ANALYSIS OF STRAIN-SOFTENING BEHAVIOR OF SOIL

by

John F. Peters, Daniel A. Leavell, Lawrence D. Johnson

Geotechnical Laboratory
U. S. Army Engineer Waterways Experiment Station
P. O. Box 631, Vicksburg, Miss. 39180

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PREFACE

This is a working document describing work performed by the U. S. Army Engineer Waterways Experiment Station (WES) under Project No. AT22, Task AO, Work Unit 004, "Finite Element Analysis of Material Susceptible to Stress Decay," sponsored by the Office, Chief of Engineers, U. S. Army. The research was accomplished in part by Dr. Yannis F. Dafalias and Dr. Leonard R. Herrmann at the University of California, Davis, under Research Contract No. DACA 39-79-M-0059. Their final report is presented in its entirety as a supporting appendix.

The research was limited in scope to the development of constitutive relationships for the mechanical behavior of dilatant materials susceptible to strain softening. Implementation of the constitutive relationships into a prototype finite element analysis was not included in the research. The research performed at WES is part of a more comprehensive independent research effort by Mr. John F. Peters which is to be presented as a Ph. D. dissertation to the University of Illinois-Chicago Circle.

The computer graphics package used for the analysis and presentation of the laboratory data was developed by Mr. Arden P. Park, Soils Research Center (SRC), Soil Mechanics Division (SMD). Mr. William L. Hanks is gratefully acknowledged for the drafting of figures, Ms. Katherine E. Grant and Vicky H. Smith for the typing of the report.

The report was prepared by Mr. Peters, Soils Research Facility (SRF), SRC, SMD, Geotechnical Laboratory (GL), WES, with the assistance of Mr. Daniel A. Leavell, SMD, and Dr. Lawrence D. Johnson, Research Group (RG), SMD. The work was performed under the supervision of Mr. Clifford L. McAnear, Chief, SMD, and Dr. William F. Marcuson III, Chief, GL.

COL John L. Cannon, CE, COL Nelson P. Conover, CE, and COL Tilford C. Creel, CE, were Commanders and Directors of WES during the conduct of the investigation and preparation of this report. Mr. Fred R. Brown was Technical Director.
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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

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PART I: INTRODUCTION

Purpose and Scope

1. The purpose of this report is to present the results of a study involving (a) development of models for simulating strain-softening behavior of soils, (b) laboratory triaxial strength tests to obtain stress-strain data on strain-softening soils, (c) verification of the models by numerical analyses using results of the laboratory strength tests, and (d) implementation of the most suitable model into a subroutine that can be incorporated into an appropriate finite element computer code.

2. Two incremental elastoplastic models for simulating the strain-softening behavior of soils were developed as part of this study. Laboratory triaxial shear strength tests were performed on two highly plastic clay soils and one low plasticity clay to determine their drained and undrained stress-strain response. The minimum requirement to evaluate parameters for the models consisted of testing soils at two values of overconsolidation ratio (OCR). The values of OCR selected for testing of each soil were 1.2 and 15, which required a minimum of four strength tests on each soil (two drained and two undrained). These two OCR's were selected to simplify evaluation of the material parameters required to use each of the models and to evaluate the model behavior under both drained and undrained conditions. The models were analyzed for relative usefulness by first determining the material parameters from drained strength data, predicting the undrained stress-strain behavior in triaxial shear, and comparing behavior predicted by the models with observed behavior for results of the undrained triaxial shear tests. Additional tests were performed at other values of OCR to determine the ability of the models to predict behavior at intermediate values of OCR.

3. The report is presented as five parts. Part I provides introductory background material on the significance of strain softening to geotechnical analysis. In particular, the role of strain softening in
establishing a correspondence between traditional limit equilibrium analyses and the finite element analyses is described. Parts II and III present the description of the testing program and a summary of test results. The formulations of the two constitutive models are outlined in Part IV, and the predictions of the models are compared with laboratory data. Part V presents conclusions on the performance of the model with recommendations for future research. Appendix A presents the formulation of constitutive relationships using the theory for elastoplastic strain-hardening materials. Appendix B discusses stress-dilatancy theories and the Hvorslev failure law. Bounding surface plasticity for cohesive soils is discussed in Appendix C, reproduced from a report by Drs. Yanis F. Dafalias and Leonard R. Herrmann. The computer code for the bounded frictional-dilatant flow is described in Appendix D, while Appendix E presents laboratory test data.

Background

4. The determination of soil strength has a predominant role in the mechanical analysis of earth structures. In part, the importance of strength stems from the widespread use of limit analyses in geotechnical design as exemplified by current practice in determination of earth pressures, bearing capacity, and slope stability. Consideration of soil deformation is generally restricted to settlement computations which are often empirically related to soil strength (e.g., Peck, Hanson, and Thorburn 1974). The use of semiempirical limit equilibrium methods has the practical benefit of reducing otherwise intractable problems to a statically determinant form which can be analyzed with relatively straightforward computations.

5. A fundamental shortcoming of limit equilibrium methods is that they ignore the deformation characteristics of the soil which has an influence on the limit load (Roscoe 1970). The importance of considering soil deformation in the determination of earth pressures was recognized by Terzaghi (1920), who noted that the constant pressure distribution could be unconservative for braced excavations since deformations are
inhibited and the full soil strength is not mobilized. Traditional design of such structures depends heavily on semiempirical pressure distributions developed from field observations and model tests (Tschebotarioff 1973), to account for the influence of deformation. Slope stability computations are similarly complicated by the deformation characteristics of the soil, since the level of mobilized shear strength can vary greatly along a potential failure surface (Peck 1967). Further, the stress-strain behavior of soil determines the postfailure deformation characteristics of a slope which can morphologically vary from that of a nonprogressive slump to a large-scale progressive flow slide.

6. Modern computer-based numerical methods have resolved many of the mathematical difficulties by incorporating the true stress-strain behavior of soil into design analyses. Unfortunately, the problems resolved by improved tractability of mathematical solutions have been replaced by difficulties in describing the mechanical behavior of soil. These difficulties arise from three areas. First, until recently, detailed constitutive laws for soil had not been available that would accurately model soil behavior and yet be of suitable form for implementation in computer codes. Second, constitutive models that provide the best simulation of soil behavior may not lead to a mathematically well posed problem when incorporated into full-scale numerical analyses. Finally, the numerous parameters required to implement complex constitutive relationships into numerical analyses tax the laboratory resources available for most routine design projects. The last consideration is particularly important since theoretical advances will not obtain their full potential in geotechnical design unless tempered by experience. For numerical analyses to achieve the level of reliability required by the designer, their routine use must be established within the design rather than the research environment.

7. Strain-softening behavior of soils poses a major problem to the development of finite element codes for routine use in geotechnical analyses. As a practical definition, strain softening can be described as a loss of shear strength after a peak strength has been reached (Prévost and Höeg 1975). The problems created by softening behavior
arise directly from mathematical aspects of the finite element methods and indirectly from the implications that softening behavior has on the generalization of constitutive relationships for soils. Importantly, strain softening is governed by the state of the stress, drainage conditions, and compactness of the soil (dry density or void ratio). Any soil can display softening behavior. Thus, in developing constitutive relationships to account for softening, consideration must be given to both the mathematical formulation of the model and the physical phenomena that the model represents.

Implication of Strain Softening to Analyses Procedures

8. Strain softening represents unstable material behavior which has special implications to both limit equilibrium and finite element analyses. In each case, the loss of shear resistance after a peak stress is reached constitutes the central difficulty in the analysis regardless of the physical mechanisms causing the softening to occur.

Limit equilibrium stability analysis

9. Limit equilibrium analyses include earth pressure, bearing capacity, and slope stability computations. Generally, these problems are made statically determinant by combining the equations of equilibrium with the failure criteria (Lee and Moore 1968), which gives a bound on the true solution. The utility of the limit equilibrium analysis lies in the fact that the upper and lower bounds solutions are sufficiently close that the true collapse load can be approximated with sufficient accuracy (Davis 1968). In general, the rigorous application of the bounding solution is difficult for many problems, and practical limit equilibrium methods involve expeditious simplifying assumptions which, from a mathematical standpoint, render the applicability of the bounding theories invalid (Lee and Moore 1968). However, experience has shown that most limit equilibrium methods currently in use do provide sufficiently accurate solutions for determinant analyses. Therefore, the discussion on the implication of strain-softening behavior on limit analyses will be approached from the standpoint of practical limit
methods rather than the mathematically elegant approach of using the bounding theories directly.

10. The basic elements of the limit equilibrium for slope stability problems are depicted in Figure 1. Generally, the location of the potential failure surface is assumed and the stability of the mass bounded by the assumed failure surface computed. By an iterative selection process, the failure surface giving the lowest factor of safety is found. Assuming that the iteration procedure used to obtain the critical failure surface is reliable, the accuracy of the method hinges on the ability to define the stresses acting on the potential failure surface. For example, the failure mass can be broken up into a series of slices and the equilibrium of each slice considered individually (Figure 1b). The problem, as presented, is statically indeterminant. However, it can be rendered statically determinant by appropriately assuming the magnitude of the forces on the sides of the slices (Lambe and Whitman 1969). Regardless of how the side forces are determined, the factor of safety of the slope is given by:

\[
F = \frac{\tan \phi \sum_{i=1}^{n} \bar{N}_i}{\sum_{i=1}^{n} W_i \sin \theta_i}
\]

where

- \( F \) = factor of safety against sliding
- \( \phi \) = effective Coulomb friction angle of soil
- \( \bar{N}_i \) = force acting on \( i^{th} \) slice normal to slip surface
- \( W_i \) = weight of \( i^{th} \) slice
- \( \theta_i \) = angle between slip surface and horizontal at \( i^{th} \) slice

and where it is assumed that \( T_i = \bar{N}_i \tan \phi \) at failure. If the soil does not soften, then any movement beyond that required to mobilize the available strength will require that additional work be performed (such as placing additional loads at the top of the slope). Thus, the factor of safety corresponding to the trial surface is bounded by the average condition indicated by Equation 1 since \( F > 1 \) implies that
A. POTENTIAL SLIDE

\[ \tan \theta \sum_{i=1}^{n} \bar{N}_i \]

\[ F = \frac{\sum_{i=1}^{n} W_i \sin \theta_i}{\sum_{i=1}^{n} W_i \sin \theta_i} \]

B. FORCES ON SLICE

\[ T_i = \bar{N}_i \tan \theta \]

C. SHEAR RESISTANCE VS DISPLACEMENT

Figure 1. Limit equilibrium analyses using method of slices
\( T_i/N_i < \tan \phi \) for all slices regardless of the actual slope of the stress-strain curve.

11. If the soil can strain soften, the accuracy of the stability calculation does not depend solely on an accurate determination of \( T_i \) and \( N_i \). For example, if at any point on the trial surface, the stress state is equal to the peak strength, any perturbation of the slope would cause the soil to unload (soften) at that point. Since Equation 1 does not differentiate between the hardening and softening portions of the stress-strain curve, \( F > 1 \) does not ensure that the slope is stable. Thus, Equation 1 cannot be related to either an upper or lower bound of \( F \).

12. The inability to relate the factor of safety to a bounding solution greatly complicates the use of limit equilibrium methods for stability analyses. If the peak strength of the soil is used, Equation 1 may greatly overestimate the factor of safety of the slope. Numerous case histories of failed slopes in soils having peak strengths significantly greater than their residual strengths (Skempton 1964; Bjerrum 1967; D'Appolonia, Alperstrin, and D'Appolonia 1967), clearly demonstrate that the peak strength does not give a correct factor of safety for strain-softening soils. On the other hand, Bjerrum (1967) found the use of residual strength to be overconservative in many cases.

13. The difficulty in choosing the correct strength value for strain-softening soils lies in the fact that limit equilibrium analyses do not account for the deformation required to mobilize the soil strength. As noted by Taylor (1948), Turnbull and Hvorslev (1967), and Bishop (1967), the state of strength mobilization on a potential failure surface is not constant. Further, Peck (1967) showed that because the strain required to obtain peak strength depends on the effective normal stress, the peak strength cannot be mobilized simultaneously at the limit condition. The maximum strength that can be mobilized along a sliding surface is less than the peak but greater than the residual strength (Peck 1967). Therefore, the appropriate strength value that should be used in a particular analysis does not depend solely on the soil properties, but rather depends on a combination of soil properties.
and the geometry, loading condition, and stratigraphy of the slope. A similar line of argument can be directed toward other limit equilibrium problems including those associated with foundation design.

**Finite element analysis**

14. The finite element method can be used to obtain solutions to stress analyses problems that fully account for the stress-strain behavior of the soil. The ability of the finite element method to model the limiting condition of the soil has been questioned on the basis that displacements must necessarily be continuous to obtain a proper solution. This requirement precludes the formation of failure planes and implies that the rupture-type failure cannot be modeled by the finite element approach.

15. The dichotomy between the limit equilibrium approach, which assumes the existence of a rupture surface, and the finite element method, which precludes the formation of the rupture surface, can be resolved when it is realized that "...both types of solutions must satisfy the same fundamental differential equations of equilibrium" (Schofield and Wroth 1968). The difference between the two types of analyses is created by the constitutive relationships which determine whether a continuous displacement field (elliptic problems) or a discontinuous field (hyperbolic problems) will prevail. The ability of the finite element method to model the limit equilibrium state depends on the behavior of the solution as the "ellipticity" of the governing equations is lost.

16. The ability of the finite element solution to model the limit equilibrium state has been discussed in detail by Rice (1976) and Prévost and Hughes (1980 and 1981). The general finite element solution can be reduced to the incremental relationship

\[
[K] \{da\} = \{df\}
\]

where

- \([K]\) = incremental coefficient matrix
- \(\{da\}\) = unknown displacement increments
- \(\{df\}\) = known body and boundary force increments

The ability to obtain a unique solution for Equation 2 depends on the
ability to invert the matrix \([K]\). Note that, by definition, at the limiting state, unlimited displacements result without an increase in load. Thus,

\[
[K] \{\text{da}\} = 0
\]  

(3)

The solution for Equation 3 is meaningful only if \([K]\) is singular (cannot be inverted) since otherwise \(\{\text{da}\} = 0\). Thus, it can be reasoned that, provided the invertibility of \([K]\) depends solely on the constitutive relationship (and not on incorrectly prescribed boundary conditions), Equation 3 describes the condition for which the limiting condition is reached.

17. The matrix \([K]\) represents an assemblage of elements with each element having a stiffness relationship given by:

\[
\]  

(4)

where

- \([K_i]\) = stiffness matrix for element \(i\)
- \(\{\text{d}e\} = [B]\{\text{da}_i\} = \text{total incremental strain}
- \(\{\text{d}\sigma\} = [D^{\text{EP}}]\{\text{d}e\} = \text{incremental effective stress}
- \([B]\) = matrix relating strain to displacement
- \([D^{\text{EP}}]\) = elastoplastic stiffness matrix
- \(\{\text{da}_i\}\) = displacements associated with \(i^{th}\) element

Provided the problem is properly formulated, the assembled \([K]\) will be nonsingular if \([D^{\text{EP}}]\) is everywhere nonsingular.

18. The incremental stiffness relationship \([D^{\text{EP}}]\) for elastic-plastic materials was derived in Appendix A as:

\[
[D^{\text{EP}}] = [D^e] - \frac{1}{L}[D^e]\{\beta\}\{N\}^T[D^e]
\]  

(5)

where

- \(L = \{N\}^T[D^e]\{\beta\} - H\)
- \(H = \text{hardening modulus}
- \(\{\beta\} = \text{unit vector of plastic strain increment direction}
- \{N\} = \text{normal to yield surface}
- \([D^e]\) = elastic stiffness matrix

As shown by Prévost and Hughes (1980 and 1981), and Lade and Nelson
(1981), $[D^{ep}]$ is singular if $H = 0$. Further, one condition for localization of shear bands (failure planes) is that $H < 0$ (Rice 1976). Thus, an important requirement for achieving realistic finite element analyses is that the constitutive relationships be based on the correct criteria for $H \leq 0$.

19. The relationship between the stiffness characteristics for each element and for the assemblage at the limiting state is shown in Figure 2. If the hardening modulus is greater than zero within all elements, then the assemblage has the capability to absorb energy from applied load increments $\{df\}$ and a unique displacement $\{da\}$ will be obtained. However, $[K]$ will be singular if the state within a contiguous band of elements is such that (a) $H < 0$ and (b) the direction of the inelastic displacements associated with each element are compatibly aligned to permit rigid body movement of a section of the assemblage. Prévost and Hughes (1980 and 1981) have shown that it is possible to obtain good estimates of both collapse loads and localization of shear zones using finite element analyses provided the correct conditions can be specified for which $H \leq 0$.

**Possible Strain-Softening Mechanisms**

20. A number of mechanisms have been proposed to explain differences between peak and residual strengths. Since the approach to modeling mechanical behavior should be directed by consideration of the mechanisms involved, a brief review of strain-softening mechanisms is of interest.

**Cementation and diagenesis**

21. If interparticle resistance is derived partly from cementation it can be expected that a falloff of strength will occur once the peak strength has been reached. Cementing at interparticle contacts can result from precipitation of free carbonates, iron oxide, alumina, and organic material (Mitchell 1976). As the interparticle bonding is destroyed by disturbance (strain) a large loss of strength results. Also as noted by Terzaghi (1955), intergranular bonding can inhibit
A. FINITE ELEMENT MESH OF SLOPE

B. STRESS-STRAIN CHARACTERISTICS

Figure 2. Finite element analysis of slope at limit for strain-softening soil
compression after deposition, tending to give the soil a metastable structure which is highly sensitive to disturbance.

22. In an attempt to classify soils with respect to their potential to fail progressively, Bjerrum (1967) emphasized the significance of diagenetic bonds. Diagenetic bonds are zones of contact between soils that might develop adhesion from molecular bonds or precipitation of cementing agents. During the process of physical and chemical weathering, these bonds can break down. Soils with the greatest potential for progressive failure are overconsolidated plastic clays with strong bonds that have been exposed to gradual disintegration by weathering. Unweathered and weathered overconsolidated plastic clays with weak or no bonds are also subject to high potential for progressive failure. Soils with the least potential for failure include unweathered plastic overconsolidated clays with strong bonds and overconsolidated clays with low plasticity.

23. Thixotropic hardening creates an effect similar to cementation and can contribute to the sensitivity of soil strength to strain. As defined by Mitchell (1976), thixotropy is an isothermal, reversible, time-dependent process occurring under conditions of constant composition and volume, whereby a material stiffens while at rest and softens or liquifies upon remolding. In addition to sensitivity, thixotropic hardening may partly explain the increase in peak strength that results from the aging of remolded specimens (i.e., time period between remolding and shearing).

Fabric change

24. Soil fabric is a commonly used (though loosely defined) term which refers to the geometrical arrangement of particles, particle groups, and pore spaces in a soil (Mitchell 1976). As strain can be conceived to result from interparticle movements within a soil mass, the soil fabric changes as the soil is loaded. For example, a remarkable association between soil texture after drying and consolidated stress state was shown by Hvorslev (1937) which clearly indicated the influence of stress history on particle arrangement. While all soils display anisotropy as a result of fabric (Oda 1981), clayey soils undergo
dramatic changes in interparticle orientation during shear. At large strains where discrete failure planes are formed, clay particles can become aligned face to face, creating a "slickenside" surface (Lambe and Whitman 1969). Once such a surface is formed, shear resistance is derived entirely from interparticle friction. The difference between the peak Coulomb friction angle and the true interparticle friction angle is generally small for soils of low plasticity. However, for highly plastic soils, the Coulomb friction angle can range from over 20 deg at peak strength to as low as 3 to 4 deg once the slickenside surface is formed.

25. A mechanism of brittle creep that can result in failure in overconsolidated soils and rocks was proposed by several authors (Lo and Lee 1973; Feda, Kamenov, and Klablena 1973; Nelson and Thompson 1977; and Cogan 1978). Failure during brittle creep is thought to be caused by deterioration of strength from the accumulation of plastic strain. Microcracks develop and coalesce into shear fracture. Rupture occurs when the accumulative strain from creep at a particular deviator stress equals the postpeak strain. Any level of shear stress above the residual shear strength will lead to creep along structural discontinuities. Prediction of the lifetime to failure in slopes was made using this mechanism (Lo and Lee 1973, Nelson and Thompson 1977).

26. An important feature of softening related to fabric change and brittle creep is that both depend on a localized zone of shear within the soil mass. Such zones are commonly observed in soil test specimens and field situations. However, in view of the discussion in paragraphs 11 to 16, localized shear zones are the result, rather than the cause of strain-softening behavior.

Creep rupture

27. Creep can be defined as continuing strain under a constant stress state. Creep is one manifestation of time-dependent behavior. It has been found that soils that demonstrate large creep strains are susceptible to creep rupture, characterized by failure under a sustained creep strain at a constant shear stress that is significantly less than the peak strength measured in a test of relatively short duration.
(Mitchell 1976). Such soils also display a dependency between peak shear strength and loading rate. While creep rupture is not equivalent to strain softening, it has been cited as a contributing factor to the progressive failure of slopes (Skempton 1964) and represents a source of mathematical instability similar to strain softening (Drucker 1964a).

**Dilatancy**

28. When a soil is loaded under drained conditions, it undergoes contraction or dilation that is related to the ratio of shear and normal stress. The relationship between the dilatancy rate and stress ratio have been proposed for cohesionless soils by a number of researchers (see Appendix B), although most theories can be stated in the same generalized conceptual form:

\[
\text{Total strength} = \text{friction component} + \text{dilatancy component} \quad (6)
\]

The significance of Equation 6 can be seen from a comparison of stress-strain behavior for dense and loose sand, as depicted in Figure 3. It is seen that the stress ratio and dilatancy rate are coupled such that the soil is dilating for stress ratios above the constant volume value and contracting for ratios below the constant volume value.

29. Schofield and Wroth (1968) present a concept for a generalized model in which soils that are dilating at the peak stress ratio tend to soften, whereas soils that tend to contract throughout loading reach their maximum strength asymptotically with no distinct peak strength. Since the strength of a soil is partly governed by void ratio, dilatancy should weaken the soil. Moreover, it should be expected that dilatancy be related to unstable behavior since no material, regardless of its makeup, can be expected to dilate unbounded under an increasing compressive stress.

30. The concept that strain softening can be related to dilatancy has special appeal as it lends to an elegant scheme by which the behavior of a material can be categorized in terms of its void ratio and mean stress state (Schofield and Wroth 1968). The stress dilatancy concept works equally well for both clays and sands, although the influence of cementation, diagenetic bonding, thixotropy, and changes in soil fabric
Figure 3. Comparison of stress-strain curve and volume change characteristics for drained tests
and creep may all be superimposed on the dilatancy-softening relationship.

**Model versus Mechanism**

31. From the preceding discussions on strain softening, two methods for modeling strain-softening behavior emerge. The first method relates the hardening-softening process explicitly to strain and gives a relationship for the hardening modulus $H$ (Equation 5) of the form:

$$H = f(S)$$

where $S$ is an increasing function of strain. The assumption of a unique relationship between $H$ and strain leads to a relatively simple model provided the function $f$ can be determined. Models based on this concept have been proposed by Rowe (1971), Prévost and Höeg (1975), Prévost (1978), and Baladi and Rohani (1979).

32. While the assumption of a unique relationship between hardening-softening and strain is computationally simple, it makes determination of material parameters difficult. By implication, such a relationship should exist for those soils where stress-strain behavior is dominated by all of the mechanisms described in the previous section except dilatancy. Unfortunately, dilatancy tends to be the dominant feature of soil behavior even for highly plastic clays. As a result, the parameters that describe the function $f(S)$ are dependent on the initial conditions and loading path. In general, models based on Equation 7 are best suited for analyses of problems in which undrained behavior can be assumed but at best these models remain highly dependent on curve-fitting of laboratory data.

33. An improvement in the modeling process can be made by recognizing that void ratio changes play a dominant role in the mechanical behavior of soil. As shown in Appendix A, the hardening modulus can be explicitly related to the dilatancy rate by the relationship:

$$H = \frac{\partial f}{\partial S} - (1 + e) \frac{\partial f}{\partial e} dV$$

where $S$, $e$, and $dV$ are the strain, void ratio, and volume change, respectively.
where

\[ f = \text{yield surface} \]
\[ = f(\sigma, S, e) \]
\[ e = \text{void ratio} \]
\[ S = \text{strain parameter} \]
\[ dV = \text{volume change increment} \]
\[ \sigma = \text{effective stress} \]

The modulus \( \partial f/\partial S \) is always positive and \( H \leq 0 \) only if the quantity \((1 + e) \partial f/\partial e \, dV\) is positive. Since \( \partial f/\partial e \) is always negative, \( H \leq 0 \) when \( dV \leq 0 \). By definition, this occurs when the soil is dilating. Thus, Equation 8 predicts that the peak strength and softening conditions occur only when the soil is dilating and that the ultimate residual state is obtained when \( \partial f/\partial S = dV = 0 \).

34. The use of Equation 8 appears to replace the problem of determining \( f(S) \) with the equally difficult problem of finding a suitable relationship for \( f(\sigma, S, e) \). However, since the rationale behind Equation 8 is more closely related to the true mechanical behavior of soil, determination of suitable mathematical forms for \( f(\sigma, S, e) \) is generally straightforward. Note that if behavior is greatly controlled by cementation, thixotropy, and fabric change, \( \partial f/\partial S \) may be negative and the resulting model may become more complex. Also, Equation 8 does not include provisions for time-dependent (viscous) effects such as creep.

35. The mechanisms modeled by Equation 8 are equally valid for both drained and undrained behavior provided some provision has been included in the problem formulation to independently account for changes in pore pressures. Methods to formulate problems for saturated soils under undrained conditions are discussed in Appendix A.
PART II: DESCRIPTION OF TESTING PROGRAM

Testing Philosophy

36. The testing program was designed to evaluate the theoretical basis for the dilatancy-softening model for soil behavior under both drained and undrained conditions. Since it has generally been observed that both the softening and dilatant tendencies are greater for soils with a high OCR, the testing program was designed to make OCR the primary independent variable. Further, based on the recommendation of Dafalias and Herrmann (Appendix C), OCR's of 1.2 and 15 were selected as the two main test values. The development of the constitutive models was contemporaneous with the laboratory testing and tests were added to the program as needed to clarify the theoretical questions.

37. To reduce the differences in physical characteristics among the specimens tested, all specimens were subjected to the same compression history prior to rebounding to achieve the final OCR. Two factors were considered important. First, the specimens should be virtually identical with respect to void ratio, water content, and preparation procedure prior to initial consolidation in the triaxial cell. Second, the maximum consolidation pressure in the triaxial cell should be sufficiently large to ensure that at least the final two consolidation increments corresponded to loading along the virgin compression curve. For all three soil types tested, reconstitution of the specimens in a slurry consolidometer to a vertical stress of 40 psi* and isotropic compression in the triaxial cell to a maximum effective pressure of 150 psi was found to meet both criteria.

38. An important theoretical question to be addressed by the research was the applicability of the stress dilatancy concept to modeling the mechanical behavior of highly plastic clays. The soils chosen for testing permitted a comparison in behavior of soils with greatly different plasticity characteristics but with the same stress history.

* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 3.
Materials Tested

39. The physical characteristics of the three materials tested are shown in Figure 4. The plasticity characteristics of the three clays are summarized in Figure 5. From comparison of the grain size curves, it can be seen that the materials can be distinguished by their clay content (material finer than 0.002 mm) with the Vicksburg silty clay having the lowest clay content (17 percent), the buckshot clay intermediate (40 percent), and the Yazoo clay the highest (77 percent). This trend is reflected in the plasticity characteristics with the Vicksburg silty clay plotting clearly within the CL range, the buckshot clay near the border between CL and CH, and the Yazoo clay at the far end of the CH portion of the chart. The activity of the clay fraction of each soil (Figure 5b) falls within the 0.76 to 1.0 range, suggesting that the plasticity of the soils is related to the percentage of clay-size particles rather than the plasticity of the clay fraction. While activity gives only a rough measure of clay mineralogy, activities greater than 0.5 suggest that a significant amount of clay minerals from the illitic and montmorillonitic groups are present.

Equipment and Procedures

Equipment

40. A schematic diagram and photograph of the equipment used for the triaxial testing are shown in Figure 6. This equipment was modified from the triaxial device described by Donaghe and Townsend (1975). Modifications to the device consisted of changes in the pressure supply and back pressure system with the mechanical operation of the loading system left intact. Chamber and back pressures were applied using compressed air controlled by pneumatic pressure regulators. All pressures were measured with a high-compliance transducer to the nearest 0.1 psi.

41. The pore pressure, back pressure, and cell pressures were measured sequentially by selecting the proper position of valve L (Figure 6a). In addition, the air pressure applied to the test chamber
a. Vicksburg silty clay

b. Buckshot clay

Figure 4. Physical characteristics of Vicksburg silty clay, buckshot clay, and Yazoo clay (Continued)
c. Yazoo clay

Figure 4. (Concluded)
Figure 5. Plasticity characteristics of Vicksburg silty clay, buckshot clay, and Yazoo clay
Figure 6. Schematics and photograph of triaxial equipment (Continued)
b. Photographs

Figure 6. (Concluded)
could be monitored independently by a pressure gage. The force applied to the piston was measured using a strain gage load cell calibrated to within 0.1 lb. Changes in sample height were measured with a dial indicator reading 0.01 mm per division and with a displacement potentiometer calibrated to the nearest 0.001 in. During shear, the load cell, pore pressure transducer, and displacement potentiometer readings were automatically plotted on x-y recorders. During saturation, volume changes were measured in a 25-cc capacity glass burette and during consolidation and drained shear with a 5-cc capacity glass burette. Both burettes were etched for reading to the nearest 0.1 cc per division.

Sample preparation

42. Test specimens were trimmed from 8-in.-diam cakes consolidated from a slurry in a one-dimensional consolidometer (Figure 7). This consolidometer was designed and fabricated at Northwestern University, Evanston, Ill. (Krizek and Sheeran 1970). Slurry preparation and placement techniques are described by Donaghe and Townsend (1975). The slurry was consolidated in stages of 2.0, 4.0, 8.0, 16, 32, and 45 psi and rebounded to a zero load in stages of 30, 15, and 0 psi. The final slurry cake (Figure 7c) was cut into 10 to 20 samples (depending on the cake height). Each sample was of sufficient size to trim a 1.4-in.-diam by 3.5-in.-length triaxial specimen. The samples were sealed and stored in a humid room until trimmed for testing. The slurry consolidated specimens were quite uniform in water content (Figure 8) and this consolidation behavior in the triaxial test was highly reproducible.

Specimen setup

43. The specimens were trimmed in a humid room, measured and weighed prior to setup. Each specimen was separated from the bottom and top platens by a 1.4-in.-diam by 0.125-in.-thick porous disk of sintered stainless steel covered by a filter paper disk cut from filter paper (Whitman No. 1 chromatography paper). Radial drainage was facilitated by covering 50 percent of the specimen's circumferential area with a filter paper cage, also cut from Whitman No. 1 chromatography paper. Two Trojan membranes were used to encase the specimens and were sealed with rubber O-rings at both the top and bottom platens.
Figure 7. Schematic and photographs of slurry consolidometer and typical slurry cake
Figure 8. Water content distributions along axis of slurry consolidated cakes
44. Prior to specimen setup, the bottom pore pressure line and porous disk were saturated. The top porous disk, pore pressure line, and 5-cc burette were left dry to facilitate evacuation of air from the specimen and filter paper. After the membranes had been placed on the specimen and sealed, the chamber was assembled and filled with de-aired water. A vacuum was then applied through the top pore pressure line. The pore pressure was monitored at the base of the specimen (setting 2 on valve L, Figure 6) until a pressure of less than -8 psi was measured. The specimen was closed off at valves C and D and valves J, H, and I opened to bring the back pressure system to equilibrium. Water was permitted to flow to the 5-cc burette and to the top pore pressure line by opening valve G. Valve F was closed and water was introduced to the specimen through valves C and D.

Back-pressure saturation and consolidation

45. The back-pressure saturation process was initiated by setting a differential pressure between cell and back pressure (valve A at setting 1). The differential pressure used during the back saturation was 5 psi for the Yazoo and Vicksburg silty clays and 10 psi for the buckshot clay. The back pressure was increased to 0 psi and the back saturation process initiated. The back pressure was increased to the final 50-psi value in increments of 5 psi. The volume in the 25-cc burette was measured at each increment. Also at each increment, the pressure differential between the cell and back pressure was checked at the transducer (settings 4 and 2 on valve L, respectively) and if necessary adjusted to the correct valve. Once the final back pressure was reached, the specimen was left overnight to achieve full saturation prior to consolidation. Since the slurry consolidated samples were nearly saturated prior to setup (S = 98 to 100 percent), the back saturation pressure was considerably in excess of that normally required (EM 1110-2-1906). However, the Vicksburg silty clay sometimes required flushing of 5 to 10 cc of water through the specimen to achieve a rapid pore pressure response. Flushing was accomplished by permitting water to flow from the 25-cc burette through the specimen into the 5-cc burette with valve G closed.
46. The specimens were consolidated at stages corresponding to effective pressures of 10, 20, 40, 80, and 150 psi. The time-volume change data were obtained for all steps to ensure the specimens were fully consolidated prior to beginning the next step. The sample height and pore pressure response (B value) were measured at the beginning of each consolidation step. The stages used for sample rebound varied depending on the final OCR valve. However, rebound increments were not permitted to exceed the back pressure (50 psi) to avoid cavitation of the pore water within the specimen.

Shearing

47. All tests were performed with a constant confining pressure and constant axial strain rate. The strain rates used were 0.005 percent/min for the Yazoo clay, 0.02 percent/min for the buckshot clay, and 0.02 percent/min for the Vicksburg silty clay.

Reduction of data

48. The variables measured directly from the test data were as follows:

Axial Strain ($\varepsilon_a$):

$$\varepsilon_a = \ln \left( \frac{L_0}{L} \right)$$

where

$L_0$ = original sample length
$L$ = length of specimen

Stress Difference ($q = \sigma_a - \sigma_r$):

$$q = \frac{P}{A_{cor}}$$

where

$P$ = axial load
$A_{cor} = \frac{A_o}{1 - \varepsilon}$
$A_o$ = initial specimen area
$\varepsilon = \frac{\Delta L}{L_0}$

Mean Effective Stress ($\bar{\sigma} = \frac{1}{3}(\bar{\sigma}_a + 2\bar{\sigma}_r)$):
\[ \bar{p} = \frac{1}{3} (q + 3\bar{\sigma}_r) \]

where
\[ \bar{\sigma}_r = \sigma_r - u \]
\[ \sigma_r = \text{cell pressure} \]
\[ u = \text{pore-water pressure} \]

**Void Ratio (e):**
\[ e = \frac{V_v}{V_s} \]

where
\[ V_v = V_{v_0} - \Delta V \]
\[ V_{v_0} = \text{initial volume of voids} \]
\[ \Delta V = \text{change in volume measured in the 5-cc burette} \]
\[ V_s = \text{volume of solids in specimen} \]

49. To facilitate determination of parameters for the constitutive models, the following numerical data were also computed:

**Total Dilatancy Rate (D):**
\[ D = 1 + \frac{1}{1 + e} \frac{de}{d\varepsilon_a} \]

where \( e = \text{current void ratio} \)

**Constant Volume Stress Difference (q_{cv}):**
\[ q_{cv} = q + 3p \left( \frac{1-D}{2+D} \right) - 3\kappa \ln 10 \frac{dp}{2D(1+e) d\varepsilon_a} \]

where \( \kappa = \text{slope of elastic rebound curve on } e-\log_{10} p \text{ plot} \)

**Hvorslev Equipment Pressure (p_e):**
\[ p_e = p_o 10 \exp \left( \frac{e - e_o}{C_c} \right) \]

where
\( e_o = \text{reference void ratio} \)
\( p_o = \text{pressure on virgin curve corresponding to } e_o \)
\[ e = \text{current void ratio} \]

\[ C_c = \text{slope of virgin compression curve on } e-\log_{10} p \text{ plot} \]

50. The equation for \( q_{cv} \) is equivalent to the energy correction proposed by Roscoe, Schofield, and Thurairajah (1963) and has important implications with respect to constitutive relationships based on either stress dilatancy theory or critical state theory. The Hvorslev equivalent pressure \( p_e \) has special implications to critical state models and cap models in general. The implications of stress dilatancy theory and the use of \( p_e \) in normalizing test data is discussed in detail in Appendix B.
PART III: LABORATORY TEST DATA

Compression Data

51. The isotropic compression characteristics of the three soils tested (Figure 9) indicate that all three samples reached the virgin compression state prior to rebounding to the final OCR values. The characteristics of the virgin curves needed to define the Hvorslev equivalent stress (equation) are tabulated below ($p_o = 1$ atm (14.7 psi)):

<table>
<thead>
<tr>
<th>Soil</th>
<th>$C_c$</th>
<th>$e_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vicksburg silty clay (CL)</td>
<td>0.15</td>
<td>0.65</td>
</tr>
<tr>
<td>Buckshot clay (CH)</td>
<td>0.31</td>
<td>0.86</td>
</tr>
<tr>
<td>Yazoo clay (CH)</td>
<td>0.63</td>
<td>1.77</td>
</tr>
</tbody>
</table>

It is seen that the virgin compression curves are flatter and possess lower $e_o$ values as the plasticity characteristics decrease; the relationship between the slope of the compression curve and plasticity is similar to trends noted by Schofield and Wroth (1968).

Shear Test Data

Stress-strain data

52. The stress-strain characteristics are depicted by relating the rates $q/2p$ to the axial strain (Figures 10-15). For the undrained tests, a plot of pore pressure $u$ as a function of strain is also shown. For drained tests, a plot of void ratio versus strain is shown. These data are tabulated in Appendix E.

