WIND SET-UP AND WAVES IN SHALLOW WATER

TECHNICAL MEMORANDUM NO. 27
BEACH EROSION BOARD
CORPS OF ENGINEERS

JUNE 1952
This paper presents the results of an analysis of certain wind, wave, and water level data obtained in Lake Okeechobee, Florida, during the passage of two hurricanes, one in August 1949 and the other in October 1950. The purpose of the analysis was to relate wind set-up and wave heights observed in Lake Okeechobee to wind velocity, fetch, water depth, and surface shape of the lake.

The data used as the basis of this paper were collected by the Jacksonville District, of the Corps of Engineers. The planning of the observational program and the development, modification, and maintenance of field instruments represented a joint effort of the Hydraulics and Hydrology Branch of the Office of Chief of Engineers, the Washington District of the Corps of Engineers, the Beach Erosion Board, the Jacksonville District of the Corps of Engineers, and U. S. Weather Bureau. The basic data collected at Lake Okeechobee and serving as the basis of the present memorandum are set forth in Project Bulletin No. 2, Waves and Wind Tides in Inland Waters, distributed by the Jacksonville District of the Corps of Engineers in June 1950 and containing the data for the August 1949 storm, and are, as yet, undistributed project bulletin containing similar data for the storm of October 1950.

The present paper was prepared by Thorndike Saville, Jr., of the Research Division of the Beach Erosion Board, who analyzed the data, and prepared the report. Mr. Saville was assisted in the computations and plotting by George Simmons, now of the U. S. Army.

The opinions and conclusions expressed by the author are not necessarily those of the Board.
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1. **Introduction.** A program of observation and study of wind set-up and wave action in inland waters has been undertaken by the Corps of Engineers as a part of a general program of engineering investigations related to the design, construction, and operation of flood control and navigation projects involving levees and channel improvements on lakes and major reservoirs. As an initial step in these studies, special observation stations have been established in Lake Okeechobee, Florida, in Fort Peck Reservoir, Montana, and in Denison Reservoir, Oklahoma. It is possible that this program may be supplemented by observations on other reservoirs, lakes, and rivers at a later date. The findings from these studies will be correlated with certain laboratory studies and special investigations, such as the determination of the effect of embankment slopes on height of wave run-up and study of slope protection requirements. This paper deals with the development of the relation between wind velocity and duration and the resulting wind set-up (sometimes known as wind tide) in shallow inland waters based on data from Lake Okeechobee.

2. **Winds, waves, and water surface elevations have been recorded for several years at a number of stations on Lake Okeechobee. The records include the two major hurricanes which occurred on 26-27 August 1949 and 17-18 October 1950.** This report deals primarily with the analysis of the data from these two storms. Locations of gages, and the type of data obtained from each, are indicated on the topographic map of the lake (figure 1).

3. **Theoretical Considerations.** Theoretical developments of a wind set-up equation have been made by Hellstrom (1), Keulegan (2) and Thijsse, among others. The mechanics of the various determinations have differed somewhat, but the resultant equations have been essentially the same, giving

\[
\frac{dh}{dx} = \frac{nF\tau_d}{\rho g D} \quad \text{or} \quad S = \frac{nF\tau_d}{\rho g D},
\]

where
- \( h \) = surface elevation;
- \( x \) = horizontal coordinate;
- \( \frac{dh}{dx} \) = slope of water surface;
- \( \tau_d \) = surface shear stress;
- \( g \) = acceleration due to gravity;
- \( \rho \) = density of water;
- \( D \) = depth of lake surface horizontal;
- \( n \) = a coefficient defined as \( n = \frac{2}{\tau_d} + 1 \) where \( \tau_d \) is the shear stress along the bottom. This coefficient has a value of 1.5 for laminar flow, and for turbulent flow will depend on the particular theory adopted.
Hellstrom, adopting the Boussinesq theory for turbulent flow, finds \( n \) to vary between 1.30 and 1.15 for moderate to large depths. Keulegan assumes an average value of \( n = 1.25 \) for his laboratory tests.

\[ S = \text{set-up, expressed as the difference in water surface elevations at windward and leeward sides of the lake;} \]
\[ F = \text{fetch length, or distance between points of maximum and minimum water surface elevation.} \]

4. The important assumptions involved in obtaining this equation are that it be for a steady state, equilibrium case with the set-up developed by the action of the wind alone, and that the set-up be small compared to the depth.

5. The surface shear is generally expressed as a function of the wind velocity,
\[ \tau_s = K \rho_a V^2 \]
where \( K \), a numerical constant \( \approx 0.003 \),
\( \rho_a \) = air density
\( V \) = wind velocity

Substituting this in the equation for set-up, then:
\[ S = \frac{K n \rho_a V^2 F}{\rho g D} \]

6. Frequently the fetch direction, the line connecting the points of minimum and maximum water surface elevation, does not coincide with the average wind direction. In these cases the use of a resultant shear force in the direction of the fetch is generally deemed best, where the resultant force \( (\tau_s') = \tau_s \cos \theta \) if \( \theta \) denotes the angle between the wind and the fetch. It is thought that, since it is the shear force that is producing the set-up the set-up is determined by a resultant force in the direction of the fetch rather than by a resultant velocity which might be said to be producing the shear force. Hence the cosine term enters the equation to the first power rather than the second power, and the set-up equation becomes
\[ S = \frac{K n \rho_a V^2 F}{\rho g D} \cos \theta \]

