Research Report 64

JULY, 1960

Temperature Distribution of an Idealized Ice Cap

by Chi Tien

U. S. ARMY SNOW ICE AND PERMAFROST
RESEARCH ESTABLISHMENT
Corps of Engineers
Wilmette, Illinois

PREFACE

This is one of a series of reports of work performed on USA SIPRE Project 022.01.036, Thermal properties of snow and ice. The purpose of this investigation is to determine the temperature distribution of an ice cap under a set of simplified assumptions. The investigation was carried out during June - August 1958 by Dr. Chi Tien for USA SIPRE's Basic Research Branch, Mr. J. A. Bender, chief.

This report has been reviewed and approved for publication by the Office of the Chief of Engineers.

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Colonel, Corps of Engineers
Director

Manuscript received 5 August 1959.

Department of the Army Project 8-66-02-400.

AD Accession No

U. S. Army Snow Ice and Permafrost Research Establishment, Corps of Engineers, Wilmette, Ill.
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The problem is analyzed mathematically, assuming that the icecap has constant physical properties, that it grows at a constant rate from an initial zero thickness, is internally static, and subject to a linear climatic change and a constant geothermal heat flux. The results are compared with direct measurements in Greenland and in the Ross Ice Shelf. The problem is treated as a Stefan-type problem, and the solution is obtained by the principle of superposition. The results indicate that the temperature at the base of the icecap increases with time, and eventually would reach the melting point of ice. Under such conditions the icecap is not resting on permafrost as suggested earlier. The predicted results agree fairly well with direct measurements for greater depths.

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2. Glaciers--Growth--Mathematical analysis

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CONTENTS

ILLUSTRATIONS

SUMMARY

The distribution of temperature throughout an idealized ice cap is studied. The idealized ice cap is considered as one with a constant growth rate, without internal movement, subject to a linear climatic change and to a constant geothermal influx. The problem is treated as a Stefan-type problem and the solution is obtained by the principle of superposition.

The results indicate that the temperature at the base of the ice cap rises with time and eventually would reach the melting point of the ice. Under such conditions, it is concluded that the ice cap is not resting on a permafrost base as suggested by previous investigation.

TEMPERATURE DISTRIBUTION OF AN IDEALIZED ICE CAP

by

Chi Tien

Introduction

The problem of temperature distribution through an ice cap has been a subject of study for many years. Previous works (Benfield, 1951; Brockamp, 1951) include the study of the effect of geothermal influx as well as internal friction on the temperature of an ice sheet. Robin (1955) deals with the case of a stable ice cap and obtains analytical expressions for both temperature and ice movement. Although Robin's results are quite good, their application seems to be somewhat restricted since: (1) the model of his "stable ice sheet" is unlikely to approximate physical reality and (2) the effect of climatic change has been neglected. This problem has also been studied by Wexler (1959) with the ice cap approximated as a semi-infinite solid.

The object of our work is to obtain the temperature distribution throughout the ice cap using a different model which takes the climatic effect into consideration. This work consists of (a) development of mathematical equations describing the physical problem, (b) solution of the differential equation, (c) discussion of results, and (d) numerical computation and comparison with evidence.

Mathematical development

The following conditions are assumed for an idealized ice cap:

(a) The ice has constant physical properties.
(b) The ice cap grows at a constant rate and the initial thickness is zero.
(c) A constant heat influx, geothermal in nature, is provided at the base of the ice cap.
(d) Surface temperature changes linearly with time.

The one-dimensional heat transfer equation can be used to describe the physical situation.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial T}{\partial t} \qquad (1)$$

for $0 < x < X_s(t)$ and $\phi(x) < t$

with α^2 = thermal diffusity of ice

T = temperature

t = time

x = distance from base

$X_s(t)$ = thickness of ice cap at any time.

The base of the ice cap is taken as the zero coordinate and $X_s(t)$ refers to the upper boundary. If the accumulation is constant as assumed, $X_s(t) = Ut$, where U is the linear rate of growth. $\phi(x)$ is the inverse function of $X_s(t)$ and in this case is simply x/U.

The boundary conditions are:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = -q/k \qquad (2)$$

where q is the geothermal influx and k is the thermal conductivity of ice. At

$$x = X_s(t), \qquad T = T_0 + a\cdot t \qquad (3)$$

where T_0 is the temperature at $t=0$ and a is the annual increase in temperature.

