DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

WAVE FORECASTING RELATIONSHIPS FOR THE GULF OF MEXICO

TECHNICAL MEMORANDUM NO. 84

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DECEMBER 1956

FOREWORD

The Beach Erosion Board is issuing a series of reports providing wave statistics for selected regions of the coastal waters of the United States. Coverage of the coastal waters of Gulf of Mexico is being provided by a series of five reports, issued as Beach Erosion Board Technical Memoranda 85-89. The need for such data is evident, and it is planned ultimately to supply it by actual wave measurements for sufficiently long periods to establish the wave climate at many locations. However, with present instrumentation and funds, and the long period of record needed to supply adequate statistical data, such a field program for complete shore line coverage does not appear feasible at the present time. The compilation of wave statistics by "hindcast" techniques, though not as exact as recorded data, nevertheless will provide the engineer with better wave data than have heretofore been available.

This report presents the methods utilized in obtaining hindcast statistical wave data for locations in the Gulf of Mexico. It demonstrates a numerical method of forecasting wave generation and propagation over a sloping bottom taking into account both generation by the wind and dissipation by bottom friction. It also shows the averaging techniques used to apply this method to statistical accumulation of hindcast data in the Gulf.

This report was prepared by Charles L. Bretschneider, a hydraulic engineer in the Research Division of the Beach Erosion Board under the supervision of Joseph M. Caldwell, Chief of the Division. At the time the report was prepared, Brigadier General Theron D. Weaver was President of the Board.

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WAVE FORECASTING RELATIONSHIPS FOR THE GULF OF MEXICO

by
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INTRODUCTION

This report describes the development and application of a method for computing wind wave data over the Continental Shelf along the United States coast of the Gulf of Mexico. Methods for forecasting deep-water wind waves, and wind waves in shallow water of constant depth, such as shallow lakes and bays, are already available (8, 9, 12,16)*. The problem of the Continental Shelf of the Gulf of Mexico, however, is more complex, since here the waves may generate originally in deep water, but then, under the continued influence of wind, are propagated shoreward over the Continental Shelf where bottom friction and type of bottom profile become important. Because of the variety of initial deep-water conditions and bottom profiles, each location requires separate treatment. A set of generalized forecasting curves is required for each location and each direction to bring the waves in over the shallow sloping bottom to the desired depth. Using deep-water forecasting relationships and taking bottom friction into account, a generalized set of dimensionless forecasting relationships is prepared for each of the five locations (Figure 1) in the Gulf of Mexico for which statistical deep-water wave data are compiled (1, 2, 3, 4, 5). The forecasting curves are intended for the most frequent minimum fetch and corresponding wind speed for various deep-water wave height ranges and average bottom conditions of various directions. For the cases of winds parallel to the coast or from land to sea the curves are applicable to all water depths. However, for the case of winds blowing from sea toward land, the forecasting relationships are satisfactory only for depths of about 20 feet or greater, although the technique has been stretched to a depth of 12 feet for cases where winds are not too high. At depths of about 20 feet or less the bottom slope changes too rapidly for the theory to apply, and longer period swell will be breaking in the surf zone, thereby obscuring the wind wave pattern. Examples on the use of the forecasting graphs are given in Appendix A.

It must be emphasized that a certain amount of averaging is required to simplify the problem, and the forecasting curves must be used with discretion for other than nearby locations, where refraction becomes important. The forecasting curves result in wave heights to within ±10 percent of the average condition, except in the extreme case of shoal areas.

*Numbers in parentheses denote references listed at end of text.

Statistical distributions of wind waves from records (not yet completely analyzed) obtained in the Gulf of Mexico agree fairly closely to those presented by Putz (6) based on wave records for the Pacific Coast of the United States. Locations selected for developing forecasting relationships and for which statistical wave data are computed are shown in Figure 1. Wave statistics for these locations are given in the following Beach Erosion Board publications.

Wave Statistics for Gulf of Mexico	Technical Memorandum No.
Off Brownsville, Texas	85
Off Caplen, Texas	86
Off Burrwood, Louisiana	87
Off Apalachicola, Florida	88
Off Tampa Bay, Florida	89

BASIC EMPIRICAL RELATIONSHIPS FOR WIND-WAVE GENERATION

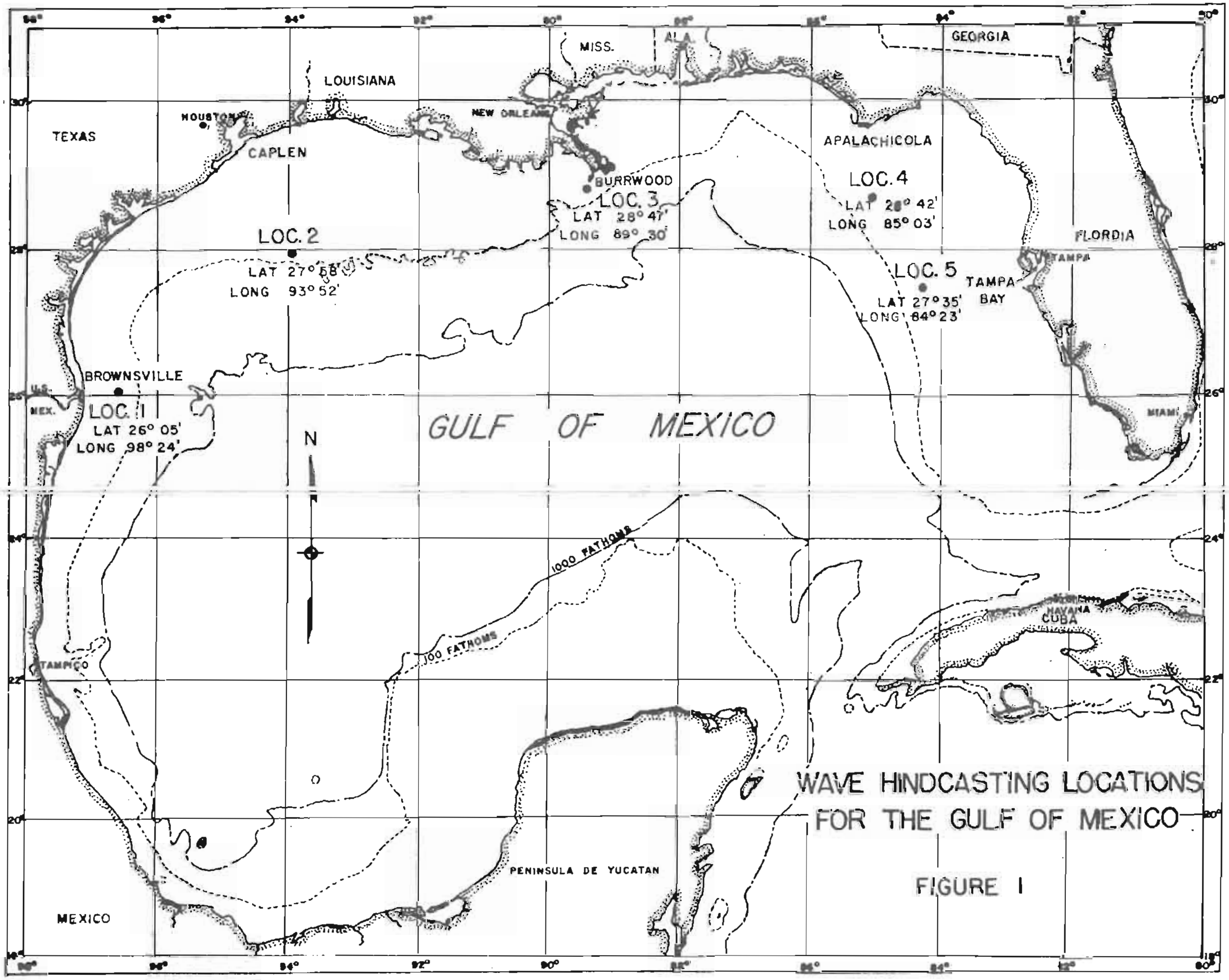
Empirical relationships for the generation of wind waves may be arrived at by use of the Buckingham Pi-theorem(7), supplemented by wave observations, provided the observations extend over a wide enough range of the variables. In general the variables to consider are as follows:

Variable	Symbol	Dimension (on the f, l, t system)
wind speed	U	l/t
wave height	H	l
wave period	T	t
wave length	L	l
wave speed	C	l/t
fetch length	F	l
wind duration	t	t
acceleration of gravity	g	l/t^2
depth of water	d	l
bottom slope	$\delta d/\delta x$	l/l
bottom friction factor	f	dimensionless
bottom percolation factor	p	l^2/t
refraction factor	K_R	dimensionless

In setting up the dimensionless relationships one can make use of the Airy wave theory (eq. 1 and 2) to eliminate the variables L and C.

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L} \qquad (1)$$

$$C^2 = \frac{gL}{2\pi} \tanh \frac{2\pi d}{L} \qquad (2)$$



WAVE HINDCASTING LOCATIONS FOR THE GULF OF MEXICO

FIGURE 1

Thus one obtains equations 3 and 4:

$$H = H\ (g, U, F, t, d, \delta d/\delta x, f, p, K_r) \tag{3}$$

$$T = T\ (g, U, F, t, d, \delta d/\delta x, f, p, K_r) \tag{4}$$

By use of the Pi-theorem it can be shown that:

$$gH/U^2 = \phi\left(\frac{gF}{U^2}, \frac{gt}{U}, \frac{gd}{U^2}, \delta d/\delta x, f, p, K_r\right)^* \tag{5}$$

$$\frac{gT}{U} = \psi\left(\frac{gF}{U^2}, \frac{gt}{U}, \frac{gd}{U^2}, \delta d/\delta x, f, p, K_r\right) \tag{6}$$

Equations 5 and 6 can be reduced to certain special cases, some of which are as follows:

a. Deep water, unlimited wind duration:

$$gH_o/U^2 = \phi_1\left(\frac{gF}{U^2}\right) \tag{5a}$$

$$gT/U = \psi_1\left(\frac{gF}{U^2}\right) \tag{6a}$$

b. Deep water, unlimited fetch length:

$$gH_o/U^2 = \phi_2\left(\frac{gt}{U}\right) \tag{5b}$$

$$gT/U = \psi_2\left(\frac{gt}{U}\right) \tag{6b}$$

c. Shallow water, impermeable flat bottom, constant bottom friction factor, and unlimited wind duration:

$$gH/U^2 = \phi_3\ (gF/U^2,\ gd/U^2) \tag{5c}$$

$$gT/U = \psi_3\ (gF/U^2,\ gd/U^2) \tag{6c}$$

d. Shallow water, impermeable flat bottom, constant bottom friction factor, and unlimited fetch length:

$$gH/U^2 = \phi_4\ (gt/U,\ gd/U^2) \tag{5d}$$

$$gT/U = \psi_4\ (gt/U,\ gd/U^2) \tag{6d}$$

*ϕ and ψ presents function notation.

e. Shallow water, impermeable flat bottom, constant bottom friction factor, unlimited fetch and unlimited wind duration:

$$gH/U^2 = \phi_5 \ (gd/U^2) \tag{5e}$$

$$gT/U = \psi_5 \ (gd/U^2) \tag{6e}$$

Expressions 5a, 5b, 6a, and 6b represent the deep-water wave generation relationships (Figure 2) of Bretschneider(8); expressions 5c, 5d, 5e, 6c, 6d, and 6e are strictly for shallow water with a bottom of constant depth, such as shallow lakes and bays.

Equations 5 and 6, applicable to the Continental Shelf of the Gulf of Mexico, are somewhat involved. One can simplify the work by considering the following:

a. In most cases wind waves in shallow water are limited either by the fetch length or the water depth at the end of the fetch, or a combination of a fetch length and water depth. Thus by introducing the concept of minimum fetch length the parameter gt/U can be eliminated.

b. If one limits the development to average conditions, the refraction coefficient k_r can be neglected temporarily, but applied later. This is perhaps more justifiable since it is difficult to determine refraction of short-crested waves of growing period under the continued influence of wind.

