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Creep of Snow under Combined Stress

by J. K. Landauer

**U. S. ARMY SNOW ICE AND PERMAFROST
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PREFACE

This work was performed by Dr. Landauer, physicist, Snow and Ice Basic Research Branch, as part of SIPRE Project 22.1-7, Triaxial mechanics of plastic deformation of snow, under supervision of Mr. J. A. Bender, Acting Chief.

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<p>AD Accession No</p> <p>U. S. Army Snow Ice and Permafrost Research Establishment, Corps of Engineers, Wilmette, Illinois CREEP OF SNOW UNDER COMBINED STRESS — J. K. Landauer</p> <p>Research Report 41, Dec 57, 12p-illus-tables, DA Proj 8-66-02-004, SIPRE Proj 22.1-7. Unclassified Report</p> <p>Laboratory experiments on the creep of sifted snow under uniaxial, hydrostatic, and confined-side compressive stresses are described, and the results are discussed on the basis of an additive theory for combined stresses. Stress (4-40 psi) and temperature (-3.6C to -13.6C) dependence was investigated for 35 samples (initial density $\cong 0.4 \text{ g/cm}^3$) subjected to uniaxial and hydrostatic stress for 200-sec time increments. Another series of 104 samples (initial densities 0.36-0.63) was subjected to uniaxial, hydrostatic, and confined-side compressive stresses until an arbitrary change in density was reached. In all cases the applied stress ultimately became a shear stress acting between grains, the open structure reacting differently to (over)</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> 1. Snow--Creep--Test results 2. Snow--Mechanical properties <ol style="list-style-type: none"> I. Landauer, Joseph K. II. U. S. Army Snow Ice and Permafrost Research Establishment 	<p>AD Accession No</p> <p>U. S. Army Snow Ice and Permafrost Research Establishment, Corps of Engineers, Wilmette, Illinois CREEP OF SNOW UNDER COMBINED STRESS — J. K. Landauer</p> <p>Research Report 41, Dec 57, 12p-illus-tables, DA Proj 8-66-02-004, SIPRE Proj 22.1-7. Unclassified Report</p> <p>Laboratory experiments on the creep of sifted snow under uniaxial, hydrostatic, and confined-side compressive stresses are described, and the results are discussed on the basis of an additive theory for combined stresses. Stress (4-40 psi) and temperature (-3.6C to -13.6C) dependence was investigated for 35 samples (initial density $\cong 0.4 \text{ g/cm}^3$) subjected to uniaxial and hydrostatic stress for 200-sec time increments. Another series of 104 samples (initial densities 0.36-0.63) was subjected to uniaxial, hydrostatic, and confined-side compressive stresses until an arbitrary change in density was reached. In all cases the applied stress ultimately became a shear stress acting between grains, the open structure reacting differently to (over)</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> 1. Snow--Creep--Test results 2. Snow--Mechanical properties <ol style="list-style-type: none"> I. Landauer, Joseph K. II. U. S. Army Snow Ice and Permafrost Research Establishment
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SUMMARY

Experiments have been performed on the creep of sifted snows under uniaxial, hydrostatic, and confined-side compressive stresses. The dependence of creep rate on stress (in the range 4 to 40 psi), time (up to several thousand seconds), temperature (between -3.6 and -13.6C) and density (from 0.36 to 0.63 g/cm³) was investigated. For one group of tests, stress was applied for constant time increments while, for another group, stress was continued until an arbitrarily selected small change in density was reached. The results are discussed on the basis of an additive theory for combined stresses.

CREEP OF SNOW UNDER COMBINED STRESS

by

J. K. Landauer

INTRODUCTION

A considerable amount of work has been done in the field of creep under combined stress, but generally on materials for which the compressibility can be neglected (Johnson, 1951; Coffin, 1954). These tests involve combinations of torsional and tensile stresses. Their applicability to snow is severely limited, however, by the great compressibility of snow. For typical creep tests, snow density may change 5 to 10% in a few hundred seconds. Bucher (1948) studied the creep of low-density snow under tension, compression, and shear, using low stresses. He developed theoretical relationships to compare his results under different stress conditions but his data are insufficient to establish a useful correlation.

