DEPARTMENT OF THE ARMY — CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

ON THE THEORY OF THE HIGHEST WAVES

TECHNICAL MEMORANDUM NO. 116

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FOREWORD

In the design of certain types of structures, the highest wave which can reach the structure for a particular set of wave conditions is frequently of importance. Certain parts of the structure may have to be designed to withstand a single highest wave incidence rather than the more commonly occurring significant or average waves. This paper presents a theoretical development for computing the properties of the highest wave. The theory is valid for all depths where the relative depth (d/L_o) is greater than about 0.04; the solution for the highest wave in deep water (as calculated by Michell and Havelock) is obtained as a special case.

This report was prepared in the Exploration and Production Research Division of the Shell Development Company in Houston, Texas, as a part of their general program of wave investigations. The author of the report, Dr. J. E. Chappelear, is a Physicist in that organization.

Because of its application to the research and investigation program of the Beach Erosion Board, and the wide interest in the description of wave phenomena in this country, this report is being published at this time in the Technical Memorandum series of the Beach Erosion Board, through the courtesy of the author and the Shell Development Company. It is hoped that dissemination of this information may serve as a stimulus and a valuable aid to workers in this country.

Views and conclusions stated in this report are not necessarily those of the Beach Erosion Board.

This report is published under authority of Public Law 166, 79th Congress, approved July 31, 1945.

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LIST OF SYMBOLS

A_j	equations (22) to (25)
b_j	equation (6)
B_j	equations (28) to (31)
C_n	equation (27)
C_0	wave velocity
d	depth
D_n	equations (36) to (38)
g	gravitational constant
H	wave height
k	wave number, $2\pi/\lambda$
$q(z)$	complex velocity, $u - iv$
r	radius vector from crest, equation (32), Figure 2
T	wave period
$u(x, y)$	horizontal velocity component
$v(x, y)$	vertical
$w(z)$	complex potential, $\phi + i\psi$
x	horizontal coordinate
y	vertical coordinate
z	$x + iy$
α	angle between r and vertical, equation (33) and Figure 2
ϵ	distance from the free surface to bottom (w plane)
ϵ'	elevation of crest in z plane
λ	wavelength
$\phi(x, y)$	velocity potential
$\psi(x, y)$	stream function
ω	angular frequency, $2\pi/T$

ON THE THEORY OF THE HIGHEST WAVES

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ABSTRACT

Following a suggestion of Michell,[3] we have made a calculation of the properties of the highest periodic gravity waves which can exist in steady, two-dimensional flow, neglecting viscosity. The "highest wave" is one satisfying the criterion of Stokes that the particle velocity at the wave crest be equal to the wave velocity. The theory is valid for all values of the parameter d/T^2 greater than 0.2 ft/sec^2. The highest wave in deep water, whose properties were first calculated by Michell and by Havelock,[5] is obtained as a special case.

INTRODUCTION

Calculation of the properties of the highest wave has been a theoretical and practical problem of considerable interest since the publication of the investigation of Gerstner.[1] Gerstner found that in water of infinite depth one could obtain an exact solution in terms of elementary functions. The wave motion is rotational and hence does not seem to be physically realizable. For purposes of the investigation described in this paper, the problem is to find an irrotational solution to the equations of hydrodynamics having the properties of a wave. It should be steady and two-dimensional, with the particle velocity at the crest equal to the wave velocity.

Stokes[2] pointed out that such a wave would have a sharp angle of 120° at the crest. Michell,[3] by means of an approximate treatment, was able to find the highest wave in deep water and suggested how to extend his results to water of finite depth. McCowan[4] found the highest wave in shallow water (the solitary wave) by a modification of his treatment of the ordinary solitary wave. Havelock[5] improved the numerical procedure of Michell and showed, by an appropriate modification of the assumed functional form of the solution, that there was a smooth transition from the highest wave to the infinitesimal waves of the Airy theory. The practical problem of prediction of wave properties in intermediate depths is reviewed by Bretschneider,[6] who also gives a summary of the literature.

This paper carries out the suggestion of Michell that his theory could be extended to water of finite depth. His numerical results are shown to be slightly inaccurate, largely as a consequence of a better approximation procedure employed here. Numerical results are obtained for values of the parameter $d/T^2 > 0.2$ ft/sec^2. The numerical results do not agree exactly with the modified solitary wave theory of Munk,[13] probably owing to the inherent inaccuracies of both calculations.

THE HIGHEST PERIODIC WAVE

The problem to be solved is the determination of the surface profile and particle velocity for the highest permanent wave in water of finite depth. A mathematical formulation suitable for our purposes can be stated rather concisely. Stoker[7] (Chapter 1) gives a much more detailed description of the general problem for waves which are not the highest.

The waves are assumed to be periodic, steady, irrotational, and two-dimensional. For the waves to be steady, there must be a steady flow superposed in order to bring the wave profile to rest. The translational velocity of the coordinate system relative to a fixed system is the wave velocity, one of the wave properties to be calculated. The flow may be represented conveniently by a complex velocity potential as a function of z, the complex position,

$$w(z) = \phi(x,y) + i\psi(x,y) \quad . \tag{1}$$

The real part of w is the velocity potential, and the imaginary part is the stream function. The derivative of w with respect to z is minus the complex velocity, whose real part is the horizontal velocity component $u(x,y)$ and whose imaginary part is the negative of the vertical velocity component $v(x,y)$.

$$\frac{dw}{dz}(z) \equiv -q(z) = -u(x,y) + iv(x,y) \quad . \tag{2}$$

It is convenient to consider the boundary value problem in the potential plane rather than in the z plane; that is, the potential is a conformal map of the z plane onto the w plane. In the w plane, the region occupied by one wave is a rectangle. The problem in the w plane is to find a periodic function satisfying the boundary condition,

$$\left[\frac{\partial}{\partial\phi}|q(w)|^4\right]_{\psi=\epsilon} = +4g \ \mathrm{Im}\ [q(w)]_{\psi=\epsilon} \quad , \tag{3}$$

the Bernoulli theorem. This form of the theorem is obtained from the usual form in Appendix I. Another boundary condition is that there is no flow through the bottom.

$$\text{Im}\ [q(w)]_{\psi=0} = 0 \quad . \tag{4}$$

For convenience, the real period of the complex velocity is chosen to be π. There, the condition of periodicity, the additional boundary condition, is

$$q(w + \pi) = q(w) \quad . \tag{5}$$

It is still necessary to consider the question of what is meant by the highest wave. Stokes suggested that the particle velocity at the crest of the highest wave should equal the wave velocity. This condition establishes an upper limit on the particle velocity, which might not occur except in very special circumstances. However, a separate investigation would be necessary to prove, either mathematically or physically, the existence of the waves treated here. For the purposes of this paper, the highest wave is defined as a wave satisfying Stokes' criterion.