Assuming then that the bottom friction factor is constant, one obtains for the generation of wind waves over the Continental Shelf:

$$gH'_o/U^2 = \phi_6 \ (gF/U^2, \ gd/U^2, \ \text{bottom profile}) \tag{5f}$$

$$gT/U^2 = \psi_6 \ (gF/U^2, \ gd/U^2, \ \text{bottom profile}) \tag{6f}$$

where d is the depth of water at station of interest, and F is the minimum fetch from deep water to $d/L = 0.5$.

The term H'_o, equivalent deep-water significant wave height*, has been introduced in place of the actual significant wave height H_s, which can be obtained from:

$$H_s = H'_o \ K_s, \text{ where} \tag{7}$$

K_s is the shoaling factor given by Figure 7, which was prepared from tables by Wiegel(10).

*The term H'_o equivalent deep-water significant wave height, is defined as that height the shallow-water significant wave would have had in deep water if unaffected by refraction or energy losses or gains.

The introduction of H'_o simplifies numerical computations required to obtain the wave height in shallow water.

Theoretical Considerations. The discussion following is limited to that necessary for the understanding of a numerical procedure for computing wind waves in shallow water. Sverdrup and Munk(11) have presented a detailed theoretical development for generation of wind waves in deep water. The basic differential equation for the steady state condition is:

$$\frac{C}{2}\frac{\partial E}{\partial x} + \frac{E}{2}\frac{\partial C}{\partial x} = R_t \pm R_n, \text{ where} \tag{8}$$

C = deep-water wave speed
E = $1/8\ \rho g H^2$, wave energy
ρ = density
g = acceleration of gravity
H = wave height
R_t = rate at which energy is being transferred from wind to wave by tangential stress
R_n = rate at which energy is being transferred from wind to wave (or wave to wind) by normal pressure

$$R_t \pm R_n = R_H + R_c, \text{ where} \tag{9}$$

R_H = rate at which energy is transformed into wave height, and

R_c = rate at which energy is transferred into wave speed.

In a report by Bretschneider and Reid(12) a detailed development is presented for the change in wave height (of swell travelling over a shallow bottom) due to bottom friction, percolation and refraction, assuming no wind effect. The basic differential equation for the steady state condition is:

$$nc\frac{\partial (bE)}{\partial x} + Eb\frac{\partial (nc)}{\partial x} = -\left[D_f + D_p\right] b \tag{10}$$

where

E = $1/8\ \rho g H^2$, wave energy
C = wave speed
n = fraction of energy travelling with wave speed
b = perpendicular distance between two wave rays
D_f = rate at which energy is being lost due to bottom friction
D_p = rate at which energy is being lost due to percolation

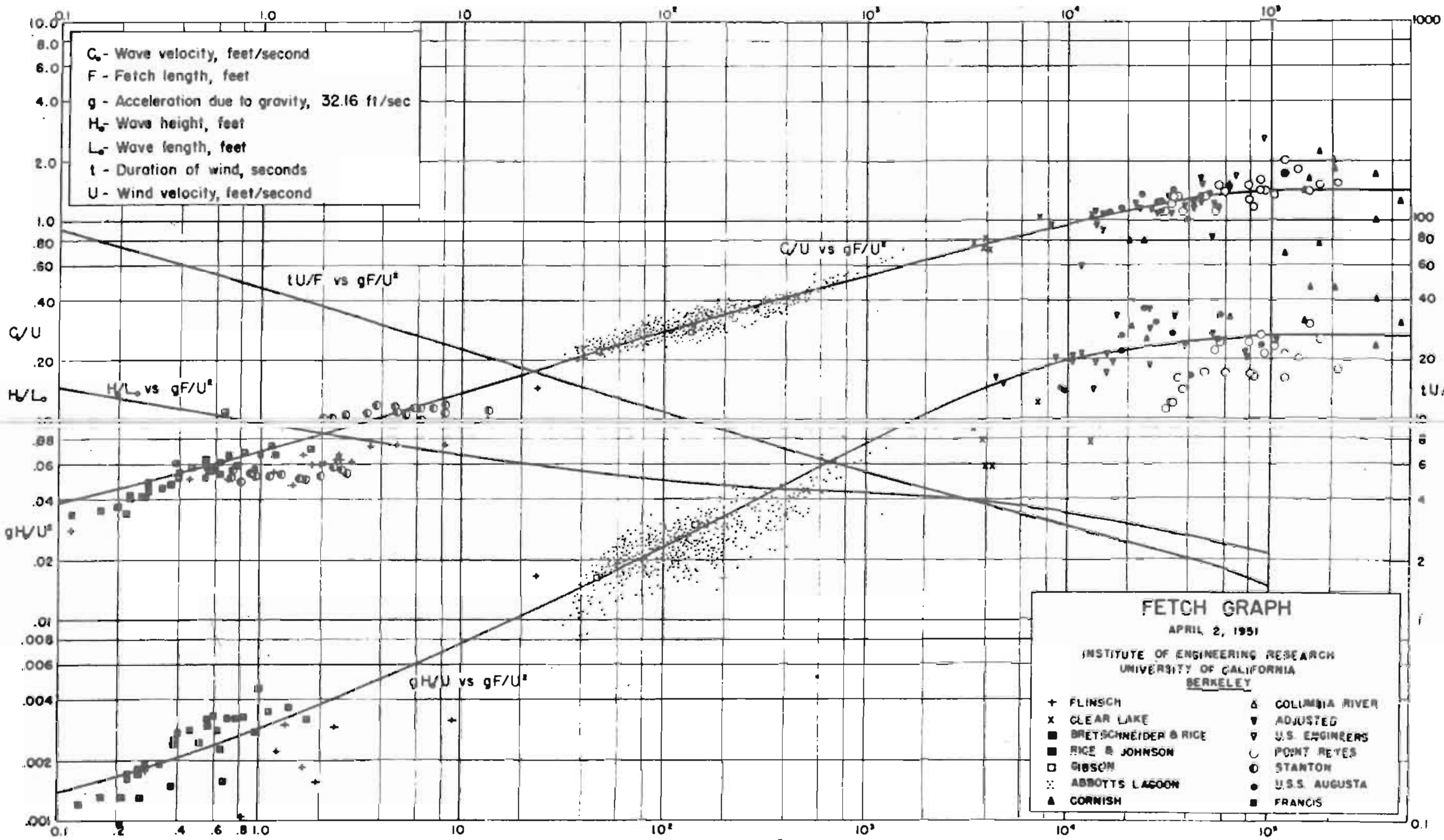
Special solutions of equation 10 are given in reference (12).

If the waves are small and consist of a uniform train being generated in shallow water one may write the following differential equation for the steady state condition, neglecting refraction and percolation:

$$\frac{d(ncE)}{dx} = R_H + R_c - D_f \qquad (11)$$

Two general cases for equation 11 exist: (a) for $D_f = 0$, which results in the deep-water wave forecasting relationships, Figure 2; and (b) for $R_H + R_c = 0$ which results in the relationship for wave energy loss (wave height reduction) due to bottom friction, Figure 3. A general solution of the above equation is difficult, because one needs to know how R_H and R_c are affected by D_f. For this report it is assumed that R_H and R_c are related to C/U in some manner (the exact manner is not important) such as that given by Sverdrup and Munk(11); and wind speed, U, and minimum fetch length,* F_{min}, are given by Figure 2. This is not strictly correct but adjustments are made in the selection of an apparent bottom friction factor which permits a computational procedure and gives agreement between observed and computed wave heights under known conditions. Furthermore, because of the nature of the problem, the closer to steady state conditions in shallow water, the less dependent is the wave period on energy exchange from wind to wave to dissipation by bottom friction. Using the above relationships and the relationships for wave energy loss by bottom friction, Figure 3, it is possible to solve equation 11 by numerical means or successive approximations. An actual mathematical solution of equation 11 is not necessary since various coefficients must be determined from empirical data, and since in general numerical means are required for a bottom of variable depth and slope. The proper procedure then is to set up a computational procedure and use observations where available, such that a comparison can be made between observations and computations. If satisfactory agreement is reached where wave data are available, then the method can be extended to locations where wave data are absent. This has been done for the special case of a flat bottom using a bottom friction factor f = .01. The resulting relationships agree quite well with wave observations from Lake Okeechobee, Florida, and the nearly flat areas of the Gulf of Mexico, as shown in Figure 4 reproduced in part from Bretschneider(13). The numerical procedures(8) used in arriving at these relationships are similar to those used on the Continental Shelf, discussed below.

*Minimum fetch and minimum duration of wind are definitions pertaining to steady state conditions. Minimum duration is that duration of wind (equal to or less than actual duration) for any particular wind speed and fetch length required for steady state generation. Minimum fetch length is that fetch length (equal to or less than actual fetch) for any particular wind speed and wind duration required for steady state generation.



