DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

# LABORATORY INVESTIGATION OF THE VERTICAL RISE OF SOLITARY WAVES ON IMPERMEABLE SLOPES

TECHNICAL MEMORANDUM NO. 33

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# FOREWORD

This paper presents the results of an experimental study of the vertical rise of solitary waves on impermeable slopes conducted in the laboratory of the Beach Erosion Board. The test program, analysis of the data, and preparation of the report were accomplished by Jay V. Hall, Jr., of the Engineering Division and George M. Watts of the Research Division.

The testing for this study was conducted shortly before the beginning of World War II and contemplated additional testing was curtailed due to the war. The experimental data presented are not complete in terms of covering the desired range of conditions, however the results obtained are believed to merit publication at this time.

Tests using oscillatory waves in connection with wave run-up on various types of impermeable shore structures are currently being made for the Beach Erosion Board at the Waterways Experiment Station of the Corps of Engineers, Vicksburg, Mississippi and additional laboratory studies of wave run-up on impermeable slopes as a result of tsunamis are to be undertaken soon by the Beach Erosion Board. The Waterways Experiment Station tests and contemplated tsunami study are in effect a continuation of the study presented herein.

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# LABORATORY INVESTIGATION OF THE VERTICAL RISE OF SOLITARY WAVES ON IMPERMEABLE SLOPES

## INTRODUCTION

1. Purpose. The maximum elevation of wave run-up is an important parameter that must be considered in the design of shore line structures. This parameter is especially significant for coastal protective works having a sloping seaward face. The condition in which the slope is blanketed with a permeable material has been studied to some extent, but little information is available for the case where the sloping surface is impermeable. This is particularly true for such waves as channel surges, flood waves, and tsunamis, all of which have essentially the nature of solitary waves. As the problem in those cases is to provide economical protection against one large wave, the major portion of the structure may not be subjected to waves of normal or even storm intensity. Factual data on the height that waves will rise on an impermeable slope will aid the coastal engineer in arriving at the maximum elevation of this type structure and help assess the volume of flooding landward of the structure during extreme wave conditions.

2. The purpose of this laboratory study is to determine and evaluate the relationships between wave height, water depth, impermeable slope angle, and the maximum elevation reached by a wave running up the slope. The investigation was carried out by employing a solitary type wave, therefore the data presented cannot necessarily be extrapolated to the case of a train of oscillatory waves. Incidental data relative to wave velocity became available during the tests.

3. Procedure. It was realized that the use of a train of oscillatory waves in this particular study would result in a complex water surface pattern throughout the wave tank, since much of the generated or incident wave energy would be reflected from the impermeable slope in the form of waves, and this condition would make it difficult to obtain reliable test measurements of the generated waves. The problem, therefore, was confined to the properties of a solitary type wave and by considering the influencing variables, the various parameters could first be derived through dimensional analysis. Utilizing the dimensionless parameters as an index, the investigation then involved evaluating these parameters by setting up the experimental apparatus for generating the solitary wave, establishing slopes, and developing the necessary instrumentation for obtaining pertinent data.

4. The laboratory apparatus permitted testing with the following ranges of conditions:

| | |
|---|---|
| Slopes - (degrees) | = 5 to 45 |
| Still water depth - (feet) | = 0.50 to 2.25 |
| Wave heights - (feet) | = 0.005 to 0.5 |

The testing program was arranged so that two of the above variables could be held constant and the third varied with each test, and wave run-up values measured for various combinations of conditions.

## DEVELOPMENT OF DIMENSIONLESS PARAMETERS

5. The variables involved in this study appeared to be:

Linear dimensions

- h = height of wave above still water immediately before becoming influenced by the slope
- d = still water depth
- R = wave run-up on slope and expressed as vertical height above still water
- L = length of wave, crest to crest
- S = slope of seaward face of structure expressed as the vertical angle in degrees between the structure face and the horizontal

Kinematic and dynamic quantities

- C = velocity of travel of wave crest
- $E_T$= total potential and kinetic energy of wave above still water

Physical properties of the fluid

- $\rho$ = density of the fluid
- $\mu$ = dynamic viscosity of the fluid

Utilizing Buckingham's Pi Theorem (1)* the following general equation can be written

$$f\ (h, d, R, L, S, C, E_T, \rho, \mu) = 0 \qquad (1)$$

If each term is expressed in dimensional units of length L, time T, and Mass M, there will be six dimensionless $\pi$ terms in the function.

$$f'\ (\pi_1, \pi_2, \pi_3, ------\pi_6) = 0 \qquad (2)$$

Selecting variables of length d, velocity C, and density , with the remaining terms appearing singly in each group with a negative exponent, it follows

$$\pi_1 = d^{x1}\ C^{y1}\ \rho^{z1}\ h^{-1} \qquad \pi_4 = d^{x4}\ C^{y4}\ \rho^{z4}\ R^{-1}$$

$$\pi_2 = d^{x2}\ C^{y2}\ \rho^{z2}\ L^{-1} \qquad \pi_5 = d^{x5}\ C^{y5}\ \rho^{z5}\ E_t^{-1}$$

$$\pi_3 = d^{x3}\ C^{y3}\ \rho^{x3}\ S^{-1} \qquad \pi_6 = d^{x6}\ C^{y6}\ \rho^{z6}\ \mu^{-1}$$

