Friction in Turbulent Flows
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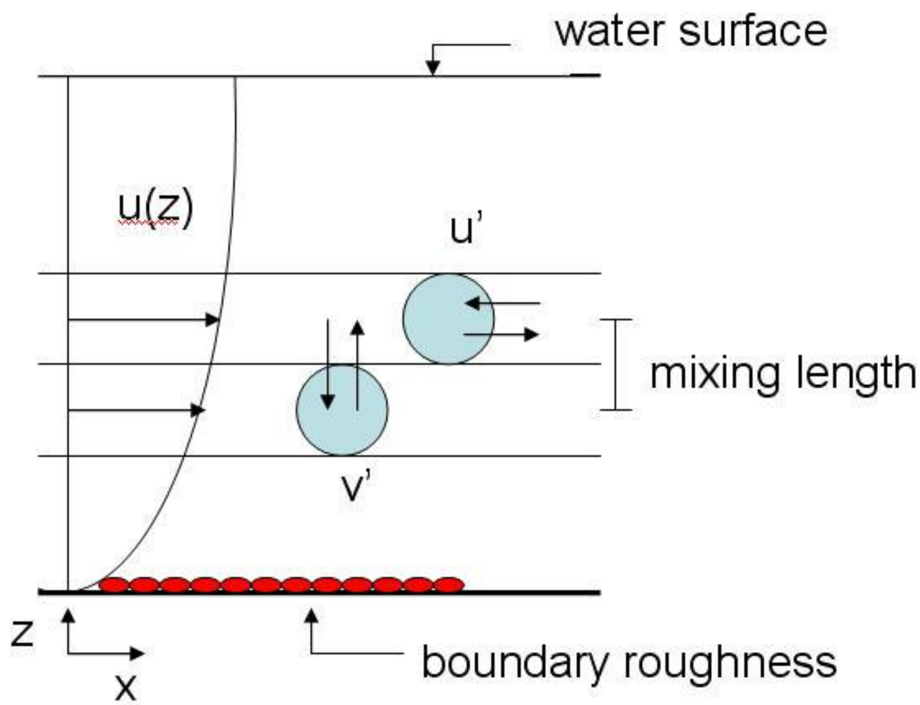
2 basic forms of friction loss – skin drag and form drag

Skin drag -- drag introduced at the interface of a solid boundary with a flowing turbulent fluid, and distributed through the flow via turbulent mixing.



Energy is transported into the flow and dissipated via the Reynolds stress

$$\tau = \rho \overline{u'v'} \tag{1}$$

How do we quantify this boundary shear?

Traditional hydraulics solutions:

Darcy-Weisbach equation:

$$\Delta H = f' \frac{v^2}{2g} \frac{L_{ch}}{R} \tag{2}$$

Chezy's equation:

$$v = C R^{1/2} S^{1/2} \tag{3}$$

Manning's equation

$$v = \frac{K_n}{n} R^{2/3} S^{1/2} \tag{4}$$

Theoretical basis of traditional hydraulic methods

A theoretical development can be used to show the physical basis and limitations of these various empirically derived methods [1]

Consider the Darcy-Weisbach equation:

$$\Delta H = f' \frac{v^2}{2g} \frac{L_{ch}}{R} \tag{5}$$

And Chezy's Equation:

$$v = C R^{1/2} S^{1/2} \tag{6}$$

Writing (5) on the form of (6), and noting that $S = \Delta H/L_{ch}$, we arrive at the following expression of (5):

$$v = 4\sqrt{2g} \frac{1}{4\sqrt{f'}} R^{1/2} S^{1/2} \tag{7}$$

Nikuradse [2] found that, for turbulent rough flow, the friction factor (f') can be expressed as a function of the dimensionless ratio of the hydraulic radius (R) to the roughness height (k):

$$\frac{1}{4\sqrt{f'}} = 1.171 + \log_{10} \frac{R}{k} \tag{8}$$

By performing a logarithmic regression of this expression, and allowing a maximum error of ±3%, the following 3 approximations of (8) are generated.