53. The relationship between volume change characteristics, strength, and $p/p_e$ is well illustrated by the data from the drained tests on the Vicksburg silty clay. Specimens CL-2 and CL-8 were tested with respective initial $p/p_e$ ratios of 0.774 and 0.878. Both of the specimens displayed contractive behavior throughout shear and neither displayed a distinct peak. By contrast, specimen CL-3 sheared at a much lower $p/p_e$ ratio of 0.097, displayed dilative behavior, and obtained
Figure 9. Isotropic compression curves for Vicksburg silty clay, buckshot clay, and Yazoo clay
Figure 10. Drained stress, strain, and void ratio data for Vicksburg silty clay (data tabulated in Appendix E)
Figure 11. Undrained stress, strain, and pore pressure data for Vicksburg silty clay (data tabulated in Appendix E)
Figure 12. Drained stress, strain, and void ratio data for buckshot clay (data tabulated in Appendix E)
Figure 13. Undrained stress, strain, and pore pressure data for buckshot clay (data tabulated in Appendix E)
Figure 14. Drained stress, strain, and void ratio data for Yazoo clay (data tabulated in Appendix E)
Figure 15. Undrained stress, strain, and pore pressure data for Yazoo clay (data tabulated in Appendix E)
its peak strength at a strain of about 1 percent. Specimen CL-4 sheared at an intermediate \( \frac{p}{p_e} \) ratio of 0.336, showed little volume change tendency, and had an initially stiff stress-strain response but with only a gradual monotonic increase in strength beyond an axial strain of 0.8 percent. Thus, the four specimens clearly show the trend from contractive to dilative behavior as the \( \frac{p}{p_e} \) ratio is decreased. Further, the increased dilative tendency is accompanied by a stiffer stress-strain response and higher peak strength.

54. The undrained behavior of the Vicksburg silty clay is similar to that for the drained tests, although the stress-strain response is much stiffer and the transition into the failure state more abrupt. Specimen CL-5 sheared at an initial \( \frac{p}{p_e} \) ratio of 0.758, displayed no definite peak strength, and was contractive as indicated by the continually increasing pore pressure during shear. Specimens CL-9 and CL-7, with respective \( \frac{p}{p_e} \) ratios of 0.020 and 0.094, displayed dilative behavior with each showing a decreasing pore pressure at an axial strain of less than 1 percent. Specimen CL-6, which had an intermediate value of \( \frac{p}{p_e} \) of 0.365, displayed a flat pore pressure response and was therefore comparable to the drained specimen CL-4. Interestingly, the stress-strain response of specimens CL-6 and CL-4 are quite similar, suggesting that the \( \frac{p}{p_e} \) ratios of 0.336 to 0.365 lie within the borderline values where neither contraction nor dilation dominate stress-strain response.

55. The behavior of the CH clays (Figures 12 to 15) is similar to that of the Vicksburg silty clay. Specimens with low \( \frac{p}{p_e} \) ratios tend to be dilative; whereas, specimens with high \( \frac{p}{p_e} \) ratios tend to be contractive. Also, dilative specimens tend to display a definite peak strength; whereas, contractive specimens approach maximum strength monotonically. Finally, stress-strain response for both drained and undrained specimens are similar except that the stress-strain response for the undrained specimens is characterized by a much stiffer initial response with a more distinct peak strength.

56. While the general interrelationship between \( \frac{p}{p_e} \), contractive-dilative tendency, and difference between peak and residual
strength is similar for the three clays tested, some differences in behavior apparently related to plasticity are notable. For example, differences between peak and residual strength were much sharper for the two CH specimens than for the CL. Specimens CL-3 (Vicksburg silty clay, p/p_e = 0.097), S-17 (buckshot clay, p/p_e = 0.121) and SY-5 (Yazoo clay, p/p_e = 0.192) all had the same stress histories. However, the more plastic clay, SY-5 (Figure 14), had the sharpest falloff in strength after the peak was reached, while the least plastic specimen, CL-3 (Figure 10), displayed a gradual strength falloff. The buckshot clay specimen S-17 (Figure 12) displayed a more distinct peak strength than the Vicksburg silty clay specimen, CL-3, but did not lose strength as abruptly as the Yazoo clay specimen. The stress-strain characteristics of the three specimens were reflected in the nature of the failure surfaces formed during shear. The Vicksburg silty clay would tend to form multiple-slip surfaces with no single zone of localized shear identified. By contrast, both the buckshot clay and Yazoo clay formed distinct failure surfaces which controlled the post-peak behavior. In particular, the Yazoo clay tended to form a distinct failure surface and the specimen could be readily broken into pieces bounded by these surfaces. Also, the texture of the failure surface in the Yazoo clay specimens was characteristic of the slickenside often observed in direct shear test specimens.

57. The correspondence between the postpeak behavior and plasticity for the three specimens tested is consistent with observations made by Lupini, Skinner, and Vaughan (1981), who showed that the characteristics of the failure surface formed in direct shear (vane shear apparatus) are dependent on the percentage of clay particles. Specimens with less than 30 percent clay fractions do not form discrete surfaces since the turbulent nature of the interparticular movements of the rotund-shape silt and sand particles precludes orientation of the platy clay particles. As the percent of clay approaches 50 percent, zones can be maintained in which highly oriented clay particles create polished slickensided surfaces. Therefore, the postpeak behavior of the specimens is partly controlled by the degree to which the clay fraction can form and maintain slickensided surfaces along bands of localized shear strain.
Stress paths

58. Two formats were used to plot the stress paths for each test, plots of the stress variables \( q/2 \) versus \( p \) and plots of the normalized variables \( q/2p_e \) versus \( p/p_e \) (Figure B26). The plots of normalized stress paths show the combined influence of stress and dilatancy on material behavior and will thus be considered in detail. The void ratio and \( p_e \) are constant for the undrained tests, thus the normalized undrained stress paths are simply scaled versions of the true stress path. The stress-dilatancy effects on stress path direction can be readily seen from the data for Vicksburg silty clay (Figures 16 and 17). For example, the normally consolidated specimen (CL-5) displayed initially contractive behavior as indicated by the negative slope of the stress path.* At a stress ratio of \( q/2p = 0.65 \), the stress path turns, indicating dilative behavior. The stress paths for the overconsolidated specimens CL-7 and CL-9 rise vertically to a stress ratio of 0.65 and then begin dilation. All stress paths appear to ultimately converge to a single path at \( p/p_e = 0.7 \). A similar trend can be seen from the data for buckshot clay (Figures 18 and 19). In particular, all stress paths appear to converge at \( p/p_e = 0.58 \), a feature not apparent on the non-normalized plot. The undrained stress paths for the Yazoo clay (Figures 20 and 21) do not display as good a correspondence between stress path direction, stress ratio, and \( p/p_e \). The analysis of the overconsolidated specimens, SY-7 and SY-9, was problematic from two standpoints. First, because of its high swell potential, the equilibrium condition for rebounded specimens was difficult to identify making it difficult to accurately estimate \( p_e \) after rebound. Second, the tendency of the specimens to form distinct shear planes upon reaching their peak strengths made the interpretation of the postpeak stress-strain response difficult.

59. The void ratio and \( p_e \) change during drained shear, thus the directions of the normalized stress paths for the drained tests are

* Note that the effective stress path for an undrained test should be vertical for an isotropic, elastic material.
Figure 16. Stress paths for drained tests on Vicksburg silty clay (data tabulated in Appendix E)
Figure 17. Stress paths for undrained tests on Vicksburg silty clay (data tabulated in Appendix E)
Figure 18. Stress paths for drained tests on buckshot clay (data tabulated in Appendix E)
Figure 19. Stress paths for undrained tests on buckshot clay (data tabulated in Appendix E)
Figure 20. Stress paths for drained tests on Yazoo clay (data tabulated in Appendix E)
Figure 21. Stress paths for undrained tests on Yazoo clay (data tabulated in Appendix E)
controlled by volume changes. The resulting normalized drained stress paths are similar to those for the undrained tests. For example, inspection of the normalized paths for the normally consolidated specimens, CL-2 and CL-8 (Figure 16), display the same contractive-dilative trend as the undrained tests. The major difference between drained and undrained data arises from the initial slope of the stress paths on the normalized plots. The stress paths for undrained tests rise vertically; whereas, the paths for drained tests have an initial slope of $\Delta q/2\Delta p = 1.5$, the direction of the true stress paths. Note that the stress ratio corresponding to the change from contractive to dilative behavior is $q/2p = 0.63$, the same as for the undrained test. The trends of the normalized stress paths for the drained test on the buckshot and Yazoo clays are similar to those for the Vicksburg silty clay except that the overconsolidated specimens, S-16 (Figure 18) and SY-4 (Figure 20), display a sharp falloff in strength caused by formation of shear planes.

60. Several important generalizations can be made from inspection of these plots. First, while the stress path directions are by no means uniquely related to initial $\frac{p}{p_e}$, the paths are bounded by a definable surface on the $q/2p_e - p/p_e$ plot. This bounding surface is the same for both drained and undrained tests. Also, the demarcation between contractive and dilative behavior occurs at a line defined by a unique stress ratio. This line is equivalent to the critical state line in $q - p$ space. Finally, it is important to note that provided a shear plane does not form, strain softening is never manifested by a reduction in $q/p_e$. Reductions in $q$ are offset by reduction in $p_e$. The initiation of softening occurs within the dilative region when the stress paths flatten to become colinear to the critical state line (Figure B15).
61. Two models were developed to describe strain-softening behavior of soil. The first model is described in Appendix C. This model was based on the boundary surface concept proposed by Dafalias and Popov (1976) and will be referred to as the bounding surface (BS) model. The second model was developed at the U. S. Army Engineer Waterways Experiment Station (WES) for this research by the authors. The WES model was based on a unified interpretation of stress-dilatancy and critical state theories and will be referred to as the bounded frictional-dilatant flow (BFDF) model. The differences between the BS and BFDF lie in details of their theoretical development. While these details have an important influence on their implementation in numerical analyses, the models are similar in many respects. In particular, both models relate strain softening to dilatancy rate. In the presentation of the two models, familiarity with procedures for formulating elastoplastic stiffness relationships is assumed. These procedures are presented in Appendix A. For consistency, the terminology used in Appendix A will be used for the description of each model.

Bounding Surface Model

62. The bounding surface concept was motivated by the observation that stress-strain curves converge at a specific bounding condition at a rate that depends on the distance of the stress point from the bound. That is, the incremental stiffness of the stress-strain response is related to the distance between the stress state and the bounding state. The advantage of the bounding surface concept lies in the ability to model inelastic response without introducing the conceptual and practical difficulties associated with defining a distinct yield surface. This aspect of the bounding surface concept is quite attractive for developing constitutive relationships for soils, but its application is
not restricted to soils. The characteristics of the model depends on the mathematical definition of the bounding surface and the metric used to define the distance from the surface. The model used in this report was developed from traditional critical state concepts as described by Dr. Y. Dafalias and Dr. L. R. Herrmann in Appendix C. A brief description of the model is presented in the following paragraphs.

63. To formulate constitutive relationships from the bounding surface concept, it is necessary to define the bounding surface, distance, and hardening law. As consistent with critical state theory, the bounding surface is defined in terms of stress state and void ratio, and movement of the surface is caused by changes in void ratio. Thus,

\[ F(\bar{1}, \sqrt{J}, e) = 0 \]

where the bar over stress quantities indicates points on \( F = 0 \). The actual stress point always lies on or within the bounding surface. The gradient vector, \( \{N\} \), is taken to be the normal to the bounding surface at the "image" point. The definition of the image point is a property of the class of material represented by the constitutive model. For the model under consideration, the image point is found by projecting a line from the stress origin, through the actual stress state onto \( F = 0 \). Thus,

\[ \bar{\sigma}_{ij} = \beta \sigma_{ij} \]

where \( \beta \) is a scalar. The normal direction is accordingly given as

\[ \{N\} = \left\{ \frac{\partial F}{\partial \bar{\sigma}_{ij}} \right\} \]  \hspace{1cm} (9)

Observe that \( \{N\} \) is equivalent to the \( L_{ij} \) tensor in Appendix C.

64. The plastic strain increment direction is assumed to be colinear with the normal direction which gives \( \{\beta\} \) to be

\[ \{\beta\} = \frac{1}{|N|} \{N\} \]  \hspace{1cm} (10)

The \( R_{ij} \) tensor in Appendix C (Equation C26a) is equivalent to the vector \( |N||\{\beta\} \). This definition of \( \{\beta\} \) corresponds to the associated flow rule.
65. The use of the associated flow rule immediately suggests suitable forms for the bounding surface. For example, the constant volume state \((\Delta e = 0)\) must correspond to the stress ratio \(M = q/p\). Or,

\[
\frac{dF}{dI} = 0
\]

at

\[
\frac{\sqrt{J}}{I} = N, \quad N = \frac{\sqrt{3}}{3} M
\]

where \(M\) is the critical state friction parameter and the factor \(\sqrt{3}/3\) relates the invariant \(I\) and \(\sqrt{J}\) to the stress parameters \(p\) and \(q\).

A second criteria dictated by the flow rule is that \(\partial F/\partial J = 0\) at the intersection of the hydrostatic axis with the bounding surface. Finally, the location of the bounding surface must be related to void ratio. These criteria are met by the following:

\[
\frac{F}{I_0^2} = \left(\frac{I}{I_0}\right)^2 + \left(\frac{R - 1}{N}\right)^2 \left(\frac{\sqrt{J}}{I_0}\right)^2 - \frac{2}{R} \frac{I}{I_0} + \frac{2 - R}{R} = 0 \quad \text{(11)}
\]

for \(\bar{I}/I_0 > R\) and

\[
\frac{F}{I_0^2} = \left(\frac{\bar{I}}{I_0}\right)^2 - \frac{1}{N^2} \left(\frac{\sqrt{J}}{I_0}\right)^2 - \frac{2}{R} \left(\frac{\bar{I}}{I_0}\right) + \frac{2}{N} \left(\frac{1}{R} + \frac{A}{N}\right) \frac{\sqrt{J}}{I_0} - \frac{2A}{RN} = 0 \quad \text{(12)}
\]

for \(\bar{I}/I_0 < R\). Equation 11 is an ellipse in the coordinates \(\sqrt{J}/I_0\) and \(I/I_0\) having an aspect ratio of \(R\). Equation 12 is a hyperbola which meets the ellipse at the combined condition

\[
\frac{I}{I_0} = R \quad \text{(13)}
\]

and

\[
\frac{\sqrt{J}}{I} = N
\]

The combination of Equations 11 and 12 forms a closed convex surface about the stress origin. Observe that this surface, combined with the definition of the image point, implies that the direction of the strain increment (hence the dilatancy rate) is uniquely defined by the ratio \(\sqrt{J}/I\); a direct correspondence with the stress-dilatancy concept is thus established.
66. The hardening rule is defined by relating $H(K_p$ in Appendix C) to the distance between the image point and the stress point. By noting that the change in void ratio is proportional to $dF/dI$, the plastic modulus can be written as

$$H = 3(1 + e) \frac{dF}{de} \frac{dF}{dI} + h p_a \left(1 + \left|N\right|^m\right) 9 \left[\left(\frac{\partial F}{\partial I}\right)^2 + \frac{1}{3} \left(\frac{\partial F}{\partial J}\right)^2\right] \left(\frac{d}{\delta_o - \delta}\right)$$

where

$$\theta = \frac{\sqrt{J}}{I}$$

$\delta = $ distance from bounding surface

$$\delta_o = \frac{1}{3} I_o$$

67. The relationship between the bounding surface location and void ratio is introduced through the preconsolidation stress, $I_o$. It is assumed that

$$\frac{dI_o}{de} = - \frac{I_o}{\lambda - \kappa}$$

where

$\lambda = $ slope of virgin compression curve or $e - \ln p$ plot

$\kappa = $ slope of elastic rebound curve on $e - \ln p$ plot

Equation 15 implies that $I_o$ is comparable to the Hvorslev parameter $p_e$, although the two are not equivalent. In fact, the use of overconsolidation stress $I_o$ as a parameter is a misinterpretation of the critical state concept since by definition, $I_o$ is the maximum past stress the soil has been subjected to. Thus, $I_o$ is uniquely defined by stress history. However, by Equation 15 it is clear that $I_o$ can change throughout loading even if the past maximum consolidation is never exceeded. By contrast, the parameter $p_e$ defines a location on the state surface and does not depend on stress history (see Appendix B). The inconsistency in relating $I_o$ to the past maximum consolidation stress does not affect the application of the model provided the initial soil conditions are interpreted such that $I_o$ is defined as a location on the virgin curve associated with the void ratio and not with the past maximum consolidation stress. This distinction is important if the soil is not consolidated under an isotropic stress condition.
68. The bounding surface model has been described in terms of behavior observed under monotonic loading in the triaxial compression test. The two key concepts that control model behavior—stress-dilatancy and state surface—can be generalized to general stress conditions as discussed in Appendix B. The analytical framework has been extended to model general stress conditions by Herrmann, Dafalias, and DeNatale (1980).

Bounded Frictional-Dilatant Flow Model

69. The BFDF model represents a direct application of stress-dilatancy theory, described in Appendix B, to define the mathematical relationships for yield function, flow rule, and hardening function. The yield function is based on the observation that shear resistance consists of a constant frictional part and dilatant part that depends on strain, void ratio, and mean stress. The following mathematical form is thus suggested:

\[ F(\mathbf{g}) - J(S, e, p) = 0 \]  \hspace{1cm} (16)

Accordingly

\[ \{N\} = \frac{\partial F}{\partial \{\sigma\}} - \frac{dJ}{dp} \frac{\partial p}{\partial \{\sigma\}} \]  \hspace{1cm} (17)

\[ H = \frac{\partial J}{\partial S} - (1 + e) \frac{\partial J}{\partial e} (\beta_1 + \beta_2 + \beta_3) \]  \hspace{1cm} (18)

The function \( F \) is defined as

\[ F(\mathbf{g}) = \frac{I_1 I_2}{I_3} \]  \hspace{1cm} (19)

where \( I_1, I_2, \) and \( I_3 \) are invariants defined in terms of principal stresses as

\[ I_1 = \sigma_1 + \sigma_2 + \sigma_3 \]
\[ I_2 = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 \]
\[ I_3 = \sigma_1 \sigma_2 \sigma_3 \]  \hspace{1cm} (20)

Equation 19 coincides with the Mohr-Coulomb failure criterion for triaxial stress conditions and thus gives the model the characteristics of an ideal frictional material. However, since the value of \( J \) is a
function of $p$, the loading direction defined by Equation 17 is influenced by mean stress. Thus, unlike most frictional models, yielding caused by increasing $p$ can be modeled without introducing an additional cap-type yield surface.

70. The functional relationship used for $J(S, e, p)$ is (see Figure D1)

$$J(S, e, p) = J_o + J_1(n_e)J_2(S)$$

(21)

where

- $J_o$ = value of $F$ for hydrostatic stress state = 9
- $J_1 = \frac{9 + 6r_e}{(1 - 2/3r_e)(1 + 4/3r_e)}$
- $r_e = \frac{A(n_e)}{B(n_e)}C(n_e)$
- $n_e = \frac{P}{p_e}$
- $A(n_e) = k_1n_e(1 - n_e^m)$
- $B(n_e) = (a_e + k_2n_e) + A(n_e)$
- $C(n_e) = a_e + k_2n_e$
- $J_2 = \frac{S}{a_1 + S}$
- $S = \sqrt{\text{tr}[d\varepsilon]^2}$

The parameters $k_1$, $k_2$, $m$, $a_e$ control the maximum value of $F(\varepsilon)$ that can be obtained for a given $n_e$. The parameter $a_1$ controls the stiffness of the strain hardening portion of $H$. The derivatives for $H$ (Equation 18) can be evaluated using chain rule differentiation by noting that

$$\frac{dn_e}{dp} = \frac{1}{p_e}$$

$$\frac{dn_e}{de} = \frac{2.302585}{C_c}$$

70. The strain increment direction $\{\beta\}$ is determined from a
nonassociated flow rule based on stress-dilatancy theory. It can be shown from geometry that the direction of the principal strain increments \{d\} is given by

\begin{align*}
    d_1 &= \sqrt{\frac{3}{4}} \sin(\mu + 30) \sin \theta + \sqrt{\frac{1}{3}} \cos \theta \\
    d_2 &= \left[ \sqrt{\frac{1}{2}} \sin (\mu + 30) - \sqrt{\frac{1}{6}} \cos (\mu + 30) \right] \sin \theta + \sqrt{\frac{1}{3}} \cos \theta \\
    d_3 &= d_2 - 2 \sin (\mu + 30) \sin \theta
\end{align*}

where

\begin{align*}
    \mu &= \text{lode angle for strain increment} \\
    \theta &= \text{dilatancy angle}
\end{align*}

The lode angle \( \mu \) is determined by assuming that the deviatoric components of the strain increment direction is normal to the surface defined by Equation 19. To obtain a relationship for \( \theta \), it is assumed that a given position of the yield surface \( F(q) \) corresponds to a particular value of \( \theta \) (see Appendix B). Thus, it is sufficient to find a relationship between \( \theta \) and \( F \) for triaxial stress conditions \( (\sigma_1 = \sigma_a, \quad \sigma_2 = \sigma_r) \) in which

\[ R = \frac{1}{4} (F - S) + \sqrt{(F - S)^2 - 16} \]

where \( R \) is the ratio \( \sigma_a/\sigma_r \). The ratio \( R \) can be related to \( \theta \) using Rowe's equation (Equation B4)

\[ R = KD \]

where \( D \) is the dilatancy rate defined previously. From the relationship given in Appendix B

\[ \cos \theta = \frac{d_\sigma + 2d_r}{\sqrt{3} \sqrt{d_\sigma^2 + 2d_r^2}} \]

Combining Equations 25 and 26

\[ \cos \theta = \frac{K - R}{\sqrt{3} \sqrt{K^2 + \frac{1}{2} R^2}} \]

where \( R \) is obtained from Equation 24.
\{\beta\} = [T] \{d\} \quad (28)

where \([T]\) relates the principal strain increment values to the reference coordinate system. While use of Equations 23 and 28 lacks the mathematical conciseness of a formulation based on a plastic potential function, it does not require significantly greater computational effort. Moreover, computing \{\beta\} from principal directions provides the flexibility of using other stress-dilatancy relationships (e.g., Tatsuoka 1976, Matsuoka and Nakai 1977, and Tokue 1979) which are usually developed in terms of principal strain increments.

73. The frictional parameter \(K\) is related to the parameter \(M\) by

\[
K = \frac{2M + 3}{3 - M} \quad (29)
\]

which establishes a correspondence between the flow rule for the BS and BFDF models. It was determined from a detailed analysis of the stress-dilatancy characteristics of the buckshot and Yazoo clays that a better approximation of the volume of change behavior could be obtained by including a cohesion term in the flow rule. Cohesion can be included into Rowe's stress-dilatancy relationship (Rowe 1963) and has been employed in semiempirical flow rules similar to the critical state energy balance relationship (Akai, Adachi, Fujimoto 1977). Cohesion was introduced into the BFDF model by the empirical relationship

\[
M = M' + \frac{c}{p} \quad (30)
\]

where \(M'\) is the true friction parameter and \(c\) is the cohesion. The apparent frictional parameter \(K\) is obtained from substituting \(M\) into Equation 29. From Equation 30, it is seen that at high levels of \(p\), cohesion has little influence on the flow rule.

Comparison to Test Data

74. The stress-strain, volume change, and pore pressure response was computed for each set of test data using the BS model (Appendix C)
and the BFDF model (Appendix D). The material parameters used for the BS model are given in Table 1 and for the BFDF model in Table 2. The parameters were determined from the drained test data and a single set of parameters was used for simulations of tests performed on each material. Thus, an important element of the comparison between observed and computed behavior is the ability to model stress-strain characteristics for a large range of stress-on-void ratios. With sufficient effort in selecting parameters, either model could be made to reproduce a particular test with considerable accuracy. However, to have the greatest utility for practical analysis, the models should be able to predict differences in behavior caused by changes in conditions.

75. The computed response for the BS model is presented in Figures 22-24 for drained behavior and Figures 25-27 for undrained behavior. It can be seen that the model gives a good qualitative representation of the observed behavior. For example, the differences in strength and volume change behavior are modeled relatively well. Also, the model tendency for greater stiffness and peak q/2p ratio is computed by the model. However, detailed comparisons are not particularly good. A problem encountered in using the model was the inability to model all conditions accurately using a single set of parameters. The behavior of the model is controlled by the location and shape of the bounding surface. The bounding surface characteristics are determined by three parameters, R, A, and M. The parameter M is determined from the stress-dilatancy characteristics; thus, only two parameters can be independently varied to obtain a better comparison with experimental data. Evidently, improvements could be made in the mathematical formulas used to define the bounding surface.

76. Improvements could also be made in the hardening law used in the BS model. A characteristic of all clays tested was the tendency for the stress-strain curve to display a distinct break at about one percent axial strain. This characteristic is especially pronounced for test S-16 (Figure 23) and tests SY-4 and SY-5 (Figure 24). Strain-hardening relationships such as Equation 14 do not model abrupt changes in the stress-strain response.
Figure 22. Computed versus observed drained behavior for bounding surface model on Vicksburg silty clay
Figure 23. Computed versus observed drained behavior for bounding surface model on buckshot clay
Figure 24. Computed versus observed drained behavior for bounding surface model on Yazoo clay
Figure 25. Computed versus observed undrained behavior for bounding surface model on Vicksburg silty clay
Figure 26. Computed versus observed undrained behavior for bounding surface model on buckshot clay
Figure 27. Computed versus observed undrained behavior for bounding surface model on Yazoo clay
77. The computed response for the BFDF model is shown in Figures 28-30 for drained behavior and Figures 31-33 for undrained behavior. In general, the comparison between the predicted and observed behavior is quite good, although the comparison is consistently better for drained tests than for undrained tests. Two aspects of the model contribute to its overall performance. First, the model is based on well established principles of traditional soil mechanics, which assures that qualitative comparison between predicted and observed behavior will be obtained regardless of the material parameters used. Second, a sufficiently large number of parameters have been introduced to model important details of stress-strain behavior. In particular, the use of non-associated flow rule is not only more consistent with observed behavior but gives independent control over yield and volume change characteristics. Importantly, the parameters required by the model can be obtained from relatively few tests.

78. Presentations of data in terms of q/2p versus strain has the important advantage that the data tend to contain more detail than plots of shear stress versus strain. For example, the peak strength is considerably enhanced when the stress-strain characteristics are expressed in terms of q/2p. However, from a design viewpoint, it is the load-carrying capacity (shear strength) and not stress ratio that is needed. The computed shear stress q/2 versus strain data is compared with test data in Figures 34 to 45. As for the previous comparisons, the BFDF model performed better than the BS model. Both models give a good prediction of the influence of void ratio, effective pressure, and drainage condition (drained versus undrained) on behavior.
Figure 28. Computed versus observed drained behavior for bounded frictional-dilatant flow model on Vicksburg silty clay.
Figure 29. Computed versus observed drained behavior for bounded frictional-dilatant flow model on buckshot clay.
Figure 30. Computed versus observed drained behavior for bounded frictional-dilatant flow model on Yazoo clay
Figure 31. Computed versus observed undrained behavior for bounded frictional-dilatant flow model for Vicksburg silty clay
Figure 32. Computed versus observed undrained behavior for bounded frictional-dilatant flow model for buckshot clay
Figure 33. Computed versus observed undrained behavior for bounded frictional-dilatant flow model for Yazoo clay
Figure 34. Computed versus observed shear stress-strain data for bounding surface model, drained tests on Vicksburg silty clay
Figure 35. Computed versus observed shear stress-strain data for bounding surface model, undrained tests on Vicksburg silty clay
Figure 36. Computed versus observed shear stress-strain data for bounding surface model, drained tests on buckshot clay
Figure 37. Computed versus observed shear stress-strain data for bounding surface model, undrained tests on buckshot clay.
Figure 38. Computed versus observed shear stress-strain data for bounding surface model, drained tests on Yazoo clay
Figure 39. Computed versus observed shear stress-strain data for bounding surface model, undrained tests on Yazoo clay
Figure 40. Computed versus observed shear stress-strain data for bounded frictional-dilatant flow model, drained tests on Vicksburg silty clay
Figure 41. Computed versus observed shear stress-strain data for bounded frictional-dilatant flow model, undrained tests on Vicksburg silty clay.
Figure 42. Computed versus observed shear stress-strain data for bounded frictional-dilatant flow model, drained tests on buckshot clay
Figure 43. Computed versus observed shear stress-strain data for bounded frictional-dilatant flow model, undrained tests on buckshot clay
Figure 44. Computed versus observed shear stress-strain data for bounded frictional-dilatant flow model, drained tests on Yazoo clay
Figure 45. Computed versus observed shear stress-strain data for bounded frictional-dilatant flow model, undrained tests on Yazoo clay.
PART V: CONCLUSIONS AND RECOMMENDATIONS

79. The two constitutive models presented in this report give a good representation of softening behavior in soils. Importantly, both models correctly predict that softening will occur in soils having high OCR. Also, both models relate softening to dilation. The two models differ primarily in the precision that could be achieved in the comparison between predicted and observed behavior.

80. From a computational standpoint, the BS model may be easier to implement in a computer code because it is formulated from an associated flow rule. However, the advantages of an associated flow rule stem from convenience rather than from necessity. A number of computational schemes exist to analyze plasticity problems formulated using a nonassociated flow rule. Also, the stress-strain response cannot be modeled for some types of loading if an associated flow rule is used. Note that the associated flow rule is not essential to the bounding surface concept and a bounding surface model based on a nonassociated flow rule could be developed (Appendix C).

81. Future research efforts should be directed toward implementation of the soil models in practical numerical analyses. Although the development of constitutive models can provide insight into the mechanical behavior of soil, the simulation of stress-strain curves does not serve as an end in itself. The ability to formulate well posed problems for engineering analysis using the constitutive models can only be fully assessed from experience gained from their use in practical computer codes.
REFERENCES


Hermann, L. R., Dafalias, Y. F., and DeNatale, J. S. 1980. "Bounding Surface Plasticity for Soil Modeling," Final report to Civil Engineering Battalion Center, Port Hueneme, Calif., by University of California, Davis.


Table 1  
Bounding Surface Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Vicksburg Silty Clay</th>
<th>Buckshot Clay</th>
<th>Yazoo Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.061</td>
<td>0.143</td>
<td>0.321</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.012</td>
<td>0.020</td>
<td>0.0825</td>
</tr>
<tr>
<td>$\nu$</td>
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<td>1.00</td>
<td>0.75</td>
</tr>
<tr>
<td>$R$</td>
<td>3.00</td>
<td>2.20</td>
<td>2.00</td>
</tr>
<tr>
<td>$A$</td>
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<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$T$</td>
<td>0.10</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>$I_2$ (psi)</td>
<td>44.1</td>
<td>44.1</td>
<td>44.1</td>
</tr>
<tr>
<td>$I_0$ (psi)</td>
<td>450.0</td>
<td>450.0</td>
<td>450.0</td>
</tr>
<tr>
<td>$m$</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$h$</td>
<td>75.0</td>
<td>80.0</td>
<td>40.0</td>
</tr>
<tr>
<td>$v$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 2  
Bounded Frictional-Dilatant Flow Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Vicksburg Silty Clay</th>
<th>Buckshot Clay</th>
<th>Yazoo Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M'$</td>
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<td>0.95</td>
<td>0.69</td>
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<tr>
<td>$c$</td>
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<tr>
<td>$B_c$</td>
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<td>3.23</td>
<td>1.587</td>
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<tr>
<td>$e_o$</td>
<td>0.67</td>
<td>0.89</td>
<td>1.77</td>
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<tr>
<td>$a_1$</td>
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<tr>
<td>$a_2$</td>
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<td>0.09</td>
<td>0.075</td>
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<tr>
<td>$t$</td>
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<td>0.60</td>
<td>0.025</td>
</tr>
<tr>
<td>$k_1$</td>
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<td>1.41</td>
<td>1.48</td>
</tr>
<tr>
<td>$k_2$</td>
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<td>-0.45</td>
<td>-0.3</td>
</tr>
<tr>
<td>$a_e$</td>
<td>1.5</td>
<td>1.0</td>
<td>0.75</td>
</tr>
<tr>
<td>$m$</td>
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<td>25.0</td>
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APPENDIX A: FORMULATION OF CONSTITUTIONAL RELATIONSHIPS USING THE THEORY FOR ELASTOPLASTIC STRAIN-HARDENING MATERIALS

1. The past decade (beginning 1970) has seen a proliferation of theories on constitutive relationships for soils. Of the many theories proposed, elastoplasticity has received the greatest attention both in theory and practice. The popularity of elastoplasticity theory is due primarily to its inherent simplicity and versatility. Also, there is a close correspondence between the fundamental plasticity concepts and strength theories commonly used by engineers. This appendix describes the formulation of constitutive relationships using the theory of elastoplasticity as it is applicable to granular materials. The description is drawn heavily from Zienkiewicz (1977), Mróz, Norris, and Zienkiewicz (1978), and Lade and Nelson (1981).

Definitions

2. To formulate constitutive relationships for inelastic materials, the following must be defined:

   a. A criteria that separates stress increments that induce inelastic strains (loading) from those that do not (unloading).
   b. The relative magnitude of inelastic strains.
   c. The relationship between the magnitude of the strain increment and stress increment.

Definition of loading

3. Loading can be defined in terms of a yield surface as:

\[ f(\{\sigma\}, \{K\}) = 0 \]  \hspace{1cm} (A1)

where \( \{\sigma\} \) is the stress state and \( \{K\} \) consists of variables that define the hardening state. For example, a yield surface proposed by Lade (1972) can be written in the form of:

References cited in this and subsequent appendices are more fully identified in the References section at the end of the main text.
\[ f(\{\sigma\} , K_w) = \frac{I_1^3}{I_3} - K_w \]  

(A2)

where

\[ I_1 = \sigma_1 + \sigma_2 + \sigma_3 \]
\[ I_3 = \sigma_2 \sigma_3 \]

\[ K_w = \text{work hardening function} \]

In Equation A2 the array \( \{K\} \) consists of a scalar work hardening term, although in general a number of parameters may be used to define the state of the material.

4. Traditionally, loading is defined as:

\[ l = \{N\}^T \{d\sigma\} \]  

(A3)

where \( \{N\} = \partial f/\partial \{\sigma\} \), whereby loading occurs when \( l \) is positive and unloading occurs when \( l \) is negative. Neutral loading implies that \( l = 0 \). Equation A3 is insufficient to define loading if strain softening is possible because the product \( l \) can be negative (decreasing \( \{d\sigma\} \)), even though by any practical definition the soil is being loaded. Herrmann and Dafalias (Appendix C) resolved this problem by defining \( l \) as:

\[ l_1 = H \frac{\partial f}{\partial \{\sigma\}} \]  

(A4)

where \( H \) is negative if the material is strain softening but positive otherwise. Singh (1972) (as described by Sandu (1981)) used a definition for loading based on the stress path produced by a "pilot" elastic increment. The pilot stress increment is computed by assuming the material to be elastic. The elastic stress increment thus obtained can be used to define loading as:

\[ l_2 = \{N\}^T \{d\sigma^e\} \]  

(A5)
which again is positive during a loading increment. While the techniques of Herrmann and Dafalias (see Appendix C) and Singh (1972) are intuitive, they provide nonambiguous definitions for loading for all cases.

Direction of plastic strain increment

5. The plastic strain increment is defined as:

\[ \{d\varepsilon^p\} = \lambda \{\beta\} \]  \hspace{1cm} (A6)

where \(\{\beta\}\) gives the relative magnitude of the strains and \(\lambda\) is a scalar multiplier that defines the strain increment magnitude. Since \(\{\beta\}\) defines direction only, it is convenient to specify

\[ \{\beta\}^T \{\beta\} = 1 \]  \hspace{1cm} (A7)

The strain increment is often related to the stress state by a plastic potential function. If the strain increments are analytic functions of stress, a scalar function \(g\) can define such that

\[ \{d\varepsilon^p\} = \lambda' \frac{\partial g}{\partial \{\sigma\}} \]  \hspace{1cm} (A8)

While the plastic potential function is an arbitrary device used to define the strain increment direction, it has an important role in the theory of plasticity. A ramification of Drucker's stability postulate is

\[ g \equiv f \]  \hspace{1cm} (A9)

which is a sufficient, though not necessary, requirement to prove a number of important limit theories. If Equation A9 is used, Equation A8 is referred to as the associated flow rule. If Equation A8 is otherwise determined, it is referred to as the nonassociated flow rule. However determined, the plastic strains must be specified such that positive
energy is dissipated during loading; that is:

$$\{\beta\}^T\{\sigma\} \geq 0 \quad (A10)$$

Equation A9 implies that \( g \) represents a surface that is convex about the stress origin. The stability postulate likewise specifies that \( f \) is convex. However, Equation A10 is a requirement not a postulate and must be true even if a nonassociated flow rule is used.

**Magnitude of Strain Increment**

6. The strain increment magnitude is related to the stress increment implicitly through a hardening relationship. The hardening relationship defines the movement of \( f \), relative to the stress coordinates, caused by changes in the parameters \{\( K \)\}. The hardening modulus is defined as:

$$H = \left\{ \frac{\partial f}{\partial \{K\}} \right\}^T \{dK\} \frac{1}{\lambda} \quad (A11)$$

If \( H \) is greater than zero, \( f \) is expanding in stress space (hardening). If \( H \) is zero, \( f \) is contracting (softening). Note that the \( H \) parameter used to define loading in Equation A4 is identical to Equation A11.