In the moving coordinate system, the velocity at the crest is zero, since the crest is part of the profile which is assumed to be steady. Stokes proved heuristically that if the zero at the crest were assumed to be a branch point (in particular he assumed that $w \propto z^{\alpha}$), the order of the branch point would be 1/3 in the w plane. Consequently, the flow in the vicinity of the crest would be the same as the flow in a corner between walls inclined at 120°. A short proof employing the notation of complex variable theory is presented in Appendix II. No exclusion is made of the possibility that the character of the complex velocity in the vicinity of the crest might be different, e.g., be proportional to $(w \log w)^2$; no proof has been found that such a behavior is impossible, but it does seem physically reasonable that such a solution would be unstable relative to the one discussed here.

The complex velocity is now limited to being a periodic function with an array of zeros, everywhere regular. The free surface will be identified with $\psi(x,y) = \epsilon$ and the bottom with $\psi(x,y) = 0$. The procedure of Stokes for the determination of the properties of waves of finite height is first to assume that the velocity can be expanded in a Fourier series along the bottom where the velocity is real. Then the velocity is extended off the real axis by analytic continuation. Finally, the unknown coefficients are calculated from Bernoulli's theorem by putting successive coefficients of $\cos n\phi$ equal to zero. That the series obtained in this fashion is convergent was first shown by Struik.[8]

Michell proposed a modification of this procedure which takes explicit account of the nature of the branch points. The Fourier series is multiplied by the 1/3 power of a periodic function of w which has simple zeros at the correct positions in the w plane. Although there are a number of possible choices, it is convenient to follow the suggestion of Michell and to put

$$q(w) = \frac{\theta_0^{1/3}(w)}{2^{1/3}}(1 + 2b_1e^{-2\epsilon}\cos 2w + 2b_2e^{-4\epsilon}\cos 4w + 2b_3e^{-6\epsilon}\cos 6w) \quad . \quad (6)$$

A constant factor with the dimensions of a velocity has been put equal to 1. The function $\theta_0(w)$ is one of Jacobi's theta functions (Erdelyi,[9] Vol. 2) and for this paper will be defined by its Fourier series

$$\theta_0(w) = 1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-2n^2\epsilon} \cos 2nw \quad . \tag{7}$$

There are first-order zeros of $\theta_0(w)$ at $w = m\pi + i(2n + 1)\epsilon$ where m and n are integers. This property can be verified by the transformation formula

$$\theta_0(w) = ie^{\frac{\epsilon}{2}+iw}\theta_1(w - i\epsilon) \quad , \tag{8}$$

where the function $\theta_1(w)$ is defined by

$$\theta_1(w) = 2e^{-\epsilon/2}\sum_{n=0}^{\infty} (-1)^n e^{-2n(n+1)\epsilon} \sin(2n + 1)w \quad . \tag{9}$$

θ_1 is periodic with period 2π and has the specified zeros. The formulas, in consequence, will be valid only in the rectangle bounded by the lines $\phi = 0$, $\psi = 0$, $\phi = \pi$, and $\psi = \epsilon$. The velocity and profile are to be continued out of this rectangle periodically. The succeeding terms in the Fourier series in equation (6) are omitted with the hope that their influence on the solution would be small. The choice of the exact form of the expansion coefficients (i.e., $2b_j e^{-2j\epsilon}$) conforms with Michell and Havelock, whose results for infinite depth ($\epsilon \to \infty$) are a special case of these formulas. It can be shown that the complex velocity, equation (6), is identical with that proposed by Michell and Havelock for $\epsilon \to \infty$.

Because of the choice of units, the numerical value of g is not given, and it must be calculated as one of the unknowns, together with b_1, b_2, and b_3. All these unknowns depend upon the parameter ϵ, which specifies the physical parameters (e.g., the ratio of the depth to the wavelength). It is reasonable that there is only one parameter, since there is presumably only one "highest" wave in a given water depth.

On the free surface,

$$w = \phi + i\epsilon \quad . \tag{10}$$

When this value of w is put into equation (6), there results

$$q(\phi + i\epsilon) = \frac{e^{-\frac{\epsilon}{6}+i\left(\frac{\phi}{3}-\frac{\pi}{6}\right)}}{2^{1/3}} \theta_1^{1/3}(\phi)\,[1 + 2b_1e^{-2\epsilon}\cos 2(\phi + i\epsilon)$$

$$+ 2b_2e^{-4\epsilon}\cos 4(\phi + i\epsilon) + 2b_3e^{-6\epsilon}\cos 6(\phi + i\epsilon)] \tag{11}$$

by the use of equation (9).

After considerable algebraic manipulation, the first four terms in the Fourier series expansion of the left-hand side of equation (3) can be calculated. They are

$$\frac{\partial}{\partial\phi}|q(\phi + i\epsilon)|^4 = \frac{4}{3}\,\frac{e^{\epsilon/6}}{2^{1/3}}\,\theta_1^{1/3}(\phi)\sum_{j=1}^{4} A_j\cos\,(2j-1)\phi \quad . \tag{12}$$

The functions A_j are abbreviations for

$$A_1 = B_0 - \frac{B_2}{4}(3\eta + 1) + B_4\eta^3 - \frac{B_6}{4}(5\eta^6 - \eta^3) + \frac{B_8}{2}(3\eta^{10} - \eta^6)$$

$$- \frac{B_{10}}{4}(7\eta^{15} - 3\eta^{10}) + B_{12}(2\eta^{21} - \eta^{15}) - \frac{B_{14}}{4}(9\eta^{28} - 5\eta^{21})$$

$$+ \frac{B_{16}}{2}(5\eta^{36} - 3\eta^{28}) + \frac{B_{18}}{4}(7\eta^{36}) \quad . \tag{13}$$

$$A_2 = -3B_0\eta + \frac{B_2}{4}(7\eta^3 + 5) - B_4(2\eta^6 + 1) + \frac{B_6}{4}(9\eta^{10} + 3\eta)$$

$$- \frac{B_8}{2}(5\eta^{15} + \eta^3) + \frac{B_{10}}{4}(11\eta^{21} + \eta^6) - B_{12}(3\eta^{28})$$