$$\text{for } \frac{R}{k} < 4.32 \qquad \frac{1}{4\sqrt{f'}} = 1.14 \left(\frac{R}{k}\right)^{1/3} \tag{9}$$

for $4.32 < \frac{R}{k} < 276 \quad \frac{1}{4\sqrt{f'}} = 1.46\left(\frac{R}{k}\right)^{1/6}$ (10)

for $\frac{R}{k} > 276 \quad \frac{1}{4\sqrt{f'}} = 2.33\left(\frac{R}{k}\right)^{1/12}$ (11)

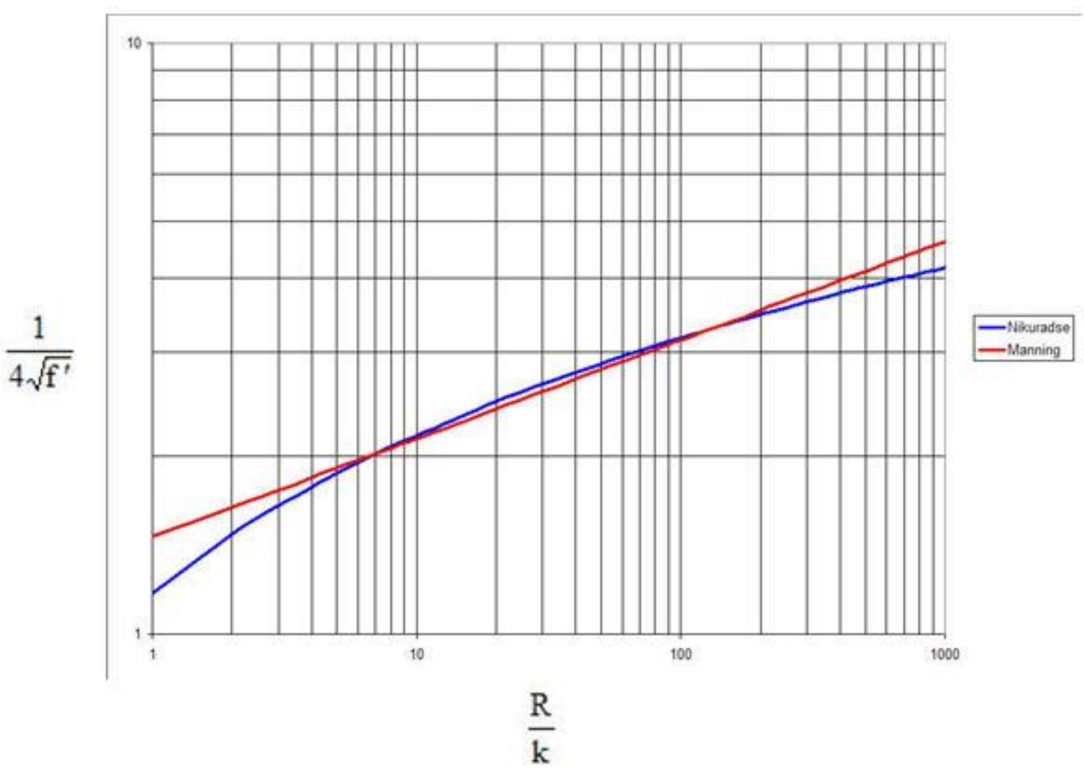
Substituting (9), (10), and (11) into (8) results in the following power formulas:

for $\frac{R}{k} < 4.32 \quad v = L\,R^{5/6}S^{1/2} \quad \text{where } L = \frac{6.46\sqrt{g}}{k^{1/3}}$ (12)

for $4.32 < \frac{R}{k} < 276 \quad v = M\,R^{2/3}S^{1/2} \quad \text{where } M = \frac{8.25\sqrt{g}}{k^{1/6}}$ (13)