Expressing each term in dimensional units of M-L-T and solving values for the exponents, there results

$$\pi_1 = d\, C^o\, \rho^o\, h^{-1} = d/h \qquad \pi_4 = d\, C^o\, \rho^o\, R^{-1} = d/R$$

$$\pi_2 = d\, C^o\, \rho^o\, L^{-1} = d/L \qquad \pi_5 = d^2\, C^2\, \rho\, E^{-1} = d^2\, C^2\, \rho/E_t$$

$$\pi_3 = d^o\, C^o\, \rho^o\, S = S \qquad \pi_6 = d\, C\, \rho\, \mu^{-1} = d\, C\, \rho/\mu$$

and

$$\pi_4 = \varphi\, (\pi_1, \pi_2, \pi_3, \pi_5, \pi_6) \tag{3}$$

Substituting, their results

$$R/d = K \left[ (h/d)^a,\ (L/d)^b,\ (S)^c,\ \left(\frac{E_t}{d^2\, C^2 \rho}\right)^d,\ \left(\frac{\mu}{d\, C\, \rho}\right)^e \right] \tag{4}$$

If it is assumed that the wave height and still water depth are the dominant characteristics of the solitary wave and the wave length is infinite, the $\pi_2$ term can be eliminated. Test data (3) (5) illustrate that for a solitary wave the velocity of wave propagation is a function of the still water depth and wave height, and the total potential and kinetic energies of the wave form above the still water level are approximately equal; these relationships being

$$C^2 = gd \tag{5}$$

$$E_T = 1.54\ \rho\, g\, h^{3/2}\, d^{3/2} \tag{6}$$

where g is the acceleration of gravity.

If equations (5) and (6) are considered in the $\pi_5$ term, the term can be reduced to $K\ (h/d)^d$ which is identical to the $\pi_1$ term; thus showing that the energy parameter is automatically handled when evaluating the h/d parameter. It is believed viscous effects would be negligible in this study which would eliminate the evaluation of the Reynolds number or $\pi_6$. With these considerations the function to be studied reduces to

$$R/d = K \left[ (h/d)^a\ (S)^c \right] \tag{7}$$

where the constant K and exponents a, and c can be evaluated by test data.

6. In evaluating the above parameters, certain relationships of a solitary wave must be considered. In general, the solitary wave is a wave of translation and can be considered essentially irrotational. If friction effects are not considered and the depth is constant, the wave has a permanent form and constant velocity in the direction of propagation with

a theoretically infinite wave length. The fluid particles are positively displaced in the passage of this type of wave; the particle motion being that of acceleration, from zero to maximum velocity (wave crest), and deceleration back to zero after passage of the wave crest. The positive horizontal displacement of the fluid particles is proportional to the wave volume. This motion differs from fluid motion in oscillatory waves in that oscillatory waves are periodic and move the fluid particles in essentially a circular or elliptical path, with no appreciable positive displacement. Various degrees of approximation have been presented in published literature concerning theoretical solutions for the actions of the solitary wave by use of the dynamical equations of motion in an infinite series. The essential solitary wave characteristics can be described by the wave height and still water depth, (2), (3), (4). These are as follows:

Wave profile

$$h_x = h \operatorname{sech}^2 \left[ \left(\frac{3h}{4d^3}\right)^{1/2} (x - Ct) \right] \tag{8}$$

where

$h_x$ = height of surface profile above still water at any point x

h = maximum height above still water

d = still water depth

x = horizontal displacement of point of measurement from vertical axis of reference

C = velocity of wave travel

t = time elapsed since passage of wave crest past vertical axis of reference

Velocity of wave propagation

$$C^2 = gd \tag{5}$$

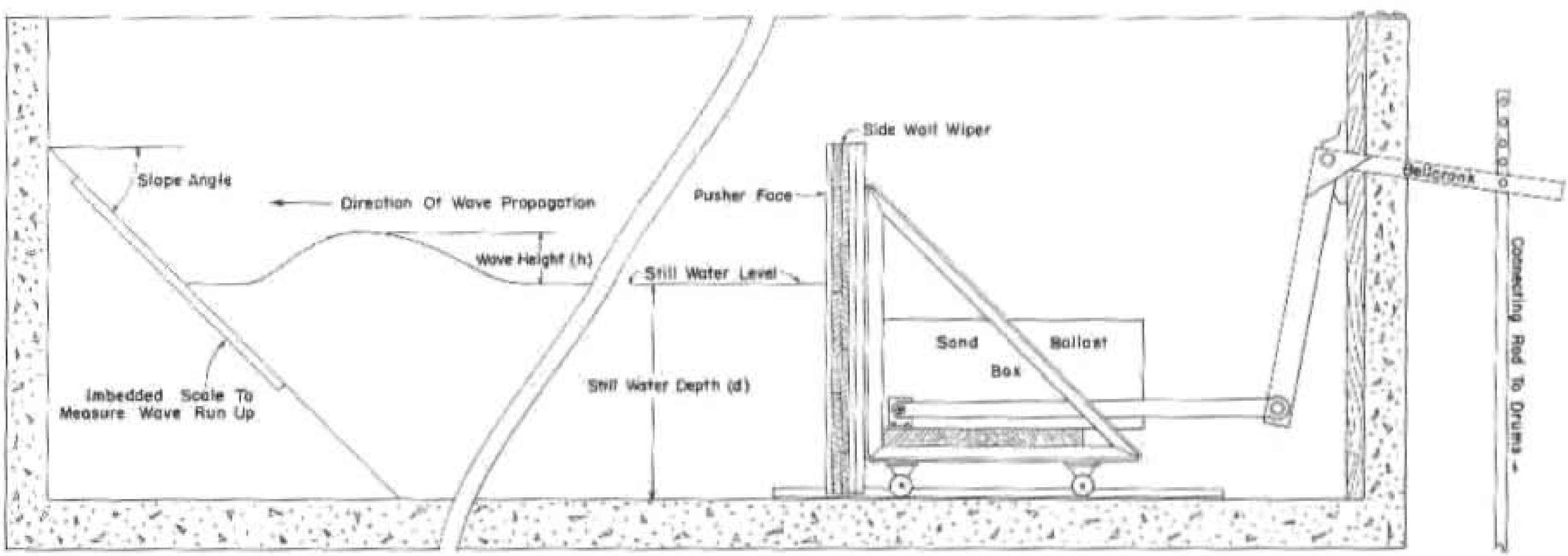
Total wave energy

$$E_T = 1.54 \ \rho g h^{3/2} \ d^{3/2} \tag{6}$$

The dimensionless parameters developed in paragraph 5 can be evaluated, by use of the above relationships and experimental data of wave height, still water depth, and wave run-up.