$$+ \frac{B_{14}}{4}(13\eta^{36} - \eta^{15}) + \frac{B_{16}}{2}(\eta^{21}) - \frac{B_{18}}{4}(3\eta^{28}) + B_{20}(\eta^{36}) \quad . \tag{14}$$

$$A_3 = 5B_0\eta^3 - \frac{B_2}{4}(11\eta^6 + 9\eta) + B_4(3\eta^{10} + 2) - \frac{B_6}{4}(13\eta^{15} + 7)$$

$$+ \frac{B_8}{2}(7\eta^{21} + 3\eta) - \frac{B_{10}}{4}(15\eta^{28} + 5\eta^3) + B_{12}(4\eta^{36} + \eta^6)$$

$$- \frac{B_{14}}{4}(3\eta^{10}) + \frac{B_{16}}{2}(\eta^{15}) - \frac{B_{18}}{4}(\eta^{21}) + \frac{B_{22}}{4}(\eta^{36}) \quad . \tag{15}$$

$$A_4 = -7B_0\eta^6 + \frac{B_2}{4}(15\eta^{10} + 13\eta^3) - B_4(4\eta^{15} + 3\eta) + \frac{B_6}{4}(17\eta^{21} + 11)$$

$$- \frac{B_8}{2}(9\eta^{28} + 5) + \frac{B_{10}}{4}(19\eta^{36} + 9\eta) - B_{12}(2\eta^3) + \frac{B_{14}}{4}(7\eta^6)$$

$$- \frac{B_{16}}{2}(3\eta^{10}) + \frac{B_{18}}{4}(5\eta^{15}) - B_{20}(\eta^{21}) + \frac{B_{22}}{4}(3\eta^{28}) - \frac{B_{24}}{2}\eta^{36} \quad . \tag{16}$$

We have employed as abbreviations

$$A'_0 = [1 + b_1^2(1 + \eta^2) + b_2^2(1 + \eta^4) + b_3^2(1 + \eta^6)] ,$$

$$A'_1 = [b_1(1 + \eta) + b_1 b_2(1 + \eta^3) + b_2 b_3(1 + \eta^5)] ,$$

$$A'_2 = [b_2(1 + \eta^2) + b_1 b_3(1 + \eta^4) + b_1^2(\eta)] ,$$

$$A'_3 = [b_3(1 + \eta^3) + b_1 b_2(\eta + \eta^2)] ,$$

$$A'_4 = [b_1 b_3(\eta + \eta^3) + b_2^2(\eta^2)] ,$$

$$A'_5 = [b_2 b_3(\eta^2 + \eta^3)] ,$$

$$A'_6 = b_3^2 \eta^3 ,$$

and

$$B_0 = A_0'^2 + 2(A_1'^2 + A_2'^2 + A_3'^2 - A_4'^2 + A_5'^2 + A_6'^2),$$

$$\frac{B_2}{4} = A'_0 A'_1 + A'_1 A'_2 + A'_2 A'_3 + A'_3 A'_4 + A'_4 A'_5 + A'_5 A'_6,$$

$$B_4 = 4(A'_0 A'_2 + \tfrac{1}{2} A_1'^2 + A'_1 A'_3 + A'_2 A'_4 + A'_3 A'_5 + A'_4 A'_6),$$

$$\frac{B_6}{4} = A'_0 A'_3 + A'_1 A'_2 + A'_1 A'_4 + A'_2 A'_5 + A'_3 A'_6 ,$$

$$\frac{B_8}{2} = 2(A'_0 A'_4 + A'_1 A'_3 + A'_1 A'_5 + A'_2 A'_6) + A_2'^2 ,$$

$$\frac{B_{10}}{4} = A'_0 A'_5 + A'_1 A'_4 + A'_1 A'_6 + A'_2 A'_3,$$

$$B_{12} = 4(A'_0 A'_6 + A'_1 A'_5 + \tfrac{1}{2} A_3'^2 + A'_2 A'_4),$$

$$\frac{B_{14}}{4} = A_1'A_6' + A_2'A_5' + A_3'A_4',$$

$$\frac{B_{16}}{2} = 2(A_2'A_6' + A_3'A_5') + A_4'^2,$$

$$\frac{B_{18}}{4} = A_3'A_6' + A_4'A_5',$$

$$B_{20} = 4(A_4'A_6' + \tfrac{1}{2}A_5'^2),$$

$$\frac{B_{22}}{4} = A_5'A_6',$$

$$\frac{B_{24}}{2} = A_6'^2,$$

and

$$\eta = e^{-4\epsilon}.$$

with the neglect of terms of higher order than $e^{-144\epsilon}$ in the Fourier series for $\theta_1(w)$. In order to obtain reasonable accuracy in computing $\theta_1(w)$ for values of ϵ as small as 0.2, it is necessary to retain terms of this order. We do not neglect products of the b_j's, as do Michell and Havelock. Consequently, the numerical results obtained here will not agree exactly with their results for the case of infinite depth.

The right-hand side of equation (3) can also be expanded in a Fourier series in the interval $0 \leq \phi \leq \pi$. The expansion

$$\sin\left[\left(2r + \frac{1}{3}\right)\phi - \frac{\pi}{6}\right] = -\frac{6\sqrt{3}}{\pi}(6r + 1)\sum_{n=0}^{\infty}\frac{\cos(2n + 1)\phi}{9(2n + 1)^2 - (6r + 1)^2} \tag{17}$$

is useful in the calculations. The first four terms are

$$\operatorname{Im} q(\phi + i\epsilon) = -\frac{e^{\epsilon/6}\theta_1^{1/3}(\phi)}{2^{1/3}} \frac{6\sqrt{3}}{\pi} \sum_{j=1}^{4} B_j \cos(2j - 1)\phi \tag{18}$$

where the functions B_j are the abbreviations,

$$B_1 = \frac{1}{8} + b_1\left[\frac{5}{16}e^{-4\epsilon} - \frac{7}{40}\right] + b_2\left[\frac{11}{112}e^{-8\epsilon} - \frac{13}{160}\right] + b_3\left[\frac{17}{280}e^{-12\epsilon} - \frac{19}{352}\right] , \tag{19}$$

$$B_2 = \frac{1}{80} + b_1\left[\frac{7}{32} - \frac{5}{56}e^{-4\epsilon}\right] + b_2\left[\frac{11}{40}e^{-8\epsilon} - \frac{13}{88}\right] + b_3\left[\frac{17}{208}e^{-12\epsilon} - \frac{19}{280}\right] , \tag{20}$$

$$B_3 = \frac{1}{224} + b_1\left[\frac{7}{176} - \frac{1}{40}e^{-4\epsilon}\right] + b_2\left[\frac{13}{56} - \frac{11}{104}e^{-8\epsilon}\right] + b_3\left[\frac{17}{64}e^{-12\epsilon} - \frac{19}{136}\right] , \tag{21}$$

and

$$B_4 = \frac{1}{440} + b_1\left[\frac{7}{392} - \frac{5}{416}e^{-4\epsilon}\right] + b_2\left[\frac{13}{272} - \frac{11}{320}e^{-8\epsilon}\right] + b_3\left[\frac{19}{80} - \frac{17}{152}e^{-12\epsilon}\right] . \tag{22}$$

