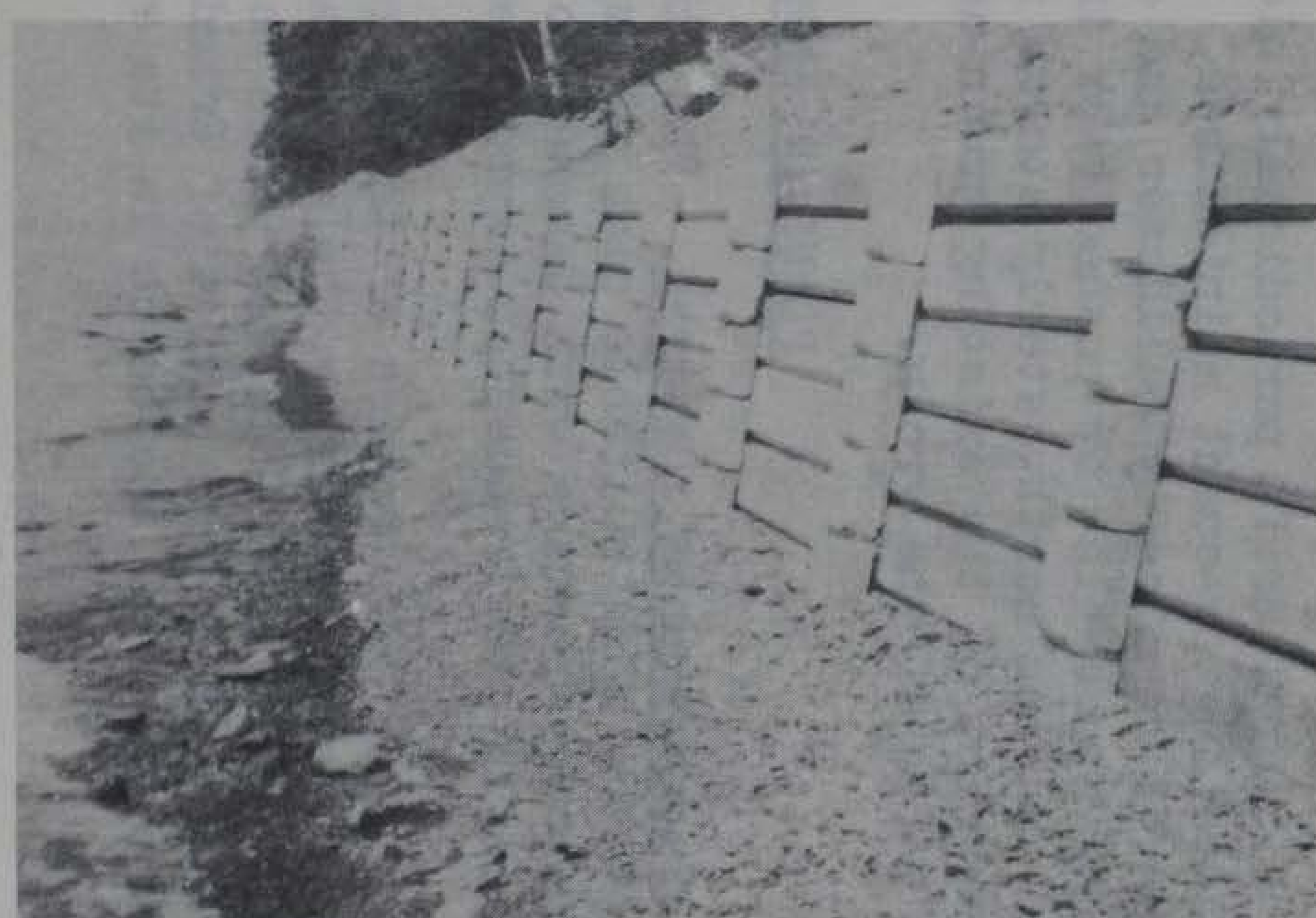
30. The loss of the protective structures resulted in erosion up to a 40-foot depth of the previously protected bluff. This erosion also resulted in the loss of the bathing beach which had extended for a length of 400 feet north of the groin, the loss of a large number of trees, and the damage of the flagstone walk leading to the clubhouse to such an extent that it was unusable.



