DEPARTMENT OF THE ARMY

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BEACH EROSION BOARD

OFFICE OF THE CHIEF OF ENGINEERS

RE-ANALYSIS OF EXISTING WAVE FORCE DATA ON MODEL PILES

TECHNICAL MEMORANDUM NO. 71

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FOREWORD

Although circular piling is a much used structural element in shore protection, harbor, and other maritime structures, it has only been in the last few years that significant advances have been made toward gaining a quantitative understanding of the forces developed by wave action against piling. Recent tests have advanced our knowledge of these forces considerably, but certain inconsistencies have, however, been observed in much of the early work. This paper presents an attempt to reconcile some of these inconsistencies by using a somewhat different method of analysis.

The author of the report, R. Curtis Crooke, is a California engineer who has made a considerable study of this subject. Because of its applicability to the general research and investigation program of the Beach Erosion Board, particularly as concerns structural design, and through the courtesy of the author, the report is being published at this time in the Technical Memorandum series of the Beach Erosion Board. Views and conclusions stated in the report are not necessarily those of the Beach Erosion Board.

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RE-ANALYSIS OF EXISTING WAVE FORCE DATA ON MODEL PILES

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All of the past published reports on wave forces contain irreconcilable inconsistencies in the methods of deriving the forces produced by the action of waves on piles and/or other structural members.

This led the author to feel that either the approach to the analysis of model tests had to be varied to give consistent and reasonable values, or full scale prototype tests had to be conducted under actual sea conditions which would give direct results.

A paper by Iversen and Balent(1)* gives the results of experimental work with flat disks and the derivation of the forces accomplished by using a single coefficient (C) representing the combined effect of drag and mass. A diagram of his test arrangement is shown in Figure 1.

Data were taken with two different size disks, 2 feet and 1 foot in diameter. Four different driving forces were applied to each disk. Table I lists the test conditions. The results of this test are shown in Figure 2.

The following is the development of the correlation modulus as used by Iversen and Balent:

Ma = (k) x (Mass of fluid displaced by the body)
Ma = added Mass

$$F - M_e A = C_D \frac{\rho}{2} V^2 S + k \rho BA \qquad (1)$$

where
F = force
M_e = Mass of object
A = Acceleration
V = Velocity
S = Area
B = Displaced Volume

The fluid which is in the field of disturbance of an object moving through the fluid flows around the object. When the relative velocity is steady, i.e., no acceleration of the body relative to the undisturbed fluid, the normal evaluation of the force existing on the body is by a drag coefficient,

$$C_D = \frac{F}{\left(\frac{\rho V^2 S}{2}\right)} \qquad (2)$$

* Numbers in parentheses refer to references on page 19.

where C_D = Drag coefficient = $\emptyset(N_R, N_F, \text{geometry})$
F = Force
ρ = Fluid density
V = Velocity
S = Area
N_R = Reynolds modulus
N_F = Froude modulus

The drag coefficients are usually determined by experiment.

The addition of an acceleration to the motion produces an added resistance which can also be developed in terms of a resistance coefficient and a correlating modulus from a consideration of the various terms of the Navier-Stokes equations. The Navier-Stokes equations written for one axis of an incompressible fluid particle are:

$$\rho \frac{Du}{Dt} = \rho X - \frac{\partial p}{\partial x} + \mu \nabla^2 u \qquad (3)$$

where u = particle velocity in the x direction,

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \qquad (4)$$

where

$$\nabla^2 = \text{the Laplace operator } \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (5)$$

The criteria for dynamical similarity may be developed from this equation. For two systems which are geometrically and dynamically similar, the ratios of the variables are:

VARIABLE		RATIO	VARIABLE		RATIO
Length	b_L	L_1/L_2	Density	b_ρ	ρ_1/ρ_2
Time	b_t	t_1/t_2	Velocity	b_v	V_1/V_2
Pressure	b_p	p_1/p_2	Acceleration	b_a	A_1/A_2
Viscosity	b_μ	μ_1/μ_2	Body Force	b_x	X_1/X_2

(6)

Equation 3 written with the subscript 1 designates the flow in system 1. Substitution of the ratios of Equation 6 give the equation for the second system. Since this dynamical system is one which has changes in velocity with respect to time, the term $\partial u/\partial t$ will be designated as an acceleration, (a).

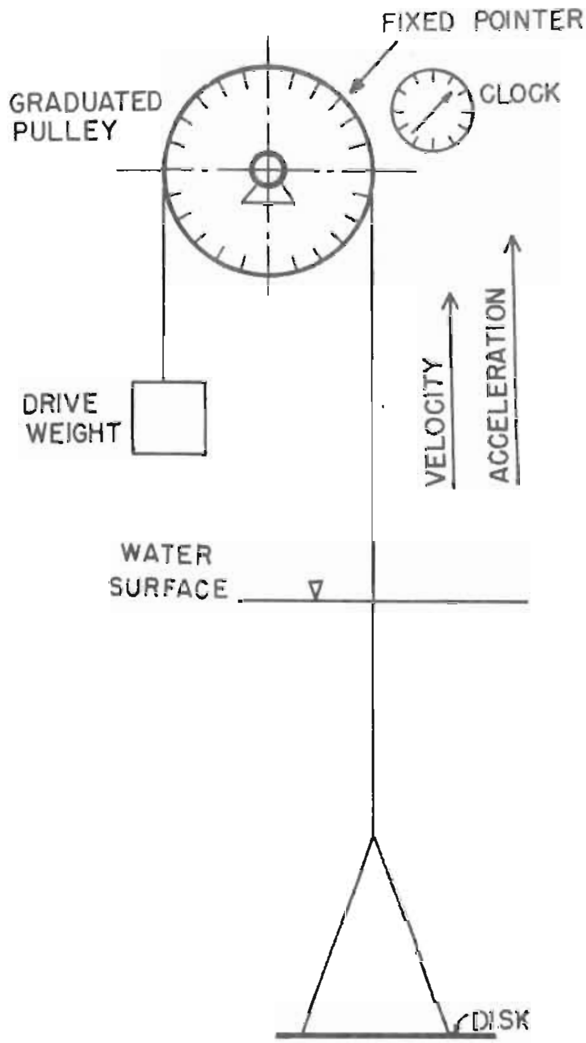


FIGURE 1. SCHEMATIC OF IVERSEN'S TEST EQUIPMENT

TABLE I

Iversen's Experimental Conditions

Run No.	Disk Diameter Ft.	Gross Driving Force Pounds	Mass of Moving Parts Slugs
29	2	4.78	1.263
30	2	6.78	1.325
31	2	8.78	1.387
32	2	10.78	1.449
34	1	2.28	0.512
35	1	3.28	0.543
36	1	4.28	0.574
37	1	5.28	0.605