**Formulation of Incremental Relationships**

7. The formulation of incremental relationships generally proceeds from the consistency relationship, which is the differential of Equation A1:

$$df = \{N\}\{d\sigma\} + \left\{ \frac{\partial f}{\partial \{K\}} \right\}^T \{dK\} \quad (A12)$$
Inserting Equation A11 into A12

\[
{N}\{d\sigma\} + \lambda H = 0 \tag{A13}
\]

Equation A13 is insufficient to relate strain increment to stress increment. However, an additional relationship between \(\{d\sigma\}\) and strain can be obtained from the assumption that total strain consists of identifiable elastic and plastic components:

\[
\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\} \tag{A14}
\]

The elastic strains are related to the stress increment by

\[
\{d\sigma\} = [D^e]\{d\varepsilon^e\} \tag{A15}
\]

where \([D^e]\) is the elastic stiffness matrix. Combining Equations A6, A14, and A15, Equation A13 can be written as

\[
{N}'\{d\varepsilon\} - \lambda\{\beta\} + \lambda H = 0 \tag{A16}
\]

Upon rearranging terms, \(\lambda\) can be computed as

\[
\lambda = \frac{1}{L} \{N\}'\{d\varepsilon^e\} \tag{A17}
\]

where

\[
L = \{N\}'[D^e]\{\beta\} - H
\]

The elastic strain can be obtained by combining Equations A6, A14, and A17 and inserting this into Equation A15 to obtain the elastoplastic stiffness relationship

\[
\{d\sigma\} = [D^{ep}]{d\varepsilon} \tag{A18}
\]
where

\[
[D_{ep}] = [D^e] - \frac{1}{L} [D^e] \{\beta\} \{N\}^T [D^e]
\]

Observe that \([D_{ep}]\) is symmetric only if the associated flow rule is assumed.

**Strain-Hardening Relationship for Soil**

8. The yield characteristics of soil can often be related to void ratio and strain. While strain is often introduced through a work hardening law, a more direct approach is to define the hardening parameters as

\[
\{dK\}^T = (dS, de)
\]

(A19)

where

\[
dS = \sqrt{\text{tr}[\varepsilon]^2}
\]

The strain parameter \(S\) and change in void ratio are computed from Equation A6 as

\[
dS = \lambda
\]

\[
de = -\lambda(1 + e)(\beta_1 + \beta_2 + \beta_3)
\]

(A20)

The hardening modulus can be determined by substituting Equation A2 into All

\[
H = \frac{\partial f}{\partial S} - (1 + e) \frac{\partial f}{\partial e} (\beta_1 + \beta_2 + \beta_3)
\]

(A21)

9. The elastoplastic incremental matrix \([D_{ep}]\) is nonsingular only if \(\{d\sigma\} \neq 0\) for all \(\{d\varepsilon\}\). To determine the conditions for which \([D_{ep}]\) is singular it is sufficient to find those cases in which
\{d\sigma\} is independent of \{d\varepsilon\}. It may be noted that the condition

$$\{N\}^T\{d\sigma\} = 0$$

(A22)

implies that the components of the stress increment vector are not mutually independent and as such cannot be uniquely related to the strain increment vector. Thus, from Equation A13 [D^EP] is singular for $H = 0$ (Prévost and Hughes 1981, Lade and Nelson 1981). Equation A22 implies that for plastic loading, the stress increment must be tangential to the yield surface. This is consistent with the definition of failure for perfectly plastic materials, since Equation A22 describes a state in which a material may undergo an arbitrary amount of strain when subjected to an increment of loading.

10. The condition for which $H \neq 0$ can be discerned from Equation 21 by noting that \(\partial J/\partial S \geq 0\) and \(\partial J/\partial e < 0\). Thus, $H$ is zero only if

$$I: \beta_1 + \beta_2 + \beta_3 < 0$$

or

$$II: \text{ both } \frac{\partial J}{\partial S} = 0 \text{ and }  \beta_1 + \beta_2 + \beta_3 = 0$$

Condition I occurs when the soil is dilating and \(\partial J/\partial S\) is relatively small as at the onset of strain softening when $H$ becomes negative. Condition II represents the ultimate state at which no further increase in shear resistance can be mobilized by strain $S$, and the material strains without volume change. The two conditions, respectively, represent the peak strengths and the critical state as defined by Roscoe, Schofield, and Thurairajah (1963). As observed in Appendix B, dilative materials tend to reach a peak strength (Condition I) and approach ultimate constant volume state (Condition II) by softening; whereas, a contractive material never displays a distinctive failure stress but asymptotically approaches Condition II.
Undrained Behavior

The equations of equilibrium must be written in terms of total stress. However, the constitutive relationships (Equation A18) are written in terms of effective stress; that is,

$$
\{d\bar{\sigma}\} = \{\bar{\sigma}^{EP}\}\{d\varepsilon\}
$$

(A23)

where the bar denotes an effective stress quantity. The relationship between effective and total stress is

$$
\{d\sigma\} = \{d\bar{\sigma}\} + u\{m\}
$$

(A24)

where $u$ is a scalar defining the magnitude of the pore pressure and

$$
\{m\}^T = (1,1,1,0,0,0)
$$

To compute the pore pressure $\{u\}$, an additional relationship is evidently required. A number of techniques have been used to model undrained behavior of soils. For example, the assumption of incompressibility of the soil-fluid mixture has been used in numerical analysis to formulate problems using the Lagrange multiplier technique (Zienkiewicz 1977), where the multiplier can be physically interpreted as the pore pressure $u$. Alternatively, by formulating problems to account for consolidation phenomena, the undrained condition can be computed as the initial condition (Zienkiewicz and Humpheson 1977). This formulation is numerically equivalent to the Lagrange multiplier method. The technique used for simulating undrained behavior in this report has been suggested by Zienkiewicz and Humpheson (1977) (also Dafalias and Herrmann, Appendix C) and is based on the assumption that

$$
du = K_w \frac{1}{n} (d\varepsilon_x + d\varepsilon_y + d\varepsilon_z)
$$

(A25)
where

\[ K_w = \text{bulk modulus of water} \]
\[ n = \text{porosity of soil} \]

Note that all of these techniques have the common characteristic that an additional relationship is developed to describe the action of the pore fluid. A special constitutive relationship is not developed for undrained behavior. Proceeding from Equation A25, the total stress relation can be written as (Zienkiewicz and Humpheson 1977):

\[
\{d\sigma\} = \left( [D^{ep}] + \frac{1}{n} K_w \{m\}\{m\}^T \right) \{d\varepsilon\} \tag{A26}
\]

This method corresponds to setting Poisson's ratio nearly equal to 0.5, a common technique used to model incompressible elastic materials. Once the strains are computed from Equation A26, the pore pressure is computed from Equation A25 and the effective stress from Equation A24. For drained conditions, \( K_w = 0 \); for undrained conditions, \( K_w = 314,000 \) psi, the bulk modulus of pure water. The computation of \( u \) is generally not affected by the value of \( K_w \) chosen, provided it is on the order of 10,000 atmospheres or greater.
APPENDIX B: STRESS-DILATANCY THEORIES AND THE HVORSLEV FAILURE LAW

1. Stress-dilatancy theory predicts that a particulate material must contract or dilate to resist applied shear stress. Critical state theory supplements the stress-dilatancy concept with the postulate that soil strength can be defined as a surface that is defined in terms of stress and void ratio. As shown in Appendix A, strain-softening behavior arises naturally from the mathematical development of the elastoplastic incremental stiffness relationships for dilatant materials because of the dependence of strength on void ratio. Critical state theory thus provides a means of accounting for strain softening without introducing new parameters into the analysis.

2. Unfortunately, several problems exist with traditional critical state theory. Some of these problems are related to its application in mathematical models rather than the theory itself and thus can be resolved through improved modeling techniques. For example, the bounding surface (BS) model described in Appendix C resolves the problem of predicting plastic strains for simple shear stress paths (dp = 0) without altering the fundamental concepts. Other problems are fundamental and cannot be resolved through improved modeling techniques. For example, the critical state theory is explicitly based on the stability postulate of Drucker (1964b) (Schofield and Wroth 1968), which implies the associated flow rule. The stability postulate in general, and the associated flow rule in particular, have been shown not to be valid for soil (Poorooshasb, Holubec, and Sherbourne 1966; Tatsuoka and Ishihara 1974; Lade and Duncan 1976). Further, the descriptions of a yield surface for sands in terms of stress and void ratio have been criticized on the basis that no unique compression curve can be defined for sands (Rowe 1971). If these criticisms are valid, use of critical state theory would be applicable only to clay, a distinction that seems artificial and appears to be contrary to observation (e.g., Lambe and Whitman 1969).

3. This appendix presents a detailed discussion of two key concepts of critical state theory, stress-dilatancy theory and the Hvorslev
failure law. It is shown that these concepts are valid for both sands and clays. Also, the stress-dilatancy phenomenon is shown to be inherent in any material that derives strength from friction. A less restrictive interpretation of stress-dilatancy theory and the state surface concept is proposed. This interpretation serves as the basis for the bounded frictional dilatant flow model (BFDF).

**Stress-Dilatancy Theory**

4. The physical components of stress-dilatancy behavior can be readily understood by considering the force-displacement relationship for a rough discontinuity such as a rock joint (Figure Bl). The relative displacement of the joint blocks is controlled by the nature of the joint surface asperities; the shearing force $T$ and normal force $N$ can be related to the geometry of the asperities by:

$$\frac{T}{N} = \tan (\phi') + i$$  \hspace{1cm} (B1)

where

- $T$ = horizontal (shearing) force
- $N$ = vertical (normal) force
- $\tan \phi'$ = coefficient of friction of rock material
- $\tan i$ = $dv/du$
- $v$ = vertical displacement
- $u$ = horizontal displacement

Equation B1 can be put into a linear form by noting that for small angles of $i$, the addition formula for tangents reduces to (Figures Blb and Blc)

$$\tan (\phi' + i) = \tan \phi' + \tan i$$

Thus, the strength of the rock joint can be simply expressed as the algebraic sum of frictional and dilatant components (Figure Bld),

$$\frac{T}{N} = \tan \phi' + \tan i$$  \hspace{1cm} (B2)
a. GEOMETRY OF ROCK JOINT

\[ \tan \theta = \frac{U}{\mu} \]

b. GEOMETRY OF IDEAL ROCK JOINT

\[ \frac{T}{N} = \tan(\phi_u + \iota) \]
\[ = \tan \phi_u + \tan \iota \]
\[ = \frac{T}{N} \tan(\phi_u) \]

\[ \phi \tan \phi_u + \tan \iota \]

\[ (-) \quad \text{EXPANSION} \quad \tan \iota \quad \text{CONTRACTION} \quad (+) \]

c. RELATIONSHIP BETWEEN STRENGTH AND DILATANCY

\[ T = N (\tan \phi_u + \tan \phi') \]

\[ N \tan \phi_u \quad N \tan \phi' \]

\[ \text{FAILURE CONDITION} \]

Figure Bl. Frictional-dilatant model of the rock joint.
Equation B2 can be used to describe the stress-dilatancy behavior of sand in the direct shear test on sand which closely simulates the action of the rock joint. In Figure B2, direct shear test data from Taylor (1948) demonstrates that the rate of volumetric expansion \( \frac{dV}{dv} \), and the strength \( \frac{\tau}{\sigma} \) are both mobilized with displacement \( u \). For dense sand, the specimen initially contracts, then expands, with the highest rate of expansion corresponding to the peak strength of the sand. A similar trend is shown for the loose sand, except that the specimen is more contractive and the strength is not defined by a pronounced peak. Both dense and loose specimens ultimately reach the same strength.

5. For the frictional parameters \( \phi, \mu \) to be a material property, the mobilization of shear strength with displacement must be related to changes in dilatancy rate, \( d \mu/du \). In Figure B3, it can be seen that the stress-dilatancy relationships (Equation B1 or B2) describe the observed behavior reasonably well for \( \phi = 25 \) deg. Also, the relationship between dilatancy rate and horizontal displacement are seen to be similar to the relationship between strength and displacement.

6. The stress-dilatancy behavior for discontinuities such as those occurring in rock joints or in the direct shear test can also be identified in soil tests that have homogeneous stress and deformation states. In Figure B4, the stress, strain, and volume change data are shown for an angular quartz sand with initial void ratios ranging from 0.812 to 1.297. The well known correspondence between void ratio, dilatancy, and strength is readily apparent. The dense sand \( (e = 0.812) \) has the greatest strength and is most dilative, while the loose sand has the lowest strength and is generally contractive. Further, the stress-strain curves for the dense sand display a peak strength and then soften to a residual value; whereas, the loose sand monotonically approaches its ultimate strength value. Note that all specimens displayed the same ultimate residual strength value.

7. The stress-dilatancy behavior for sand in the triaxial test can be analyzed by a relationship similar to that used for the direct shear test:
Figure B2. Data from direct shear test on sand (Taylor, 1948)
Figure B3. Stress-dilantancy relationships for sands in direct shear test (data from Taylor, 1948)
Figure B4. Stress-strain and volume change data for sand (data from Koerner, 1968)
\[ \tan \alpha = M_{cv} - \cot \theta \]  

(B3)

where \( M_{cv} \) is an empirical friction parameter and \( \alpha \) and \( \theta \) are invariants of strain increment and stress defined in Table B1. Equation B3 has a form similar to Equation B2. However, while Equation B2 is a linearized form of the theoretically derived relationship \( \frac{T}{N} = \tan(\phi + \mu) \), Equation B3 does not have a similar correspondence to a rigorous stress-dilatancy relationship.

8. A stress-dilatancy plot for the triaxial data is shown in Figure B5. The empirical Equation B3 gives a good representation of the stress-dilatancy relationship for all but the initial strains, partly a result of not separating elastic and plastic strains before computing \( \cot \theta \).

9. Stress-dilatancy theories have been proposed by a number of researchers including Rowe (1962), Horne (1965), Newland and Alley (1957), Roscoe, Schofield, and Thurairajaha (1963), Matsuoka (1974), and Tokue (1979). While all of these theories are successful at describing the stress-dilatancy phenomenon, only the theories of Horne (1965), Matsuoka (1974), and Tokue (1979) were conceived for describing the stress-dilatancy phenomenon in general stress states.

10. The two most common theories in current use for analysis of triaxial test data are due to Rowe (1962), who derived the relationship:

\[ \frac{\sigma_a}{\sigma_r} = K \left( 1 - \frac{d\varepsilon_v}{d\varepsilon_a} \right), \quad w = 60^\circ \]

\[ = \frac{1}{K} \left( 1 - \frac{d\varepsilon_v}{d\varepsilon_a} \right), \quad w = 0 \]  

(B4)

where

\[ K = \text{Rowe's frictional parameter} \]

\[ \tan (w + 30^\circ) = \left( \frac{1}{\sqrt{3}} \right) \left[ (2 - b)/b \right] \]

\[ b = (\bar{\sigma}_2 - \bar{\sigma}_3)/(\bar{\sigma}_1 - \bar{\sigma}_3) \]

and Roscoe, Schofield, and Thurairajah (1963) who proposed:
Table B1

Invariant Relationships for Stress and Strain Increment Representation

**Stress**

\[
|\sigma| = \sqrt{T^2 + \sigma_p^2}
\]

\[
T = \frac{1}{\sqrt{3}} \sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2}
\]

\[
\sigma_p = \frac{1}{\sqrt{3}} (\sigma_1 + \sigma_2 + \sigma_3)
\]

\[
\tan \alpha = \frac{T}{\sigma_p}
\]

\[
\tan \left( w + \frac{\pi}{6} \right) = \frac{1}{\sqrt{3}} \left( \frac{2 - b}{b} \right)
\]

\[
b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}
\]

**Strain Increment**

\[
|d\varepsilon| = \sqrt{d\varepsilon^2 + dv^2}
\]

\[
d\varepsilon = \frac{1}{\sqrt{3}} \sqrt{(d\varepsilon_1 - d\varepsilon_3)^2 + (d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 + d\varepsilon_3)^2}
\]

\[
dv = \frac{1}{\sqrt{3}} (d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3)
\]

\[
\cot \theta = \frac{dv}{d\varepsilon}
\]

\[
\tan \left( w + \frac{\pi}{6} \right) = \frac{1}{\sqrt{3}} \left( \frac{2 - d}{d} \right)
\]

\[
d = \frac{d\varepsilon_2 - d\varepsilon_3}{d\varepsilon_1 - d\varepsilon_3}
\]

**Plastic Work Increment**

\[
dW^p = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3
\]

\[
= |\sigma| |d\varepsilon| [\cos \alpha \cos \theta + \sin \alpha \sin \theta \cos (w - \mu)]
\]

\[
= |\sigma| |d\varepsilon| \cos \rho
\]

**Strain Measure**

\[
S = \int (d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2)^{1/2}
\]
Figure B5. Stress-dilantancy data for triaxial tests on sand (data from Koerner, 1968)
\[
\frac{q}{p} = M - \frac{3}{2} \frac{d\varepsilon_v}{d\varepsilon_a - d\varepsilon_r} \tag{B5}
\]

where

\[q = \bar{\sigma}_a - \bar{\sigma}_r\]

\[p = \frac{1}{3}(\bar{\sigma}_a + 2\bar{\sigma}_r)\]

\[M = \text{critical state frictional parameter}\]

11. Equation B4 was derived from particulate mechanics and though controversial has fundamental significance. Equation B5 is analogous to Equation B2 and is based on a heuristic comparison to Taylor's energy separation computation. However, Equation B5 was shown by Rowe (1964) to be an approximation to Equation B4 and has the added advantage of being easily generalized into an invariant form. For example, Equation B5 is equivalent to Equation B3 since:

\[
\frac{q}{p} = \frac{3}{\sqrt{2}} \tan \alpha
\]

\[
\frac{3}{2} \frac{d\varepsilon_v}{d\varepsilon_a - d\varepsilon_r} = \frac{2\sqrt{2}}{3} \cot \theta
\]

Moreover, Equation B3 can be generalized into an invariant form by including the influence of \(\omega\):

\[
\tan \alpha_{cv} = M_{cv}(\omega) - \frac{\cot \theta}{\cos (\omega - \mu)} \tag{B6}
\]

Since the frictional parameter \(M(\omega)\) is assumed to be a function of \(\omega\) only, its value can be determined by plotting stress states on the \(\pi\) plane (Figure B6) at which \(\cot \theta = 0\) (constant volume state). From Figure B6, it can be seen that the relationship between \(\omega\) and the peak strength is a reflection of the relationship between the constant volume state, \(\tan \alpha_{cv}\) and \(\omega\). Further, the value of \(M_{cv}(\omega)\) is essentially the same for both dense and loose sand.

12. The dependence between \(M_{cv}\) and \(\omega\) implies that \(M_{cv}\) does not take on the same value in triaxial compression (\(\omega = 60\) deg) and
Figure B6. Stress states at constant volume shear from true triaxial test data.
triaxial extension ($\omega = 0$ deg). The difference can be readily explained from Equation B4. For constant volume ($d\varepsilon_v = 0$):

$$M_{cv}^{C} = \sqrt{2} \frac{K - 1}{K + 2}, \; \omega = 60^\circ$$

$$M_{cv}^{E} = \sqrt{2} \frac{K - 1}{2K + 1}, \; \omega = 0$$

13. From Equation B7, it is seen that $M_{cv}^{C}$ would be the same in both compression and extension only if $K = 1$, which represents the case where the soil particles are frictionless. The ratio of $M_{cv}$ in compression and extension is given by:

$$B = \frac{M_{cv}^{E}}{M_{cv}^{C}} = \frac{K + 2}{2K + 1}$$

(B9)

Since $K$ typically ranges from 3.0 to 4.0 for sand, it is expected that $M_{cv}^{E}$ will be about 70 percent of $M_{cv}^{C}$, a value consistent with the data shown in Figure B6.

14. Equation B9 has some interesting implications on the general relationship between $M_{cv}$ and $\omega$. For example, $M_{cv}(\omega)$ can be related to a plastic potential function since the plastic potential function and $M_{cv}(\omega)$ must coincide on the $\pi$ plane when $\cot \theta = 0$. To satisfy the requirement that plastic strains dissipate positive energy, the plastic potential function must be convex about the stress origin. Also, for isotropy, the tangent to $M_{cv}(\omega)$ must be perpendicular to the $\sigma_1$ and $\sigma_2 = \sigma_3$ axes. These restrictions, combined with Equation B9, limit the plastic potential and $M_{cv}(\omega)$ to the region shown in Figure B7.

15. The similarity in trends between the constant volume stress condition and peak strength suggests that the peak strength corresponds to a peak value of dilatancy rate ($\cot \theta$). Such a correspondence is likewise illustrated in Figure B8 which shows that the peak value of dilatancy is independent of the Lode angle $\omega$. Importantly, the invariance of $\theta$ to the Lode angle persists throughout the stress-strain curve as illustrated in Figure B9. Thus, the relationship between the Lode angle and strength is a direct result of $M(\omega)$. 

B13
REGION BOUNDING ADMISSIBLE PLASTIC POTENTIAL FUNCTION

RESTRICTIONS TO PLASTIC POTENTIAL:

<table>
<thead>
<tr>
<th>RESTRICTION</th>
<th>JUSTIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. POINTS a AND c</td>
<td>CORRESPONDS TO ROWE'S STRESS-DILATANCY EQUATION FOR CONSTANT VOLUME</td>
</tr>
<tr>
<td>2. ORTHOGONALITY WITH oa AND oc</td>
<td>ISOTROPY</td>
</tr>
<tr>
<td>3. SEGMENTS cd, da, ab, AND bc</td>
<td>CONVEXITY</td>
</tr>
</tbody>
</table>

Figure B7. Limits imposed on plasticity and stress-dilantancy theories
Figure B8. Dilatancy ($\theta$), strength ($\gamma$), and constant volume stress angle ($\gamma_{cv}$) as a function of lode angle ($\omega$)
Figure B9. Stress parameters ($\gamma$) and dilantancy parameters ($\theta$) versus strain (data from Lade, 1972)
16. Another ramification of the stress-dilatancy relationship as depicted in Figure B7 and the strength-dilatancy relationship shown in Figure B8 is that the plastic potential function will be geometrically similar in the \( \tau \) plane. This supports the general observation that the associated flow rule gives a good approximation of the direction of the deviatoric portion of the strain increment (Lade 1972, Vermeer 1978). However, stress-dilatancy theory also implies that the associated flow rule will not give a good approximation of the volumetric strain increment component. The reason for this is fundamental as can be readily observed from consideration of rock joint behavior.

17. Equations B1 and B2 are derived from the equilibrium condition for the rock joint and can likewise be used to express the failure condition,

\[
F = \frac{T}{N} - \tan (i_{\text{max}} + \phi_{\mu}) \quad (B10)
\]

However, the flow rule is also contained in the equilibrium equation since at failure

\[
\frac{dv}{du} = \tan \, i_{\text{max}} \quad (B11)
\]

Equation B11 can be used to develop a plastic potential function by noting that the displacement vector forms a constant angle \( (\phi_{\mu}) \) with the normal of the sliding surface. The plastic potential function derived from this condition is shown in Figure B10 (Roberds and Einstein 1978), where it is seen that the plastic potential function is not geometrically similar to the failure criteria. Thus, the associated flow rule of plasticity theory is not applicable to the behavior of the simple rock joint system.

18. More complex ideal frictional particulate systems analyzed by Parkin (1965), Rowe (1962), and Thornton (1979) similarly demonstrate nonassociated behavior. For example, the assumption that failure is determined by a maximum rate of dilatancy \( D_{\text{max}} \) would imply a failure criteria based on Rowe's equation to be of the form
Figure B10. Failure criteria, plastic potential, and hardening relationship for a rock joint
\[ \frac{\sigma_a}{\sigma_r} = K D_{\text{max}} \]

\[ \sigma_r = \text{constant} \]

which corresponds to the Mohr-Coulomb criterion. However, the plastic potential that corresponds to the stress-dilatancy relationship (Equation B4) has the form (Barden and Khayatt 1966)

\[ g = \frac{\sigma^K}{\sigma_r} \]

which has a bullet shape similar to Figure B10a.

Hvorslev Failure Law

19. The characterization of soil has traditionally been based on a dichotomous approach whereby clays are described in terms of overconsolidation ratio (OCR) and sands are described by relative density. It is well known that dense sands behave similarly to overconsolidated clays, and loose sand behaves similarly to normally consolidated clay. Further, neither the OCR nor \( D_r \) gives a complete picture of soil behavior. The OCR has no meaning for compacted soils and \( D_r \) is based on arbitrary limits of void ratio. The use of \( D_r \) is further complicated by the influence of effective stress on strength and dilatancy characteristics.

20. Hvorslev (1937) proposed a Mohr-Coulomb failure law that incorporates both void ratio and stress

\[ \frac{\tau_f}{\sigma_e} = c_e + \frac{\sigma_f}{\sigma_e} \tan \phi_e \]

where

- \( \tau_f \) = shear stress at failure
- \( \sigma_e \) = equivalent normal stress
  \[ = \sigma_a \exp \left[ (e_o - e)/\lambda \right] \]
\[ \sigma_a = \text{atmospheric pressure} \]
\[ \lambda = \text{slope of compression curve on } e-\log \sigma \text{ plot} \]
\[ c_e = \"true\" \text{ cohesion} \]
\[ \sigma_f = \text{normal stress at failure} \]
\[ \phi_e = \"true\" \text{ angle of friction} \]

The Hvorslev failure law thus defines the failure state as a surface expressed in terms of \( q_f \), \( p_f \), and \( e \) (Roscoe, Schofield, and Wroth 1957), which plots as a family of parallel lines in \( e-\log p \) space.

21. While the Hvorslev theory has generally been applied to the study of shear strength of clays, the appendant concept of "state-lines" in \( e-\log p \) space has seen more general applications in the modern soil mechanics theories. The existence of a unique constant volume (critical state) failure line in \( e-\log p \) space is a fundamental postulate of critical state theory (Schofield and Wroth 1968). Further, the existence of parallel compression curves, similar to those shown in Figure B11a, is predicted by critical state theory. Casagrande's concept of the critical void ratio was investigated in some detail by Castro (1969), who demonstrated that sand specimens tending to dilate at failure could be distinguished from those tending to contract by a unique line in the \( e-\log p \) plot. A similar conceptual model was proposed by Egan and Sangrey (1978) in which a family of lines on the \( e-\log p \) plot could be used to represent the behavior of soil under monotonic and cyclic loadings. Nearly all constitutive models based on a cap-shaped surface implicitly assume the existence of state-lines which correspond to the compression curve for proportional loading.

22. The application of Equation B16 to cohesionless soils requires some interpretation of the cohesion term \( c_e \). It was observed by Rowe (1971) that if dilatancy is accounted for, the strength does not include the cohesion component \( c_e \). The relationship between dilatancy and \( c_e \) is readily seen from the critical state relationship by rewriting Equation B16 in terms of the invariants in Table B1:

\[
\frac{T_f}{\sigma_e} = c_e + \frac{\sigma_f}{\sigma_p} \tan \rho \tag{B17}
\]
A. SAMPLE STATE DEFINED BY $\sigma_{P_e}$

B. STRESS-DILATANCY RELATIONSHIPS AT PEAK FAILURE

Figure B11. Interpretation of Hvorslev's failure law based on stress-dilatancy theory
Since the flow rule (Equation B6) is valid for all stress states, the stress-dilatancy relationship at failure is given by:

\[
\frac{T_f}{\sigma_p} = M_{cv} + \psi
\]  

(B18)

where \( \psi \) is the maximum dilatancy rate \((-\cot \theta)\). By comparison of Equations B17 and B18, it can be summarized that the cohesion is in fact a measure of the dilatancy term \( \psi \) (Figure B11b). This is consistent with the general observation that overly consolidated soils (which have low \( p/p_e \) ratios) tend to be dilative at failure.

23. A common criticism of the state surface concept for sand is the lack of a well defined \( e - \log \bar{p} \) relationship (Rowe 1971). Empirical equations describing the compressibility of sands are usually void ratio-dependent power curves and not the unique \( e - \log \bar{p} \) relationship used for clays. However, this criticism stems from a misunderstanding of the difference between the virgin compression line and a state surface. When viewed as a state surface, the virgin isotropic compression line represents the limiting stress state that can be reached under a stress path with \( \sigma_1 = \sigma_2 = \sigma_3 \). As illustrated in Figure B12, the limiting state concept remains valid for sand even though individual compression curves do not follow the linear \( e - \log \bar{p} \) line.

24. To extend the Hvorslev concept for application to general mathematical analysis, it is useful to consider the entire stress path on the Hvorslev plot and not merely the peak stress. The stress paths from two undrained tests on Carrollton Bend sand by Torrey (1981) are shown in Figure B13. Both sand specimens were consolidated anisotropically to an axial stress of 4.0 kg/cm\(^2\) at a stress ratio \((\sigma_a/\sigma_r)\) of 2.0. The specimen prepared at a void ratio of 0.927 exhibited contractive behavior up to 5 percent strain where its ultimate undrained strength was achieved. The second specimen prepared at a void ratio of 0.878 was contractive-dilative and had considerable dilative strength at strain beyond 5 percent. A comparison of the two stress paths on the Hvorslev plot reveals that the initial conditions of the two specimens are quite
Figure B12. Behavior of clay and sand in isotropic compression
Figure B13. Comparison of stress paths of Carrollton Bend sand on $p - q$ and $p/p_e - q/p_e$ plots (data from Torrey, 1981)
differ with respect to their $p/p_e$ ratio, with the specimen having the higher ratio also being more contractive. The series of stress paths shown in Figure B14 further illustrates this trend. It would be expected that specimens with $p/p_e$ ratios greater than 0.3 would display little dilative behavior, while specimens with $p/p_e$ ratios less than 0.1 would display little contractive behavior.

25. Pender (1976) developed a constitutive model in which the undrained stress paths were described in terms of the $p/p_e$ ratio. This concept applies to drained behavior equally well. Since the void ratio changes during a drained test, the stress paths in drained tests display a pattern similar to those of undrained tests (Figure B15). Thus, the use of the Hvorslev coordinates provides a means of separating the effects of changing void ratio on stress-strain response. For example, the key concept of the bounding surface model (Appendix C) is a capped-shaped bounding surface, which serves as a reference state for material behavior. Changes in the position of the bounding surface are controlled by volumetric strain. As shown in Figure B16, the shape and location of the bounding surface would be fixed when plotted in Hvorslev coordinates; the normalized stress paths would asymptotically approach, but never cross, the surface. The Hvorslev plot can therefore be used to infer the most suitable bounding surface shape.

Concluding Remarks

26. Three important conclusions can be made from the data presented in this appendix:

a. The fundamental concepts of stress-dilatancy theory that have been established for the direct shear and triaxial tests can be extended to general stress conditions. Importantly, general models that require parameters only from standard triaxial tests (e.g., Lade 1972) are given strong theoretical basis.

b. The influence of stress and void ratio on dilatant strength can be accounted for by generalizing the Hvorslev failure law.

c. Both sand and clay can be analyzed by a single unified theory if the virgin compression curve is treated as a limiting state.
Figure B14. Undrained stress paths of Carrollton Bend sand in $p - q$ and $p/p_e - q/p_e$ coordinates (data from Torrey, 1981)

NOTE: 1. $e_0$ DETERMINED FROM TRIAXIAL CONSOLIDATION CURVE WITH $\sigma_a/\sigma_r = 2.0$.
2. ALL STRESS PATHS TERMINATE AT 7 PERCENT AXIAL STRAIN.
Figure B15. Correspondence between stress paths and stress-strain curves to Hvorslev's failure line
Figure B16. Depiction of bounding surface model on Hvorslev's coordinates
APPENDIX C: BOUNding surface plasticity for cohesive soils by Yanis F. Dafalias and Leonard R. Herrmann

Background

This Appendix contains a report prepared by Professors Yanis F. Dafalias and Leonard R. Herrmann of the University of California at Davis, Calif. The report describes a constitutive model capable of simulating strain softening behavior of soils. The work documented by this report was carried out under contract No. DACA 39-79-M-0059. Subsequent work on a more comprehensive version of the bounding surface model is documented in a report by Herrmann et al. (1980) prepared for the Civil Engineering Laboratory, Naval Construction Battalion Center, Port Hueneme, Calif.
SUMMARY OF PROJECT

The project consisted of a theoretical and numerical investigation of the application of bounding surface plasticity theory for the characterization of the stress-strain properties of clay soils. The major accomplishments of the project include: 1) the further developments of the bounding surface plasticity model for clay soils to include tensile stress states, 2) the successful comparison of model predictions to available experimental results, 3) the development of a simple computer program for use in comparing model predictions to homogeneous test results, and 4) the preparation of a property's subroutine for incorporating the model into existing finite element analyses for earth structures.

Figure 6.15. Departure of bounding surface model on Hoek-Brown coordinates.
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1. INTRODUCTION

In the field of Geotechnical Engineering it is well recognized that if the major developments in numerical analysis (e.g., finite element procedures) of the past 20 years are to be effectively used, comprehensive constitutive equations for soils must be developed. While the past decade has seen a flurry of activity in this area, no truly comprehensive theory has emerged. The difficulty lies with the inherent inelasticity and variability of soils.

One of the most important recent developments in the field of material characterization is the formulation of bounding surface plasticity theory. Bounding surface plasticity theory would appear to be capable of describing many of the features of soil behavior which have heretofore defied description by other theories. The work done to date, that is described in the following sections, tends to confirm this assessment. It is the purpose of this report to describe these recent advancements.*

2. BOUNDING SURFACE PLASTICITY THEORY

The development of the theory and its application to soils is described in detail in Appendix I.

* Included (for completeness) in this description is work done under University sponsorship prior to the initiation of this project and work done subsequent to its completion under NSF sponsorship.
3. NUMERICAL IMPLEMENTATION

3.1 Incrementalization of the bounding surface plasticity model

The bounding surface plasticity model for cohesive soils is particularly easy to incorporate into numerical stress analysis procedures for earth structures. In order to avoid having to deal with separate formulations for drained and undrained conditions, it is convenient to express them in a common form (the numerical consequences of this step are discussed in a later section). This can be accomplished if the slight compressibility of the soil skeleton and the pore water is recognized. Thus, the pore water pressure $p$ can be written in terms of the bulk modulus $\Gamma$ of the skeleton and the pore water, and the resulting volume change $\varepsilon_{kk}$, i.e.,

$$p = \Gamma \varepsilon_{kk}$$

For drained conditions $\Gamma = 0$. Using the above expression, the total stress increment $\Delta \sigma_{ij}$ is given by the expression:

$$\Delta \sigma_{ij} = \dot{\sigma}_{ij} + \Gamma \varepsilon_{kk} \delta_{ij}$$

Using eq. (4.11) of Appendix I to eliminate $\dot{\sigma}_{ij}$ and factoring out the strain increment gives:

$$\dot{\sigma}_{ij} = D_{ijkl} \varepsilon_{kl}$$  \hspace{1cm} (1)

*Note: The space in front of the eq. numbers of Appendix I stands for the letter I.*
where

\[
D_{ij\ell} = G(\delta_{\ell i} \delta_{\ell j} + \delta_{\ell k} \delta_{\ell l}) + (T + K - \frac{2}{3}G)\delta_{ij} \delta_{\ell\ell} - \left[ \frac{3K}{\partial T} \right] \delta_{ij} \delta_{\ell\ell} \\
+ \frac{G}{\sqrt{J}} \frac{\partial F}{\partial \sqrt{J}} \delta_{ij} \delta_{\ell\ell} \left[ \frac{3K}{\partial T} \right] \delta_{ij} \delta_{\ell\ell} + \frac{G}{\sqrt{J}} \frac{\partial F}{\partial \sqrt{J}} \delta_{ij} \delta_{\ell\ell} \left[ \frac{3K}{\partial T} \right] \delta_{ij} \delta_{\ell\ell} \right] \frac{L}{Kp + 9K(\frac{\partial F}{\partial T})^2} + \frac{G}{K} \frac{\partial F}{\partial T} \delta_{ij} \delta_{\ell\ell} \frac{L}{Kp + 9K(\frac{\partial F}{\partial T})^2} \frac{L}{Kp + 9K(\frac{\partial F}{\partial T})^2} \frac{L}{Kp + 9K(\frac{\partial F}{\partial T})^2} (2)
\]

The quantity \( L \) is unity for loading and zero for unloading; its value is conveniently determined by the sign of \( \hat{\beta} \) (see eqs. (6.4) and (3.2) of Appendix I for a definition of \( \beta \));

\[
L = \begin{cases} 1 & \text{if } \hat{\beta} \leq 0 \\ 0 & \text{if } \hat{\beta} > 0 \end{cases} \quad (3)
\]

Eq. (1) relates the tensor components of stress and strain. For the purpose of finite element analyses, it is more convenient to express this relationship in matrix form, i.e.,

\[
\left[ \sigma \right] = \left[ D \right] \left[ \varepsilon \right] \quad (4)
\]

\( ([ ]^T \) is the matrix transpose)

\[
\left[ \sigma \right]^T = (\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}) \\
\left[ \varepsilon \right]^T = (\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}) \quad (5)
\]

The tensor components of shear strain \( \varepsilon_{ij} \) are one-half of the engineering components \( \gamma_{ij} \). The \( [D] \) matrix is expressed in terms of the components of the \( D_{ijk\ell} \) tensor as follows (because of the symmetry of the stress tensor, interchanging \( i \) and \( j \) or \( k \) and \( \ell \) in eq. (2) results in the same quantity):

\[
[D] = \begin{bmatrix}
D_{1111} & D_{1122} & D_{1133} & D_{1112} & D_{1113} & D_{1123} \\
D_{2222} & D_{2233} & D_{2212} & D_{2213} & D_{2223} & D_{2223} \\
D_{3333} & D_{3312} & D_{3313} & D_{3312} & D_{3323} & D_{3323} \\
D_{1212} & D_{1213} & D_{1212} & D_{1313} & D_{1323} & D_{1323} \\
D_{2323} & D_{2323} \end{bmatrix} \quad (symm) \quad (6)
\]

C6
In order to be able to use eq. (4) in a finite element program, it must be expressed in an iterative-incremental form. Consider the Nth step of an incremental analysis, i.e., the solution has been found at N-1, and it is now desired to calculate the incremental change that will give the solution at N. Because of the nonlinear nature of elastic-plastic behavior, iteration is in general required to establish the incremental change (analyses which do not use iteration are discussed later). In the K-1 iteration of the process, the estimates of the stress and strain states at N are given by the expressions:

\[ [\sigma]_{N, K-1} = [\sigma]_{N-1} + [\Delta\sigma]_{N, K-1} \]  
\[ [\varepsilon]_{N, K-1} = [\varepsilon]_{N-1} + [\Delta\varepsilon]_{N, K-1} \]  

The iteration process is continued until some specified convergence criterion is satisfied or until a specified maximum number of iterations have failed to yield convergence (an indication of possible failure and/or unstable behavior).

