ADAPTIVE HYDRAULICS

A TWO-DIMENSIONAL MODELING SYSTEM
DEVELOPED BY THE COASTAL AND HYDRAULICS LABORATORY
ENGINEER RESEARCH AND DEVELOPMENT CENTER
A PRODUCT OF THE SYSTEM-WIDE WATER RESOURCES PROGRAM

USERS MANUAL

Second Order Temporal Terms

AdH REV 5939

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Introduction

Recently, tests were performed which analyzed the capabilities of the Adaptive Hydraulic Model (AdH) (Chapman 2005). It is widely known that first order backward schemes are dissipative. This often is not a problem in a forced system with a relatively long wave. But this test involves up to 55 cycles of refection with no forcing; and the first order scheme was not up to the task. It should be said that all numerical methods have some form of numerical dissipation which is often used to stabilize the scheme. However for best results in terms of accuracy, the numerical dissipation is reduced as much as possible. In an effort to decrease the numerical dissipation in AdH, it was decided that the option of secondorder accurate temporal terms should be available to the user as well as the first order accurate temporal terms that were used during the tests performed by Chapman. That is to say, terms in the form:

$$\frac{dh}{dt} \approx \frac{h^{n+1} - h^{n}}{dt}$$

would now be replaced by approximations in the form:

$$\frac{dh}{dt} \approx \frac{(\frac{3}{2}h^{n+1} - \frac{1}{2}h^{n}) - (\frac{3}{2}h^{n} - \frac{1}{2}h^{n-1})}{dt}$$

The increase in accuracy of these terms has much improved the previous numerical dissipation issues. This is shown using a Slosh test that was performed in the original report by Chapman (2005) and then addressed here using the improved 2nd order accurate temporal scheme.

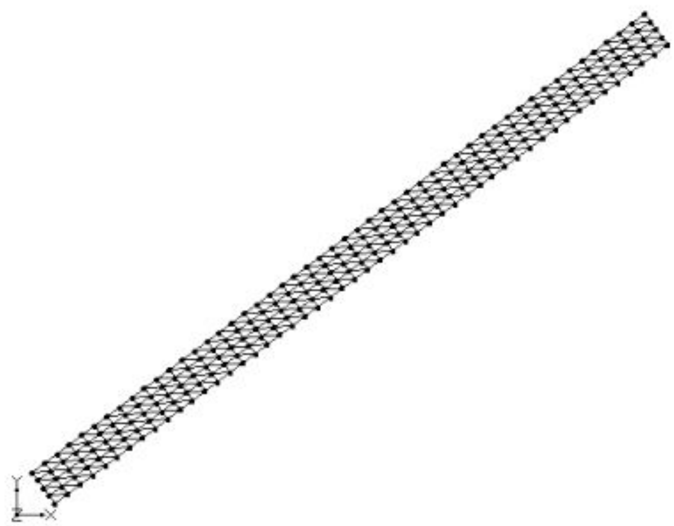
The numerical scheme was further enhanced by developing a form in which the user could choose between the two schemes or even a fractional amount of each with the use of the variable tau_temporal. The variable tau_temporal is controlled via the **OP TEM** card in AdH. The final form of the temporal scheme is given by:

$$\frac{dh}{dt} \approx \alpha \frac{(\frac{3}{2}h^{n+1} - \frac{1}{2}h^{n}) - (\frac{3}{2}h^{n} - \frac{1}{2}h^{n-1})}{dt} + (1-\alpha)\frac{h^{n+1} - h^{n}}{dt}$$

where α is tau_temporal. The possible values for α range from 0 to 1.0. Therefore, when α is input by the user as zero, the scheme will be the original first order accurate scheme. However, when α has a value of 1.0, then the resulting scheme is second order accurate.

Example problem: Slosh test

The computational grid, at right, was used to perform the slosh test on AdH. The rectangular mesh is 1000m long and 50m wide. Each cross section consists of five grid nodes which are approximately 12.5m apart. There are 50 grid nodes in the streamwise direction, which are approximately 20.4m apart. The simulations were performed without friction in order to investigate the full effect of the dissipative tendencies of the model. The Manning's N value was therefore approximately 0. The initial water surface elevation increased linearly from 4.9920m upstream to 5.0080m downstream (Figure 7.1). Consequently the magnitude of the displacement at both ends is 0.008 in the negative and positive directions respectively. As ADH is used to model the system, the fluid appears to "slosh" back and forth.