Let $\theta = T - T_0$, eq 1 becomes

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial\theta}{\partial t} . \qquad (4)$$

The free-boundary value problem is solved by the principle of superposition as outlined in the following section.

Method of solution

Assuming that $\theta = \theta_1 + \theta_2$, it is obvious that both θ_1 and θ_2 would satisfy a differential equation of the form of eq 4.

For θ_1, we impose the following conditions:

$$\frac{\partial^2\theta_1}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial\theta_1}{\partial t} \qquad (5)$$

for $0 < x < Ut$

$$\text{and } \left.\frac{\partial\theta_1}{\partial x}\right|_{x=0} = -(q/k + a/U) \qquad (6)$$

$$\theta_1 = 0 \quad \text{at} \quad t < \phi(x) \qquad (7)$$

$$\theta_1 = 0 \quad \text{at} \quad x = X_s(t) \qquad (8)$$

$$\frac{\partial\theta_1}{\partial x} = 0 \quad \text{at} \quad t \leq \phi(x) \text{ and } x > 0 . \qquad (9)$$

The condition set by eq 9 is purely arbitrary; its sole purpose is to make the solution possible. It implies that no thermal energy escapes from the ice cap. This is rather unlikely during the earlier period of growth; however, the situation might be approximately correct after the ice cap reaches a certain thickness.

For the second part, we have

$$\frac{\partial^2\theta_2}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial\theta_2}{\partial t} \qquad (10)$$

and

$$\theta_2 = 0 \quad \text{at} \quad x = 0 \qquad (11)$$

$$\theta_2 = a \cdot t \quad \text{at} \quad x = X_s(t) . \qquad (12)$$

It can be verified very easily that the combined boundary conditions of θ_1 and θ_2 are equivalent to those postulated for θ.

The solution of eq 5 is obtained through the use of Laplace transforms. The transform of $\theta_1(x, t)$ is defined as (Evans, et al., 1950):

$$\overline{\theta}_1(x, s) = \int_{\phi(x)}^{\infty} \theta_1(x, t) \exp(-st)\, dt .$$

The transform of eq 5 becomes

$$\frac{\partial^2}{\partial x^2}\overline{\theta}_1(x, s) = \frac{s}{\alpha^2}\overline{\theta}_1(x, s) . \qquad (13)$$

The solution of eq 13 is of the form:

$$\overline{\theta}_1(x, s) = K_1(s) \exp\left(-\frac{\sqrt{s}}{\alpha}x\right) + K_2(s) \exp\left(\frac{\sqrt{s}}{\alpha}x\right) . \qquad (14)$$

Applying the boundary conditions as expressed in eq 6 and 8 in their transformed forms, we have:

$$\theta_1(x, s) = \left(\frac{q}{k} + \frac{a}{U}\right) \alpha \frac{\sinh \frac{\sqrt{s}}{\alpha}[X_s(t) - x]}{s[\sqrt{s} \cosh \frac{\sqrt{s}}{\alpha} X_s(t)]} . \qquad (15)$$

By the use of the inversion integral theorem $\theta_1(x,t)$ is given by (Churchill, 1944):

$$\theta_1(x,t) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \left(\frac{q}{k} + \frac{a}{U}\right) \alpha \frac{\sinh \frac{\sqrt{\lambda}}{\alpha}[X_s(t) - x]}{\lambda[\sqrt{\lambda} \cosh \frac{\sqrt{\lambda}}{\alpha} X_s(t)]} \exp(\lambda t)\, d\lambda . \tag{16}$$

The integrand is analytic everywhere over the half plane of the complex domain except at points $\lambda = 0$, and $\lambda = -\frac{\alpha^2}{X_s^2(t)}(n + \frac{1}{2})^2 \pi^2$; $n = 0, 1, 2, 3 \ldots\ldots$ The residue of $\lambda = 0$ is simply

$$\left(\frac{q}{k} + \frac{a}{U}\right) \alpha \frac{1}{\alpha}[X_s(t) - x] = \left(\frac{q}{k} + \frac{a}{U}\right)[X_s(t) - x]. \tag{17}$$

