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October 2020

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A Measurement System for the Study of Nonlinear Propagation Through Arrays of Scatterers

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Final report

Approved for public release; distribution is unlimited.

Prepared for US Army Corps of Engineers
Washington, DC 20314

Under Project AB2, Program Element Number 611102

Preface

This study was conducted for the U.S. Army Corps of Engineers under Project Number AB2, Program Element 611102, AMSCO Code 611102AB200. The technical monitor was Mr. Ritchie L. Rodebaugh, Technical Director for Geospatial Research and Engineering.

The work was performed by the Signature Physics Branch of the Research and Engineering Division, U.S. Army Engineer Research and Development Center, Cold Regions Research and Engineering Laboratory (ERDC-CRREL) and the Ecological Processes Branch of the Installations Division, Construction Engineering and Research Laboratory (ERDC-CERL).

At the time of publication, Dr. M. Andrew Niccolai was Branch Chief, and Mr. J. D. Horne was Division Chief, ERDC-CRREL. Dr. Chris Rewerts was Branch Chief and Ms. Michelle Hanson was Division Chief, ERDC-CERL. The Deputy Director of ERDC-CRREL was Mr. David B. Ringelberg, and the Director was Dr. Joseph L. Corriveau. The Acting Deputy Director of ERDC-CERL was Ms. Michelle Hanson and the Acting Director was Dr. Kirankumar Topudurti

This paper was originally published as a proceeding of the 21st International Symposium on Nonlinear Acoustics (ISNA), International Conference on Nonlinear Elasticity in Materials, Santa Fe, New Mexico, July 9-13, 2018.

The Commander of ERDC was COL Teresa A. Schlosser and the Director was Dr. David W. Pittman.

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ABSTRACT

Various experimental challenges exist in measuring the spatial and temporal field of a nonlinear acoustic pulse propagating through an array of scatterers. Probe interference and undesirable high-frequency response plague typical approaches with acoustic microphones, which are also limited to resolving the pressure field at a single position. Measurements made with optical methods do not have such drawbacks, and schlieren measurements are particularly well suited to measuring both the spatial and temporal evolution of nonlinear pulse propagation in an array of scatterers. Herein, a measurement system is described based on a z-type schlieren setup, which is suitable for measuring axisymmetric phenomena and visualizing weak shock propagation. In order to reduce directivity and initiate nearly spherically-symmetric propagation, laser induced breakdown serves as the source for the nonlinear pulse. A key component of the schlieren system is a standard schliere, which allows quantitative schlieren measurements to be performed. Sizing of the standard schliere is aided by generating estimates of the expected light refraction from the nonlinear pulse, by way of the forward Abel transform. Finally, considerations for experimental sequencing, image capture, and a reconfigurable rod array designed to minimize spurious wave interactions are specified.

1. INTRODUCTION

A z-type schlieren system is suitable for measuring axisymmetric phenomena and visualizing weak shock propagation in an array of scatterers. The measurement principle of this system is based on the refraction of collimated light that is partially blocked by an image cutoff [1, pp. 32-37]. Perturbations in the index of refraction causes light refraction within the test section and a corresponding change in light intensity on the imaging plane [2]. In order to relate light intensity changes to refraction angles, quantitative schlieren measurements rely upon a standard schliere (e.g. a refracting lens). Selection of the standard schliere is aided by estimates of refraction angles, which are based upon a forward Abel transform of the pressure gradient field derived from weak shock theory.

Additional experimental considerations include the acoustic source, experimental sequencing, image capture, and a reconfigurable rod array designed to minimize spurious wave interactions. Laser-induced breakdown of air is an ideal acoustic source for nondestructive laboratory scale measurements. Preliminary measurements of this type of source serves as input for the weak shock theory. Important considerations on experimental sequencing include sources of timing jitter and a strategy to mitigate changes in the shockwave propagation speed. Finally, a remark on a reconfigurable rod array will be presented.

2. ACOUSTIC SOURCE: LASER-INDUCED BREAKDOWN

Laser-induced breakdown of air is a highly repeatable, spherically symmetric, nonlinear acoustic source [3, 4, 5]. The beam of a high-powered laser is focused such that the light intensity exceeds the threshold of breakdown, which is on the order of 10^{15} W/m². At this intensity level an electron cascade process forms a plasma that rapidly heats the surrounding air volume. This rapid volume expansion leads to an acoustic shockwave.