Even though rate independent behavior is being considered, it is convenient to think in terms of the time history of the quantities involved. If eq. (4) is integrated from time \( t_{N-1} \) to \( t_N \), it yields:

\[ \int_{t_{N-1}}^{t_N} [\dot{\sigma}] \, dt = \int_{t_{N-1}}^{t_N} [D] \, [\dot{\varepsilon}] \, dt \]

or \[ [\Delta\sigma]_N = \int_{t_{N-1}}^{t_N} [D] \, [\dot{\varepsilon}] \, dt \]

A trapezoidal formula is used to approximate the integral:
Because \([D]_N\) is a function of the stress and strain states at \(N\) (see eq. (2)), it is necessary to base its value on the stress and strain calculations of the previous iteration (see eqs. (7) and (8)); denote the predicted value as \([D]_{N,K}\).

The equation resulting from using this estimate in eq. (11) is used to relate the estimates of \([\Delta \sigma]_N\) and \([\Delta \varepsilon]_N\) for iteration \(K\), i.e.,

\[
[\Delta \sigma]_{N,K} = [D]_{N,K} [\Delta \varepsilon]_{N,K}
\]

where

\[
[D]_{N,K} = \frac{1}{2} \left\{ [D]_{N-1} + [D]_{N,K} \right\}
\]

Eq. (12) is the incremental constitutive equation for iteration \(K\) of increment \(N\). The matrix \([D]_{N-1}\) is calculated using the stress and strain states found at \(N-1\) and \([D]_{N,K}\) is based upon the \(K-1\) estimate of the stress and strain vectors for \(N\). For the first iteration \((K-1=0)\) of all increments beyond the first \((N>1)\), the estimates of \([\Delta \sigma]\) and \([\Delta \varepsilon]\) are usually derived from the values found in the previous increment (and perhaps on a knowledge of the ratio of the current applied load increment to the previous one). For the first iteration of the first increment, it is usually desirable to arbitrarily assume very small increments for the normal stresses and strains (e.g., \(\Delta \varepsilon_x = \Delta \varepsilon_y = \Delta \varepsilon_z = 10^{-5}\), \(\Delta \sigma_x = \Delta \sigma_y = \Delta \sigma_z = 0.001\) \(p_a\); \(p_a =\) atmospheric pressure).

Using \([\sigma]_{N-1}\) and \([\varepsilon]_{N-1}\), the components of \([D]_{N-1}\) are calculated directly from eq. (2); similarly, \([D]_{N,K}\) is calculated using the values of \([\sigma]_{N,K-1}\) and \([\varepsilon]_{N,K-1}\) (see eqs. (7) and (8)). The calculations for \([D]_{N-1}\) and \([D]_{N,K}\) are straightforward and only a few steps need elaboration.
The parameter $\theta$ of eq. (5.8) of Appendix I is undefined for a zero value of the first effective stress invariant ($I^*$). The numerical problems associated with a zero or near zero value of $I^*$ are avoided by arbitrarily replacing the value of $|I^*|$ by $10^{-4} P_a$ for such cases. When $\theta = 0$, eq. (7.2) is undefined; for near zero values of $\theta$ this problem is avoided by replacing $|\theta|$ by $10^{-5}$. In general, these steps have no effect on the calculated properties.

As the soil state approaches the bounding surface, a stress state outside of the surface may be predicted for a particular iteration. Because such a prediction has no meaning, the state is instead assumed to fall on the surface, i.e., $\beta$ (see eq. (6.4)) is restrained to be $\geq 1.0$ and $\frac{\delta}{\delta_o}$ (see eq. (4.9)) is restrained to be $\geq 0$.

At any instant, the size of the bounding surface is determined by the value of $I_o$ (see Figure L2). The differential change in $I_o$ is given by eq. (6.2), i.e.,

$$dI_o = \frac{1+e_o}{\lambda-wK} I_o (d\epsilon_{kk} - \frac{1}{3K} dI^*)$$

Dividing by $I_o$ and integrating the resulting expression for increment $N$ gives:

$$I_{o_{N,K}} = \frac{1+e_o}{\lambda-wK} \left( \int \frac{dI_o}{I_o} = \int \frac{\epsilon_{kk}^{N,K}}{I_o} - \int \frac{1}{3K} dI^* \right)$$

The first two integrals may be evaluated exactly while the third is approximated by the trapezoidal rule:

$$\ln \left( \frac{I_{o_{N,K}}}{I_{o_{N-1}}} \right) = \frac{1+e_o}{\lambda-wK} \left[ \Delta \epsilon_{kk}^{N,K} - \frac{1}{6} \left( \frac{1}{K_{N-1}} + \frac{1}{K_{N,K}} \right) \Delta I^* \right]$$

or

$$I_{o_{N,K}} = I_{o_{N-1}} \exp \left( \frac{1+e_o}{\lambda-wK} \left[ \Delta \epsilon_{kk}^{N,K} - \frac{1}{6} \left( \frac{1}{K_{N-1}} + \frac{1}{K_{N,K}} \right) \Delta I^* \right] \right)$$
At a particular point in an analysis if the stress state is one of pure hydrostatic compression, i.e., \( \sigma_x = \sigma_y = \sigma_z = \sigma_0 \) and \( \tau_{xy} = \tau_{yz} = \tau_{xz} = 0 \) (\( \sqrt{J} = 0 \) and \( I^* = 3\sigma_0 \)); such a condition often occurs at the beginning of a triaxial test) and if the subsequent history is such that \( \sqrt{J} \neq 0 \), the next one or two increments should be rather small. These increments should be small in order that eq. (11) accurately models the quite severe nonlinearities introduced by the hardening function for states where \( \sqrt{J} \approx 0 \) (see eq. (7.1)).

### 3.2 Properties subroutine (CLAY) for finite element programs

A properties subroutine (FORTRAN IV) CLAY has been prepared. This subroutine evaluates the incremental stress-strain properties given by the bounding surface plasticity model for cohesive soils; the subroutine is based on the incremental-iterative approach discussed in the previous section. A listing of the subroutine is given in Appendix III. The subroutine is intended for incorporation into new or existing finite element programs for the analysis of earth structures. The subroutine is designed for use in finite element programs that account for inelastic behavior in an incremental-iterative manner; its adaption for use with non-iterative programs is also discussed. The properties relate the strain and "total" stress increments and can be used for either drained or undrained conditions.

While, for drained conditions, the subroutine may be used with any type of element, its use for undrained conditions has certain limitations. The usual statement of soil properties for undrained conditions leads to incompressible soil behavior. Incompressible material behavior requires a special finite element
formulation [15,26] which is not available in most programs for earth structures. In order to avoid the need for the special finite element formulation, the slight compressibility of the soil skeleton and the pore water has been included, thus, leading to "nearly incompressible" behavior. Considerable care must be exercised in the finite element analysis of nearly incompressible materials in order to avoid excessive round-off error. Either the special formulation for incompressible material [15] should be used or low order numerical integration of the element stiffness matrix is required [30] (e.g., four point integration for a 2nd order two-dimensional isoparametric element). Triangular elements (and quadrilateral elements made up of two or four triangular elements) which use exact integration should be avoided.

For each iteration of each increment and for all points (e.g., element centers or quadrature points) in the body where the incremental properties are required, the parent finite element program will call the subroutine. The call to the subroutine is as follows:

\[
\text{CALL CLAY (IDIM, INC, ITNO, PROP, STOR, SIGB, DSIG, DEPM, D, GAM)}
\]

The quantities IDIM, INC and ITNO are integer variables, GAM is a floating point variable, and PROP, STOR, SIGB, DSIG, DEPM and D are floating point arrays respectively dimensional (14), (6), (6), (6), (6) and (6,6). With the exception of IDIM which is discussed in a later paragraph, the arguments in the call are described below:

INC: Increment number (the first increment must be numbered 1)

ITNO: Iteration number (the first iteration of each increment must be numbered 1)
PROP: An array containing the values of the material parameters which describe the soil at the point in the structure for which the incremental properties are sought. The parent finite element program must read and store the values of the soil parameters for each different type of soil in the earth structure and then for each call of CLAY present the appropriate values for the element in question. The soil parameters are stored in the array in the following order (the significance of the various parameters is discussed in detail in the Appendix I which describes the model; a brief description is also given in Appendix II): $\lambda, \kappa, M, R, A, T, I_1, I_0, m, h, e, \nu, \Gamma, p_a$. That is, PROP(1) = $\lambda$, PROP(2) = $\kappa$, etc. The initial value of $I_0$ (see Section 6 of Appendix I) is denoted by $I_{0,1}$. The value of atmospheric pressure $p_a$ is expressed in the particular units selected for the analysis.

STOR: This array is used to store certain quantities (e.g., current value of $I_{0,1}$) which vary with the current state of the soil and thus for a given step in the analysis are unique to the point in the earth structure that is under consideration. The values in STOR must be stored (after each call to CLAY) by the parent finite element program for each point in the earth structure for which the incremental properties are needed (e.g., element centers). Prior to each call (with the exception of $N=1, K=1$) to CLAY, the appropriate values for the point in question are retrieved from storage (e.g., from a two-dimensional array or a disk file which stores the values for each element in the system) and are presented to the subroutine in the call.

SIGB: $[\sigma]_{N-1}$, i.e., values of stress at the beginning of the increment.

DSIG: $[\Delta \sigma]_{N,K-1}$, i.e., the estimate (supplied by the parent finite element program) of the stress increment.
DEPM: $[\Delta \epsilon]_{N,K-1}$, i.e., the estimate (supplied by the parent finite element program) of the strain increment.

D: $[\bar{D}]_{N,K}$, i.e., the estimate of the incremental stress-strain properties calculated by the subroutine and supplied to the parent finite element program.

GAM: parameter for use in calculating pore water pressure, i.e., in the parent finite element program the pore water pressure "p" can be calculated from the expression $p_N = \text{GAM} \left( \epsilon_{xN} + \epsilon_{yN} + \epsilon_{zN} \right)$.

Subroutine CLAY computes the three-dimensional incremental properties for a cohesive soil for drained ($\Gamma = 0$) or undrained conditions ($\Gamma \neq 0$). The ordering of the stress and strain components in the $[\sigma]$ and $[\epsilon]$ vectors are indicated by eq. (5). The subroutine can also be used to supply properties for two-dimensional finite element analyses; the procedure for its use in such cases and the value of the parameter IDIM in the subroutine call are described in the following paragraphs.

Axisymmetric Analysis (IDIM=3): The ordering of the stress and strain components are as follows $(\sigma_r, \sigma_\theta, \sigma_z, \tau_{r\theta}, 0.0, 0.0)$ and $(\epsilon_r, \epsilon_\theta, \epsilon_z, \gamma_{r\theta}, 0.0, 0.0)$. The indicated zero values must be supplied by the parent finite element program for the $[\sigma]_N$, $[\Delta \sigma]_{N,K}$ and $[\Delta \epsilon]_{N,K}$ matrices. The incremental properties of interest are in the upper-left 4 x 4 corner of the 6 x 6 D array returned by the subroutine.

Plane Stress (IDIM=3): The ordering of the unknowns is $(\sigma_x, \sigma_y, 0.0, \tau_{xy}, 0.0, 0.0)$ and $(\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, 0.0, 0.0)$. It is to be noted that the subroutine can only be used to supply properties for plane stress finite element analyses that calculate values for the thickness strain $\epsilon_z$. 
Plane Strain: The subroutine can be used to supply properties for plane strain finite element analyses in two different ways. For plane strain programs which calculate values for the stress \( \sigma_z \) normal to the plane of the body, IDIM is given the value of 3 and the ordering of the stress and strain vectors are \((\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, 0.0, 0.0)\) and \((\varepsilon_x, \varepsilon_y, 0.0, 0.0, 0.0, 0.0)\) respectively. For plane strain analyses that do not calculate the value of \( \sigma_z \), IDIM is given the value of 2 and the stress and strain vectors are \((\sigma_x, \sigma_y, \tau_{xy}, 0.0, 0.0, 0.0)\) and \((\varepsilon_x, \varepsilon_y, 0.0, 0.0, 0.0, 0.0)\) respectively. The coefficients in the D array are appropriately arranged in each case.

In the above discussion, it is assumed that the parent finite element program iterates within each increment. The subroutine can also be used with programs that do not iterate (of course, in general, considerably smaller increments are required and greater error is to be expected); the following additional steps are required.

In the formation of each element stiffness matrix, the subroutine is called (if it is desired to use incremental properties as predicted by the stress and strain states at \(N-1\) then \([\Delta \sigma]_N\) and \([\Delta \varepsilon]_N\) are assigned zero values) and incremental properties \([\Delta D]_{N_0}\) are calculated; these properties must be stored by the parent program. After the finite element solution is completed for the increment in question, the subroutine is again called* for each point where the properties are required (the calculated stress and strain increments are supplied for \([\Delta \sigma]_N\) and \([\Delta \varepsilon]_N\)); denote the revised incremental properties as \([\Delta D]_{N_1}\). A stress correction matrix \([\Delta \sigma_c]_N\) is calculated (and stored):

\[
\text{This step may be done at the time of forming the element matrices for increment \(N+1\), thus, doing away with the necessity of storing \([\Delta \sigma_c]_N\); If incremental properties as predicted by the stress state at the beginning of the increment are being used (i.e., \([\Delta \sigma]=0\) and \([\Delta \varepsilon]=0\) are supplied to CLAY), then only one call is required, the right-hand side of eq. (14) becomes}
\]

\[
\frac{1}{2} \left\{ [\Delta D]_{N-1} - [\Delta D]_{N_0} \right\} [\Delta \varepsilon]_N\text{; this expression follows directly from eq. (13).}
\[ [\Delta \sigma_c]_N = \{[\bar{D}]_{N+1} - [\bar{D}]_{N_0}\} [\Delta \varepsilon]_N \]  

(14)

In the next increment (N+1), the subroutine is again called to yield a first estimate of the incremental properties \([\bar{D}]_{N+1_0}\). The incremental stress-strain equations are then written to include the stress correction matrix from the previous increment, i.e.,

\[ [\Delta \sigma]_{N+1} = [\bar{D}]_{N+1_0} [\Delta \varepsilon]_{N+1} + [\Delta \sigma_c]_N \]

The stress correction matrix \([\Delta \sigma_c]_N\) contributes to the element load matrix.

3.3 Program (EVAL) to evaluate test results

In the fitting of a material model to experimental measurements, a means must be available for using the model to predict the results of simple homogeneous tests; program EVAL has been written for this purpose. Program EVAL when combined with subroutine CLAY can be used for predicting the behavior of homogeneous samples of cohesive soils (characterized by the bounding surface plasticity model) subjected to arbitrary homogeneous stress and strain histories.

The specified stress and strain histories are broken into "history segments". Within each history segment, a consistent combination of six stresses and strains are prescribed (i.e., the histories of \(\varepsilon_x\) or \(\sigma_x\), \(\varepsilon_y\) or \(\sigma_y\), \(\varepsilon_z\) or \(\sigma_z\), \(\gamma_{xy}\) or \(\tau_{xy}\), \(\gamma_{xz}\) or \(\tau_{xz}\), and \(\gamma_{yz}\) or \(\tau_{yz}\)). The particular combination of stresses and strains which are prescribed may differ for each history segment. For example, the analysis of a plane strain test might involve two segments, i.e., at the end of
the first segment $\sigma_{x_1} = \sigma_c$, $\sigma_{y_1} = \sigma_c$, and $\varepsilon_{z_1} = 0$ might be prescribed to simulate the application of a confining pressure, while at the end of the second $\sigma_{x_2} = \sigma_c$ and $\varepsilon_{y_2} = \varepsilon_0$, $\varepsilon_{z_2} = 0$ could be prescribed to simulate a subsequent specified strain of $\varepsilon_0$. Each history segment is broken into increments (the relative size of the increments for a given segment is controlled by the specified value of D) with iteration conducted within each increment. Instructions for preparing the input data for EVAL are given in Appendix II; a listing of the program is given in Appendix IV.

The analysis conducted by EVAL is in essence a one-element finite element analysis of a homogeneous body. At the end of each increment, the increment number, the six strains, the six stresses, the pore water pressure (labeled U), and the number of iterations required for convergence are printed.

When convergence is not achieved within the specified maximum number of iterations, the program prints the relative difference (between the last two iterations) in the norms for the stress and strain increments, and then proceeds to the next analysis.
4. IDENTIFICATION AND CALIBRATION OF THE MATERIAL CONSTANTS

Unless the several parameters which describe a material model can be evaluated for the particular material at hand the model will be of little value. One of the most appealing features of the bounding surface plasticity model for soils is its simplicity and hence the relative ease with which the parameters may be determined for a particular soil. For this purpose, the set of soil parameters will be divided into two groups.

(i) Old material constants.

This group includes the elastic constants $K$ (or $\kappa$) and $G$ (or Poisson's ratio $\nu$), and the critical state soil mechanics material constants $\lambda$, $R$ and $M = \sqrt{3}N$. Their determination follows well known methods although there is still considerable discussion for $K$ and $G$. Here $K$ is defined by means of eq. (6.3) in terms of $\kappa$, and $G$ is computed from $K$ and a constant $\nu$. The objections raised on the basis of energy dissipation \cite{31} against such a dependence of $G$ on $\kappa$ do not apply straightforward here since no purely elastic range exists in the usual sense for $\theta > 0$. However, there may be still some cyclic stress paths near $\theta = 0$ causing problems and further investigation will be necessary.

(ii) New material constants

The new constants are $I_2$, $A$, $m$, $h$ and $T$. The $I_2$ is not related to the bounding surface concept and refers to a better description of the elastic response near the origin. It can be usually taken equal to the atmospheric pressure $p_a$. The $A$ defines the shape of the hyperbolic part of the bounding surface for heavily overconsolidated states, and it can also be considered as an appropriate parameter for a classical yield surface formulation improving the surface shape.

The role of $m$ has been explained in Appendix I in connection with the role of $|M/n|$ or $|N/\theta|$. It was found that $m = 0.2$ can be used for most
clays. For this value, the first bracket of eq. (7.2) varies between 2.58 and 1, as \( \theta \) changes from \( N/10 \) to 0, a variation which is negligible compared to the values of \( K_p \) and the other quantities entering eq. (7.2). The \( h \) is the most important material constant defining the response for stresses within the surface which is the salient feature of the bounding surface concept.

The following concrete steps are now suggested for the calibration of \( h \) and \( A \), once the old material constants have been defined and \( I_k = p_a, m = 0.2 \).

1. Obtain the experimental curves (\( q - p', q - \epsilon_1, p - \epsilon_1 \), etc) for deviatoric loading of an isotropically overconsolidated sample at an overconsolidation ratio between 1 and 0, preferably at \( OCR = 3R/(2R + 1) \). Determine \( h \) by curve fitting the experimental data using the developed incremental relations and Program EVAL (see Section 3.3). This can be done by a trial-and-error process observing that increasing \( h \) implies stiffer response. For this range of \( OCR \), the \( A \) does not appear in the equations.

2. Obtain the experimental curve for deviatoric loading after heavy overconsolidation, that is \( OCR \geq 5 \) at least. With \( h \) known from step 1, \( A \) is determined similarly by a trial-and-error numerical process. Increasing \( A \) implies a "flatter" hyperbola with a stiffer response and reduced dilatation, while a very small \( A \) can make the hyperbola almost identical with the critical state line for material with small cohesion.

It is important to emphasize that the response to cyclic loading is obtained on the basis of the above state dependent formulation as a sequence of monotonic loading/unloading events without introducing any additional cyclic empirical parameter.
The parameter $T$ is defined from $I_t = T I_o$ where $I_t$ represents the tensile strength of the soil, see Fig. I-2, Appendix I. It is taken in this report equal to 0.08. Its value is sufficient to define ellipse 2, Fig. I-2, whose equations are built in the properties subroutine (PROP). It must be mentioned however that negative, $I < 0$, does not appear in the fitting of experimental results (see section 6), although some of the normal stress components may be negative (tensile).

The following two sets of material constants will be used subsequently:

a. Set No. 1

- $M = 3 \sqrt[3]{N} = 1.05$
- $\kappa = 0.05$
- $\lambda = 0.14$
- $\nu = 0.15$
- $R = 2.72$
- $m = 0.20$
- $h = 44$
- $A = 0.30$

b. Set No. 2

- $M = 3 \sqrt[3]{N} = 0.95$
- $\kappa = 0.05$
- $\lambda = 0.26$
- $\nu = 0.15$
- $R = 2$
- $m = 0.20$
- $h = 58$ (or 20)
- $A = 0.30$

Some of the above constants were obtained by the calibration procedure outlined above in connection with experimental data as it will be shown subsequently.

5 MODEL BEHAVIOR IN THE TRIAXIAL SPACE

The response of the model under monotonic and cyclic loading is subsequently presented in the triaxial space $p^* - q$, by using the general formulation of section 4.5 of Appendix I.
5.1 Monotonic loading

Using the set No. 1 of material constants the drained and undrained behavior of the model with increasing q up to critical failure at OCR = 1, 1.2, 2, 5, 10 and 20 for initial void ratios e_0 = 0.94, 0.95, 0.97, 1.02, 1.06 and 1.08 correspondingly, is shown in Figures 1 and 2 respectively where also the initial position of the bounding surface is marked. Stress or strain increments are imposed, and the model incremental response is obtained from the developed constitutive relations and the use of program EVAL.

The smooth undrained stress paths, Fig. 2a, show a p' reduction indicating plastic consolidation within the bounding surface at the very early stages of loading. In a classical yield surface formulation, the stress paths would "shoot" upwards until they reach the surface first and then plastic loading occurs. The same "smoothness" reflects in all other curves, without sharp transitions from elastic to elasto-plastic behavior. For OCR = 5, 10, 20 the response is of particular interest. For drained or undrained cases the stress path crosses the critical state line becoming associated with the contracting hyperbolic part of the bounding surface where K_p ≤ 0 but K_p is still positive. Eventually δ becomes small enough and K_p negative enough to yield K_p = 0 according to eq. (7.2). The locus of all these points where K_p = 0 marks the so-called "failure envelope," although failure has not occurred yet, lying between the critical state line and the initial bounding surface, Figs. 1a and 2a. The behavior subsequently differs.

For drained loading K_p becomes negative indicating a falling q-ε_1 curve Fig. 2b. Simultaneously dilatation, which has begun the moment the stress crossed the critical state line while q was still increasing, becomes predominant and this is shown in the ε_p-ε_1 curve especially for OCR = 10, 20. The stress eventually falls back on the critical state line together with the contracting bounding surface and critical failure occurs.
It is interesting to study now the undrained stress path in connection with its differential eq. (4.14), where $I^\tau$, $J^\tau$ correspond to $p^\tau$, $q$. When the stress is on the critical line, the "image" point is point C, Fig. 2 of Appendix I, where $(\partial F/\partial \tau^\tau) = 0$ thus $(\tau^\tau)/I^\tau = \infty$ and the path crosses vertically the line. When eventually $K_p = 0$, observe from eqs. (4.7) and (4.14) that this implies motion along a near neutral loading direction while negative pore-water pressure develops, Fig. 2c, and the bounding surface contracts. The stress path moves towards increasing mean effective stress, Fig. 2a, until it reaches the critical state line almost horizontally and then critical failure occurs. This is the well known stabilizing effect of negative pore-water pressure increase. Accordingly, instability is not pronounced as in the drained case and this is shown in Fig. 2b. These observations on eq. (4.14) explain also the characteristic "hook" of the stress path.

5.2 Cyclic loading

Using the set No. 2 of soil parameters with $h = 20$ and $e_o = 1$, the undrained cyclic deviatoric loading for 8 cycles and for amplitudes $q/p_o = .25$ and $q/p_o = .42$ yields the soil response as shown in Fig. 3. Observe the progressive motion of the effective stress towards the critical state line and the simultaneous expansion of the bounding surface shown by discontinuous lines. At the end of the 8th cycle and for $q/p_o = .25$ the stress state has not reached the critical state line. On the other hand for $q/p_o = .42$ four cycles suffice to bring the soil to a state where the effective stress loop does not progress anymore while pore water pressure increases and decreases cyclically with zero mean net increase, and axial strain $e_1$ accumulates continuously. For failure a final $q$ increase is necessary.

A problem here is that the stress state will be brought to the critical line (not necessarily with failure) for any amplitude of $q$ if a sufficient number
of cycles is applied. Exactly the opposite problem may arise for a bounding/yield surface formulation [19] with kinematic hardening, i.e. the stress loops stabilize when the stress alternates between the points of zero gradient component along \( p' \) of the yield surface, and this may be undesirable for the particular amplitude of \( q [24] \). In the present formulation the corresponding problem can be easily remedied, introducing the concept of the elastic nucleus, Fig. 3. The elastic nucleus defines a domain of purely elastic response (recall that irreversible plastic deformation is responsible for the phenomenon of cyclic mobility) within the bounding surface, in the sense that \( K_p = \infty \) inside the nucleus. Its growth can depend on the magnitude of the plastic void ratio \( e^{n} \). As the nucleus size increases, the lower amplitude loops will eventually enter its domain with full stabilization (no accumulation of \( \varepsilon^1 \)) while the higher amplitude loops will still be capable of reaching the critical state line. Observe that the elastic nucleus is not the same concept as that of a yield surface (no consistency condition, no loading-unloading criterion etc.).

As a matter of fact, it is not necessary to explicitly write an equation for the growth of the elastic nucleus. Its effect can be conveniently built into the form of Eq. (7.2) interpreted in the triaxial space as follows. With \( \mathbf{r} = \mathbf{OA}, \) Fig. 3, a purely elastic response i.e. \( K_p = \infty \) can be assumed whenever \( \delta = \Delta K > \mathbf{r}/s \) where \( s > 1 \) is a stabilization factor possibly a function of the state. This can be achieved by substituting for \( \delta_0 - \delta \) in Eq. (7.2), the quantity \( \left< p_p - (s p_0/\mathbf{r}) \delta \right> \) if \( \delta_0 = p_0 \). Then, whenever \( \delta > \mathbf{r}/s \) the brackets yield a zero value and \( K_p = \infty \). Observe that \( K_p + \infty \) in a continuous way as \( \delta \rightarrow \mathbf{r}/s \). Further investigation of this modification is necessary.

6 COMPARISON WITH EXPERIMENTS

The experimental results of soft clay response under undrained monotonic deviatoric loading in compression are shown by corresponding symbols in Fig. 4.
and 5 for OCR = 1, 1.2, 2, 5, 8 and 12 with initial void ratios $e_0 = 0.93, 0.95, 0.97, 0.95, 0.95$ and 0.95 respectively, as reported by Banerjee and Stipho$^{[2, 3]}$. The classical parameters $M, \kappa, \lambda, \nu$ and $R$ are taken from the above references ($\nu$ was changed to 0.15 from the suggested value 0.30), and $m$ was put equal to 0.20. According to step 1 of the suggested calibration procedure of section 5 the experimental curves for OCR = 1.2, Fig. 4, were used to obtain $h = 44$. Observe the internal consistency of the model shown by the good fit of all three curves simultaneously corresponding to OCR = 1.2 in Fig. 4. Subsequently, the curves for OCR = 5, Fig. 5, were used to obtain $A = 0.30$ (again one curve would suffice). This yields the set No. 1 of material constants and the predictions and comparison with experimental data for all OCR are shown in Figs. 4, 5 by curved lines.

The soil parameters set No. 2 with $h = 58$ and $e_0 = 1$ is used to predict the undrained kaolin response under cyclic deviatoric loading as reported experimentally by Wroth and Loudon$^{[29]}$. Again the classical critical state parameters are taken from the above work. The comparison of calculated versus experimental behavior for the undrained stress path is shown in Fig. 6a. The $q-\varepsilon_1$ and $p-\varepsilon_1$ curves are shown in Figs. 6b, 6c but no experimental data were available per comparison. In all the above, the values of $h$ are different than the ones reported in $^{[13]}$. This is because eq. (7.2) has been slightly modified and $h$ is non-dimensionalized by introducing $p_a$.

The bounding surface plasticity model for cohesive soils may be used for all possible stress states. In order to investigate its applicability to stress states which include tensile components, the hollow cylinder test conducted by Al-Hussaini and F.C. Townsend $^{[1]}$ was analyzed. Unfortunately, the experimental information required for calibrating the model for the particular soil used in
the test is not available, and thus, some rather arbitrary assumptions had to be made; as a consequence, any conclusions drawn must be regarded as tentative.

For this study, it was decided to consider only one of the three soils tested by Al-Hussaini and Townsend, i.e., the "Vicksburg buckshot clay" (denoted as the "CH" soil). It was decided to use the results of the "unconfined compression test" to tentatively calibrate the model and then to attempt to predict the results of the Hollow Cylinder Test. It is recognized that at best, the results of this comparison must be regarded as very preliminary; the problems are that the unconfined compression test does not provide enough information for calibrating the model, the values of the modulus measured from these two tests appear to be contradictory (this fact was noted by Al-Hussaini and Townsend), and uncertainties concerning the degree of drainage of the sample exist (a fact also noted by Al-Hussaini and Townsend).

Values of modulus of \( E = 3200 \) psi and Poisson's ratio of \( 0.2 \pm 0.3 \) were measured in the unconfined compression test; it was assumed that these values reflect the elastic behavior of the soil. Because in the consideration of compressive stress states for other soils, low values of Poisson's ratio were indicated, a value of \( 0.2 \) was initially selected for the CH soil. From Al-Hussaini and Townsend's report, a value of \( 0.85 \) was given for the void ratio. Because the mean pressure is small for the unconfined compression test, the stress state is in the region where the model assumes a constant bulk modulus, i.e., \( K = \frac{(1+\varepsilon)}{3\kappa} \times 44.1 \) psi (i.e., \( I \leq 3 \) Pa). Using the values of \( E = 3200 \) psi and \( \nu = 0.2 \), a value of \( K = 1800 \) psi was calculated; using this value and \( e = 0.85 \) a value of \( \kappa = 0.015 \) was found. In the absence of additional experimental evidence for the CH soil, values of \( M = 1.05 \), \( \lambda = 0.25 \), \( R = 2.72 \), \( A = 0.06 \), \( h = 30 \), and \( m = 0.2 \) were used; these are typical values found for other clay soils (because the definitions of some of the parameters have been revised, these
values differ somewhat from those cited in a preliminary report of October 1979). Al-Hussaini and Townsend found that the compressive strength of the CH soil was \( \approx 23 \) psi; using this value and \( R = 2.72 \), it was estimated that \( P_{oi} \approx 70 \) psi (hence, \( l_{oi} \approx 210 \) psi). After the first analysis of the unconfined compression test, the values of \( \kappa, h \) and \( v \) were revised to .011, 680, and .15 respectively. While it is not clear whether the tests should be considered as drained or undrained, it appears probable that they most nearly correspond to drained conditions (no attempt was made to saturate the samples). Thus, the first series of analyses were done assuming drained conditions.

For the above values of the model parameters, the predictions for the unconfined compression test are plotted on a reproduction of Figure 18 of Al-Hussaini and Townsend's report and are displayed herein as Figure 7. Al-Hussaini and Townsend's experimental curves are labeled CH-1 and CH-2, the model predictions using the above parameters are labeled \( \kappa = .011 \). In view of the necessity of using assumed values for a number of the parameters of the model, the agreement is good.

The analysis of the Hollow Cylinder Tensile Test presents some particular difficulties; these problems stem from the fact that the sample is not in a homogeneous stress state (this fact was discussed by Al-Hussaini and Townsend), and uncertainties exist concerning the degree of drainage. The following expressions were used for the average radial and circumferential stresses and strains (the notation is the same as in Al-Hussaini and Townsend's report).

\[
\begin{align*}
\sigma_r &= \frac{p_i + p_o}{2}, \\
\sigma_t &= \frac{p_0 r_o - p_i r_i}{r_o - r_i}, \\
\varepsilon_r &= \frac{u_o - u_i}{r_o - r_i}, \\
\varepsilon_\theta &= \frac{u_o + u_i}{r_o + r_i}
\end{align*}
\] (15)

C25
The test was conducted under assumed plane strain conditions, i.e., $\varepsilon_z=0$. Utilizing the plasticity model (and the previously stated values for the material parameters) the histories of $\varepsilon_r$ and $\varepsilon_\theta$ were predicted and thence, the histories of $u_0$ and $u_1$ were calculated.

The results predicted by the analysis are given in Figure 8 which is a reproduction of Figure 15 from Al-Hussaini and Townsend's report. The results obtained using the model parameters found from analyzing the unconfined compression test are labeled $\kappa = 0.11$; the comparison with the experimental results are seen to be rather poor. The poor correlation would appear to result from one or more of the following reasons: i) the approximation involved in assuming the stress state to be homogeneous, ii) the assumption of drained conditions, iii) use of an inappropriate "hardening function" or "bounding surface" in the model, iv) incorrect choice of the several parameters which describe the model, and/or v) the apparent difference in the measured elastic modulus of the soil for the two test conditions. While insufficient information exist to investigate all these factors in detail, preliminary evaluations of some of the probable causes were made.

The extreme stress and strain conditions present at the inner and outer surfaces of the cylinder were analyzed; the results were similar to those obtained for the average conditions assumed in eq. (15) and thus, it was concluded that the inhomogeneous nature of the test was not the probable cause of the discrepancy.

A consideration of a limited range of values for the hardening parameters and different shapes of the bounding surface (as controlled by the parameter $A$) also failed to resolve the discrepancy in the results of the two tests.

The analyses were repeated assuming undrained conditions. In the hollow cylinder test, this led to a calculation of a tensile axial stress beyond a certain point in the test. Because it is unreasonable to assume that such axial tensile
stresses can be developed by the testing apparatus, when that point was reached
in the analysis, the plane strain condition of $e_z=0$ was replaced by a condition
of $\sigma_z=0$. The results for undrained conditions are shown in Figure 9 and 10.

The assumption of undrained conditions had little effect on the "fit" to the
unconfined compression test data while improving the predictions for the hollow
cylinder test. Sufficient information, however, does not exist to permit one to
decide if such an assumption is at all warranted.

The most likely cause for the discrepancy remains the unexplained
difference in the moduli measured in the two test; this is illustrated by considering
the results obtained by changing the value of $\kappa$ to more nearly agree with the
apparent elastic modulus observed in the Hollow Cylinder Test, i.e., $\kappa = .003$.
Using this value of $\kappa$, the analyses were repeated (see Figures 7 and 8). Now
whereas the hollow cylinder results are in better agreement, the unconfined
compression test results show considerable disagreement.

It would appear that a final determination of the reason for the discrepancy
will require further experimental investigation, e.g., hollow cylinder tests on a
soil whose material parameters have been established by a series of triaxial
tests. Such tests should be conducted under either drained or undrained conditions
(for undrained conditions, the average pore water pressure should be measured)
and measurement should be made of the axial force required to maintain plane
strain conditions (this measurement would give one additional quantity that could
be compared to the model predictions).

7 CONCLUSION AND FUTURE IMPROVEMENTS

In this report a general aspect of the concept of the bounding surface
was presented, supplemented by a procedure to construct bounding surface models
in general. The combined bounding/yield surface models$^{[10, 16, 19]}$ can be
included within the framework of the general aspects presented in section 3 of Appendix I, and further elaboration on this can be found elsewhere [19]. The general equations of soil plasticity were developed within a bounding surface formulation. Attention was subsequently given in constructing within the critical state soil mechanics framework a simple model in invariant stress space for clays. No yield/loading surface is explicitly introduced and a simple "radial" rule associates stress points within the bounding surface with their "images" on the surface. The state and the functioning of the model are defined only in terms of the stress and the plastic void ratio change. These properties provide certain attractive features of simplicity which can be very important from the point of view of the numerical analysis for large systems.

Despite its simplicity and with the exception of the capability to account for anisotropy, the model can describe realistically the soil response under different monotonic and cyclic loading conditions at any OCR, including unstable behavior and cyclic mobility with further potential to include tension response. Comparison with experiments demonstrates these properties.

The present formulation introduces only two new parameters \( h \) and \( m \) associated with the general functioning of the model. A third parameter \( A \) aims at improving the shape of the used bounding surface but it is not essential to the general concept (other shapes can be used). Methods for the calibration of \( h \), \( A \) are proposed and applied.

Future improvements of the model can be summarized as follows

1. Introduction of the third stress invariant by means of the "Lode" angle in order to describe the different properties observed for triaxial extension.

2. Consideration of initial and induced anisotropy. This can be achieved by means of proper stress invariants and a non-associated flow rule varying with anisotropy.
3. More detailed consideration of the response under cyclic loading by means of the concept of the elastic nucleus (section 5.2).

As a final conclusion, perhaps the value of this presentation can be embodied in the demonstrated simple idea that any sound classical yield surface soil plasticity model can be easily transformed into a corresponding and more flexible bounding surface model.