Some uniaxial tests have been performed on snow (Yosida and Huzioka, 1954; Landauer, 1955). They indicate a flow law for creep varying from a linear law at low stresses to a power relationship at high stresses. In-place compactive tests have been run on natural snow (Kondrat'eva, *et. al.*, 1949; Fujimoto, 1950; Kojima, 1955) but these are difficult to analyze because of the variable snow conditions.

The present experiments were run under three different stress conditions: (1) uniaxial compressive; (2) confined-side compactive; and (3) hydrostatic, all at constant stress between 4 and 40 psi. For the uniaxial case, both vertical and lateral creep were determined; under the confined-side compactive stress, vertical creep and lateral stress were measured; and change in volume was determined for the hydrostatic tests. Tests were run at various temperatures from -3.6 to -13.6C and for snow densities between 0.36 and 0.63 g/cm³. For each experimental condition, several samples were measured to obtain an average value.

EXPERIMENTAL

General

The snow used for the tests reported here was natural snow that had been stored at low temperatures (-25 to -50C) for periods ranging from a few months to a few years. Grain size and shape characteristic of these snows varied, depending on the original deposition and the conditions of storage. In all cases, the stored natural snow was mixed and sieved into suitable containers and allowed to harden for periods of 1 to 4 weeks. Most of the hardening process occurred during the first week and tests performed after this did not involve an appreciably varying hardness (Bender, 1957).

Because of differences in the natural snows used, the densities after sifting varied between 0.36 and 0.5 g/cm³. Higher-density snows were obtained by compacting freshly sieved snows and allowing them to harden in the same manner as for uncompacted snows. No change in density occurs during the hardening process.

The tests were performed in cold rooms where the temperatures varied less than 1C. For some experiments the temperatures were controlled to ± 0.1 C by placing the experimental apparatus in a constant temperature inclosure.

Apparatus

The uniaxial and confined-side experiments were performed on an electrically operated press. Stress was maintained approximately constant by an on-off action of the press. Actually, therefore, a roughly sinusoidal stress was applied. The amplitude of the sinusoidal component was a few percent of the applied stress, and it was determined that the resulting deformations were the same as if a truly constant stress was maintained.

Vertical stress was measured using a 500-lb Baldwin Type U load cell. Vertical strain was measured with a millimeter scale for large deformations and a 0.001 in. Starrett dial gage for small deformations.

The lateral expansion accompanying vertical compaction for uniaxial stress was measured using four 0.001 in. dial gages at the center of each of the sides of the specimen (Fig. 1).

Figure 1. Apparatus for uniaxial tests. Four dial gages measure lateral strain.

Figure 2. Confined side compactive apparatus shown removed from press.

The confined-side compactive experiments were run in square cross-section steel prisms lined with a thin coat of Teflon to reduce friction. Inside dimensions were 5.75 x 5.75 cm x 12.7 cm high. The apparatus is shown in Figure 2. One side had a circular hole 3.60 cm in diameter in the center. A 3.46 cm diam pressure plate fitting coaxially into this hole was used to measure the lateral stress. Force was measured with a 5 kg Dillon dynamometer. A fine screw arrangement allowed the lateral pressure plate to be kept flush with the inner wall of the steel cylinder. For this purpose, a 0.0001 in. Starrett dial gage was used as reference indicator.

Hydrostatic stress was applied using compressed air and was measured with a 200-lb Baldwin SR-4 pressure cell. The snow sample rested on a porous base and was enclosed in a snugly fitting thin rubber envelope extending over the junction with the base (Fig. 3). A brass can was used to enclose the pressure system. A manometer arrangement served to measure the air expelled from the snow through the porous base and thus provided a measure of the deformation.

Procedure

A large number of uniaxial and hydrostatic experiments (group I) were made with cylindrical snow samples 5.8 cm diam by 18.9 cm high. In these tests deformation was measured at each of a series of increasing stresses for constant time increments. Stresses of

Figure 3. Hydrostatic apparatus with pressure can removed. The sample is enclosed in a rubber membrane.

about 4, 10, 16, 22, and 28 psi were applied successively for several hundred seconds each. The total deformation of a sample was usually less than 10% and most of this occurred at the higher stresses. These experiments were performed on snows near 0.4 g/cm^3 density and at various temperatures between -3.6 and -13.6°C .