7. It may be assumed that the set-up at a particular spot may be modified by the topographic configuration of the lake as it approaches that point or that a convergent effect would occur where the wind blows from a wider portion of the lake towards a narrower portion causing the actual set-up to be higher than that expected; similarly a divergence might be expected if the wind direction were reversed. This convergence or divergence, generally would be of a very minor magnitude, but might reach an appreciable value in the case of a triangular shaped lake with the wind blowing toward (or away from) an apex. Hellstrom (1) has mentioned this probability, and personnel of the Jacksonville District of the Corps of Engineers have devised a method of computing this convergence or divergence based on the ratio of the width at the leeward end of the lake to the average
width (3). However, when the leeward end of the lake narrows to or nearly to a point so that there is little or no width, the method breaks down. If this convergence factor is denoted by \( N \), then the final set-up equation becomes \( S = AN \frac{2B}{\rho} \cos \theta \), where \( A = \frac{K_0 A}{\rho} \). The term \( A \) will be approximately constant for all conditions.

8. It is obvious that steady state conditions generally do not exist over a lake, as the wind velocity and direction vary at different points across the lake, and also change with time at each point. As the wind direction changes, the fetch and average depth along the fetch also change. The primary difficulty involved, therefore, in the application of the set-up equation to open bodies of water, is the choice of suitable average values for the parameters involved so that the average values selected will adequately represent the cumulative effect of the actual values.

9. Method of Computation. There appear to be two different concepts for determining average values which might be utilized for each parameter at any one time. One of these is based on the assumption that the action of each strip of water is influenced primarily by the wind and the effect of the bottom in that strip alone, and is essentially independent of the wind and bottom forces acting on other areas of the lake; this would lead to average values being obtained from the actual values observed along the fetch alone. The other is based on the assumption that the set-up at any one point is due to the action of the entire water mass as a whole, and, as such, is not produced by the wind and bottom forces acting along the fetch alone, but is a cumulative effect of all these forces acting on the entire body of water. In this case, average values would be computed from the actual values observed over the entire lake surface. Both of these averages have been computed.

10. The average depth over the entire lake is obtained by dividing the volume of water in the lake by the surface area, and will be constant for any one storm if the inflow and outflow of the lake are assumed to be negligible or compensating. The average depth over the fetch is obtained by dividing the cross-sectional area along the fetch by the fetch length, and will change with time as the fetch changes. Both methods assume that the actual lake bottom may be replaced by a hypothetical level bottom such that, in the case of average lake depths, the volume of the lake remains the same, and, in the case of average fetch depths, the cross-sectional area under the fetch is the same.

11. To determine the average wind velocities, the hydrometeorological section of the Weather Bureau analyzed the anemometer record charts for ten-minute average velocities throughout the storm; these were then plotted at each station on a series of lake maps, and lines of equal velocity (isovels) and vectors showing direction were drawn on each map. These maps were drawn at one hour intervals during the greater part of the storm, and at half-hour intervals during the more critical portions of the storm. The average velocity over the entire lake surface was obtained by planimeter,
and since the velocity appears as a squared term in the set-up equation, the average value was assumed to be given by the equation

\[ v = \sqrt{\frac{d^2 + b^2 + c^2 + \ldots}{d^2 + b^2 + c^2 + \ldots}} \]

where \( v_1 \) is the velocity appropriate to the planimetered area \( A_1 \). The average velocity over the fetch was obtained from the equation

\[ v = \sqrt{\frac{d^2 + b^2 + c^2 + \ldots}{d^2 + b^2 + c^2 + \ldots}} \]

where \( v_1 \) is the velocity appropriate to the short segment, \( F_1 \), of the fetch. The average, or resultant, direction of the wind was obtained by visual inspection of the flow pattern, and the angle between this average direction and the fetch obtained by protractor.

12. It is evident that the set-up produced is the result of the wind forces acting over a certain period of time and that it takes the lake a certain definite time interval to fully react to the forces applied. Study of the differences in times of definite changes in the wind and the corresponding lake level patterns show the lake to have approximately a one-hour lag behind the wind. This is in agreement with conclusions reached by Haurwitz (4) and the personnel of the Jacksonville District for this same storm period. Hallstrom gives an approximate method for computing the time necessary for equilibrium conditions to be reached if a lake, initially with a level surface, is subjected to a wind of constant velocity. Applying this method to Lake Okeechobee, for a fetch of 35 miles and a wind speed of 60 miles per hour, it would take about 1.6 hours to approach an equilibrium state (i.e. when 95% of the energy required for steady motion has been developed). This figure also is in general agreement with those given by Proudman and Doodson (5).

13. In the case of Lake Okeechobee, however, the wind is a gradually increasing one, rather than one of constant speed, so that for any one value of wind speed the initial lake level would already be partly deformed, and the corresponding time to reach an equilibrium state would be less than that computed. Hence the average effective values producing set-up at any one time have been computed as averages over the preceding hour.