## APPARATUS AND PROCEDURE

7. Wave Tank. The wave tank used for this investigation was a concrete tank 85 feet long, 14 feet wide and 4 feet deep. The wave generator was located at one end of the tank and the impermeable slope at the other. Figure 1 is a schematic diagram of the laboratory apparatus.




SCHEMATIC DIAGRAM OF WAVE TANK AND APPURTENANCES

Fig. 1

8. A pusher type wave generator was constructed for this study which produced a positive uniform horizontal motion in any depth of water. A water ballast tank mechanically linked through a bell crank to the pusher was raised by a winch to a specific elevation. When the brake was tripped the downward motion of the drums transmitted through the linkage arrangement imparted a horizontal movement of the pusher face. Waves of various heights could be generated and reproduced by adjustment of the weight and fall of drums.

9. The impermeable slope was constructed of 1/4-inch masonite on a wooden support which included tension and compression members to check a "breathing" type of movement of the slope under wave action. All surface joints on the slope, and the space between the slope and walls of the wave tank were filled with an okum compound and covered with a rubber varnish to insure water tight conditions. A system of hand screws attached to the slope permitted adjustment of the various slope angles and also adjustment of the slope to maintain a constant distance between the wave generator face and the point of intersection of the still water level with the slope. The toe of the slope was secured to the tank floor before each test.

10. Wave velocity measurements were made by use of a key positioned at each end of the tank and auxiliary equipment. The closure of the key by the wave energized a solenoid which operated a stop watch. Wave heights at the toe of the slope were recorded by visual observation as the wave passed an observation window equipped with an etched scale. Still water depth measurements were made with point gages. The limit of wave run-up on the slope was read visually from a scale laid along the slope and the vertical rise on the slope was computed therefrom. Adjustments of the still water level values were made because of modification of levels by displacement of the pusher face, however the maximum correction was only in the order of one percent.

11. <u>Test Conditions and Procedure</u>. The slope angles selected were 5, 10, 15, 25, and 45 degrees. Still water depths were 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, and 2.25 feet. The throws on the pusher, to create wave heights up to 0.5 foot, were 0.50, 0.75, 1.00, and 1.25 feet. Combinations of the above values of variables required approximately 160 tests. Basically the test procedure was:

(a) adjustment of slope angle and position;

(b) adjustment of throw on wave generator;

(c) adjustment of still water level;

(d) setting of instruments for automatic recording of data pertinent to wave velocity;

(e) release of brake to generate wave;

(f) Observation of wave height and wave run-up.

TABLE 1
OBSERVED DATA AND VALUES OF h/d AND R/d

h = wave height          d = still water depth          R = wave run-up   (all in feet)

**5° Slope**

| h | d | R | h/d | R/d |
|---|---|---|---|---|
| .312 | 2.25 | .913 | .139 | .406 |
| .242 | 2.25 | .761 | .107 | .338 |
| .172 | 2.25 | .545 | .076 | .242 |
| .112 | 2.25 | .400 | .050 | .178 |
| .312 | 2.00 | .922 | .156 | .461 |
| .242 | 2.00 | .738 | .121 | .367 |
| .172 | 2.00 | .547 | .086 | .274 |
| .092 | 2.00 | .392 | .046 | .196 |
| .302 | 1.75 | .907 | .178 | .518 |
| .232 | 1.75 | .735 | .132 | .420 |
| .172 | 1.75 | .571 | .098 | .326 |
| .107 | 1.75 | .391 | .061 | .223 |
| .272 | 1.50 | .813 | .181 | .542 |
| .202 | 1.50 | .637 | .134 | .424 |
| .212 | 1.50 | .645 | .141 | .430 |
| .142 | 1.50 | .472 | .095 | .314 |
| .092 | 1.50 | .321 | .061 | .214 |
| .262 | 1.25 | .749 | .210 | .599 |
| .212 | 1.25 | .646 | .170 | .517 |
| .152 | 1.25 | .488 | .122 | .390 |
| .092 | 1.25 | .314 | .074 | .252 |
| .212 | 1.00 | .603 | .212 | .603 |
| .172 | 1.00 | .528 | .172 | .528 |
| .112 | 1.00 | .360 | .112 | .360 |
| .062 | 1.00 | .221 | .062 | .221 |
| .192 | 0.75 | .509 | .256 | .680 |
| .156 | 0.75 | .442 | .208 | .590 |
| .152 | 0.75 | .419 | .203 | .559 |
| .117 | 0.75 | .378 | .156 | .208 |
| .102 | 0.75 | .329 | .136 | .438 |
| .092 | 0.75 | .327 | .123 | .436 |
| .062 | 0.75 | .203 | .083 | .271 |
| .222 | 0.50 | .556 | .444 | 1.112 |
| .152 | 0.50 | .360 | .304 | .720 |
| .122 | 0.50 | .296 | .244 | .593 |
| .102 | 0.50 | .294 | .204 | .588 |
| .067 | 0.50 | .189 | .134 | .378 |