Because of the relatively large change in density between the beginning and end of these experiments, a second set of experiments (group II) was performed at constant stress with the deformation continuing until the snow had densified a given small amount. In these experiments, initial densities were from 0.37 to 0.63 g/cm^3 . The temperature was maintained at -10°C . For the uniaxial and confined-side experiments the square prismatic samples ($5.75 \times 5.75 \times 12.7 \text{ cm}$) were used, but the circular cylinders were again used for the hydrostatic experiments.

For all of the experiments the snow was brought to temperature several hours prior to testing. For the uniaxial and hydrostatic tests, the snow sample was removed from its container and tested immediately. The snow for the confined-side experiments was not removed from the container, but loosened slightly to break any side bonding.

Usually a small initial pressure was applied (less than 10% of the test pressure) to assure good seating and provide an adequate zero point from which to measure deformation.

THEORETICAL RELATIONS

Additive theory

On the basis of an additive concept, deformation under combined stress is the sum of the deformations which would occur for separate uniaxial stresses. If this assumption is correct, deformation under combined stress can be predicted from uniaxial experiments. As will be seen later, the uniaxial creep law for snow can be taken as

$$\dot{\epsilon}_{zu} = k\sigma_z(1 + a\sigma_z^2) \quad (1)$$

where $\dot{\epsilon}_{zu}$ and σ_z are the compressive strain rate and stress in the z -direction and k is a constant depending on temperature, density, time, and snow-grain characteristics.

Snow under uniaxial compression undergoes horizontal expansion. The ratio

$$-\frac{\dot{\epsilon}_{zu}}{\dot{\epsilon}_{xu}} = m \quad (2)$$

is a measure of this effect. $1/m$ is the visco-plastic analogue of Poisson's ratio.

For combined stress the strain rate in the i -direction, using eq 1 and 2, is

$$\dot{\epsilon}_i = k\sigma_i(1 + a\sigma_i^2) - \frac{k\sigma_j}{m}(1 + a\sigma_j^2) - \frac{k\sigma_k}{m}(1 + a\sigma_k^2) \quad (3)$$

where j and k are directions normal to i . Under uniaxial compression, the lateral strain rate, $\dot{\epsilon}_{xu}(= \dot{\epsilon}_{yu})$, is simply

$$\dot{\epsilon}_{xu} = -\frac{k\sigma_z}{m}(1 + a\sigma_z^2). \quad (4)$$

For hydrostatic stress, the strain rate, $\dot{\epsilon}_{zh} (= \dot{\epsilon}_{xh} = \dot{\epsilon}_{yh})$ is, using eq 3,

$$\dot{\epsilon}_{zh} = \frac{m-2}{m} k \sigma_z (1 + a \sigma_z^2). \quad (5)$$

In the case of confined-side compaction

$$\dot{\epsilon}_{xc} (= \dot{\epsilon}_{yc}) = 0.$$

For this case, using eq 3, the ratio of vertical to lateral stress is found to be

$$\frac{\sigma_z}{\sigma_x} = (m-1) \frac{1 + a \sigma_x^2}{1 + a \sigma_z^2}. \quad (6)$$

For a linear creep law this reduces to

$$\frac{\sigma_z}{\sigma_x} = m-1.$$

Creep in the z -direction should be less for the confined-side than for the uniaxial case. From eq 3 and 6, the strain rate is

$$\dot{\epsilon}_{zc} = \frac{(m-2)(m+1)}{m(m-1)} k \sigma_z (1 + a \sigma_z^2). \quad (7)$$

Creep rates for uniaxial, hydrostatic, and confined-side stresses can be compared on the basis of the additive concept. The ratios of the creep rates are

$$\frac{\dot{\epsilon}_{zu}}{\dot{\epsilon}_{zh}} = \frac{m}{m-2} \quad (8)$$

and

$$\frac{\dot{\epsilon}_{zu}}{\dot{\epsilon}_{zc}} = \frac{m(m-1)}{(m-2)(m+1)}. \quad (9)$$

Relationship between tests run for constant change in density and tests for constant time

Group I experiments were performed for constant time increments. An average velocity was determined, which was the total strain divided by the time, t , during which constant stress was applied. Thus, a generalized creep curve of the form;

$$\frac{\Delta z}{zt} = ABST \quad (10)$$

was found, where $\underline{A}$ is a constant depending on density, temperature, and grain characteristics, $\underline{B}$ is a constant of the stress condition (i.e., the factor to represent combined stress behavior which is equal to unity for the uniaxial case), $\underline{S}$ is the uniaxial stress function, and $\underline{T}$ is a function depending only on time.

