Local Spatial Dispersion for Multiscale Modeling of Geospatial Data

Exploring Dispersion Measures to Determine Optimal Raster Data Sample Sizes

S. Bruce Blundell and Nicole M. Wayant

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S. Bruce Blundell and Nicole M. Wayant
Geospatial Research Laboratory
U.S. Army Engineer Research and Development Center
7701 Telegraph Road
Alexandria, VA 22315-3864

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Abstract

Scale, or spatial resolution, plays a key role in interpreting the spatial structure of remote sensing imagery or other geospatially dependent data. These data are provided at various spatial scales. Determination of an optimal sample or pixel size can benefit geospatial models and environmental algorithms for information extraction that require multiple datasets at different resolutions. To address this, an analysis was conducted of multiple scale factors of spatial resolution to determine an optimal sample size for a geospatial dataset. Under the NET-CMO project at ERDC-GRL, a new approach was developed and implemented for determining optimal pixel sizes for images with disparate and heterogeneous spatial structure. The application of local spatial dispersion was investigated as a three-dimensional function to be optimized in a resampled image space. Images were resampled to progressively coarser spatial resolutions and stacked to create an image space within which pixel-level maxima of dispersion was mapped. A weighted mean of dispersion and sample sizes associated with the set of local maxima was calculated to determine a single optimal sample size for an image or dataset. This size best represents the spatial structure present in the data and is optimal for further geospatial modeling.
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Preface

This study was conducted for the Geospatial Research Laboratory (GRL) under PE 62784/Project 855/Task 22, “NET-CMO.” The technical monitor was Ms. Nicole Wayant.

The work was performed by the Data and Signature Analysis Branch (TR-S) of the TIG Research Division (TR), U.S. Army Engineer Research and Development Center, Geospatial Research Laboratory (ERDC-GRL). At the time of publication, Ms. Jennifer L. Smith was Chief, TR-S; Ms. Martha Kiene was Chief, TR; and Mr. Terrance Westerfield, TV-T was the Technical Director for ERDC-GRL. The Deputy Director of ERDC-GRL was Ms. Valerie L. Carney and the Director was Mr. Gary Blohm.

COL Teresa A. Schlosser was Commander of ERDC, and Dr. David W. Pittman was the Director.
1 Introduction

1.1 Background

Terrestrial features in remotely sensed imagery or geospatial data have inherent and quantifiable spatial variability and heterogeneity. The spatial resolution of a remotely sensed image represents the scale of sensor observations on the land surface (i.e. the pixel size). Other types of spatially sampled environmental data (e.g. precipitation) can be represented in gridded or raster form. The selection of an appropriate scale depends on the type of information desired as well as the size and variability of the land phenomena under examination. In modeling processes on the Earth’s surface, the spatial resolution must be considered. If the process is affected by detail at a finer scale than provided by the data, the model’s output will be misleading (Goodchild 2011).

The relationship between the size of objects or features in an image and spatial resolution helps determine the spatial structure of the image. Fine resolution, relative to scene object size, results in high correlation of neighboring pixels, reducing the local spatial variance. Large pixel size, relative to scene objects, results in a mixing of response from different kinds of objects, also depressing local variance. The pixel size that results in a maximum variance would then best capture the spatial variation in the image (Rahman et al. 2003; McCloy and Bøcher 2007). As will be seen in this study, this general principle may not hold for images with heterogeneous spatial structure having a broad range of spatial frequency of variation for image objects.

Understanding how an image and the features in it change as the spatial resolution changes may allow for more efficient information extraction. Yet, there is no generally recognized procedure for choosing an optimal pixel size. Curran (2000) suggests that in classifying land cover by remote sensing, results that are more accurate will be obtained if the spatial resolution is similar to the size of a typical field. Using the local variance approach, Woodcock and Strahler (1987) examined simulated images of discrete land surface objects and found that a peak in a plot of local variance against pixel size occurs at a range of ½ to ¾ of the object size. Rahman et al. (2003) concluded that half the sample size associated with the lowest peak was optimal for the vegetation types under study with
hyperspectral data, as it agreed with semivariogram analysis and retained characteristic spatial variation. This approach would seem to agree with sampling theory as applied to digital images (Richards and Jia 1999). However, McCloy and Bøcher (2007) found that the sample size associated with the first trough in the local variance function is associated with the highest classification accuracy for agricultural and forested scenes with 3-band imagery in the visible range. Choosing the sample size associated with the peak introduced additional within-class variance, lowering the classification accuracy.

Selection of a particular pixel for terrestrial land surface mapping or modelling represents a sampling strategy in remote sensing. Careful selection of an optimal sample size can enhance the precision of measurement (Atkinson and Curran 1995). Marceau et al. (1994) defined optimal spatial resolution as the sampling unit that corresponds to the scale and aggregation level characteristic of the geographical entity under consideration. The authors define the aggregation level as the degree to which an earth surface feature is an assemblage of sub-elements. For example, a tree is an aggregate of leaves, branches, and bark in a particular arrangement. For a remote sensing image, the sampling unit is represented by the resolution cell.