14. Comparison of Theory and Observations. The factor \( AN \) in the set-up equation, computed by substituting average values from the data on the 1949 storm in the set-up equation, is found to be remarkably constant at a value of approximately \( 3.4 \times 10^{-6} \), the extreme limits being \( 3.14 \times 10^{-6} \) and \( 3.45 \times 10^{-6} \) (see table 1). Keulegan's experimental work at the Bureau of Standards has indicated that, at least for models in small closed channels, the value of \( A \) should be about \( 3.3 \times 10^{-6} \). The agreement between these two values is notable, particularly in view of the greatly different conditions involved in the two cases. If Keulegan's value is accepted as the correct one, then the convergence factor \( N \) equals 1.03. This is an extremely small value, particularly in view of the fact that the maximum rise in water surface elevation occurs essentially in the apex of a triangular embayment, and therefore it might be expected that the convergence, or shape factor, would be large although volume computations which would also take into account the extreme narrowness of the upwind end of the lake tend to show a convergence factor of the order of only 1.01 - 1.02. Small though it is,
however, with the maximum set-up reaching almost 19 feet it makes a difference of 0.55 feet in the predicted maximum value. With such a small value, percentage-wise at least, it is difficult to say definitely that convergence does, or does not, occur, since errors of several per cent may be introduced in the computation of the appropriate average effective values. These errors have been eliminated as much as possible by having the values obtained independently by two different computers. In most cases these values were almost identical, but in some cases differences of as much as two miles per hour in the computations of wind velocity resulted. In all cases, an average of the two values obtained by the different computers was taken as more closely approximating the true value.

15. Curves comparing the observed values of set-up (S₀) in the 1949 storm with those predicted by the formula using values of the parameters developed over the entire lake are shown in figure 2. Shown also are curves showing the set-up predicted if the convergence or cosine θ terms are omitted, as well as one showing values obtained if no time average is introduced and the water surface is assumed to react completely and instantaneously to the forces applied. The values of the parameters used are tabulated in Table 1. The validity of the use of time average values of the parameters is demonstrated quite clearly by the plots on Figure 2. Also the choice of a one-hour period for averaging the parameter values was apparently justified, as the observed and predicted curves are in close agreement, at least for that part of the storm before the wind shift. Actually, observed set-up values obtained after the wind shift (of about 180°) are not strictly comparable with values predicted by using the set-up equation as steady state conditions are no longer approximated. The set-up at that time is due not only to the wind forces, but also to forces set up by the rotational effect of the wind and the water, and to a seiche action. Haurwitz (4) has shown that, in such a case, the set-up may be greatly different from the steady state condition, depending on the relation between the seiche period of the lake and the speed of rotation of the storm. Before the wind shift occurs, however, steady state conditions are approximated, and the agreement of predicted and observed values is much better than might be expected, showing a maximum difference of about 4 per cent.

16. The agreement at the higher values of set-up is especially noteworthy as the basic assumption that the set-up is small compared to the depth is no longer valid. Perhaps the continued agreement in this range may be partly explained by the use of average values of depth over the lake. If h denotes the water surface displacement at any point, then the average value of D + h is very nearly the same as the average value of D used in the equation, provided the surface area of the lake does not change appreciably. Then the substitution of the more correct term (D + h) for D would not change the results significantly.

17. Curves for the 1949 storm showing predictions based on values averaged over the fetch rather than over the lake are given in figure 3. Values of the parameters are tabulated in Table 2. It may be seen that
although the agreement between predicted and observed values of set-up is not bad, it is not nearly as good as that obtained when average lake values were used. The closest agreement occurs when the cosine $\theta$ term is neglected, which would tend to indicate that the same set-up will occur regardless of the angle of the wind with the fetch. The same disagreement in values after the wind shift is obtained here as was observed with the lake average values.

18. Possible Method for Correcting for Marsh Effect. Similar computations have been made for the 1950 storm and are shown in figures 4 (lake average values) and 5 (fetch average values); the parameters are tabulated in tables 3 and 4. The direction of the wind was such that during most of the first part of the storm the rise in water elevation occurred over a marshy area covered with thick grasses and other vegetation. This vegetation produces a damping effect on the set-up reducing the total set-up due to the increased friction losses occurring as the water moves through the grasses, and to the reduction in surface shear between the water and the wind for those times when the grasses protrude above the water surface. This reduction in set-up is clearly shown by the curves. It may be supposed that a correction factor, $C_m$, can be applied to the predicted set-up, $S$, to obtain the observed set-up $S_0$, such that $S_0 = S(1-C_m)$.

In general, $C_m$ will decrease with some function of the depth over the marshy area which can probably be expressed dimensionlessly as a ratio of the depth, or the set-up, and some friction length parameter of the marsh vegetation. This correction factor $C_m$ has been determined from the data and plotted against the predicted values of set-up (as for practical use it is more convenient to use the predicted values than the observed). These plots are shown in figure 6 (lake average values) and figure 7 (fetch average values). Since only the one storm and area were involved, no attempt could be made to determine the friction length parameter approximate to these grasses. A sharp break in the curve is observed when a set-up of 9.5 feet is obtained, as at this point, with increasing set-up the correction factor drops rapidly toward zero. It may be that when the set-up reaches this figure, there is enough depth of water over the vegetation for the set-up to take place essentially independently of the vegetation, and the full set-up results. This might occur if the bottom return flow occurs above the tops of the vegetation, and the water included within the vegetation is still and not a part of the circulatory system of water flow resulting from the set-up. It is probable, however, that this correction factor is also dependent on some term embodying the ratio of that portion of the marsh area included in the fetch to the entire length of the fetch. Then, when the fetch lies outside the marsh area, after some appropriate lag in time, full set-up would be reached. This postulation may also be an explanation of the sharp change in the correction factor at the higher set-up stage.