**10° Slope**

| h | d | R | h/d | R/d |
|---|---|---|---|---|
| .322 | 2.25 | .969 | .143 | .430 |
| .242 | 2.25 | .728 | .107 | .323 |
| .172 | 2.25 | .500 | .076 | .222 |
| .162 | 2.25 | .516 | .072 | .229 |
| .112 | 2.25 | .329 | .050 | .146 |
| .302 | 2.00 | 1.080 | .151 | .540 |
| .242 | 2.00 | .706 | .121 | .353 |
| .182 | 2.00 | .509 | .091 | .254 |
| .172 | 2.00 | .509 | .086 | .254 |
| .112 | 2.00 | .308 | .056 | .154 |
| .292 | 1.75 | .969 | .167 | .553 |
| .232 | 1.75 | .710 | .132 | .405 |
| .192 | 1.75 | .937 | .110 | .535 |
| .182 | 1.75 | .517 | .104 | .296 |
| .172 | 1.75 | .514 | .098 | .294 |
| .112 | 1.75 | .330 | .064 | .188 |
| .267 | 1.50 | .889 | .178 | .592 |
| .252 | 1.50 | .889 | .168 | .592 |
| .192 | 1.50 | .647 | .128 | .431 |
| .142 | 1.50 | .457 | .095 | .304 |
| .132 | 1.50 | .457 | .088 | .304 |
| .102 | 1.50 | .304 | .068 | .203 |
| .262 | 1.25 | .839 | .210 | .671 |
| .202 | 1.25 | .641 | .162 | .513 |
| .142 | 1.25 | .477 | .114 | .382 |
| .092 | 1.25 | .293 | .074 | .235 |
| .257 | 1.00 | .836 | .257 | .836 |
| .252 | 1.00 | .807 | .252 | .807 |
| .142 | 1.00 | .488 | .142 | .488 |
| .137 | 1.00 | .473 | .137 | .473 |
| .102 | 1.00 | .679 | .102 | .679 |
| .082 | 1.00 | .261 | .082 | .261 |
| .222 | 0.75 | .719 | .296 | .960 |
| .182 | 0.75 | .585 | .243 | .780 |
| .122 | 0.75 | .425 | .163 | .567 |
| .082 | 0.75 | .251 | .109 | .335 |
| .212 | 0.50 | .625 | .424 | 1.250 |
| .162 | 0.50 | .460 | .324 | .920 |
| .102 | 0.50 | .312 | .204 | .624 |
| .067 | 0.50 | .243 | .134 | .486 |