TABLE II

Experimentally Determined Coefficient of Mass

Average Values, Average Deviation and Range

$C_M = 1.96 \pm 0.25 \quad (1.15 - 2.83)$

Experimentally Determined Coefficient of Drag

Average Values, Average Deviation and Range

$C_D = 2.03 \pm 0.40 \quad (0.98 - 3.50)$

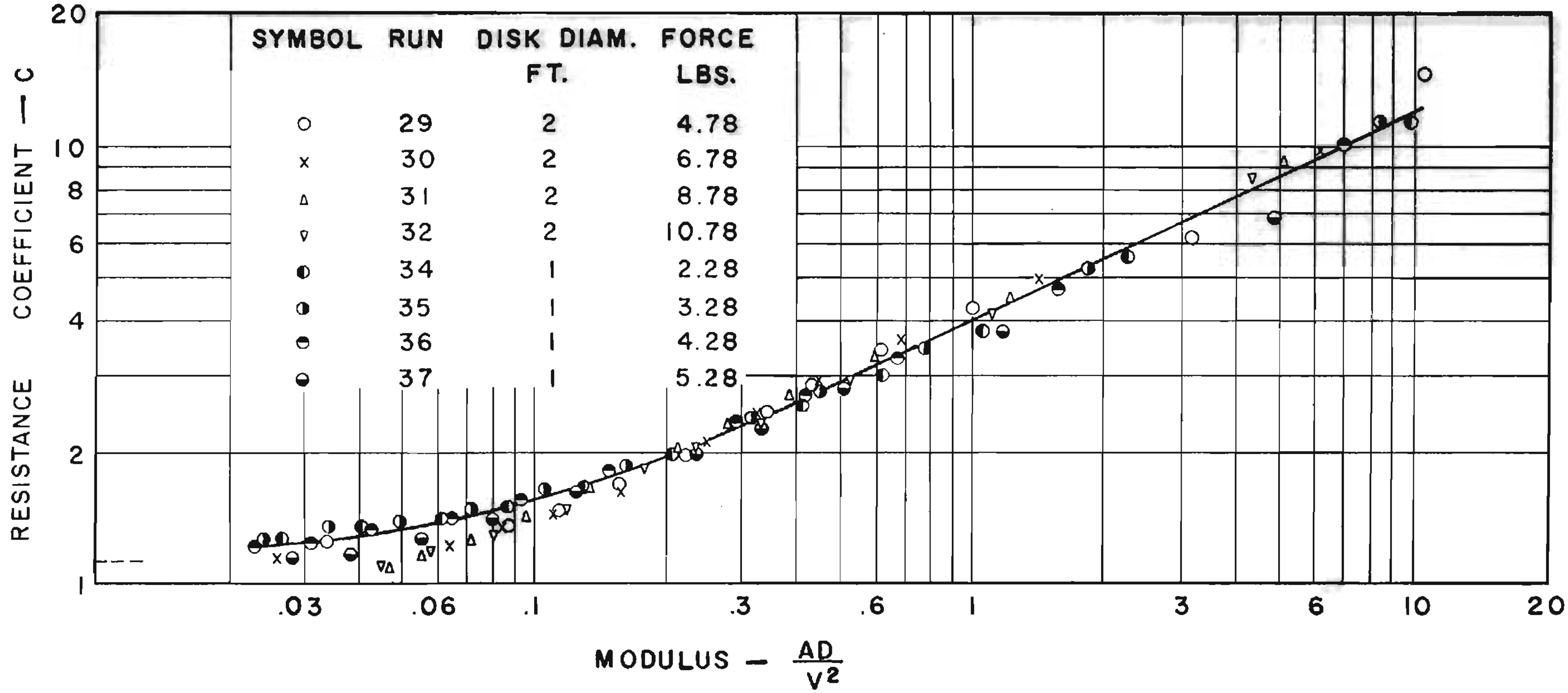


FIGURE 2. "C" vs. AD/V^2 FOR DISKS AS MEASURED BY IVERSEN

Then from equations (3) and (6),

$$b_\rho b_a \rho_2 A_2 + b_\rho \frac{b_v^2}{b_L}\left(\rho_2 u_2 \frac{\partial u_2}{\partial x_2} + \ldots\ldots\right)$$

$$= b_\rho b_x \rho_2 X_2 - \frac{b_p}{b_L}\frac{\partial p_2}{\partial x_2} + b_\mu \frac{b_v}{b_L^2}(\mu_2 \nabla^2 u_2) \qquad (7)$$

This expression must be the same, for dynamical and geometrical similarity, as Equation (3) written directly for the second system.

$$\rho_2 A_2 + \rho_2 \left(u_2 \frac{\partial u_2}{\partial x_2} + \ldots\right) = \rho_2 X_2 - \frac{\partial p_2}{\partial x_2} + \mu_2 \nabla^2 u_2 \qquad (8)$$

From Equations 7 and 8:

$$b_\rho \; b_a = b_\rho \frac{b_v^2}{b_L} = b_\rho \; b_x = \frac{b_p}{b_L} = b_\mu \frac{b_v}{b_L^2} \qquad (9)$$

I II III IV V

Each of the terms of Equation 9 represent force ratios which can be designated as those due to:

I Local Inertia
II Convective Inertia
III Gravity
IV Pressure
V Viscosity

For systems where the gravity and viscous fields are negligible, only I, II and IV need to be considered. The pressures are due to the object influence. An integration of pressures on the body will result in the resistance to motion of the body; hence, the local and convective inertias can be used to define the conditions under which the pressure forces are similar.

Thus:

$$b_\rho \; b_a = b_\rho \frac{b_v^2}{b_L} = \frac{b_p}{b_L} \qquad (10)$$

$$\frac{b_a \, b_L}{b_v^2} = 1 = \frac{b_p}{(b_L \, b_\rho \, b_v^2/b_L)} \qquad (11)$$

In the application of these ratios to the two dynamical systems any corresponding velocity or acceleration which defines the motion may be taken to evaluate the ratio between the systems. In the case of an object moving through a stationary fluid, the velocity and acceleration of the object relative to the fluid at rest define the motion.

Hence: $$\frac{p_2}{\rho_2 V_2^2} = \frac{p_1}{\rho_1 V_1^2}\left(\frac{A_2 L_2}{V_2^2} \Big/ \frac{A_1 L_1}{V_1^2}\right) \qquad (12)$$

Also: $$F = \int_0^S p\,dS \propto pL^2 \qquad (13)$$

when F is the force on the object and p is the pressure at the boundary of the object of area S

$$\frac{F_2}{\rho_2 L_2^2 V_2^2} = \frac{F_1}{\rho_1 L_1^2 V_1^2}\left(\frac{A_2 L_2}{V_2^2} \Big/ \frac{A_1 L_1}{V_1^2}\right) \qquad (14)$$

when geometrical and dynamical similarity exists, the ratio

$$\frac{A_2 L_2}{V_2^2} \Big/ \frac{A_1 L_1}{V_1^2} = 1 \qquad (15)$$

then:

$$\frac{F}{\rho L^2 V^2} = \phi\left(\frac{AL}{V^2}\right) = C \qquad (16)$$

Equation 16 under the conditions previously stated thus gives the correlating function for the resistance coefficient under accelerated motion. If the viscosity and gravity effects are not negligible, a similar analysis shows

$$C = \phi\left(\frac{AL}{V^2}, \frac{VL\rho}{\mu}, \frac{V^2}{gL}\right) \qquad (17)$$

where:

$\frac{AL}{V^2}$ = Iversen's Modulus

$\frac{VL\rho}{\mu}$ = Reynolds' Number

$\frac{V^2}{gL}$ = Froude's Modulus

It has been conceived that the Iversen approach could be applied to the derivation of the forces produced by wave action and has been so applied in this paper. Before giving this analysis in detail it has been considered advisable to take a cursory survey of the problem as handled by other analysts.

Munk in his original work(2) proposed using the maximum velocity under the crest to determine the Reynolds number and to use the same velocity in the force equation with the corresponding steady state coefficient of drag. This gave a force that was maximum at the crest and went to zero at the still water level ($\theta = 90^\circ$).