The time dependent function was found to be well represented by

$$\epsilon = \frac{\Delta z}{z} \propto t^r. \quad (11)$$

The average strain rate is, therefore;

$$\frac{\overline{\Delta z}}{zt} \propto t^{r-1} \equiv T \quad (12)$$

while the strain rate, $\dot{\epsilon}$, is

$$\dot{\epsilon} \propto rt^{r-1}. \quad (13)$$

Thus, in terms of strain rate, the generalized creep curve for constant time becomes

$$\dot{\epsilon}_t = ABSrT. \quad (14)$$

Owing to changing density during deformation, it is reasonable to compare creep rates at a chosen density as done for group II experiments. To attain a given change in density (initial densities being equal) it is necessary to produce a larger $\underline{z}$ -axis strain for the uniaxial case than for the confined-side or hydrostatic cases. The $\underline{z}$ -axis strains to reach constant density will be in the ratio

$$\epsilon_u : \epsilon_c : \epsilon_h = 1 : \frac{m-2}{m} : \frac{m-2}{3m}. \quad (15)$$

To express the constant-time creep curve (eq 14) in terms of creep rates at constant density, it is first necessary to express the curve at constant $\underline{z}$ -axis strain. Since the time to attain a given strain is

$$t_\epsilon = (\epsilon/ABS)^{1/r}, \quad (16)$$

the creep rate at constant $\underline{z}$ -axis strain becomes

$$\dot{\epsilon}_\epsilon = (ABS)^{1/r} r \epsilon^{(r-1)/r}. \quad (17)$$

At constant density the ratios of the strain rates under confined-side and hydrostatic stresses to strain rate under an equal uniaxial stress are

$$\left(\frac{\dot{\epsilon}_c}{\dot{\epsilon}_u}\right)_\gamma = B_c^{1/r} \left(\frac{m-2}{m}\right)^{(r-1)/r}$$

and

$$\left(\frac{\dot{\epsilon}_h}{\dot{\epsilon}_u}\right)_\gamma = B_h^{1/r} \left(\frac{m-2}{3m}\right)^{(r-1)/r}$$

From eq 8 and 9 the values of B can be taken as $B_c = \frac{(m-2)(m+1)}{m(m-1)}$

and $B_h = \frac{m-2}{m}$. The above equations can be rewritten:

$$\left(\frac{\dot{\epsilon}_c}{\dot{\epsilon}_u}\right)_\gamma = \frac{m-2}{m} \left(\frac{m+1}{m-1}\right)^{1/r}$$

and

$$\left(\frac{\dot{\epsilon}_h}{\dot{\epsilon}_u}\right)_\gamma = \left(\frac{m-2}{m}\right)^3 (1-r)/r.$$

(18)

RESULTS AND DISCUSSION

Group I experiments

Group I experiments provide information on the stress and temperature dependence of the uniaxial and hydrostatic cases. Thirty five samples were tested. Typical creep curves for the two stress conditions are shown in Figure 4. An average strain rate was determined for each pressure by dividing the total strain by the time increment. A few tests at 300 and 500 sec increments were converted to the predominant 200 sec basis by eq 12, using $r = 0.5$. The volume deformation determined for the hydrostatic experiments were converted to z -axis strain rates by dividing the volume strain rates by 3.

Either of two stress functions can be used to represent these data. A very good fit of the data is obtained using a hyperbolic sine stress dependence; a slightly inferior fit is obtained from the sum of linear and cubic stress terms. In Figure 5, the experimental results at -8.4°C are compared with hyperbolic sine and linear plus cubic stress curves. Some deviation occurs at the higher stresses because of the continuously increasing density of the sample.

Figure 4.
Deformation vs time,
group I test at -13.55°C .
Numbers on curves
show stress in psi.

Figure 5.
Empirical creep curve,
group I test at -8.4°C
using coefficients from
Table I.

The coefficients, $\underline{C}$, for a hyperbolic sine creep law:

$$\left(\frac{\Delta z}{zt}\right)_{200} = C \sinh \frac{\sigma}{\sigma_0} \quad (19)$$

are given in Table I. For both uniaxial and hydrostatic experiments σ_0 is a constant equal to 10 psi.