The residues at the other points are given by the expression

$$\left(\frac{q}{k} + \frac{a}{U}\right) \alpha \left.\frac{\sinh \frac{\sqrt{\lambda}}{\alpha}[X_s(t) - x]}{\frac{d}{d\lambda}[\lambda\sqrt{\lambda} \cosh \frac{\sqrt{\lambda}}{\alpha} X_s(t)]}\right|_{\lambda = \frac{-\alpha^2}{X_s^2(t)}(n + \frac{1}{2})^2\pi^2} . \tag{18}$$

The final expression results in the form:

$$\theta_1(x,t) = \left(\frac{q}{k} + \frac{a}{U}\right)\left\{X_s(t) - x + 8\frac{X_s(t)}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(2n+1)^2}\right.$$
$$\left.\sin\left[\frac{(2n+1)}{2}\frac{X_s(t) - x}{X_s(t)}\pi\right] \exp\left[-\left(\frac{2n+1}{2}\right)^2 \frac{\alpha^2\pi^2}{X_s^2(t)} t\right]\right\}. \tag{19}$$

The solution of θ_2 is simply

$$\theta_2 = \frac{a}{U}x . \tag{20}$$

Combining θ_1 and θ_2, the temperature distribution throughout the ice cap is found to be

$$T = T_0 + \frac{a}{U}x + \left(\frac{a}{U} + \frac{q}{k}\right)\left\{X_s(t) - x + 8\frac{X_s(t)}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(2n+1)^2}\right.$$
$$\left.\sin\left[\frac{(2n+1)}{2}\frac{X_s(t) - x}{X_s(t)}\pi\right] \exp\left[-\left(\frac{2n+1}{2}\right)^2 \frac{\alpha^2\pi^2}{X_s^2(t)} t\right]\right\}, \tag{21}$$

and $X_s(t) = Ut$ for $x \leqslant X_s(t)$.

Discussion of results

Base temperature. One question of interest is to find the temperature history at the lower boundary of the ice cap. Because of the effect of geothermal influx and, to a much less degree, of climatic change, it is reasonable to believe that the base temperature would increase with time and eventually reach the melting point of ice corresponding to existing conditions,* and that any further introduction of geothermal heat will cause the base ice to melt. The temperature distribution given by eq 21 will not be applicable when the base ice is melting, as both boundaries of the ice cap are moving; different boundary conditions should be incorporated in eq 1 to obtain the proper solution. From eq 21 the expression for base temperature, T_b, is given by

* Considering the decrease in melting point due to hydrostatic pressure.

$$T_b \cong T_0 + \left(\frac{q}{k} + \frac{a}{U}\right)[X_s(t)]\left\{1 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left[-\left(\frac{2n+1}{2}\right)^2\frac{\alpha^2\pi^2}{X_s^2(t)}t\right]\right\}$$

$$= T_0 + \left(\frac{q}{k} + \frac{a}{U}\right)[X_s(t)]\left\{1 - \frac{8}{\pi^2}\left[\exp\left(-\frac{1}{4}\frac{\alpha^2\pi^2}{X_s^2(t)}t\right) + \frac{1}{9}\exp\left(-\frac{9}{4}\frac{\alpha^2\pi^2}{X_s^2(t)}t\right) + \ldots\ldots\right]\right\}. \quad (22)$$

$$T_b \cong T_0 + X_s(t)\left[\frac{q}{k} + \frac{a}{U}\right]\left[1 - \frac{8}{\pi^2}\exp\left(-\frac{1}{4}\frac{\alpha^2\pi^2}{X_s^2(t)}t\right)\right]. \quad (22a)$$

Figure 1 gives the estimated base temperature history of the ice cap at Byrd Station, Antarctica.

Minimum temperature. From eq 21, it can be shown that for a fairly large value of $X_s(t)$, the temperature gradient is positive at the upper part and negative near the base. This naturally leads to the conclusion that a minimum temperature exists somewhere in between. By differentiating eq 21 with respect to x, for a given thickness, the location of the minimum temperature points are given by:

$$\frac{a}{U} + \left(\frac{q}{k} + \frac{a}{U}\right)\left\{-1 + 8\frac{X_s(t)}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{(2n+1)^2}\frac{(2n+1)}{2}\frac{(-1)}{X_s(t)}\pi\right.$$

$$\left.\cos\left[\frac{(2n+1)}{2}\frac{X_s(t)-x}{X_s(t)}\pi\right]\exp\left[-\left(\frac{2n+1}{2}\right)^2\frac{\alpha^2\pi^2}{X_s^2(t)}t\right]\right\} = 0 \quad (23)$$