Note: For deep water only

FIGURE 2

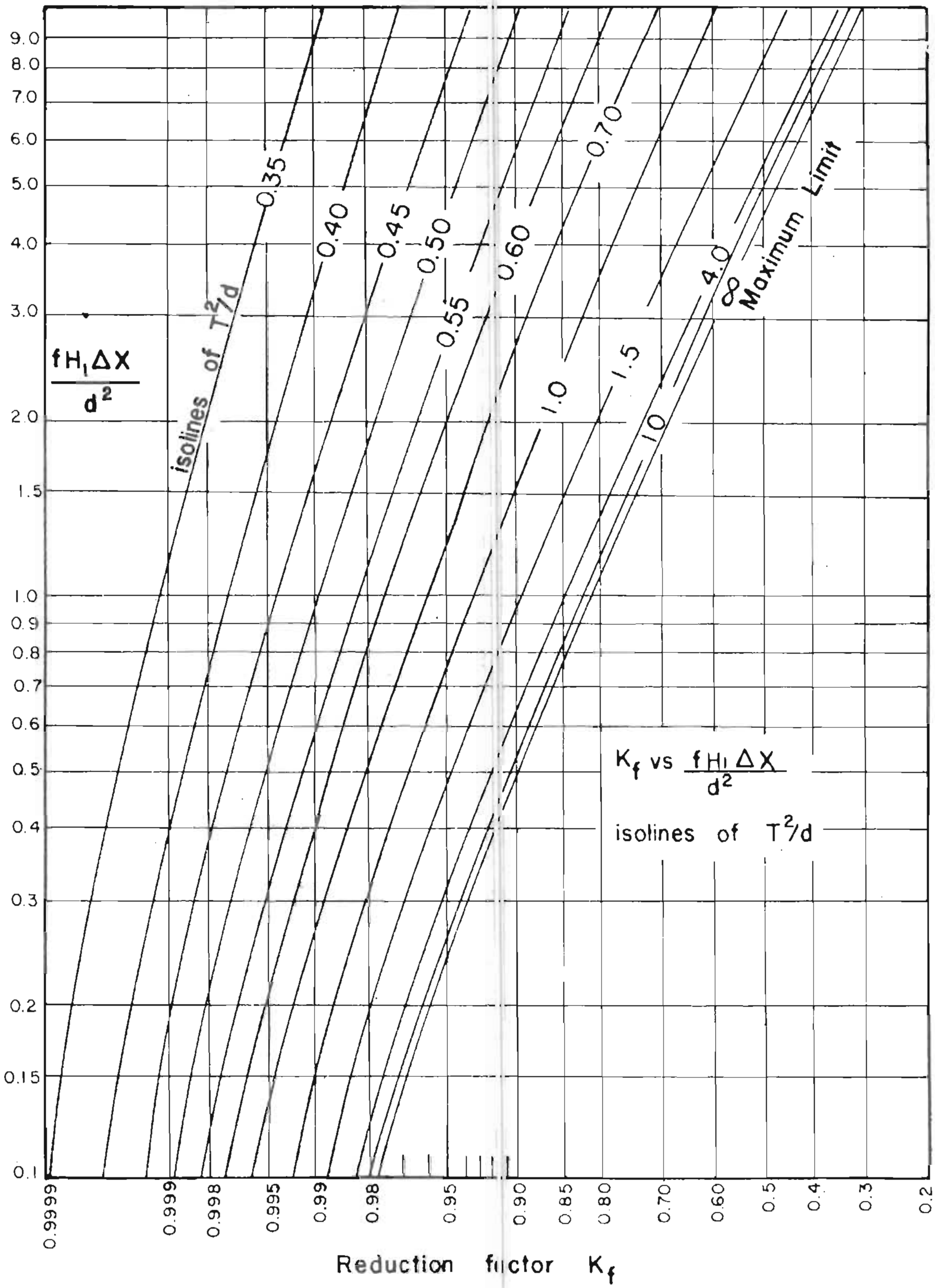


FIGURE 3. RELATIONSHIP FOR FRICTION LOSS OVER A BOTTOM OF CONSTANT DEPTH

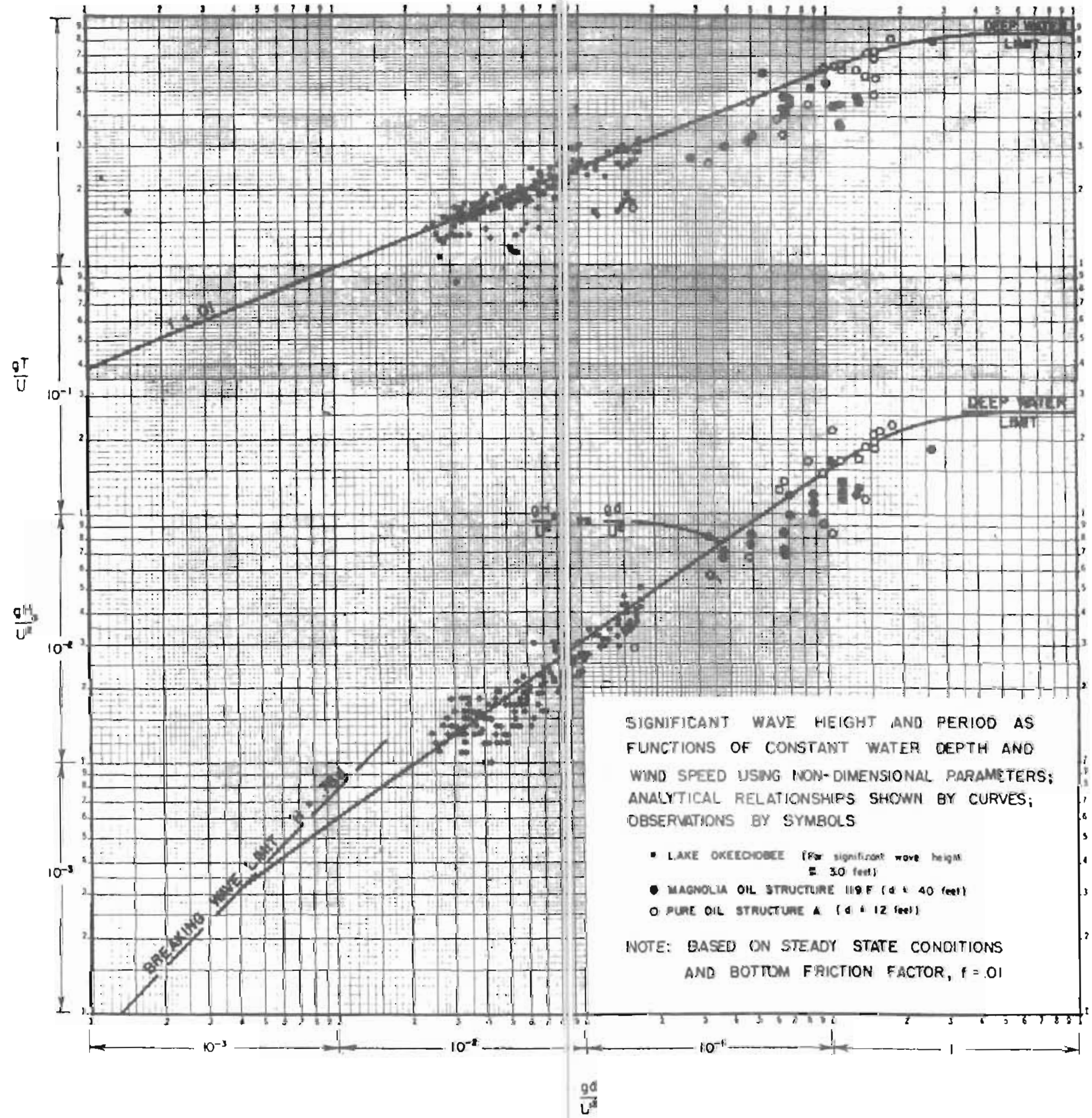

DEEP WATER LIMIT
f = .01
DEEP WATER LIMIT
$\frac{gH}{U^2}$ vs $\frac{gd}{U^2}$
BREAKING WAVE LIMIT H = .78d
$\frac{gT}{U}$
$\frac{gH_s}{U^2}$
10^{-1}
10^{-2}
10^{-3}
10^{-3}
10^{-2}
10^{-1}
1
$\frac{gd}{U^2}$
SIGNIFICANT WAVE HEIGHT AND PERIOD AS FUNCTIONS OF CONSTANT WATER DEPTH AND WIND SPEED USING NON-DIMENSIONAL PARAMETERS; ANALYTICAL RELATIONSHIPS SHOWN BY CURVES; OBSERVATIONS BY SYMBOLS
• LAKE OKEECHOBEE (For significant wave height ≥ 3.0 feet)
● MAGNOLIA OIL STRUCTURE 119 F (d ≈ 40 feet)
○ PURE OIL STRUCTURE A (d ≈ 12 feet)
NOTE: BASED ON STEADY STATE CONDITIONS AND BOTTOM FRICTION FACTOR, f = .01

FIGURE 4

Numerical Procedure. A procedure for computing wind waves over a bottom of constant depth and also over a bottom of constant slope has previously been described(9). In general the procedure for a bottom of constant depth can be applied to winds blowing parallel to the coast and the procedure for a bottom of constant slope can be applied to winds blowing perpendicular to the coast. Through continued use of the above techniques, it was found that the procedure for bottom of constant depth could be used satisfactorily over a bottom of constant slope, provided the bottom slope was segmented into equal steps, each having an average constant depth. Computations based on a flat bottom relationship are much less involved than for a sloping bottom. For this reason the bottom profiles are segmented into increments of constant depth.

Figures 2 and 3 are those required to predict the changing wave characteristics as the wave moves over the shallow slope. Figure 2 is used to determine curves of H_o and T_s versus fetch length for various wind speeds, and in effect represents wind-wave energy generation. Figure 3 is used to compute a reduction factor K_f to be applied to H_o to account for the dissipation of wave energy due to bottom friction.

In regard to wave generation by onshore winds there are two conditions to consider. First, the initial deep-water waves generated may be propagated shoreward as swell under the continued influence of the generating winds; and second, regeneration of wind waves is constantly taking place all along the fetch over the Continental Shelf. Swell will feel bottom far from shore and commence losing energy at an early stage, whereas wind waves with shorter period continue to grow and do not feel bottom until they are sufficiently large and are nearing the coast. In the breaker zone one would observe both swell and wind waves, and the present method of computation cannot be applied here. The steps are as follows: *

a. A wind speed is selected, and a graph of H_o and T versus fetch distance is computed from Figure 2. Figure 5 is an example for a 26-knot wind and unlimited wind duration.

b. The minimum fetch length, F_{min}, is selected and H_o and T is determined at $F = F_{min}$. The deep-water wave length is given by $L_o = 5.12\ T^2$, and the waves will begin to feel bottom at a depth $d = L_o/2$. This is the initial point from which to begin computations shoreward.

c. The bottom profile along the fetch toward the location of interest is determined. The traverse is segmented into at least ten equal increments, ΔF, of about 5 to 10 miles in length, depending on the bottom slope and width of Continental Shelf. Figure 6 is a typical example.

d. An average depth, d_{ave}, is determined over each increment (Figure 6).

* See Bretschneider (20) for an alternative method for computing wind waves for high wind speeds and short fetches.