Concrete Wall North Of Existing Groin

June 1951

November 1952



Concrete Crib Wall

August 1947

One Section Remaining, November 1952

FIGURE F-1

31. Plans and Profiles. During 1952, 14 profiles were obtained in the study area extending from 150 feet landward of the water's edge to the 24-foot depth curve. The first four profiles were located in the 2 1/2 miles south of the park area, 7 profiles were located in the park, and 3 in the mile north of the park. The full lengths of the profiles are shown on Plate F-2. The following table shows the average slopes of beach and bottom.

TABLE F-4

AVERAGE BEACH SLOPES

Location	Average elevation of 1st change in slope, feet	Slope landward of change	Slope lakeward of change to about elev. 240.0 feet
South of park	246	1 on 8	1 on 60
In park	243	1 on 7	1 on 40
North of park	244	1 on 35	1 on 40

32. Shore Line and Offshore Changes. No earlier surveys are available which could be oriented with sufficient accuracy to make a comparison of the bluff, beach and offshore areas with those of the survey of 1952. However, erosion of the bluff was sufficiently severe to cause the State to expend over $293,000 between 1929 and 1948 in an attempt to halt it. As previously stated during the period of high lake stages in 1952, loss of up to 40 feet depth of bluff occurred in the area for a distance of 2,800 feet north of the groin, and lesser erosion occurred from this point to the park's northerly boundary.

33. Littoral drift periodically blocks the mouth of Grindstone Creek necessitating frequent maintenance. The material, consisting of gravel and cobbles is removed by bulldozer and either has been placed on the beach north of the outlet or pushed back into the swamp to widen the barrier beach. During northwesterly storms some sand and gravel is carried over the low part of the existing groin and assists in supplying the narrow beach north of the groin.

PLANS OF IMPROVEMENT

34. Improvements Desired. The State of New York requested the development of the most suitable plan of improvement for the protection of the entire park frontage with beaches to be restored in certain areas. Specifically the requested improvements include:

a. Protection of the shore for approximately 450 feet on each side of the existing groin by the restoration and improvement of the former bathing beach to sufficient width to protect the shore from erosion and by stabilization of the outlet of Grindstone Creek which is considered essential to improvement of the beach in its vicinity.

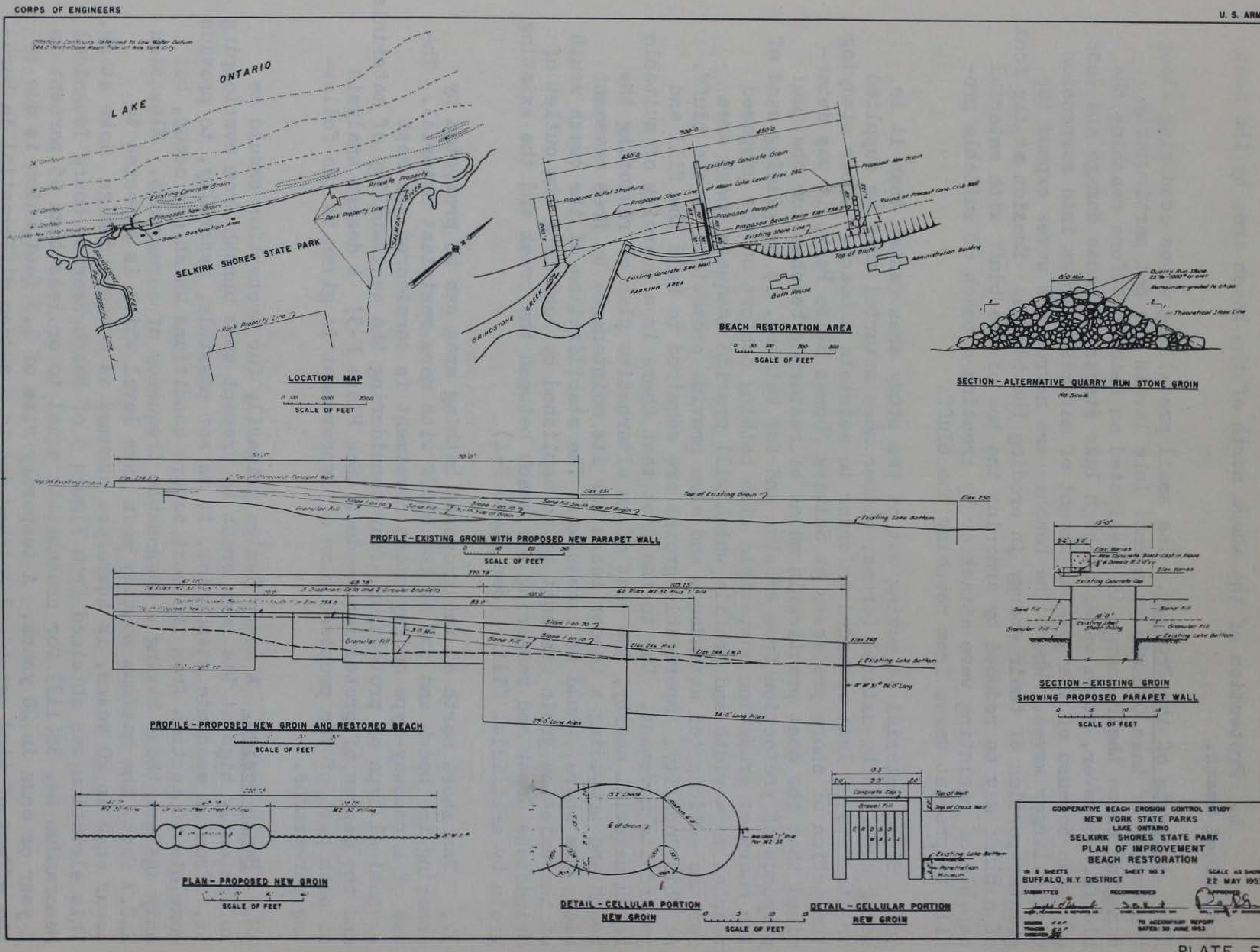
b. Protection of the shore north of the beach area by the most economical means.