**15° Slope**

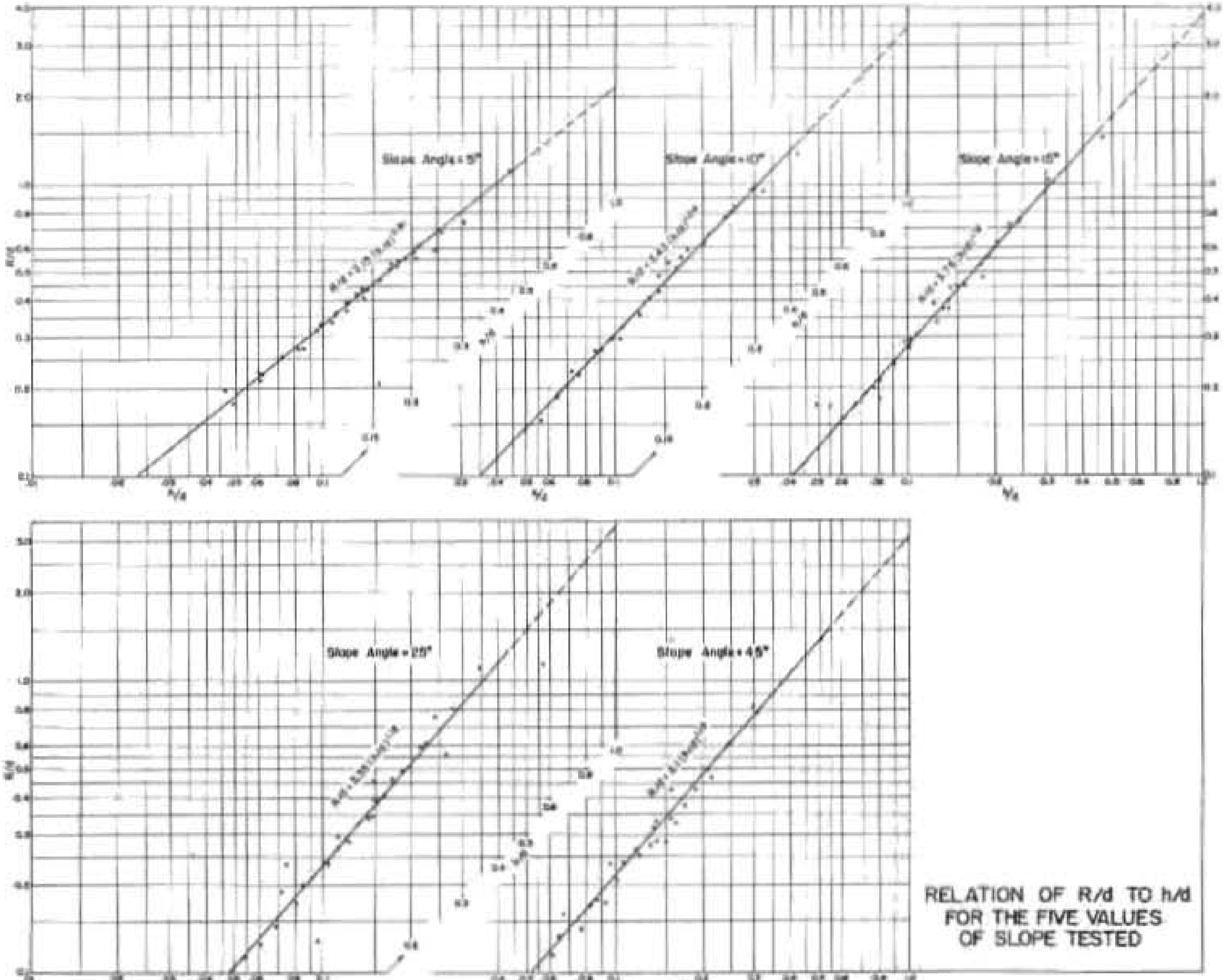
| h | d | R | h/d | R/d |
|---|---|---|---|---|
| .302 | 2.25 | .848 | .134 | .377 |
| .232 | 2.25 | .644 | .103 | .286 |
| .182 | 2.25 | .476 | .081 | .211 |
| .172 | 2.25 | .447 | .076 | .199 |
| .162 | 2.25 | .435 | .072 | .193 |
| .122 | 2.25 | .388 | .054 | .173 |
| .112 | 2.25 | .388 | .050 | .173 |
| .312 | 2.00 | .901 | .156 | .450 |
| .252 | 2.00 | .673 | .126 | .336 |
| .182 | 2.00 | .479 | .091 | .240 |
| .122 | 2.00 | .311 | .061 | .156 |
| .322 | 1.75 | .925 | .181 | .529 |
| .242 | 1.75 | .686 | .139 | .392 |
| .177 | 1.75 | .492 | .101 | .281 |
| .172 | 1.75 | .505 | .098 | .288 |
| .122 | 1.75 | .311 | .070 | .178 |
| .282 | 1.50 | .841 | .188 | .561 |
| .222 | 1.50 | .664 | .148 | .442 |
| .162 | 1.50 | .459 | .108 | .306 |
| .102 | 1.50 | .268 | .068 | .179 |
| .282 | 1.25 | .880 | .225 | .704 |
| .222 | 1.25 | .660 | .178 | .528 |
| .172 | 1.25 | .466 | .138 | .373 |
| .102 | 1.25 | .228 | .081 | .183 |
| .242 | 1.00 | .739 | .242 | .739 |
| .202 | 1.00 | .601 | .202 | .601 |
| .142 | 1.00 | .440 | .142 | .440 |
| .102 | 1.00 | .272 | .102 | .272 |
| .232 | 0.75 | .763 | .310 | 1.018 |
| .182 | 0.75 | .569 | .243 | .759 |
| .142 | 0.75 | .421 | .189 | .561 |
| .077 | 0.75 | .220 | .102 | .294 |
| .232 | 0.50 | .719 | .464 | 1.438 |
| .152 | 0.50 | .486 | .304 | .973 |
| .112 | 0.50 | .362 | .224 | .724 |
| .102 | 0.50 | .311 | .204 | .622 |
| .062 | 0.50 | .194 | .124 | .388 |

**25° Slope**

| h | d | R | h/d | R/d |
|---|---|---|---|---|
| .322 | 2.25 | .751 | .143 | .334 |
| .252 | 2.25 | .570 | .112 | .254 |
| .182 | 2.25 | .391 | .081 | .174 |
| .122 | 2.25 | .254 | .054 | .113 |
| .312 | 2.00 | .782 | .156 | .391 |
| .242 | 2.00 | .570 | .121 | .285 |
| .172 | 2.00 | .401 | .086 | .200 |
| .122 | 2.00 | .254 | .061 | .127 |
| .262 | 1.75 | .803 | .150 | .459 |
| .232 | 1.75 | .570 | .132 | .326 |
| .182 | 1.75 | .412 | .104 | .236 |
| .122 | 1.75 | .254 | .070 | .145 |
| .282 | 1.50 | .740 | .188 | .493 |
| .212 | 1.50 | .518 | .141 | .347 |
| .152 | 1.50 | .359 | .101 | .239 |
| .112 | 1.50 | .354 | .075 | .236 |
| .272 | 1.25 | .697 | .265 | .557 |
| .202 | 1.25 | .507 | .162 | .406 |
| .152 | 1.25 | .349 | .122 | .279 |
| .222 | 1.00 | .612 | .222 | .612 |
| .172 | 1.00 | .465 | .172 | .465 |
| .112 | 1.00 | .296 | .112 | .295 |
| .072 | 1.00 | .190 | .072 | .190 |
| .212 | 0.75 | .602 | .283 | .803 |
| .162 | 0.75 | .444 | .216 | .592 |
| .112 | 0.75 | .296 | .149 | .395 |
| .072 | 0.75 | .148 | .096 | .198 |
| .282 | 0.50 | .908 | .564 | 1.130 |
| .172 | 0.50 | .549 | .344 | 1.110 |
| .122 | 0.50 | .380 | .244 | .760 |
| .072 | 0.50 | .169 | .144 | .338 |