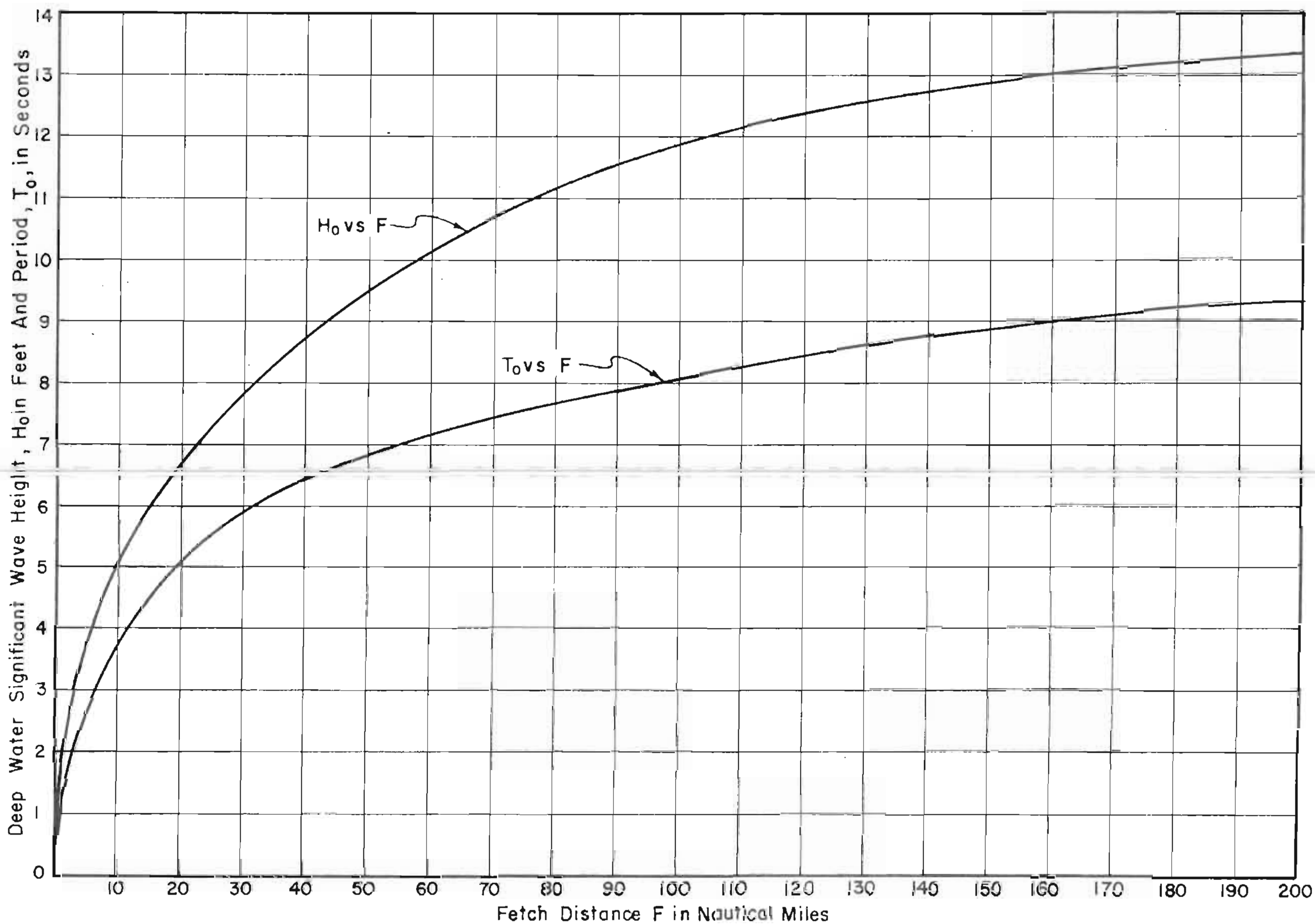


FIGURE 5. DEEP WATER WAVE GENERATION FOR WIND SPEED OF 26 KNOTS AND UNLIMITED DURATION

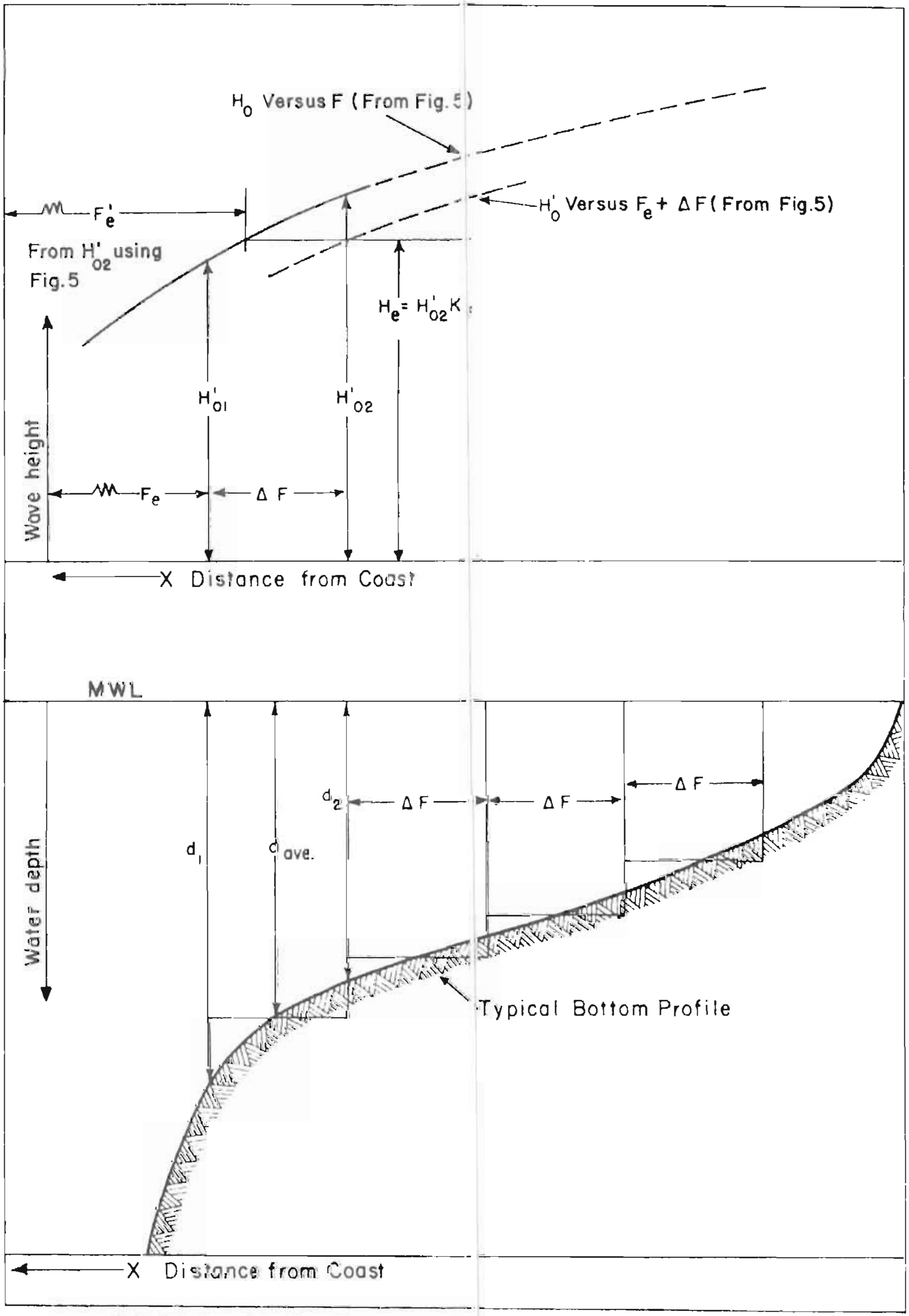


FIGURE 6. SCHEMATIC DIAGRAMS ILLUSTRATING PROCEDURE FOR COMPUTING WIND WAVES IN SHALLOW WATER

e. A deep water wave height, H_o and wave period T is determined at the beginning of the first increment of ΔF using Figure 5 and $F = F_{min}$.

f. This value of H_o is then assumed to travel over the increment ΔF as swell taking bottom friction into account. This is done by use of Figure 3. The quantity $\frac{fH\Delta X}{d^2}$ is determined using bottom friction factor f = .01 based on Lake Okeechobee studies and wave data in shallow water of the Gulf of Mexico; $H = K_r K_s H_o$, where K_s is the shoaling factor (obtained from Figure 7); K_r is the refraction coefficient over the increment ΔF; $\Delta X = 6080 \Delta F$, where ΔF is in nautical miles; $d = d_{ave}$, average depth over the increment ΔF; and T^2/d is computed using the average significant period over the increment and the average depth, d_{ave}. K_f is read from Figure 3 and the actual significant height is equal to $H_s = H_o K_f K_s K_r$.

g. An equivalent deep-water wave height H'_o is obtained from $H'_o = K_f H_o$.

h. Using H'_o and Figure 5 for example, an equivalent deep-water fetch length F'_e is obtained. For the case of regeneration of wind waves one also obtains an equivalent deep-water period, T_o.

i. An equivalent deep-water wave height is determined at the end of the second increment using $F = F'_e + \Delta F \leqq F_{min}$, using Figure 5 for example. For the case of regeneration of wind waves one also obtains an equivalent deep-water period.

j. With the average wave height $1/2\ (H'_{o1} + H'_{o2})$ steps f, g, h, and i are repeated. (This gives the swell height when the wave period T is held constant; T is given for U at $F = F_{min}$.) Using the average wave period $1/2\ (T_{o1} + T_{o2})$ steps f, g, h, and i are repeated. This gives the regenerated wind-wave height. The above procedure is used for all except the last increment or until the wave breaks, whichever occurs first. Usually the last increment cannot be treated by the above method since here the bottom slope increases too rapidly. The above procedure can be used for depths from deep water to about 20 feet, but has been applied to depths of 12 feet, when the winds are not too great.

k. The maximum wave height for all except the last increment is computed from

$$H_{max} = H_s \left[\frac{145\ gd}{U^2}\right]^{0.1} \pm 10\% \leqq .78d \leqq 1.87\ H_s \qquad (12)$$

The above formula is obtained from a report (13) based on wind-generated waves recorded in the Gulf of Mexico and hurricane wind-generated waves recorded in Lake Okeechobee, Florida.

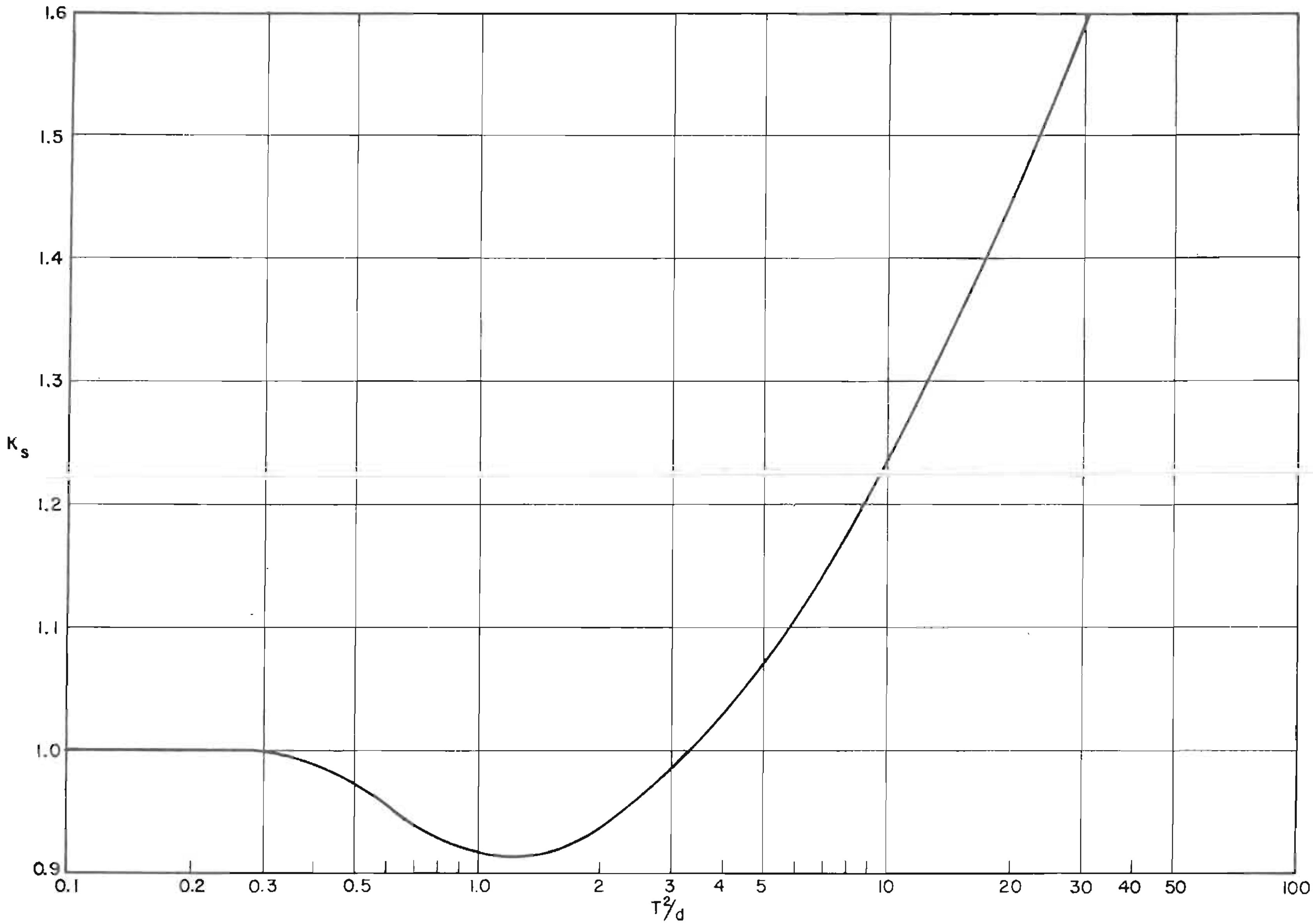


FIGURE 7. SHOALING COEFFICIENT K_s Vs T^2/d

1. Maximum waves over the last increment and the location of breaking are obtained by plotting $H_b = .78d$ seaward from the coast. A continuation of the smooth curve of H_{max} obtained in (k) is made until it intersects the curve of $H_b = .78d$.

In general the swell heights (under the influence of wind) and the regenerated wind-wave heights in intermediate to shallow water will be about the same, except in the breaking zone where the swell having a larger shoaling factor, becomes dominant, and the wind waves will be superimposed thereon. The reason swell heights (under the influence of wind) are about the same as the regenerated wind waves in intermediate to shallow water is that dissipation of wave energy by bottom friction tends to become independent of wave period. This can be seen from Figure 2 where the isolines become crowded for large values of T^2/d. It is for this reason that one can use the wave-period relationships given for deep water when computing generation of wind waves in shallow water.