The next step is to consider an object moving with constant acceleration in a fluid. This is slightly more complicated because there are not only drag or shear forces, but also inertia forces. At this point the agreement between various investigators breaks down.

The work by Morison(3) showed the interpretation of Munk to be an oversimplification of the problem. This was obvious from the fact that the measurements of wave force on a pile showed that the maximum force did not occur at the crest, but occurred before the passage of the crest at a variable phase angle which depended upon the distance above the bottom and the diameter of the pile.

Morison(3) developed a force equation containing two terms. One term contained wave and pile constants, the velocity squared term and drag coefficient; the second term contained wave and pile constants, the acceleration and coefficient of mass. The two terms of the equation are 90° out of phase with each other, hence the requirements that the maximum force was out of phase with the crest and varied with depth and pile diameter were met.

Morison's method of determining the value of the coefficients is as follows: The force or moment on the pile is measured during the passage of the waves. Also, the wave profile as it passes the pile is measured. With this data it is possible to solve the force equation for the value of the coefficient of drag when the crest and trough pass the pile ($\theta = 0$, $\theta = 180^\circ$) and to solve the force equation for the coefficient of mass when both of the still water levels pass the pile ($\theta = 90^\circ$, $\theta = 270^\circ$). The coefficients can be solved only for these four points in the wave cycle. The value of the coefficients is checked by holding them constant throughout the wave cycle and the corresponding calculated force curve is compared with the measured force curve.

In all of Morison's work no satisfactory explanation for the variation in the values of the coefficients has been given. In one of Morison's reports(4) he gave a curve of the coefficient of drag vs. the instantaneous Reynolds Number (Figure 3). The correlation was not good

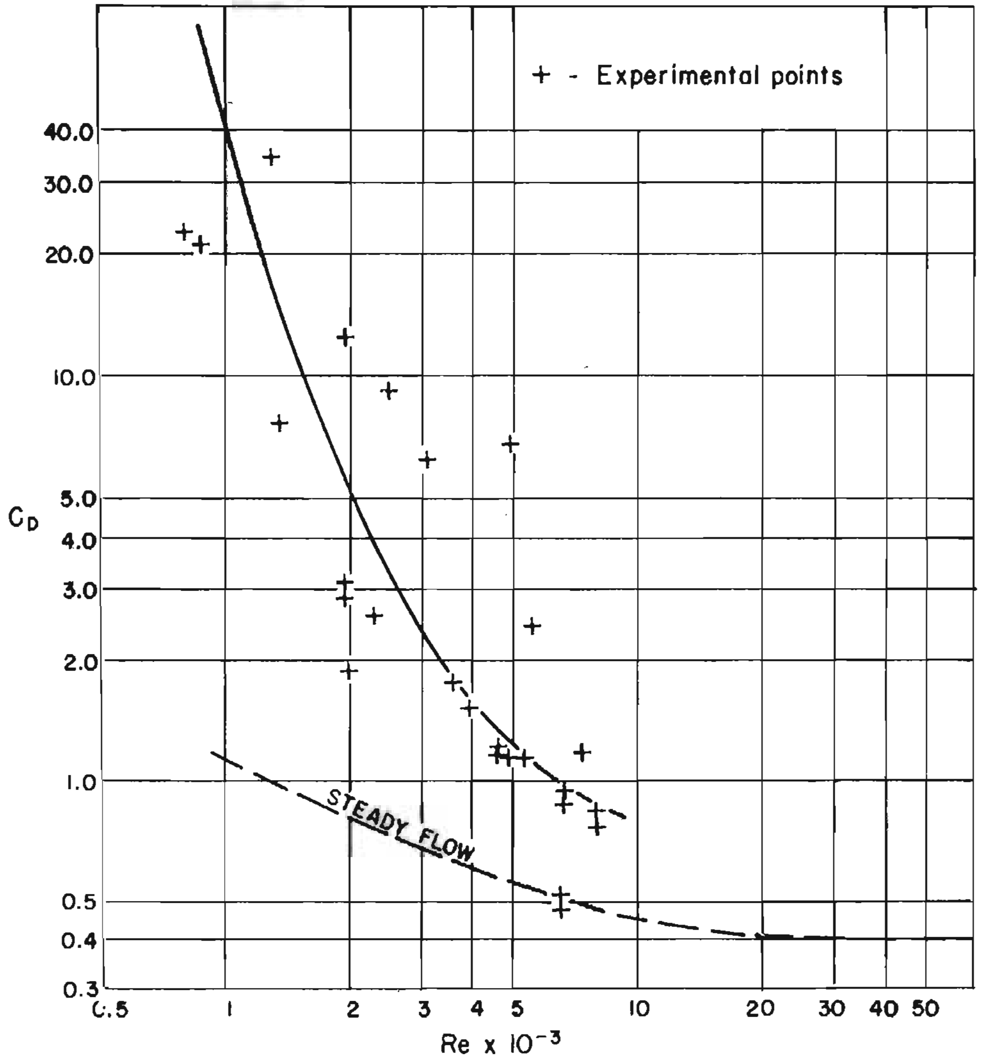


FIGURE 3.

COEFFICIENT OF DRAG vs. REYNOLD'S NUMBER FOR A SPHERE IN OSCILLATORY FLOW AS DETERMINED BY MORISON and the data were all for very low values of Reynold's Number. He states that no correlation for the coefficient of mass has been found. Hence, in all of Morison's work the values of the coefficients have been taken as averages of all measurements. (See Table 2 for the average values over the full length of the pile and through the complete wave cycle.

From communications with R. L. Wiegel of the Wave Research Projects at the University of California, Berkeley, it was learned that a rough correlation of the coefficient of drag vs. instantaneous Reynolds' Number has been obtained in the analysis of the current prototype test data, but nothing in the way of a correlation has been attained for the coefficient of mass.

This is as one would expect, for Iversen(1) in conducting tests on a disk under constant acceleration, had the following to say: "Published experimental results, mostly with oscillating systems with small amplitudes of motion, show an added mass constant that is higher than that derived from potential flow with values that are dependent upon the fluid and the object size. A few previous experiments on resistance in unidirectional accelerated motion indicate that the added mass is variable and depends upon the state of motion". G. P. Weinblum(5) had the following to say: "This means that for a given speed and acceleration the added mass of a body may vary with the kind of motion; for instance, assume different values for a translation, a free or a forced oscillation in the same direction. Under these circumstances, the question whether and to what extent the concept of hydrodynamic masses can still be maintained in the case of an accelerated motion of the body on a free surface, appears justified. We shall here anticipate the answer: The concept remains quite suitable; however, the quantities in question can be functions of certain variables, so that they lose their simple geometrical character." Brahmig(6) said: "For an exact understanding of the forces or loads acting on the body, a knowledge of hydrodynamic inertia effects, and in individual cases their numerical magnitudes is indispensable. In oscillatory phenomena in particular the effects of the size of the oscillating mass on both the frequency and amplitude is worthy of note. -- Whereas the calculated hydrodynamic mass depends only on shape, its values vary with flow conditions in a real eddying medium. The virtual mass of completely submerged or floating bodies in translational motion is determined experimentally by measuring the force of acceleration and the acceleration itself. Since frictional resistance varies with time in accelerated motion, it is impossible to separate the two components. Hence, it is not possible to prove that the pure hydrodynamic inertial resistance is a function of the acceleration as it is suspected to be. --Since, however, the flow pattern about an oscillating body and concurrently the magnitude of the entrained mass of the medium changes not only with respect to frequency, but also with respect to amplitude, the determination of the apparent mass by the method of free vibrations is inherently unreliable."