While increasing the resolution of geospatial data can provide more information about its intrinsic subtle patterns, it can also make it more difficult to model them accurately due to noise (Costanza and Maxwell 1994). Rahman et al. (2003) assessed image spatial structure of similar vegetation by analyzing the mean local variance of pixel values at varied spatial resolutions. The authors found that a maximum value for this function may be related to an optimum pixel size for the segmentation of a particular land surface process or feature type. Two competing concerns are involved: finding a balance between reducing the correlation among neighboring pixels having sizes smaller than the spatial structure, and reducing effects of different spatial objects intermixed within a given pixel (pixel mixing). The balance between these concerns is obtained by finding the sample size associated with the maximum mean local variance of a feature when plotted against pixel size (Woodcock and Strahler 1987). This size will be tuned to the particular spatial structure of scene elements that make up the feature or features under investigation.
Whatever the strategy for choosing an optimal pixel size, the approach may be suited to an image or image subset containing a dominant feature with a characteristic spatial structure, but may not be effective for a scene with multiple disparate features (e.g. mixed land cover types). In that case, the spatial variance may not show a clear maximum as a function of spatial resolution due to the mixing of a range of frequencies of spatial variation in the scene. In this work, we build upon the literature to provide a method to optimize the sample size of a geospatial dataset. Unlike previous research, this method provides an optimal pixel size regardless of the heterogeneity of the land surface features or geospatial data under consideration.

1.2 Objectives

This study is directly supporting the New and Enhanced Tools for Civil-Military Operations (NET-CMO) project at the Engineer Research and Development Center – Geospatial Research Laboratory (ERDC-GRL). NET-CMO is concerned with the prediction and mapping of the spread of disease across space and time by mosquito vectors. In epidemiology, land cover, as derived from remote sensing, can be a critical variable in assessing vector density and risk of disease (Curran et al. 2000). The algorithms employed in the NET-CMO project require disparate spatial datasets with a wide range of spatial resolutions that must be reconciled through multiscale modeling techniques. Accordingly, the objective is to devise a semi-automated method and workflow to determine the optimal sample size for geospatial analysis and modelling within this project. To address this objective, the role of local variance was examined in the estimation of an optimal sample size for spatial data containing environmental information. In this work, the term “sample size” is used to indicate the pixel size that results from a particular level of image resampling. A methodology was devised to select a spatial resolution that will maximize the strength of the relationship between the sampled data in an image and the biophysical variables of interest.

1.3 Approach

For any image or image subset with a relatively uniform spatial structure or frequency of variation, measures of local variance often reach a maximum at some level of resampling, and this sample size may be optimal for various image or data processing functions. For many real-world raster datasets with a heterogeneous spatial structure, this ideal situation may not hold. When this happens, local variance can lose its
value as an indicator for optimal sample size. In light of these difficulties, a new approach was designed to the problem through multidimensional analysis of resampled images with increasingly coarser spatial resolution. A three-dimensional image space of resampled images was created and Local Spatial Dispersion (LSD) throughout this space was calculated. As used here, dispersion is a measure of the statistical distribution of image values in some local neighborhood. Dispersion is represented by two dispersion statistics: Local Spatial Variance (LSV) and Mean Absolute Deviation (MAD). This “LSD space” was then optimized to create a set of LSD local maxima, representing a subset of all LSD space locations. Rather than seeking an elusive single maximum value of the local variance function, the image sample sizes associated with the set of LSD space locations of the local maxima in a weighted mean formulation were used to arrive at an optimal sample size for the image under study. In this way, the locality of variance throughout the multidimensional image space is preserved and used to compute an optimal sample size for the dataset.
2 Methods

In pursuit of our objectives, an algorithmic approach was developed that addressed the creation of a spatial data model and a set of methods for performing internal calculations to arrive at an optimal sample size. These methods were required to compute and optimize LSD within the model before calculating the optimal sample size based on the discovered set of local maxima. A graphical user interface was then created in the MATLAB software environment to perform the calculations and display the results of LSD optimization.

2.1 Overview

Consider a three-dimensional model for spatial data with the image space (row and column) comprising two independent dimensions and a third dimension represented by the pixel aggregation level of the dataset’s native resolution, or original sample size (Figure 1). The LSD of image values for a particular dataset will vary throughout this spatial data model and occupies what may be called “LSD space”.

![Figure 1. Spatial data model.](image)

LSD tells us how the local distribution of image values change as the image location changes. A good choice for measuring LSD is LSV, but there are other options, such as MAD. MAD is similar to LSV except the absolute value is taken of the residuals rather than squaring them. Whatever the dispersion statistic used, LSD values will vary across any particular image according to the distribution of feature objects and their associated spatial frequencies in the image or dataset. They will also vary in the orthogonal direction for sample size at any image location. LSD can then be represented in general as the trivariate function
LSD = f(r,c,s)

where

\[ r = \text{row} \]
\[ c = \text{column} \]
\[ s = \text{sample size}. \]

This multidimensional function cannot be easily represented visually and must be estimated numerically. However, LSD space can be imagined as a series of undulating surfaces stacked on top of each other, one for each level of pixel aggregation of the spatial data’s native resolution. Any one of these surfaces may have local maxima or minima due to the response of LSD to particular spatial frequency regimes encountered at different locations in LSD space. The authors are particularly interested in finding a set of LSD local maxima, located throughout the multidimensional model, for a chosen image or spatial dataset. A spatial dispersion maximum can occur at a particular pixel aggregation level for a particular uniform feature with an associated spatial frequency. The total set of these LSD maxima values may allow us to determine an optimal sample or support size for more efficient image segmentation, or provide a basis for determining a uniform support size required by higher-level multiscale modeling algorithms for spatial data.

2.2 Algorithmic approach

The first step in the process is to create the spatial data model by populating it with resampled versions of the original image or spatial dataset with progressively lower spatial resolution. To do this, a resampling method must be chosen to create these pixel aggregation levels calculated by a neighborhood function. Three resampling methods were allowed: block processing of the mean of each set of neighborhood values, bilinear interpolation, and bicubic interpolation.