19. Another sharp change in the slope of this curve seems to be indicated at a set-up of about 2.4 feet, though this may be just scatter of the plotted points. However, a sharp change would be expected somewhere in this range, since at some value of set-up the tops of the vegetation
disappear beneath the water, and the surface shear between the wind and the water changes abruptly to its full value.

20. It is recognized that the data used in obtaining this correction factor ($C_m$) are entirely from one storm, and that the apparent straight line fit may be completely coincidental due to a rather fortuitous choice of the variables. It is hoped that additional data from future, smaller storms over the same area may strengthen the basis for the curve.

21. Values of set-up obtained by using correction values ($C_m$) obtained from this curve are also shown in figures 4 and 5. Good agreement is shown between the predicted and observed values, with, for fetch average values, the best agreement again being obtained when the cosine $\theta$ term is omitted. Unfortunately, the effect of the marsh prevents any exact determination of the divergence, but a value for $N$ may be determined for the time (toward the peak of the storm) when the set-up appears to be taking place essentially independently of any effect from the vegetation. This value obtained by the Jacksonville method would be 0.96, which at least is of the order of magnitude to be expected. Curves drawn on the basis of no divergence factor as well as for this value are shown. The use of a different value for the divergence would change the slope and intercept of the marsh correction curve slightly, but a similar straight line curve would result.

22. After the wind shift for the 1950 storm a discrepancy is apparent between predicted and observed set-up that is even greater than that indicated for the 1949 storm. This, as in the 1949 storm, is undoubtedly due to non-uniform conditions.

23. Possible Method for Computing the Rise in Water Level at the Leeward End of the Lake. It is usually the rise in water level at the leeward end of the lake that is of major interest, rather than the total set-up between the two ends of the lake. Greger, Justin and Hinds (6) have studied the reported set-up on numerous lakes and arrived at an average value of the ratio of the rise of the lake at the leeward end of the lake to the total set-up of about 0.571. Study of the Lake Osechobee data indicates that this ratio is not constant, but varies somewhat with the nearshore slope and the amount of set-up. If it is assumed that the water surface slope may be sufficiently approximated by a straight line and that the bottom profile of the lake may be idealized as shown in the sketch below, then an expression for the percentage of total set-up occurring above or below mean lake level at either end of a rectangular lake is easily obtained. In the sketch the triangular area $A_1 = \frac{h_1 c}{2}$ and $A_2 = \frac{h_2}{2}$ (L-c).
24. Since the amount of water in the lake (and hence in the cross-section) is presumed to remain the same, \(h_1 = h_2\) and \(h_2 = \frac{c}{h_1} \frac{L - c}{L - c}
\)

But \(\frac{c + h_2 \cos \alpha_i}{h_i} = \frac{c + h_2 \cos \alpha_i}{h_i} \frac{(H-h_2)}{h_i} \cos \alpha_i + b + (H-h_2) \cos \alpha_i + (h_i + h_2) \cos \alpha_i \)

Making use of the equality \(L-b = H (c \cos \alpha_i + c \cos \alpha_i)\), this may be solved for \(c\):

\[
c = \frac{h_i L - h_i h_2 (c \cos \alpha_i + c \cos \alpha_i)}{h_i + h_2}
\]

Then \(\frac{h_2}{h_1} = \frac{c}{L-c} = \frac{h_i}{h_2} \left(\frac{L-h_2 \phi}{L-h_2 \phi}\right)\)

where \(\phi = (c \cos \alpha_i + c \cos \alpha_i)
\]

25. Since the set-up, \(S = h_1 + h_2\), a quadratic equation in \(h_1\) or \(h_2\) alone may be obtained.

\[
h_1^2 \phi + (2L - S \phi) h_1 - SL = 0
\]

resulting in \(h_1 = \frac{S \phi - 2L \pm \sqrt{4L^2 - 4SL \phi}}{2\phi}\)

and

\[
h_2^2 \phi - (2L + S \phi) h_2 + SL = 0
\]

For \(h_1 > 0\), it must be associated with the plus sign; then, since \(S = h_1 + h_2\), \(h_2\) must be associated with the minus sign. It may be noted that for the limiting case of vertical sides, these equations reduce to \(h_1 = h_2 = \frac{S}{2}\).