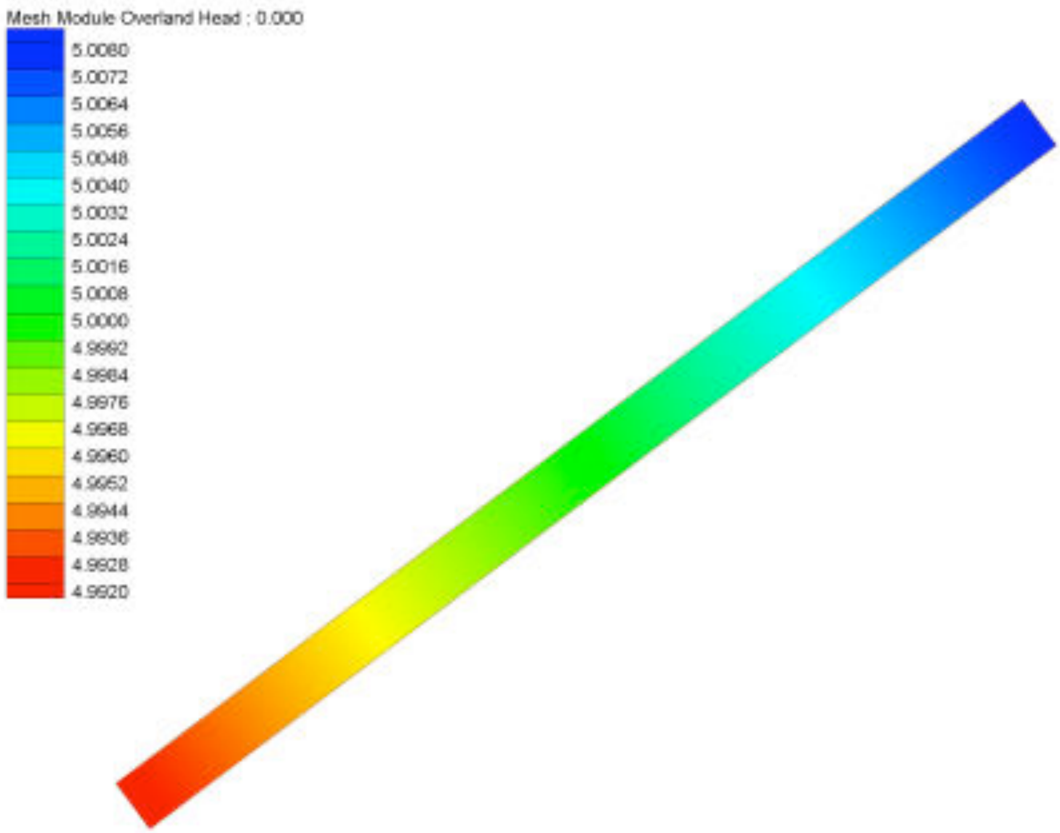


Figure 7.1: Second Order Transport Water Surface Elevation for Slosh test

In order to simulate the same conditions of the previous report, an 8 second time step was used in the first test. Seen in Figure 7.2 are the results from Chapman (2005). The x-axis gives the grid node and the y-axis the displacement from the still water horizontal position of 5.0m. As time increases, the surface displacement decreases until finally settling at a value of zero within 30 minutes. In a truly nondissipative system, the surface displacement should continue to rise and fall for an infinite amount of time. As no numerical technique is completely lacking in dissipation, it is expected that any model undergoing this test will display some dissipative tendencies, most likely settling at 0 at extremely long times. However the purpose of this test is to show the obvious decrease in artificial dissipation and therefore increase in accuracy provided by the new 2nd order temporal scheme.