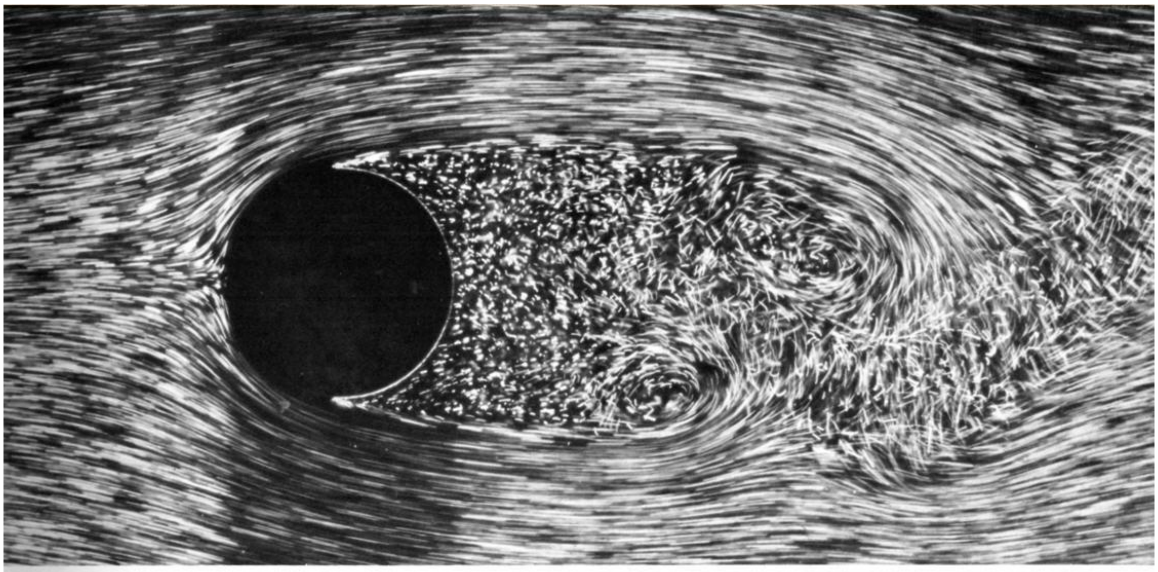
for $\frac{R}{k} > 276 \quad v = N\,R^{7/12}S^{1/2} \quad \text{where } N = \frac{13.18\sqrt{g}}{k^{1/12}}$ (14)

Note that (13) is Manning's Equation, with M substituted for K_n/n.



So Manning's equation is shown to be valid, but only for a limited range of the roughness ratio.

<u>Form drag –</u> drag induced by flow separation around an object intruding into the flow, resulting in a net momentum loss



47. Circular cylinder at $R=2000$**.** At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

Form drag can be expressed as a fraction of the momentum flux:

$$\tau = \frac{1}{2} C_D \rho u |u| \tag{15}$$

<u>Friction in estuaries:</u>

Often, equations such as Manning’s equation are used liberally, to describe not only all ranges of skin friction, but also to describe form friction phenomena, such as flow through emergent wetland vegetation.

This extends the equation beyond the limits of its validity.

Flow though “idealized” emergent vegetation

Form drag approach [3]:

$$\tau = \frac{1}{2} C_D \rho u |u| \tag{16}$$

$$C_D = C_{D.S} dm\delta \tag{17}$$

d = the water depth,
m= the density of the stems (stems per unit surface area)
δ = the diameter of each stem

