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Plastic Deformation of Thick-Walled Snow- Ice Cylinders under Hydrostatic Pressure

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PREFACE

This is one of a series of reports of work performed on USA SIPRE task 022.01.039 (formerly 022.01.008) Mechanics of deformation of snow and ice. The purpose of the investigation reported here is to simulate, on a very small scale, the deformation of a tunnel in glacier ice.

The investigation was carried out by Dr. Jellinek, contract scientist. The experiments were performed by Mr. G. M. Walker, research assistant. The work was performed for the Basic Research Branch, Mr. James A. Bender, chief.

This report has been reviewed and approved for publication by the Office of the Chief of Engineers.

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CONTENTS

	Page
Preface -----	ii
Summary -----	iv
Experimental -----	1
Materials -----	1
Apparatus -----	1
Experimental results -----	1
Deformation under constant hydrostatic pressure -----	1
Deformation as a function of hydrostatic pressure -----	1
Deformation as a function of temperature at constant pressure -----	1
Discussion -----	1
References -----	7

ILLUSTRATIONS

Figure	Page
1. $\log V_0/V_t$ as a function of time for a snow-ice cylinder under 7.03 kg/cm ² hydrostatic pressure -----	3
2. $\log V_0/V_t$ as a function of time for a snow-ice cylinder under 14.06 kg/cm ² hydrostatic pressure ----	3
3. $\log V_0/V_t$ as a function of time for a snow-ice cylinder under increasing hydrostatic pressure -----	3
4. $\log V_0/V_t$ as a function of time and temperature, snow-ice cylinder -----	3
5. Rate constants k as a function of circumferential stress -----	6
6. Rate constants k as a function of $\sinh \sigma_c/\sigma_0$ -----	6
7. Evaluation of energy activation from experiments shown in Figure 4 -----	6

SUMMARY

Experiments have been carried out on the plastic deformation of thick-walled snow-ice cylinders under hydrostatic pressure as a function of pressure and temperature. At constant circumferential stress and temperature the natural strain rate of closure is a constant. This constant varies with the circumferential stress according to a sine function and is exponentially dependent on temperature with an energy of activation of 14.1 kcal/mole at an average circumferential stress of 3.1 kg/cm². The mechanism of the plastic flow is in agreement with a mechanism proposed previously, that is, flow between grain boundaries takes place.

PLASTIC DEFORMATION OF THICK-WALLED SNOW-ICE CYLINDERS UNDER HYDROSTATIC PRESSURE

by

H. H. G. Jellinek

Experiments on the deformation of thick-walled snow-ice cylinders, closed at one end, were initiated by Higashi (1959). The present report deals with a continuation of this work considering only the purely plastic range of snow-ice. The rate of closure of a cylindrical hole was studied as a function of circumferential stress and temperature.

EXPERIMENTAL

Materials.

Snow-ice was prepared by sieving a large amount of snow into distilled water and letting it freeze in flexible polyethylene containers at -5°C . The snow-ice was fine-grained with grain sizes ranging from about 1 to 2 cm diam.

Apparatus.

The apparatus was similar to that used by Higashi (1959); however some modifications were made. The electronic pressure gage was replaced by a direct reading air pressure gage, the funnel for filling the apparatus was replaced by a cylindrical container, whose contents could be emptied into the apparatus by applying gas pressure. Cast steel tubing was substituted for the copper tubing and reducing valves were installed (McMaster and Carr valves: Carbon steel gage 600 lb, steam pressure 4000 WOG). The base of the hydrostatic pressure tank was provided with a cylindrical copper wall forming a cup, which facilitated the mounting of the specimens. These were prepared on the lathe at -10°C and mounted as described in Higashi's report. All snow-ice cylinders had similar dimensions within small limits: height 9.1 cm, outer diameter 2.6 cm, diameter of cavity 0.484 cm, length of cavity 7.73 cm.

EXPERIMENTAL RESULTS

Deformation under constant hydrostatic pressure (-4.5°C)

Figure 1 shows $\log V_0/V_t$ plotted against time for a snow-ice cylinder kept under constant hydrostatic pressure of 7.03 kg/cm^2 . V_0 is the volume of the cylindrical cavity at time $t=0$ and V_t its volume at time t . Figure 2 shows a deformation curve for a snow-ice cylinder under a constant hydrostatic pressure of 14.06 kg/cm^2 .

Deformation as a function of hydrostatic pressure (-4.5°C)

Figure 3 gives $\log V_0/V_t$ values as a function of time for two snow-ice samples. The hydrostatic pressure was increased during the course of the experiments from 2.11 to 7.03 kg/cm^2 .

Deformation as a function of temperature at constant pressure (10.55 kg/cm^2)

A snow-ice cylinder was kept under a constant hydrostatic pressure of 10.55 kg/cm^2 and the temperature was lowered from -5.5°C to -20°C . Temperatures were kept approximately constant for 100 to 200 hr at each of the following temperatures: -5.5°C , -10.0°C , -16.0°C , and -20.0°C . The entire experiment lasted 800 hr. Figure 4 shows the $\log V_0/V_t$ values plotted against time for time intervals during which the temperatures were constant.