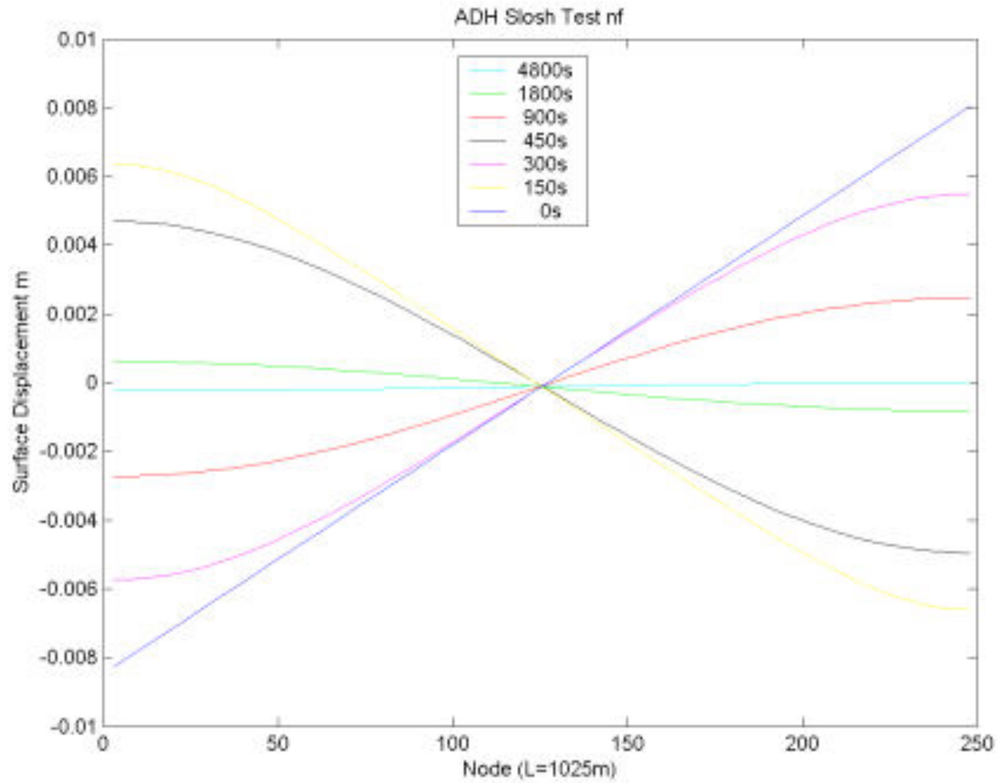


Figure 7.2: ADH slosh test performed by Chapman (2005) with 8 second time step.

In Figure 7.3, the slosh test is shown with results from the second order scheme. As in Figure 7.2, the x-axis gives the grid node and the y-axis the displacement from the still water horizontal position of 5.0m. The apparent step-like characteristic of the lines show that across the channel cross-section the water surface elevation remains constant. From this figure it is possible to see that there is a definite decrease in the dissipative effects of the scheme. However this concept is perhaps better visualized in Figure 7.4 which shows a comparison of the two different schemes. In this figure a time series is shown of the surface displacement at the first grid node only. Initially, as expected, the surface displacement is -0.008 for both schemes because this is at the furthest upstream position. As time progresses, the displacement level continues to rise and fall as the water sloshes back and forth for the 2nd order scheme. However the 1st order scheme quickly drops to zero.