35. Analysis of the Problem. The small prevailing waves occurring along the shores of Lake Ontario at normal lake levels are not particularly damaging since their energy is dissipated on beaches before reaching the bluffs. However, storm waves at high lake stages do cause damage and loss of beach because of the greater amount of material thrown into suspension by the larger waves. Where the beaches are narrow the waves expend the greater portion of their energy in eroding the bluffs. Erosion at the foot of a bluff may be reduced by armoring the toe of the bluff with material capable of resisting wave attack or by creating a beach of suitable profile to prevent waves from reaching the bluff.

36. Since material is moving out of the study area faster than it is being supplied by natural forces, either more material must be supplied or its rate of loss retarded in order to maintain a beach. After studying the action of shore processes at Selkirk Shores State Park, it was determined that the most practicable means of restoring beaches for the dual purposes of protection from erosion and for bathing, is by placing sand of the coarsest gradation compatible with bathing comfort on the proposed beaches to a width and profile which will provide adequate beach area for the anticipated attendance and also provide protection to the bluff. At this locality, impermeable groins are required to hold the fill and retard its movement. Due to the fact that there is a scarcity of suitable beach material nearby, the cost of an alternative plan of restoring the beach and providing a feeder beach for its maintenance by free movement along the shore, would be excessive. The stabilization of the beach south of the existing groin can best be accomplished by confining the outlet of Grindstone Creek and restoring the beach between the creek and the existing groin by artificial fill. (See Plate F-4)

37. The shore north of the proposed bathing area can be protected by armoring the beach and toe of the bluff with more resistant material. The use of a roadway-type quarry stone revetment is considered the most economical type of protection after considering the availability of materials and the degree of protection needed. (See Plate F-3) A design analysis of each part of the proposed plan of improvement is given in the following paragraphs.

38. Beach Design. A beach design primarily for protection should be sufficiently high at the berm crest to prevent wave uprush from overtopping it. From an economic standpoint, it is not possible, generally, to provide complete protection from the most severe conditions which may occur, but only against those having a reasonable frequency of occurrence. Elevation 248.7 feet, the maximum monthly mean lake level which is expected to occur once in 20 years, is therefore assumed as a basic design lake stage. This elevation was obtained from Figure 5 of Beach Erosion Board Technical Memorandum No. 38(113) for probability equal to or less than 95 percent in 1 year or once in 20 years. A temporary rise of 1.3 feet, which is due to the effect of wind setup and seiches due to a sudden differential in barometric pressures over the lake surface was used. This temporary rise was