A pile in ocean waves is by far the most complicated fluid flow problem because there is not only unsteady motion but oscillatory motion. For any one wave the frequency along the length of the pile is constant, but the amplitude, hence instantaneous velocities and acceleration, vary with the depth and the phase angle. Of course, from wave to wave, all of the conditions vary. Considering the preceding discussion it should be apparent that the values of the coefficients of mass and drag should be variable quantities from wave to wave, phase angle in the wave, and position vertically along the pile. This has not been the case as previously considered, as all published work assumes these coefficients to be constant.

The approach of Iversen has been applied to published data of Morison[7,4] and to unpublished data of Morison. The test conditions, wave characteristics, and values of the single coefficient, correlating modulus and Reynold's Numbers are given for all the available data in Table III.

The data consist of horizontal wave forces on horizontal cylinders, vertical wave forces on horizontal cylinders, horizontal wave forces on spheres, and horizontal forces on vertical cylinders. These data are suitable for this type of analysis because the vertical dimension of the test object is small compared to the water depth and the wave characteristics. This means that the values of particle velocity and acceleration can be assumed to be constant over the vertical length of the test segment without introducing any sizeable error.

The results of the analysis are shown in Figure 4 where all of the data have been plotted. Three different curves have been drawn through the data representing the conditions for the sphere, the horizontal cylinder and the vertical cylinder. With the amount of data available it would seem as though three different curves exist and this is what one would expect for the flow pattern will differ for each of the three conditions.

It will be seen in Figure 4 that a very good correlation appears to exist in the available data. It is conceived that a Reynold's Number effect will appear at the lower values of the correlating modulus AD/V^2 where the velocity term predominates over the acceleration. While the model data have an upper limit of Reynold's Number of about 5×10^3 there does appear to be some effect present as can be seen from the data on the vertical cylinder where AD/V^2 is less than approximately 10. It looks as though for any constant value of AD/V^2 as the Reynold's Number is increased, the value of "C" will decrease. This would give a family of curves in the lower range of AD/V^2 which would tend to approach each other at some larger value of AD/V^2.

TABLE III

BASIC DATA

Run No.	Object	Orientation	Dia Ft	Wave Height Ft	Wave Period Sec.	Wave Length Ft	SWL Depth Ft	Object Depth Ft	Re No.	Coeff C	$\frac{AD}{V^2}$
1	Cyl	Hor/Hor	0.083	0.615	1.200	6.172	1.543	0.489	$2.1\ 10^3$	6.44	2.38
2	"	"	"	0.620	1.196	6.378	1.543	0.490	$2.7\ 10^3$	3.43	1.46
3	"	"	"	0.601	1.200	6.216	1.545	0.490	$1.6\ 10^3$	8.00	4.05
4	"	"	"	0.613	1.200	6.353	1.528	0.978	$5.4\ 10^3$	1.24	0.08
5	"	"	"	0.618	1.183	6.259	1.550	0.980	$4.6\ 10^3$	2.11	0.40
6	"	"	"	0.630	1.183	6.548	1.524	0.979	$4.0\ 10^3$	2.62	0.77
7	"	"	"	0.324	0.783	3.204	1.503	0.985	$1.4\ 10^3$	10.2	4.62
8	"	"	"	0.331	0.775	3.167	1.485	0.985	$1.4\ 10^3$	10.0	4.60
9	"	Hor/Ver	"	0.568	1.183	6.452	1.537	0.474		2.36	0.46
10	"	"	"	0.540	1.183	6.452	1.527	0.990		1.36	0.13
16	Sphere	Hor.	0.125	0.209	1.150	5.913	1.328	0.125	$2.2\ 10^2$	636	340
17	"	"	"	0.346	1.183	6.085	1.331	"	$2.0\ 10^3$	13.6	5.55
18	"	"	"	0.249	1.483	8.696	1.328	"	$2.8\ 10^3$	5.25	1.93
19	"	"	"	0.241	0.850	3.732	1.328	"	$4.4\ 10^1$	11,416	9,100
20	"	"	"	0.257	0.717	3.468	1.451	"	$9.8\ 10^1$	1,392	1,820
21	"	"	"	0.217	2.133	14.770	1.328	"	$2.8\ 10^3$	3.83	1.49
22	"	"	"	0.217	2.133	14.770	1.328	"	$4.5\ 10^3$	1.75	0.54
23	"	"	"	0.273	2.167	15.000	1.328	"	$5.1\ 10^3$	1.67	0.39
11a	Cyl	Hor/Ver	0.042	0.188	0.96	4.77	1.92	0.59	$8.9\ 10^0$	9,278	4,000
11b	"	"	0.083	"	"	"	"	0.59	$1.7\ 10^1$	24,778	7,778
11c	"	"	0.167	"	"	"	"	0.59	$3.6\ 10^1$	48,000	15,778
12a	"	"	0.042	"	"	"	"	0.70	$1.2\ 10^1$	7,750	2,438
12b	"	"	0.083	"	"	"	"	0.70	$2.4\ 10^1$	15,938	4,875
12c	"	"	0.167	"	"	"	"	0.70	$4.7\ 10^1$	30,125	9,812
13a	"	"	0.042	"	"	"	"	0.80	$1.5\ 10^1$	6,320	1,760
13b	"	"	0.083	"	"	"	"	0.80	$3.0\ 10^1$	12,800	3,440
13c	"	"	0.167	"	"	"	"	0.80	$5.9\ 10^1$	22,160	6,920
14a	"	"	0.042	"	"	"	"	0.90	$1.8\ 10^1$	4,639	1,333
14b	"	"	0.083	"	"	"	"	0.90	$3.5\ 10^1$	3,750	2,667
14c	"	"	0.167	"	"	"	"	0.90	$7.1\ 10^1$	16,000	5,361
15a	"	"	0.042	"	"	"	"	1.00	$2.4\ 10^1$	2,609	844
15b	"	"	0.083	"	"	"	"	1.00	$4.7\ 10^1$	5,984	1,672
15c	"	"	0.167	"	"	"	"	1.00	$9.5\ 10^1$	11,250	3,375
16a	"	"	0.042	"	"	"	"	1.10	$3.0\ 10^1$	1,870	610
16b	"	"	0.083	"	"	"	"	1.10	$5.9\ 10^1$	4,150	1,220
16c	"	"	0.167	"	"	"	"	1.10	$1.2\ 10^2$	7,200	2,420
17a	"	"	0.042	"	"	"	"	1.20	$3.6\ 10^1$	1,607	486
17b	"	"	0.083	"	"	"	"	1.20	$7.1\ 10^1$	3,428	964
17c	"	"	0.167	"	"	"	"	1.20	$1.4\ 10^2$	6,171	1,943
18a	"	"	0.042	"	"	"	"	1.30	$4.5\ 10^1$	1,023	354
18b	"	"	0.083	"	"	"	"	1.30	$8.8\ 10^1$	2,468	700
18c	"	"	0.167	"	"	"	"	1.30	$1.8\ 10^2$	4,800	1,409
19a	"	"	0.042	"	"	"	"	1.40	$6.0\ 10^1$	647	220
19b	"	"	0.083	"	"	"	"	1.40	$1.2\ 10^2$	1,390	435
19c	"	"	0.167	"	"	"	"	1.40	$2.4\ 10^2$	2,898	875