**45° Slope**

| h | d | R | h/d | R/d |
|---|---|---|---|---|
| .332 | 2.25 | .636 | .147 | .282 |
| .242 | 2.25 | .460 | .107 | .204 |
| .172 | 2.25 | .318 | .076 | .141 |
| .122 | 2.25 | .212 | .054 | .098 |
| .312 | 2.00 | .654 | .156 | .322 |
| .242 | 2.00 | .513 | .121 | .256 |
| .172 | 2.00 | .354 | .086 | .177 |
| .122 | 2.00 | .230 | .061 | .115 |
| .302 | 1.75 | .654 | .172 | .374 |
| .232 | 1.75 | .478 | .132 | .273 |
| .162 | 1.75 | .301 | .092 | .172 |
| .112 | 1.75 | .230 | .064 | .132 |
| .282 | 1.50 | .636 | .188 | .424 |
| .207 | 1.50 | .424 | .138 | .283 |
| .152 | 1.50 | .318 | .101 | .212 |
| .097 | 1.50 | .195 | .065 | .130 |
| .262 | 1.25 | .619 | .210 | .495 |
| .192 | 1.25 | .424 | .154 | .340 |
| .132 | 1.25 | .301 | .106 | .241 |
| .102 | 1.25 | .212 | .082 | .170 |
| .242 | 1.00 | .601 | .242 | .601 |
| .212 | 1.00 | .460 | .212 | .467 |
| .117 | 1.00 | .265 | .117 | .265 |
| .067 | 1.00 | .159 | .067 | .159 |
| .222 | 0.75 | .661 | .296 | .803 |
| .182 | 0.75 | .460 | .243 | .613 |
| .102 | 0.75 | .247 | .136 | .330 |
| .072 | 0.75 | .177 | .096 | .236 |
| .252 | 0.50 | .654 | .504 | 1.308 |
| .152 | 0.50 | .389 | .304 | .778 |
| .072 | 0.50 | .212 | .154 | .424 |
| .067 | 0.50 | .159 | .134 | .318 |




RELATION OF R/d TO h/d FOR THE FIVE VALUES OF SLOPE TESTED

Fig. 2

## ANALYSIS OF RESULTS

12. Wave Run-up. The data for all test runs on each slope angle are given in Table 1. The tabulation includes the measured wave height above still water h, the measured still water depth d, the vertical component of wave run-up R, and the various dimensionless ratios needed to investigate the relationships developed in paragraph 5.

13. Figure 2 shows plots of the relationship $R/d = K(h/d)^a$ for each slope used. The data as plotted on log-log paper indicate that a straight line can be drawn through the points. The equation of this line is of the form as the above relationship. Table 2 gives the constant K and the exponent a for this relationship as associated with each slope angle. Although the arithmetical differences of K with various slope angles are not great, there is sufficient evidence to indicate that the slope influences the relationship. This evidence can also be noted in the small changes of the

TABLE 2
Relationship $R/d = K(h/d)^a$

| Slope Angle | | | |
|---|---|---|---|
| degrees | tangent | K | a |
| 5 | 0.087 | 2.15 | 0.810 |
| 10 | 0.176 | 3.43 | 1.042 |
| 15 | 0.268 | 3.75 | 1.120 |
| 25 | 0.466 | 3.35 | 1.123 |
| 45 | 1.000 | 3.10 | 1.150 |

exponent a for the various slope angles. Figure 3 is a plot of the values of K and a against the tangent of the slope angle. As can be seen there appears to be a change in relationship near a slope angle of 12 degrees. For the limits of slope angle between 5 and 12 degrees

$$K = 11\ S^{0.67} \quad (9)$$

$$a = 1.9\ S^{0.35} \quad (10)$$

and for the limits of slope angle between 12 and 45 degrees

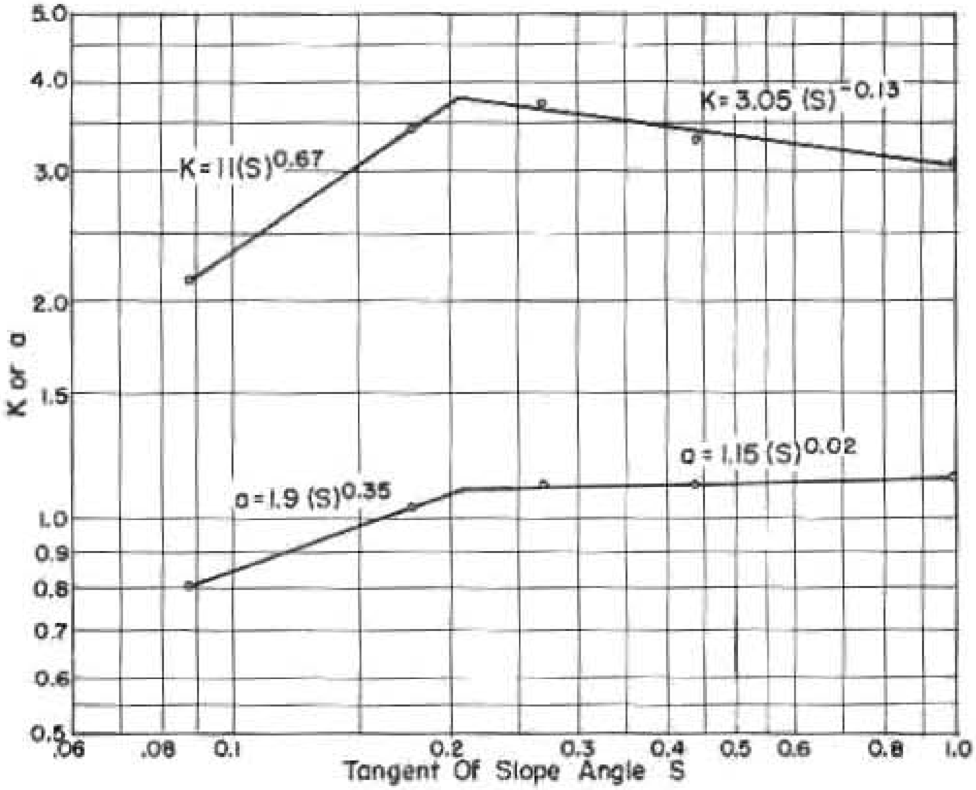
$$K = 3.05\ S^{-0.13} \quad (11)$$

$$a = 1.15\ S^{0.02} \quad (12)$$

When these values are substituted into the derived relationship there results for 5 to 12 degrees