CORPS OF ENGINEERS
U. S. ARMY
LAKE ONTARIO
SELKIRK SHORES STATE PARK
GRINDSTONE CREEK
SALMON RIVER
Existing Concrete Groin
Proposed New Groin
Beach Restoration Area
Private Property
Park Property Line
LOCATION MAP
SCALE OF FEET
Proposed Outlet Structure
Proposed Shore Line
Proposed Parapet
Proposed Beach Berm
Existing Shore Line
Existing Concrete Sea Wall
Parking Area
Bath House
Administration Building
Top of Bluff
BEACH RESTORATION AREA
SCALE OF FEET
Quarry Run Stone
Theoretical Slope Line
SECTION - ALTERNATIVE QUARRY RUN STONE GROIN
No Scale
Top of Existing Groin
Granular Fill
Sand Fill
Existing Lake Bottom
PROFILE - EXISTING GROIN WITH PROPOSED NEW PARAPET WALL
SCALE OF FEET
Existing Concrete Cap
Existing Steel Sheet Piling
SECTION - EXISTING GROIN
SHOWING PROPOSED PARAPET WALL
SCALE OF FEET
Granular Fill
Sand Fill
Existing Lake Bottom
PROFILE - PROPOSED NEW GROIN AND RESTORED BEACH
SCALE OF FEET
PLAN - PROPOSED NEW GROIN
SCALE OF FEET
DETAIL - CELLULAR PORTION
NEW GROIN
SCALE OF FEET
Concrete Cap
Gravel Fill
CROSS WALL
Top of Wall
Top of Cross Wall
Existing Lake Bottom
DETAIL - CELLULAR PORTION
NEW GROIN
COOPERATIVE BEACH EROSION CONTROL STUDY
NEW YORK STATE PARKS
LAKE ONTARIO
SELKIRK SHORES STATE PARK
PLAN OF IMPROVEMENT
BEACH RESTORATION
BUFFALO, N.Y. DISTRICT
22 MAY 1953
TO ACCOMPANY REPORT
DATED: 30 JUNE 1953

PLATE F-4

obtained from data on Figure 6 of Beach Erosion Board Technical Memorandum No. 38(113) which indicates a frequency of occurrence of once in 12 months. This was added to the static lake stage to arrive at a design lake stage or still water level of 250.0 feet. From Plate D-11 when the lake is at design stage, a 4.5-foot wave would break in about 6 feet of water at the low water datum shore line (elevation 244.0 feet). It can be expected to to expend its energy in uprush before reaching an elevation equal to its own height above design lake stage on a beach slope of about 1 on 10, or before reaching elevation 254.5 feet. This elevation is used as the design beach berm elevation, and its adequacy is confirmed by the elevation of existing beach berms formed by waves during the recent period of record high lake stages.

39. A beach slope depends upon the size and density of material on the beach and upon the characteristics of the wave action. The best indication of the probable slope of an artificial beach is that assumed by similar material on natural beaches at the locality. It is assumed that the coarsest sand, compatible with bathing comfort, would be used for beach fill to reduce losses from littoral movement. In general such sand would have grain sizes ranging from 0.4 to 2.0 millimeters in diameter. Material of this gradation can reasonably be expected to remain on a 1 on 10 slope lakeward of the beach berm to the junction of the slope with the existing bottom.

40. The width of a bathing beach from the shore line at mean lake stage to the bluff should be sufficient to accommodate the number of bathers expected to use it; consequently, a bathing beach is frequently wider than a beach designed primarily for protection. The proposed width of the beach area north of the existing groin will provide 63,000 square feet of area between mean lake level and the toe of the bluff, and the southerly beach area will contain 54,000 square feet. With an allowance of 75 square feet per person the proposed beaches will accommodate approximately 840 and 720 bathers, respectively. The design beach profile commencing at the bluff would include a level berm from 40 to 70 feet wide at elevation 254.5 feet and a beach slope below the crest of the berm of 1 on 10. The shore line of the proposed beach is expected to assume an alignment parallel to the alignment of the berm of the existing impounded beach south of the existing groin.