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Figure 1a. Drained behavior of the model at different OCR
Figure 1b. Drained behavior of the model at different OCR
Figure 1c. Drained behavior of the model at different OCR
Figure 2a. Undrained behavior of the model at different OCR
Figure 2b. Undrained behavior of the model at different OCR
Figure 2c. Undrained behavior of the model at different OCR
Figure 3. Undrained cyclic behavior of the model for different cyclic stress amplitudes
Figure 4. Predictions of the model for lightly overconsolidated clay (experimental data from Banerjee and Stipho[2])
Figure 4 (continued)

<table>
<thead>
<tr>
<th>OCR</th>
<th>1</th>
<th>1.2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$ (kPa)</td>
<td>365</td>
<td>365</td>
<td>386</td>
</tr>
</tbody>
</table>

Undrained Test Compression

Deviatoric Stress $\sigma' / p_0$

Axial Strain $\varepsilon_{1\%}$
Undrained Test Compression

PORE WATER PRESSURE \( p \) (kPa)

OCR 1 1.2 2
\( p_0 \) (kPa) 365 365 386

Predicted
Measured

AXIAL STRAIN \( \varepsilon_1 \% \)

Figure 4 (continued)
Figure 5. Predictions of the model for heavily overconsolidated clay (experimental data from Banerjee and Stiphos)

OCR: 5 8 12
p₀ (kPa): 380 386 414

Predicted: 
Measured: 

AXIAL STRAIN $\varepsilon_{1%}$
Figure 5 (continued)

Unsaturated response of specimens 44s and 22p:

Predicted

Measured

Undrained Test Compression

AXIAL STRAIN $\epsilon_1\%$

Figure 5 (continued)
Figure 6a. Theory versus experiments for undrained cyclic loading (experimental data from Wroth and Loudon[28])

\[ R = 2 \]
\[ M = 0.95 \]
\[ \kappa = 0.05 \]
\[ \lambda = 0.26 \]
\[ h = 58 \]
\[ m = 0.20 \]
Figure 6b. Theory versus experiments for undrained cyclic loading (experimental data from Wroth and Loudon [28]).
Figure 6c. Theory versus experiments for undrained cyclic loading (experimental data from Wroth and Loudon[28]).
Figure 7. Stress–Strain Curves for CH Soil in the Unconfined Compression Test (assumed drained conditions)
Figure 8. Radial Pressure–Deformation Curves for CH Soil in the Hollow Cylinder Tensile Test – $P_0 = 2$ PSI (assumed drained conditions)
Figure 9. Stress—Strain Curves for CH Soil in the Unconfined Compression Test (assumed undrained conditions)
Experimental Results

Figure 10. Radial Pressure–Deformation Curves for CH Soil in the Hollow Cylinder Tensile Test—$P_0 = 2$ PSI (assumed undrained conditions)
INTRODUCTION *

A common shortcoming of many stress-strain laws for soils is that they are applicable only to loading conditions of a rather specific nature. This weakness becomes particularly keen when an artificial distinction is made between monotonic and cyclic loading for practical purposes. For example, the traditional studies of cyclic loading response deal primarily with gross overall soil behavior under specifically chosen cyclic conditions, and the corresponding constitutive relations are useless for monotonic or interchange of monotonic and cyclic loading. But what is, after all, a cyclic loading but a sequence of monotonic ones? This shows the necessity to develop the constitutive laws within a more fundamental framework, such that they are applicable to monotonic or cyclic, drained or undrained, or any other form of loading conditions in order to be of value for the analysis of soil structures under complex loading.

The classical mathematical theory of plasticity provides such a framework and great advances were made in the last 25 years, especially after the establishment of the critical state theory. [25] Still, however, some very important aspects of soil behavior, mainly in relation to the cyclic response, cannot be adequately described. The principal reason is that the classical concept of a yield surface provides little flexibility in describing the change of the plastic modulus with loading.

*Note: The notation .1 is used in place of 1.1 to number the first section of this appendix, and the same applies to .2, .3, etc.
directions and implies a purely elastic stress range contrary to the reality for many soils. The last feature is responsible for the inability of the classical theory to predict even qualitatively strain accumulation for drained, or pore-water pressure built-up for undrained cyclic deviatoric loading within a stress domain which has been defined as elastic.

The need for new concepts in plasticity became a necessity. Eisenberg and Phillips [14] introduced the concept of loading surfaces distinct from the yield surface and Mroz [17] introduced the field of workhardening moduli which had similarities to, but it is not equivalent to the sublayer models. Mroz's idea was originally applied to metal plasticity and subsequently to soils by Prevost [22] and Mroz, Norris and Zienkiewicz [18]. The endochronic concept [27] presents a totally novel approach which has been applied to soils by Bazant [4] and Valanis [28].

Among the new concepts is that of the "Bounding Surface" originally introduced by Dafalias and Popov [8,10] and Dafalias [5], and independently by Krieg [16] in conjunction with an enclosed yield surface for metal plasticity. The concept and the name were motivated by the observation that the stress-strain curves converge with specific "bounds" at a rate which depends on the distance of the stress point from the bounds. The original bounding surface model has been extended to include materials with a vanishing yield surface [9,11], rate effects [12] and other aspects of material behavior [20]. The bounding surface bears a similarity with the outer surface used in the field of workhardening moduli formulation [17], but it is not equivalent and in general provides a simpler constitutive model. In addition, the concept of the bounding surface occupies a more fundamental position in the development of plasticity theory due to its interpretation in terms of micro-mechanics [21]. The salient features of a bounding surface formulation are that plastic deformation may occur for stress states within the surface, and the possibility to have a very flexible variation of the plastic modulus. These features yield
definite advantages over a classical yield surface formulation, particularly with regards to soil plasticity.

A bounding/yield surface plasticity formulation for soils was fully developed by Mroz, Norris and Zienkiewicz [19] within the triaxial space of critical state soil mechanics; the reader is referred to this reference for a comprehensive discussion on the physical meaning of the bounding surface in relation to the soil structure. Two different direct bounding surface formulations within the framework of critical state soil plasticity were also presented qualitatively by Dafalias for the case of zero elastic range [6] and quasi-elastic range [7]. The latter was fully developed and applied to clays by Dafalias and Herrmann [13] in the triaxial space.

This Appendix presents a generalization and further analysis of the previous work [13] in a general stress space by means of stress invariants.

It should be mentioned finally, that in this presentation it is attempted not only to introduce a new soil plasticity model with a definite form and certain applications, but perhaps more important, it is attempted to present a new framework within which many different constitutive models for soils can be developed by employing the concept of the bounding surface.

.2 GENERAL FORMULATION OF ELASTOPLASTICITY FOR SOILS

In this section a general formulation of rate independent elasto-plasticity for soils will be summarized. The role of the bounding surface in this formulation will be presented in the following section. Since the constitutive relations refer to the deformation of the soil skeleton, the state of the material is defined in terms of the effective stress $\sigma_{ij}^*$ and plastic internal variables $q_n$ accounting for the past loading history. The $q_n$ are usually scalar or second order tensor quantities such as the plastic work, the plastic strains, etc. With $p$ denoting the pore-water pressure, the total stress $\sigma_{ij}$ is related to $\sigma_{ij}^*$ and $p$ by:
\[ \sigma_{ij} = \sigma_{ij}^e + \delta_{ij} \rho \]  \hspace{1cm} (2.1)

with \( \delta_{ij} \) being the Kronecker delta. Observe that the deviatoric components \( s_{ij} \) and \( s_{ij}^e \) are equal. If now \( \varepsilon_{ij} \), \( \varepsilon_{ij}^e \), \( \varepsilon_{ij}^p \) are the total, elastic and plastic strains respectively and a dot indicates the rate, the relation

\[ \dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p \]  \hspace{1cm} (2.2)

is assumed. Small strain formulation is presented henceforth, but the extension to large deformations is straightforward if a proper stress measure and a conjugate rate of deformation measure for which the decomposition (2.2) holds true are employed.

The elastic incremental constitutive relations are given by

\[ \dot{\varepsilon}_{ij} = C_{ijkl} \dot{s}_{kl}^e, \quad \dot{\varepsilon}_{ij}^p = E_{ijkl} \dot{\varepsilon}_{kl}^p \]  \hspace{1cm} (2.3)

where \( C_{ijkl}, E_{ijkl} \) are the tensors of elastic compliance and moduli respectively, being in general functions of the state with \( E_{ijkl} C_{klpq} = \delta_{ip} \delta_{jq} \).

The plastic constitutive relations require the definition of the direction (or vector) of plastic loading \( L_{ij} \) and the plastic modulus \( K_p \), both functions of the state, which in turn determine the loading function \( L \) as

\[ L = \frac{1}{K_p} L_{ij} \dot{\sigma}_{ij}^p \]  \hspace{1cm} (2.4)

Plastic loading, unloading and neutral loading occur when \( L > 0 \), \( L < 0 \) and \( L = 0 \), respectively. The inclusion of \( K_p \) in \( L \) allows for the description of unstable behavior when both scalar quantities \( L_{ij} \dot{\sigma}_{ij}^p \) and \( K_p \) are negative but \( L > 0 \). Then, assuming linear dependence of \( \dot{\varepsilon}_{ij} \) on \( \dot{\sigma}_{ij} \) for rate independence (homogeneity of order one would guarantee rate independence for a more general development), considering
\( \varepsilon^{ij} \) as one of the \( q_n \) but distinguishing it for emphasis, and imposing the requirement of continuous material response with respect to a changing direction of \( \dot{\varepsilon}_{ij} \) across neutral loading \([10]\), the constitutive relations are given by

\[
\dot{\varepsilon}_{ij} = < L > R_{ij} \quad (2.5a)
\]

\[
\dot{q}_n = < L > r_n \quad (2.5b).
\]

where the brackets \(< >\) define the operation \(<z> = z^* (z)\), \( h^* \) being the heaviside steep function, and \( R_{ij}, r_n \) are functions of the state. The \( R_{ij} \) is usually assumed to be the gradient of a plastic potential. In classical plasticity, the \( L_{ij} \) is defined as the gradient of the yield surface \( f = 0 \) and \( K_P \) is obtained by means of the consistency condition \( \dot{f} = 0 \).

The inversion of eqs. (2.2), (2.3), (2.4) and (2.5) yields

\[
L = \frac{L_{km} E_{k'm}}{K_P + E_{abcd} L_{ab} R_{cd}} \varepsilon_{mn} \quad (2.6)
\]

\[
\dot{\sigma}_{kk} = E_{km} [\dot{\varepsilon}_{ij} - < L > R_{ij}] \quad (2.7)
\]

Eq. (2.6) offers an easy computation of \( L \) when \( K_P = 0 \) at the initiation of unstable behavior.

For undrained conditions and incompressible fluid and solid phases, the internal constraint \( \dot{\varepsilon}_{kk} = 0 \) must be satisfied, which by means of eqs. (2.2), (2.3a) and (2.5a) yields

\[
\dot{\varepsilon}_{kk} = \dot{\varepsilon}_{kk}^{\varepsilon} + \dot{\varepsilon}_{kk}^{\sigma} = [C_{kki} + (1/K_P) L_{ij} R_{kk}] \dot{\sigma}_{ij} = 0 \quad (2.8)
\]
where of course for \( L \leq 0 \), \( \dot{\epsilon}_{kk} = 0 \). Eq. (2.8) is a differential equation for the undrained effective stress path which is defined by the tensor inner product of \( \dot{\sigma}_{ij} \) and the bracketed quantity. The effective stress is not independently controlled any more, and the constitutive relations should be expressed in terms of the total strain and stress rates. Using eqs. (2.1), (2.4), (2.6) and (2.8), one has

\[
\begin{align*}
L &= \frac{C_{aabb} L_{ij} - L_{qq} C_{kkij}}{K_p C_{rrss} + K_m m_{nn}} \\
\dot{\sigma}_{ij} &= \frac{L_{kk} E_k \dot{e}_{mm}}{K_p + E_{abcd} l_{ab} R_{cd}} \dot{e}_{mm} \quad (2.9)
\end{align*}
\]

\[
\dot{p} = \frac{1}{C_{aabb}} \left[ R_{kk} \langle L \rangle + C_{kkij} \dot{s}_{ij} \right] + \frac{1}{3} \dot{e}_{mm} m = \frac{1}{3} E_{kkij} \left[ R_{ij} \langle L \rangle - \dot{\epsilon}_{ij} \right] + \frac{1}{3} \dot{e}_{mm} \quad (2.10)
\]

with \( s_{ij}, e_{ij} \) the deviatoric stress and strain components. Observe that \( \dot{s}_{ij} \) can be used instead of \( \dot{\epsilon}_{ij} \) in (2.9). The constitutive relations (2.5a) and (2.7) hold true for undrained loading with \( L \) defined from (2.9) and \( \dot{\epsilon}_{ij} \) substituting \( \dot{s}_{ij} \) since \( \dot{\epsilon}_{kk} = 0 \). The total mean normal stress rate \((1/3) \dot{\sigma}_{mm}\) is indeterminate (incompressibility) and effects only the pore-water pressure rate \( \dot{p} \) through eq. (2.10).

### 3 THE BOUNDING SURFACE

A novel approach of defining the key quantities \( L_{ij} \) and \( K_p \) is presented in the following, which is not subjected to the limitations of a yield surface plasticity formulation as explained in the introduction. The previous loading history, expressed quantitatively by means of the values of \( q_n \), determines a "Bounding Surface" in stress space, Fig. 1, analytically described by

\[
F (\bar{\sigma}_{ij}, q_n) = 0 \quad (3.1)
\]
where a bar over stress quantities indicates points on \( F = 0 \). The actual stress point \( \sigma_{ij}^* \) lies always within or on the bounding surface. To each \( \sigma_{ij}^* \), a unique "image" point \( \bar{\sigma}_{ij}^* \) on \( F = 0 \) is defined according to a specific rule which is part of the constitutive relations. It is possible to define different rules according to the material under consideration. One such rule which was found to provide the attractive features of simplicity and predictive capability for applications to clays is the simple "radial" rule: assuming that the origin \( O \) lies always within a convex bounding surface, \( \bar{\sigma}_{ij}^* \) is defined as the intersection of \( F = 0 \) and the straight line connecting the origin with \( \sigma_{ij}^* \), Fig. 1. Analytically, this can be expressed by

\[
\bar{\sigma}_{ij}^* = \beta(\sigma_{ij}^*, q_n) \sigma_{ij}^* \quad (3.2)
\]

with \( \beta \) obtained from \( F(\beta, \sigma_{ij}^*, q_n) = 0 \). Only for the origin \( \sigma_{ij}^* = 0 \) and \( \bar{\sigma}_{ij}^* \) is undefined without any further consequence. It is important to emphasize that the following general formulation does not depend on the particular rule chosen to associate the "image" to the actual stress point. The use of eq. (3.2), whenever necessary, will be explicitly stated.

The direction of plastic loading \( L_{ij} \) at \( \sigma_{ij}^* \) is defined as the gradient of \( F \) at the "image" point \( \bar{\sigma}_{ij}^* \), i.e.,

\[
L_{ij} = \frac{\partial F}{\partial \bar{\sigma}_{ij}^*} \quad (3.3)
\]

For any stress rate \( \dot{\sigma}_{ij}^* \) causing plastic loading, a corresponding "image" stress rate \( \dot{\bar{\sigma}}_{ij}^* \) occurs through the hardening of \( F = 0 \) by means of the \( q_n \). Thus, the following three key equations complete the bounding surface formulation:
1. The loading function \( L \), eq. (2.4), is defined in terms of \( \tilde{L} \), eq. (3.3), the two stress rates \( \dot{\sigma}_{ij} \) and \( \ddot{\sigma}_{ij} \), and two plastic moduli: the actual one \( K_p \) associated with \( \dot{\sigma}_{ij} \) and a "bounding" plastic modulus \( \overline{K}_p \) associated with \( \ddot{\sigma}_{ij} \) as follows:

\[
L = \frac{1}{K_p} \frac{\partial F}{\partial \dot{\sigma}_{ij}} \dot{\sigma}_{ij} = \frac{1}{\overline{K}_p} \frac{\partial F}{\partial \ddot{\sigma}_{ij}} \ddot{\sigma}_{ij}
\]  

(3.4)

2. The "bounding" plastic modulus \( \overline{K}_p \) is obtained from the consistency condition \( F = 0 \). Using eqs. (2.5b), (3.1) and the second part of (3.4), one has

\[
\overline{K}_p = -\frac{\partial F}{\partial q_n} q_n
\]  

(3.5)

3. A state dependent relation \( \hat{K}_p \) between \( K_p \) and \( \overline{K}_p \) is established as a function of the distance \( \delta = \left[ (\sigma_{ij}^\ast - \sigma_{ij}) (\sigma_{ij}^\ast - \sigma_{ij}) \right]^{1/2} \) between the current stress state and its "image", i.e.,

\[
K_p = \hat{K}_p (\overline{K}_p, \delta, \sigma_{ij}^\ast, q_n)
\]  

(3.6)

such that \( \hat{K}_p > \overline{K}_p \) for \( \delta > 0 \) (the stress point inside the bounding surface) and \( \hat{K}_p = \overline{K}_p = K_p \) for \( \delta = 0 \) (the stress point on the bounding surface, identical with its "image").

The last eq. (3.6) embodies the meaning of the bounding surface concept. It allows for plastic deformation to occur for points within the surface at a progressive rate which depends on \( \delta \). The closer is the stress point to the surface, the smaller is the \( K_p \) approaching the corresponding \( \overline{K}_p \) and the greater is the plastic strain rate for a given stress rate. The stress \( \sigma_{ij}^\ast \) may eventually reach the bounding surface in the course of plastic loading as it can be seen from eq. (3.4) where the projection of \( \dot{\sigma}_{ij}^\ast \) on the gradient of \( F = 0 \) is greater than the corresponding projection of \( \ddot{\sigma}_{ij}^\ast \) since \( K_p > \overline{K}_p \). The stress point remains on \( F = 0 \) if loading continues, and upon unloading it detaches from \( F = 0 \) moving inwards and so forth.
As a result of the definition of $L_{ij}$ and $L$, it follows that at each point $\sigma_{ij}^*$, a surface homeothetic to the bounding surface with respect to the origin is indirectly defined, shown by the dashed curve in Fig. 1, which determines all the paths of neutral loading emanating from $\sigma_{ij}^*$. This surface defines a quasi-elastic domain, but it is not a yield surface since the stress point may move first elastically inwards and then cause plastic loading before it reaches this surface again. It is not a loading surface either, since no associated consistency condition is required. As a matter of fact it never enters the present formulation explicitly.

Finally, it is worth mentioning that a classical yield surface formulation can be obtained easily in the limit if the functional dependence of $K_p$, eq. (3.6), on $\delta$ is such that $K_p = \hat{K}_p = \infty$ for $\delta > 0$ and $K_p = \hat{K}_p + \overline{K}_p$ as $\delta = 0$. Then the bounding surface becomes a yield surface in the classical sense. More important this leads to the inverse conclusion that any classical yield surface formulation can be transformed easily into a bounding surface formulation by identifying the yield surface as the corresponding bounding surface and using the set of the three key equations (3.4), (3.5), (3.6) together with a proper association of an "image" stress point to any actual stress point within or on the surface.

4 ISOTROPIC SOILS

The formulation so far has been very general. As a matter of fact it can be applied to any material and it is only the concept of the effective stress under undrained conditions which makes it appropriate for soils in particular, i.e., eqs. (2.8), (2.9), (2.10). Any type of material symmetries can be incorporated by properly defining the elastic moduli, eq. (2.3), and rendering $F$ a function of proper invariant quantities of $\bar{\sigma}_{ij}$ and $q_n$, eq. (3.1). Further consideration will be restricted to isotropic soils. Assuming that elastic isotropy is not altered by plastic deformation, the elastic moduli are given by
where the bulk modulus $K$ and the shear modulus $G$ are independent of $q_n$ but can depend on the isotropic stress invariants. Eq. (2.3) can now be written

$$
\sigma_{kk}^\ast = 3 K \epsilon_{ii}^\ast, \quad s_{ij} = 2 G \epsilon_{ij}^\ast
$$

Subsequently, compressive stress and strain are considered positive.

Isotropy requires that $F = 0$ must be a function of the basic and mixed isotropic invariants of $\sigma_{ij}$ and $q_n$. This does not exclude, however, the possibility to describe developed plastic anisotropy with respect to subsequent reference states by a proper choice of $q_n$. For example if the bounding surface undergoes general kinematic hardening, this is obtained by choosing some of the $q_n$ to be the coordinates of its center thus providing a built-in feature which generates anisotropy in the course of plastic deformation. [5,10,19] Here a simpler model will be suggested.

It will be first assumed that $F = 0$ depends only on the basic isotropic invariants of $\sigma_{ij}$ and $q_n$ (not the mixed). Furthermore, the stress dependence will be restricted to the first effective stress invariant $I^\ast$ and the square root of the second deviatoric stress invariant $J^{\ast \ast}$, defined as

$$
I^\ast = \text{tr} \sigma_{ij}^\ast = \sigma_{ii}^\ast, \quad \sqrt{J} = \left[ \frac{1}{2} s_{ij} s_{ij} \right]^{\frac{1}{2}}
$$

A more general model with an asymmetric bounding surface around the hydrostatic axis $J^{\ast \ast} = 0$ would require the introduction of the third stress invariant.

It will also be assumed that the bounding surface undergoes isotropic and kinematic hardening along the hydrostatic axis, described by one single scalar $q_n$ which measures the plastic volumetric strain $\epsilon_{kk}^\ast$. If $e$ is the total void ratio, it follows
\[ \dot{e} = -(1 + e_0) \dot{e}_{kk} = -(1 + e_0) \dot{e}'_{kk} - (1 + e_0) \dot{e}''_{kk} = \dot{e}' + \dot{e}'' \] (4.4)

with \( e_0 \) the initial void ratio corresponding to the reference configuration with respect to which strains are measured (for natural strains: \( e_0 = e \)). One can easily now identify the quantities \( \dot{e}' = -(1 + e_0) \dot{e}'_{kk} \) and \( \dot{e}'' = -(1 + e_0) \dot{e}''_{kk} \) as the elastic and plastic void ratio rates respectively, and choose \( \dot{e}'' \) as the only \( q_n \).

Thus, eq. (3.1) becomes

\[ F (\bar{I}^*, \sqrt{J}, e^{**}) = 0 \] (4.5)

where a bar indicates again stress invariants on \( F = 0 \).

Using eqs. (3.3), (3.4), (4.3) and (4.5) and assuming the associated flow rule \( L_{ij} = R_{ij} \), a straightforward computation yields

\[ L_{ij} = R_{ij} = \frac{\partial F}{\partial \bar{T}^*} \delta_{ij} + \frac{1}{2\sqrt{J}} \frac{\partial F}{\partial \sqrt{J}} \tilde{s}_{ij} \] (4.6)

\[ L = \frac{1}{K_p} \left[ \frac{\partial F}{\partial \bar{T}^*} \bar{T}^* + \frac{\partial F}{\partial \sqrt{J}} \sqrt{J} \right] = \frac{1}{K_p} \left[ \frac{\partial F}{\partial \bar{T}^*} \bar{T}^* + \frac{\partial F}{\partial \sqrt{J}} \sqrt{J} \right] \] (4.7)

where the relation \((\tilde{s}_{ij} \tilde{s}_{ij}/2\sqrt{J}) = (s_{ij} s_{ij}/2\sqrt{J}) = \sqrt{J}\) pertaining to the "radial" rule of associating \( \tilde{\sigma}_{ij} \) with \( \sigma_{ij} \) is used in order to obtain the form of the first bracket in eq. (4.7). The form of \( L \) can easily be interpreted as indicating loading whenever the inner product of the rate of the stress invariants \( \bar{T}^*, \sqrt{J} \) with the gradient of \( F \) in invariant space divided by \( K_p \) is positive. The plastic strain rate is given from eqs. (2.5a), (4.6) and (4.7). Non-associated flow rules can also be used by defining \( R_{ij} \) otherwise.

Writing now, according to eqs. (2.5b) and (4.4), \( \dot{e}'' = r <L> = -(1 + e_0) R_{kk} <L> \), \( r \) can be easily identified and with \( R_{kk} \) expressed from eq. (4.6) the expression (3.5) yields
Observe form (4.8) that with \( \frac{\partial F}{\partial e_T} > 0 \), the bounding plastic modulus \( \bar{K}_p \) is positive (consolidation), negative (dilatation) or zero (unrestricted shear flow) according to the value of \( \frac{\partial F}{\partial e_T} \). Correspondingly, the bounding surface expands, contracts or does not harden. This is a particularly interesting property which allows the easy incorporation of the present formulation into a critical state framework. It is a drawback of the above formulation that developed anisotropy cannot be accounted for. On the other hand, the simplicity involved and the fact that many soils can be adequately described by monitoring only \( e_T \), plus all the advantages obtained by using a bounding surface, renders the present formulation a useful constitutive model. Developed anisotropy can still be incorporated in terms of a varying non-associated flow rule but this will be presented elsewhere.

The changes of \( \bar{K}_p \) on \( F = 0 \) reflect into the values of the actual plastic modulus \( K_p \) by means of eq. (4.6). Here the following form of this equation will be assumed

\[
K_p = \bar{K}_p + H (1, \sqrt{J}, e_T) \frac{\delta}{\delta_0 - \delta} \quad (4.9)
\]

where \( H \) is a positive "shape" hardening function of the state, \( \delta \) is the distance between actual and "image" stress points in either the stress or stress invariants space, and \( \delta_0 \) is a properly chosen reference stress or distance in the corresponding space such that \( \delta_0 - \delta > 0 \) and \( \delta_0 / \delta \) remains invariant from the space used to calibrate \( H \). The exact definition of \( H \) will require the identification and experimental determination of certain material parameters. The \( H \) and the associated parameters constitute the "new" elements of the present formulation with regard to classical yield surface formulations, and are intimately related to the soil response for states within \( F = 0 \) (overconsolidation). For \( H \to \infty \) observe that \( K_p \to \infty \), and \( K_p = \bar{K}_p \) only for \( \delta = 0 \).
In this case, the bounding surface behaves as a yield surface. It is possible to have $K_p < 0$ and $K_p > 0$ if $\delta$ is large enough which allows the description of an initially rising stress-strain curve as the stress point approaches the contracting bounding surface ($K_p < 0$); and the subsequent unstable falling curve behavior when eventually $\delta$ becomes small enough to have both $K_p$, $K_p < 0$ as in the case of heavily overconsolidated clays.

Using now eqs. (4.1) and (4.6), the inverse relations (2.6), (2.7) become

$$L = \frac{3K \frac{\partial F}{\partial I}: \varepsilon_{kk} + \frac{G}{\sqrt{3}} \frac{\partial F}{\partial J} \varepsilon_{pq} \varepsilon_{pq}}{K_p + 9K \left(\frac{\partial F}{\partial I^+}\right)^2 + G \left(\frac{\partial F}{\partial J}\right)^2}$$

$$\varepsilon_{ij}^* = 2G \varepsilon_{ij}^* + (K - \frac{2G}{3}) \delta_{ij}^* - \langle L \rangle \left[3K \frac{\partial F}{\partial I^+}: \delta_{ij}^* + \frac{G}{\sqrt{3}} \frac{\partial F}{\partial J} \varepsilon_{ij}^* \right]$$

Similarly, the undrained relations (2.9) and (2.10) become

$$L = \sqrt{\frac{3K \frac{\partial F}{\partial I^+}}{J}} = \frac{G \frac{\partial F}{\partial I^+}}{\sqrt{3}} \frac{\partial F}{\partial J} \varepsilon_{rs} \varepsilon_{rs}$$

$$\varepsilon_{ij}^* = 3K \frac{\partial F}{\partial I^+} \langle L \rangle + \frac{1}{3} \varepsilon_{ij}^*$$

where again $\varepsilon^*$ is indeterminate. With $L$ defined from (4.12) and $R^*$ from (4.6), $\varepsilon_{ij}^*$ is obtained from (2.5a). Also, with $\varepsilon_{kk} = 0$ and $L$ from (4.12), eq. (4.11) yields $\varepsilon_{ij}^*$ in the inverse-undrained formulation. Observe that the loading condition $L \geq$ depends on $\sqrt{3}$ only, since $I^*$ is not controlled independently. Finally, eq. (2.8) yields for the undrained invariant stress path the differential equation
\[
\frac{\sqrt{J}}{J} = - \left[ \frac{\partial F/\partial T^*}{\partial F/\partial \sqrt{J}} + \frac{K_p}{\sqrt{K} (\partial F/\partial T^*)(\partial F/\partial \sqrt{J})} \right]
\] (4.14)

For any given increment \( \dot{\sigma}_{ij} \, dt \) or \( \dot{e}_{ij} \, dt \), the soil response is fully determined from the above incremental relations. Observe that the material state expressed in terms of the seven quantities \( \sigma_{ij} \), \( e^{-} \) suffices to define completely these relations beginning with the determination of the "image" stress point \( \bar{\sigma}_{ij} \), eq. (3.2) or any other rule, and subsequently applying in a straightforward way the corresponding expressions.

5 RELATIONS BETWEEN QUANTITIES IN INVARIANT AND TRIAXIAL SPACES

The soil response under triaxial loading conditions can be obtained as a special case of the general development. Therefore, there is no need to recast the incremental relations in terms of the triaxial variables. However, since the material parameters associated with eq. (4.9) will be determined from triaxial experiments within a "unit normal" formulation to be defined subsequently, there is a need to relate the plastic moduli and the distances \( \delta \), \( \delta_0 \) between invariant and triaxial spaces.

The usual stress and strain triaxial measures \(^{25}\) \( p', q \) and \( \varepsilon_p, \varepsilon_q \) are given in terms of the principal stresses and strains by

\[
p' = \frac{1}{3} (\sigma_1' + 2\sigma_3') \quad , \quad q = \sigma_1' - \sigma_3' = \sigma_1^* - \sigma_3^* \quad (5.1a)
\]

\[
\varepsilon_p = \varepsilon_1 + 2\varepsilon_3' \quad , \quad \varepsilon_q = \frac{2}{3} (\varepsilon_1 - \varepsilon_3) \quad (5.1b)
\]

The sets of the triaxial and invariant stress measures are related by

\[
1' = 3p' \quad , \quad \sqrt{J} = (1/\sqrt{3}) |q|
\] (5.2)
and corresponding relations hold between the partial derivatives with respect to the two sets. The bounding surface in triaxial space is described by \( F^* = 0 \) where

\[
F^* (T^*, \sqrt{J}, e^{*}) = F(3\bar{p}^*, (1/\sqrt{3}) |\bar{q}|, e^{*}) = 0
\]  
(5.3)

The components \( n_p, n_q \) of the unit normal to \( F^* = 0 \) are defined by

\[
n_p = \frac{1}{g} \frac{\partial F^*}{\partial p^*}, \quad n_q = \frac{1}{g} \frac{\partial F^*}{\partial q^*}, \quad n_p^2 + n_q^2 = 1
\]  
(5.4)

where

\[
g^* = \left( \frac{\partial F^*}{\partial p^*} \right)^2 + \left( \frac{\partial F^*}{\partial q^*} \right)^2
\]  
\[
= g \left( \frac{\partial F}{\partial T} \right)^2 + \frac{1}{3} \left( \frac{\partial F}{\partial \sqrt{J}} \right)^2
\]  
(5.5)

Using now eqs. (4.6), (4.7), (5.2), the basic constitutive law (2.4), (2.5a) in the "unit normal" triaxial space formulation is given by

\[
L^* = g^* L = \frac{1}{K_p^*} (n_p^* \dot{p}^* + n_q^* \dot{q}^*) = \frac{1}{K_p^*} (n_p^* \dot{p} + n_q^* \dot{q})
\]  
(5.6a)

\[
\dot{\varepsilon}_p = <L^*> n_p^*, \quad \dot{\varepsilon}_q = <L^*> n_q^*
\]  
(5.6b)

where the triaxial moduli \( K_p^*, K_p^* \) are related to \( K_p^*, K_p^* \) by

\[
K_p = g^* K_p^*, \quad K_p^* = g^* K_p^*
\]  
(5.7)

The triaxial moduli \( K_p^*, K_p^* \) have the proper dimension of stress as it can be seen from eqs. (5.6), and this is a definite advantage when a relation similar to eq. (4.9) is established between them from triaxial experiments.

Attention will be focused on the definition of \( \delta_0 \), eq. (4.9). For further use, we introduce the quantities
\[ \theta = \sqrt{\frac{1}{2}}, \quad n = \frac{1}{p}, \quad n^2 = 27 \theta^2 \quad (5.8) \]

Using (5.2), (5.2) and (5.8), the distance between actual and "image" stress points in the invariant and triaxial spaces, denoted by \( \delta \) and \( \delta^* \) respectively, are expressed by

\[ \delta = (\beta - 1)(1 + \theta^2)^{1/2} 1^- \quad (5.9a) \]
\[ \delta^* = (\beta - 1)(1 + n^2)^{1/2} p^- = (1/3) (\beta - 1)(1 + 27\theta^2)^{1/2} 1^- \quad (5.9b) \]

Obtaining the relation (4.9) between \( K_p^* \) and \( K_p^* \) requires the definition of a reference distance \( \delta_0^* (p^*, q, e^{-}) \) in triaxial space. Generalizing this relation to the invariant space by means of eq. (5.7), a corresponding reference distance \( \delta_0 \) must be defined in such a way that the ratio \( \delta/\delta_0^* \) remains invariant under the transformation (5.2). In other words, one must have \( \delta_0/\delta = \delta_0^*/\delta^* \) which by means of eqs. (5.2) and (5.9) yields:

\[ \delta_0 = 3 \left( \frac{1 + \theta^2}{1 + 27\theta^2} \right)^{1/2} \delta_0^* (1^*/3, \sqrt{3}J, e^{-}) \quad (5.10) \]

The conclusion of this section can be summarized as follows: establishing for the triaxial space a relation of the form

\[ K_p^* = K_p^* + H^* (p^*, q, e^{-}) \frac{\delta^*}{\delta_0^* - \delta^*} \quad (5.11) \]

the corresponding relation for the invariant space is given by eq. (4.9), where \( \delta_0 \) is defined by eq. (5.10), \( \delta \) is defined by eq. (5.9a) and according to eq. (5.7) \( H \) is given by
The important quantity therefore which is to be determined from experimental data is the material function \( H^* \) (or \( H \)). It must be emphasized on the basis of the above conclusion that in order to calibrate the different material constants of \( H^* \) from triaxial experiments, the general formulation of section 5 can be used in combination with eqs. (4.9) and (5.12) without recasting the constitutive relations in terms of the triaxial variables.

6 SPECIFIC FORM OF THE BOUNDING SURFACE

Subsequently the development will focus on clay soil behavior. Postponing the determination of \( H \) and \( \delta_0 \) for the next section, specific analytical expressions for the bounding surface will be given here together with the corresponding expressions for the \( \delta F/\delta \overline{T}' \), \( \delta F/\delta \sqrt{3} \) and the \( K_p \) from eq. (4.8). Extending the ideas of the critical state soil mechanics from the triaxial to the invariant space, the bounding surface is shown eloquently in Fig. 2 intersecting the projection of the critical state line, which has a constant slope \( N \), at point C where \( \delta F/\delta \overline{T}' = 0 \) and the \( I' \) axis at points where \( \delta F/\delta \sqrt{3} = 0 \). The intersection with the positive \( I' \) axis is denoted by \( I_0' \). It follows immediately from (5.2) that \( N=(1/3 \sqrt{3})M \) with M the slope of the critical state line in triaxial space.

The dependence of \( F = 0 \) on \( \varepsilon'' \) is introduced by means of the dependence of \( I_0 \) on \( \varepsilon'' \), which accounts simultaneously for isotropic and kinematic hardening along the \( I' \) axis. The center of the bounding surface, which is the projection \( I_1 \) of point C on \( I' \), is related to \( I_0 \) by \( I_1 = I_0/R \) with R being a material constant. The value of R has been taken in the past equal to 2.72 (natural logarithm)\(^{25}\) or 2\(^{23}\), but any other value in the range \( 1 < R < \infty \) may be suitable for particular soils. Let \( \kappa \) and \( \lambda \) denote the slopes of the rebound and isotropic consolidation curves in
the e-ln\( I \) plot (same as in the e-lnp' plot). Assuming that there is a limit value \( I' \geq I_L \) (usually taken equal to one atmosphere) such that for \( I' < I_L \) the relation between \( I' \) and the elastic part of the void ratio \( e' \) changes continuously from logarithmic to linear in order to prevent excessive softening of the elastic stiffness around \( I' = 0 \) for cohesive soils, one has

\[
\frac{I'}{e'} = -\frac{<I' - I_L^1 I_L^2> + I_L^1 I_L^2}{\kappa}, \quad \frac{I_o}{e} = -\frac{I_o}{\lambda} \tag{6.1}
\]

The sought expression \( I_o/e' \) follows immediately from \( e' = e - \dot{e} \) and eq. (6.1):

\[
\frac{I_o}{e'} = -\frac{I_o}{\lambda - \omega \kappa} \quad \text{with} \quad \omega = \frac{I_o}{<I_o - I_L^1 I_L^2> + I_L^1 I_L^2} \tag{6.2}
\]

The first part of eq. (6.1) in combination with eqs. (4.2), (4.4) yields an expression of the bulk modulus \( K \) in terms of \( \kappa \) as

\[
K = \frac{(1 + e_o) (<I' - I_L^1 I_L^2> + I_L^1 I_L^2)}{3\kappa} \tag{6.3}
\]

Using the radial rule, eq. (3.2), it will be found convenient to substitute \( \beta \) by a function \( \gamma(\theta) \) such that

\[
\dot{T} = \beta \dot{\gamma} = \gamma(\theta) I_o \tag{6.4}
\]

and of course \( \dot{T}^2 = \dot{\gamma} \dot{T} = \dot{\gamma}(\theta) I_o \). It is instructive at this point to express the distance \( \delta \), eq. (5.9a), in terms of \( \gamma \) instead of \( \beta \) by

\[
\delta = (1 + \theta^2)^{\frac{1}{2}} [\gamma(\theta) I_o - I' \tag{6.5}
\]

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With these preliminaries the expression for \( F = 0 \) and the related quantities are defined as follows:

(i) For \( 0 \leq \theta \leq N \) (Ellipse 1)

The bounding surface is the ellipse 1, Fig. 2, defined by

\[
\frac{F}{I_o^2} = \left( \frac{T^*}{I_o} \right)^2 + \left( \frac{R-1}{N} \right)^2 \left( \frac{\sqrt{3}}{I_o} \right)^2 - \frac{2}{R} \frac{T^*}{I_o} + \frac{2-R}{R} = 0
\]  \hspace{1cm} (6.6)

and with \( x = \theta/N \) it follows

\[
\gamma(\theta) = \frac{1+(R-1)}{R} \frac{[1+R(R-2)x^2]^{1/2}}{[1+x^2+R(R-2)x^2]}
\]  \hspace{1cm} (6.7)

\[
\frac{\partial F}{\partial T^*} = 2I_o \left( \gamma - \frac{1}{R} \right)
\]  \hspace{1cm} (6.8a)

\[
\frac{\partial F}{\partial \gamma} = 2I_o \theta \gamma \left( \frac{R-1}{N} \right)^2
\]  \hspace{1cm} (6.8b)

\[
\bar{K}_p = \frac{1+e_0}{\lambda-\omega R} \frac{12I_o^3}{R} \left( \gamma - \frac{1}{R} \right) (\gamma + R - 2)
\]  \hspace{1cm} (6.9)

where for the last equation use of eqs. (4.8), (6.2), (6.4) and (6.8a) was made. It can easily be shown that as \( \theta \) varies from 0 to \( N \), \( \bar{K}_p \) remains non-negative varying continuously from \( \bar{K}_p = (1+e_0)12I_o^3(R-1)^2/(\lambda-\omega R)^2 \) to \( \bar{K}_p = 0 \).