Figure 1 shows the averaged waveform of 128 laser-induced shockwaves measured approximately 16.4 cm from the shock center. Shock waves were generated by a Nd:YAG laser (Quantel, Brilliant Eazy, 1064 nm, 5-6 ns pulse, 352 mJ, 5.20 mm beam diameter), measured with a baffled 1/4" acoustic microphone (Brüel & Kjær Type 4938), and recorded on an oscilloscope (Tektronix, TDS 2024B). The protective grid of the microphone was removed and a 3" diameter circular baffle was affixed to the microphone in order to delay the onset of edge diffraction [6]. The average peak is 2000 Pa and the positive phase duration is approximately 20 μ s. It can be expected that the peak pressure measured by an 1/8" microphone would be 4000 Pa, based on measurements by Qin and Attenborough [4]. This expected peak pressure and measured positive phase duration serve as input to the weak shock theory.

3. SHOCKWAVE INDUCED LIGHT REFRACTION

A. SHOCKWAVE PROPAGATION

The solution to the nondimensional lossless Burgers equation, for spherically symmetric propagation, is the Earnshaw solution [7],

$$\begin{aligned} V &= G(\phi), \\ \phi &= y + \sigma V, \end{aligned} \tag{1}$$

where $V = (p/p_0)(r/r_0)$ is the nondimensional shock amplitude, $\sigma = \beta r_0 \ln(r/r_0)p_0/(\rho_0 c_0^3 t_0)$ is the nondimensional distance, $y = \tau/t_0$ is the nondimensional retarded time, and ϕ is the Earnshaw phase variable. The acoustic pressure $p(\tau, r)$ is a function of retarded time $\tau = t - (r - r_0)/c_0$, and range r , where t is time, r_0 a reference range, and $c_0 = 343$ m/s is the small-signal speed of sound. Parameters in both

Figure 1: Shock waveform average from 128 measurements of laser induced breakdown. Vertical resolution is limited by the 8 bit digitizer in the oscilloscope. The origin of the time axis corresponds to the time of laser emission. Hence, the average shockwave speed from its origin to the microphone is approximately 371 m/s.

the nondimensional distance and nondimensional retarded time are the peak pressure at r_0 , p_0 , coefficient of nonlinearity, $\beta = 1.2$, ambient air density, ρ_0 , and positive phase duration t_0 .

Let the initial condition for Eq. (1) be a modified Friedlander waveform,

$$V(y) = (1 - y)e^{-by}, \quad (2)$$

where b is a decay constant. Using weak shock theory, the nondimensional amplitude, distance, and retarded time can be solved for in terms of the Earnshaw phase variable at the shock front (denoted with subscript s),

$$V_s = \frac{\sqrt{1 + 2\sigma(e^{-b\phi_s} + b - 1)} - 1}{\sigma b}, \quad (3)$$

$$\sigma = \frac{2e^{b\phi_s} [1 - b(1 - \phi_s) + (b - 1)e^{b\phi_s}]}{[b(1 - \phi_s)]^2}, \quad (4)$$

$$y_s = \phi_s - \frac{2 [1 - b(1 - \phi_s) + (b - 1)e^{b\phi_s}]}{b^2(1 - \phi_s)}. \quad (5)$$

Blackstock [1983] arrived at a solution for $b = 1$, however, Eqs. (3–5) are generalized for $b \geq 1$.

B. LIGHT REFRACTION THROUGH A SHOCKWAVE

At an instant in time, light refraction through a shockwave is dependent upon the index of refraction gradient [1, pp. 25-27],

$$\varepsilon_x(x) = \frac{1}{n_0} \int_{-\infty}^{\infty} \frac{\partial n(x, z)}{\partial x} dz, \quad (6)$$

where ε_x is the angle of refraction, $n_0 = 1.000292$ is the ambient index of refraction, the fluctuating index of refraction $n = kp/c_0^2$ is proportional to the acoustic pressure [2], and $k = 2.25 \times 10^{-4} \text{ m}^3/\text{kg}$ is the Gladstone-Dale coefficient. The collimated light beam is parallel to the z -axis and light refraction transverse to the light beam in the x -axis is considered (i.e. $y = 0$). Changing the variable of integration from z to r , the light refraction angle due to the gradient of acoustic pressure is,

$$\varepsilon_x(x) = \frac{2k}{n_0 c_0^2} \int_x^{\infty} \frac{\partial p(r)}{\partial x} \frac{r}{\sqrt{r^2 - x^2}} dr. \quad (7)$$