The percentage of set-up occurring at the leeward end can be expressed as a function of \(2L/S\) :

\[
\frac{h_1}{S} = \frac{1}{2} \left(1 - \frac{L/\phi}{S/2} \sqrt{\left(\frac{L/\phi}{S/2}\right)^2 + 1}\right)
\]

This function is shown as a graph in figure 8. From it, values of \(h_1/S\) have been computed for the first half of each storm, and are compared in figure 9, with observed values. The values of the necessary parameters are tabulated in Table 5. Also shown on the graphs are the average values obtained from each storm and the average value given by Justin, Creager, and Hinds. As may be seen from the graphs, the observed points are fitted much better by the derived curves than by any constant average value. This is particularly true for the 1950 storm. It should be noted that considerable scatter should be expected at times of small set-up, since the values of set-up were determined only to the nearest tenth of a foot (for a
two-foot total set-up, an error of 0.05 feet in the partial set-up results in a change of 0.025 in the observed h/b ratio). The predicted and observed set-ups at the leeward end of the lake are compared in figure 2; also shown are values computed by use of constant percentage values. As may be seen from this curve, the predicted and observed values agree extremely well except for the final value at 2130. It is suspected that this may be partly due to the wind shift occurring at that time. This shift may have already produced non-equilibrium conditions, giving a somewhat lower set-up at the leeward end than might otherwise be expected. This is partly substantiated by the continued decrease in proportionate set-up at the leeward end as the shift continues; at 2200 the rise in water level was only 48.1 per cent of the total set-up.

26. Possible Dependence of Set-up on Form Resistance of the Waves. Keulegan, in his experiments, found that for small closed channels the shear stress is not independent of the length of the channel if waves are produced on the water surface, but that an additional stress is added due to the form resistance of the waves. This would result in the addition of an extra term to the set-up equation, which, for Keulegan's experiments became \( S = 3.3 \times 10^{-6} \sqrt{f \varepsilon_3 (D/\bar{E})^{3/2}} \frac{v^2 R}{D} \). Despite the generation of waves of appreciable size (up to 7 feet) during the more violent part of the storm on Lake Okeechobee, the data do not indicate the necessity of any such additional term. It is thought that either the effect of the form resistance of the waves becomes gradually less important with increased length of fetch, until at some undetermined point it becomes essentially negligible, or possibly that the form resistance term is introduced into the tank tests by the restriction of the area of air flow, which would not be observed in nature.

27. Conclusion. If the dimensions of the set-up equation are considered as miles per hour for velocity, miles for fetch, and feet for depth and set-up, then \( S = 1.165 \times 10^{-3} \frac{v^2 R}{D} \cos \theta \). It will be noted that this coefficient \((1.165 \times 10^{-3})\) is almost identical with that \((1.25 \times 10^{-3})\) of the so-called Zuiderzee formula developed some years ago from data for the Zuiderzee. This close agreement between coefficients applicable to such widely varying conditions as those in Keulegan's model experiments, the Zuiderzee, and Lake Okeechobee, would seem to indicate that this is a wind set-up formula of general application. It is hoped that further checks on its applicability may be made on other inland waters, particularly on those of considerably greater depth than that of Lake Okeechobee and the Zuiderzee.
Legend:
- observed = $S_D$
- $3.30 \times 10^{-6} \sqrt{\frac{g}{\rho}}$
- $3.30 \times 10^{-6} \sqrt{\frac{g}{\rho}} \cos \theta$
- $3.30 \times 10^{-6} \sqrt{\frac{g}{\rho}} N \cos \theta$
- $3.30 \times 10^{-6} \sqrt{\frac{g}{\rho}} \cos \theta$
- For best fit, $N = 1.03$

Values of $\theta$ and $V$ averaged over previous hour
Values of $\theta$ and $V$ averaged over a ten minute period only

Total Set-up

Rapid Wind Shift

SET-UP AT LEEWARD SIDE ONLY

Legend:
- observed $h_1$
- predicted $h_1$ for $N = 1.03$
- $0.553 \ S_P$ (Average)
- $0.571 \ S_P$ (Creager)
- predicted $h_1$ for $N = 1.00$

North Wind

Wind Direction

26 August 1949
LAKE OKEECHOBEE—1949 HURRICANE
Lake Average Values (depth & velocity averaged over entire lake)

27 August 1949
FIGURE 2
LAKE OKEECHOBEE—1949 HURRICANE

Fetch average values (depth & velocity averaged over fetch)

Figure 3

For best fit, \( N = 1.05 \)
\( N_i = 1.08 \)
LEGEND:
- observed = $S_0$
- $3.3 \times 10^{-6} \sqrt{F/gD}$
- $3.3 \times 10^{-6} \sqrt{F/gD} \cos \theta$
- $3.3 \times 10^{-6} \sqrt{F/gD} \sin \phi$
- $3.3 \times 10^{-6} \sqrt{F/gD} \cos \theta \cos \phi$
- $3.3 \times 10^{-6} \sqrt{F/gD} \cos \theta \sin \phi$

$N = 0.96$

↑ North Wind

LAKE OKEECHOBEE—1950 HURRICANE

Lake average values (depth & velocity averaged over entire lake)

17 October 1950
18 October 1950

Figure 4
LEGEND:

- observed = $S_0$
- $3.3 \times 10^{-6} \frac{V}{F/gD}$
- $3.3 \times 10^{-6} \frac{V}{F/gD} \cos \theta$
- $3.3 \times 10^{-6} \frac{V}{F/gD} N \cos \theta$
- $3.3 \times 10^{-6} \frac{V}{F/gD} N$
- $3.3 \times 10^{-6} \frac{V}{F/gD} N \cos \theta$
- $3.3 \times 10^{-6} \frac{V}{F/gD} N \cos \theta$
- $N = 0.96$