or

$$\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos\left[\left(n+\frac{1}{2}\right)\frac{X_s(t)-x}{X_s(t)}\pi\right]\exp\left[-\left(\frac{2n+1}{2}\right)^2\frac{\alpha^2\pi^2}{X_s^2(t)}t\right]$$

$$= 1 - \frac{(a/U)}{(a/U)+(q/k)} = \frac{(q/k)}{(a/U)+(q/k)}. \quad (23a)$$

Since

$$\cos\left[\left(n+\frac{1}{2}\right)\frac{X_s(t)-x}{X_s(t)}\pi\right] = (-1)^n\sin\left(n+\frac{1}{2}\right)\frac{x\pi}{X_s(t)},$$

by substitution we have

$$\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^{2n}}{(2n+1)}\sin\left(n+\frac{1}{2}\right)\frac{x\pi}{X_s(t)}\exp\left[-\left(\frac{2n+1}{2}\right)^2\frac{\alpha^2\pi^2}{X_s^2(t)}t\right]$$

$$= \frac{(q/k)}{(a/U)+(q/k)}. \quad (24)$$

The expression may or may not yield a realistic solution, depending on the magnitude of the exponential term. This can be shown very easily, for since $X_s(t) = Ut$, the exponent equals $-\left(\frac{2n+1}{2}\right)^2\frac{\alpha^2\pi^2}{U^2t}$. For a certain physical situation with all the constants a, U, given at the beginning stage of growth, it is possible that $\exp\left[-\left(\frac{2n+1}{2}\right)^2 \frac{a^2\pi^2}{U^2 t}\right]$ is too small to satisfy the condition as given by eq 24. However, when the minimum temperature does exist, the following expression gives an approximate location for it:

$$\frac{x}{X_s(t)} = \frac{2}{\pi} \sin^{-1}\left[\frac{\pi}{4} \exp\left(\frac{1}{4} \frac{a^2\pi^2}{X_s^2(t)} t\right)\right]. \tag{25}$$

This expression is obtained by taking only the first term of the infinite series and also noting $q/k >> a/U$ which makes the right side approximately equal to unity.

Effect of rate of growth: From eq 21 it is obvious that a slower rate of growth would tend to give a higher temperature for the upper part of the ice cap; however, the increase due to the second term will be more than compensated by the first term (negative) of the finite series.

Effect of geothermal influx. A large value of geothermal heat q, tends to increase the ice cap temperature as observed from eq 21. But this effect will diminish with decrease in depth for a given thickness (or increase in x since depth is equal to $X_s(t) - x$). This is demonstrated in Figure 2.

Comparison with observed results

Recent field work conducted by USA SIPRE (Langway, 1958) in Greenland gives limited information on temperature distribution throughout an ice cap. The temperature readings were taken to a depth of 1400 ft out of a total thickness of approximately 7000 ft. The other pertinent information is estimated as follows:

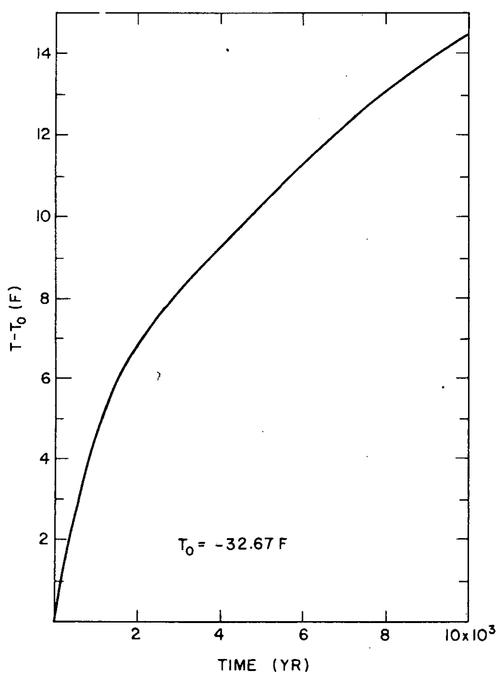

Figure 1. Estimated base temperature vs time at Byrd Station.