DISCUSSION

The deformation curve in Figure 2 shows the usual three stages of creep: a primary, secondary, and tertiary creep. This corresponds to the behavior of most materials. However, there is a difference in the present case as the deformation for the thick-walled ice cylinders does not proceed to destruction but to complete

closure of the cavity; this means to a complete cessation of deformation. This is indicated in Figure 2, where the creep eventually slows down as the cavity becomes small.

This report is concerned with the purely plastic flow component, the secondary or steady state; this component is not recoverable after the pressure is released but does not include the tertiary creep, which is a different process and seems to be connected with the destruction of the snow-ice structure.

The rate of decrease in volume of the cavity for the minimum creep may be assumed to obey an expression as follows:

$$-\frac{dV_t}{dt} = kV_t \quad \text{or} \quad -\frac{dV_t}{V_t dt} = k \quad (1)$$

where V_t is the volume of the cavity at time t and k is a rate constant; $dV_t/V_t dt$ is the natural volume strain rate. The rate constant k depends on the structure and history of the particular sample being investigated and on temperature and circumferential stress σ_c . The latter is responsible for the closure of the cavity and is given by:

$$\sigma_c = \frac{p}{\ln \frac{b}{r_t}} \quad (2)$$

where p is the hydrostatic pressure, b the outside radius of the sample, and r_t the radius of the cavity at time t . Vertical compression will be negligible because of the thick walls of the cylinders. Integration of eq 1 leads to

$$2.303 \log \frac{V_0}{V_t} = kt. \quad (3)$$

Hence, a straight line should be obtained for the steady state creep by plotting $\log V_0/V_t$ against t provided the temperature and circumferential stress remain constant and provided that the hydrostatic pressure be kept relatively small so that the tertiary stage does not interfere appreciably. Then k can be derived from the slope of the straight line multiplied by 2.303. Eq 1 can also be expressed in terms of the radius r_t of the cavity.

$$-\frac{d(\pi r_t^2 h)}{\pi r_t^2 h dt} = k \quad \text{or} \quad -\frac{dr_t}{r_t dt} = \frac{k}{2} \quad (4)$$

where h is the height of the cavity. By integration of (4) one obtains,

$$\ln \frac{r_0}{r_t} = \frac{kt}{2}. \quad (5)$$

Figure 1 shows that eq 1 holds over a time interval of 500 hr; actually the tertiary creep already interferes at larger time intervals - otherwise the slope should decrease rather than increase because the circumferential stress σ_c decreases with decreasing cavity radius at constant hydrostatic pressure.

Figure 2 shows that for higher stresses, the region of steady state creep becomes rather short. Figure 3, which represents a study of the deformation at various hydrostatic pressures, shows that for relatively short time intervals

Figure 1. $\text{Log } V_0/V_t$ as a function of time for a snow-ice cylinder under 7.03 kg/cm^2 hydrostatic pressure at -4.5°C .

Figure 2. $\text{Log } V_0/V_t$ as a function of time for a snow-ice cylinder under 14.06 kg/cm^2 hydrostatic pressure.

Figure 3. $\text{Log } V_0/V_t$ as a function of time for a snow-ice cylinder under increasing hydrostatic pressure at -4.5°C .

Figure 4. $\text{Log } V_0/V_t$ as a function of time for a snow-ice cylinder. Temp decreased from -5.5 to -20.0°C . Temp indicated kept constant for 100 to 200 hr.

straight lines are obtained at various pressures. The rate constants and the corresponding hydrostatic pressures are:

p (kg/cm ²):	2.11	3.17	4.78	7.03
k (hr ⁻¹):	2.60 x 10 ⁻³	3.74 x 10 ⁻³	5.25 x 10 ⁻³	8.61 x 10 ⁻³

The k values are plotted against the circumferential stress σ_c in Figure 5. The k values plotted against $\sinh \sigma_c / \sigma_0$ - where σ_0 is a constant - give a good straight line (Fig. 6) indicating that the rate constants can be expressed by

$$k = A \sinh \frac{\sigma_c}{\sigma_0} = A \sinh \frac{P}{\sigma_0 \ln(b/r_t)} \quad (6)$$

For the calculation of σ_c , average radii for each constant pressure interval were employed.