Next, compute the local dispersion at each cell location for each resampled image in the spatial data model. As mentioned previously, common techniques for this purpose include LSV and MAD. The result is a set of LSD “images,” each one having the spatial resolution of the resampled image from which it was created. However, in order to proceed further with the matrix algebra necessary to find the set of LSD maxima in this multidimensional space, the spatial data model must have uniform
granularity along all three orthogonal directions \( (r,c,s) \). This is required in order to have a uniform distribution of LSD values. Due to the nature of the pixel aggregation process, this granularity decreases in the sample size \( (s) \) direction. Therefore, an interpolation scheme must be applied to each LSD image to achieve the same spatial resolution as the original image. Nearest neighbor interpolation will be employed for this purpose. This will result in a uniform distribution of LSD values throughout LSD space.

In order to find the complete set of LSD maxima, the multidimensional function \( LSD = f(r,c,s) \) must be optimized. Optimization strategies can be classified into two groups depending on whether they require evaluation of derivatives. Direct methods are those that do not require such calculations; gradient methods are those that do (Chapra and Canale 2002). A gradient method involving a matrix of second-order partial derivatives known as the Hessian matrix will be employed. This symmetric matrix will have one row and column for each independent variable. The procedure will be as follows: the original dataset will first be successively re-sampled to provide the sample size dimension in LSD space. At each level of resampling, LSD will be calculated for each row, column location. The elements of the Hessian matrix \( H \) will then be evaluated by finite-difference approximation for each LSD space location. For each 3-element vector \( \mathbf{x} = \mathbf{x}(r,c,s) \) in LSD space, a particular series of determinants will be computed based on three subsets of \( H \). These determinants will allow us to test the Hessian for a property known as negative definiteness. Describing this property involves consideration of other concepts in linear algebra and they will not be pursued further here. For the purposes of this report, it can be shown that \( \mathbf{x} \) will be a local maximum of \( f(r,c,s) \) if \( H(\mathbf{x}) \) is negative definite. The test for negative definiteness will involve the second partial derivatives with respect to each variable, as well as the mixed partials with respect to any two of the three variables.

The result will be a set of \( \mathbf{x} \) vectors that provides the sample size, \( s \), associated with each local maximum of \( f(r,c,s) \) in the dataset. A weighted average of these values can then be made based on the pixel aggregation level associated with the sample size. This average will be taken as the optimal sample size for the image or dataset under study.
2.3 Algorithm development

2.3.1 Hessian matrix optimization

Let us first define the Hessian matrix for the three independent variables: row \((r)\), column \((c)\), and sample size \((s)\):

\[
H_{rcs} = \begin{bmatrix}
h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial^2 f}{\partial r^2} & \frac{\partial^2 f}{\partial r \partial c} & \frac{\partial^2 f}{\partial r \partial s} \\ \frac{\partial^2 f}{\partial c \partial r} & \frac{\partial^2 f}{\partial c^2} & \frac{\partial^2 f}{\partial c \partial s} \\ \frac{\partial^2 f}{\partial s \partial r} & \frac{\partial^2 f}{\partial s \partial c} & \frac{\partial^2 f}{\partial s^2}
\end{bmatrix}
\]

In order to find the Hessian for each \(x\) vector, each element must be evaluated numerically. A finite divided-difference approximation method will be used for this purpose. The values of \(x\) in the row, column, and sample dimensions will be perturbed by some small fractional value, \(\delta\), to generate the partial derivatives. \(\delta\) cannot be too small or too large. Too small a value may not provide enough variation in the variable to capture the functional trend at that location. Too large a value may cause excess inaccuracy in the estimate for the derivative. Nominally, each \(\delta\) increment in LSD space can be taken as an adjoining raster grid cell (one pixel) along one of the orthogonal axes \(r,c,s\).

In employing the divided-difference method to approximate the partial derivatives, one can normally choose from equations for a “forward,” “centered,” or “backward” sampling scheme for the \(\delta\) increment. Since the centered difference equations are considered a more accurate representation of the derivative, this approach will be used to estimate the Hessian matrix elements. This requires adding and subtracting \(\delta\) for each independent variable in the approximation equations, maintaining a consistent approach. However, because we cannot sample outside image boundaries, Hessian elements for pixels within a distance \(\delta\) of the \(r,c\) edges for each image will not be able to be estimated. Normally, this limitation would also apply along the \(s\) axis as well. However, because any higher resolution images with sample sizes between \(s = 1\) and \(s = \delta\) may contain a large amount of LSD local maxima information, these images
will be retained by substituting delta increments that yield LSD samples in the positive s direction.

The result of these divided-difference calculations is an estimated Hessian matrix for each location in LSD space. The centered approximation equations for the 9 Hessian elements $h_{ij}$ ($i=1,2,3; j=1,2,3$) are provided below. If assumed that the partials are continuous in the region surrounding each location, $x$, in LSD space, the mixed partials will be equivalent, e.g. $\partial^2 f / \partial r \partial c = \partial^2 f / \partial c \partial r$.