Figure 2: Simulated light refraction through shock waves as a function of normalized displacement. Reference values for peak pressure, range, and positive phase duration are: $p_0 = 4000$ Pa, $r_0 = 16.35$ cm, and $t_0 = 20$ μ s. The modified Friedlander decay constant is (a) 1.0, and (b) 1.2. A refraction angle of 5 μ rad is approximately 1 arcsecond.

By the chain rule $(\partial p / \partial x) = (\partial p / \partial r)(x / r)$. An expression for the pressure gradient, valid up to the shock front, follows from the Earnshaw solution, Eq. (1),

$$\frac{\partial p}{\partial r} = -\frac{p}{r} \left[1 + \left(\frac{\partial y}{\partial r} + G \frac{\partial \sigma}{\partial r} \frac{r}{1 + \sigma G B} \right) \right], \quad (8)$$

where $B(\phi) = [b + (1 - \phi)^{-1}]$. By the following change of variables, $\rho = r^2$, $\xi = x^2$, $F(\rho = r^2) = f(r) = r^{-1} \partial p / \partial r$, and $E_x(\xi = x^2) = \varepsilon_x(x)$, Eq. (7) becomes

$$E_x(\xi) = \frac{k \xi^{1/2}}{n_0 c_0^2} \int_{\xi}^{\infty} \frac{F(\rho)}{\sqrt{\rho - \xi}} d\rho, \quad (9)$$

where the integral is a modified forward Abel transform of $F(\rho)$ [8, pp. 262–266]. The integral is evaluated by making a linear interpolation of $F(\rho)$ over discrete intervals of ρ , evaluating the integral at each interval, and summing over all integral contributions.

Figure 2 shows predictions of light refraction angles based upon Eq. (9). For $b = 1.0$, and a propagation time of $t = 16t_0$, the maximum refraction angle is 17.0 μ rad (3.50 arcsecond) and the minimum refraction angle is -7.81 μ rad (-1.61 arcsecond). At a propagation time of $t = 40t_0$ the maximum refraction angle is 10.4 μ rad (2.15 arcsecond) and the minimum refraction angle is -6.94 μ rad (-1.43 arcsecond). The convolutional nature of the Abel transform causes negative light refraction contributions to be spread over displacements exceeding the spatial extent of a shock.

Quantifying the expected light refraction angles informs the choice of a standard schliere. Considering the small angle approximation for a lens with focal length f , the refraction angle of parallel light, at a radial distance r_l from the lens center, will be $\varepsilon = r_l / f$ [9]. The maximum refraction angle for such a lens will be at the lens periphery where $r_l = R$, and R is the radius of the lens. Considering a lens with a diameter of 25.4 mm, and specifying a maximum refraction angle of 17.0 μ rad requires the focal length to be 747 m.

4. EXPERIMENTAL SEQUENCING AND IMAGE CAPTURE

A z-type schlieren system constructed with two 317.5 mm parabolic mirrors is used to visualize the shock wave propagation through the scatterer array. Time-resolved schlieren image sequences of the propagating shock wave are captured with a high-speed framing camera (Kirana, SIL-7000-00-H01), which

Figure 3: Schematic of the velocity trap timing method for schlieren image acquisition. Here the camera will begin capturing the image sequence at $t_i = t_2 + \frac{d_2}{d_1} (t_2 - t_1)$.

records 180 924-by-728 pixel frames at a programmable rate up to 5 million frames per second. The frame rate and exposure may be adjusted to accommodate the field of view and propagation interval of interest. For example, circumscribing the test area within the field of view produces a $413 \mu\text{m}/\text{pixel}$ resolution, which a shockwave propagating at 371 m/s will traverse at a rate of $1.11 \mu\text{s}$ per pixel.