TABLE III

BASIC DATA

Run No.	Object	Orientation	Dia Ft	Wave Height Ft	Wave Period Sec	Wave Length Ft	SWL Depth Ft	Object Depth Ft	Re No.	Coeff C	$\frac{AD^2}{V}$
20a	Cyl	Hor/Ver	0.042	0.188	0.96	4.77	1.92	1.50	$8.0\ 10^1$	394	14.3
20b	"	"	0.083	"	"	"	"	1.50	$1.6\ 10^2$	359	281
20c	"	"	0.167	"	"	"	"	1.50	$3.2\ 10^2$	2057	566
21	"	"	0.042	"	"	"	"	0.59	$6.6\ 10^1$	174	72.9
22	"	"	"	"	"	"	"	0.90	$2.3\ 10^2$	29	7.4
23	"	"	"	"	"	"	"	1.00	$2.5\ 10^2$	35	9.4
24	"	"	"	"	"	"	"	1.10	$2.8\ 10^2$	23	5.9
25	"	"	"	"	"	"	"	1.20	$3.1\ 10^2$	21	5.5
26	"	"	"	"	"	"	"	1.30	$3.5\ 10^2$	18	4.9
27	"	"	"	"	"	"	"	1.40	$2.8\ 10^2$	31	9.3
28	"	"	"	"	"	"	"	1.50	$7.3\ 10^2$	6.6	1.1
29	"	"	0.083	"	"	"	"	0.80	$2.1\ 10^2$	272	36.4
30	"	"	0.167	"	"	"	"	1.40	$1.1\ 10^2$	150,250	11,000
31	"	"	0.042	0.454	0.98	4.97	2.00	0.72	$6.6\ 10^1$	386	182
32	"	"	"	"	"	"	"	0.80	$7.8\ 10^1$	464	140
33	"	"	"	"	"	"	"	1.00	$1.1\ 10^2$	168	85.7
34	"	"	"	"	"	"	"	1.40	$2.3\ 10^2$	371	21.1
35	"	"	0.083	"	"	"	"	0.72	$1.3\ 10^2$	1,268	360
36	"	"	"	"	"	"	"	0.80	$1.5\ 10^2$	856	277
37	"	"	"	"	"	"	"	1.00	$2.2\ 10^2$	521	170
38	"	"	"	"	"	"	"	1.20	$3.5\ 10^2$	265	84.9
39	"	"	"	"	"	"	"	1.40	$5.6\ 10^2$	130	41.8
40	"	"	"	"	"	"	"	1.60	$9.3\ 10^2$	47.4	19.4
41	"	"	0.167	"	"	"	"	0.72	$2.6\ 10^2$	2,820	725
42	"	"	"	"	"	"	"	0.80	$3.1\ 10^2$	2,020	557
43	"	"	"	"	"	"	"	1.00	$4.5\ 10^2$	1,290	376
44	"	"	"	"	"	"	"	1.20	$7.0\ 10^2$	670	171
45	"	"	0.042	0.194	0.96	4.82	1.91	0.59	$8.9\ 10^1$	170	41.1
46	"	"	"	"	"	"	"	0.70	$9.5\ 10^1$	183	41.0
47	"	"	"	"	"	"	"	0.80	$1.1\ 10^2$	164	34.6
48	"	"	"	"	"	"	"	0.90	$1.1\ 10^2$	143	35.7
49	"	"	"	"	"	"	"	1.00	$1.2\ 10^2$	135	31.1
50	"	"	"	"	"	"	"	1.10	$2.5\ 10^2$	31.9	8.2
51	"	"	"	"	"	"	"	1.20	$1.9\ 10^2$	72	16.6
52	"	"	"	"	"	"	"	1.30	$3.6\ 10^2$	24.6	5.8
53	"	"	"	"	"	"	"	1.40	$8.4\ 10^2$	5.5	0.49
54	"	"	"	"	"	"	"	1.50	$9.2\ 10^2$	4.8	0.54
55	"	"	0.083	"	"	"	"	0.59	$2.4\ 10^1$	1280	380
56	"	"	"	"	"	"	"	0.70	-- -	00	00
57	"	"	"	"	"	"	"	0.80	$5.9\ 10^1$	3,320	920
58	"	"	"	"	"	"	"	0.90	$6\ 10^0$	342,000	104,000
59	"	"	"	"	"	"	"	1.00	$6\ 10^0$	383,000	114,000
60	"	"	"	"	"	"	"	1.10	$2.5\ 10^2$	257	70
61	"	"	"	"	"	"	"	1.20	$7.7\ 10^1$	2,325	730
62	"	"	"	"	"	"	"	1.30	$1.2\ 10^1$	143,250	40,000

TABLE III

BASIC DATA

Run No.	Object	Orientation	Dia Ft	Wave Height Ft	Wave Period Sec	Wave Length Ft	SWL Depth Ft	Object Depth Ft	Re No.	Coeff C	$\frac{AD}{V^2}$
63	Cyl	Hor/Ver	0.083	0.194	0.96	4.82	1.91	1.40	1.3 10^2	1,358	372
64	"	"	"	"	"	"	"	1.50	1.6 10^2	947	262
65	"	"	0.167	"	"	"	"	0.59	4.7 10^1	22,300	7,600
66	"	"	"	"	"	"	"	0.70	5.9 10^1	2,530	835
67	"	"	"	"	"	"	"	0.80	3.2 10^2	793	260
68	"	"	"	"	"	"	"	0.90	1.1 10^2	9,500	2,562
69	"	"	"	"	"	"	"	1.00	3.7 10^2	668	225
70	"	"	"	"	"	"	"	1.10	3.6 10^1	82,800	28,444
71	"	"	"	"	"	"	"	1.20	1.3 10^2	9,640	2,900
72	"	"	"	"	"	"	"	1.30	1.2 10^2	11,210	3,250
73	"	"	"	"	"	"	"	1.40	7.1 10^2	365	99.2
74	"	"	"	"	"	"	"	1.50	7.6 10^2	395	98.8
75	"	"	"	"	0.94	4.57	1.89	0.59	4.2 10^1	43,100	14,700
76	"	"	"	"	"	"	"	0.70	5.2 10^1	24,800	8,250
77	"	"	"	"	"	"	"	0.80	6.3 10^1	18,467	6,133
78	"	"	"	"	"	"	"	0.90	8.0 10^1	14,375	5,100
79	"	"	"	"	"	"	"	1.00	1.0 10^2	10,271	3,300
80	"	"	"	"	"	"	"	1.10	1.3 10^2	5,992	2,183
81	"	"	"	"	"	"	"	1.20	1.6 10^2	4,542	1,558
82	"	"	"	"	"	"	"	1.30	2.0 10^2	3,455	1,090
83	"	"	"	"	"	"	"	1.40	2.8 10^2	2,184	696
84	"	"	"	"	"	"	"	1.50	3.6 10^2	1,530	466
85	"	"	"	"	"	"	"	1.60	4.8 10^2	987	302
86	"	"	0.083	"	"	"	"	0.59	2.1 10^1	22,500	7,350
87	"	"	"	"	"	"	"	0.70	2.6 10^1	12,850	4,225
88	"	"	"	"	"	"	"	0.80	3.1 10^1	10,733	3,067
89	"	"	"	"	"	"	"	0.90	4.0 10^1	8,850	2,550
90	"	"	"	"	"	"	"	1.00	5.0 10^1	5,514	1,650
91	"	"	"	"	"	"	"	1.10	6.4 10^1	3,483	1,091
92	"	"	"	"	"	"	"	1.20	8.2 10^1	2,542	779
93	"	"	"	"	"	"	"	1.30	1.0 10^2	1,764	545
94	"	"	"	"	"	"	"	1.40	1.4 10^2	1,018	348
95a	"	"	0.042	0.188	0.96	4.77	"	0.59	—	—	—
95b	"	"	0.083	"	"	"	"	0.59	—	—	—
95c	"	"	0.167	"	"	"	"	0.59	1.5 10^3	5.6	0
96a	"	"	0.042	"	"	"	"	0.70	4.2 10^2	2.2	0
96b	"	"	0.083	"	"	"	"	0.70	8.3 10^2	1.6	0
96c	"	"	0.167	"	"	"	"	0.70	1.7 10^3	5.4	0
97a	"	"	0.042	"	"	"	1.92	0.80	4.6 10^2	2.6	0
97b	"	"	0.083	"	"	"	"	0.80	9.1 10^2	4.0	0
97c	"	"	0.167	"	"	"	"	0.80	1.8 10^3	4.0	0
98a	"	"	0.042	"	"	"	"	0.90	5.1 10^2	2.1	0
98b	"	"	0.083	"	"	"	"	0.90	1.0 10^3	3.3	0
98c	"	"	0.167	"	"	"	"	0.90	2.0 10^3	1.6	0
99a	"	"	0.042	"	"	"	"	1.00	5.6 10^2	1.7	0
99b	"	"	0.083	"	"	"	"	1.00	1.1 10^3	1.7	0