Now equation (3) reads

$$0 = \sum_{j=1}^{4} \left[A_j - \frac{18\sqrt{3}}{\pi} g B_j\right] \cos(2j - 1)\phi , \tag{23}$$

When the coefficients of the various cosines are put separately to zero, four equations in the four unknowns result; all the equations contain the parameter ϵ. The solutions to these equations as functions of ϵ are given in Table 1.

TABLE 1

ϵ*	b_1	b_2	b_3	g	d/λ	H/d	d/T^2 ft/sec^2	H/T^2 ft/sec^2	$\frac{2\pi\lambda}{gT^2} = \frac{2\pi C_0^2}{g\lambda}$	ϵ'/λ	E
0.2	.496	.222	.0890	5.39	.0666	.870	.1990	.1729	.5830	.1178	.261
$0.2\mathring{3}$	.430	.174	.0631	$4.9\overline{5}$	.0775	.821	.2524	.2074	.6351	.1320	.159
$0.2\mathring{6}$	.377	.137	.0456	4.58	.0885	.786	.3111	.2444	.6854	$.146\overline{5}$	.069
0.3	.3317	.1100	.03371	4.291	.0996	.7582	.3735	.2834	.7321	.1609	.057
0.4	.2339	$.0606\overline{5}$	.01573	3.671	.1328	.6932	.5822	.4038	.8558	.2034	.041
0.5	$.170\overline{5}$	.03684	.00894	3.298	.1659	.6386	.8098	.5171	.9526	.2439	.067
0.6	.1281	.02480	.00606	3.062	.1988	.5871	$1.04\overline{5}$	.6134	1.0258	.2823	.053
0.7	.09949	.01848	.00469	2.911	.2314	.5386	1.280	.6894	1.0792	.3187	.050
0.8	.08019	.01504	.00397	2.811	.2638	.4939	1.511	.7461	1.1176	.3541	.046
0.9	.06716	.01309	.00356	2.745	.2961	.4535	1.736	.7876	1.1444	.3883	.043
1.0	.05839	.01195	.00331	2.700	.3282	.4175	1.956	.8163	1.1634	.4218	.042
2.0	.04076	.01007	.00286	2.615	.6471	.2205	3.986	.8787	1.2012	.7434	.038
3.0	.04043	.01004	$.0028\overline{5}$	2.614	$.965\overline{5}$	.1479	5.947	.8797	1.2019	1.0618	.039

*The values of ϵ which were used were correct to eight decimals. Thus 0.2 means 0.20000000, and $0.2\mathring{3}$ means 0.23333333.

The final step in the problem is the integration to obtain z as a function of w,

$$z(w) = \int \frac{dw}{\frac{dw}{dz}(w)} \quad . \tag{24}$$

This integration was performed numerically. Since the line $\psi = \epsilon$ is the free surface, a parametric representation of the free surface was obtained directly from equation (24). Because $q(w)$ has a branch point at $w = i\epsilon$ (the wave crest), it was convenient to perform the integration directly by means of a power series expansion in the neighborhood of this point. The numerical values obtained are listed in Table 2. The values have been normalized by dividing by the wavelength.