(ii) For \( N \leq \theta < +\infty \) (Hyperbola).

It is possible to extend ellipse 1 to this range as shown by the dashed curve CE in Fig. 2 using all the previous equations with the restriction \( R \geq 2 \) since the origin must lie within or on the bounding surface which crosses the origin for \( R = 2 \).
It was found however, that the prediction of the model was unsatisfactory for deviatoric loading of heavily overconsolidated clays suggesting a curve with a shape more parallel to the critical line OC, which is tangent to the ellipse at C. This would yield the advantage of having the possibility to choose $R < 2$ if necessary, at the expense of one additional parameter for the new curve. A hyperbola is therefore proposed whose apex C is at a distance D from its center G and its asymptote is parallel to OC. Assuming $D = A/l_0$, the equation of the hyperbola is defined in terms of one additional material parameter $A$ by

$$\frac{F}{l_0^2} = \left(\frac{T-\ell_0}{l_0}\right)^2 - \frac{1}{N^2} \left(\frac{\sqrt{3}}{l_0}\right)^2 - \frac{2}{R} \frac{T}{l_o} + \frac{2}{N} \left[\frac{1}{R^2} + \frac{A}{N}\right] \sqrt{\frac{3}{l_0}} - \frac{2A}{RN} = 0 \quad (6.10)$$

and with $y = RA/N$ one has (recall $x = \theta/N$):

$$\gamma(\theta) = \frac{x-1+xy-[(x-y-1)^2 + (x^2-1)y^2]^{1/2}}{R(x^2-1)} \quad (6.11)$$

$$\frac{\partial F}{\partial \ell} = 2 \frac{l_0}{R} \left(\gamma - \frac{1}{R}\right) \quad (6.12a)$$

$$\frac{\partial F}{\partial \sqrt{J}} = \frac{2}{N} \left[\frac{A + (N/R) - \theta}{N} \gamma\right] \quad (6.12b)$$

$$\bar{K}_{p} = \frac{1 + e_o}{\lambda - \omega \xi} \frac{12 \frac{l_0^3}{R}}{R} \frac{(y - \frac{1}{R})}{R} \left[\frac{1-x-xy}{\gamma} + \frac{2A}{N}\right] \quad (6.13)$$

It can be shown that as $x$ varies from 1 to $+\infty$ ($\theta$ varies from $N$ to $+\infty$), $\bar{K}_{p}$ is negative decreasing from 0 at point C to $\bar{K}_{p} = -(1 + e_o) \frac{12 \frac{l_0^3}{R}}{R} \frac{(y - \frac{1}{R})}{R} \left[\frac{(2A/N) - (1 + y) \left[1 + y - (1 + y^2)^{1/2}\right]}{(1/R)}/(\lambda - \omega \xi)\right]^{1/2}$ at point B.
(iii) For $-\infty < \theta \leq 0$ (Ellipse 2)

In order to describe the material behavior in tension\[1\], it is possible to extend the bounding surface into the $I'' < 0$ range as a smooth curve tangent to the hyperbola at point B, Fig. 2, and intersecting the $I''$ axis at point $I_t$ with a tangent parallel to $\sqrt{J}$ axis. The $I_t$ measures the tensile strength of the soil. Such an extension as a second ellipse, Fig. 2, was proposed by Dafalias and Herrmann\[13\] but other shapes may also be suitable. In view of insufficient evidence for the tensile behavior, further discussion is postponed with the observation that the material response will be described by equations similar to the ones developed in the previous cases, with attention given in securing the continuity of $R_p$ at point B.

### 7 SHAPE HARDENING FUNCTION

It is only left to specify the shape hardening function $H$ and the reference distance $\delta_0$ of eq. (4.9), in terms of the state and certain material constants. Recalling eq. (5.11), the following form of $H^*$ is proposed for the triaxial space

$$H^* = h p_a \left( 1 + \frac{|M|}{|\eta|}^m \right) \quad (7.1)$$

where $p_a$ is the atmospheric pressure providing the proper stress units and $h, m$ are dimensionless material constants. The introduction of the absolute value of the ratio $M/\eta$ does not allow plastic deformation to occur within $F = 0$ for $\eta = 0$ (zero deviatoric stress) rendering $H^*$, and by consequence $K_p^*$, infinite except when $\delta^* = 0$ where of course one must define $\lim \left[ \frac{|M/\eta|^m}{\eta^m} \delta^* \right] = 0$ as $\eta, \delta^* \to 0$. This can be easily achieved numerically by changing $\eta$ to $\eta + \varepsilon$ with $\varepsilon$ a very small positive number. A small value of $m$ eliminates the influence of $|M/\eta|$ for $\eta > 0$, and it is the shape hardening constant $h$ which bears mainly the responsibility for the material response within the bounding surface.
If \( p_0 \) is the point of intersection of \( F^* = 0 \) with the \( p^* \) axis measuring the amount of past preconsolidation, the choice \( \delta_o^* = p_0 = l_o/3 \) is plausible. One can easily establish the condition \( M \leq [R(R-1)]^{1/2} \) in order to guarantee that \( p_0 - \delta^* \geq 0 \). Other choices of \( \delta_o^* \) are possible but will not be presented here. Recalling the discussion of section 5 and using eqs. (4.9), (5.2), (5.5), (5.8), (5.12) and (7.1), the following relation is established between \( K_p \) and \( \overline{K}_p \) in invariant stress space:

\[
K_p = \overline{K}_p + \nu P_a \left[ 1 + \frac{N^m}{\delta} \right] \left[ 9 \left( \frac{\partial F}{\partial T^*} \right)^2 + \frac{1}{3} \left( \frac{\partial F}{\partial J^*} \right)^2 \right] \frac{\delta - \delta_o}{\delta_o - \delta} \tag{7.2}
\]

with \( \delta \) given by eq. (6.5) and \( \delta_o \) by eq. (5.10) in which \( \delta_o^* = p_0 = l_o/3 \). Observe that all the quantities entering the above relation are function of the state \( \mathbf{I}^*, \mathbf{J}^{1/2}, e^{**} \) and two material constants \( m \) and \( h \).
Figure I-1. Schematic illustration of the bounding surface in a general stress space
Figure 1-2. The bounding surface in the space of stress invariants
Appendix II: Input Instructions for EVAL

I. Heading Information

Card 1 (12A6)

Col. 1-72: TITLE—any information that is to be printed as a title for the analysis

II. Material Parameters (detailed definitions are given in previous sections)

Card 2 (8E10.3)

Col. 1-10 $\lambda$ Slope of isotropic consolidation line in $e-\ln p'$ plot
11-20 $\kappa$ Slope of elastic rebound line in $e-\ln p'$ plot
21-30 $M$ Slope of critical state line in triaxial space
31-40 $R$ parameters describing shape of bounding surface (see Figure I.2 of Appendix I, $T = 1/T_o$)
41-50 $A$
51-60 $T$
61-70 $I_2$ Transitional value (between compressive and tensile zones of behavior) of first stress invariant—suggested range of value = $0.5P_a + 3P_a$
71-80 $I_o$ Initial value of effective preconsolidation stress invariant (3 times preconsolidation pressure)

Card 3 (6E10.3)

Col. 1-10 $m$
11-20 $h$
21-30 $e_o$ Initial void ratio
31-40 $v$ Poisson's ratio
41-50 $\Gamma$ Bulk modulus for soil skeleton and pore water ($\Gamma=0$ for drained conditions); if no information is available, it is suggested that a value of 20,000 $P_a$ be used.
51-60 $P_a$ Atmospheric pressure
III. Iteration information

Card 4 (I5, E10.2)

Col. 1-5  ITMAX: Max. no. of iterations per increment (typically a value of 5-10 should be used)

Col. 6-15 ERMAX: Max permitted relative difference \( \frac{N_K - N_{K-1}}{N_K} \) for the norms of the incremental stress and strain vectors (e.g. for \( \Delta \sigma \), \( N = \sum_{i=1}^{6} |\Delta \sigma_i| \)). Typically, values of .05 to .01 should be used. If convergence does not occur in ITMAX iterations, the program prints a message and then continues to the next problem. Typically values of .05 to .01 should be used.

IV. Initial conditions

Card 5 (2E10.3)

Col. 1-10  \( \sigma_{c_0} \): Initial hydrostatic confining pressure*

11-20  \( P_a \): Atmospheric pressure

V. Description of M history segments

For each of the M history segments, one card (6(I1,E9.2), I5, E10.2) is required:

Col 1: \( IC_1 = \left\{ \begin{array}{l} 0 - \sigma_{x_M} \\ 1 - \varepsilon_{x_M} \end{array} \right\} \) is specified

2-10: \( V_1 = \text{value of} \left\{ \begin{array}{l} \sigma_{x_M} \\ \varepsilon_{x_M} \end{array} \right\} \) for \( IC_1 = \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \)

---

*The strains produced by the application of \( \sigma_{c_0} \) are not calculated. It is assumed that \( \sigma_{c_0} \) is applied under drained conditions (regardless of value of \( \Gamma \)). If it is desired to calculate the strains due to \( \sigma_{c_0} \) and/or to apply \( \sigma_{c_0} \) under undrained conditions, then \( \sigma_{c_0} \) is set equal to zero and the confining pressure is applied in history segment 1.
11: \[ IC_2 = \begin{cases} 0 - \sigma_{y_M} \\ 1 - \varepsilon_{y_M} \end{cases} \] is specified

12-20: \[ V_2 = \text{value of} \begin{cases} \sigma_{y_M} \\ \varepsilon_{y_M} \end{cases} \text{ for } IC_2 \begin{cases} 0 \\ 1 \end{cases} \]

21: \[ IC_3 = \begin{cases} 0 - \sigma_{z_M} \\ 1 - \varepsilon_{z_M} \end{cases} \] is specified

22-30: \[ V_3 = \text{value of} \begin{cases} \sigma_{z_M} \\ \varepsilon_{z_M} \end{cases} \text{ for } IC_3 \begin{cases} 0 \\ 1 \end{cases} \]

31: \[ IC_4 = \begin{cases} 0 - \tau_{xy_M} \\ 1 - \gamma_{xy_M} \end{cases} \] is specified

32-40: \[ V_4 = \text{value of} \begin{cases} \tau_{xy_M} \\ \gamma_{xy_M} \end{cases} \text{ for } IC_4 \begin{cases} 0 \\ 1 \end{cases} \]

41: \[ IC_5 = \begin{cases} 0 - \tau_{xz_M} \\ 1 - \gamma_{xz_M} \end{cases} \] is specified

42-50: \[ V_5 = \text{value of} \begin{cases} \tau_{xz_M} \\ \gamma_{xz_M} \end{cases} \text{ for } IC_5 \begin{cases} 0 \\ 1 \end{cases} \]

51: \[ IC_6 = \begin{cases} 0 - \tau_{yz_M} \\ 1 - \gamma_{yz_M} \end{cases} \] is specified

52-60: \[ V_6 = \text{value of} \begin{cases} \tau_{yz_M} \\ \gamma_{yz_M} \end{cases} \text{ for } IC_6 \begin{cases} 0 \\ 1 \end{cases} \]

Note: \( \sigma_{x_M} \) is the value of \( \sigma_x \) at the end of segment \( M \), etc. Thus, when \( IC_{1M} = 0 \), the value of \( \sigma_{x_M} - \sigma_{x_{M-1}} \) is applied during the loading segment. \( \sigma_{x_{M-1}} \) is the value of \( \sigma_x \) calculated (\( IC_{1M-1} = 1 \)) or specified (\( IC_{1M-1} = 0 \)) at the end of segment \( M-1 \).
61-65: NINC - number of increments into which segment M is to be divided.

66-75: D - Increment ratio for specified quantities (e.g., if $IC_1 = 0$ then $\Delta \sigma_x / \Delta \sigma_{x-1} = D$; $\Delta \sigma_x$ is the increment of $\sigma_x$ applied during increment $N$ of loading segment $M$, etc.). A value of 1.0 gives equally spaced increments for the segment.

VII. End Card (II)

Col 1 - punch 9

The above sequence I + VII of cards is repeated for analysis 2 etc.
APPENDIX III

SUBROUTINE CLAY(IDIM,INC,ITNO,PROP,STOR,SIGB,
1   DSIG,DEPM,C,GAM)

SUBROUTINE TO EVALUATE YANNIS DAFALIAS' BOUNCING
SURFACE PLASTICITY MODEL FOR CLAY -- UNIVERSITY OF
CALIFORNIA, DAVIS -- SUBROUTINE PREPARED BY L.R. HERRMANN.

DIMENSION PROP(14),STOR(6),SIGB(6),DSIG(6),DEP(6),C(6,6),
1   SB(3,3),SF(3,3),DEPM(3),DEPM(6),SIGFE(6)
DATA II/11,22,33,12,13,23/
DATA OLTA/1.0,3.0,1.0,3.0,1.0,3.0/

CHANGE MATRIX COMPONENTS OF STRAIN TO TENSOR COMPONENTS.

DO 40 I=1,6
40 DEP(I)=DEPM(I)
GAM=PROP(13)
IF(ITNO.GT.1) GO TO 100
IF(INC.GT.1) GO TO 50

STOR(1)=PROP(8)
STOR(3)=0.5*(SIGB(1)+SIGB(2))
STOR(4)=0.01*PROP(14)
STOR(5)=0.0
GO TO 100

UPDATE HISTORY

STOR(1)=STOR(2)
STOR(3)=STOR(3)+STOR(4)
STOR(5)=STOR(5)+STOR(6)

CONVERT FROM PLANE STRAIN TO 3-D

IF(IDIM.EQ.3) GO TO 200
SIGB(4)=SIGB(3)
SIGB(3)=STOR(3)
DSIG(4) = DSIG(3)
DSIG(3) = STOR(4)
DEP(4) = DEP(3)
DEP(3) = 0.0
DO 110 I = 5, 6
SIGB(I) = 0.0
DSIG(I) = 0.0
110 DEP(I) = 0.0

DETERMINE 3-D INCREMENTAL PROPERTIES

200 DO 204 I = 4, 6
204 DEP(I) = DEP(I) * 0.5
YS = PROP(4) * PROP(5) * 3.0 * SQRT(3.0) / PROP(3)
XIB = 0.0
XIF = 0.0
DDIL = 0.0

CALCULATE EFFECTIVE STRESS INVARIANTS AND DISTORTIONAL STRESS

DO 205 I = 1, 3
DDIL = DDIL + DEP(I)
XIB = XIB + SIGB(I)
205 XIF = XIF + SIGB(I) + DSIG(I)
DO 215 N = 1, 6
I = II(N) / 10
J = MOD(II(N), 10)
SB(I, J) = SIGB(N) * XIB * DLTA(I, J) / 3.0
SB(J, I) = SB(I, J)
SF(I, J) = SIGB(N) + DSIG(N) * DLTA(I, J) * XIF / 3.0
215 SF(J, I) = SF(I, J)
UB = STOR(5)
XIB = XIB + UB * 3.0
XIF = XIF - (UB + GAM * DDIL) * 3.0
STOR(6) = GAM * DDIL
SRTJB = 0.0
SRTJF = 0.0
DO 220 I = 1, 3
DO 220 J = 1, 3
SRTJB = SRTJB + SB(I, J) * SB(I, J)
220 SRTJF = SRTJF + SF(I, J) * SF(I, J)
SRTJB = SQRT(0.5 * SRTJB)
SRTJF = SQRT(0.5 * SRTJF)

AVOID ZERO MEAN PRESSURE

DU1 = 0.0001 * PROP(14)
IF(ABS(XIB), GT, DU1) GO TO 227
DU = XIB
XIB = DU1
IF(DU .LT. 0.0) XIB = -DU1
227 IF(ABS(XIF), GT, DU1) GO TO 230
DU = XIF
XIF = DU1
IF(OU.LT.0.0) XIF=-DU1

230 CONTINUE

C CALCULATE ELASTIC PROPERTIES

CBULK=2.0*GAM
DU1=(1.0+PROP(11))/3.0/PROP(2)
DU2=1.5*(1.0-2.0*PROP(12))/(1.0+PROP(12))
DU=XIB
IF(DU.LT.PROP(7)) DU=PROP(7)
BB=DU1*DU
GB=DU2*BB
DU=XIF
IF(DU.LT.PROP(7))DU=PROP(7)
BF=DU1*DU
GF=DU2*BB
DO 235 M=1,6
I=II(M)/10
J=MOD(II(M),10)
DO 235 N=M,6
K=II(N)/10
L=MOD(II(N),10)
DU1=DLTA(K,I)*DLTA(L,J)+DLTA(K,J)*DLTA(I,L)
C(M,N)=((GB+GF)*DU1*0.5+0.5*(BB+BF+CBULK*2.0*(GB+GF)/3.0)*
1
DU1)*DLTA(I,J)*DLTA(K,L)
235 C(N,M)=C(M,N)

C CALCULATE SIZE OF BOUNDING SURFACE

OMEG=1.0
IF(XIOB.LT.PROP(7)) OMEG=XIOB/PREP(7)
DU=(1.0+PROP(11))/(PROP(1)-PROP(2)*OMEG)
XIOB=STOR(1)
XIOF=XIOB*EXP(DU*(DDIL-1.0/BB+1.0/BF)*(XIF-XIB)/6.0))
STOR(2)=XIOF

C CALCULATE BOUNDING SURFACE PROPERTIES

THB=SRTJF/XIB
THF=SRTJF/XIF
CALL BOUND(PROP,YS,THB,XIOB,XIB,GAMB,DFIB,DFJB,XKSB)
CALL BOUND(PROP,YS,THF,XIOF,XIF,GAMF,DFIF,DFJF,XKSF)
BSB=GAMB*XIOB/XIB
IF(BSB.LT.1.0) BSB=1.0
BSF=GAMF*XIOF/XIF
IF(BSF.LT.1.0) BSF=1.0
DU1=(GAMB*XIOB*XIB)*SQR1(1.0+27.0*THB*THB)
DB=DU1/(XIOB=DU1)
IF(DB.LT.0.0) DB=0.0
DU1=(GAMF*XIOF*XIF)*SQR1(1.0+27.0*THF*THF)
DF=DU1/(XIOF=DU1)
IF(DF.LT.0.0) DF=0.0

C CALCULATE PLASTIC MODULUS
c
XN=PROP(3)/3.0/SQRT(3.0)
XMS=PROP(9)
H=PROP(10)*PROP(14)
DU1=0.00001
DU=ABS(TH8)
IF(DU_.LT_.DU1) DU=DU1
XKB=XKB+H*DB *(1.0+(XN/DU)**XMS)*(9.0*DFIB**2+DFJB**2/3.0)
DU=ABS(THF)
IF(DU_.LT_.DU1) DU=DU1
XKF=XKF+H*OF *(1.0+(XN/DU)**XMS)*(9.0*DFIF**2+DFJF**2/3.0)

C CHECK FOR LOADING AND UNLOADING

WHEN SOME DISTANCE FROM BOUNDING SURFACE COMPARE
ESTIMATED STRESS STATES AT BEGINING AND END OF
INCREMENT

SL=1.0
IF(BSF .GT. BSB) SL=0.0

WHEN NEAR BOUNDING SURFACE COMPARE STRESS AT BEGINING
OF INCREMENT TO WHAT HAVE AT END IF USE THE
ESTIMATED STRAIN INCREMENT TO CALCULATE AN ELASTIC
STRESS INCREMENT

IF(BSB .GT. 1.05) GO TO 355
DO 270 I=1,6
DU=SIGB(I)
DU=DO 270 I=1,3
260 DU=DU+C(I,J)*DEP(J)*C(I,J+3)*DEP(J+3)*2.0
270 SIGFE(I)=DU
XIFE=SIGFE(1)+SIGFE(2)+SIGFE(3)
RTJFE=0.0
DO 280 I=1,3
RTJFE=(SIGFE(I)-XIFE/3.0)**2+RTJFE
280 RTJFE=2.0*SIGFE(I+3)**2+RTJFE
RTJFE=SQRT(0.5*RTJFE)
DU=0.0001*PROP(14)
IF(ABS(XIFE) .LT. 0.001) GO TO 290
DU=XIFE
XIFE=DU
IF(DU .LT. 0.0)XIFE=-DU
290 THFE=RTJFE/XIFE
CALL BOUND(PROP,YS,THFE,XIFE,XIFG,GAMEF,DFIF,DFJF,XKSF)
BSFE=GAMFE*XIFG/XIFE
SL=1.0
IF(BSF .GT. BSB)SL=0.0
355 CONTINUE

C CALCULATE PLASTIC PORTION OF INCREMENTAL PROPERTIES

IF(SL .EQ. 0.0) GO TO 410
DO 400 M=1,6
I=II(M)/10
J=MOD(II(M),10)
DO 400 N=M,6
K = II(N)/10
L = MOD(II(N), 10)
DU1 = 3.0*BB*DFIB
DU2 = 0.0
IF(SRTJB .NE. 0.0) DU2 = GB*DFJB/SRTJB
DU3 = KF + 9.0*BB*DFIB*2 + GB*DFJB*2
DU = -0.5*SL*(DU1*DLTA(I, J) + DU2*SB(I, J)) * (DU1*DLTA(K, L) + 1
DU2*SB(K, L))/DU3
DU1 = 3.0*BF*DFIF
DU2 = 0.0
IF(SRTJF .NE. 0.0) DU2 = GF*DFJF/SRTJF
DU3 = KF + 9.0*BF*DFIF*2 + GF*DFJF*2
DU = DU - 0.5*SL*(DU1*DLTA(I, J) + DU2*SF(I, J)) * (DU1*DLTA(K, L) + 1
DU2*SF(K, L))/DU3
C(M, N) = DU + C(M, N)
400 C(N, M) = C(M, N)
410 CONTINUE
IF(IDIM .EQ. 3) RETURN
C
CONVERT 3-D PROPERTIES TO PLANE STRAIN
C
SIG8(3) = SIG8(4)
SIG8(4) = 0.0
DSIG(3) = DSIG(4)
DSIG(4) = 0.0
DU = 0.0
D0 420 I = 1, 4
D0 430 I = 1, 3
C(I, 3) = C(I, 4)
430 C(I, 4) = 0.0
STOR(4) = DU
RETURN
END
SUBROUTINE BOUND(PROP, YS, TH, XI0, XI, GAM, DFJ, DFI, XKS)
SUBROUTINE TO EVALUATE RELATIONSHIP OF STRESS STATE
TO BOUNDING SURFACE
C
DIMENSION PROP(14)
XN = PROP(3)/3.0/SQRT(3.0)
R = PROP(4)
OMEG = 1.0
IF(XI0 < PROP(7)) OMEG = XI0/PROP(7)
DUS = 12.0*(1.0+PROP(11))/(PROP(1)-PROP(2)*OMEG)
IF(XI, LT, 0.0) GO TO 300
IF(TH .GT. XN) GO TO 200
C
NORMAL CONSOLIDATION ZONE
C
X = TH/XN
DU = R*(1.0 + X*X + R*(R-2.0)*X*X)
GAM=(1.0+(R=1.0)*SQRT(1.0+R*(R=2.0)*(X*X)))/DU
DFI=2.0*XIO*(GAM=1.0/R)
DFJ=2.0*XIO*TH*GAM*((R=1.0)/XN)**2
XKS=DUS*XIO**3*(GAM=1.0/R)*(GAM=R=2.0)/R
RETURN

C OVERCONSOLIDATED ZONE

A=PROP(5)
X=TH/XN
DU=1.0*X*(1.0+YS)
GAM=((DU+SQRT((X-YS=1.0)**2+(X*X=1.0)*YS*YS)))/(R*(X*X=1.0))
DFI=2.0*XIO*(GAM=1.0/R)
DFJ=2.0*XIO*(A+XN/R=TH*GAM)/XN/XN
XKS=DUS*XIO**3*(GAM=1.0/R)*(DU*GAM+2.0*A/XN)/R
RETURN

C TENSION ZONE

T=PROP(6)
Y=(1.0+YS)*SQRT(1.0+YS*YS)-(1.0+YS*YS)
XNT=XN*(Y=RT)*SQRT( Y*(Y=2.0*RT) /((1.0+YS*YS))/(R*RT)**2)
RT=Y/R/RT
XIO=TI*Y*XIO/(Y=2.0*R/RT)
XT=TH/XNT
DU=RT*(1.0+XT*XT)RT*(RT=2.0)*XT*XT)
GAM=(1.0-(RT=1.0)*SQRT(1.0+RT*(RT=2.0)*XT*XT)))/DU
GAM=GAMT*XIO/XIO
DFI=2.0*XIO*(GAM=1.0/RT)
DFJ=2.0*XIO*TH*GAM*((RT=1.0)/XNT)**2
XKS=DUS*XIO**3*(GAM=1.0/RT)*(GAM+RT=2.0)/RT
RETURN
END
APPENDIX IV

***EVAL**** PROGRAM TO PREDICT HOMOGENEOUS TEST RESULTS
PREPARED BY L. R. HERRMANN AT UNIV OF CALIF DAVIS

DIMENSION PROP(14), STOR(6), SIGB(6), DSIG(6), DEP(6), C(6, 6),
1 EPB(6), TITLE(12), ICOD(6), V(6), DV(6), S(6, 6), R(6), RP(6)
XXX=1.0E+25
1 READ(5, 800, END=700) TITLE
800 FORMAT(12A6)
WRITE(6, 900) TITLE
900 FORMAT(1H1/5X, 12A6///)

READ MATERIAL PROPERTIES

READ(5, 801) (PROP(I), I=1, 14)
801 FORMAT(8E10.3)
WRITE(6, 901) (PROP(I), I=1, 12), PROP(14)
901 FORMAT(5X, "CLAY PROPERTIES"/15X, "LAMBDA =", E9.3/15X,
3 "10 =", E9.3/15X, "HARDENING PARAMETERS:"/19X,
5 E9.3/15X, "POISSON'S RATIO =", E9.3/15X,
6 "ATMOSPHERIC PRESSURE =", E9.3///)
IF PROP(13).EQ.0.0 WRITE(6, 904)
904 FORMAT("**** DRAINED CONDITIONS *****"///)
IF PROP(13).NE.0.0 WRITE(6, 905) PROP(13)
905 FORMAT(5X, "**** UNDRAINED CONDITIONS — THE COMBINED ",
1 "SKELETON AND WATER BULK MODULUS =", E9.3///)

READ ITERATION INFORMATION

READ(5, 802) ITMAX, ERMAX
802 FORMAT(IS, E10.2)
WRITE(6, 902) ITMAX, ERMAX
902 FORMAT(5X, "ITMAX =", I3/5X, "ERMAX =", E9.3)

INITIALIZATION

NO=0
DO 100 I=1, 6
SIGB(I)=0.0
DSIG(I)=0.0
EPB(I)=0.0
100 DEP(I)=0.0
READ(5, 803) CONFIN, ATMO
803 FORMAT(2E10.3)
WRITE(6, 903) CONFIN
903 FORMAT(5X, "INITIAL CONFINING PRESSURE =", E9.3///)
DO 110 I=1, 3
SIGB(I)=CONFIN

C86
DSIG(I)=0.01*ATMO
110 DEP(I)=0.00001

LOADING SEGMENT LOOP

WRITE(6,910)
115 READ(5,804) (ICOD(I),V(I),I=1,6),NUM,D
804 FORMAT(6(I1,E9.1),I5,E10.2)
IF(ICOD(I).GT.1) GO TO 600
IF(D.EQ.0.0) D=1.0

DETERMINE FIRST INCREMENTS

DU1=NUM
DU1=1.0/NUM
IF(D.NE.1.0) DU1=(1.0-D)/(1.0-D**NUM)
DO 120 I=1,6
DU=V(I)=SIGR(I)
IF(ICOD(I).EQ.1) DU=V(I)-EPB(I)
120 DV(I)=DU*DU1

INCREMENT LOOP

DO 500 INC=1,NUM

ITERATION LOOP

DO 400 ITNO=1,ITMAX

FIND INCREMENTAL PROPERTIES

NJ=NO+1
CALL CLAY(3,NJ,ITNO,PROP,STOR,SIGB,DSIG,DEP,C,GAM)

FORM AND MODIFY STIFFNESS

DO 160 I=1,6
DO 140 J=1,6
140 S(I,J)=C(I,J)
R(I)=DV(I)
IF(ICOD(I).EQ.0) GO TO 160
R(I)=DV(I)*XXX
S(I,I)=XXX
160 CONTINUE

SOLVE FOR STRAIN INCREMENT

DO 200 I=1,6
DU=1.0/S(I,I)
R(I)=R(I)*DU
DO 165 J=1,6
165 S(I,J) = S(I,J) * DU
   IF(I,EQ.0) GO TO 200
   I = I + 1
   DO 175 J = II, 6
   DU = - S(J,I)
   DO 170 K = 1, 6
170 S(J,K) = S(J,K) + DU * S(I,K)
175 R(J) = R(J) + DU * R(I)
200 CONTINUE
   I = 6
   DO 210 II = 1, 5
   I = I + 1
   IL = I + 1
   DO 210 J = IL, 6
210 R(I) = R(I) - S(I,J) * R(J)
   CALCULATE STRESS INCREMENT
   DO 240 I = 1, 6
   DU = 0.0
   DO 230 J = 1, 6
230 DU = C(I,J) * R(J) + DU
240 RP(I) = DU
   COMPUTE ERROR NORMS AND UPDATE INCREMENTAL ESTIMATES
   ERSIG = 0.0
   EREP = 0.0
   DU1 = 0.0
   DU2 = 0.0
   DO 260 I = 1, 6
260 ERSIG = ERSIG + ABS(RP(I) - DSIG(I))
   EREP = EREP + ABS(R(I) - DEP(I))
   DU1 = DU1 + ABS(RP(I))
   DU2 = DU2 + ABS(R(I))
   DSIG(I) = RP(I)
   DEP(I) = R(I)
260 ERSIG = ERSIG / DU1
   EREP = EREP / DU2
   CHECK FOR CONVERGENCE
   IF(ERSIG .LE. EMAX .AND. EREP .LT. EMAX) GO TO 420
400 CONTINUE
   WRITE(6, 907) NO + 1, ERSIG, EREP
907 FORMAT(*3X, "CONVERGENCE DID NOT OCCUR FOR INCREMENT", I3, 
   1 " --- ERSIG =", E9.3, " AND EREP =", E9.3/)
410 READ(5, 804) KKK
   IF(KKK .GT. 1) GO TO 600
   GO TO 410
   UPDATE TOTAL VALUES
   NO = NO + 1
DO 430 I=1,6
DV(I)=DV(I)+0
SIGB(I)=SIGB(I)+DSIG(I)
430 EPB(I)=EPB(I)+DEP(I)
U=GAM*(EPB(I)+EPB(2)+EPB(3))
C C C
PRINT INCREMENTAL VALUES
C
WRITE(6,908) NO,(EPB(I),I=1,6),(SIGB(I),I=1,6),U,ITNO
908 FORMAT(1X,I3,1P6E9.1,7E10.2,1X,I3)
500 CONTINUE
WRITE(6,602)
602 FORMAT(/)
GO TO 115
600 CONTINUE
GO TO 1
700 STOP
END

(b) explicate the work of the computational model into a numerical analysis

code for solution of classical boundary value problems. The second

treatment of problems is important since different algorithms are currently in use for

incorporating computational solutions into classical boundary value problems. For example, the elastic

distributed stiffness matrix for a frictional mate-

rial is neither symmetric nor necessarily positive definite. Then,

when problems are formulated using the finite element method it is often

computationally advantageous to use algorithms that do not directly em-
ploy the elastic-plastic stiffness matrix (e.g., Zienkiewicz and Roache 1977).
For versatility, the elastic strain increment direction $\mathbf{\varepsilon}^p$ and the

loading direction $\mathbf{f}$ are computed in separate subroutines, which are

called by the subroutine that formulates the elastic-plastic

2. The subroutines used in the code can be classified into three
categories: program control, utility computations, and model-specific computations. Program
control subroutines read data, control the incremental

integration, and write the results. The utility computations are general

routines to perform operations such as solution of linear simultaneous

equations, computation of principal stress values, and computation of

products of matrices and vectors. Model-specific computations include

operations which are specifically required for the model. A brief de-

scription of each subroutine is given in Table 3.
APPENDIX D: COMPUTER CODE FOR THE BOUNDED FRICTIONAL-DILATANT FLOW MODEL

General Description of Computer Code

1. This appendix presents the computer code used to simulate the mechanical behavior of a single homogeneous element of soil based on the bounded frictional-dilatant flow model. The code is written as a one-element finite element program and is thus similar to programs used by Lade (1972) and Dafalias and Herrmann (Appendix C). However, the code is highly fragmented into subroutines to (a) allow for changes in the particular functional forms used to describe $F$, $J$, and $\frac{\sigma_p}{\sigma_e}$, and (b) expedite the use of the constitutive model into a numerical analysis code for solution of full-scale boundary value problems. The second factor is important since different algorithms are currently in use for incorporating constitutive relationships into boundary value problems. For example, the elastic-plastic stiffness matrix for a frictional material is neither symmetric nor necessarily positive definite. Thus, when problems are formulated using the finite element method it is often computationally advantageous to use algorithms that do not directly employ the elastoplastic stiffness (e.g., Zienkiewicz and Humpheson 1977). For versatility, the plastic strain increment direction $\{\varepsilon\}$ and the loading direction $\{N\}$ are computed in separate subroutines, which are called by the subroutine that formulates the elastic-plastic stiffness.

2. The subroutines used in the code can be classified into three groups: program control, utility computations, and model-specific computations. Program control subroutines read data, control the incremental integration, and write the results. The utility computations are general routines to perform operations such as solution of linear simultaneous equations, computation of principal stress values, and computation of products of vectors and matrices. Model-specific computations include operations which are specifically required for the model. A brief description of each subroutine is given in Table D1.
Table D1
Subroutine Description

<table>
<thead>
<tr>
<th>Subroutine Name</th>
<th>Operation Performed</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAIN</td>
<td>Reads data file, specifies each increment of applied stress or strain, writes out results</td>
</tr>
<tr>
<td>MARCH</td>
<td>Integrates stress-strain curve using modified Runge-Kutta marching scheme</td>
</tr>
<tr>
<td>EPSTIF</td>
<td>Formulates elastoplastic stiffness</td>
</tr>
<tr>
<td>UPLO</td>
<td>Factors incremental stiffness matrix into upper-lower triangular form</td>
</tr>
<tr>
<td>RESOLVE</td>
<td>Resolves factored matrix to obtain unknown strain increments</td>
</tr>
<tr>
<td>SETIT</td>
<td>Modifies stiffness and stress increment matrix to account for specified values of strain increment</td>
</tr>
<tr>
<td>CHAC</td>
<td>Determines invariants and principal values of second-order tensor</td>
</tr>
<tr>
<td>COORD</td>
<td>Constructs relationship to transform principal values of second-order tensor to general coordinate system</td>
</tr>
<tr>
<td>PROD</td>
<td>Finds trAB where A and B are second-order tensors</td>
</tr>
<tr>
<td>MULT</td>
<td>Finds product of two matrices</td>
</tr>
<tr>
<td>DIRECT</td>
<td>Finds strain increment direction $\mathbf{b}^k$</td>
</tr>
<tr>
<td>NORM</td>
<td>Finds normal to $\mathbf{F}(\sigma^k)$</td>
</tr>
<tr>
<td>RADIUS</td>
<td>Finds tan $\alpha$ for given $\mathbf{F}$ and $\mathbf{1}$</td>
</tr>
<tr>
<td>MOVEIT</td>
<td>Determines $\sigma^k$</td>
</tr>
<tr>
<td>DJ</td>
<td>Finds differentials $\partial J/\partial S$, $\partial J/\partial \varepsilon$, and $\partial J/\partial \sigma_p$ for a given $\mathbf{F}^k$</td>
</tr>
<tr>
<td>ELASTO</td>
<td>Determines elastic stiffness components</td>
</tr>
<tr>
<td>WATER</td>
<td>Add stiffness of pore fluid to elastoplastic stiffness relationship to model undrained conditions</td>
</tr>
<tr>
<td>RESET</td>
<td>Updates location of apparent hydrostatic axis or identifies development of a new nested yield surface</td>
</tr>
<tr>
<td>SURFACE</td>
<td>Evaluates $\mathbf{F} = I_1 I_2 / I_3$</td>
</tr>
</tbody>
</table>
3. The computer code has the facility to model behavior under complex loads such as unload-reload loops and cyclic loading, a feature not pertinent to the modeling of strain-softening behavior. The response to complex load paths is controlled by a kinematic hardening model described in detail by Peters (1982). Subroutines RADIUS, MOVEIT, and RESET are used to model the kinematic hardening process. These subroutines play no role in the computations presented in this report.