**Centered Divided Difference:**

\[ h_{11} = \partial^2 f / \partial r^2 = \frac{[f(r+\delta r,c,s) - 2f(r,c,s) + f(r-\delta r,c,s)]}{(\delta r)^2} \]

\[ h_{22} = \partial^2 f / \partial c^2 = \frac{[f(r,c+\delta c,s) - 2f(r,c,s) + f(r,c-\delta c,s)]}{(\delta c)^2} \]

\[ h_{33} = \partial^2 f / \partial s^2 = \frac{[f(r,c,s+\delta s) - 2f(r,c,s) + f(r,c,s-\delta s)]}{(\delta s)^2} \]

\[ h_{21} = \partial^2 f / \partial r \partial c = \frac{[f(r+\delta r,c+\delta c,s) - f(r+\delta r,c-\delta c,s) - f(r-\delta r,c+\delta c,s) + f(r-\delta r,c-\delta c,s)]}{4\delta r \delta c} \]

\[ h_{31} = \partial^2 f / \partial r \partial s = \frac{[f(r+\delta r,c,s+\delta s) - f(r+\delta r,c,s-\delta s) - f(r-\delta r,c,s+\delta s) + f(r-\delta r,c,s-\delta s)]}{4\delta r \delta s} \]

\[ h_{32} = \partial^2 f / \partial c \partial s = \frac{[f(r,c+\delta c,s+\delta s) - f(r,c+\delta c,s-\delta s) - f(r,c-\delta c,s+\delta s) + f(r,c-\delta c,s-\delta s)]}{4\delta c \delta s} \]

where

\[ h_{12} = h_{21}; \quad h_{13} = h_{31}; \quad \text{and} \quad h_{23} = h_{32}. \]

The next step in the process is to test each Hessian for the property of negative definiteness. Every location, $x$, in LSD space for which $H(x)$ is negative definite will define a local maximum for $f(r,c,s)$. To perform this test, first find the determinants of three subset matrices $H_1, H_2, H_3$ of the Hessian, starting from the upper left position ($h_{11}$). These are:

\[ H_1 = \begin{bmatrix} h_{11} \end{bmatrix} \quad (\text{a 1x1 matrix}) \]
\[
\det(H_1) = h_{11} = \frac{\partial^2 f}{\partial r^2}
\]

\[
H_2 = \begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix} \quad \text{(a 2x2 matrix)}
\]

\[
\det(H_2) = h_{11} h_{22} - h_{12} h_{21}
\]

\[
= \frac{\partial^2 f}{\partial r^2} \frac{\partial^2 f}{\partial c^2} - \frac{\partial^2 f}{\partial r \partial c} \frac{\partial^2 f}{\partial c \partial r}
\]

Under the assumption that the partials are continuous in the region surrounding location \(x\) in LSD space,

\[
\det(H_2) = \frac{\partial^2 f}{\partial r^2} \frac{\partial^2 f}{\partial c^2} - \left(\frac{\partial^2 f}{\partial r \partial c}\right)^2
\]

\[
H_3 = H \quad \text{(the full 3x3 matrix)}
\]

\[
\det(H_3) = h_{11} h_{22} h_{33} - h_{11} h_{23} h_{32} - h_{12} h_{21} h_{33} + h_{12} h_{23} h_{31} + h_{13} h_{21} h_{32} - h_{13} h_{22} h_{31}
\]

\[
\det(H_3) = \frac{\partial^2 f}{\partial r^2} \frac{\partial^2 f}{\partial c^2} \frac{\partial^2 f}{\partial s^2} - \frac{\partial^2 f}{\partial r^2} \frac{\partial^2 f}{\partial c \partial s} \frac{\partial^2 f}{\partial s \partial c} - \frac{\partial^2 f}{\partial r \partial c} \frac{\partial^2 f}{\partial c \partial r} \frac{\partial^2 f}{\partial s \partial r} + \frac{\partial^2 f}{\partial r \partial c} \frac{\partial^2 f}{\partial c \partial s} \frac{\partial^2 f}{\partial r \partial s} + \frac{\partial^2 f}{\partial r \partial s} \frac{\partial^2 f}{\partial c \partial r}
\]

Again, assuming that the partials are continuous in the local region,

\[
\det(H_3) = \frac{\partial^2 f}{\partial r^2} \frac{\partial^2 f}{\partial c^2} \frac{\partial^2 f}{\partial s^2} - \frac{\partial^2 f}{\partial r^2} \frac{\partial^2 f}{\partial c \partial s} \frac{\partial^2 f}{\partial s \partial c} - \frac{\partial^2 f}{\partial r \partial c} \frac{\partial^2 f}{\partial c \partial r} \frac{\partial^2 f}{\partial s \partial r} + \frac{\partial^2 f}{\partial r \partial c} \frac{\partial^2 f}{\partial c \partial s} \frac{\partial^2 f}{\partial r \partial s} + \frac{\partial^2 f}{\partial r \partial s} \frac{\partial^2 f}{\partial c \partial r}
\]

The following conditions are necessary and sufficient for \(H(\mathbf{x})\) to be negative definite:

\[
\det(H_1) < 0
\]

\[
\det(H_2) > 0
\]

\[
\det(H_3) < 0
\]
This test is applied to every location vector \( x \) in LSD space, ultimately transforming LSD space into a “local maximum” space. \( x \) is a local maximum of \( f(r,c,s) \) wherever \( H(x) \) is negative definite. The output from these operations is, in theory, the set of optimal sample sizes associated with the subset of \( x \) vectors defined by the negative definiteness property of \( H(x) \) across the image or spatial dataset as determined by the LSD approach. These may be mapped to particular feature objects in the data with relatively uniform spatial frequencies to determine the optimal sample sizes generated by different features. If a single optimal sample size for the full dataset is desired, a weighted mean may be taken of the full set of derived sample sizes.