TABLE 2

ε*																				
0.20	$2x/\lambda$	1.0000	.9468	.8948	.844$\bar{5}$	.795$\bar{5}$	.7469	.6978	.6474	.5954	.5424	.4894	.4374	.3866	.3363	.2854	.2323	.1747	.1082	0
	$2Y(x)/\lambda$	.1199	.1190	.1170	.1149	.1136	.1136	.1150	.1172	.1192	.1201	.1199	.1191	.1191	.1212	.1265	.136$\bar{5}$	.1528	.1789	.2356
$0.2\dot{3}$	$2x/\lambda$	1.0000	.9484	.8974	.8472	.7979	.7487	.6992	.6489	.5976	.5457	.4937	.4421	.3910	.3400	.2880	.2339	.175$\bar{5}$	.108$\bar{5}$	0
	$2Y(x)/\lambda$	.1366	.1362	.1351	.1339	.1332	.1334	.1344	.1359	.1374	.1385	.1390	.1396	.1412	.1448	.151$\bar{5}$	.1626	.1799	.2067	.2639
$0.2\dot{6}$	$2x/\lambda$	1.0000	.9493	.8991	.8493	.8000	.7508	.7012	.6512	.6004	.5492	.4978	.4464	.3951	.3434	.2907	.2357	.1765	.1089	0
	$2Y(x)/\lambda$	.1538	.153$\bar{5}$	.1529	.1522	.1520	.1523	.1533	.1546	.1561	.157$\bar{5}$	.1589	.1606	.1636	.1685	.1766	.1890	.2073	.2350	.2930
0.30	$2x/\lambda$	1.0000	.9502	.9005	.8512	.8021	.7531	.7038	.6541	.6038	.5531	.5021	.4509	.3994	.3474	.2942	.2386	.1788	.1105	0
	$2Y(x)/\lambda$	.1708	.1707	.1703	.1701	.1701	.1707	.1717	.1731	.1748	.1767	.1789	.1818	.1860	.1923	.2016	.2151	.2344	.2629	.3218
0.40	$2x/\lambda$	1.0000	.9517	.9035	.8552	.8071	.7587	.7102	.6613	.6119	.5621	.5117	.4607	.4090	.3563	.3021	.2453	.1841	.1140	0
	$2Y(x)/\lambda$	.2227	.2228	.2230	.2234	.2242	.2256	.2274	.2298	.2328	.236$\bar{5}$	.2411	.2468	.2540	.2635	.2759	.2924	.3145	.3453	.4068
0.50	$2x/\lambda$	1.0000	.9527	.9053	.8579	.810$\bar{5}$	.7628	.7149	.6666	.6178	.5684	.5184	.4676	.4158	.3629	.3081	.2505	.1883	.1168	0
	$2Y(x)/\lambda$	.2759	.2761	.2767	.2777	.2793	.281$\bar{5}$	.2843	.2880	.292$\bar{5}$	.2978	.3044	.3123	.3219	.3338	.3486	.3673	.391$\bar{5}$	.4242	.4878
0.60	$2x/\lambda$	1.0000	.9532	.9065	.8597	.8127	.7655	.7180	.6702	.6217	.5726	.5229	.4722	.4205	.3674	.3123	.2543	.1914	.1190	0
	$2Y(x)/\lambda$	.3312	.3315	.3324	.3339	.3361	.3390	.3428	.3476	.3532	.3601	.3682	.3777	.3891	.4028	.4193	.4396	.4653	.499$\bar{5}$	.5646
0.70	$2x/\lambda$	1.0000	.9537	.9073	.8608	.8142	.7673	.7201	.672$\bar{5}$	.6243	.575$\bar{5}$	.5259	.4753	.4237	.3705	.3153	.2569	.1936	.1205	0
	$2Y(x)/\lambda$	.3886	.3890	.3900	.3919	.3946	.3982	.4027	.4082	.4149	.4228	.4320	.4428	.455$\bar{5}$	.4704	.4882	.5097	.5364	.5715	.6379
0.80	$2x/\lambda$	1.0000	.9539	.9078	.8616	.8151	.768$\bar{5}$	.7215	.6741	.6260	.5774	.5279	.477$\bar{5}$	.4258	.3726	.3173	.2588	.1952	.1215	0
	$2Y(x)/\lambda$	.4477	.4481	.4494	.4515	.4545	.4586	.4637	.4698	.4772	.4859	.4959	.5076	.5211	.5369	.5556	.5778	.6053	.6412	.7083
0.90	$2x/\lambda$	1.0000	.9541	.9081	.8620	.8158	.7692	.7224	.6751	.6272	.5787	.5293	.4789	.4273	.3741	.3187	.2601	.1963	.1223	0
	$2Y(x)/\lambda$	.5081	.5086	.5099	.5123	.5156	.5200	.5254	.5321	.5400	.5491	.5597	.5720	.5861	.6026	.6218	.6447	.6726	.7090	.7767
1.00	$2x/\lambda$	1.0000	.9542	.9083	.8623	.8162	.7697	.7230	.6758	.6290	.5795	.5302	.4799	.4283	.3751	.3197	.2610	.1970	.1228	0
	$2Y(x)/\lambda$	.5695	.5700	.5715	.5740	.577$\bar{5}$	.5821	.5878	.5948	.6029	.6125	.6235	.6362	.6508	.6676	.6873	.7105	.7389	.7756	.8436
2.00	$2x/\lambda$	1.0000	.9544	.9087	.8629	.8169	.7707	.7241	.6771	.6295	.5811	.5320	.4818	.4303	.3770	.3216	.2627	.1984	.1238	0
	$2Y(x)/\lambda$	1.2015	1.2020	1.2037	1.2064	1.2103	1.2154	1.2217	1.2293	1.2383	1.2486	1.2604	1.2740	1.2894	1.3071	1.3276	1.3516	1.3807	1.4181	1.4869
3.00	$2x/\lambda$	1.0000	.9544	.9087	.8629	.8170	.7708	.7242	.6771	.6295	.5812	.5320	.4818	.4303	.3771	.3216	.2627	.198$\bar{5}$	.1238	0
	$2Y(x)/\lambda$	1.8381	1.8386	1.8403	1.8430	1.8469	1.8520	1.8584	1.8659	1.8749	1.8852	1.8971	1.9107	1.9261	1.9439	1.9643	1.9884	2.017$\bar{5}$	2.0548	2.1236

*See footnote, Table 1.

The integration was continued from the trough vertically to the bottom and then along the bottom to a point directly underneath the crest, a path shown in Figure 1.

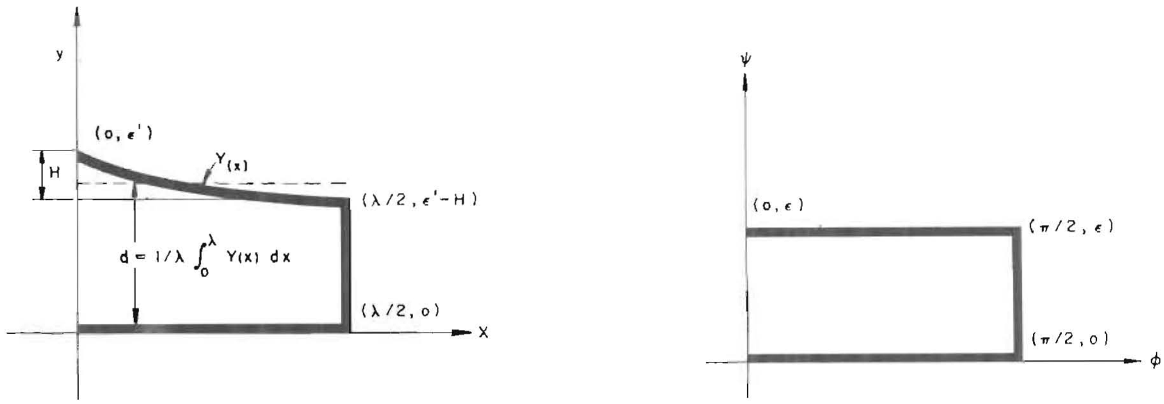

Figure 1

The depth, which is the average distance from the profile to the bottom,

$$d = \frac{1}{\lambda}\int_0^{\lambda} Y_{(x)}dx \quad , \tag{25}$$

and the wave height, which is the vertical distance from crest to trough,

$$H = Y(0) - Y\left[\frac{\lambda}{2}\right] \quad , \tag{26}$$

were computed. The ratios d/λ and H/d are given in Table 1.