As an example, the above procedure has been used to compute wind waves from deep water over the bottom profile perpendicular to shore off Caplen, Texas, using U = 26 knots and F_{min} = 200 nautical miles (over deep water to d/L = 0.5). The results of these computations are given in Table 1 and Figure 8. This particular profile was selected since it consists of several different locations having the same depth. It is quite an irregular bottom profile, others being less irregular. For example, from Table 1 and Figure 8 there are two 40-foot depths, one for which H'_o = 10.9 feet and the other H'_o = 8.7 feet, the average value of H'_o = 9.8 was used to compute gH'_o/U^2 versus gd/U^2 and gF/U^2 used in Figure 11. This shows that one can expect forecasts on the order of ± 10% of the values obtained from the forecasting graphs, discussed below.

GENERALIZED FORECASTING CURVES FOR THE GULF OF MEXICO

In general, for any location on the Continental Shelf, three special cases for wind-wave generation exist: (a) winds blowing parallel to the coast, (b) winds blowing from land to sea and (c) winds blowing from sea to land. A set of generalized forecasting curves has been prepared based on the above numerical procedures. These forecasting curves are discussed below:

a. Case I, Winds Blowing Parallel to the Coast. In this case, except where very irregular bottom topography exists, the best approach is to use the flat bottom relationships of gH'_o/U^2, versus gd/U^2 and gF/U^2 given in Figure 9.

b. Case II, Winds Blowing from Land to Sea. In a report by Bretschneider and Thompson(14) it was shown that for most offshore winds, waves are generated which do not feel the bottom. This is

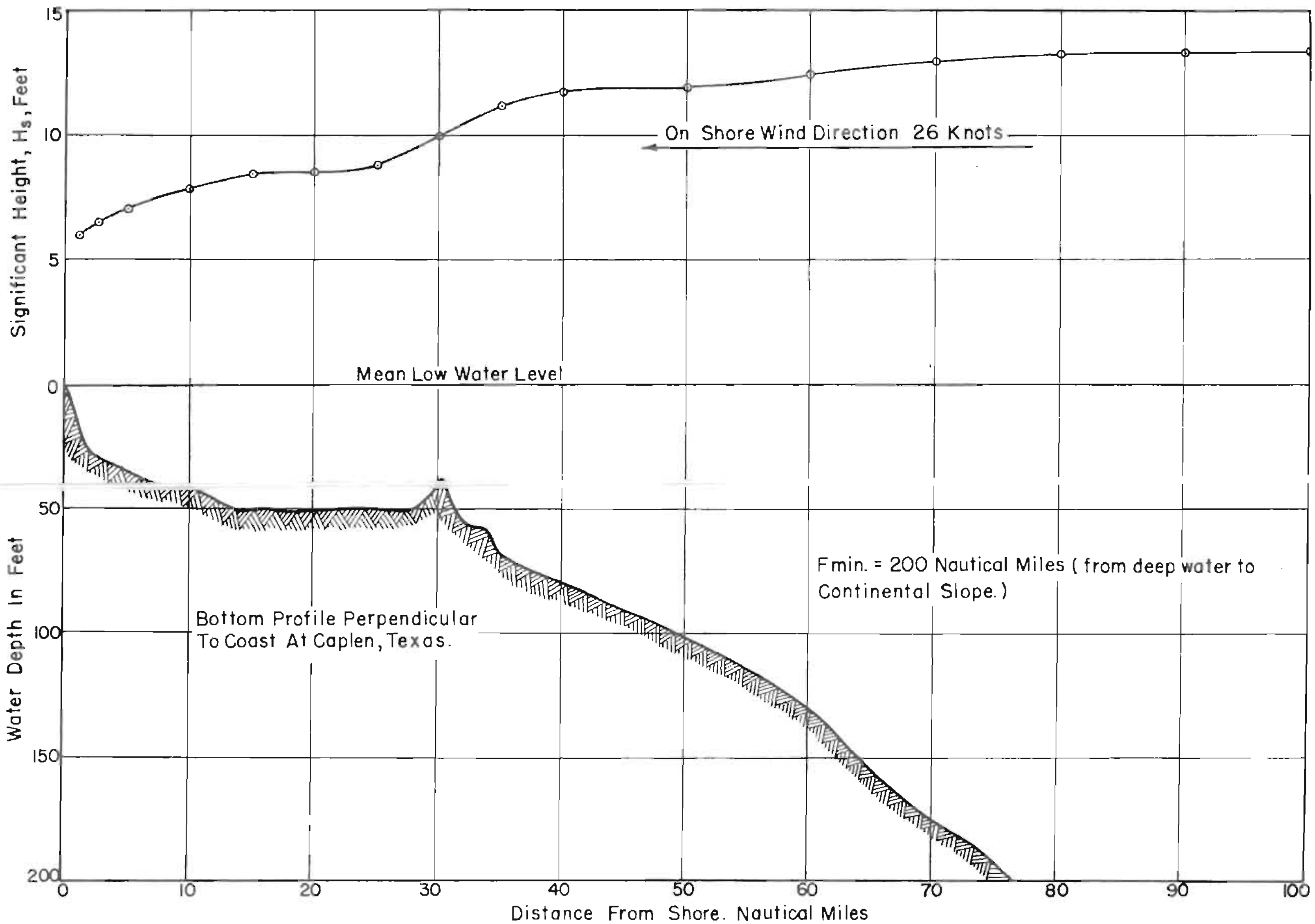


FIGURE 8. WIND-WAVE FORECAST FOR A 26- KNOT WIND BLOWING PERPENDICULAR TO COAST AT CAPLEN, TEXAS

TABLE I

SUMMARY OF COMPUTATIONS FOR EXAMPLE OF WIND-WAVE GENERATION OVER THE CONTINENTAL SHELF

Line		Increment	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	X*	n.m.	100	90	80	70	60	50	40	35	30	25	20	15	10	5	2.5	1.25	0
2	F_{min}	n.m.	200	200	200	190	187	182	155	136	112	74	53.5	48.5	45	39.5	29.5	25.2	
3	ΔF	n.m.	10	10	10	10	10	10	5	5	5	5	5	5	5	5	2.5	1.25	1.25
4	d_1	feet	deep	300	230	190	175	130	100	80	67	40	50	51	51	38	34	28	20
5	d_2	feet	300	230	190	175	130	100	80	67	40	50	51	51	38	34	28	20	0
6	d_{ave}	feet	deep	265	210	182.5	152.5	115	90	73.5	51.5	45	50.5	51	44.5	36	31	24	
7	H_{o1}	feet	13.3	13.3	13.3	13.29	13.28	13.21	12.94	12.66	12.19	10.89	9.67	9.35	9.09	8.65	7.72	7.30	
8	H_{o2}	feet	13.3	13.3	13.3	13.30	13.30	13.29	13.00	12.76	12.31	11.11	10.01	9.67	9.43	9.03	8.00	7.42	
9	H_{ave}	feet	13.3	13.3	13.3	13.295	13.29	13.25	12.97	12.71	12.25	11.00	9.84	9.51	9.26	8.84	7.86	7.36	
10	T_{o1}	sec	9.4	9.4	9.4	9.3	9.3	9.24	8.94	8.50	8.25	7.52	6.94	6.76	6.6	6.38	5.82	5.52	
11	T_{o2}	sec	9.4	9.4	9.4	9.4	9.4	9.40	9.00	8.76	8.35	7.62	7.10	6.94	6.8	6.60	6.00	5.62	
12	$T_{o\ ave}$	sec	9.4	9.4	9.4	9.35	9.35	9.32	8.97	8.63	8.30	7.57	7.02	6.85	6.7	6.49	5.91	5.57	
13	$\frac{T^2}{d}$	$sec^2/feet$	deep	.333	.42	.48	.572	.757	.894	1.01	1.335	1.29	.975	.920	1.01	1.17	1.12	1.29	
14	$K_{s\ ave}$		1.0	.996	.985	.975	.960	.932	.921	.918	.914	.913	.919	.920	.918	.913	.914	.913	
15	$\frac{fH\Delta X}{d^2}$		small	.114	.180	.212	.334	.567	.438	.656	1.285	1.51	1.09	1.02	1.30	1.89	1.14	.89	
16	K_f		1.0	1.00	.9993	.9982	.9932	.974	.974	.955	.885	.870	.933	.940	.917	.855	.912	.92	
17	H'_{o2}	feet	13.3	13.3	13.29	13.28	13.21	12.94	12.66	12.19	10.89	9.67	9.35	9.09	8.65	7.72	7.30	6.83	
18	F_e	n.m.	200	200	190	187	182	155	136	112	74	53.5	48.5	45	39.5	29.5	25.2	21.5	
19	gH'_o/U^2 ($H'_o = H'_{o2}$)		.221	.221	.221	.221	.219	.215	.210	.202	.181	.161	.155	.151	.144	.128	.121	.113	
20	gd/U^2 ($d = d_2$)		4.98	3.82	3.16	2.91	2.16	1.66	1.33	1.11	.664	.831	.847	.847	.631	.565	.465	.322	

* X is distance from shore.

because the fetch length, increasing seaward, is limiting. As the fetch length gets longer the wave period gets longer, but the water depth becomes greater. This would indicate that for offshore winds, one may use the deep-water wave forecasting curves, Figure 2. Actually Figure 9 can also be used, since the lines of constant gF/U^2 terminate on the right hand side of Figure 5 at deep water.

c. Case III, Winds Blowing from Sea to Land. This is perhaps the most complex situation for wave generation, and no one set of curves can be developed similar to those for Cases I and II. Since each location has a different bottom profile leading shoreward from various directions, a set of graphs would be required for each direction for each station. This becomes impracticable just as the numerical procedure discussed in the previous section becomes impracticable when one requires a multitude of forecasts. For this reason one generalized forecasting graph for onshore winds is prepared for each of five locations in the Gulf of Mexico. The forecasting curves are computed by the procedure outlined above based on the following conditions:

(1) The most frequent minimum fetch length was selected for each wind speed range and onshore direction, based on wind and fetch statistics using three years of weather maps from the files of A. H. Glenn and Associates[19]. The most frequent fetch length associated with a particular wind and direction was determined from

$$F_{min} = \frac{F_1 t_1 + F_2 t_2 + F_3 t_3 + \dots + F_n t_n}{t_1 + t_2 + t_3 + \dots t_n} \qquad (13)$$

where F_1 is the minimum fetch length and t_1 the duration of a particular wind and direction over this fetch, etc.

(2) For each location an average bottom profile was determined based on three shoreward bottom profiles, forming the cone of 45° on each side of the profile perpendicular to the coast. The profile perpendicular to the coast was given double weight in determining the average. This was done for each location, resulting in five average bottom profiles. Each of the average bottom profiles was divided into short increments, and an average depth over each increment was determined. Computation of wind waves were then made as outlined above.

(3) Values of H'_o, F_{min}, d_F and U were tabulated, d_F = depth at end of fetch F where H'_o applies. From the above, values of gH'_o/U^2, gF/U^2 and gd/U^2, were determined where F is same as F_{min} and d same as d_F. A generalized plot was made of gH'_o/U^2 versus gd/U^2 and isolines of gF/U^2. This required a certain amount of smoothing to obtain the best overall picture. Scatter of the data, however, was not significant, being less than ±10% of the final selected curves. The

resulting families of curves for each location are given in Figures 10, 11, 12, 13 and 14 respectively for Brownsville, Texas; Caplen, Texas; Burrwood, Louisiana; Apalachicola, Florida; and Tampa, Florida. These figures can be used for forecasting wind waves for onshore winds perpendicular to the coast $\pm 45^{\circ}$.

These forecasting curves are used to compute the shallow water wave statistics given in references (1, 2, 3, 4, 5) making use of the duration of individual wind speed ranges and the corresponding most frequent minimum fetches.

It must be emphasized that certain assumptions were required to obtain the forecasting curves for wave statistics purposes, and that individual forecasts may deviate by $\pm 10\%$. It is therefore, necessary, that any individual forecast take these factors into consideration. Since the curves have not yet been used for daily forecasts, it is difficult to recommend any modification except that the following suggestions seem in order;

(1) For onshore winds perpendicular to the coast one might multiply values of H'_o by 1.1, where H'_o is determined from the proper figure.