41. Groin Design. Although the predominant direction of littoral current is from south to north there are frequent reversals in the study area. The proposed beach area is small and even a low rate of transport would soon denude the beach of sand. This would necessitate frequent costly replenishment. Groins are considered necessary to compartment the beach and prevent undue loss of the expensive sand fill in either direction. Since the proposed beach will extend a distance of 450 feet on both sides of the existing groin, one new groin is required in addition to the jetties at the creek outlet. The existing groin will have to be altered to improve its effectiveness in retaining the proposed beach fill. The southerly 450 feet of beach would then be contained between the north outlet structure and the existing groin, while the northerly 450 feet would be contained by the existing groin and the proposed new groin. The new groin which will have an over-all

length of approximately 220 feet should start at the base of the bluff. If the shore to the north is protected by revetment as proposed, it is not considered necessary to extend the groin into the bluff to allow for future erosion. The top of the landward 70 feet of the groin should be at elevation 255.5 feet. This groin is located at the downdrift end of the beach and the additional foot of height above the proposed beach berm would provide added insurance against loss of sand by overtopping the groin. The groin should extend 150 feet lakeward from the level section, with its top having a uniform slope to elevation 248.0 feet at the outer end. The outer end of the groin would be approximately coincident with the outer edge of the proposed beach fill. The elevation of the lakeward end of the groin would be higher than necessary from a protection standpoint, but is so designed to eliminate costly underwater construction. If the lake level is sufficiently low at the time the groin is constructed to allow the outer end to be lowered to elevation 246.0 feet without additional cost, it may be so constructed. However, from the standpoint of safety to small boat navigation it is desirable to have the end of the groin above water. At an elevation of 248.0 feet it would be submerged by static lake stages, about once in 4 or 5 years. Consideration was given to both steel sheet piling and quarry-run stone for the construction of the groin. Estimates of first costs of the stone groin are over 10 percent higher than for the steel groin.

42. The steel sheet pile groin consists of an inner 43-foot section and an outer 109-foot section of cantilever-type wall. Details of the groin are shown on Plate F-4. Cellular construction is used where the difference in elevation between the surface of the proposed beach fill on the south side of the groin and the existing lake bottom on the opposite side will be greater than 7.8 feet. A cantilever-type groin has insufficient strength against bending in this 69-foot section.

43. A stone groin should have the same top elevation and profile as the steel sheet pile groin. The trapezoidal cross section should be approximately 8 feet wide at the top with side slopes of 1 on 2. It would be constructed of quarry-run stone ranging in size from chips to 1,000-pound stone, with sufficient stone of sizes between these limits to make the finished structure practically impermeable. It is expected that the stone will be end-dumped from trucks and that a crane would be used for trimming the slopes and segregating, insofar as practicable, the larger stone to the outside limits of the cross section. Plans for both the steel sheet pile and stone groin are shown on Plate F-4.

44. Because the anticipated height of the beach berm south of the existing groin will be higher than the present elevation of the groin, the landward 70 feet of the existing groin should be raised to elevation 254.5 feet. Lakeward of this level 70-foot section, an additional 70 feet should be raised having its top on a 1 on 20 slope down to elevation 251.0 feet. A one-foot step at the end of this second 70-foot section would meet the top of the existing groin. The existing groin has a 2-foot thick concrete deck 15 feet wide; however, it is not considered necessary to raise the entire width of the groin, a 3-foot wide parapet wall of concrete along the southerly side of the groin would be sufficient to retain the sand fill.

Plans for the alteration of the existing groin are shown on Plate F-4.

45. Revetment Design. Criteria for the design of a stone revetment are that the protective belt and individual stones shall be of such dimensions that wave action will not displace the individual cover stones, or reach the bluff and remove fine materials by leaching. The Iribarren formula for the design of rubble-mound structures, as modified by Hudson for use of dimensionally homogeneous units in the American system, is considered to be the best guide available in determining the size of stones necessary to prevent displacement by wave action. The following general and specific assumptions were made in determining values for substitution in the formula:

a. A static lake stage of 248.7 feet. This elevation may be expected to occur once in 20 years.

b. A temporary rise of 1.3 feet, which may be expected to occur approximately once a year, based on records at Oswego, New York.

c. A 3-foot, 6-second wave could reach the toe of the structure after scour had occurred in front of the structure to elevation 246.0 feet providing 4 feet of water when the lake is at the design stage of 250.0 feet.

d. A design slope for the revetment of 1 on 2.

e. A unit weight of 165 pounds per cubic foot for stone.

Substituting these data in the modified Iribarren formula indicates that the weight of the individual face stone should be not less than 200 pounds however, because of ice action it is considered advisable that at least 25 percent of the stone weigh 500 pounds or more. The remaining 75 percent should be well graded from 500 pounds to chips, to form an impermeable structure.