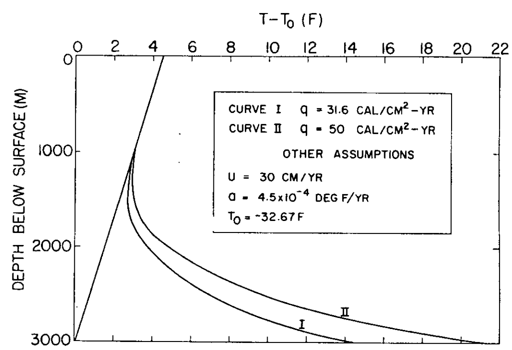

Figure 2. Effect of geothermal influx on temperature distribution.

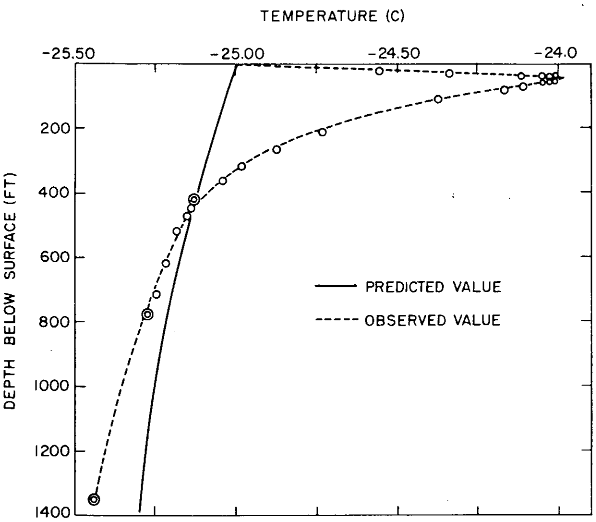

Figure 3. Comparison between predicted and observed temperature - depth results for ice cap at Site 2, Greenland.

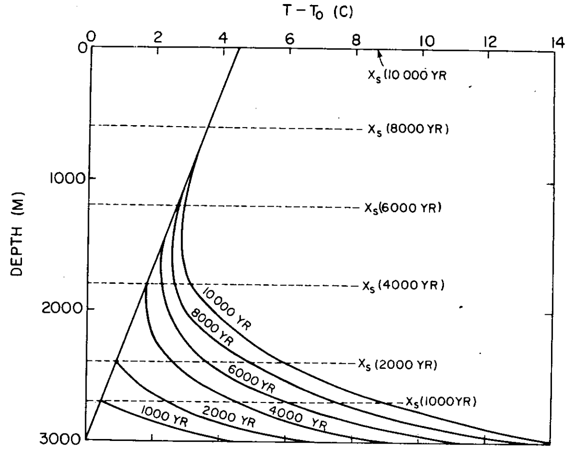

Figure 4. Estimated temperature distribution at Byrd Station, Antarctica.

Rate of growth: 41.5 g of water equivalent per yr.

Mean annual temperature (1957): -25C.

Geothermal influx: 3.16×10^5 cal /m²-yr.

Temperature increases at a rate of 4.5C/10,000 yr.

With this information, temperature distribution is computed from eq 21 and compared with the observed readings (Figure 3). The deviation for the first several hundred feet is understandable. In this analysis it is assumed that we have an annual ice accumulation and all the physical properties throughout the ice cap are constant, while in reality growth takes place as snowfall with a consolidation process in subsequent years. The physical properties vary appreciably in the first several hundred feet, as reported by recent investigation (Langway, 1958).

For greater depths, predicted results agree with the observed ones fairly well, especially if we make certain adjustments of the value of the surface temperature (for example, if we make T = -25.2C instead of T = -25C). However, it should be pointed out that agreement or lack of it does not prove or disprove the validity of this work since it is only the lower part of the ice cap that is closely approximated by the model.

This hypothesis is at least partially verified by the results obtained from a deep hole drilled in the Ross Ice Shelf, Antarctica (Ragle, et al., 1960). The temperature readings were taken down to a depth of 300 m. The other information is estimated as follows:

$U = 30$ cm/yr

$a = 4.5 \times 10^{-5}$ C/yr

$q = 3.16 \times 10^5$ cal./m²-yr.

Complete temperature profiles at 1000, 2000, 4000, 6000, 8000, and 10,000 yr after the formation of the ice cap have been estimated and presented in Figure 4 and Table I. The temperature profile at 10,000 yr (corresponding to the present time) is compared with observed results as shown in Figure 5.