TABLE III

BASIC DATA

Run. No.	Object	Orientation	Dia Ft	Wave Height Ft	Wave Period Sec	Wave Length Ft	SWL Depth Ft	Object Depth Ft	Re No.	Coeff C	$\frac{AD}{V^2}$
99c	Cyl	Hor/Ver	0.167	0.188	0.96	4.77	1.92	1.00	$2.3\ 10^3$	—	0
100a	"	"	0.042	"	"	"	"	1.10	$6.4\ 10^2$	2.3	0
100b	"	"	0.083	"	"	"	"	1.10	$1.3\ 10^3$	1.4	0
100c	"	"	0.167	"	"	"	"	1.10	$2.5\ 10^3$	--	0
101a	"	"	0.042	"	"	"	"	1.20	$7.1\ 10^2$	2.3	0
101b	"	"	0.083	"	"	"	"	1.20	$1.4\ 10^3$	1.7	0
101c	"	"	0.167	"	"	"	"	1.20	$2.8\ 10^3$	1.4	0
102a	"	"	0.042	"	"	"	"	1.30	$8.0\ 10^2$	1.8	0
102b	"	"	0.083	"	"	"	"	1.30	$1.6\ 10^3$	0.9	0
102c	"	"	0.167	"	"	"	"	1.30	$3.2\ 10^3$	0.9	0
103a	"	"	0.042	"	"	"	"	1.40	$9.2\ 10^2$	1.5	0
103b	"	"	0.083	"	"	"	"	1.40	$1.8\ 10^3$	0.7	0
103c	"	"	0.167	"	"	"	"	1.40	$3.7\ 10^3$	0.7	0
104a	"	"	0.042	"	"	"	"	1.50	$1.0\ 10^3$	2.4	0
104b	"	"	0.083	"	"	"	"	1.50	$2.0\ 10^3$	1.1	0
104c	"	"	0.167	"	"	"	"	1.50	$4.0\ 10^3$	1.1	0
105	"	"	0.042	0.194	0.96	4.82	1.91	1.60	$1.1\ 10^3$	2.96	0.26
106	"	"	0.042	"	"	"	"	1.70	$1.2\ 10^3$	2.8	0.37
107	"	"	0.083	"	"	"	"	1.80	$1.3\ 10^3$	20	5.1
108	"	"	0.042	0.277	1.22	5.81	1.00	0.59	$1.5\ 10^3$	1.94	0.22
109	"	"	"	"	"	"	"	0.70	$1.7\ 10^3$	2.41	0.15
110	"	"	"	"	"	"	"	0.80	$1.8\ 10^3$	2.45	0.11
111	"	"	"	"	"	"	"	0.90	$1.6\ 10^3$	3.49	0.28
112	"	"	0.083	"	"	"	"	0.59	$2.8\ 10^3$	4.7	0.69
113	"	"	"	"	"	"	"	0.70	$3.5\ 10^3$	2.9	0.15
114	"	"	"	"	"	"	"	0.80	$3.3\ 10^3$	4.0	0.37
115	"	"	"	"	"	"	"	0.90	$3.7\ 10^3$	3.0	0.13
116	"	"	0.167	"	"	"	"	0.59	$2.9\ 10^3$	29.7	8.1
117	"	"	"	"	"	"	"	0.70	$3.8\ 10^3$	18.9	4.8
118	"	"	"	"	"	"	"	0.80	$2.7\ 10^3$	40.7	10.6
119	"	"	0.042	0.452	0.97	4.92	2.00	0.72	$6.5\ 10^2$	5.2	1.52
120	"	"	"	"	"	"	"	0.80	$4.7\ 10^2$	16.7	3.48
121	"	"	"	"	"	"	"	1.00	$1.0\ 10^3$	3.2	0.50
122	"	"	"	"	"	"	"	1.20	$1.2\ 10^3$	3.1	0.51
123	"	"	0.083	"	"	"	"	0.72	$4.2\ 10^2$	134.6	37.0
124	"	"	"	"	"	"	"	0.80	$2.9\ 10^2$	360	95.5
125	"	"	"	"	"	"	"	1.00	$3.2\ 10^2$	279	78.7
126	"	"	"	"	"	"	"	1.20	$9.7\ 10^2$	40.9	10.2
127	"	"	"	"	"	"	"	1.30	$9.1\ 10^2$	60.7	14.8
128	"	"	"	"	"	"	"	1.40	$2.2\ 10^3$	11.3	2.9
129	"	"	"	"	"	"	"	1.50	$3.6\ 10^3$	6.9	1.1