In the z plane the velocity can be represented by a complex Fourier series,

$$q(z) = -C_0 + \sum_{n=1}^{j} C_n \cos nkz \quad , \tag{27}$$

where k is the wave number defined as

$$k = 2\pi/\lambda \quad . \tag{28}$$

The number of terms in the series j was left variable in order that the series could be used to fit the various velocity fields with a preassigned accuracy. The velocity was computed at a large number of points along the path shown in Figure 1, excluding the immediate vicinity of the crest. The coefficients C_n were calculated to give the best fit at these points in the sense of least squares. The values of the C_n are listed in Table 3 as

TABLE 3

ϵ	$-C_1/C_0$	$-C_2/C_0$	$-C_3/C_0$	$-C_4/C_0$	C_5/C_0	D_1	D_2	D_3
0.20	.1549	$.9174(10)^{-1}$	$.4140(10)^{-1}$	$.2711(10)^{-1}$	$.5259(10)^{-2}$	1.949	5.722	76.19
0.2$\overset{\circ}{3}$	.1600	$.8133(10)^{-1}$	$.3124(10)^{-1}$	$.1433(10)^{-1}$	$.2344(10)^{-2}$	1.940	5.451	82.74
0.2$\overset{\circ}{6}$	.1648	$.7185(10)^{-1}$	$.2222(10)^{-1}$	$.7370(10)^{-2}$		1.9131	5.098	79.16
0.30	.1698	$.6220(10)^{-1}$	$.1594(10)^{-1}$	$.3986(10)^{-2}$		1.881$\bar{5}$	4.7371	72.067
0.40	.1718	$.3760(10)^{-1}$	$.5605(10)^{-2}$	$.7540(10)^{-3}$		1.7887	3.8094	50.248
0.50	.1605	$.2205(10)^{-1}$	$.2199(10)^{-2}$			1.7191	3.1579	35.626
0.60	.1454	$.1204(10)^{-1}$	$.8845(10)^{-3}$			1.6692	2.7159	26.878
0.70	.1267	$.6745(10)^{-2}$	$.390\bar{5}(10)^{-3}$			1.6352	2.4209	21.682
0.80	.1082	$.3842(10)^{-2}$	$.1895(10)^{-3}$			1.6120	2.2240	18.536
0.90	$.9106(10)^{-1}$	$.2245(10)^{-2}$	$.9773(10)^{-4}$			1.5964	2.0927	16.581
1.00	$.7597(10)^{-1}$	$.1347(10)^{-2}$	$.520\bar{5}(10)^{-4}$			1.5860	2.0044	15.336
2.00	$.1067(10)^{-1}$	$.1850(10)^{-4}$	$.1261(10)^{-6}$			1.5649	1.8295	13.031
3.00	$.1445(10)^{-2}$	$.3363(10)^{-6}$	$.3131(10)^{-9}$			1.564$\bar{5}$	1.8263	12.994

ratios C_n/C_0. C_0 is defined as the wave velocity, following Stokes. No more than six coefficients were needed for any value of ϵ.

With the wave velocity and wavelength known, the period T can be calculated. Then the dimensionless ratio d/gT^2 was obtained, and hence H/gT^2 and λ/gT^2. The values of d/T^2 and H/T^2 in units of ft/sec^2 were obtained by multiplication by g (32.2 ft/sec^2). They are listed in Table 1, together with $2\pi\lambda/gT^2$.

Within a distance of about 1/36 of a wavelength from the crest, many more terms would be required in equation (27) because of the rapid variation of the velocity. However, in this neighborhood an exact expression can be obtained. Expansion of equation (6) in a power series and term-by-term integration yield

$$z - i\epsilon' = B_0'(w - i\epsilon)^{2/3} + B_1'(w - i\epsilon)^{5/3} + B_2'(w - i\epsilon)^{8/3} + \cdots \qquad (29)$$

where the B_j' are various functions of b_j and ϵ. The distance from the bottom to the crest in the z plane is ϵ'. Then $w - i\epsilon$ can be obtained from equation (17) as a function of $z - i\epsilon'$ by some algebraic manipulation. The result is

$$w - i\epsilon = \frac{(z - i\epsilon')^{3/2}}{B_0''}\left[1 + B_1''(z - i\epsilon')^{3/2} + B_2''(z - i\epsilon')^3 + \cdots\right] \qquad (30)$$

where the B_j'' are various functions of the B_j'. The velocity is then obtained by differentiation,

$$\frac{q(z)}{C_0} = D_0\left(\frac{z - i\epsilon'}{-i\lambda}\right)^{1/2}\left[1 + D_1\left(\frac{z - i\epsilon'}{-i\lambda}\right)^{3/2} + D_2\left(\frac{z - i\epsilon'}{-i\lambda}\right)^3 + \cdots\right], \qquad (31)$$

where the D_n are various functions of b_j and ϵ given in equations (36), (37), and (38).

Transformation to a stationary coordinate system can be accomplished by adding 1 to the complex velocity and putting $x = x' - C_0 t$. The introduction of polar coordinates, as is shown in Figure 2, yields

$$r = \frac{\sqrt{(x_0' - ct)^2 + (\epsilon' - y)^2}}{\lambda} \tag{32}$$

and

$$\alpha = \tan^{-1} \frac{x' - C_0 t}{\epsilon' - y} \quad . \tag{33}$$

The radius vector from the crest, r, is measured in units of the wavelength. The angle α between the vertical and r is measured positive counterclockwise. The distance from the crest to the ocean bottom measured in units of the wavelength is just ϵ'.

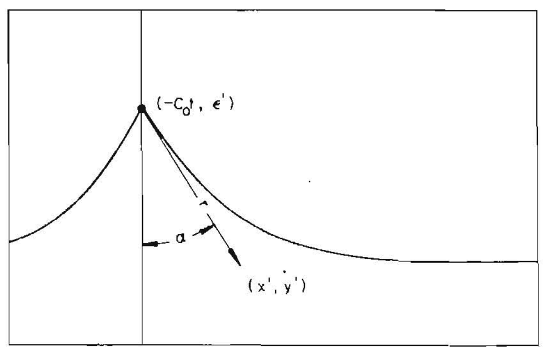

Figure 2

The velocity components are given by equation (2). They are

$$\frac{u}{C_0} = 1 - D_0 r^{1/2} \left[\sqrt{2} \cos \frac{\alpha}{2} - 2D_1 r^{3/2} \cos 2\alpha + \sqrt{2}\, D_2 r^3 \cos \frac{7\alpha}{2}\right] \tag{34}$$

and

$$\frac{v}{C_0} = D_0 r^{1/2}\left[\sqrt{2}\sin\frac{\alpha}{2} - 2D_1 r^{3/2}\sin 2\alpha + \sqrt{2}\, D_2 r^3 \cos\frac{7\alpha}{2}\right] , \quad (35)$$

where

$$D_0 = \frac{1.5}{\sqrt{2}}\left(\frac{2}{3}EF\right)^{3/2}\frac{\lambda_0^{1/2}}{C_0} , \quad (36)$$

$$D_1 = \frac{1.2}{\sqrt{2}}\left(\frac{2}{3}EF\right)^{3/2}\left(\frac{1}{3}+\frac{H}{F}\right)\lambda_0^{3/2} , \quad (37)$$

and

$$D_2 = 4.5\left(\frac{2}{3}EF\right)^3\left[\frac{11}{25}\left(\frac{1}{3}+\frac{H}{F}\right) - \frac{1}{4}\left(\frac{1}{18}+\frac{H}{3F}+\frac{G}{F}-\frac{1}{8}\frac{K}{J}\right)\right]\lambda_0^3 . \quad (38)$$