(2) For onshore winds blowing at 45° to the coast one might multiply values of H'_o by 0.9. (The above rules of thumb need to be checked against observations.)

(3) Allowance should be made for location ahead of and behind various shoal areas.

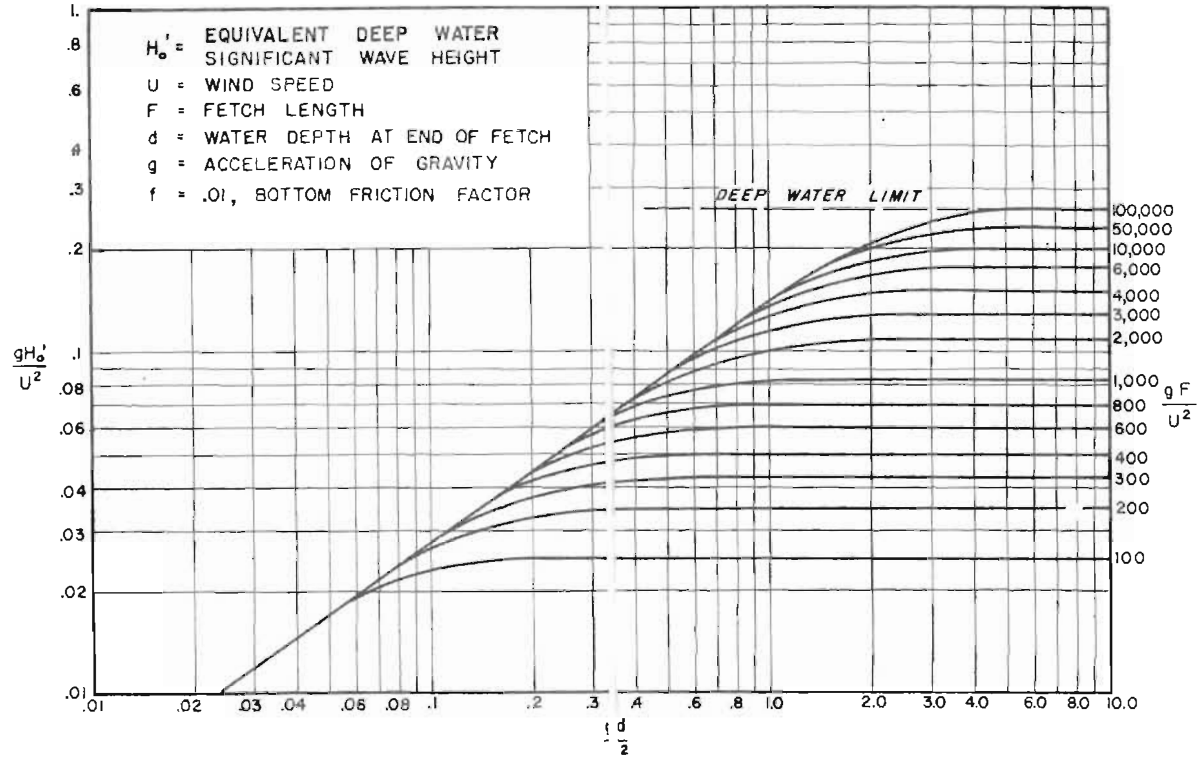
DETERMINATION OF SIGNIFICANT WAVE PERIOD

No generalized forecasting curves are presented for obtaining the significant wave period. Although a wave period is necessarily used and obtained in the computations for wave height it is not necessarily the significant wave period, although it may be close to the significant period. However, the significant wave period has a "definite significant relationship" to the significant height. It is shown by Bretschneider(20) for deep-water wind waves for $gF/U^2 \lessapprox 10,000$ that the average value of the significant period is related to the significant height by:

$$H_o/T_o^2 = 0.22 \tag{14}$$

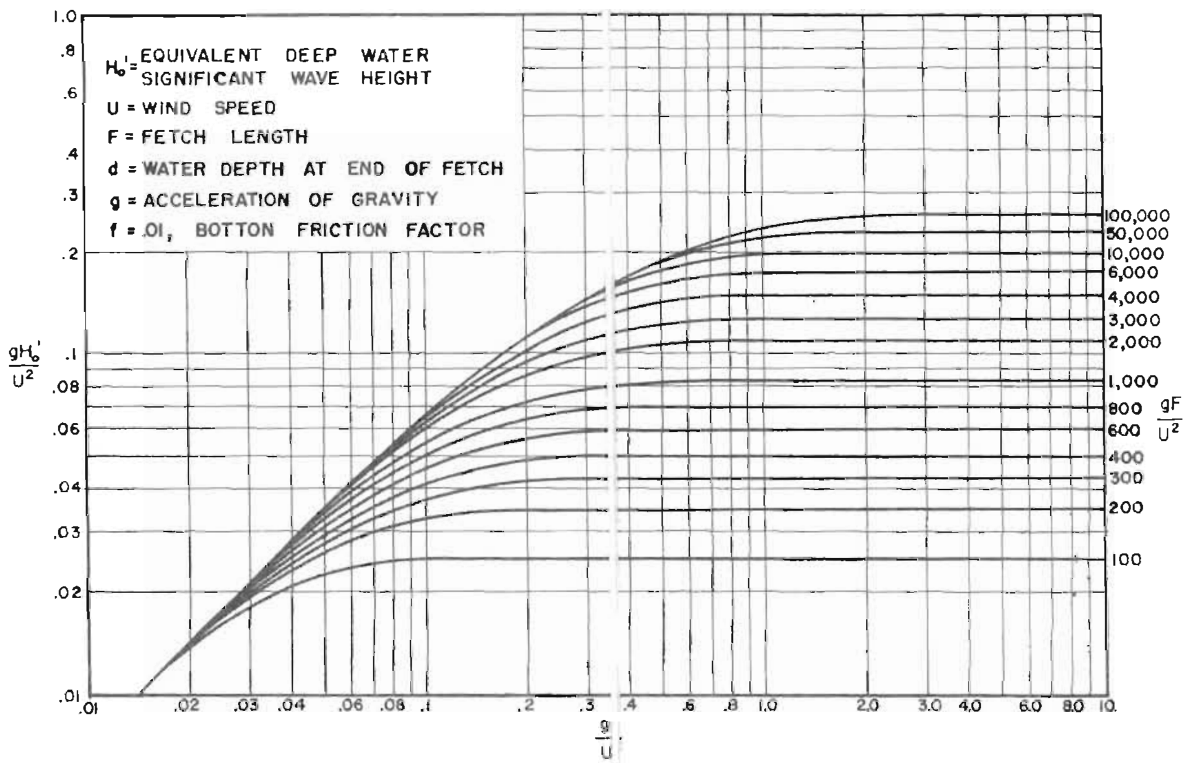
This expression is quite significant since it agrees with Jasper(21), wherein one may determine the average value of $H_o/T_o^2 \cong 0.20$, based on deep-water wave observations in the North Atlantic. From Figure 3 one may determine for shallow water (gd/U^2 = .01 to 1.0):

$$H'_o/T^2 = .17 \pm 20\% \tag{15}$$



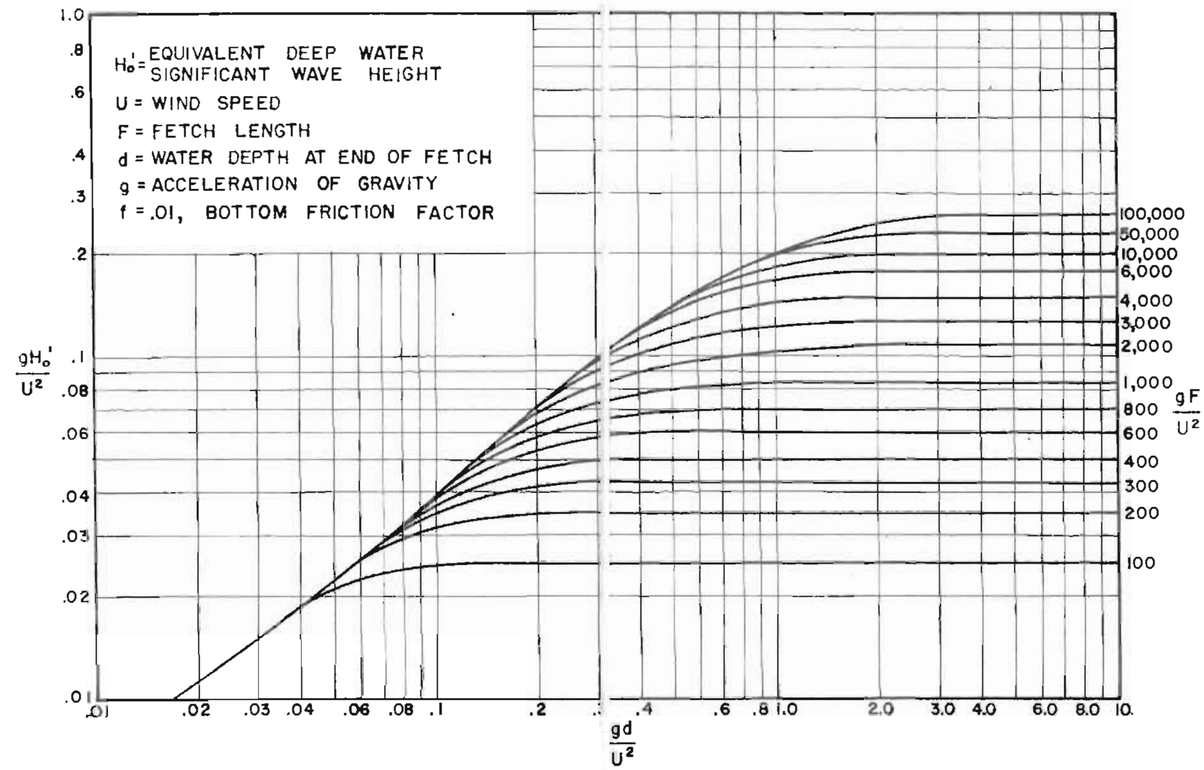
GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT DEPTH FOR UNLIMITED WIND DURATION REPRESENTED AS DIMENSIONLESS PARAMETERS