46. Since the existing beach consists of gravel and cobbles, the revetment stone may be placed directly on the beach without a filter blanket of finer material. No filter is necessary against the bluff face because the revetment itself is impermeable. The top width of the revetment should be about 10 feet to allow trucks to use it during construction and also to provide sufficient material to allow flattening of the slope after construction without exposing the bluff to wave action. The lakeward face of the revetment should be constructed roughly on a 1 to 2 slope. The top elevation should be 254.5 feet. (Design lake stage 250.0 feet plus 150 percent of the height of a 3-foot wave which is capable of reaching the toe).

47. During the study, consideration was given to the use of a cobblestone beach as an alternative to the quarry-run stone revetment for protection of the same reach of shore, a large stock pile of screenings consisting of gravel and cobbles being available at a sand and gravel pit about 15 miles from the park. The size of material varies from 1/4-inch up to about 8 inches. A large proportion of this stock pile is 3 to 4

inches but contains only a small percent of larger material. Fines less than 3 inches could be readily eliminated by screening. Because of the small proportion of large material, periodic replenishment would be required. In view of the higher maintenance cost of the cobble beach and the relatively small difference in first costs the quarry stone revetment is considered more economical.

48. Grindstone Creek Outlet Design - During severe storms from the west and northwest the mouth of the creek becomes blocked by a gravel ridge which forms across the mouth due to wave action and littoral drift. The obstruction forces the waters of the creek to turn abruptly northward and follow along in front of the sea wall thus preventing a protective beach from forming in front of the sea wall south of the existing groin. To prevent the mouth from being blocked the stream should be carried across the beach into the lake to a depth where littoral drift is less pronounced. The use of a buried pipe culvert was considered but was rejected since construction would have to be done under water by a diver or in the dry by cofferdamming. Construction would be costly as would any maintenance if the pipe became blocked.

49. Further study suggested a solution by means of an outlet structure which would serve the dual purpose of providing an outlet for the creek and acting as a groin for the improvement of the adjacent beach area. The proposed outlet structure consists of two steel sheet pile jetties constructed of single rows of piling set 20 feet apart and tied together with top bracing. The location of the outlet was chosen at the point where the natural stream first approaches the barrier beach, which is also a satisfactory location considering the desirable spacing of groins for protection of the beach. The elevation of the lake bottom at the lakeward end of the jetties is approximately 240.0 feet where the depth of water is 6 feet at mean lake stage. The inner end of the jetties are at the landward edge of the barrier beach. To prevent flanking of the inner ends, short training walls are provided by flaring the opening at an angle of 45 degrees to form an entrance to the channel.

50. The top elevation of the jetties, which was determined by the anticipated height of the beach berm should be at elevation 254.5 feet which would be maintained throughout their length to reduce filling with sand and gravel from wave wash over the structure and to provide for possible future extension.

51. An attempt was made to proportion the size of the opening in the outlet to discharge the maximum runoff without excessive pending in the swamp area and still provide reasonable velocities in the outlet during normal and minimum flows. It is anticipated that flood flows will scour the opening so that only infrequent clearing will be required.

52. The following analysis was made to determine the probable maximum runoff and the dimensions of the outlet from the standpoint of discharge capacity. Topographic maps and field inspections made of the 50 square-mile

drainage area show it to be almost entirely of cultivated or uncultivated farm land and woodland. It is estimated that a 2.7-inch rainfall may be expected to occur over the area with a frequency of once in 5 years. Because data are not available for determining an accurate hydrograph, the rational method is used in computing the probable rate of runoff. It is realized that the rational method is not ordinarily applicable to a drainage area of this large a size, but, for the purpose intended in the instant case, it is considered appropriate. Substituting, then, in the formula $Q = C\ i\ A$, when $C = 0.17$, $i = 2.7/24$, and $A = 50 \times 640$, Q is determined to be 600 cfs.

53. The outlet channel would be 20 feet wide, and it is proposed that excavation would be on a straight bottom grade from elevation 244.0 feet at the inlet to approximately elevation 240.0 feet at the outlet. It is probable that some erosion at the inlet end will occur during periods of high discharge and the channel will stabilize at some lower elevation. The wall, therefore, have been structurally designed to be stable with the channel bottom at elevation 241.0 feet at the inlet end.

54. The landward portion of the beach area between the outlet structure at Grindstone Creek and the existing wall will be built up with available cobble and gravel to elevation 254.5 to minimize erosion during high flow conditions in the creek.

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