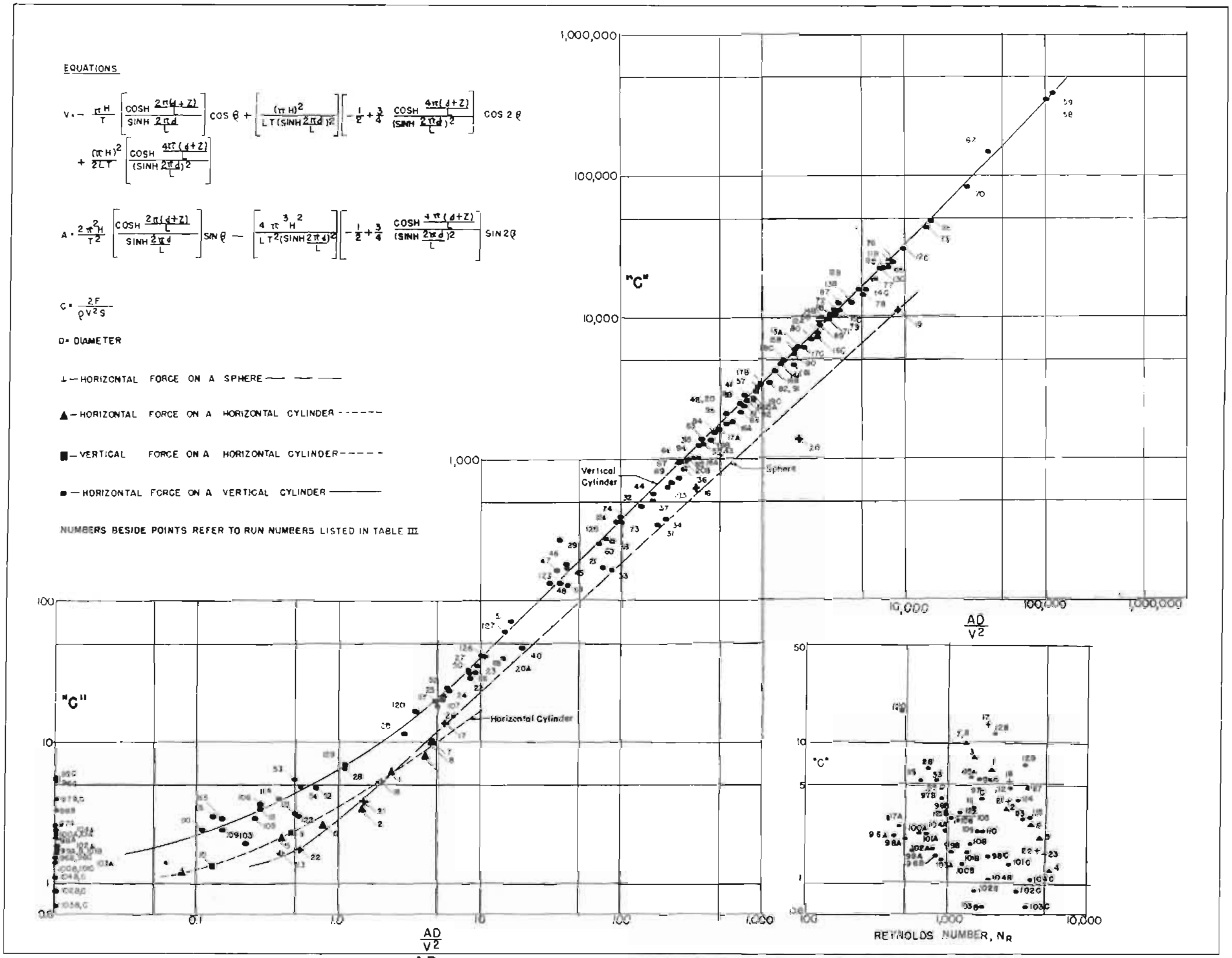


FIGURE 4. "C" VERSUS $\frac{AD}{V^2}$ FOR MODEL CYLINDERS AND SPHERES IN OSCILLATORY FLOW

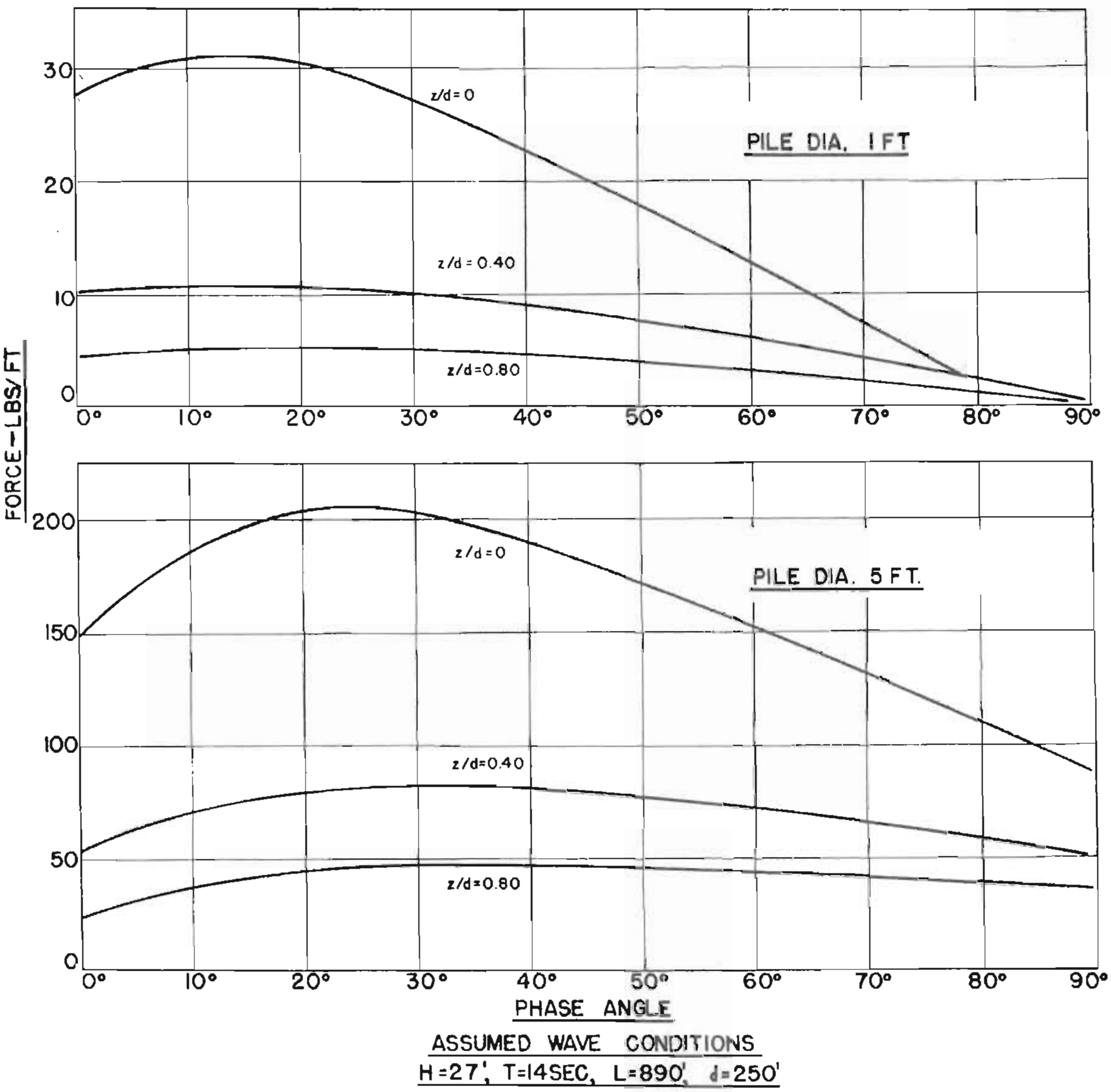
There is considerable scatter in the points plotted as AD/V^2 approaches zero, (or the points determined for the forces directly under the wave crest). It is impossible to say what the cause is except that all of these data were taken at one time and there is the possibility that there is an experimental error.

The values of the coefficient "C" for all points below $AD/V^2 = 10$ are plotted against Reynold's Number in the insert of Figure 4, and it can be seen that there is a great deal of scatter. Considering only the points labeled 95c through 103c (points where phase angle equals zero and the acceleration terms equal zero) it will be seen that there is a general decrease in the values of the single coefficient "C" as the Reynold's Number is increased. For the remainder of the points one would not expect a correlation (such as in steady state conditions) between the single coefficient "C" and the Reynold's Number, because in these cases the acceleration term is also present.