The abbreviations used in equations (36), (37), and (38) are listed below:

$$J = (1 - 3e^{-4\epsilon} + 5e^{-12\epsilon} - 7e^{-24\epsilon} + 9e^{-40\epsilon} - 11e^{-60\epsilon})$$
$$K = (1 - 27e^{-4\epsilon} + 125e^{-12\epsilon} - 343e^{-24\epsilon} + 729e^{-40\epsilon} - 1331e^{-60\epsilon})$$
$$E = (J)^{1/2}$$
$$F = 1 + a'\cosh 2\epsilon + b'\cosh 4\epsilon + c'\cosh 6\epsilon$$
$$G = -2(a'\cosh 2\epsilon + 4b'\cosh 4\epsilon + 9c'\cosh 6\epsilon)$$
$$H = 2(a'\sinh 2\epsilon + 2b'\sinh 4\epsilon + 3c'\sinh 6\epsilon)$$
$$a' = 2b_1e^{-2\epsilon}$$
$$b' = 2b_2e^{-4\epsilon}$$
$$c' = 2b_3e^{-6\epsilon} .$$

The functions D_1, D_2, and D_3 are given in Table 3. λ_0 is the wavelength in appropriate units.

DISCUSSION OF RESULTS

The current engineering practice for prediction of properties of waves is summarized by Bretschneider.[8] The material presented there is partially empirical and partially theoretical, with various ingenious extrapolation techniques being used for predictions in regions where there are no available data.

Figures 3, 4, and 5 give a comparison between certain wave properties calculated here and those given by Bretschneider. The calculated values are in all cases shown as solid lines, and the values which have been read off Bretschneider's charts are shown as dashed lines. A consideration of the figures indicates that there is small reason to prefer either as a representation of the experimental data.

The case of very large depth was investigated by putting $\epsilon = 10$ into the equations resulting from equation (23). The solutions were the same as in the case $\epsilon = 3$, within an accuracy of three significant figures. There are three other numerical treatments of the problem, those of Michell,[3] Havelock,[5] and Yamada.[11] The ratios $C_0^2/g\lambda$ (the dimensionless wave velocity) and H/λ (the steepness) are compared in Table 4.

TABLE 4

	Michell	Havelock	Yamada	Chappelear
$C_0^2/g\lambda$	0.191	0.191	0.1899	0.1913
H/λ	0.142	0.1418	0.1412	0.1428

The wave corresponding to the smallest value of ϵ can be compared to the highest solitary wave; this has been investigated by McCowan[4] and

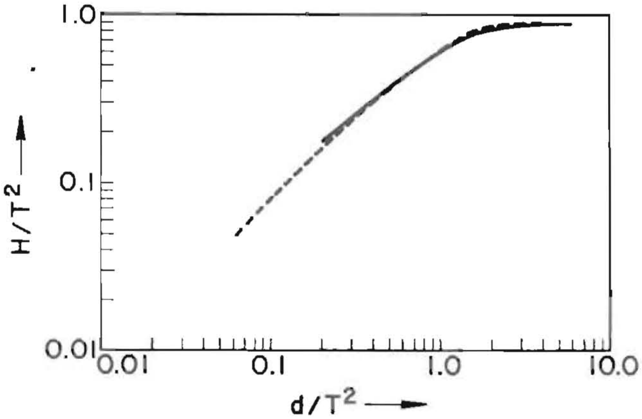

Figure 3

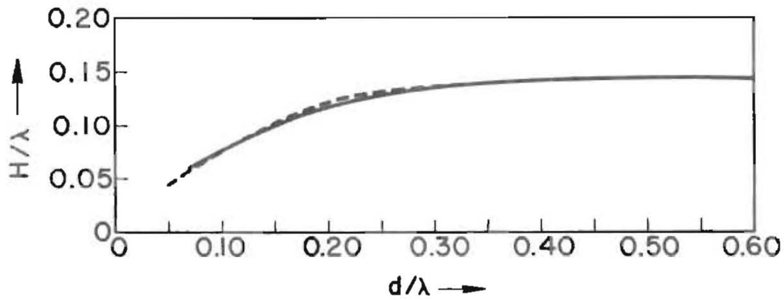

Figure 4

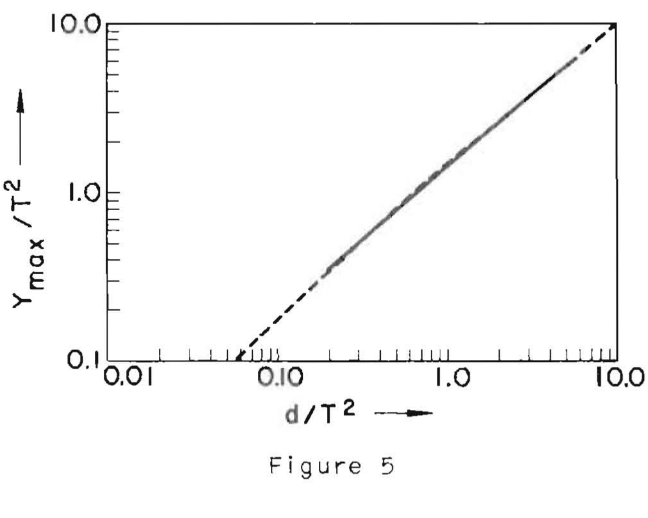

Figure 5

TABLE 5

	McCowan	Yamada	Chappelear
C_0^2/gH	2	1.999	1.602
H/d	0.780	0.8234	0.8696

Yamada.[12] In Table 5 the dimensionless ratios C_0^2/gH and H/d are listed for comparison. It is apparent that the value $\epsilon = 0.2$ is still rather far from the solitary wave.

As has been pointed out by Bretschneider, it is rather surprising that the breaking index curve calculated here lies above that predicted for small values of d/T^2. His curve in this region is taken directly from the modified solitary wave theory of Munk, and it might well be argued that the highest wave in any depth should be the (modified) solitary wave.