In this treatment, the mean of the set of sample sizes associated with the set of LSD local maxima determined by the above procedure will be weighted by the LSD value associated with each local maximum. Because every location in the dataset’s LSD space is investigated for a possible local maximum, this single average sample size will be implicitly weighted by the area of individual feature objects that generate similar optimal sample sizes due to a relatively uniform spatial frequency response in the data.

2.3.2 Peakedness and optimal sample size

This complete set of local maxima may not be of uniform quality in terms of the robustness of each maximum found for \( LSD = f(r,c,s) \). That is, there may be some very weak or “shallow” maxima that are barely included in the set because they meet the requirements for negative definiteness near the limits of precision for the floating point numbers used in the calculations. These maxima may have spurious accuracies and may not represent the spatial frequencies of the underlying image or spatial data feature. It may be useful, therefore, to apply a threshold to exclude these lower-quality maxima. The term “peakedness” will be used to describe the strength or quality of the LSD local maximum.

The peakedness of each local maximum will be calculated using the Laplacian of the function \( LSD = f(r,c,s) \) evaluated at each point determined by the Hessian matrix calculations. From vector analysis, the Laplacian is a term that means the “divergence of the gradient” of a scalar function, and is itself a scalar quantity. For a local maximum of a multivariate function, the Laplacian will be a negative number. The more “peaked” the local maximum, the more negative the number. In this way, the range of Laplacian values can be calculated for the initial full set of
local maxima, and then a chosen threshold can be applied expressed as a percentage of that range to include only those maxima with Laplacian values more negative than the threshold. The full set of local maxima in LSD space is equivalent to a threshold of zero.

For the purposes here, the scalar function is $LSD = f(r,c,s)$. The Laplacian at any point $(r,c,s)$ is then given by

$$\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial c^2} + \frac{\partial^2 f}{\partial s^2}$$

Fortunately, these second-order partial derivatives were already estimated numerically when calculating the Hessian matrix for each location in LSD space, and comprise the principal diagonal of the matrix. They are now available to calculate the Laplacian for the set of local maxima determined by the Hessian matrix analysis. To do this, the trace (the sum of elements of the principal diagonal) is found of each Hessian matrix $tr(H_{rcs})$ in LSD space. The full range of Laplacian values, or peakedness, in LSD space can then be found.

The final step in this process is the calculation of optimal sample size. Using peakedness, the effect of different thresholds on the process of finding an optimal sample size for the whole image can be explored. The optimal size is defined as the mean of the set of sample sizes associated with the LSD space locations of the set of local maxima after applying a chosen Laplacian threshold, if desired. This mean is weighted by the number of local maxima and their associated LSD values at each sample size. It is given by

$$S_{opt} = \frac{\sum_{i,j=1}^{n,m} LSD(lmax_{i,j})S_i}{\sum_{i,j=1}^{n,m} LSD(lmax_{i,j})}$$

where

- $S_{opt}$ = optimal sample size
- $i$ = resampled image number
- $n$ = total number of resampled images
- $m$ = total number of LSD local maxima in resampled image $i$
- $LSD(lmax_{i,j})$ = for image $i$, the LSD value for each $j$ of $m$ local maxima with peakedness above a given threshold
- $S_i$ = sample size of image $i$
2.4 Graphical user interface development

Using the approach described above, a workflow of procedures and user-defined parameters was created to process an image or spatial dataset for determination of a useful optimal sample size. These procedures were coded into a series of MATLAB functions and developed a Graphical User Interface (GUI) to call these functions, create and optimize the LSD space, calculate the optimal sample size, and generally streamline the process for the user. The GUI was named the LSD Analysis Tool (Figure 2). The flow of required steps are numbered in order, and allow the user some control over the process by changing default parameters.

Figure 2. Local spatial dispersion analysis tool.

The steps required by the LSD Analysis Tool are the following:

1. Select image or spatial dataset in GeoTIFF format. If an image, it should consist of a single spectral band or panchromatic values.
2. Set native resolution Ground Sample Distance (GSD) and units. Normally, this is automatically read from the GeoTIFF header. Default units are in meters.
3. Define subset. The user can display and process a subset of the original image by declaring the vertical and horizontal offsets from the upper left corner of the image, and the height and width of the subset. Default values are for the entire image.

4. Select output file to hold a text summary of processing output.

5. Create LSD space. LSD is calculated as a neighborhood function to find residuals between each kernel element and the mean value of the kernel throughout each resampled image. The user has control over the size of the kernel (default is 3x3 pixels) as well as the choice of dispersion statistic: LSV or MAD. As with LSV, MAD computes residuals, but takes their absolute value rather than squaring them. The number of resampled images created is controlled by the maximum percentage of edge pixels (default 5%). The higher this number, the more images can be created. There are three choices for the image resample method: pixel block mean value (the default), bilinear interpolation, and bicubic interpolation. After these parameters are chosen, the ‘RUN’ button is depressed to create the LSD space from the sequence of resampled images.

6. Optimize LSD space. Here, the user can choose to set a minimum peakedness threshold (default 25% of the peakedness range). The finite difference delta (default 1) is the pixel interval used in the finite difference equations needed for Hessian optimization. The optimization process begins on depressing the ‘RUN’ button. The number of LSD local maxima found is then shown, both with and without the peakedness threshold. Finally, the relevant optimal sample sizes are displayed.