4. The computations are controlled by subroutine MARCH which computes the increments of effective stress, strain, and pore pressure associated with each specified total stress and strain increment. The main program is written as needed by the user to model a particular type of test. An example of a main program to model a triaxial test is listed on pages D5 and D6. The computer code for the subroutines is given on pages D7-D15.

5. Material parameters for the model (Figure D1) fall within two groups: those determined directly from laboratory tests and those estimated from trial curve-fitting. The values of $K$, $B_c$, $e_o$, and $a_1$ can be readily determined from isotropic and triaxial shear testing data. These parameters have the greatest control on model behavior. The parameters $a_2$ and $t$ provide additional "shaping" to the stress-strain curve and can generally be estimated after a few trial calculations. The parameters $a_e$, $K_1$, $K_2$, and $m$ control the shape of the surface bounding the stress paths on the $q/p_e - p/p_e$ plot and initial estimates of their values can be made using these plots. However, because the limiting surface is never intercepted by the stress path, revised estimates must be made using a trial and error procedure.

6. Note that the second hardening parameter $a_2$ is inserted into Equation 18 (main text) when the ratio

$$\frac{F(q^k)}{J_1 \frac{S}{a_1 + S}} - 9$$

exceeds the parameter $t$. This gives a more realistic hardening relationship for soils that display distinct breaks in the stress-strain response.
Figure D1. Parameters for the bounded frictional-dilatant flow (BFDF) model
Example of a Main Program
10
DIMENSION SIG<6>,SIG0(6),SIGC(6)
20
DIMENSION EPS(6),BINC(6),ICON(6),DEP(6),DSIG(6)
30
DIMENSION NUM<10),SIZEC10)
40
DATA SIGC/3*l.Or3*0.0/
50
nATA SIG0/3*1•0•3*0,0/
60
DATA SIG/3*10.0,3*0.0/
70
DATA EPSrBINC/6*0.0r6*0.0/
80
DATA ICON/0, 5*1/
90
D~TA NUM/1000r1900,800,1200,800,1400r1000,400,200•1000/
100
rATA SIZE/O.OOOOl,0.0001r-0.00001r0,00001,-0.00001r0.00001,
110& -o .oooot.o.oooot , -o .oo oo1.o . oooo1/
120
COMMON /PROP/DMAX,EREFrASl,AS2rTHRESH,CE,TROrXM,BCrEO,ROWE,DRAGrXKAPrATM
130
COMMON /STA TE/ ISO,£,F<10),STRESS<6•10)
COMMON /FLUID/BW,U
140
!SOC
160
CALL FPARAM<1r132)
170
9
CONTINUE
EF'S(1)=0.0
180
190
EF'S<2>=0.0
200
EF'SC3>=0.0
210
AS1=0.04
':'?0
AS2=0.075
230
THRESI-!=0.025
240
CE=0.75
TR0=-0,3
250
DMAX=100,0
260
270
XM=l.O
280
BC=1.587
EREF=1.30
290
300
ROWE=1.90
DRAG=7.0
305
310
XKAP=0.04
320
ATM= 14.7
330
DO 20 I=1•6
STRESS<I•1>=SIGO<I>
335
STRESS(I,~>=SIGC<I>
340 20
350C
READC12•1•END=909) LNB,£,PCrBWrBLODErNC
360
SIGCl>=PC
370
380
SIG<2>=SIG<1>
SIG<3>=SIG<1>
390
400
£0=1.77
EOUT =E
410
BW=BW*ATH
420
430C
440
IS0=1
F<1> =9 .0
450
U=O ,
460
S=O.O
470
480C
FORMAT(V)
490 1
WRITE C6, 2 >
500
FORHAT<I////)
510 2
520C
WRITE<6,1>'PROF'ERTIES:'
530
WRITE<6•1>'AS1= 'rAS1r' AS2= '•AS2r' THRESHOLD= 'rTHRESH
540
WRITE<6r1) KAPPA= ',XKAP
550
WRITE<6r1)
I
560
1
WRITEC6r1)'CE=
,CEr'
TAN RHO= ' rTRO
570
WRITE<6r1) XH= 1 rXM•' DHAX= rDHAX
580
WRITE(6rl> 1 1
590
EO= 1 rEO
WRITE ( 6, 1 ) I BC= I , BC, I
600
WRITE(6,1)' I
610
rDRAG
WRITE<6•1> 'R-CONS TANT VOLUME= ',ROWE,' COHESION=
620
WRITE<6d)' I
630
•
WRITE<6r1)'INITIAL CONDiriONS:'
640
WRITE(6,1)'Sl0C= ',PCr' VOIDS RATIO= '•E
650
WRITE<6,1)'PWP= '•U
660
WRIT£(6.1) I I
670
WRITE(6,1>'HISC:'
680
WRITE<6,1>'ATHOSPHERIC PRESSURE= 'rATH
690
WRITE<6rl)'WATER COMPRESSIBILITY= ' ,BW
700
WRITE(6r1>' '
710
7~0
WRITEC6r1000)
730
WRITE(6,2000)
740 1000 FORHAT<3Xr'S'r6Xr'SIG1'r5X,'SIG2',5Xr'SIG3',6Xr'UW'r6X,'EP\'r 5X r 'FP2'
750& , sx ,'EP3 ,4X,'VOIDS' ,JX, 'P /P£',9X•'X',7X•'Y',6X,'EXT'r5X,'EYT',3X,'ISO ' >
760 2000
FORMAT<1Xr5('-'),JXr4(6('-'),3X>r5(5('-'),3X>,4Xr4(5('-')r3X,

--

1
1

1

1

1

1

DS

I


770% \) *1x,3(') \)
780C
790C NOW COMPUTE
800  \)FX=0.0
810  XC=0.0
820  U10=0.95*PC
830  NSKIP=10
840  WRITE=0
850  NEXT=0
860  DO 15 I=1,NC
870  NINC=NUM(CIC)
890  call MARCHCIC(SIGC1, SIGC2, SIGC3, BINC, ICON, DEP, DSIG)
910  DO 10 I=1,NINC
920  BINCC1>=SIZECIC>
950  NRITE=NRITE+1
970  call MARCHCSIG(SIGC1, SIGC2, SIGC3, BINC, ICON, DEP, DSIG)
980  EOUT=EOUT - (1. + EOUT) *(DEP(1) + DEP(2) + DEP(3))
990  EPS(1)=EPS(1) + 100.*DEP(1)
1000 EPS(2)=EPS(2) + 100.*DEP(2)
1010 EPS(3)=EPS(3) + 100.*DEP(3)
1020 DS=DEP(1)**2 + DEP(2)**2 + DEP(3)**2
1030 S=S + 100.*SORT(DS)
1040 IF(U10.EQ.U100) GO TO 19
1050 NEXT=NEXT + NSKIP
1060 19 CONTINUE
1070 TQ=(SIGC1)-SIGC2)**2 + (SIGC3)-SIGC2)**2 + (SIGC1)-SIGC3)**2
1080 PE=ATM*10.0**BC*(EO-E)
1090 P=0.33333333*(SIGC1) + SIGC2 + SIGC3)
1100 TALPH=0.3333333*SQRT(TO)/P
1110 X=0.4082*(SIGC2) - SIGC3)/P
1120 Y=0.2357*(2.*SIGC1) - SIGC2 - SIGC3)/P
1130 EX=0.7071*(EPSC2) - EPSC3)
1140 EY=0.4082*(2.*EPSC1) - EPSC2 - EPSC3)
1150 P1=P/PE
1160 WRITEC6r3000) SrSIGC1rSIGC2rSIGC3),1JrEPSC1rEPSC2rEPSC3)
1170 3000 FORMATC1X,F5.2r3Xr4CF6.1,3X),3(F5.1,3X>,2<F5.3,3X),2(F7.3r1X)rl3)
1180 2CF5.3r3X)
1210 10 CONTINUE
1220C
1230 11 CONTINUE
1250 3 FORMAT(/)
1260 15 CONTINUE
1270C
1280 909 STOP
1300 END
Listing of Computer Subroutines

1200C**********HARCH**********
1210C SUBROUTINE TO INTEGRATE ONE STEP OF STRESS-STRAIN RELATIONSHIP
1220C
1230DIMENSION SIG(6),SIG0(6),SIGC(6),BINC(6)
1240DIMENSION ICON(6),DSIG(6),UEPC(6)
1250DIMENSION SC(6),SOC(6),SC(6)
1260DIMENSION STIFC(6,6),D(6,6),XNC(6)
1270DIMENSION DELAS(6),DM(6,6)
1280C
1290COMMON /STATE/ISO,E,F(I),STRESS(6+10)
1300COMMON /PROP/DMAX,ER,AS1,AS2,THRESH,CE,TRO,XM,BC,EO,ROWE,DRAG,PKAP,ATM
1310COMMON /FLUID/ BW,U
1320C
1330C ZERO MEAN ELASTIC-PLASTIC STIFFNESS
1340DO11I=1,6
1350DO11J=1,6
1360DM(I,J)=0.0
1370CONTINUE
1380C
1390DO10I=1,6
1400S(I)=SIG(I)
1410S0(I)=STRESS(I,ISO)
1420SC(I)=STRESS(I,ISO+1)
143010CONTINUE
1440E1=E
1450FT=F(ISO)
1460FN=F(ISO+1)
1470U1=U
1480C
1490C FIND ELASTIC STRESS INCREMENT
1500CALL ELASTO(SIG,D)
1510CALL WATER(D)
1520CALL SETIT(D,BINC,ICON,STIF,DEP,6)
1530CALL UPLDO(STIF,6,DET)
1540CALL RESOLVE(STIF,DEP,6)
1550C FIND STRESS
1560CALL MULT(D,DEP,DELAS,6,6)
1570DUE=(DEP(1)+DEP(2)+DEP(3))*BW
1580DELAS(1)=DELAS(1)-DUE
1590DELAS(2)=DELAS(2)-DUE
1600DELAS(3)=DELAS(3)-DUE
1610C
1620C FIND ELASTIC-PLASTIC INCREMENT
1630THE=1.0
1640DO90I0=1,2
1650CALL EPSTIF(ISO,DELAS,S,S0,SC,FN,E1,D,XN,BULK,LOAD)
1660CALL WATER(D)
1670FACT=1.0-THE
1680DO12I=1,6
1690DO12J=1,6
1700DM(I,J)=THE*DM(I,J)+FACT*DM(I,J)
1710C
1720CALL SETIT(DM,BINC,ICON,STIF,DEP,6)
1730CALL UPLDO(STIF,6,DET)
1740CALL RESOLVE(STIF,DEP,6)
1750CALL MULT(DM,DEP,DSIG,6,6)
1760DU=(DEP(1)+DEP(2)+DEP(3))*BW
1770DSIG1=DSIG1-DU
1780DSIG2=DSIG2-DU
1790DSIG3=DSIG3-DU
1800C
1810C COMPUTE VARIABLES FOR NEXT INC
1820DF=0.0
1830VOL=0.0
1840DO20I=1,3
1850S(I)=SIG(I)+DSIG(I)
1860SC13=SIG(I)+DSIG13
1870VOL=VOL+DEP(I)-0.3333333*DSIG(I)/BULK
1880DF=DF+DSIG(I)*XN(I)+DSIG13*XN(I3)
1890CONTINUE
1900C
1910DE=-(1.0+E)*VOL
1920E1=E+DE
1930U1=U+DU
1940CALL RESET(LOAD,FT,FN,DSIG,S,S0,SC)
1950THE=0.5
196090CONTINUE
1970C

D7
1900C CONVERT BACK
1910 DO 40 I=1,6
1920 SIG(I)=S(I)
1930 STRESS(I,ISO)=S0(I)
1940 STRESS(I,ISO+1)=SC(I)
1950 40 CONTINUE
1960 E=E1
1970 F(ISO)=FT
1980 F(ISO+1)=FN
1990 U=U1
2000 RETURN
2010 END
2100C
2110C*************UPLO************
2120C SUBROUTINE TO FACTOR A INTO L*U
2130 SUBROUTINE UPLO(A,N,DET)
2140 DIMENSION A(N,N)
2150C FIND TERMS FOR J=1
2160 DO 10 I=2,N
2170 10 A(I,J)=A(I,J)/A(I,I)
2180C WORK BY COLUMN J, DON'T TOUCH ROW I=1
2190 DO 20 J=2,N
2200C WORK EACH ROW ELEMENT TO DIAGONAL
2210 DO 30 I=2,J
2220 SUM=0.0
2230 DO 40 K=1,I-1
2240 40 SUM=SUM+A(I,K)*A(K,J)
2250 30 A(I,J)=A(I,J)-SUM
2260C UPDATE DETERMINANT
2270 DET=DET*A(J,J)
2280C NOW WORK BELOW DIAGONAL
2290 IL=J+1
2300 IF(J.EQ.N) GO TO 20
2310 DO 50 IL=IL,N
2320 J1=J-1
2330 SUM=0.0
2340 DO 60 K=1,J1
2350 60 SUM=SUM+A(I,K)*A(K,J)
2360 50 A(I,J)=A(I,J)-SUM/A(J,J)
2370 20 CONTINUE
2380 RETURN
2390 ENDSUBROUTINE UPLO
2400C*************RESOLVE************
2410C SUBROUTINE FOR RESOLUTION OF L*U*X=B, WITH L*U=B
2420C NOW WORK BELOW DIAGONAL
2430 IL=J+1
2440 IF(J.EQ.N) GO TO 20
2450 DO 50 IL=IL,N
2460 J1=J-1
2470 SUM=0.0
2480 DO 60 K=1,J1
2490 60 SUM=SUM+A(I,K)*A(K,J)
2500 50 A(I,J)=A(I,J)-SUM/A(J,J)
2510 20 CONTINUE
2520 RETURN
2530 ENDSUBROUTINE RESOLVE
2540C*************SETIT************
2550C SUBROUTINE TO MODIFY A & B FOR PRESCRIBED VALUES OF X
2560C IN A*X=B
2570C SUBROUTINE SETIT(A,B,ICON,XK,F,N)
2580 DIMENSION A(N,N),B(N),ICON<N>,XK<N,N>,F<N>
2590C SOLVE TOWARDS FOR I*Y=B
2600 DO 10 I=1,N
2610 SUM=0.0
2620 DO 11 NI=I,N
2630 11 I=I+1
2640 B(I)=B(I)-SUM
2650 10 CONTINUE
2660C NOW GO UP FOR UX=Y
2670 DO 20 NI=1,N
2680 SUM=0.0
2690 DO 21 I=1,N
2700 21 I=I+1
2710 SUM=SUM+A(I,J)*B(J)
2720 20 CONTINUE
2730 RETURN
2740 ENDSUBROUTINE SETIT
SUBROUTINE CHAC(SIG,S)
DIMENSION SIG(6),S(3)
ATM=1.0

3000 FIND STRESS INVARIENTS
3010 X1=SIG(1) + SIG(2) + SIG(3)
3020 X2=SIG(1)*SIG(2) + SIG(1)*SIG(3) + SIG(2)*SIG(3)
3030 S4=SIG(4)*SIG(4)
3040 S5=SIG(5)*SIG(5)
3050 S6=SIG(6)*SIG(6)
3060 X2=X2 - (S4 + S5 + S6)
3070 X3=SIG(1)*SIG(2)*SIG(3) + 2.0*SIG(4)*SIG(5)*SIG(6)
3080 X3 = X3 - (SIG(1)*S5 + SIG(2)*S6 + SIG(3)*S4)
3090 FIND DEVIATORIC INVARIENTS
3100 T2=0.33333333*X1*X1
3110 S1 = SIG(1)
3120 S2 = SIG(2)
3130 S3 = SIG(3)
3140 RETURN
3150 CONTINUE
3160 Y2=SQRT(T2)
3170 Y3 = 0.074074074*X1*X1 - 0.3333333333*X1*X2 + X3
3180 FIND LODE ANGLE
3190 IF(ABS(CANG).GT.0.09999999) CANG=0.99999999
3200 ANG = 0.3333333333*ARCOS(CANG)
3210 C1 = 0.3333333333*X1
3220 C2 = 1.15470054*Y2
3230 AND FINALLY PRINCIPAL STRESSES
3240 S(1) = C2*COS(ANG) + 1.0471976 + C1
3250 S(2) = C2*COS(ANG) + 1.0471976
3260 S(3) = -C2*COS(ANG) + C1
3270 RETURN
3280 CONTINUE
3290 RETURN

3300 FIND PRINCIPAL STRESSES AND INVARIENTS
3310 IF(ABS(SIG(4)).GT.0.0001*ATM) GO TO 10
3320 IF(ABS(SIG(5)).GT.0.0001*ATM) GO TO 20
3330 IF(ABS(SIG(6)).GT.0.0001*ATM) GO TO 30
3340 CONTINUE
3350 CONTINUE
3360 CONTINUE
3370 CONTINUE
3380 CONTINUE
3390 CONTINUE
3400 CONTINUE
3410 CONTINUE
3420 CONTINUE
3430 CONTINUE
3440 CONTINUE
3450 CONTINUE
3460 CONTINUE
3470 CONTINUE
3480 CONTINUE
3490 CONTINUE
3500 CONTINUE

END
DO 30 J = 1, 3
A1 = SIG(2) - SIG(J) + SIG(3) - SIG(J) + SIG(5) - SIG(5)
B1 = SIG(5) * SIG(6) - SIG(4) * SIG(3) - SIG(3) - SIG(J)
C1 = SIG(4) * SIG(5) - SIG(6) * SIG(2) - SIG(2) - SIG(J)
XL = A1 * A1 + B1 * B1 + C1 * C1
T(1,J) = A1 * A1 / XL
T(2,J) = B1 * B1 / XL
T(3,J) = C1 * C1 / XL
CONTINUE
RETURN
END
C*************DIREC*************
C SUBROUTINE TO FIND UNIT VECTOR OF STRAIN INC
SUBROUTINE DIRECCSTRU,F,SX,SX2,SY,SZ,S,DF,BETA,T
DIMENSION SC3,TDCR<3>,DF<3>,BETAC6),T(6,3)
COMMON /PROP/DMAX,EREF,AS1,AS2,THRESH,AE,TROT,TH,XKAP,AlM
C COMPUTE DILATANCY RATE
ROS = 3.*CROWE - 1.)/CROWE + 2.)
P = 0.33333333333 * XJ1
IF(P.LE.ATM) P = ATM
R05 = R05 + DRAG/P
ROPE = 2.*ROS + 3.)/(3, - ROS)
CJOOC
F = SIG(5) LE 5, 01
R = 0.25*<~5 + SORF<~~*F5 - t6.0>>
ENOM = StmT < ROPE*fWf.'E
CTHE = 0.57735*S<1>*<1> - R)/[LEOM
SIHE = SQRT(1. - CTHE*CTHE>
STRESS ANGLE
SMAG = SQRT(SC1)*S<1> + 5(2)*S<2> + SC3)*SC3))
CALPH = 0.57735*<S<1> + 5(2) + SC3))/SMAG
SALPH = SQRT(1.0 - CALPH*CALPH>)
ANGLE WRT NORMAL
X = S<2> - S<3>
Y = 2.*S<1> - S<2> - S<3>
DFX = DF<2> - DF<3>
DFY = 2.*DF<1> - DF<2> - DF<3>
XY = SQRT(0.5*X*X + 0.1666667*Y*Y, 0.00000001
DFL = SORTCO.S*DFX*DFX + 0.16666667*DFY*DFY, 0.00000001
CDIF = (0.5*X*DFX + 0.16666667*Y*DFY,XY*DFL
DFX = 0.70711*DFX/DFL
DFY = 0.40825*DFY/DFL
ANZCOSINE
FRIC = CALPH*CTHE + SALPH*STHE*CDIF
IF(R.LT.1.0000001) FRIC = 1.0
FSQ = FRIC*FRIC
TF = SORT(1.0 - FSQ)/FSQ
TF = TF*ST
FRIC = SORT(1.0/1.0 + TF*TF))
NOW RELATE TO COORDINATES OF PI PLANE
SY = 0.707106781*(1.7320508*SY/SZ - 1.0)
SZ = 1.22474487*(1.29099445*SY/SZ - 1.0)
SRL = SORT(SX*SY + 5YSY) + 0.00000001
CDIF = (SXSDFX + SY*DFY)/SRL
FIND COSSINE THETA
IF(SZ,GE.0.9999999) SZ = 0.9999999
SX = SORT(S1. - SZ*SZ)
XL = SX*SR*CDIF*CDIF
XL = XL1 + SZ*SZ
X1 = SORT(XL1)
ARG = XL - FRIC*FRIC
IF(ARG.LT.0.) ARG = 0.
CE = (FRIC*SZ - XL1*SORT(ARG))/XL
4100C
IF(CE.GT.0.1) CE=1.
RADIUS=SORT(1.0 - CE*CE)
VOL=0.5773503*CE
DEV1=0.8164966*DFY
DEV2=0.7071068*DFX - 0.4082483*DFY
DEV3=DEV2 - 1.4142136*DFX
RADIUS=SORT(1.0 - CE*CE)
VOL=0.5773503*CE
DEV1=0.8164966*DFY
DEV2=0.7071068*DFX - 0.4082483*DFY
DEV3=DEV2 - 1.4142136*DFX
DEL<1>=RADIUS*DEV1 + VOL
DEL<2>=RADIUS*DEV2 + VOL
DEL<3>=RADIUS*DEV3 + VOL
CONVERT TO XYZ COORDINATES
CALL MULT(T*DEL+BETA*6,3)
RETURN
END

**********NORM**********
SUBROUTINE TO FIND NORMAL TO YIELD SURFACE
DIMENSION SC3,SC3,DF3,DXYC6,2),DXYC6,2),XNC6)
FIND YIELD STATE
CALL SURFACECS,X1,X2,X3,SMP)
FIND DIRECTION OF YIELD CENTER MOVEMENT
SY = 0.40825*SC1 - SCC2 - SCC3)
SX = 0.70711*CSC1 - SCC2 - SCC3)
SRL=SORT(SY*SY + SX*SX + 0.0000001)
SY=SY/SRL
SX=SX/SRL
FIND RATE OF YIELD CENTER MOVEMENT
CALL RADIUSCSMPrSXrSYrRAD,DFDRC)
DRCDF=1.0/(DFDRC + 0.0001)
FIND DIRECTION OF YIELD NORMAL
DFC1>=CXJ2 + XJ1*<S<2>+SC3))/XJ3
DF<2>=<XJ~ + XJ1*CSC1) + S<3>) - SMP*SC1>*SC3))/XJ3
DFC3>=CXJ2 + XJ1*CSC1) + SC2)> - SMP*S<1>* S C2))/XJ3
XN(1)>=DS1*DFC1> + DS2*DF<2> + DS3*DFC3)
XNCCI>=DRCDF*CD SC 1*DFC1> + DSC 2 *DFC2> + DSC3*DFC3>
CONTINUE
RETURN
END

**********SURFACE**********
SUBROUTINE TO EVALUATE YIELD FUNCTION
DIMENSION S(3)
S1=0.8164979*DXY(I,1)
SC1=0.8164979*DXYC(I,1)
S2=0.4082484*DXY(I,1) + 0.70711*DXYC(I,2)
SC2=0.4082484*DXYC(I,1) + 0.70711*DXYC(I,2)
S3=S2 - 1.41421*DXYC(I,2)
SC3=SC2 - 1.41421*DXYC(I,1)
XN(I)>=S1*DF(1) + DS2*DF(2) + DS3*DF(3)
XNC(I)>=DRCDF*(SC1*DF(1) + DSC2*DF(2) + DSC3*DF(3))
DO 10 I=1,6
CONTINUE
RETURN
END

SUBROUTINE TO FIND NORMAL TO YIELD SURFACE
DIMENSION S(3),SC,DXY,DXYC,DF,XN,XNC,SMP,DFDRC
FIND YIELD STATE
CALL SURFACE(S,X1,X2,X3,SMP)
FIND DIRECTION OF YIELD CENTER MOVEMENT
SY = 0.40825*(SC1 - SC2 - SC3)
SX = 0.70711*(SC2 - SC3)
SRL=SORT(SY*SY + SX*SX + 0.0000001)
SY=SY/SRL
SX=SX/SRL
FIND RATE OF YIELD CENTER MOVEMENT
CALL RADIUS(SMP,SX,SY,RAD,DFDRC)
DRCDF=1.0/(DFDRC + 0.0001)
FIND DIRECTION OF YIELD NORMAL
DF(1)=XJ2 + XJ1*(S(2)+S(3)) - SMP*S(2)*S(3))/XJ3
DF(2)=XJ2 + XJ1*S(1) + S(3)) - SMP*S(1)*S(3))/XJ3
DF(3)=XJ2 + XJ1*S(1) + S(2)) - SMP*S(1)*S(2))/XJ3
PUT INTO XYZ COORDINATES
DO 10 I=1,6
CONTINUE
RETURN
END

SUBROUTINE TO EVALUATE YIELD FUNCTION
DIMENSION S(3)
S1=0.8164979*DXY(I,1)
SC1=0.8164979*DXYC(I,1)
S2=0.4082484*DXY(I,1) + 0.70711*DXYC(I,2)
SC2=0.4082484*DXYC(I,1) + 0.70711*DXYC(I,2)
S3=S2 - 1.41421*DXYC(I,2)
SC3=SC2 - 1.41421*DXYC(I,1)
XN(I)>=S1*DF(1) + DS2*DF(2) + DS3*DF(3)
XNC(I)>=DRCDF*(SC1*DF(1) + DSC2*DF(2) + DSC3*DF(3))
SUBROUTINE TO INVERT F = G(R)

SUBROUTINE RADIUS(F,sx,sy,R,DFDRC)

ERR=0.00005

IF NEAR HYDROSTATIC STRESS, R=0

R=0.0

IF(F.LT.9.000001) GO TO 20

SINCE F COINCIDES WITH MOHR-COULOMB STATE

USE ARGYRIS INTERPOLATION FOR FIRST APPROXIMATION

FIRST FIND STRESS RATIO ASSOCIATED WITH F

F1=F-5.0

RATIO=0.25*(F1 + SQRT(F1^2 - 16.0))

AND RADIUS ASSOCIATED WITH R AT SX=0

R=1.4142*(RATIO-1.0)/(RATIO+2.0)

FIND FACTOR FOR SY=0

FAC=(RATIO + 2.0)/(2.0*RATIO + 1.0)

COSINE TERM FOR INTERPOLATION

CS=SY*(4.0*SY*SY-1.0)

IF CS NEAR 1.0, FIRST GUESS IS CORRECT

IF(CS.GT.0.999) GO TO 20

D=(1+FAC) - (1-FAC)*CS

GO AHEAD AND INTERPOLATE

R=2.0*FAC*R/D

IF ABS(CS) NEAR 1.0 INTERPOLATED VALUE CORRECT

IF(ABS(CS).GT.0.999) GO TO 20

TO IMPROVE SOLUTION SOLVE CUBIC

Y=A*X**3 + B*X**2 + C

FIND CONSTANTS A, B, AND C

A=F*0.816497*(0.16666667*SY*SY-0.5*SX*SX)*SY

B=0.866025 - 0.288675*F

C=0.19245*F-1.732051

USE NEWTONS METHOD TO ITERATE FOR ACCEPTABLE SOLUTION

DO 10 I=1,100

D1=R*(3.0*A*R + 2.0*B)

D2=R*R*(A*R + B) + C

COR=D2/D1

R=R-COR

IF(ABS(COR).LE.ERR) GO TO 20

CONTINUE

COMPUTE DERIVATIVE OF F W.R.T. R

DO 10 I=1,6

VP(I) = T(I,1) + T(I,2) + T(I,3)

VY(I)=T(I,1) + 2.0*T(I,2)

5720 CONTINUE

RETURN

END

SUBROUTINE TO FIND STRESS STATE W.R.T. ANISOTROPY

SUBROUTINE MOVEIT(IT,SIG0,SIGC,SIG,S,T,R,CX,CY,C1,GAM,DXY,DXYC)

DIMENSION SIG(6),SIGC(6),SIG(6),T(6,6)

DIMENSION VP(6),VY(6),VX(6),S(3)

DIMENSION DXY(6,2),DXYC(6,2),GAM(6)

FIND ORIENTATION OF X-Y-Z W.R.T. SIG0

DO 10 I=1,6

VF(I)=T(I,1) + T(I,2) + T(I,3)

VY(I)=T(I,1)

VX(I)=T(I,1) + 2.0*T(I,2)

5720 CONTINUE

RETURN

END

SUBROUTINE TO FIND STRESS STATE W.R.T. SIG

CALL PROD(SIG,SIG0,C1,RS,RO)

CALL PROD(SIGC,SIG0,CC,RC,RO)

CALL PROD(SIG,VY,CY,RS,RY)

CALL PROD(SIGC,VY,CYC,RC,RY)
CALL PROD(SIG*VX,CX*RS*RX)

CALL PROD(SIGC*VX,CX*RC*RX)

SX=1.581132*(CX/C1 - CX/C1)

SY=1.224749*(CY/C1 - CY/C1)

RSQR=SX*SX + SY*SY

R=SQRT(RSQR)

SZ=0.5773503

FIND Principal Stress Directions

S(1) = 0.816497*SY + SZ

S(2) = SZ - 0.408249*SY + 0.70711*SX

S(3) = S(2) - 1.41421*SX

IF(IT.EQ.1) RETURN

NORMAL DIRECTIONS

RSO=RO*F*:S*C1

RSO2=RSO*RSO

D1=1.2247449*RO/(RY*RSO2)

D2=1.581132*RO/(RX*RSO2)

RSOC=RO*RC*CC

RSOC2=RSOC*RSOC

D1C=-1.2247449*RO/CRY*RSOC2)

D2C=-1.581132*RO/CRX*RSOC2>

RCC=RC*CC

ARG=1.0I*CC*CC>-1.0

IF(ARG.LT.0.0> ARG=0.0

XMAG=CC*SORT(ARG + 0.0000001)/RC

XMAG=1.0/XMAG

DO 20 I=1,6

DXY(I,1)=D1*(RSO*UY(I) - RS*RY*CY*SIGO(I))

DXY(I,2)=D2*(RSO*UX(I) - RS*RX*CXC*SIGO(I))

DXYC(I,1)=D1C*(RSOC*UY(I) - RC*RY*CYC*SIGO(I))

DXYC(I,2)=D2C*(RSOC*UX(I) - RC*RX*CXC*SIGO(I))

GAM(I)=(SIGO(I)//RO - SIGC(I)/RCC)*XMAG

CONTINUE

RETURN

END

**********PROD**********

SUBROUTINE TO FIND COSINE BETWEEN TWO STRESS VECTORS

DIMENSION A(6),B(6)

SUM1=0.0

SUM2=0.0

SUM3=0.0

DO 10 I=1,3

J=I+3

SUM1=SUM1 + A(I)*A(I) + 2.0*A(I)*A(J)*A(J)

SUM2=SUM2 + B(I)*B(I) + 2.0*B(I)*B(J)*B(J)

SUM3=SUM3 + A(I)*B(I) + 2.0*A(I)*B(J)

CONTINUE

SUM1=SQRT(SUM1)

SUM2=SQRT(SUM2)

C=SUM3/(SUM1*SUM2)

RETURN

END

**********MULT**********

SUBROUTINE TO PERFORM A=B*X

DIMENSION A(M,N),X(N),B(M)

DO 10 I=1,M

SUM=0.0

DO 20 J=1,N

SUM=SUM + A(I,J)*X(J)

CONTINUE

RETURN

END
SUBROUTINE TO COMPUTE DIFFERENTIALS FOR DJ
SUBROUTINE DJ(XMOB,FS,XJ1,E,G1,G2,G3,SN)
COMMON /PROP/DMAX,EREF,AS1,AS2,THRESH,CE,TRO,XM,BC,EO,ROWE,DRAG,XKAP,ATM

SUBROUTINE TO COMPUTE LIMITING PRESSURE
SM IS HVORSLEV EQUIVALENT MEAN PRESSURE
SME=ATM*10**BC*EO-1)

COMMON /PROP/DMAX,EREF,AS1,AS2,THRESH,CE,TRO,XM,BC,EO,ROWE,DRAG,XKAP,ATM

COMPUTE PARAMETERS FOR LIMITING STRESS RATIO
FAC=XM*ALOG(XNE)
XNE1=EXP<FAC>
XNE2=1.5*(XNE1-1)/(XNE1+2)
RAT=EREF/EO
XNEL=RAT*RAT*ALOG10<XNE>
XNE=10**XNEL

RMAX=ROWE*DMAX
RM=1.5*(RMAX-1)/(RMAX+2)
XA=XM*XNE*1.0-XNE1)
XR=CE+TRO*XNE+XA
CE=CE/XNE+TRO
RMAX=XA*XC/XB
DENOM=9.0

FMAX=9.0*(6.0*RMAX)/(DENOM-1.0)

DFDNE=DFDR*DRDNE

COMPUTE MOBILIZATION
XMOB=(SN-9.0)/FMAX

IF(XMOB>0.0)

S=AS1*XMOB*(1.0-XMOB)
DS=XM+AS1
R00=IF(XMOB.GT.THRESH) AS1=AS2
DS=DS*DS

COMPUTE GRADIENTS
G1=FMAX*AS1/DS
G2=3.02585*BC*XNE*DFDNE*XMOB
G3=DFDNE*XMOB/XJ1E

RETURN
END

SUBROUTINE TO FILL ELASATIC MATRIX
SUBROUTINE ELASTO(SIG,D)
DIMENSION SIG(6),D(6,6)

COMMON /PROP/DMAX,EREF,AS1,AS2,THRESH,CE,TRO,XM,BC,EO,ROWE,DRAG,XKAP,ATM

DIMENSION SIG(6),D(6,6)

RETURN
END

FOR NOW ASSUME ISOTROPIC ELASTIC W/ PR=0.0
BULK MODULUS SPECIFIED
BULK=0.7675284

RETURN
END

E=3.0*BULK

RETURN
END

D14
SUBROUTINE TO FIND ELASTO PLASTIC STIFFNESS

SUBROUTINE EPSTIF(ISORDELASrSIGrSIGOrSIGCrFNrErDrXNrBULKrLOAD>

DIMENSION DELASC6)rSIGC6),SIG0<6)rSIGCC6)rDELC3)rSC3)

DIMENSION DC6r6>rDXYC6r2>rGAMC6) 7380

DIMENSION B1(6)rB2C6>

DIMENSION BETAC6>rXN<6)rXNC<6)rTC6r3),DFC3)rSCC3>

COMMON /PROP/DMAXrEREFrAS1rAS2rTHRESHrAErTROrXMrBCrEOrROWErDRAGrXKAPrATM

FIND ELASTIC PART

CALL ELASTOCSIGrD)

BULK=DC1,1>tDC2r2>tDC3r3>t2.0*CDC1r2)tDClr3>tDC2r3))

BULK=0.1111111111*BULK

FIND APPARENT STRESS DIRECTION

CALL CHAC<SIGOrS)

CALL COORDCSIGOrSrT)

CALL MOVEITClrSIGOrSIGOrSIGCrSCrTrRrSXrSYrSZrGAMrDXYrDXYC>

CALL MOVEITC2rSIGOrSIGCrSIGrSrTrRSrSXrSYrSZrGAMrDXYrDXYC>

XJ1=SIGC1) t SIGC2) t SIG<3>

FIND STRAIN INC, NORMAL VECTOR, AND DIFFERENTIALS DJ

CALL NORMCSrSC,DXYrDXYCrDFrXNrXNCrSMPrDRCDF>

CALL DJCXMOBrST,XJ1rEtG1rG2rG3rSMP>

CALL DIRECCSTrXJl,SMP,SXrSYrSZrSrDFrBETArT>

COMPUTE EFFECTS OF MOVEMENT OF YIELD CENTER

CALL PRODCXNCrGAMrFCrTlrT2>

FC=1.0- FC*Tl*T2

INCLUDE EFFECTS OF DJ/DXJl

XN<1>=XN<1> - FC*G3

XN<2>=XNC2> - FC*G3

XNC3>=XNC3) - FC*G3

CHECK FOR UNLOADING

LOAD=l

RETURN

CONTINUE

H=G1 - (1,0 t E>*G2*<BETAC1) t BETAC2> + BETAC3>)

XL=DC*X1*X2 ~ FC*H

CENTER

H=G1 - (1,0 t E>*G2*<BETAC1) t BETAC2> + BETAC3>)

XL=DC*X1*X2 ~ FC*H

OUTER PRODUCT AND COMPLETE DEP

DO 30 I=1r6

DO 30 J=1r6

DCirJ>=DCirJ) - B1CI>*B2CJ)/XL

RETURN

END

SUBROUTINE TO INCLUDE CONSTITUTIVE BEHAVIOR OF FLUID

SUBROUTINE WATERCD>

DIMENSION DC6r6)

COMMON /FLUID/BWrU

DO 10 I=1r3

DO 10 J=lr3

<10 , J ) = DC I ' J

RETURN

RESET

SUBROUTINE TO UPDATE HARDENING STATE

SUBROUTINE RESETCLOADrFTrFNrDSIGrTSIGrSIGrSIGOrSIGC>

DIMENSION TSIGC6)rSIGOrSIGOrSIGOrSIGOrSIGC)

DIMENSION TSIG(6)rSIG(6),SIG0(6)rSIG(6),S1(3),S3(3),T(6,3)

DIMENSION XS(6),XNC(6),SC(3)

DIMENSION DXYS(6,2),DXYC(6,2),GAM(6),DSIG(6)

COMMON /STATE/ ISOrErFC10>rSTRESSC6r10)

E1=E

CAN ONLY NEST 9 SURFACES

IF(ISO.GT.9) RETURN

CALL CHAC<SIG0,S1)