FIGURE 9



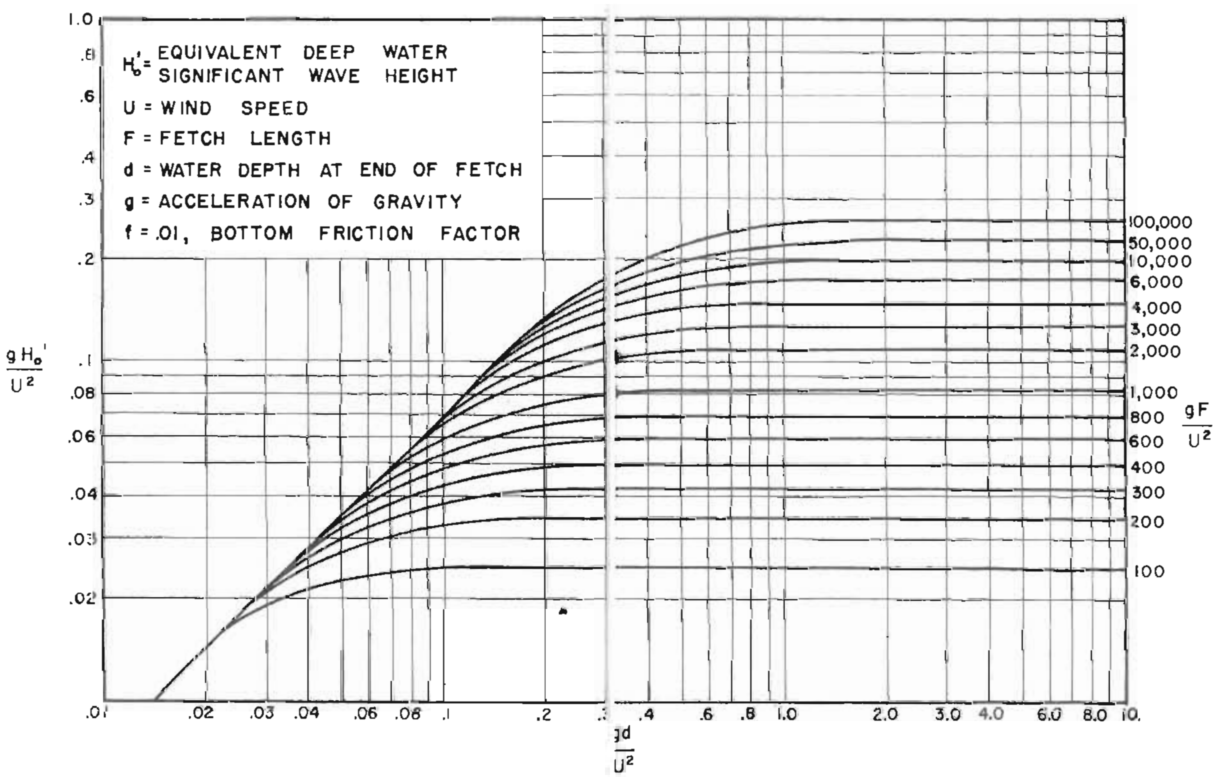
GENERATION OF ONSHORE WIND WAVES OFF BROWNSVILLE, TEXAS BASED ON AVERAGE CONDITIONS

FIGURE 10



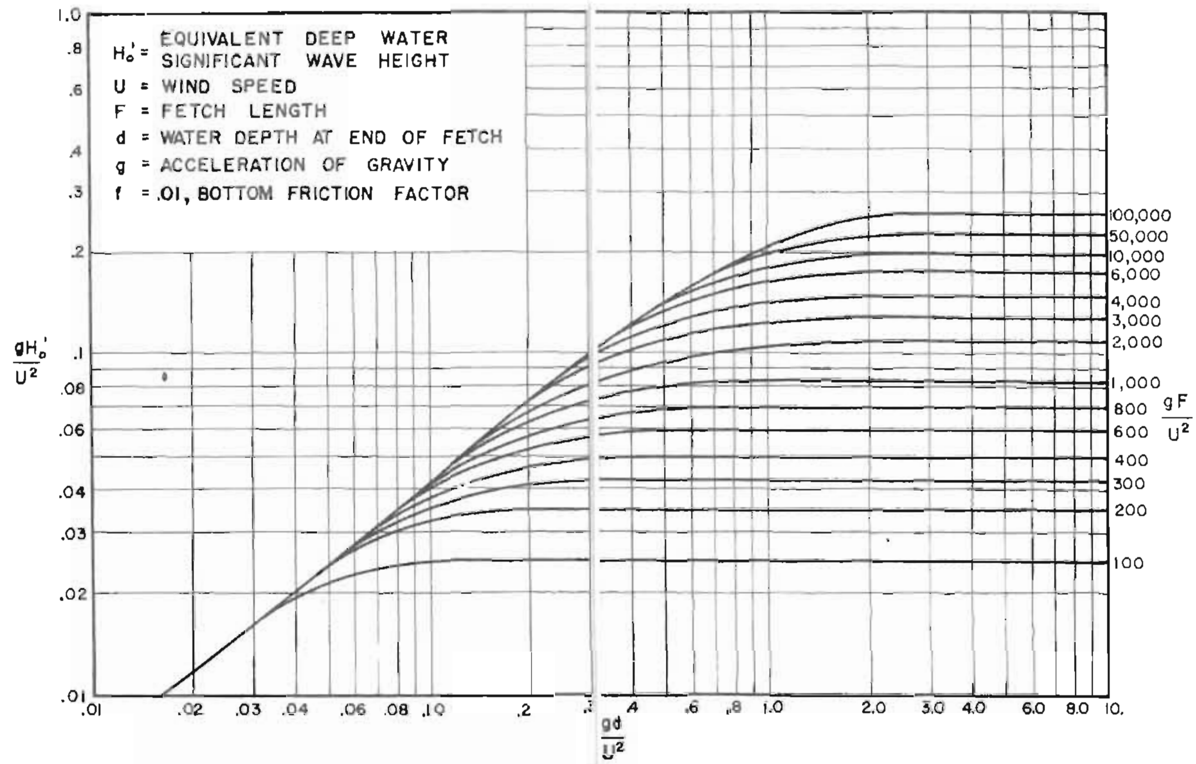
GENERATION OF ONSHORE WIND WAVES OFF CAPLEN, TEXAS
BASED ON AVERAGE CONDITIONS

FIGURE 11



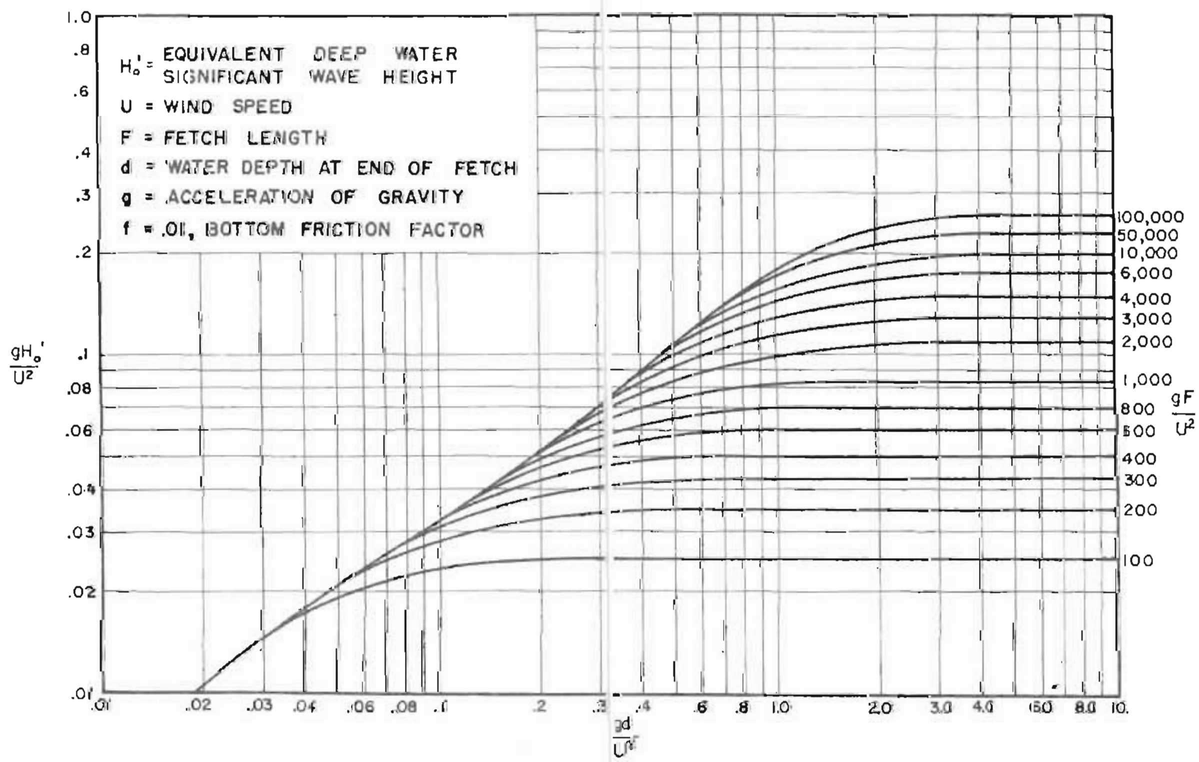
GENERATION OF ONSHORE WIND WAVES OFF BURRWOOD, LOUISIANA
BASED ON AVERAGE CONDITIONS

FIGURE 12



GENERATION OF ONSHORE WIND WAVES OFF APALACHICOLA, FLORIDA
BASED ON AVERAGE CONDITIONS

FIGURE 13



GENERATION OF ONSHORE WIND WAVES OFF TAMPA BAY, FLORIDA
BASED ON AVERAGE CONDITIONS

FIGURE 14

Using expressions 14 and 15 it follows:

$$T_o = 2.13\sqrt{H_o} \quad \text{for deep water} \qquad (16)$$

$$T_s = 2.43\sqrt{H'_o} \pm 10\% \quad \text{for shallow water} \qquad (17)$$

Once the significant wave heights of wind waves have been obtained it is possible to obtain approximately the significant wave period directly from the above equations.

DISTRIBUTION FUNCTIONS FOR WIND WAVES FOR THE GULF OF MEXICO

The preceding material is used for obtaining the significant wave height and period. Actual use of forecasts made from relationships presented in this report, as well as the hindcast wave data presented in the Beach Erosion Board Technical Memorandums 85, 86, 87, 88, and 89 must take into account the distribution of all individual waves making up the wave train. Wind wave records* obtained in the Gulf of Mexico appear to have a distribution of heights and periods very close to those of Putz(6), based on Pacific Ocean wave data. Therefore, Figures 15 and 16 reproduced from Putz(6) can be used temporarily to obtain the distribution of heights and periods, respectively, once the significant waves are forecast. Relationships between the significant wave height, H_s; average wave height, H_{ave}; average of highest ten percent, H_{10}; and the maximum wave, H_{max}; have been established by Munk(16), Wiegel(15), and Siewell(17), based on the analysis of wave records. Snodgrass(18) summarizes the average values as follows:

$$H_{max}/H_s = 1.87 \pm 20\% \qquad (18)$$

$$H_{1/10}/H_s = 1.29 \pm 10\% \qquad (19)$$

$$H_{ave}/H_s = .65 \pm 10\% \qquad (20)$$

Based on the analysis of wind wave records in shallow water, Bretschneider(13) finds that:

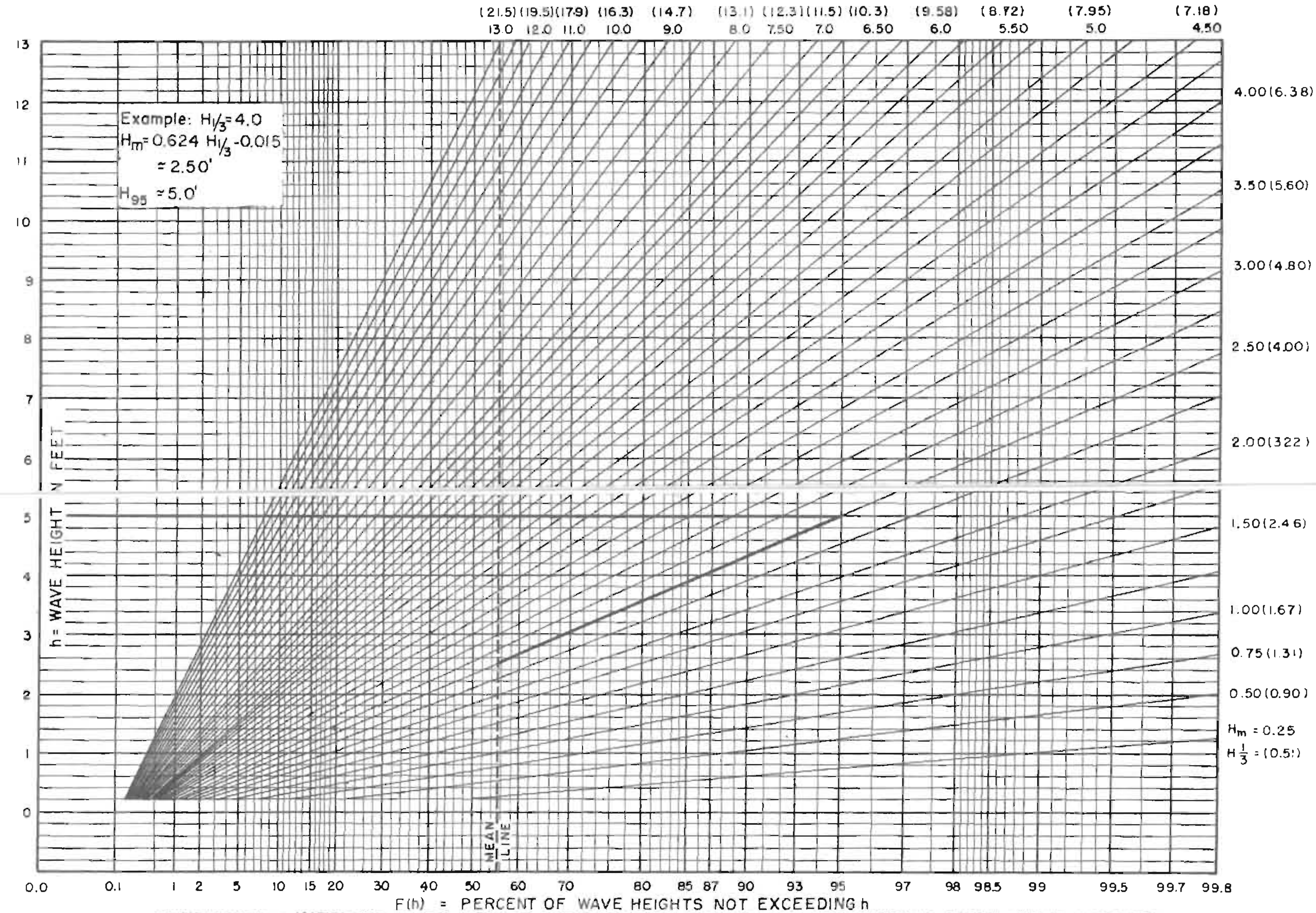
$$H/H_{max} = \left[145\,\frac{gd}{U^2}\right]^{.1} \pm 10\% \text{ for } \frac{gd}{U^2} \leqq 2.5 \qquad (21)$$

$$H_{1/10}/H_s = 1.23 \pm 10\% \qquad (22)$$

Additional analysis of these same data gives:

$$H_{ave}/H_s = .675 \pm 10\% \qquad (23)$$

*Analysis of all wave records from the Gulf of Mexico has not yet been completed and the results are therefore not presented at this time.



PREDICTION LINES FOR WAVE HEIGHT DISTRIBUTION FUNCTIONS FOR VARIOUS MEAN WAVE HEIGHTS
(SIGNIFICANT WAVE HEIGHT $H_{1/3}$ SHOWN IN PARENTHESES)

(Putz, 1950)

FIGURE 15

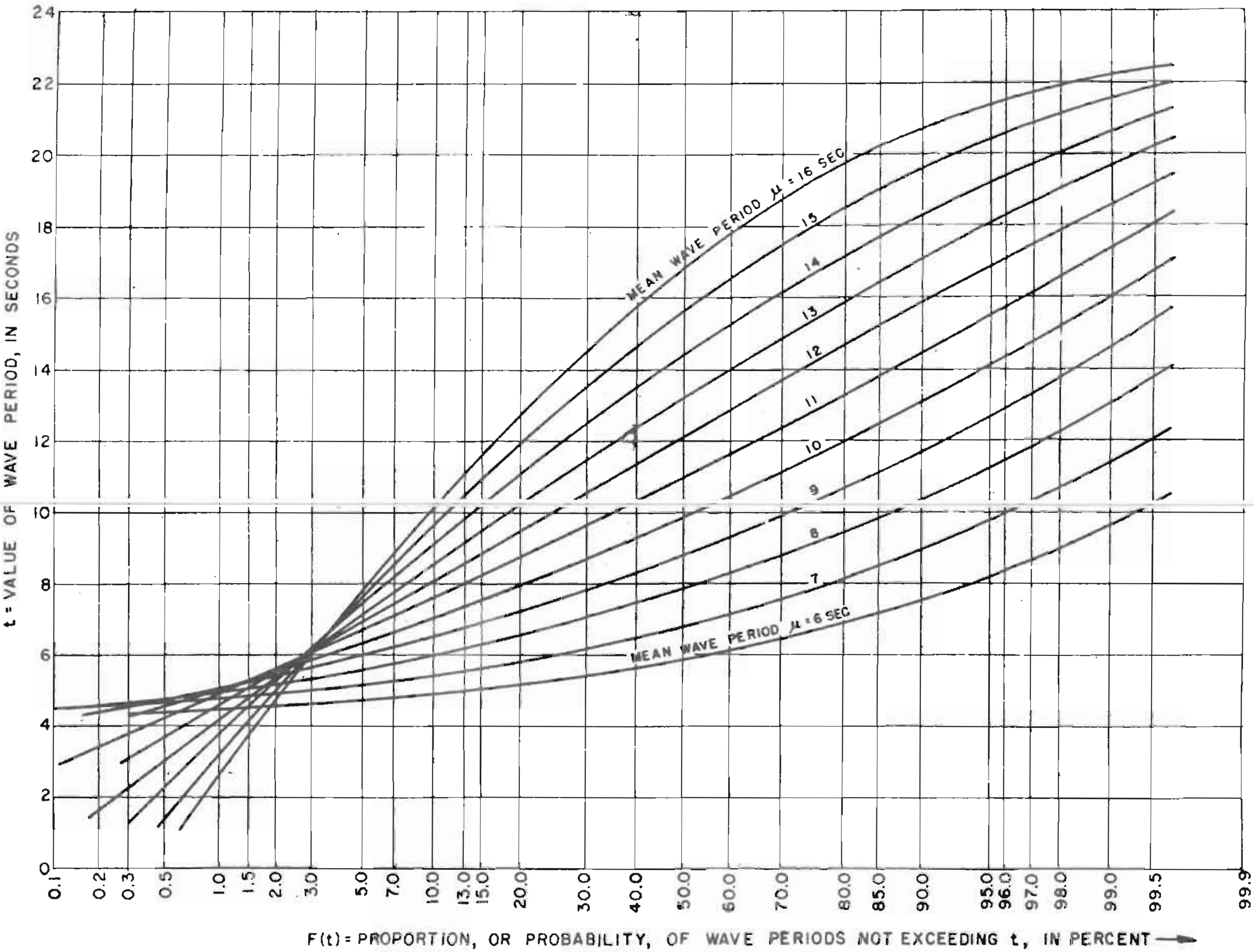


FIGURE 16. PREDICTION CURVES FOR WAVE-PERIOD DISTRIBUTIONS FOR VARIOUS MEAN WAVE PERIODS

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APPENDIX A

EXAMPLE FORECAST

Determine the significant wave height, H_s; maximum probable wave height, H_{max}; average of highest ten percent of waves, $H_{1/10}$; and the average wave, H_{ave}; for the following conditions:

(a) Location off Caplen, Texas
(b) Minimum fetch length of 200 nautical miles (1,216,000 feet)
(c) Unlimited wind duration
(d) Wind speed 26 knots (44 feet per/sec)
(e) Wind direction, SE
(I) for deep water
(II) at a water depth of 40 feet
(III) at a water depth of 20 feet
(IV) at a water depth of 10 feet

Solution:

$$gF/U^2 = \frac{32.2 \times 1,216,000}{44^2} = 20,300$$

I - for deep water

$$gd/U^2 \geq 6.6$$

$$\frac{gH'_o}{U^2} = .22 \text{ (Fig. 7)}$$

$$H'_o = H_s = \frac{.22\ (44)^2}{32.2} = 13.3 \text{ feet}$$

$$T_s = 2.13\ \sqrt{13.3}\ (\pm 10\%) = 7.75 \text{ seconds} \pm 10\%$$

$$H_{max} = 13.3 \times 1.87 = 24.9 \text{ feet } \pm 20\% \text{ (equation 18)}$$

$$H_{1/10} = 13.3 \times 1.29 = 17.2 \text{ feet } \pm 10\% \text{ (equation 19)}$$

$$H_{ave} = 13.3 \times .64 = 8.52 \text{ feet } \pm 10\% \text{ (equation 20)}$$

II - for d = 40 feet

$$gd/U^2 = \frac{32.2 \times 40}{(44)^2} = .668$$

$$\frac{gH'_o}{U^2} = .175 \text{ (Fig. 11)}$$

$$H'_o = \frac{.175\ (44)^2}{32.2} = 10.5 \text{ feet (Note } H'_o = 10.9 \text{ and } 8.7 \text{ from Table 1)}$$

$$T_s = 2.43\ \sqrt{10.5} \pm 10\% = 7.9 \text{ sec} \pm 10\% \text{ (equation 17)}$$

$$T_s^2/d = \frac{(7.9)^2}{40} = 1.5$$

$$K_s = .919 \text{ (Fig. 7)}$$

$$H_s = 10.8 \times .92 = 9.85 \text{ feet}$$

$$H_{max} = 9.85 \left[145 \times .66\right]^{0.1} = 9.85 \times 1.58 = 15.5 \text{ feet} \pm 10\% \text{ (equation 21)}$$

$$H_{1/10} = 9.85 \times 1.23 = 12.1 \text{ feet} \pm 10\% \text{ (equation 22)}$$

$$H_{ave} = 9.85 \times .675 = 6.65 \text{ feet} \pm 10\% \text{ (equation 23)}$$

III - for d = 20 feet

$$\frac{gd}{U^2} = \frac{32.2 \times 20}{(44)^2} = .33$$

$$\frac{gH'_o}{U^2} = .108 \text{ (Fig. 11)}$$

$$H'_o = .108\ \frac{(44)^2}{32.2} = 6.5 \text{ feet (Note: } H'_o = 6.83 \text{ Table 1)}$$

$$T_s = 2.43\ \sqrt{6.5} \pm 10\% = 6.10 \text{ seconds} \pm 10\% \text{ (equation 17)}$$

$$T_s^2/d = 1.92$$

$$K_s = .929 \text{ (Fig. 7)}$$

$H_s = 6.3 \times .929 = 5.85$ feet

$H_{max} = 5.85 \left[145 \times .334\right]^{0.1} = 5.85 \times 1.47 = 8.6$ feet $\pm 10\%$ (equation 21)

$H_{1/10} = 5.85 \times 1.23 = 7.20$ feet $\pm 10\%$ (equation 22)

$H_{ave} = 5.85 \times .675 = 3.96$ feet $\pm 10\%$ (equation 23)

IV - for d = 10 feet*

$\frac{gd}{U^2} = \frac{32.2 \times 10}{(44)^2} = .167$

$\frac{gH'_o}{U^2} = .06$ (Fig. 1)

$H'_o = .06 \frac{(44)^2}{32.2} = 3.6$ feet

$T_s = 2.43 \sqrt{3.6 \pm 10\%} = 4.60$ seconds $\pm 10\%$ (equation 17)

$L_o = 5.12 \left[4.15\right]^2 = 108$ feet

$d/L_o = 10/108 = .0925$

$K_s = .939$ (Fig. 7)

$H_s = .939 \times 3.6 = 3.4$ feet

$H_{max} = 3.4 \left[145 \times .167\right]^{.1} = 3.4 \times 1.375 = 4.8$ feet $\pm 10\%$ (equation 21)

$H_{1/10} = 3.4 \times 1.23 = 4.2$ feet $\pm 10\%$ (equation 22)

$H_{ave} = 3.4 \times .675 = 2.3$ feet $\pm 10\%$ (equation 23)

*Note: For this particular depth the answers are only approximate, since the theory is extended too far into the steeper zone of the shoreward portion of the Continental Shelf. In fact, these waves may well be obscured by longer period swell breaking in the surf zone. The predominant period of the swell will be on the order of 7.9 seconds as given in the first example.

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