Representation of the data in a creep curve of the form:

$$\left(\frac{\Delta z}{zt}\right)_{200} = k\sigma(1 + a\sigma^2) \quad (20)$$

is advantageous for some applications. The values of the constants, k , are shown in Table I. The constant, a , does not vary appreciably from the value 3.0×10^{-3} and k has been calculated on that basis.

Comparison of the values of $\underline{C}$ at different temperatures leads to a value of the activation energy for creep. Figure 6 is a plot of $\underline{C}$ vs inverse absolute temperature. An average activation energy of 13,400 cal/mol is found. The differences between hydrostatic and uniaxial activation energies are probably less than the experimental error.

Group II experiments

One hundred and four samples prepared in six batches were run to the strains indicated in Table II. Uniaxial samples were tested first, the $\underline{m}$ number calculated and then, using eq 15, limits for confined-side and hydrostatic experiments were determined. The dense snows required a smaller change in density than the light snows in order to run the experiments over comparable times. However, for a particular batch, all samples were run to the same final density. The strain rates and parameters $\underline{m}$ and $\underline{r}$ were determined at this final density. Results are compiled in Table III.

Stress dependence for group II tests

Stress dependence can be derived from the tests at 10, 15, and 20 psi at initial density of 0.369 g/cm^3 . Uniaxial, confined-side, and hydrostatic data indicate a creep rate varying as the cube of the stress. The difference between this result and that obtained for constant time tests rests on several factors. First, reference to eq 17 shows that the exponent of the stress function in the case of constant strain should be a factor of $1/r$ times the exponent of the function for constant time. Second, increasing density during the course of group I tests tends to reduce the strain rates for the higher stresses. This makes the effective value of the exponent lower than it would be for group II tests involving small total deformation. Third, the linear term in group I stress functions would become fairly inconspicuous for the somewhat higher stresses used in group II experiments.

Variability

Considerable variability exists between samples of the same density prepared at different times. Less but still a large variability exists between samples prepared under identical conditions at the same time. Samples produced under identical conditions may differ in creep rate by 20 to 30% and occasionally by 100% or more. The parameters $\underline{r}$ and $\underline{m}$ show similar variability.

Figure 6. The coefficient, C , for the hyperbolic sine creep curve plotted vs inverse absolute temperature.

When snows are prepared at different times out of similar but not identical snows, and aged under roughly similar, but again not identical, conditions of time and temperature, the differences in measured creep parameters can be as great as a factor of 10 although the densities are nearly equal.

The densities of samples prepared under identical conditions vary several percent but no correlation is apparent between these small variations and the variations in creep behavior. Apparently, the differences in creep are the result of differences in the aging process. This process is the change from a loose aggregate of angular snow particles to a bonded aggregate of rounded particles. The rate and degree of bonding depend to a large extent on temperature variations which are not well controlled during the aging process. Variability in creep is due, therefore, to differences in the bonding structure of the aged snow. Since variations in creep are large for constant density, the dominant creep mechanism is probably intergranular rather than intragranular, for the former is strongly dependent on the structure but the latter would be structure-independent.

Figure 7. Variation of average values of time dependent parameter $\bar{r}$ with initial density for group II tests.

Table I. Constants for 200 sec average $\bar{z}$ -axis strain rates represented by creep curves of the form $(\bar{\Delta z}/z)_{200} = C \sinh(\sigma/\sigma_0)$ and $(\bar{\Delta z}/z)_{200} = k\sigma(1+a\sigma^2)$ for group I tests under uniaxial and hydrostatic stresses. The constants have been calculated using $\sigma_0 = 10$ psi and $a = 3 \times 10^{-3} \text{ psi}^{-2}$.