CALL COORD(SIG0,S1,T)
0110C IS A NEW SURFACE BEING FORMED
0130 IF (LOAD.NE.0) GO TO 100
0140   ISO=ISO + 1
0150   FT=F (ISO)
0160 DO 10 I=1,6
0170   SIGO(I)=SIGC(I)
0180   STRESS(I,ISO+1)=TSIG(I)
0190 10 SIGC(I)=TSIG(I)
0200 RETURN
0210C
0220 100 CONTINUE
0230C RETURN IF ISOTROPIC
0250 IF (ISO.EQ.1) GO TO 40
0260C CHECK TO SEE IF STRESS IS OUTSIDE ISOTROPIC BOUNDING SURFACE
0280C CALL MOVEIT (1, STRESS (1,1), STRESS (1,1), SIG, S, T, RS, SX, SY, SZ, GAM, DXY, DXYC)
0300 CALL SURFACE (S, XJ1, XJ2, XJ3, SMP)
0305 XI=SIG (1) SIGC (2) SIGC (3)
0310 CALL DJ (XJ1, XJ2, XJ3, EI, G1, G2, G3, SMP)
0330 IF (XMOB.LT.0.98) GO TO 50
0340 ISO=1
0350 DO 60 I=1,6
0360   SIGC(I)=1.0
0370 CONTINUE
0380 50 ISO=ISO
0390 CONTINUE
0400C CHECK IF STRESS IS OUTSIDE OF CURRENT BOUNDING SURFACE
0420C NIX=0
0430 N=ISO-1
0440 NISO=ISO
0450 DO 70 K=1,N
0460 CALL MOVEIT (1, STRESS (1,NISO-1), STRESS (1,ISO), SIG, S, T, RS, SX, SY, SZ, GAM, DXY, DXYC)
0480 CALL SURFACE (S, XJ1, XJ2, XJ3, SMP)
0490 IF (SMP.LT.F(NISO)) GO TO 70
0500 NIX=1
0510 FT=SMP
0520 DO 20 I=1,6
0530   SIGC(I)=STRESS(I,ISO+1)
0540 CONTINUE
0550 20 NISO=ISO-1
0560 IF (NIX.NE.0) RETURN
0570C MOVE CENTER OF CURRENT YIELD SURFACE
0590C CALL MOVEIT (1, SIGO, SIGC, SC, T, RS, CX, CY, CZ, GAM, DXY, DXYC)
0610C CALL MOVEIT (2, SIGO, SIGC, TSIG, S, T, R, SX, SY, SZ, GAM, DXY, DXYC)
0620C CALL NORM (S, SC, DXY, DXYC, EI, G1, G2, G3, SMP, DRCDF)
0630C CALL PROD (SIGO, SIGC, CA, TO, TC)
0640C CALL PROD (XNC, GAM, DDP, T1, T2)
0650C CALL PROD (XN, BSIG, DFP, R1, R2)
0660C DJ1=DFP*R1*R2/(1.0-DRP*T1*T2)
0670C DR = DJ1*DRCDF
0680C IF (D16) RETURN
0690C
0700 DO 30 I=1,6
0710 30 SIGC(I)=STRESS (I,ISO+1) + DR*GAM (I)
0720C CONTINUE
0730 40 CONTINUE
0740C FIND FINAL VALUE OF F
0760C CALL MOVEIT (1, SIGO, SIGC, SIG, S, T, RS, CX, CY, CZ, GAM, DXY, DXYC)
0770C CALL SURFACE (S, XJ1, XJ2, XJ3, FN)
0780C RETURN
0800 END

D16
APPENDIX E: LABORATORY TEST DATA

The analyzed laboratory test data are presented for each specimen, designated as:

Vicksburg Silty Clay - CL
Buckshot Clay - S
Yazoo Clay - SY

The test results for each specimen also contain the initial specimen characteristics, test consolidation history, pretest conditions, and test description. Parameters shown in table headings correspond to those presented in paragraphs 48 and 49.
SPECIMEN CHARACTERISTICS:

<table>
<thead>
<tr>
<th>DIAMETER</th>
<th>W/C</th>
<th>VOID RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.390</td>
<td>24.3</td>
<td>0.539</td>
</tr>
<tr>
<td>HEIGHT</td>
<td>GS</td>
<td>MAT TYPE</td>
</tr>
<tr>
<td>3.499</td>
<td>2.69</td>
<td>CL</td>
</tr>
<tr>
<td>PE</td>
<td>160.0</td>
<td>CC = 0.164</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CR = 0.020</td>
</tr>
</tbody>
</table>

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.539 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUND TO A VOID RATIO OF 0.539 BY EFFECTIVE PRESSURE OF 125.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

<table>
<thead>
<tr>
<th>DIAMETER</th>
<th>BACK PRESSURE</th>
<th>VOID RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.345</td>
<td>50.0</td>
<td>0.539</td>
</tr>
<tr>
<td>HEIGHT</td>
<td>CELL PRESSURE</td>
<td>B VALUE</td>
</tr>
<tr>
<td>3.431</td>
<td>175.0</td>
<td>0.950</td>
</tr>
</tbody>
</table>

TEST DESCRIPTION:

DRAINED R TEST
FINAL W/C = 18.0%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>VOID RATIO</th>
<th>AXIAL STRAIN %</th>
<th>PORE PRESSURE PSI</th>
<th>Q/2 PSI</th>
<th>P PSI</th>
<th>Q/P</th>
<th>Q/2FE</th>
<th>P/FE</th>
<th>QCV/2PE</th>
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</thead>
<tbody>
<tr>
<td>0.538</td>
<td>0.6</td>
<td>0.6</td>
<td>0.15</td>
<td>0.26</td>
<td>0.53</td>
<td>0.030</td>
<td>0.320</td>
<td></td>
</tr>
<tr>
<td>0.534</td>
<td>0.3</td>
<td>0.3</td>
<td>0.46</td>
<td>0.199</td>
<td>0.863</td>
<td>0.516</td>
<td>0.113</td>
<td></td>
</tr>
<tr>
<td>0.538</td>
<td>0.7</td>
<td>0.7</td>
<td>0.03</td>
<td>0.226</td>
<td>0.849</td>
<td>0.433</td>
<td>0.365</td>
<td></td>
</tr>
<tr>
<td>0.527</td>
<td>1.23</td>
<td>1.23</td>
<td>0.62</td>
<td>0.256</td>
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<td>0.379</td>
<td>0.526</td>
<td></td>
</tr>
<tr>
<td>0.523</td>
<td>1.79</td>
<td>1.79</td>
<td>0.90</td>
<td>0.338</td>
<td>0.754</td>
<td>0.489</td>
<td>0.522</td>
<td></td>
</tr>
<tr>
<td>0.519</td>
<td>2.18</td>
<td>2.18</td>
<td>0.36</td>
<td>0.154</td>
<td>0.856</td>
<td>0.100</td>
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<tr>
<td>0.515</td>
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<td>2.66</td>
<td>0.46</td>
<td>0.199</td>
<td>0.863</td>
<td>0.516</td>
<td>0.113</td>
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<td>3.35</td>
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<td>0.509</td>
<td>0.373</td>
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<tr>
<td>0.506</td>
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<td>4.20</td>
<td>0.97</td>
<td>0.352</td>
<td>0.723</td>
<td>0.457</td>
<td>0.611</td>
<td></td>
</tr>
<tr>
<td>0.500</td>
<td>5.33</td>
<td>5.33</td>
<td>1.06</td>
<td>0.369</td>
<td>0.966</td>
<td>0.431</td>
<td>0.753</td>
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<tr>
<td>0.496</td>
<td>6.38</td>
<td>6.38</td>
<td>1.11</td>
<td>0.378</td>
<td>0.677</td>
<td>0.447</td>
<td>0.716</td>
<td></td>
</tr>
<tr>
<td>0.490</td>
<td>8.01</td>
<td>8.01</td>
<td>1.20</td>
<td>0.392</td>
<td>0.655</td>
<td>0.415</td>
<td>0.882</td>
<td></td>
</tr>
<tr>
<td>0.486</td>
<td>11.47</td>
<td>11.47</td>
<td>1.29</td>
<td>0.416</td>
<td>0.646</td>
<td>0.416</td>
<td>0.971</td>
<td></td>
</tr>
<tr>
<td>0.485</td>
<td>14.16</td>
<td>14.16</td>
<td>1.33</td>
<td>0.432</td>
<td>0.652</td>
<td>0.433</td>
<td>0.972</td>
<td></td>
</tr>
</tbody>
</table>

** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN: CL-3

SPECIMEN CHARACTERISTICS:

<table>
<thead>
<tr>
<th>DIAMETER = 1.388</th>
<th>W/C = 24.2</th>
<th>VOID RATIO = 0.674</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEIGHT = 3.150</td>
<td>GS = 2.69</td>
<td>Mat Type = CL</td>
</tr>
<tr>
<td>PE = 104.0 AT VOID RATIO = 0.564</td>
<td>CC = 0.152</td>
<td>CR = 0.020</td>
</tr>
</tbody>
</table>

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.534 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 0.564 BY EFFECTIVE PRESSURE OF 10.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

<table>
<thead>
<tr>
<th>DIAMETER = 1.347</th>
<th>BACK PRESSURE = 50.0</th>
<th>VOID RATIO = 0.563</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEIGHT = 3.126</td>
<td>CELL PRESSURE = 60.0</td>
<td>B VALUE = 0.560</td>
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</table>

TEST DESCRIPTION:

DRAINED R TEST
FINAL W/C = 22.4%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>VOID RATIO</th>
<th>AXIAL STRAIN %</th>
<th>PORE PRESSURE PSI</th>
<th>G/2 PSI</th>
<th>P PSI</th>
<th>Q/P</th>
<th>Q/2FE PSI</th>
<th>P/FE</th>
<th>DCC/2FE</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.564</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
<td>0.097</td>
<td>-0.068</td>
<td>0.929</td>
<td></td>
</tr>
<tr>
<td>0.564</td>
<td>0.06</td>
<td>0.0</td>
<td>1.4</td>
<td>10.9</td>
<td>0.26</td>
<td>0.014</td>
<td>0.105</td>
<td>-0.537</td>
<td>0.508</td>
</tr>
<tr>
<td>0.564</td>
<td>0.10</td>
<td>0.0</td>
<td>2.8</td>
<td>11.9</td>
<td>0.47</td>
<td>0.027</td>
<td>0.114</td>
<td>0.049</td>
<td>1.076</td>
</tr>
<tr>
<td>0.564</td>
<td>0.22</td>
<td>0.0</td>
<td>3.5</td>
<td>12.3</td>
<td>0.57</td>
<td>0.033</td>
<td>0.118</td>
<td>-0.153</td>
<td>0.541</td>
</tr>
<tr>
<td>0.563</td>
<td>0.32</td>
<td>0.0</td>
<td>5.4</td>
<td>13.6</td>
<td>0.80</td>
<td>0.051</td>
<td>0.129</td>
<td>-0.176</td>
<td>0.415</td>
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<tr>
<td>0.562</td>
<td>0.48</td>
<td>0.0</td>
<td>8.7</td>
<td>15.8</td>
<td>1.10</td>
<td>0.082</td>
<td>0.148</td>
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<td>0.962</td>
</tr>
<tr>
<td>0.562</td>
<td>0.55</td>
<td>0.0</td>
<td>9.8</td>
<td>16.5</td>
<td>1.18</td>
<td>0.091</td>
<td>0.154</td>
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<td>1.076</td>
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<tr>
<td>0.562</td>
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<td>12.2</td>
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<td>0.115</td>
<td>0.170</td>
<td>0.013</td>
<td>1.281</td>
</tr>
<tr>
<td>0.566</td>
<td>1.29</td>
<td>0.0</td>
<td>16.0</td>
<td>20.6</td>
<td>1.55</td>
<td>0.158</td>
<td>0.204</td>
<td>0.093</td>
<td>1.488</td>
</tr>
<tr>
<td>0.568</td>
<td>1.51</td>
<td>0.0</td>
<td>16.5</td>
<td>21.0</td>
<td>1.57</td>
<td>0.167</td>
<td>0.213</td>
<td>0.108</td>
<td>1.478</td>
</tr>
<tr>
<td>0.573</td>
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<td>0.0</td>
<td>17.5</td>
<td>21.7</td>
<td>1.62</td>
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<td>0.238</td>
<td>0.147</td>
<td>1.418</td>
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<tr>
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<td>17.8</td>
<td>21.9</td>
<td>1.63</td>
<td>0.216</td>
<td>0.266</td>
<td>0.167</td>
<td>1.418</td>
</tr>
<tr>
<td>0.587</td>
<td>4.51</td>
<td>0.0</td>
<td>17.8</td>
<td>21.9</td>
<td>1.63</td>
<td>0.242</td>
<td>0.297</td>
<td>0.199</td>
<td>1.320</td>
</tr>
<tr>
<td>0.596</td>
<td>6.58</td>
<td>0.0</td>
<td>17.5</td>
<td>21.6</td>
<td>1.61</td>
<td>0.275</td>
<td>0.340</td>
<td>0.258</td>
<td>1.280</td>
</tr>
<tr>
<td>0.605</td>
<td>8.68</td>
<td>0.0</td>
<td>16.6</td>
<td>21.0</td>
<td>1.57</td>
<td>0.295</td>
<td>0.374</td>
<td>0.269</td>
<td>1.173</td>
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<tr>
<td>0.611</td>
<td>11.48</td>
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<td>15.9</td>
<td>20.6</td>
<td>1.54</td>
<td>0.314</td>
<td>0.406</td>
<td>0.296</td>
<td>1.160</td>
</tr>
<tr>
<td>0.619</td>
<td>16.26</td>
<td>0.0</td>
<td>15.4</td>
<td>20.3</td>
<td>1.52</td>
<td>0.340</td>
<td>0.448</td>
<td>0.332</td>
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</table>

** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN: CL-4

SPECIMEN CHARACTERISTICS:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>W/C</th>
<th>Void Ratio</th>
<th>MAT Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.395</td>
<td>24.0</td>
<td>0.664</td>
<td>CL</td>
</tr>
<tr>
<td>Height</td>
<td>2.69</td>
<td>0.534</td>
<td>CC = 0.140 CR = 0.020</td>
</tr>
<tr>
<td>PE = 149.0 AT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEIGHT = 3.475</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.526 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 0.534 BY EFFECTIVE PRESSURE OF 50.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Back Pressure</th>
<th>Void Ratio</th>
<th>B Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.358</td>
<td>50.0</td>
<td>0.534</td>
<td>0.950</td>
</tr>
<tr>
<td>Height</td>
<td>3.382</td>
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<td></td>
</tr>
</tbody>
</table>

TEST DESCRIPTION:

DRAINED R TEST
FINAL W/C = 19.9%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>Void Ratio</th>
<th>Axial Strain %</th>
<th>Pore Pressure PSI</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>OCV/2PE</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.534</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>50.0</td>
<td>0.0</td>
<td>0.336</td>
<td>-0.409</td>
<td>0.548</td>
<td></td>
</tr>
<tr>
<td>0.534</td>
<td>0.03</td>
<td>0.0</td>
<td>1.8</td>
<td>51.2</td>
<td>0.07</td>
<td>0.012</td>
<td>0.343</td>
<td>-0.180</td>
<td>0.796</td>
</tr>
<tr>
<td>0.533</td>
<td>0.21</td>
<td>0.0</td>
<td>8.1</td>
<td>55.4</td>
<td>0.29</td>
<td>0.054</td>
<td>0.368</td>
<td>-0.306</td>
<td>0.733</td>
</tr>
<tr>
<td>0.532</td>
<td>0.53</td>
<td>0.0</td>
<td>23.6</td>
<td>65.7</td>
<td>0.72</td>
<td>0.154</td>
<td>0.428</td>
<td>-0.017</td>
<td>0.859</td>
</tr>
<tr>
<td>0.531</td>
<td>0.98</td>
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<td>34.8</td>
<td>73.2</td>
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<td>0.223</td>
<td>0.469</td>
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</tr>
<tr>
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<td>44.7</td>
<td>79.8</td>
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<td>0.488</td>
<td>0.274</td>
<td>0.881</td>
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<tr>
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<td>50.1</td>
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<td>0.492</td>
<td>0.301</td>
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<tr>
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<td>86.4</td>
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<td>0.313</td>
<td>0.496</td>
<td>0.307</td>
<td>0.964</td>
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<td>6.76</td>
<td>0.0</td>
<td>58.5</td>
<td>89.0</td>
<td>1.31</td>
<td>0.333</td>
<td>0.506</td>
<td>0.327</td>
<td>0.973</td>
</tr>
<tr>
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<td>9.84</td>
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<td>63.4</td>
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<td>0.364</td>
<td>0.530</td>
<td>0.341</td>
<td>1.067</td>
</tr>
<tr>
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<td>11.29</td>
<td>0.0</td>
<td>64.6</td>
<td>93.1</td>
<td>1.39</td>
<td>0.381</td>
<td>0.548</td>
<td>0.360</td>
<td>1.062</td>
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<tr>
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<td>15.17</td>
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<td>66.2</td>
<td>94.1</td>
<td>1.41</td>
<td>0.420</td>
<td>0.598</td>
<td>0.389</td>
<td>1.107</td>
</tr>
<tr>
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<td>0.455</td>
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</table>

** MEASUREMENTS IN INCHES AND PressURES IN PSI
SPECIMEN: CL-5

SPECIMEN CHARACTERISTICS:

DIAMETER = 1.387
HEIGHT = 3.115
PE = 165.0 AT VOID RATIO = 0.505

W/C = 23.6
6S = 2.69
MAT TYPE = CL
CC = 0.145
CR = 0.020

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.504 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 0.505 BY EFFECTIVE PRESSURE OF 125.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

DIAMETER = 1.338
HEIGHT = 3.075
BACK PRESSURE = 50.0
CELL PRESSURE = 175.0
VOID RATIO = 0.505
B VALUE = 0.940

TEST DESCRIPTION:

UNDRAINED R TEST
FINAL W/C = 19.7%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>VOID RATIO</th>
<th>AXIAL STRAIN %</th>
<th>FORE PRESSURE PSI</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>G/2PE</th>
<th>P/PE</th>
<th>QCV/2PE</th>
<th>D</th>
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** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN: CL-6

SPECIMEN CHARACTERISTICS:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>W/C</th>
<th>Void Ratio</th>
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<tbody>
<tr>
<td>1.395</td>
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<table>
<thead>
<tr>
<th>Height</th>
<th>GS</th>
<th>Mat Type</th>
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<tbody>
<tr>
<td>3.501</td>
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<table>
<thead>
<tr>
<th>P.E</th>
<th>Void Ratio</th>
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<tbody>
<tr>
<td>137.0</td>
<td>0.519</td>
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<table>
<thead>
<tr>
<th>C.C</th>
<th>C.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.145</td>
<td>0.020</td>
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TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.509 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 0.519 BY EFFECTIVE PRESSURE OF 50.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Back Pressure</th>
<th>Void Ratio</th>
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<tbody>
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<td>1.350</td>
<td>50.0</td>
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<table>
<thead>
<tr>
<th>Height</th>
<th>Cell Pressure</th>
<th>B Value</th>
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<tbody>
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<td>3.459</td>
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TEST DESCRIPTION:

UNDRAINED R TEST
FINAL W/C = 20.0%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>Void Ratio</th>
<th>Axial Strain</th>
<th>Pore Pressure</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>QCV/2 PE</th>
<th>D</th>
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<tbody>
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<td>0.208</td>
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<td>1.05</td>
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<td>0.255</td>
<td>0.466</td>
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** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
**SPECIMEN: CL-7**

---

**SPECIMEN CHARACTERISTICS:**

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<th>Void Ratio</th>
<th>Mat Type</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>3.500</td>
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<td></td>
</tr>
</tbody>
</table>

**FE = 106.0 AT VOID RATIO = 0.527**

**CR = 0.020**

---

**TEST CONSOLIDATION HISTORY:**

Consolidated to a Void Ratio of 0.500 by an Effective Pressure of 150.0
Consolidated to a Void Ratio of 0.527 by Effective Pressure of 10.0

---

**PRE-TEST CONDITIONS:**

<table>
<thead>
<tr>
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<th>Height</th>
<th>Back Pressure</th>
<th>Void Ratio</th>
</tr>
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</tbody>
</table>

**CELL PRESSURE = 60.0**

**B VALUE = 0.990**

---

**TEST DESCRIPTION:**

Undrained R Test

**FINAL W/C = 20.1%**

---

**ANALYZED TEST DATA**

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<tr>
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<th>Axial Strain %</th>
<th>Pore Pressure</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2FE</th>
<th>P/PE</th>
<th>QCv/2FE</th>
<th>D</th>
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****Measurements in Inches and Pressures in PSI**
**SPECIMEN: CL-8**

**SPECIMEN CHARACTERISTICS:**

- **Diameter:** 1.394
- **W/C:** 23.6
- **Void Ratio:** 0.698
- **Height:** 3.504
- **GS:** 2.69
- **Material Type:** CL
- **PE:** 170.0 at Void Ratio = 0.504
- **CC:** 0.141
- **CR:** 0.020

**TEST CONSOLIDATION HISTORY:**

Consolidated to a Void Ratio of 0.504 by an effective pressure of 150.0. Rebounded to a Void Ratio of 0.504 by effective pressure of 150.0. Consolidated with an effective stress ratio 1.00 at 50.0 back pressure.

**PRE-TEST CONDITIONS:**

- **Diameter:** 1.346
- **Back Pressure:** 50.0
- **Void Ratio:** 0.504
- **Height:** 3.434
- **Cell Pressure:** 200.0
- **B Value:** 0.960

**TEST DESCRIPTION:**

DRAINED R TEST

**FINAL W/C:** 18.6%

**ANALYZED TEST DATA**

<table>
<thead>
<tr>
<th>Void Ratio</th>
<th>Axial Strain</th>
<th>Fore Pressure</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>QCV/2FE</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.504</td>
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<td>0.0</td>
<td>0.0</td>
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**Measurements in inches and pressures in PSI**
SPECIMEN: CL-9

SPECIMEN CHARACTERISTICS:

| DIAMETER   | 1.402 |
| W/C       | 23.5 |
| HEIGHT    | 3.510 |
| Void Ratio | 0.645 |
| PE        | 49.0 |
| Void Ratio | 0.581 |

TEST CONSOLIDATION HISTORY:

Consolidated to a void ratio of 0.508 by an effective pressure of 1.50.0
Rebounded to a void ratio of 0.581 by effective pressure of 1.0
Consolidated with an effective stress ratio 1.00 at 50.0 back pressure

PRE-TEST CONDITIONS:

| DIAMETER | 1.373 |
| HEIGHT   | 3.510 |
| Back Pressure | 80.0 |
| Void Ratio   | 0.581 |
| Cell Pressure | 81.0 |
| B Value     | 1.000 |

TEST DESCRIPTION:

UNDRAINED R TEST
Final W/C = 22.0%

| VOID RATIO | 0.581 | 0.581 | 0.581 | 0.581 | 0.581 | 0.581 | 0.581 | 0.581 | 0.581 | 0.581 |
| AXIAL STRAIN | 0.0 | 0.14 | 0.29 | 0.43 | 1.15 | 1.43 | 2.16 | 3.63 | 5.11 | 7.39 |
| %         | 0.0 | 0.50 | 0.40 | 0.30 | 0.50 | 0.90 | 2.10 | 4.70 | 7.00 | 9.60 |
| PORE PRESSURE | 0.0 | 1.0 | 1.4 | 1.7 | 3.3 | 4.7 | 6.6 | 10.4 | 13.5 | 16.8 |
| PSI       | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0/2       | 0.0 | 0.8 | 1.2 | 1.5 | 3.3 | 4.2 | 6.6 | 10.4 | 13.5 | 16.8 |
| P         | 1.0 | 1.1 | 1.4 | 1.7 | 3.7 | 4.7 | 7.5 | 12.6 | 17.0 | 21.3 |
| Q/P       | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Q/2PE     | 0.020 | 0.017 | 0.024 | 0.031 | 0.068 | 0.087 | 0.135 | 0.213 | 0.275 | 0.344 |
| P/PE      | 0.014 | 0.022 | 0.028 | 0.035 | 0.076 | 0.097 | 0.153 | 0.250 | 0.346 | 0.445 |
| OCV/2PE   | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN: 5-16

SPECIMEN CHARACTERISTICS:

\[
\begin{align*}
\text{DIAMETER} & = 1.400 \\
\text{W/C} & = 32.6 \\
\text{HEIGHT} & = 3.500 \\
\text{GS} & = 2.69 \\
\text{PE} & = 153.0 \text{ AT VOID RATIO} = 0.588 \\
\text{CC} & = 0.339 \\
\text{CR} & = 0.050
\end{align*}
\]

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.585 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 0.588 BY EFFECTIVE PRESSURE OF 125.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

\[
\begin{align*}
\text{DIAMETER} & = 1.314 \\
\text{HEIGHT} & = 3.337 \\
\text{BACK PRESSURE} & = 50.0 \\
\text{VOID RATIO} & = 0.588 \\
\text{CELL PRESSURE} & = 175.0 \\
\text{B VALUE} & = 0.980
\end{align*}
\]

TEST DESCRIPTION:

DRAINED R TEST
FINAL W/C = 20.4%

ANALYZED TEST DATA

| VOID RATIO | AXIAL STRAIN % | PORE PRESSURE | \(O/2\) | \(P\) | \(Q/2\) | \(P/PE\) | \(Q/CU/2PE\) | D |
|------------|----------------|--------------|--------|------|--------|-----------|--------------|
| 0.587      | 0              | 0.0          | 0.1250 | 0.0  | 0.587  | 0.811     | 0.786        |
| 0.587      | 0.03           | 0.0          | 0.1304 | 0.12 | 0.590  | 0.845     | 0.734        |
| 0.586      | 0.15           | 0.0          | 0.1365 | 0.29 | 0.594  | 0.894     | 0.816        |
| 0.585      | 0.30           | 0.0          | 0.1417 | 0.35 | 0.661  | 0.910     | 0.404        |
| 0.585      | 0.39           | 0.0          | 0.1439 | 0.39 | 0.683  | 0.920     | 0.115        |
| 0.584      | 0.63           | 0.0          | 0.1455 | 0.42 | 0.722  | 0.924     | 0.736        |
| 0.581      | 1.27           | 0.0          | 0.1483 | 0.47 | 0.722  | 0.924     | 0.702        |
| 0.577      | 2.03           | 0.0          | 0.1505 | 0.51 | 0.733  | 0.916     | 0.720        |
| 0.568      | 4.22           | 0.0          | 0.1554 | 0.59 | 0.740  | 0.888     | 0.735        |
| 0.563      | 5.67           | 0.0          | 0.1577 | 0.62 | 0.747  | 0.867     | 0.767        |
| 0.555      | 8.08           | 0.0          | 0.1607 | 0.67 | 0.747  | 0.837     | 0.809        |
| 0.551      | 9.49           | 0.0          | 0.1622 | 0.69 | 0.747  | 0.822     | 0.826        |
| 0.547      | 10.81          | 0.0          | 0.1634 | 0.70 | 0.747  | 0.809     | 0.847        |
| 0.543      | 12.84          | 0.0          | 0.1644 | 0.72 | 0.747  | 0.790     | 0.846        |
| 0.536      | 16.62          | 0.0          | 0.1664 | 0.75 | 0.747  | 0.763     | 0.945        |
| 0.534      | 19.06          | 0.0          | 0.1670 | 0.75 | 0.747  | 0.755     | 0.937        |
| 0.532      | 21.33          | 0.0          | 0.1670 | 0.75 | 0.747  | 0.744     | 0.941        |
| 0.529      | 24.08          | 0.0          | 0.1670 | 0.75 | 0.747  | 0.731     | 0.942        |

** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN CHARACTERISTICS:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (W/C)</td>
<td>1.385</td>
</tr>
<tr>
<td>Height (GS)</td>
<td>3.500</td>
</tr>
<tr>
<td>Void Ratio (PE)</td>
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</tr>
<tr>
<td>CC</td>
<td>0.346</td>
</tr>
<tr>
<td>CR</td>
<td>0.050</td>
</tr>
</tbody>
</table>

TEST CONSOLIDATION HISTORY:

Consolidated to a Void Ratio of 0.584 by an effective pressure of 150.0
Rebounded to a Void Ratio of 0.684 by effective pressure of 10.0
Consolidated with an effective stress ratio of 1.00 at 50.0 back pressure.

PRE-TEST CONDITIONS:

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<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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<tr>
<td>Back Pressure</td>
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<tr>
<td>Void Ratio (PE)</td>
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<tr>
<td>B Value</td>
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TEST DESCRIPTION:

Drained Test
Final W/C = 27.0%

ANALYZED TEST DATA

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<th>Axial Strain %</th>
<th>Pore Pressure PSI</th>
<th>Q/2 PSI</th>
<th>P PSI</th>
<th>Q/P</th>
<th>Q/2PE PSI</th>
<th>P/PE</th>
<th>QCV/2PE PSI</th>
<th>D</th>
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</thead>
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<td>0</td>
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<td></td>
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** Measurements in inches and pressures in psi.
SPECIMEN: S-18

SPECIMEN CHARACTERISTICS:

DIAMETER = 1.417
W/C = 32.2
HEIGHT = 3.500
GS = 2.69
PE = 82.0 AT VOID RATIO = 0.688

VOID RATIO = 0.893
MATERIAL TYPE = CH
CC = 0.321
CR = 0.050

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 0.593 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 0.688 BY EFFECTIVE PRESSURE OF 10.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

DIAMETER = 1.362
HEIGHT = 3.377
BACK PRESSURE = 50.0
CELL PRESSURE = 60.0
VOID RATIO = 0.688
B VALUE = 0.970

TEST DESCRIPTION:

UNDRAINED R TEST
FINAL W/C = 26.2%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>VOID RATIO</th>
<th>AXIAL STRAIN %</th>
<th>PORE PRESSURE</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>QCV/2PE</th>
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** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN: S-19

SPECIMEN CHARACTERISTICS:

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<tr>
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<tbody>
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<td>Diameter (in.)</td>
<td>1.392</td>
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<tr>
<td>Height (in.)</td>
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<tr>
<td>W/C</td>
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</tr>
<tr>
<td>Void Ratio</td>
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<tr>
<td>Mat Type</td>
<td>CH</td>
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TEST CONSOLIDATION HISTORY:

Consolidated to a Void Ratio of 0.566 by an effective pressure of 150.0
Rebounded to a Void Ratio of 0.560 by effective pressure of 125.0
Consolidated with an effective stress ratio 1.00 at 50.0 back pressure

PRE-TEST CONDITIONS:

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<tr>
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<td>Void Ratio</td>
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<td>Cell Pressure</td>
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TEST DESCRIPTION:

Undrained R Test
Final W/C = 22.2

ANALYZED TEST DATA

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<th>PORE PRESSURE PSI</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>QCV/2PE</th>
<th>D</th>
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<td>45.5</td>
<td>89.5</td>
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<td>0.563</td>
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<td>46.0</td>
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<td>0.556</td>
<td>0.294</td>
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<td>87.9</td>
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<td>0.553</td>
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<td>87.8</td>
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<td>0.290</td>
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** Measurements in inches and pressures in PSI
SPECIMEN: S-20

SPECIMEN CHARACTERISTICS:

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<th>Nat Type</th>
<th>CC</th>
<th>CR</th>
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<td></td>
<td></td>
<td></td>
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TEST CONSOLIDATION HISTORY:

Consolidated to a Void Ratio of 0.603 by an effective pressure of 150.0, rebounded to a Void Ratio of 0.623 by effective pressure of 65.0, consolidated with an effective stress ratio 1.00 at 50.0 back pressure.

PRE-TEST CONDITIONS:

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<th>Void Ratio</th>
<th>R Value</th>
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</tr>
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TEST DESCRIPTION:

Undrained R Test
Final W/C = 23.6%

ANALYZED TEST DATA

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<th>Void Ratio</th>
<th>Axial Strain</th>
<th>Pore Pressure %</th>
<th>P</th>
<th>Q/F</th>
<th>Q/2FE</th>
<th>P/PE</th>
<th>GCU/2PE</th>
<th>D</th>
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<td>0.0</td>
<td>0.650</td>
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<td>0.011</td>
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<td>0.04</td>
<td>0.011</td>
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<td>0.535</td>
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** Measurements in Inches and Pressures in PSI
SPECIMEN: SY-4

SPECIMEN CHARACTERISTICS:

| DIAMETER = 1.381 | W/C = 73.7 | VOID RATIO = 2.061 |
| HEIGHT = 3.500  | GS = 2.75  | MAT TYPE = H |
| PE = 145.0 AT VOID RATIO = 1.060 | CC = 0.760 | CR = 0.190 |

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 1.020 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUNDED TO A VOID RATIO OF 1.060 BY EFFECTIVE PRESSURE OF 125.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

| DIAMETER = 1.200 | BACK PRESSURE = 50.0 | VOID RATIO = 1.060 |
| HEIGHT = 3.080  | CELL PRESSURE = 175.0 | B VALUE = 0.950 |

TEST DESCRIPTION:

DRAINED R TEST
FINAL W/C = 36.0%

ANALYZED TEST DATA

<table>
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<tr>
<th>VOID RATIO</th>
<th>AXIAL STRAIN %</th>
<th>PORE PRESSURE PSI</th>
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<th>P</th>
<th>Q/P</th>
<th>Q/2PSI</th>
<th>P/FE</th>
<th>G/CV/2PE</th>
<th>D</th>
</tr>
</thead>
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** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN: SY-5

SPECIMEN CHARACTERISTICS:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>W/C</th>
<th>Void Ratio</th>
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<tbody>
<tr>
<td>1.400</td>
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<tr>
<td>Height</td>
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TEST CONSOLIDATION HISTORY:

Consolidated to a Void Ratio of 1.040 by an effective pressure of 150.0.
Rebounded to a Void Ratio of 1.370 by effective pressure of 10.0.
Consolidated with an effective stress ratio 1.00 at 50.0 back pressure.

PRE-TEST CONDITIONS:

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<th>Void Ratio</th>
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TEST DESCRIPTION:

DRAINED R TEST
Final W/C = 49.6%

ANALYZED TEST DATA

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<th>Pore Pressure</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>QC/2PE</th>
<th>D</th>
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** Measurements in inches and pressures in psi.
SPECIMEN CHARACTERISTICS:

<table>
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<tr>
<th>DIAMETER</th>
<th>W/C</th>
<th>PE</th>
<th>AT VOID RATIO</th>
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<td>1.380</td>
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HEIGHT = 3.500

PE = 145.0 AT VOID RATIO = 1.060

VOID RATIO = 2.020

MAT TYPE = CH

CC = 0.750

CR = 0.190

TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 1.050 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUND TO A VOID RATIO OF 1.060 BY EFFECTIVE PRESSURE OF 125.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

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<th>BACK PRESSURE</th>
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HEIGHT = 3.040

CELL PRESSURE = 175.0

B VALUE = 1.000

TEST DESCRIPTION:

UNDRAINED R TEST

FINAL W/C = 38.9%

ANALYZED TEST DATA

<table>
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<th>VOID RATIO</th>
<th>AXIAL STRAIN</th>
<th>PORE PRESSURE</th>
<th>Q/2</th>
<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>OCV/2PE</th>
<th>D</th>
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** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
SPECIMEN:  SY-7

SPECIMEN CHARACTERISTICS:

<table>
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<th>Void Ratio = 1.970</th>
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</tr>
<tr>
<td>PE = 100.0 AT VOID RATIO = 1.390</td>
<td>CC = 0.720</td>
<td>CR = 0.150</td>
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TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 1.053 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUND TO A VOID RATIO OF 1.390 BY EFFECTIVE PRESSURE OF 10.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

| Diameter = 1.299 | Back Pressure = 50.0 | Void Ratio = 1.390 |
| Height = 3.261  | Cell Pressure = 60.0  | B Value = 1.000   |

TEST DESCRIPTION:

UNDRAINED R TEST
FINAL W/C = 50.4%

ANALYZED TEST DATA

<table>
<thead>
<tr>
<th>Void Ratio</th>
<th>Axial Strain %</th>
<th>Pore Pressure PSI</th>
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<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
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** MEASUREMENTS IN INCHES AND PRESSURES IN PSI
**SPECIMEN CHARACTERISTICS:**

- **DIAMETER**: 1.398
- **W/C**: 73.3
- **VOID RATIO**: 2.020
- **HEIGHT**: 3.500
- **GS**: 2.75
- **MAT TYPE**: CH
- **PE**: 80.0 at Void Ratio = 1.250
- **CC**: 0.750
- **CR**: 0.190

**TEST CONSOLIDATION HISTORY:**

- Consolidated to a Void Ratio of 1.080 by an effective pressure of 150.0
- Rebounded to a Void Ratio of 1.250 by effective pressure of 30.0
- Consolidated with an effective stress ratio 1.00 at 50.0 back pressure

**PRE-TEST CONDITIONS:**

- **DIAMETER**: 1.267
- **BACK PRESSURE**: 75.0
- **VOID RATIO**: 1.250
- **HEIGHT**: 3.170
- **CELL PRESSURE**: 105.0
- **B VALUE**: 0.970

**TEST DESCRIPTION:**

- **UNDRAINED R TEST**
- **FINAL W/C**: 44.9%

**ANALYZED TEST DATA**

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<th>P</th>
<th>Q/P</th>
<th>Q/2PE</th>
<th>P/PE</th>
<th>QC/2PE</th>
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**MEASUREMENTS IN INCHES AND Pressures IN PSI**

E19
SPECIMEN: SY-9

SPECIMEN CHARACTERISTICS:

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<th>Diameter</th>
<th>W/C</th>
<th>Void Ratio</th>
<th>Height</th>
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TEST CONSOLIDATION HISTORY:

CONSOLIDATED TO A VOID RATIO OF 1.100 BY AN EFFECTIVE PRESSURE OF 150.0
REBOUND TO A VOID RATIO OF 1.700 BY EFFECTIVE PRESSURE OF 1.0
CONSOLIDATED WITH AN EFFECTIVE STRESS RATIO 1.00 AT 50.0 BACK PRESSURE

PRE-TEST CONDITIONS:

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TEST DESCRIPTION:

UNDRAINED R TEST
FINAL W/C = 60.5%

ANALYZED TEST DATA

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** Measurements in Inches and Pressures in PSI