$$R/d = 11\ S^{0.67}\ (h/d)^{1.9S^{0.35}} \quad (13)$$

$$R/d = 3.05\ S^{-0.13}\ (h/d)^{1.15S^{0.02}} \quad (14)$$




RELATIONS OF K AND a TO TANGENT OF SLOPE ANGLE

Fig. 3

Although it cannot be stated precisely why this change takes place at approximately the 12 degree slope, it is probable that the flatter slopes introduce frictional effects between the toe of the slope and the intersection of the still water level with the slope face which reduce the wave run-up. When the slope is steepened, the bottom distance between the toe of the slope and the intersection of the still water level with the slope is reduced, which tends to reduce the frictional effects. For wave run-up on steeper slopes, the weight and inertia of the wave becomes the predominant factors rather than the bottom frictional effects.

14. In connection with another study (5), tests were made to study the reflection of energy of a solitary wave from permeable slopes and as a phase of this study some tests were made with impermeable slopes. The impermeable slope tests were not extensive, however there was an indication that the reflected energy did not follow the same basic law for all slope angles. Indications were that the relationship changed near the 37-degree slope. The data of these tests and of the report herein cannot be readily interplotted to shed more light on this fact, however, the two sets of data illustrate that there is a change of relationship with slope which is probably due to a change in the proportion of energy lost by friction. It then appears that in this investigation for the range of conditions tested, the dissipation of wave energy is at a minimum around the 12 degree slope; consequently the highest values of the vertical component of wave run-up will occur.

15. It is to be noted that if the relationship $R/d = K(h/d)^{a}$ is multiplied by d/h there results

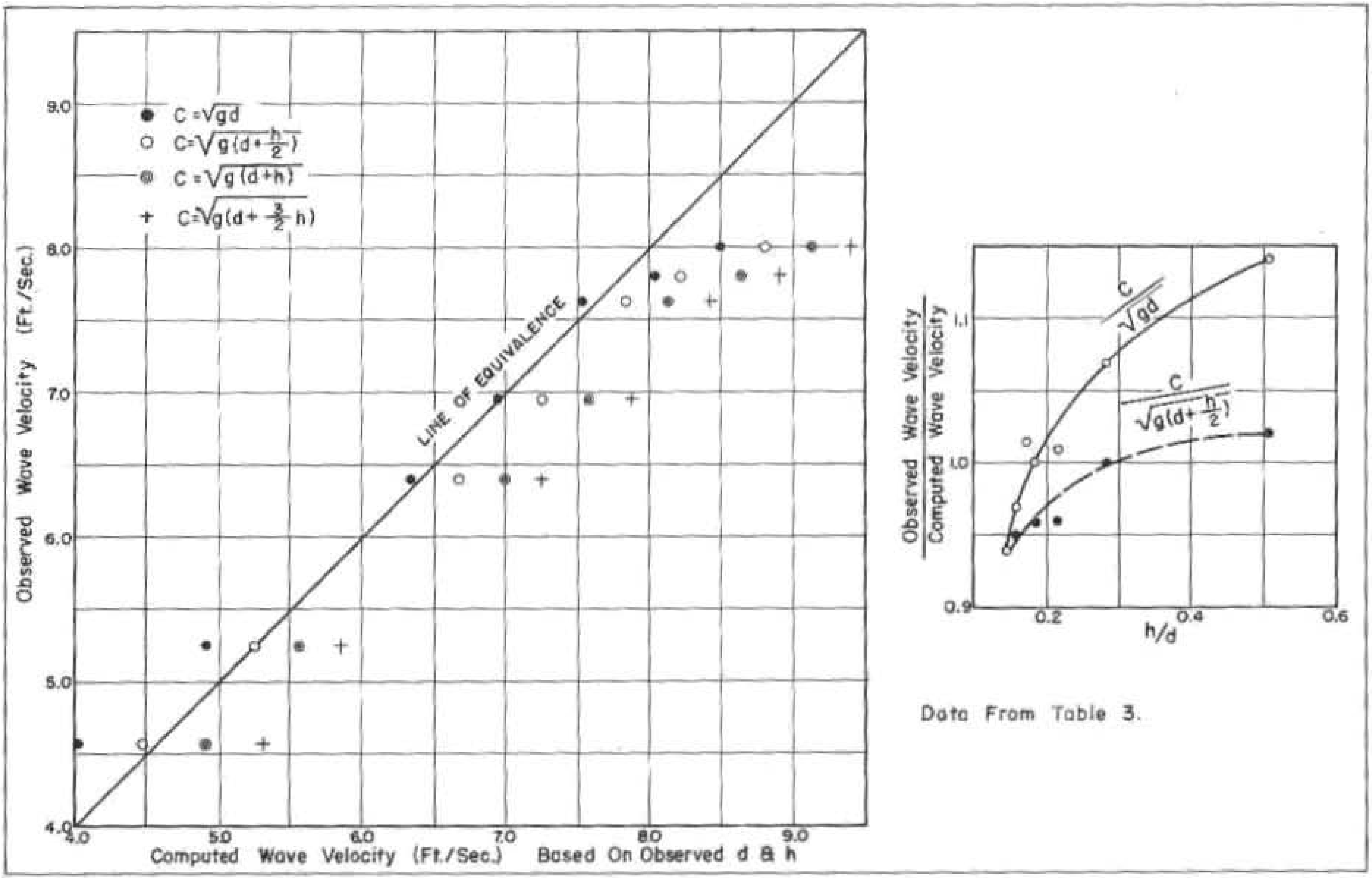
$$R/h = K(h/d)^{a-1} \qquad (15)$$

If the evaluated constant and exponent as found in the preceding paragraph are substituted, there results

$$\text{for 5 to 12 degree slope } R/h = 11(S)^{0.67}\,(h/d)^{1.9(S)^{0.35}-1} \qquad (16)$$

$$\text{for 12 to 45 degree slope } R/h = 3.05(S)^{-0.13}(h/d)^{1.15(S)^{0.02}-1} \qquad (17)$$

