UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

STABLE CHANNEL PROFILES

Hydraulic Laboratory Report No. Hyd-325

ENGINEERING LABORATORIES BRANCH

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Subject: Stable channel profiles

INTRODUCTION

In order to deal intelligently with certain problems of maintenance of channels in erodible materials, it is important to know what shape and dimensions a channel should have in order to avoid changes of cross section due to scour or sediment deposition. A knowledge of stable shapes finds application to the problem of designing canals in earth and to the problems of river regimen. The computations of stable shape patterns described herein have been made at the request of Mr. E. W. Lane who has also laid down the basis for the computations.

BASIS OF COMPUTATIONS

The problem of finding a stable channel shape is here based on the following assumptions:

a. The particle is held against the bed by the component of the submerged weight of the particle acting normal to the bed.

b. At the edge of the stream the particles are at the angle of repose under the action of gravity.

c. At the center of the stream the drag force of the flowing water holds the particles at the point of incipient instability.

d. At points between the edge and the center the particles are held at the point of incipient instability by the resultant of the gravity component of the particle's submerged weight and the drag force of the flowing water.

In dealing with the drag force of the flowing water, two assumptions have been used as follows:

e. The drag force acting on an area of the bed of the stream is produced by the component of the weight of the water directly above the area, which acts in the direction of flow. This component is equal to the above weight multiplied by the stream gradient.

f. The drag force acting on a particle is proportional to the square of the mean velocity in the stream at the point where the particle is located.

Use of assumption (e) permits a simpler treatment than is possible with assumption (f) but does not permit the lateral transfer of drag due to velocity gradients in the horizontal direction to be taken into account. This factor can be included in the computations when assumption (f) is used.

NOTATION

A represents the cross sectional area of the channel

$a = \frac{T}{2}$ or half the top width

C, Chezy's coefficient

D, depth of an infinitely wide channel

f, longitudinal slope or friction slope

m, factor of proportionality

n, coefficient of roughness

P, perimeter

Q, quantity of flow (cubic ft/sec)

q, drag force on a surface of a lamina of unit length

$r = \frac{A}{P}$, the hydraulic radius

$S = \tan \theta$, the transverse slope at the outer edge of the channel

T, top width

$U = V^2$

V, mean velocity of the lamina at a distance x from the center of a channel

V_c, mean velocity of the flow in an infinitely wide channel of depth $D = y_o$

V_m, mean velocity of the flow in a channel of finite width and of depth y_o

V_o, mean velocity of the lamina at the center of a channel

W, submerged weight of a particle

x, distance measured horizontally from the center of a channel

y, depth of a channel at the distance x from the center; or, in the evaluation of m, y represents the distance from the top of the channel of depth D to a horizontal lamina of thickness dy

y_o, depth of a channel at the center

α, a variable

β, a variable

$\eta = \frac{y}{y_o}$

θ, angle of repose for the material

$\xi = \frac{x}{y_o}$

ρ, density of water

τ_a, an average drag force

τ, maximum allowable drag force for soil in lb/ft^2

ϕ, angle the tangent at x makes with the horizontal

STABLE CHANNEL SHAPE WITH DRAG PROPORTIONAL TO DEPTH

The type of stable channel studied herein has the property that all the particles of its bed are in a state of incipient instability under the action of the forces acting on them. At the edge the drag force is zero; and incipient instability is due to the force of gravity acting on particles resting on the slope S, which is at the angle of repose. At this point the ratio of the component of force down the slope to the component normal to the slope is S. At the middle of the channel, where the slope $\frac{dy}{dx}$ is zero, the particles are held in the state of incipient instability by the drag force of the flowing water. In this case, also, the ratio of the force along the bed to the force acting on the particle in the direction normal to the bed is S. At all other points the resultant of the drag force and the gravitational component along the bed act on the particle, and the ratio of this resultant to the gravitational force component normal to the bed must also be S. These relations may be expressed mathematically in the following way. If W represents the submerged weight of the particle, then, with the notation of Figure 1, the force component normal to the bed is W cos ϕ. The gravitational component along the bed is W sin ϕ and the drag force of the flowing water per unit of area of the bed is

$$WS \cdot \frac{y}{y_o} \cos\phi$$

Then the requirement that the condition of incipient instability shall prevail everywhere is

$$\sqrt{\frac{W^2 \sin^2\phi + W^2 S^2 \frac{y^2}{y_o^2} \cos^2\phi}{W^2 \cos^2\phi}} = S \qquad \ldots \quad (1)$$