Hence, $\boldsymbol{\tau} \propto$ d

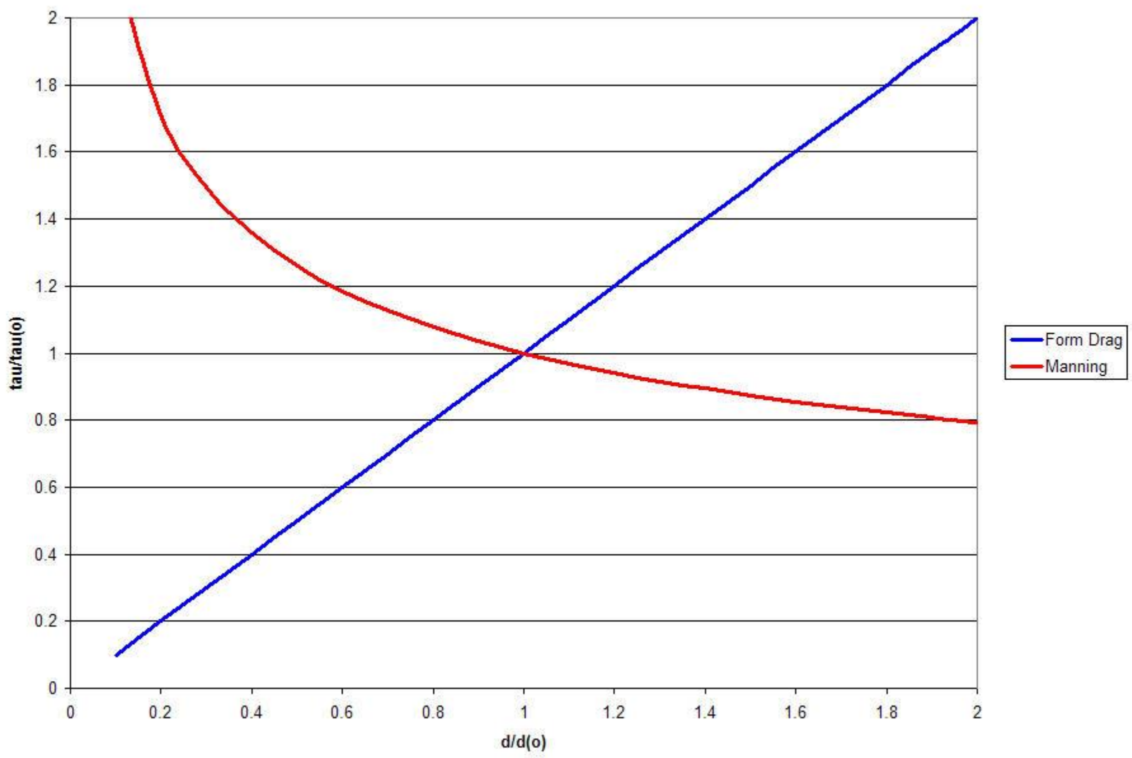
Manning's equation approach

Assuming a wide channel with R = d, and combining (13) with the standard expression for the bed shear stress (i.e. $\tau = \rho g d S$) we get the following:

$$\tau = \frac{\rho g}{M^2}\frac{v^2}{d^{1/3}} \quad \text{where } M = \frac{8.25\sqrt{g}}{k^{1/6}}$$

Hence, $\boldsymbol{\tau} \propto 1/d^{1/3}$

So if we derive our roughness coefficients for one depth of flow, and the depth changes, only the form drag coefficients can be expected to still be valid.



Hence, Skin drag relationships (such as Manning's equation) are not appropriate for use in characterizing form drag

References

1. Christensen, B. A., "Manning's n For Cast-In-Place Concrete Pipe; Discussions by Bent A. Christensen, and R. Sakthivadivel, S. Seetharaman and M.V. Somasundaram", Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 96, No. HY3., March, 1970.

2. Nikuradse, J., "Stromungsgesetze in rauhen Rohren", VDI Forschungsheft 361, Berlin, 1933.

3. Walton R., and Christensen, B. A. (1980). "Friction factors in storm surges over inland areas" *J. Waterw. Port, Coastal Ocean Div., Am. Soc. Civ. Eng.*, 106(2), 261-271.