For the particular sample in question we have:

$$k = 4.15 \times 10^{-3} \sinh \frac{\sigma_c}{2.0} \text{ hr}^{-1}.$$

For low stresses eq 6 reduces to

$$k = \frac{A}{\sigma_0} \sigma_c \quad (7)$$

In other words, Newtonian flow results. Hence, for low stresses eq 4 can be written as follows:

$$\dot{\epsilon}_r = - \frac{dr_t}{r_t dt} = \frac{A}{2\sigma_0} \sigma_c \quad (8)$$

In order to evaluate the viscosity of the sample, the strain rate $\dot{\epsilon}_r$ has to be converted into a shear strain rate $\dot{\gamma}$ and the stress σ_c into a shear stress τ_c . The conversions are given according to Nye (1953) by the following expression:

$$\tau_c = \frac{1}{\sqrt{3}} \sigma_c \quad \text{and} \quad \dot{\gamma}_r = \sqrt{3} \dot{\epsilon}_r \quad (9)$$

Generally, we have for Newtonian shear rate

$$\dot{\gamma}_r = \frac{\tau_c}{\eta}$$

where η is the viscosity coefficient. Further,

$$\dot{\epsilon}_r \sqrt{3} = \frac{\sigma_c}{\eta \sqrt{3}} \quad (10)$$

or

$$\dot{\epsilon}_r = \frac{\sigma_c}{3\eta}$$

Comparing eq 8 and 10 gives

$$\eta = \frac{2\sigma_0}{3A}.$$

For our experiment (Fig. 3), a viscosity coefficient $\eta = 1.1 \times 10^{11}$ poises is obtained. This value may be compared with values of about 10^{14} poises found from experiments on tensile elongation of snow-ice cylinders (Jellinek and Brill, 1956).

The dependence of strain rate on temperature is usually given by an exponential function. In the present case we have for the natural volume strain rate:

$$-\frac{dV}{V} \frac{dt}{dt} = k = A' \exp(-E/RT) = A \sinh \frac{\sigma_c}{\sigma_0} \quad (11)$$

σ_c should be constant. A and probably σ_0 will be functions of the absolute temperature T , E is the energy of activation, A' a constant, and R the ideal gas constant. For low stresses, eq 11 is given by

$$A' \exp(-E/RT) = \frac{2\sigma_c}{3\eta}$$

or

$$\eta = \frac{2\sigma_c}{3A'} \exp(-E/RT) \quad (12)$$

The experiment shown in Figure 4, where the rate constant k was determined as a function of temperature, was evaluated according to eq 11 by plotting $\log k$ against $1/T$ (Fig. 7). The energy of activation derived from the straight line (least squares) amounts to 14.1 kcal/mole. A value previously found (Jellinek and Brill, 1956), which lies in the Newtonian region, amounted to 16.1 kcal/mole. There is general evidence (Glen, 1955) that the energy of activation increases with stress. The condition that σ_c is constant over the temperature range is not quite fulfilled. The radius of the σ_c cavity changed during the experiment from $r_0 = 0.242$ cm to $r_t = 0.206$ cm at $t = 800$ hr, covering a range of circumferential stresses from 3.52 to 2.78 kg/cm². In order to avoid this source of error, A and σ_0 will have to be determined for each temperature, then k can be calculated for constant σ_c . It should be mentioned that Nakaya (1958) found, by a vibrational method, energies of activation varying from 12.7 to 18.7 kcal/mole for various kinds of ice. Raraty and Tabor (1958) report an energy of activation of 12 kcal/mole. Glen's value of 32 kcal/mole was discussed in a previous paper (Jellinek and Brill, 1956). Kauzmann (1941) elaborated a theory of plastic flow which is based on the theory of absolute chemical reaction rates. This theory leads to a rate expression for the plastic shear rate:

$$\dot{\gamma} = N \frac{2\lambda}{L} \frac{kT}{h} \exp(\Delta S/R) \exp(-\Delta H/RT) \sinh \left(\frac{A l q \tau}{kT} \right) \quad (13)$$

For small stresses this reduces again to Newtonian flow:

$$\dot{\gamma} = \frac{N 2 \lambda A l q \tau}{L h} \exp(\Delta S/R) \exp(-\Delta H/RT)$$

or as $\dot{\gamma} = \frac{\tau}{\eta}$ one obtains

$$\frac{1}{\eta} = \frac{N 2 \lambda A l q}{L h} \exp(\Delta S/R) \exp(-\Delta H/RT) \quad \text{and} \quad \eta = B \exp(+\Delta H/RT) \quad (14)$$

$\dot{\gamma}$ is the shear strain rate, N the number of flow units in the grain boundary regions of the snow-ice per unit volume of the specimen, λ/L the shear strain for each unit of flow, k Boltzmann's constant, T the absolute temperature, h Planck's constant, R the ideal gas constant, ΔS the entropy and ΔH the energy of activation for the flow process, A the cross-sectional area of the flow unit, l the length of the flow unit, q a constant and τ the shear stress applied to the specimen.

Figure 5. Rate constants k as a function of circumferential stress σ_c (see Fig. 3).

Figure 6. Rate constants k as a function of $\sinh \sigma_c / \sigma_0$ (see Fig. 5).

Figure 7. Evaluation of energy activation from experiments shown in Figure 4.

In general, a straight line should be obtained by plotting

$$\ln \frac{\dot{\gamma}}{T \sinh \frac{A \tau}{kT}} \quad \text{against} \quad \frac{1}{T}.$$

However to do this at least two determinations at two different stresses would have to be carried out at each temperature in order to evaluate the expression under the hyperbolic sine.

The experiments agree well with the interpretation of the plastic flow process given in a previous paper (Jellinek and Brill, 1956) representing flow between grain boundaries.

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