7. Plot results. The user can display the processed image at any sample size and has nine plotting options to display the results of processing for the chosen parameters. The various plotting options are grouped in relation to sample size, LSD space, and image space. These plots show the behavior and distribution of computed local maxima and LSD values, and may offer clues to exploring other parameter options with repeated experimentation. The available plots are:

   a. Mean and median LSD vs. sample size
   b. Number of LSD local maxima vs. sample size (semilog plot)
   c. Mean peakedness vs. sample size
   d. Point cloud distribution of local maxima in LSD space
   e. Scatter plot of all LSD space values vs. peakedness
   f. Histogram of peakedness for all local maxima
   g. Scatter plot of local maxima for a chosen sample size
   h. Heat map of LSD values for a chosen sample size
   i. Histogram of LSD values for a chosen sample size
The LSD Analysis Tool is designed to allow the user to quickly assess spatial datasets of wide-ranging spatial structure for optimal sample size by applying this novel approach of weighted means of distributed dispersion local maxima. Trial-and-error runs can be performed easily, using different combinations of user-controlled parameters. For example, by changing the peakedness threshold and plotting results, the user can explore the distribution of local maxima in relation to known features across the image space for a chosen sample size, as well as in the orthogonal sample size dimension.
3 Data

To test this algorithmic approach, multiscale processing was performed on several examples of geospatial data: a WorldView2 image over Florida processed as Normalized Difference Vegetation Index values (Figure 3) and three environmental datasets from Cambodia for tree cover (Figure 4), population density (Figure 5), and precipitation (Figure 6). These datasets have a wide disparity of spatial resolutions: 1.3 m, 30 m, 991 m, and 4954 m, respectively. The Cambodia datasets figured prominently in the NET-CMO research effort on mosquito-borne disease modeling.

Figure 3. WorldView2 image (Florida).

Figure 4. Tree cover data (Cambodia).
Figure 5. Population density data (Cambodia).

Figure 6. Precipitation data (Cambodia).
4 Results

Statistics from the multiscale optimization processing of each dataset are provided in Table 1, including the optimal sample size results with and without the peakedness threshold. Example peakedness thresholds were chosen for each dataset to display an appreciable fraction of the total number of local maxima. The number of pixels available in each original image acts as an upper limit on the number of resampled images that can be created for optimization. This is controlled by varying the maximum percentage of edge pixels. For consistency in comparison, all processing was performed with the following parameters in common: computation kernel size, 3x3; resample method, pixel block mean value; LSD statistic, Mean Absolute Deviation (MAD); and finite difference equation delta value in pixels, 1.

Table 1. Dataset parameters and optimal sizes.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>native res. (m)</th>
<th>rows</th>
<th>cols.</th>
<th># images</th>
<th>peak thresh. (%)</th>
<th># local maxima thresh.</th>
<th># local maxima no thresh.</th>
<th>optimal size (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Florida WorldView2</td>
<td>1.3</td>
<td>908</td>
<td>1304</td>
<td>28</td>
<td>25</td>
<td>311534</td>
<td>707546</td>
<td>3.71</td>
</tr>
<tr>
<td>Cambodia tree cover</td>
<td>29.7</td>
<td>1470</td>
<td>3277</td>
<td>21</td>
<td>20</td>
<td>81936</td>
<td>3026162</td>
<td>50.1</td>
</tr>
<tr>
<td>Cambodia population</td>
<td>991</td>
<td>197</td>
<td>395</td>
<td>29</td>
<td>1</td>
<td>6601</td>
<td>40344</td>
<td>2180</td>
</tr>
<tr>
<td>Cambodia precipitation</td>
<td>4954</td>
<td>40</td>
<td>80</td>
<td>7</td>
<td>15</td>
<td>868</td>
<td>1480</td>
<td>10984</td>
</tr>
</tbody>
</table>

The goal of the optimization process is to arrive at an optimal sample size for the dataset, whether or not a peakedness minimum threshold has been set. The plotting options in the LSD Analysis Tool offers the means to explore how changing different parameters can affect the size of the LSD space created, as well as the number, size, and distribution of the set of local maxima within that space as it relates to features within the dataset. In the following sections, selected examples are presented of plotting options available to the user that provide insight into the optimization process.
4.1 Florida WorldView2 dataset

For this high-resolution dataset depicting a mix of canopy, linear features, and open ground, Figure 7 shows a plot of the mean and median of the chosen LSD statistic (in this case, MAD) for each resampled image. Both these measures of central tendency reach a maximum at a sample size of about 6.5 m. This value agrees well with the optimal sizes given by the LSD optimization process.

Figure 8 shows the frequency distribution of local maxima across the series of resampled images. The maxima become less frequent in the resample size dimension, except for a slight increase at the first resample size of 2.6 m. This image contains the highest fraction of local maxima. The thresholded subset of these is depicted in Figure 9, showing their distribution across the image resampled to 2.6 m. It is apparent that they are spatially associated with different features in the images, such as the pattern of canopy and the edges of the canal in the lower left.

Figure 7. Mean, median MAD vs. sample size (WorldView2 dataset).
The full distribution of thresholded local maxima in LSD space is shown as a point cloud in perspective view in Figure 10. Note the influence of the image’s linear features in the vertical distribution of local maxima.
4.2 Cambodia tree cover dataset

Figure 11 shows a plot of the MAD mean and median for each resampled image. In this case, their plots do not reach a local maximum against resample size, so they do not give an indication of an optimal size. In spite of this, the LSD optimization method provides optimal sizes of 97 m (unthresholded) and 50 m (thresholded) for a native resolution of 30 m.
Figure 12 shows a heat map of MAD values at the sample size 89 m. This sample size is the closest in the series to the calculated optimal size of 97 m. Figure 13 depicts the distribution of thresholded local maxima, derived from the MAD heat map distribution, for the 89 m resampled image. It is apparent that the local maxima arrange themselves at locations where there are sudden changes in MAD values across the image space as seen in the heat map.
Figure 13. Local maxima distribution, sample size 89 m (Cambodia tree cover dataset).