In order to determine if the shape of the curve of the single coefficient C vs. AD/V^2 is correct to give the desired results as to phase angle vs. depth and pile diameter, the prototype conditions shown in Figure 5 were assumed, and the force curve calculated for two diameter piles at three depths of submergence. In these calculations the force was calculated using the curve for the horizontal force on a horizontal cylinder shown on Figure 4 to obtain the appropriate values for the coefficient. The results are given in Figure 5 which shows the increase in phase shift with depth and the increase in phase shift with pile diameter. It will be noted in Figure 5 that the values of AD/V^2 for the assumed prototype wave conditions are of the same order of magnitude as the model results. This means that it should be possible to obtain model and prototype data which will cover the same range of AD/V^2. Thus, less data will be needed to either define the curve or to eliminate the usefulness of the method. That is, one is not faced with all model data at one end of a curve and all prototype at the other end as is the case of the correlation of the drag coefficient with Reynold's Number.

This type of correlation with only one coefficient for use in the wave force equation gives rise to values of the coefficient that are dependent upon the velocity and the acceleration, both of which vary over the length of the pile and the depth of submergence. This is as it should be. While the exact curves cannot be defined with the limited amount of data on hand, it can be said that the shape of the curve is correct, as it duplicates the physical conditions that have been measured.

At this point it would appear that the Iversen approach is the correct one to be used for wave force studies. What is needed now is some prototype data covering larger Reynold's Numbers and different wave conditions. There is every reason to believe that the prototype data



Values of "C" and $\frac{AD}{V^2}$ for Assumed Conditions

Phase Angle	Pile Dia = 1 Ft						Pile Dia = 5 Ft					
	Z/d = 0		Z/d = 0.40		Z/d = 0.80		Z/d = 0		Z/d = 0.40		Z/d = 0.80	
θ	C	AD/V^2	C	AD/V^2	C	AD/V^2	C	AD/V^2	C	AD/V^2	C	AD/V^2
5°	0.95	0.007	0.98	0.013	1.00	0.018	1.08	0.035	1.19	0.065	1.25	0.090
10°	1.00	0.015	1.03	0.025	1.09	0.037	1.21	0.075	1.33	0.125	1.50	0.185
15°	1.05	0.023	1.10	0.040	1.17	0.058	1.32	0.120	1.53	0.200	1.74	0.290
20°	1.10	0.032	1.17	0.056	1.23	0.081	1.44	0.160	1.70	0.280	2.00	0.405
25°	1.11	0.044	1.20	0.074	1.30	0.108	1.60	0.220	1.90	0.370	2.28	0.540
30°	1.15	0.057	1.29	0.096	1.41	0.141	1.72	0.285	2.15	0.480	2.60	0.705
60°	1.88	0.346	2.60	0.544	2.70	0.761	4.4	1.73	6.0	2.72	7.50	3.80
90°	16.0	10.14	80.0	60	220	204	67	50.7	410	300	1500	1020

Notes: See Table III and Figure 4 for explanation of symbols
The curve for horizontal forces on horizontal cylinders of Figure 4 were used to determine C

FIGURE 5 · FORCE PER FOOT OF PILE LENGTH VS. PHASE ANGLE VS. DEPTH VS. PILE DIAMETER FOR ASSUMED PROTOTYPE WAVE CONDITIONS will follow the same trends and will completely define the relationships between the single coefficient "C", the correlating modulus AD/V^2 and Reynold's Number. This will be true whether the waves are deep water waves or shallow water waves as long as the appropriate theories are used for computing the velocities and accelerations.

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APPENDIX

In order to check equation (17) the method of dimensional analysis has been used.

Dimensional analysis is a method by which a partial knowledge of a physical situation may be capitalized and put into available form. The kind of partial knowledge necessary is a knowledge of the general nature of the fundamental equations which govern the system and in addition, the nature of the boundary conditions which, together with the equations, determine the detailed solution in any special case. It is not required that the equations should be actually written out in detail; in fact, the utility of the method is largely in its application to problems so complicated that the fundamental equations could not be actually written down as, for example, in most practical problems of hydraulics.

Dimensional solutions do not yield numerical answers but they provide the form of the answer so that every experiment can be used to the fullest advantage in determining a general empirical solution.

Dimensional analysis rests on the basic principle that every equation which expresses a physical relationship must be "dimensionally homogeneous"; that is, that an equality can exist only between like quantities. This restriction, with the requirement that the ratio between two solutions must not change when the units used to express the magnitudes of the variables are altered, limits the form of physical equations by requiring that the dimensional variables involved can enter only in groups which are products of powers.

The problem at hand is to derive the modulus of which the single coefficient (C) is a function.

It is assumed that the wave force (F) caused by ocean waves on a pile is a function of:

$$\text{Force} = \phi\,(\text{length, viscosity, density, velocity, gravity, acceleration}) \qquad (18)$$

or:

$$F = \sum C \rho^{a} \; L^{b} \; V^{c} \mu^{d} \; g^{e} \; A^{f} \qquad (19)$$

where:

F = Force = MLT^{-2} and in units of Mass (M), Length (L) and Time (T).

ρ = Density = ML^{-3}

$$L = \text{length} = L \qquad (20)$$

$$V = \text{velocity} = LT^{-1}$$

$$\mu = \text{viscosity} = ML^{-1}T^{-1}$$

$$g = \text{gravity} = LT^{-2}$$

$$A = \text{acceleration} = LT^{-2}$$

and

a, b, c, d, e, f are unknown powers.

Substituting (20) into (19):

$$MLT^{-2} = (ML^{-3})^a \; (L)^b \; (LT^{-1})^c \; (ML^{-1}T^{-1})^d \; (LT^{-2})^e \; (LT^{-2})^f \qquad (21)$$

Grouping the terms:

$$(M) \quad 1 = a + d$$

$$(L) \quad 1 = -3a + b + c - d + e + f \qquad (22)$$

$$(T) \quad -2 = -c - d - 2e - 2f$$

from which

a = 1 - d
b = 2 - d + e + f
c = 2 - d - 2e - 2f
d -- cannot be determined (23)
e -- cannot be determined
f -- cannot be determined

Substituting (23) into (19):

$$F = \Sigma C \; \rho^{(1-d)} L^{(2-d+e+f)} V^{(2-d-2e-2f)} \; \mu^d \; g^e \; A^f \qquad (24)$$

which reduces to:

$$F = \rho L^2 V^2 \; \Sigma C \, \rho^{(-d)} L^{(-d+e+f)} \; V^{(-d-2e-2f)} \mu^d \; g^e \; A^f \qquad (25)$$

which, upon gathering of terms, gives:

$$F = \rho L^2 V^2 \; \Sigma C \left(\frac{\rho VL}{\mu}\right)^{-d}, \left(\frac{V^2}{\frac{L}{g}}\right)^{-e}, \left(\frac{AL}{\frac{2}{V}}\right)^{f} \qquad (26)$$

from which:

$$C = \phi \left[\left(\frac{\rho VL}{\mu}\right), \left(\frac{V^2}{gL}\right) \quad \left(\frac{AL}{V^2}\right) \right] \qquad (17)$$

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