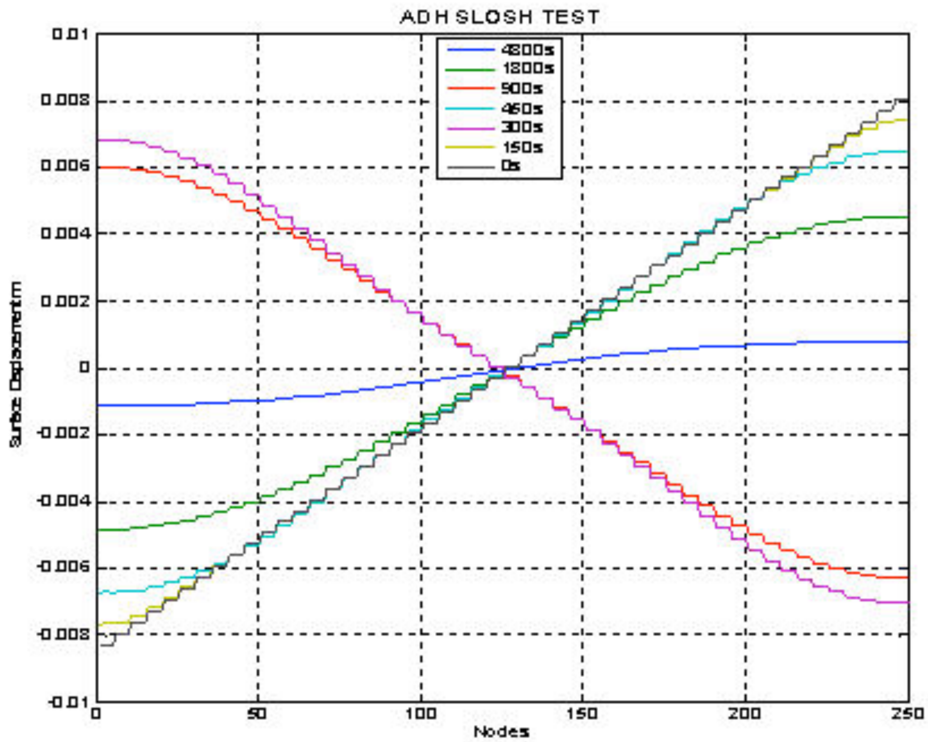


Figure 7.3: Slosh test using 2nd order accurate temporal terms.

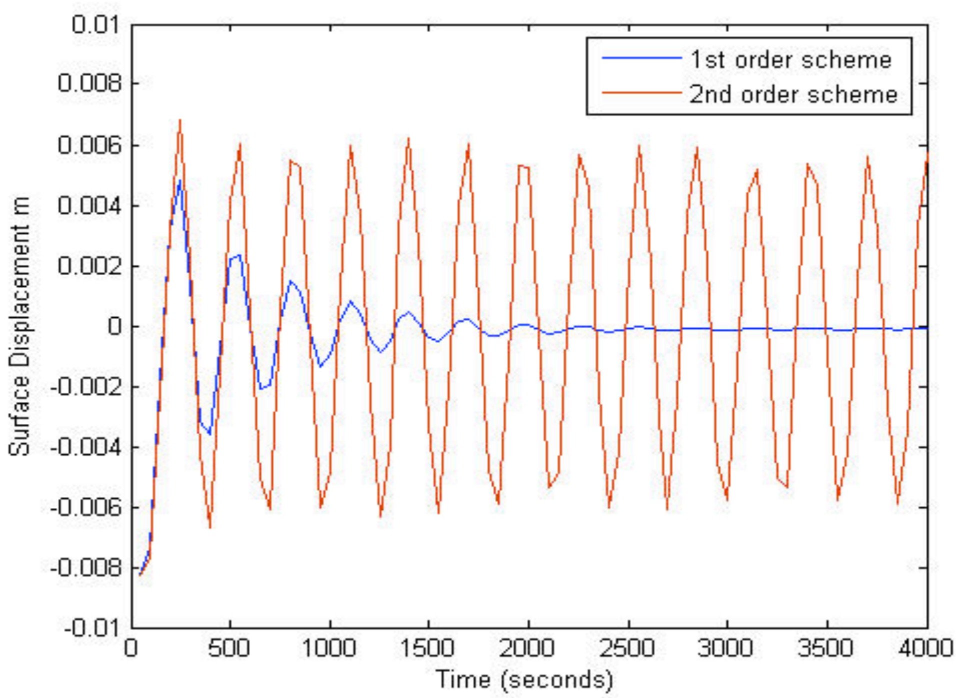


Fig
ure 7.4: Comparison of surface displacement obtained using 1st and 2nd order accurate temporal schemes.