16. Wave velocity. Comparisons of measured and computed wave velocities based on the measured still water level, d, and wave heights above still water, h, are given in Table 3. Data from this table are shown graphically in Figure 4. In comparing and computed velocities, the Lagrange velocity law (3) $C = \sqrt{gd}$ indicates good agreement for velocities between 6 and 8 feet per second. Below 6 feet per second this law requires a factor to provide agreement between observed and computed values this factor being a function of the wave height. The limits of d and h in the wave tank would not permit the generation of waves with velocities much greater than 8 feet per second. For wave velocities less than 6 feet per second the more exact relation $C = \sqrt{g(d + h/2)}$ appears to provide the best agreement, and holds true until the velocity is in the order of 4 feet per second which is the lower limit of the data. At this value, it can be noted that the




COMPARISON OF OBSERVED AND COMPUTED WAVE VELOCITIES
Fig. 4

relation $C = \sqrt{g(d+h)}$ approaches the line of equivalence. These relationships and test data indicate that if the curvature of the wave surface is small, the velocity of propagation is closely related to $\sqrt{gd}$ whereas if the curvature or wave height is large this relationship must be adjusted to include the effect of wave height. In terms of error in using these relationships for this investigation it can be seen by another plot in Figure 4 that for h/d values between 0.25 and 0.50 the maximum error in using $C = \sqrt{g(d+h/2)}$ is $\pm$ 2.5 percent, and for h/d values between 0.15 and 0.24 the maximum error in using $C = \sqrt{gd}$ is $\pm$ 5.0 percent.

TABLE 3
COMPARISON OF MEASURED WAVE PROPAGATED
VELOCITY (C) AND COMPUTED VELOCITY BASED ON MEASURED h AND d

| Measured h ft. | Measured d ft. | Measured C ft./sec. | $C^2$ ft²/sec² | $C^2 = gd$ | $C^2 = g(d+\frac{h}{2})$ | $C^2 = g(d+h)$ | $C^2 = (d+\frac{3}{2}h)$ |
|---|---|---|---|---|---|---|---|
| 0.332 | 2.25 | 8.00 | 64.00 | 72.5 | 77.7 | 83.2 | 88.5 |
| 0.312 | 2.00 | 7.80 | 60.90 | 64.4 | 67.8 | 74.4 | 79.5 |
| 0.302 | 1.75 | 7.62 | 58.10 | 56.3 | 61.2 | 66.0 | 71.0 |
| 0.282 | 1.50 | 6.96 | 48.50 | 48.3 | 52.8 | 57.3 | 62.0 |
| 0.272 | 1.25 | 6.40 | 41.00 | 40.2 | 44.6 | 49.0 | 52.3 |
| 0.212 | 0.75 | 5.25 | 27.60 | 24.1 | 27.5 | 30.9 | 34.3 |
| 0.252 | 0.50 | 4.57 | 20.85 | 16.1 | 20.1 | 24.2 | 28.2 |

SUMMARY

17. The general equation for wave run-up as derived in paragraph 5 is

$$R/d = K \left[ (h/d)^a,\ (L/d)^b,\ (S)^c,\ \left(\frac{E_T}{d^2 C^2 \rho}\right)^d,\ \left(\frac{\mu}{d\ c\rho}\right)^e \right]$$

Preliminary investigations and an analysis of the quantities involved as related to test data, suggested that only two of the above parameters need be considered; namely,

$$R/d = K \left[ (h/d)^a,\ (S)^c \right]$$

The data indicated that there was relationship common to all factors involved; however the relationship was not constant within the limits tested. The relationships determined for different slope angles were:

a. between 5 and 12 degrees

$$R/d = 11(S)^{0.67} \quad (h/d)^{1.9(S)^{0.35}}$$

b. between 12 to 45 degrees

$$R/d = 3.05(S)^{-0.13} \ (h/d)^{1.15(S)^{0.02}}$$

18. In this investigation it was found that measured velocities of wave propagation agreed with computed velocities if adjustments were made for various limits of h/d values. The relationships giving the best agreement are as follows:

$C = \sqrt{gd}$ for h/d values between 0.15 and 0.24;

$C = \sqrt{g(d + h/2)}$ for h/d values between 0.25 and 0.50.

## CONCLUDING REMARKS

19. The relationships developed in this report are based on the results of tests utilizing a solitary or translatory type wave. The extrapolation of these results to the case of progressive oscillatory waves would be questionable.

20. No scale relationship was used in this study, however prototype assumptions can be made and applied to the developed relationships with the result being, it is believed, in the right order of magnitude. For example, for an essentially impervious earth dike with a slope of 1 on 1.5 and an h/d value of 0.5, it can be computed from the last equation in paragraph 17 that R = 2.92h. The function is such that the wave run-up decreases slightly as the slope approaches 1 on 1.

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