LAKE OKEECHOBEE—1950 HURRICANE

17 October 1950
Fetch average values (depth & velocity averaged over fetch alone)
16 October 1950
Figure 5
PERCENTAGE OF SET-UP OCCURRING AT LEEWARD SIDE OF LAKE

\[ h_i = \frac{S}{2} - \frac{1}{\phi} + \sqrt{\left(\frac{1}{\phi}\right)^2 + \left(\frac{S}{2}\right)^2} \]

where \( S = h_i + h_2 \)

\[ \phi = \frac{\text{ctn} \alpha_1 + \text{ctn} \alpha_2}{H} \cdot \frac{b}{H} \]

\[ h_2 = \frac{S}{2} + \frac{1}{\phi} - \sqrt{\left(\frac{1}{\phi}\right)^2 + \left(\frac{S}{2}\right)^2} \]

Figure 8
PORTION OF TOTAL SET-UP OCCURRING AT LEEWARD END OF LAKE

Value given by Creager = 0.571
Average value = 0.553

Value given by Creager = 0.571
Average value = 0.566

Figure 9
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Note: The table continues with values for higher declination angles, but is truncated here.
PART TWO - WAVES IN SHALLOW WATER

1. Introduction. In addition to the wind and water stage recorders discussed in Part One of this report, step-resistance wave gages were installed at seven lake stations, and wave records were taken for one minute out of every ten for the duration of the storms. Lake stations 12, and 14 were in operation during the 1949 storm, and stations 10, 11, 12, 14, 15, 16 and 17 during the 1950 storm. Gage locations are shown on the topographic map (Figure 1). The records obtained were analyzed by the Jacksonville District (3) for maximum height, significant height (the average of the higher third), and significant period for each one minute strip.

2. Correlation Between Maximum and Significant Wave Heights. The excellent correlation between the height of maximum waves and the average height of the higher third was noteworthy. The average value of this ratio for all gages was 1.37 (see Figure 10). Almost all the individual points were plus or minus 20 per cent of the average, which is surprisingly good considering the many factors which conceivably could build up a maximum wave. This value should be compared with that of 1.37 ± 20% found by Wieglet for ocean waves along the Pacific coast (7) and that of 1.61 for the Pearson Type III frequency function model determined by Putz (8) to most nearly fit the statistical frequency distribution of observed ocean wave heights. The difference between these values is probably attributable to the depth difference between the shallow water encountered in Lake Okeechobee and the deep water of the ocean, but may be due, at least in part, to the fetch limitations imposed on the maturing of the wave spectra by the short lengths of Lake Okeechobee. Experimental results of wind generated waves obtained in a small wave channel by the University of California (9) tend to indicate a value for this ratio of the order 1.2 to 1.45. Unfortunately there was not enough variance in the depths at the points where wave data were obtained in Lake Okeechobee to indicate reliably any dependence of this ratio on depth. This excellent correlation between the maximum and significant wave heights seems to imply that a statistical grouping of waves is generated by a storm, and that this grouping is somewhat similar, though of a different absolute value, whether the storm occurs over a large deep area, as the ocean, or a small shallow area, as Lake Okeechobee.

3. Method of Wave Analysis. In the further analysis of the Lake Okeechobee data it was thought that since any attempt at prediction of wave characteristics must, of necessity, consider average wind and wave conditions, some type of time average wave should be used in the analysis rather than the actual wave occurring at any particular time. This would in effect, eliminate the effect of gustiness on the generation of the wave. This was done by plotting wave height and period against time for each station, which resulted in a very jagged curve; a smooth, average curve was then drawn through this set of points, as shown in Figure 11 for lake station 16 during the 1950 storm. The wave characteristics applicable at any given time were then considered to be given by the smooth, average curve rather than by the actual observed data.
4. As with the data for wind set-up, the wave data during and immediately following the rapid wind shift has been discarded. Since there was no method for determining the direction from which the recorded waves were coming, there was no certainty as to what was the area of generation. The wave might have been traveling in the direction of the wind and hence generated in the short fetch indicated at that time, but it might equally well have been generated in the rear portion of some previous fetch, and be just reaching the point at that time. Hence the fetch and wind velocities applicable to the generation of these waves could not be determined. Likewise omitted were those waves which had an appreciable part of their fetch in the marsh area, as it was thought that the waves generated there might be considerably influenced by the marsh grasses, and hence not comparable to the other waves.

5. Depth-Length Relation of the Okeechobee Data. Sverdrup and Munk (10) have previously considered the question of the generation of waves in deep water. Their theoretical curves have recently been slightly revised by Bretschneider (11) in the light of observed data on the generation of waves in deep water gathered over the past three years by the University of California. However, as the maximum depth in Lake Okeechobee is about 14 feet with an average depth of about 3 feet for these two storms, most of the observed Okeechobee wave data fall into the shallow water classification. The frequency distribution for values of the depth-length ratio \( d/L_o \) for the 1950 storm is shown in figures 12a and b. In Figure 12a the ratio is based on the depth at the gage, while in Figure 12b it is based on the average depth over the entire fetch distance. As may be seen, this does not vary the distribution a great deal. For the vast majority of the observations this ratio falls within the range 0.04 to 0.24 with the maximum number of observations being in the 0.08 to 0.12 range for both cases. Values for the 1949 storm are not shown, but the distribution is similar.