Convergence test

In addition to the dissipation test, a convergence test (see Figure 7.5) was performed by Chapman (2005), which showed that when using relatively small time steps of 0.25 and 0.125 seconds, the resulting solutions did not converge. Figure 7.6 shows the solutions obtained when using the 2nd order accurate temporal scheme. The solutions appear to have converged to one time step independent solution.

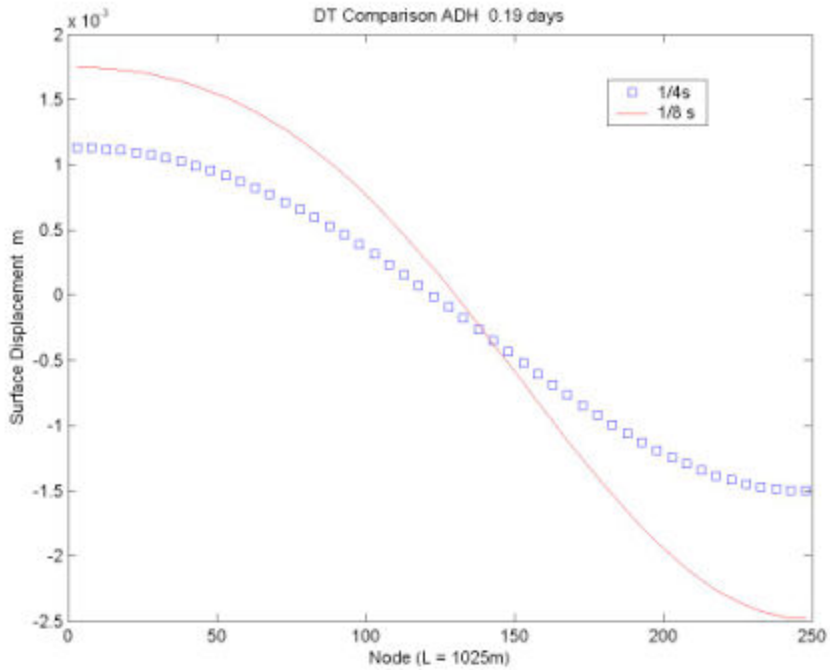


Figure 7.5: AdH time step comparison at 0.19 days into the simulation using 1st order accurate temporal scheme performed by Chapman(2005).

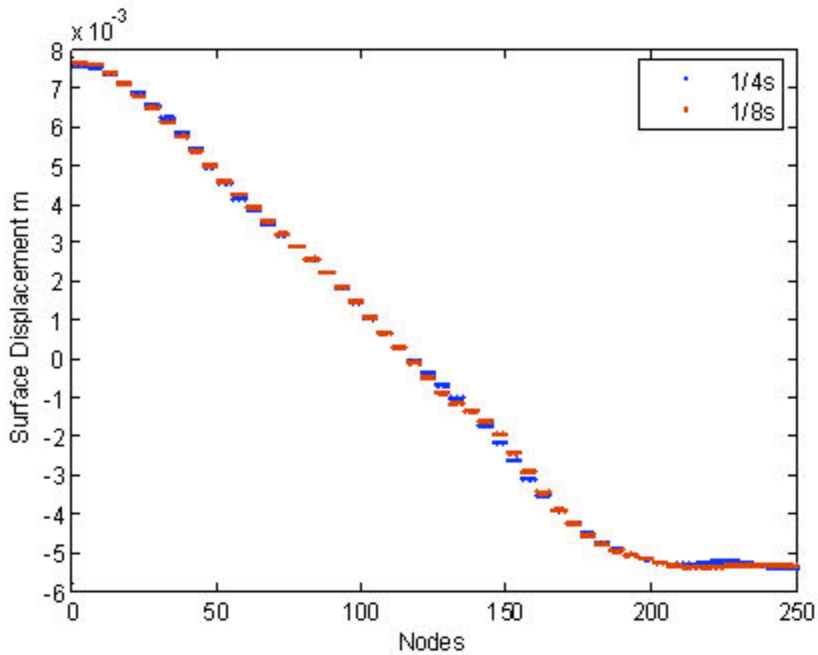


Figure 7.6: ADH time step comparison at 0.19 days into the simulation using second order accurate temporal scheme.