Batch	Snow density (g/cm ³)	Temp. (C)	Stress condition	t, Time at each stress (sec)	No. of samples averaged	$C \times 10^6$ (sec ⁻¹)	$k \times 10^6$ (sec ⁻¹)
1	0.420	- 4.2	h	500	2	8.0	0.8
1	0.420	- 8.8	h	500	2	5.3	0.45
1	0.420	-12.2	h	500	2	3.3	0.35
2	0.422	- 4.2	u	300	1	16.4	1.3
2	0.422	- 4.2	u	200	4	22.7	2.3
3	0.420	- 4.2	u	200	3	18.5	1.5
3	0.420	- 4.2	h	200	3	8.0	1.0
4	0.380	- 3.6	u	200	2	80.5	7.4
4	0.380	- 3.6	h	200	2	20.0	1.8
4	0.380	- 5.1	u	200	3	54.0	4.5
4	0.380	- 5.1	h	200	3	18.0	1.65
4	0.380	- 8.4	u	200	3	34.4	3.3
4	0.380	- 8.4	h	200	2	10.6	1.0
4	0.380	-13.55	u	200	2	32.2	3.0
4	0.380	-13.55	h	200	1	8.8	0.80

Table II. Limits of deformation and density change for group II experiments. The average density for each batch is shown.

Batch	Deformation				
	γ_0 (g/cm ³)	Δz_u (in.)	Δz_c (in.)	Δv_h (cm ³)	$\Delta \gamma$ (g/cm ³)
6	0.369	240×10^{-3}	160×10^{-3}	15.7	0.012
5	0.451	240×10^{-3}	160×10^{-3}	15.7	0.015
10	0.477	150×10^{-3}	125×10^{-3}	12.5	0.012
8	0.503	60×10^{-3}	50×10^{-3}	5.0	0.006
9	0.503	150×10^{-3}	125×10^{-3}	13.0	0.015
7	0.626	50×10^{-3}	33×10^{-3}	3.3	0.004

Table III. Results of group II tests. The indicated errors are mean deviations.

Batch	Initial density γ_0 (g/cm ³)	σ_z (psi)	$\dot{\epsilon}_z \times 10^{-7}$ (sec ⁻¹)	m	r_z	r_x	No. Averaged
A. Uniaxial tests							
5	0.457±0.002	17.5	180±17	4.3±0.5	0.45±0.04	0.29±0.02	3
6	0.370±0.001	10	220±40	5.2±0.1	0.45±0.01	0.30±0.04	2
6	0.372±0.003	15	550±10	3.1±0.1	0.49±0.05	0.40±0.04	2
6	0.371±0.007	20	1400±0	3.8±0.3	0.64±0.07	0.53±0.07	2
7	0.620	20	1.4	4.6	0.09	0.02	1
7	0.619	30	2.2	5.2	0.12	0.05	1
7	0.625±0.005	40	4.4±1.0	3.7±1.1	0.15±0.01	0.10±0.06	6
8	0.507±0.004	20	11±4	6.7±0.1	0.19±0.01	0.04±0.01	5
9	0.502±0.004	20	250±30	5.0±1.3	0.45±0.01	0.25±0.04	7
10	0.476±0.006	20	300±30	4.5±0.4	0.42±0.04	0.20±0.03	5
B. Confined-side compactive tests				σ_z/σ_x			
5	0.458±0.002	17.5	150±10	22±7	0.42±0.07		3
6	0.371±0.009	10	98±22	34±10	0.50±0.04		3
6	0.371±0.006	15	670±197	23±7	0.46±0.08		3
6	0.375±0.008	20	1600±230	25±6	0.60±0.02		3
7	0.610±0.002	40	2.5±0.5	46±10	0.15±0.01		3
8	0.496±0.008	20	12±0.7	39±10	0.24±0.02		8
9	0.497±0.009	20	130±40	20±10	0.45±0.04		7
10	0.478±0.004	20	220±60	25±14	0.41±0.01		6
C. Hydrostatic tests							
5	0.445±0.007	16.7±0.6	1130±43		0.49±0.15		9
6	0.367±0.004	9.2±0.5	430±100		0.34±0.06		3
6	0.360±0.002	15.4±0.7	2250±100		0.61±0.12		4
7	0.634±0.007	40.0±1.2	3.4±0.05		0.15±0.04		7
8	0.513±0.012	19.0±0.8	2.6±0.1		0.38±0.01		5
9	0.516±0.014	19.5±0.9	430±110		0.34±0.07		3
10	0.476±0.018	20.1±0.5	350±110		0.25±0.06		3

Time dependence

Creep curves for snow show a continuously decreasing creep rate which can be represented, with fair accuracy, by a function of the form $\epsilon \propto t^r$. The dependence of r on density is shown in Figure 7, where the average values are plotted for the six batches of snow used in group II tests.