In order that each image sequence can be easily analyzed with respect to all others, the camera capture timing should be controlled to frame the same interval of shock wave propagation for each test. This requires minimizing jitter within the trigger sequence for camera capture. Although the Nd:YAG Q-switch output trigger was shown to be synchronous with laser induced breakdown using a photodiode, using the propagation distance to delay the camera capture would produce unacceptable jitter due to non-negligible changes in sound speed. Even for a purely linear wave propagating over a 1.0 m distance in a 21°C room at 344 m/s , a change in room temperature by 1°C would produce an approximate $4.9 \mu\text{s}$ change in the time of arrival. For the field of view, previously described, this change would shift the wave front in the propagation sequence by nearly 5 pixels.

For this reason, an alternative trigger method using the “velocity trap” feature of the Kirana camera was selected. In this configuration, illustrated in Fig. 3, two microphones are placed at known positions along the line of propagation between the point of laser induced breakdown and the point of interest in the schlieren test section. By specifying the microphone positions and an estimate of the shock wave speed in the configuration software, the delay between the shock wave rising edges on each microphone is used to compute the true propagation speed on the fly. Image sequence capture then is timed to begin when the wave front arrives at the desired position in the test section. In this way, the jitter due to small changes in sound speed between tests is minimized.

5. CONCLUSION

Source characteristics, expected light refraction, and timing jitter are evaluated for a quantitative schlieren measurement system. A Nd:YAG laser will satisfy the need for a nonlinear acoustic source. Weak shock propagation induces light refraction on the order of $\pm 10 \mu\text{rad}$. This informs the selection of a standard schlieren. Timing jitter due to temperature changes can be on the order of one (1) μs , which can be mitigated by using a “velocity trap.” Finally, a reconfigurable rod array may be realized by using honeycomb paneling, which will serve to suspend rods parallel to the collimated light beam and allow passage of light through both ends of the test section.

ACKNOWLEDGMENTS

We thank Michael B. Muhlestein (U.S. Army Engineer Research Development Center) for comments on a draft of this proceedings paper.

This study was conducted for the Office of the Assistant Secretary of the Army for Acquisition, Logistics, and Technology (ASA(ALT)) under Research, Development, Test, and Evaluation Program Element 61102T2400, Signature Physics and Terrain State Basic Research. Permission to publish was granted by the Director, Cold Regions Research and Engineering Laboratory.

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REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
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1. REPORT DATE (DD-MM-YYYY) October 2020		2. REPORT TYPE Final		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE A Measurement System for the Study of Nonlinear Propagation Through Arrays of Scatterers				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER 611102	
6. AUTHOR(S) Carl R. Hart and Gregory W. Lyons				5d. PROJECT NUMBER AB2	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army Engineer Research and Development Center Cold Regions Research and Engineering Laboratory 72 Lyme Road Hanover, NH 03755 Construction Engineering and Research Laboratory 2909 Newmark Drive Champaign, IL 61822				8. PERFORMING ORGANIZATION REPORT NUMBER ERDC MP-20-2	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) US Army Corps of Engineers Washington, DC 20314				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES This MP was originally published as a proceeding of the 21st International Symposium on Nonlinear Acoustics (ISNA), International Conference on Nonlinear Elasticity in Materials, Santa Fe, New Mexico, July 9-13, 2018. Funding under AMSCO Code 611102AB200					
14. ABSTRACT Various experimental challenges exist in measuring the spatial and temporal field of a nonlinear acoustic pulse propagating through an array of scatterers. Probe interference and undesirable high-frequency response plague typical approaches with acoustic microphones, which are also limited to resolving the pressure field at a single position. Measurements made with optical methods do not have such drawbacks, and schlieren measurements are particularly well suited to measuring both the spatial and temporal evolution of nonlinear pulse propagation in an array of scatterers. Herein, a measurement system is described based on a z-type schlieren setup, which is suitable for measuring axisymmetric phenomena and visualizing weak shock propagation. In order to reduce directivity and initiate nearly spherically-symmetric propagation, laser induced breakdown serves as the source for the nonlinear pulse. A key component of the schlieren system is a standard schliere, which allows quantitative schlieren measurements to be performed. Sizing of the standard schliere is aided by generating estimates of the expected light refraction from the nonlinear pulse, by way of the forward Abel transform. Finally, considerations for experimental sequencing, image capture, and a reconfigurable rod array designed to minimize spurious wave interactions are specified.					
15. SUBJECT TERMS Nonlinear acoustics, nonlinear elasticity, physical acoustics					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER (include area code)