Since they were thresholded at 20% of the peakedness range, it is apparent that the remaining maxima represent a small fraction of the total. Table 1 shows that this figure is 81936/3026162 or 2.7%. Of these, 6,068 local maxima are found at sample size 89 m, but this is sufficient to reveal their distribution according to the change of variance across the image space.

Figure 14. Peakedness histogram (Cambodia tree cover dataset).
4.3 Cambodia population density dataset

This dataset contains a large body of water within which there are no values. Higher population densities surround the lake and line the watercourses that empty into it. As reported in Table 1, the calculated thresholded and unthresholded optimal sample sizes are 2,180 and 3,212 m, respectively, given the native spatial resolution of 991 m. Figure 15 shows an upward trend in the MAD mean and median plots for the lower sample sizes in the series of 29 images, along with the computed optimal sizes of 2,180 m and 3,212 m for thresholded and unthresholded peakedness, respectively.

Figure 15. Mean, median MAD vs. sample size (Cambodia population dataset).

Figure 16 shows the MAD heat map for the resample size 2,972 m, the size closest to the unthresholded optimal value. The full point cloud distribution of thresholded local maxima in LSD space derived from the MAD values is shown in Figure 17. However, in this view, we are looking straight down along the sample size axis at the local maxima found in the entire resampled image series.
Figure 16. MAD heat map, sample size 2,972 m (Cambodia population dataset).

Figure 17. Local maxima distribution in LSD Space, Vertical View (Cambodia population dataset).

Figure 18 shows a scatter plot of MAD values for all local maxima in LSD space, plotted against their peakedness values. This plot gives the user a sense of how the maxima are distributed across the peakedness range as well as the range of dispersion from which they were derived. Figure 19 is a plot of mean peakedness for each image in the resample series, showing that it is highest at the original spatial resolution and then drops down to a relatively constant value as sample size increases.
Figure 18. Scatter plot of MAD vs. peakedness (Cambodia population dataset).

Figure 19. Mean peakedness for resampled images (Cambodia population dataset).
4.4 Cambodia precipitation dataset

For this dataset with the largest native resolution of 4,954 m, the plot of the MAD mean and median for each resampled image is shown (Figure 20). In this case, their plots not only do not reach a local maximum against resample size, but also continue an upward trend through the resample size series. Yet, the LSD optimization approach still provides reasonable optimal sizes of 11,572 m (unthresholded) and 10,984 m (thresholded).

![Mean, median MAD vs. sample size](image.png)

The sample size in the resampled image series closest to the calculated optimal sizes is 9,909 m. Figure 21 is a histogram of the frequency of MAD values in the image with that spatial resolution, showing a maximum at a MAD value of about 10-12 m. A MAD heat map is provided for sample size 9,909 m in Figure 22. Here, one can see that the higher dispersion values are associated with transition zones with higher spatial frequencies in the original image. Finally, Figure 23 shows a perspective view of the point cloud of thresholded local maxima throughout LSD space. Their distribution appears more homogeneous at higher levels in the space.
Figure 21. MAD value frequency histogram, sample size 9,908 m (Cambodia precipitation dataset).

Figure 22. MAD heat map, sample size 9,908 m (Cambodia precipitation dataset).
Figure 23. Local maxima distribution in LSD space (Cambodia precipitation dataset).
5 Discussion

In order to test this particular optimization approach, the algorithms that implement it, and the LSD Analysis Tool, a small suite of images were selected that represented a wide range of native resolutions, feature data types, and spatial frequency regimes. Optimal sample sizes were successfully calculated in all cases that scaled well with initial resolutions as shown in Table 1. To maintain a degree of consistency, the following processing parameters were kept constant for all four datasets: computation kernel size, resample method, LSD statistic, and the delta interval for the finite difference equations. At the time of writing, it is not known how modification of these processing parameters would affect results in terms of computed optimal sample sizes or local maxima distributions.

The results showed that optimal sizes for thresholded peakedness were always slightly less than those that were unthresholded. The separation depends on the choice of threshold. This suggests that local maxima with lower values of peakedness are more concentrated near the top of LSD space, increasing the representation of smaller resample sizes in the weighting process. In fact, it was found that the mean peakedness for each image in LSD space was highest for the original resolution of each dataset in the study. This result, as depicted in Figure 19, is typical.

In this methodology, optimal sample size results are driven by the number and distribution of LSD local maxima as well as the LSD values associated with each local maximum. If a peakedness threshold is chosen, the set of local maxima is first winnowed by a minimum peakedness value. Whatever final set of maxima is used for optimization, they end up arranged in LSD space according to feature locations at each sample size and define the patterns of changing spatial frequencies therein (Figures 9, 13, and 17).

The setting of a peakedness threshold can be a useful tool for exploring the distribution and peakedness of the local maxima set in LSD space by examination of various plotting options in the LSD Analysis Tool. A threshold is required if the retention of only high-value LSD optima for optimal sample size calculations is indicated. However, a general strategy has not been identified for choosing a threshold and, absent a supporting rationale for its use, we recommend selecting the unthresholded optimal size as a default procedure.
Dispersion heat maps may be useful in depicting the pattern of subtle
changes of spatial frequency inherent in the data (Figures 12, 16, and 22).
These maps may show structure not easily gleaned from a casual
examination of the original spatial data. Figure 16 shows small
concentrations of population density within the general region of higher
dispersion values around the large lake as seen in Figure 5. This pattern is
reflected in the local maxima distribution map of Figure 17. The
distribution clearly associates with population density around the lake and
along several watercourses that empty into it.