6. Comparison with Deep-Water Relationships. It is not to be expected that the Sverdrup-Munk equations for wave generation in deep water should be exactly applicable to Lake Okeechobee. As a first step, however, it is worth considering the relationship which these data bear to the curves determined for deep water conditions by Sverdrup and Munk. The relationship between wind and wave characteristics is best shown by dimensionless representation. For those conditions where the wave characteristics are controlled by fetch rather than by wind duration, the data for deep water conditions reduces to two diagrams, one representing wave height and the other representing wave period. These may be expressed by representing the dimensionless variables \( gH/v^2 \) and \( gT/v \) as functions of \( gF/v^2 \), where \( H \) is the significant height, \( T \) is the wave period, \( F \) is the fetch, \( V \) is the wind velocity, and \( g \) is the acceleration of gravity. These parameters have been calculated in Table 6 for the Okeechobee data, and are shown graphically in figures 13 and 14.

7. There appear to be several ways in which the wind velocity applicable to wave growth can be chosen. It can be taken as the velocity at the gage, or an average velocity over the fetch; also it can be taken
as the value at a particular time, or as an average value over some time
interval. It seems logical to suspect that a value averaged over both
fetch and some time interval would be more applicable to an analysis of
the generation of the observed waves, particularly when it is remembered
that the observed values of the wave are, in themselves, averages. For
the graphs shown in figures 13 and 14, the time interval over which the
wind velocity should be averaged has been determined as the time that it
takes the wave group to travel over the fetch, and it can be computed as
the quotient of the fetch distance and the group velocity of the wave train.
The group velocity can be determined from the observed period of the waves
at the gage. Since the energy is propagated forward at the group velocity
of the wave train, the time interval determined from the group velocity is
thought to be more applicable than one determined from the wave velocity.
However, plots similar to those in figures 13 and 14 have also been made
for wind velocities averaged over the time interval determined from the
wave velocity; also for velocities averaged over an arbitrary one-hour
interval, and for velocities at the time of the wave observations. Plots
were also made for values averaged over the full fetch and over arbitrary
5 and 10-mile fetches. No significant differences were observed between
any of the series of different plots, and the only ones presented here
are those shown in figures 13 and 14, which, it is believed have a more
physically sound basis.

8. Theoretical curves for wave generation in deep water given by
Sverdrup and Munk and a revised period curve given by Bretschneider are
also shown in figures 13 and 14. It may be noted that the observed points
on the period graph (figure 13) group rather closely about the Sverdrup-
Munk curve, but fall somewhat below the Bretschneider curve; those on the
height graph (figure 14) consistently fall below the deep water curve.
This decrease in wave height and period below the values that would be
computed for deep water with the same wind conditions is to be expected,
and is due both to the effect of depth on the wave velocities, and to
the loss of energy due to friction along the bottom. It may be seen that
the effect of shallow water conditions is somewhat greater for wave
height than for wave period.

9. It is thought that the plotted points lying above the Bretschneider
curves are not too reliable as all of these points were obtained from
station 15 where the fetches were less than one mile. Due to the
variability in direction of wave travel, the fetch lengths applicable to
the observed waves at station 15 probably lie along a direction at some-
what of an angle to the wind, and are considerably longer than if measured
in the direction of the wind. The use of a longer fetch obtained from
considerations of this variability in wave direction would serve to move
these points horizontally to the right, and would probably result in the
points falling below the deep water curves.

10. The points have been segregated according to the station at
which they were observed, and it may be seen that, for any single station,
the points fall rather closely about a curve parallel to the Sverdrup-Munk
curve. These curves for each station are much more clearly defined in the period graph than in the height graph, as might be expected from the large number of factors (particularly refraction) affecting wave height and the small time interval (1 minute) over which the record is obtained. On both graphs the order of the stations is the same, the only significant difference being that, while the curve for station 17b lies below that of station 16b on the period graph, their relative positions are reversed on the height graph. One possible explanation is the refraction conditions at station 17, which would tend to produce higher waves at this station than might be expected from the wind conditions alone. These curves then, would tend to indicate that the complete shallow water solution would result in a family of curves parallel to the deep water curve and which would depend on some additional factor involving a depth parameter. These curves would approach that for the deep water conditions as a limit.

11. Consideration of Possible Depth Parameters. It was thought that the depth parameter differentiating the different curves in a family would be one of the obvious dimensionless numbers, d/H, d/L, d/F, or gd/V^2 where d is the depth (whether the depth at the gage, or the average depth over the fetch), L is the wave length, H is the wave height, and g, F, and V are as before. An investigation of these parameters, however, showed that none of these appears to adequately define the curves. Thijssen (12) has shown that the results of studies of the generation of waves in shallow water carried out by the hydrotechnical laboratory at Delft may be expressed dimensionlessly as a family of curves of the parameters gd/V^2. This family of curves is shown in figure 15, with the observed values of gd/V^2 given beside each plotted point of the Okeechobee data. Two values of gd/V^2 are given, one representing the depth observed at the gage, and the other representing an average depth observed at the gage, and the other representing an average depth along the fetch. As may be readily seen, the the general trend of values is correct, but the scatter of the points about the curves is extremely large. For the Okeechobee data the best definition appears to occur with the parameter dF/d, where dF is the average depth over the fetch.