At least three different explanations might be advanced for the apparent anomaly. First, there is of course no mathematical reason why there cannot be two solutions to the equations of hydrodynamics for waves in this region. In this case the difference is real, and some additional theoretical or experimental investigation of the question of stability is necessary. Second, there is a possibility that either theory is wrong in this region. The modified solitary wave then could associate the wrong wavelength with the wave. The waves in question are for the smallest three values of ϵ, for which the equations are the least accurate. Although the final answers should be correct to two significant figures, some chance collection of numerical errors might have brought this result about. Finally, the functional forms chosen to represent the velocity might not be applicable. This is indicated by the fact that the two waves corresponding to the smallest values of ϵ have very small secondary humps. Furthermore, the numerical determination of b_1, b_2, b_3, and g was rendered more difficult by an apparent lack of stability of the equations to the iterative scheme used in their solution; that is, it was necessary to resort to averaging of successive iterations to control the oscillations. Even when the averaging procedure was used, it was not possible to solve the equations with unlimited accuracy. Since such oscillations are not normally present in well-formulated physical problems, it might be concluded that the problem is not formulated correctly. Whether any of these proposed explanations is correct cannot be answered here.

COMMENTS AND CONCLUSIONS

A brief discussion of the use of the tables should clarify the procedure necessary to calculate approximately the properties of any particular wave. It is first necessary to select the depth and the period. Then the wave height, the wavelength, and ϵ are found by interpolation on Table 1. The profile is obtained by interpolation on ϵ in Table 2. The velocity components are calculated from the equations given below.

$$\frac{u(x' - C_0 t, y)}{C_0} = \sum_{n=1}^{j} \frac{C_n}{C_0} \cos n(kx' - \omega t)\cosh nky \quad . \qquad (39)$$

$$\frac{v(x' - C_0 t, y)}{C_0} = \sum \frac{C_n}{C_0} \sin n(kx' - \omega t)\sinh nky \quad . \qquad (40)$$

We have put

$$\omega = 2\pi/T \qquad (41)$$

and

$$C_0 = \lambda/T \quad . \qquad (42)$$

Equations (41) and (42) follow directly from equation (36) by omitting the term corresponding to the uniform flow (C_0) and replacing x by $x' - ct$.

At the crest, the velocity vector has the components (C_0,0), as is indicated by equation (39). For distances from the crest less than about 1/36 a wavelength, equations (34) and (35) are used to calculate the velocity components. Equations (32) and (33) yield r and α, and D_0, D_1, and D_2 are obtained from Table 3. The ratio of ϵ' to ϵ is found in Table 1.

It would be very interesting to demonstrate experimentally that the wave predicted here theoretically exists. It is unlikely that conditions would occur at sea so that this would be possible; however, experiments in a wave tank should in principle be able to produce these waves. If not, some insight would be gained into how to modify our theoretical procedure to predict this limitation.

Havelock found an extension of the procedure of Michell for connecting the highest waves with the finite amplitude waves of Stokes. He employed the same functional form to represent the velocity, but satisfied Bernoulli's equation on a line $\psi = \beta$, where $|\beta| < |\epsilon|$.

The branch points in the velocity were put above the wave. The velocity potential could then be expanded in a Fourier series convergent everywhere within the wave; the results for coefficients of the various terms agreed with those coefficients calculated by the procedure of Stokes within the limit of accuracy of the calculations.

The procedure of Havelock was applied to the waves predicted here, but the resulting formulas were so formidable that they were useless for practical purposes. It is suggested that the results of Dee,[10] who has extended the Stokes-Struik theory to the fifth order of approximation, be used for calculating all except the highest waves.

There seems to be no convenient way to estimate the errors made in the various approximations, since the correct solution is not available. The solutions would be exact if they did not violate the Bernoulli theorem to a certain extent. As a measure of the error, it is convenient to employ the ratio of the maximum fluctuation in the energy due to the errors to the difference in potential energy between crest and trough.

$$\text{Measure of error} \sim \frac{\Delta(gY + \frac{1}{2}u^2 + \frac{1}{2}v^2)}{gH} = E \quad .$$

This measure of the error would be zero for the exact solution, and presumably could be improved by addition of terms in the series. E is given as a function of ϵ in Table 1.

An inspection of Table 2, which gives the profile, indicates that the four smallest waves have a slight secondary crest in the position expected for the trough. The rise is at most about 5 percent of the total wave height. It is reasonable to assume that this effect is not real but is due to the omission of higher harmonies from the assumed form for the complex velocity, equation (6). The measure of error, E, is also largest for these waves, although perhaps not excessive except for the two smallest values of ϵ.

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APPENDIX I

The Bernoulli theorem is usually written

$$\frac{1}{2} |q(w)|^2 + g \text{ Im } z(w) = \text{const.} \tag{I-1}$$

on the free surface, $w = \phi + i\epsilon$. The partial derivative of equation (I-1) with respect to ϕ is

$$|q(\phi + i\epsilon)| \frac{\partial}{\partial\phi} |q(\phi + i\epsilon)| + g\frac{\partial y}{\partial\phi}(\phi, \epsilon) = 0 \quad . \tag{I-2}$$

Now

$$\frac{\partial y}{\partial\phi} = \frac{1}{|q|^2} \frac{\partial\phi}{\partial y} = -\frac{1}{|q|^2} \text{ Im } q \quad . \tag{I-3}$$

Substitution of equation (I-3) into equation (I-2) yields

$$\frac{\partial}{\partial\phi} |q(\phi + i\epsilon)|^4 = +4g \text{ Im } q(\phi + i\epsilon) \quad . \tag{I-4}$$

APPENDIX II

It is assumed that the largest term in an expansion of $q(w)$ about the crest is proportional to w^{δ}.

$$q(w) = Aw^{\delta} \quad . \tag{II-1}$$

Then an integration gives

$$z = -\frac{1}{A(\delta - 1)w^{\delta-1}} \quad . \tag{II-2}$$

The origin of coordinates may be shifted to the crest. Then the free surface is $w = \phi$. The Bernoulli theorem yields

$$|A|^4 \frac{\partial}{\partial\phi} \phi^{4\delta} + \phi^{\delta} \operatorname{Im} A = 0 \quad . \tag{II-3}$$

In order to satisfy this equation, it is necessary that

$$\delta = \frac{1}{3} \quad . \tag{II-4}$$

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1. Wave height
2. Wave theory

I. Chappelear, J. E.
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