Perspective view plots of the point cloud of local maxima in LSD space can
demonstrate how they are associated with features at various sample sizes
(Figures 10 and 23). This association may extend well into the upper
reaches of LSD space as vertical features (Figure 10), or appear to acquire
a more homogeneous distribution at some point above the lowest sample
sizes (Figure 23).

The WorldView2 dataset was the only one showing a distinct maximum (at
a sample size of 6-7 m) of the mean and median for the LSD function of
sample size (Figure 7). Tree cover dominates this image. As suggested by
previous research, this LSD maximum may reflect the average size of
individual canopies visible in the image, and may be most sensitive to the
image’s predominant spatial variation. Earlier researchers have considered
the LSD maximum, where it exists, in different ways in light of their image
analysis objectives. These results indicate an optimal sample size of 4.8 m,
slightly less than that indicated by the LSD function maximum.

Results from the WorldView2 dataset suggest that this optimization
approach is in general agreement with previous work on the interpretation
of the LSD function maximum for a particular sample size. As shown in
Figures 7 and 8, it is found that a somewhat smaller sample size than that
indicated by the LSD function maximum is optimal, and may be driven by
the preponderance of local LSD maxima at lower sample sizes.

Results show that this optimization technique for a multidimensional LSD
function successfully processes the inherent dispersion of image data with
heterogeneous spatial structure. Most importantly, it provides an optimal
sample size whether or not a maximum for the mean LSD function of
sample size exists (Figures 11, 15, and 20).
6 Summary and Conclusions

The spatial characteristics of continuously varying phenomena on the Earth’s surface directly inform remotely sensed data or other types of environmental information collected in a geospatial context. The spatial domain or structure of this data can be used to optimize its interpretation or extraction of spatial information. Effective mapping or modeling of spatially dependent information requires capturing the spatial variation patterns of features of interest. A key consideration in image analysis is the relationship between spatial resolution and the spatial frequency structure of features found in the image data.

In this work, this relationship was examined through a multiscale modeling approach to determine an optimal sample size for raster images containing remotely sensed or other environmental data with variable spatial structure. Resampling an image dataset in this way can increase the efficiency of image processing functions, such as feature segmentation or of geospatial models, such as that employed in the NET-CMO project at ERDC-GRL. Four image datasets were analyzed with disparate native resolutions collected over Florida and Cambodia. These datasets depict a variety of environmental feature data with heterogeneous spatial structure. In each case, a multidimensional dispersion space was created from which sets of local maxima were extracted. These local maxima were used in a weighted mean formulation to compute optimal sample sizes that did not depend on the single-variable functional relationship between mean dispersion and resample size. This approach captures the locality of variance in heterogeneous spatial datasets rather than relying on an overall mean dispersion value for each resampled image.

A useful tool and user interface was created, the LSD Analysis Tool, to exercise our algorithmic approach and allow a user to process a dataset while in control of particular processing parameters. Various plotting options display relationships among LSD values, local LSD maxima, maxima peakedness, and LSD space locations. These output features and level of user control provide for repeated experimentation and a better understanding of the spatial structure of the data.

The authors believe that this multiscale modeling approach to optimizing sample size is an effective and robust method as applied to geospatial data.
References


# Acronyms and Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>ERDC</td>
<td>Engineer Research and Development Center</td>
</tr>
<tr>
<td>GRL</td>
<td>Geospatial Research Laboratory</td>
</tr>
<tr>
<td>GSD</td>
<td>Ground Sample Distance</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>LSD</td>
<td>Local Spatial Dispersion</td>
</tr>
<tr>
<td>LSV</td>
<td>Local Spatial Variance</td>
</tr>
<tr>
<td>MAD</td>
<td>Mean Absolute Deviation</td>
</tr>
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<td>NET-CMO</td>
<td>New and Enhanced Tools for Civil-Military Operations</td>
</tr>
<tr>
<td>USACE</td>
<td>U.S. Army Corps of Engineers</td>
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Local Spatial Dispersion for Multiscale Modeling of Geospatial Data: Exploring Dispersion Measures to Determine Optimal Raster Data Sample Sizes

S. Bruce Blundell and Nicole M. Wayant

Geospatial Research Laboratory
U.S. Army Engineer Research and Development Center
7701 Telegraph Road
Alexandria, VA 22315-3864

Headquarters, U.S. Army Corps of Engineers
Washington, DC 20314-1000

Scale, or spatial resolution, plays a key role in interpreting the spatial structure of remote sensing imagery or other geospatially dependent data. These data are provided at various spatial scales. Determination of an optimal sample or pixel size can benefit geospatial models and environmental algorithms for information extraction that require multiple datasets at different resolutions. To address this, an analysis was conducted of multiple scale factors of spatial resolution to determine an optimal sample size for a geospatial dataset. Under the NET-CMO project at ERDC-GRL, a new approach was developed and implemented for determining optimal pixel sizes for images with disparate and heterogeneous spatial structure. The application of local spatial dispersion was investigated as a three-dimensional function to be optimized in a resampled image space. Images were resampled to progressively coarser spatial resolutions and stacked to create an image space within which pixel-level maxima of dispersion was mapped. A weighted mean of dispersion and sample sizes associated with the set of local maxima was calculated to determine a single optimal sample size for an image or dataset. This size best represents the spatial structure present in the data and is optimal for further geospatial modeling.