12. The deep water curves can be represented very closely by straight lines in this region (40 < gF/V^2 < 3000) as can the curves for each station. The intercepts are given by \( gT/V \) \((gF/V^2)^{0.26} \) for the period curve and \( gH/V^2 \) \((gF/V^2)^{0.58} \) for the height curves; the intercept values of straight lines parallel to the deep water curves through each individual observation have been plotted against the value of dF/d in figures 16 and 17. Also shown are the average values for each station. While this appears to give the best definition of all the depth parameters tried, it leaves much to be desired, particularly when it is remembered that the points determined from station 15 are not very reliable.

13. Relation of Maximum Possible Significant Wave to Depth. The Jacksonville District has found that the significant wave height does
not exceed 0.59 times the water depth (3). The averaged heights used in this analysis do not exceed about 0.5 times the water depth, with average wind velocities approaching 80 miles per hour. It is thought, therefore, that for design purposes, at least for depths up to about 20 feet, this ratio of 0.6 may be used to obtain the maximum significant wave height which should be protected against.

14. Relation Between Wave Age and Wave Steepness. Also of some interest is the relationship between "wave age" and "wave steepness". The term wave age, as previously used by Sverdrup and Munk, is the ratio of wave velocity to wind speed (c/V); wave steepness is the wave height-length ratio (H/L). A plot of these two terms is shown in Figure 18 with the curve as derived by Sverdrup and Munk shown for comparative purposes. The fact that the Lake Okeechobee data lies consistently below the Sverdrup-Munk curve is due, to a large extent, to the fact that the wave height from the Okeechobee data is consistently considerably smaller than that which would be obtained from the Sverdrup-Munk curves for a given value of the term gT/V^2; hence the computed values of wave steepness are smaller than the Sverdrup-Munk data. This is compensated for to some extent because, due to the shallow water, the values of wave length and velocity are somewhat smaller than the deep-water values, which tends to make the computed values of wave steepness larger and wave age smaller than the deep-water values associated with the Sverdrup-Munk wave. The latter effect is, however, minor compared to that of the height difference.

15. Future Work. A further theoretical analysis along the lines of the basic Sverdrup-Munk theory for the generation of waves in deep water is contemplated. This analysis will introduce the actual shallow water velocities with their dependency on depth into the equation rather than the simplified deep water velocities which have no dependency on depth. It is hoped that a complete solution to the equations then obtained may be possible, and that by using the data contained herein to obtain the necessary numerical constants, the determination of an accurate method of forecasting waves in shallow water areas can be accomplished.
RELATIONSHIP OF MAXIMUM WAVE TO SIGNIFICANT WAVE HEIGHT

FIG. 10-a

FIG. 10-b

FIG. 10-c

FIG. 10-d

FIG. 10-e

FIG. 10-f

STA. 6 - 1950
STA. 4 - 1950
STA. 8 - 1949
STA. 10 - 1950
STA. 17 - 1950
STA. 12 - 1950
STA. 12 - 1950

MAXIMUM WAVE HT. (FT.)

SIGNIFICANT WAVE HT. (FT.)

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SIGNIFICANT WAVE HT. (FT.)
RELATIONSHIP OF MAXIMUM WAVE TO SIGNIFICANT WAVE HEIGHT FOR ALL STATIONS
HURRICANE DATA FOR LAKE STATION **16, LAKE OKEECHOBEE**

**OCTOBER 17-18, 1950**

**WIND DIRECTION AND VELOCITY**

- **Wind Velocity (mph)**
  - October 17 - October 18
  - Maximum velocity: 58 mph

**Elevation (Feet MSL)**

- October 17 - October 18
- Maximum elevation: 15.6 Feet MSL

**WAVE HEIGHT**

- October 17, 1950 - October 18, 1950
- Maximum wave height: 4.8 Feet
- Significant wave height: 3.8 Feet
- Significant wave height: 3.2 Feet

**MAXIMUM WAVE HEIGHT, SIGNIFICANT WAVE HEIGHT AND SIGNIFICANT PERIOD**

**Fig. 11**
LAKE OKEECHOBEE, 1950 HURRICANE
DISTRIBUTION OF DEPTH-LENGTH RATIO \( \frac{d}{L_0} \)
(109 Observations)

Based on depth at gage

0900 thru 1400 (after wind shift)
1800 thru 0500 (before wind shift)

Based on average depth over fetch

\( \frac{d}{L_0} \)

Fig. 12a

Fig. 12b
REFERENCES


(3) District Engineer, Jacksonville, Fla. - Preliminary Report, Freeboard Determinations, Lake Okeechobee, Fla. 1951.

(4) Haurwitz, B - The Slope of Lake Surfaces Under Variable Wind Stresses. Beach Erosion Board, TM #25, 1951.