Table I. Estimated temperature distribution of the ice cap, Ross Ice Shelf, Antarctica. $T - T_0$ (C)

$\frac{X_s(t)-x_s}{X_s(t)}$	$X_s(t)=3000$ m $t=10,000$ yr	$X_s(t)=2400$ m $t=8000$ yr	$X_s(t)=1800$ m $t=6000$ yr	$X_s(t)=1200$ m $t=4000$ yr	$X_s(t)=600$ m $t=2000$ yr	$X_s(t)=300$ m $t=1000$ yr
0.1	4.05	3.24	-	-	1.07	0.735
0.2	3.6	2.88	2.16	1.7	1.323	1.05
0.3	3.21	2.52	-	1.85	1.61	1.365
0.4	2.96	2.57	2.24	2.06	1.95	1.71
0.6	3.08	2.98	3.	3.11	2.94	2.47
0.8	6.03	5.95	5.73	5.36	4.43	3.39
1.0	14.43	13.08	11.28	9.4	7.06	4.49

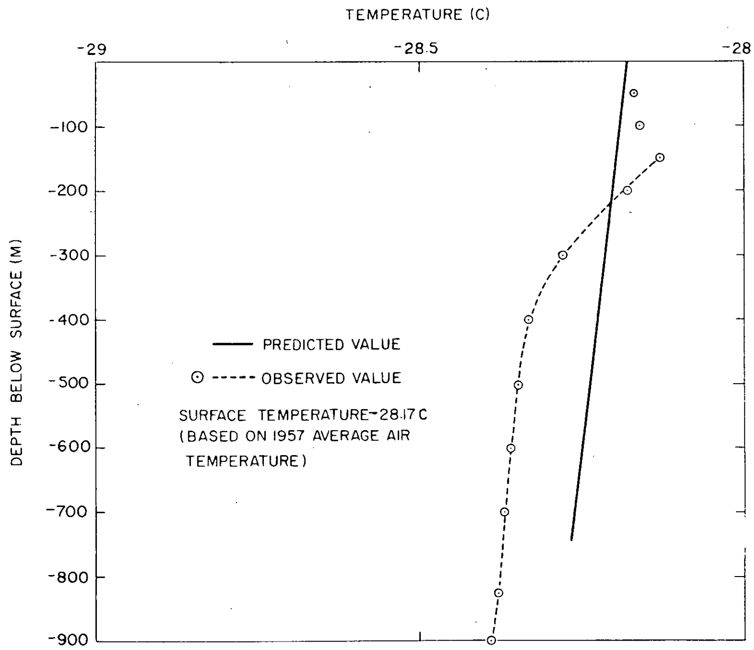

Figure 5. Comparison between predicted and observed temperature vs depth results, Byrd Station, Antarctica.

Although both comparisons seem to indicate favorable agreement with observed results, the proof is far from conclusive for the reason stated before. It is hoped that the current deep drilling project undertaken by USA SIPRE will yield a complete temperature profile and thus give a better basis for comparison.

REFERENCES

Benfield, A. E., (1951) The temperature in an accumulating snowfield, Monthly Notices of the Royal Astronomical Society, Geophysical Supplement, vol. 6, no. 3, p. 139-147.

Brockamp, B., (1951) Nachtrag zu den wissenshaftlichen Ergebnissen der Deutschen Grönlandexpedition Alfred Wegener, (Supplement to the scientific results of the German Greenland Expedition of Alfred Wegener), Neues Jahrbuch für Geologie and Paläontologie, Bd. 93, Ht. 2, p. 177-232. (German text).

Churchill, R. V., (1944) Modern operational mathematics in engineering, 1st Ed. McGraw-Hill Book Co.

Evans, G. W. Jr.; Isaacson, E.; and MacDonald, J. K. L.; (1950) Stefan-like problems, Quart. App. Math., vol. 8, no. 3, p. 312-319.

Langway, C. C. Jr., (1958) A 400-meter deep core in Greenland, Journal of Glaciology, vol. 3, p. 217.

Ragle, R. H.; Hansen, B. L.; Gow, A.; and Patenaude, R. W.; (1960) Deep core drilling in the Ross Ice Shelf, Antarctica, U. S. Army Snow Ice and Permafrost Research Establishment, Corps of Engineers, Technical Report 70.

Robin, G. de Q., (1955) Ice movement and temperature distribution in glaciers and ice sheets, Journal of Glaciology, vol. 2, p. 523.

Wexler, H., (1959) Geothermal heat and glacial growth, Journal of Glaciology, vol. 3, p. 420-425.

GPO 803984-3