Consider deformation of snow at low stress to be essentially intergranular. As density increases, each particle has more neighbors and more points of contact. For the same externally applied stress, the stress acting on the contacts between particles decreases as the number of contact points per particle increases. At low densities this stress is large, resulting in predominantly visco-plastic flow. At higher densities, the stress is small and elastic deformation becomes significant. A value of r equal to unity, as predicted for low densities by extrapolation in Figure 7, is correct for viscous flow. The value of r at high density approaches zero as for purely elastic behavior. Therefore, the observed creep-rate time dependence is mainly due to increasing density and is not the approach to an equilibrium condition.

Activation energy

The activation energy for creep of about 13,400 cal/mol, determined from the group I tests, agrees adequately with the value 14,000 cal/mol previously reported (Landauer, 1955) for uniaxial compression of snow between -5°C and -20°C.

The value 16,100 cal/mol for polycrystalline ice reported by Jellinek and Brill (1956) is based on the purely viscous (and therefore time-independent) component of creep.

Glen (1955) obtained a value of 31,800 cal/mol from the minimum creep rates of polycrystalline ice at -1.5°C , -6.7°C and -12.8°C . The reliability of his activation energy is strongly dependent on the use of a single value at -12.8°C , and therefore a liberal possible error could be assigned to his value.

The activation energy for snow based on creep rates at constant time may not be directly compared with the values based on time-independent creep rate values for ice. From eq 17, the activation energy determined at constant strain is increased by a factor of $1/r$ over the value at constant time. This would increase the observed value for snow near 0.4 g/cm^3 density by a factor of about 2.

Because of the change in density of snow during creep, no steady state creep occurs and the creep rate decreases continuously. In terms of an actual creep mechanism, such as described by Weertman (1957), the pertinent activation energy is for steady state conditions where the production and blocking of imperfections has reached equilibrium. The activation energy for snow, comparable with the steady state value, will lie between the constant time and constant strain values of 14,000 to 27,000 cal/mol. Assuming the activation energy for creep is the same for snow as for ice, the present data do not settle the discrepancy between the values of Jellinek and Brill and those of Glen.

Relations between creep rates under different stress conditions

The ratios of creep rates for confined-side and hydrostatic stress conditions to uniaxial creep rates were determined for the group II tests. In Table IV, average values for tests performed at the same stress are compared to those calculated for constant time and constant change in density. Observed values are seen to fall between the calculated ones in five cases, with the remainder scattered above and below these limits.

From these results it is concluded that the present method of analysis, although leading to some very rough agreement, is not adequate for a complete understanding of triaxial behavior of snow.

Density dependence

Creep rate is a very strong function of density, as can be seen from results in Table III. No analytical representation of the density function has been attempted because of differences in aging of the snow from batch to batch.

Variation of the parameter $\underline{m}$ with density is erratic, as shown in Table IV, and does not agree well with the average curve determined by Bader (USA SIPRE, 1951). However, the values fall within his range of scatter. Differences in the bonding structure of the snows prevent an undistorted picture of the variation of $\underline{m}$ with density.

Lateral stress for confined-side tests

The lateral stress developed under confined-side conditions is much lower than expected. Observed ratios of vertical to lateral stress vary considerably but are always greater than 20:1. Eq 4, based on the additive theory, predicts ratios of from approximately 3:1 to 6:1. Side friction can be eliminated as a possible cause for this discrepancy, for tests were run which showed that the stress reaching the bottom of the sample was within a few per cent of the applied vertical stress.

An explanation for the low lateral stress is based on the open structure of the snows used. The tendency for snow to expand when subjected to uniaxial compression is erratic, as indicated by the parameter $\underline{m}$. This expansion can take place inwardly almost as easily as outwardly and a small lateral stress is sufficient to prevent outward expansion.

Table IV. Comparison of ratio of compactive to uniaxial and hydrostatic to uniaxial strain rates with values calculated on constant time and constant density change basis. Average values for group II tests are given.

Y ₀ Initial density (g/cm ³)	m	r	$\dot{\epsilon}_{zc}/\dot{\epsilon}_{zu}$			$\dot{\epsilon}_{zh}/\dot{\epsilon}_{zu}$		
			Observed	Calc. Constant time	Calc. Constant density change	Observed	Calc. Constant time	Calc. Constant density change
0.369	4.2	0.51	0.94	0.85	1.4	3.0	0.52	1.5
0.451	4.3	0.47	0.83	0.86	1.4	6.3	0.54	1.9
0.477	4.5	0.38	0.73	0.88	1.8	1.2	0.56	3.3
0.503	6.7	0.26	1.09	0.95	2.2	0.23	0.70	16
0.503	5.0	0.43	0.52	0.90	1.5	1.72	0.60	2.6
0.626	3.7	0.15	0.58	0.80	18	0.80	0.46	235

CONCLUSIONS

Deformation of snow occurs predominantly by intercrystalline sliding. The magnitude of the creep rate depends on the bonding structure between grains. Because of the relative openness of the snows studied, there is no difference in the mode of deformation for uniaxial, confined-side, or hydrostatic stress conditions. In all cases the applied stress ultimately becomes a shear stress acting between individual grains.

The concept of combined stresses producing deformations which are the same as would occur for separate uniaxial stresses is not valid. Openness of structure allows deformation to take place under combined stresses in a somewhat different manner than for uniaxial stress. In particular, the tendency for uniaxial specimens to expand laterally when compressed is easily overcome by small lateral stresses which alter the directed motion to random displacements. The *z*-axis strain rates, measured at a particular density as in group II tests, are not strongly dependent on the type of stress applied. This means that strain rate is a function of the state of the material but does not depend on other simultaneously applied stresses.

Further investigation must be made in order to determine the relationship between combined stress behavior in the laboratory and under natural conditions. The slow process of compaction occurring in the névé regions of glaciers may not follow the behavior of the faster laboratory tests.

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<p>AD Accession No</p> <p>U. S. Army Snow Ice and Permafrost Research Establishment, Corps of Engineers, Wilmette, Illinois CREEP OF SNOW UNDER COMBINED STRESS — J. K. Landauer</p> <p>Research Report 41, Dec 57, 12p-illus-tables, DA Proj 8-66-02-004, SIPRE Proj 22.1-7. Unclassified Report</p> <p>Laboratory experiments on the creep of sifted snow under uniaxial, hydrostatic, and confined-side compressive stresses are described, and the results are discussed on the basis of an additive theory for combined stresses. Stress (4-40 psi) and temperature (-3.6C to -13.6C) dependence was investigated for 35 samples (initial density $\cong 0.4 \text{ g/cm}^3$) subjected to uniaxial and hydrostatic stress for 200-sec time increments. Another series of 104 samples (initial densities 0.36-0.63) was subjected to uniaxial, hydrostatic, and confined-side compressive stresses until an arbitrary change in density was reached. In all cases the applied stress ultimately became a shear stress acting between grains, the open structure reacting differently to (over)</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> 1. Snow--Creep--Test results 2. Snow--Mechanical properties <ol style="list-style-type: none"> I. Landauer, Joseph K. II. U. S. Army Snow Ice and Permafrost Research Establishment 	<p>AD Accession No</p> <p>U. S. Army Snow Ice and Permafrost Research Establishment, Corps of Engineers, Wilmette, Illinois CREEP OF SNOW UNDER COMBINED STRESS — J. K. Landauer</p> <p>Research Report 41, Dec 57, 12p-illus-tables, DA Proj 8-66-02-004, SIPRE Proj 22.1-7. Unclassified Report</p> <p>Laboratory experiments on the creep of sifted snow under uniaxial, hydrostatic, and confined-side compressive stresses are described, and the results are discussed on the basis of an additive theory for combined stresses. Stress (4-40 psi) and temperature (-3.6C to -13.6C) dependence was investigated for 35 samples (initial density $\cong 0.4 \text{ g/cm}^3$) subjected to uniaxial and hydrostatic stress for 200-sec time increments. Another series of 104 samples (initial densities 0.36-0.63) was subjected to uniaxial, hydrostatic, and confined-side compressive stresses until an arbitrary change in density was reached. In all cases the applied stress ultimately became a shear stress acting between grains, the open structure reacting differently to (over)</p>	<p>UNCLASSIFIED</p> <ol style="list-style-type: none"> 1. Snow--Creep--Test results 2. Snow--Mechanical properties <ol style="list-style-type: none"> I. Landauer, Joseph K. II. U. S. Army Snow Ice and Permafrost Research Establishment
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