Since

$$-\frac{dy}{dx} = \tan\phi$$

This relation can be put in the form

$$\left(\frac{dy}{dx}\right)^2 + S^2 \frac{y^2}{y_o^2} = S^2 \qquad \ldots \quad (2)$$

A solution of this differential equation which satisfies the condition that $y = y_o$ when $x = 0$ is

$$y = y_o \cdot \cos \frac{S}{y_o} x \qquad \ldots \quad (3)$$

Then it may be concluded that a simple cosine curve represents the shape of a stable channel under the conditions specified in assumption (e).

STABLE CHANNEL WITH A DRAG FORCE PROPORTIONAL TO THE SQUARE OF THE VELOCITY AND NO LATERAL TRANSFER OF DRAG

In this case the drag force term is assumed to have the form:

$$WS\frac{V^2}{V_o^{\,2}}$$

where V represents the mean velocity in the stream at the distance x from the center, and V_o represents the mean velocity at the center. Then the condition that the particles everywhere be on the verge of motion is:

$$\sqrt{\frac{W^2\sin^2\phi + W^2S^2\left(\frac{V^2}{V_o^{\,2}}\right)^2}{W^2\cos^2\phi}} = S \qquad \ldots \quad (4)$$

But since $\tan\phi = -\frac{dy}{dx}$

and $$\sin\phi = \frac{-\left(\frac{dy}{dx}\right)}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}, \quad \cos\phi = \frac{1}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \qquad \ldots \quad (5)$$

the above expression can be put in the form:

$$\left(\frac{dy}{dx}\right)^2 + S^2\left(\frac{V}{V_o}\right)^4 \left[1 + \left(\frac{dy}{dx}\right)^2\right] = S^2 \qquad \ldots \quad (6)$$

In order to obtain an expression for the velocities, we may apply the Chezy formula

$$V = C\sqrt{rf} \qquad \ldots \quad (7)$$

to compute the flow through an element such as the shaded element of Figure 1. Since the assumption of no lateral transfer of drag is equivalent to an assumption of no drag forces on the sides of the element, the hydraulic radius r may be computed by dividing the area of the element ydx by the length of bed in contact with its base. This length is

$$dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Then

$$r = \frac{y}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \qquad \ldots \quad (8)$$

and from (7)

$$V^2 = C^2 rf = \frac{C^2 yf}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \qquad \ldots \quad (9)$$

Since

$$V_o^2 = C^2 y_o f$$

then, by division

$$\frac{V^2}{V_o^2} = \frac{y}{y_o\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \qquad \ldots \quad (10)$$

If this expression is substituted into Equation (6), it becomes:

$$\left(\frac{dy}{dx}\right)^2 + S^2\frac{y^2}{y_o^2} = S^2$$

This is identical with Equation (2) and, of course, has the same solution, namely:

$$y = y_o \cos\frac{S}{y_o}x$$

It may be concluded therefore, that if the lateral transfer of drag is neglected, both assumptions (e) and (f) lead to the same shape for the stable channel. If this seems surprising, it should be recalled that the Chezy formula itself is derived by equating a frictional drag force proportional to the square of the velocity to the component of the weight of the water acting in the direction of flow. Therefore, if the Chezy relation is used with the above restrictions to determine the relation between velocity and drag, assumptions (e) and (f) become identical.

STABLE CHANNEL SHAPE WITH LATERAL DRAG TRANSFER ACCOUNTED FOR

Before the effect of lateral drag transfer can be introduced into the equations, it will be necessary to develop some means of computing it. In the present case it will be assumed that the shear force acting on any section through the stream is proportional to the gradient of the square of the velocity. The factor of proportionality, here called m, will be determined by computing the mean velocity in an infinitely wide channel according to these relations and then determining the factor m by comparison with the Chezy formula. The modifications needed to introduce the effect of lateral drag transfer can be made as soon as the factor m is evaluated. The detail of the evaluation of m is as follows:

In a channel of depth D and having a friction slope f, such as is shown in Figure 2, the shear on a unit area of the plane at a distance y below the top is

$$\tau = m\frac{dU}{dy} \qquad \ldots \quad (11)$$

where $U = V^2$ and V represents the stream velocity at y. The gravitational component acting on the volume included between unit areas of the planes y and $y + dy$ is

$$\rho f dy$$

where ρ represents the weight of a unit volume of water. Then if this gravitational component is to be held in balance by the difference between the drag forces on the top and bottom of the element:

$$-\frac{d}{dy}\left(m\frac{dU}{dy}\right) = \rho f \qquad \ldots (12)$$

In setting up this expression, forces in the downstream direction have been considered positive. By integration

$$-m\frac{dU}{dy} = \rho f y + C_1$$

and

$$-mU = \frac{\rho f y^2}{2} + C_1 y + C_2$$

where C_1 and C_2 are constants.

If

$$-m\frac{dU}{dy} = \rho f D \quad \text{at} \quad y = D$$

$$U = 0 \quad \text{at} \quad y = D$$

then

$$C_1 = 0 \quad \text{and} \quad C_2 = -\frac{\rho f D^2}{2}$$

Hence

$$mU = \frac{\rho f D^2}{2} - \frac{\rho f y^2}{2}$$

or

$$U = \frac{\rho f}{2m}(D^2 - y^2) \qquad \ldots (13)$$

and $\frac{dV}{dx}$

$$V = \sqrt{\frac{\rho f}{2m}} \sqrt{D^2 - y^2} \qquad \ldots (14)$$

The mean velocity is

$$V_{mean} = \frac{1}{D}\int_0^D Vdy = \frac{1}{D}\sqrt{\frac{\rho f}{2m}}\left[\frac{y}{2}\sqrt{D^2 - y^2} + \frac{D^2}{2}\sin^{-1}\frac{y}{D}\right]_0^D$$

or

$$V_{mean} = \sqrt{\frac{\rho f}{2m}} \quad \frac{\pi D}{4} \qquad \ldots (15)$$

A rearrangement which will permit a comparison with the Chezy formula is

$$V_{mean} = \sqrt{\frac{\rho D \pi^2}{32m}} \quad \sqrt{Df} \qquad \ldots (16)$$

For this case Chezy's formula has the form:

$$V_{mean} = C\sqrt{Df} \qquad \ldots (17)$$

Then by comparison

$$C = \sqrt{\frac{\rho D \pi^2}{32m}} \qquad \ldots (18)$$

and it follows that:

$$m = \frac{\rho D \pi^2}{32C^2} \qquad \ldots (19)$$

It will now be possible to modify Equation (9) by adding to the original gravitation driving force the difference of the frictional drag forces on the two sides of the shaded element of Figure 1. The drag force acting on the side of the shaded element nearest to the center of the channel is, for a unit length of channel,

$$q = y\tau \qquad \ldots (20)$$

and acts in the downstream direction if the velocity gradient $\frac{dV}{dx}$ is negative. Under the same conditions the force on the side away from the origin is given by a similar expression, but here the force acts upstream. The difference between the two forces is

$$dq = \frac{d}{dx}(y\tau)\,dx \qquad \ldots \quad (21)$$

Modification of Equation (9) will be most easily made if this difference is expressed in terms of an equivalent channel gradient. To do this set

$$f_1 \rho y dx = \frac{d}{dx}(y\tau)\,dx$$

Then

$$f_1 = \frac{1}{\rho y}\frac{d}{dx}(y\tau) \qquad \ldots \quad (22)$$

and the equation as modified to account for the lateral transfer of drag becomes

$$V^2 = \frac{C^2 y}{\sqrt{1 + (\frac{dy}{dx})^2}}(f + f_1) \qquad \ldots \quad (23)$$

$$V^2 = \frac{C^2 y}{\sqrt{1 + (\frac{dy}{dx})^2}}\left[f + \frac{m}{\rho y}\frac{d}{dx}(y\,\frac{dV^2}{dx})\right] \qquad \ldots \quad (24)$$

Since

$$V_c = C^2 y_o\, f \qquad \ldots \quad (25)$$

$$\frac{V^2}{{V_c}^2} = \frac{y}{y_o\sqrt{1 + (\frac{dy}{dx})^2}}\left[1 + \frac{m}{\rho f\, y}\frac{d}{dx}\,(y\,\frac{dV^2}{dx})\right]$$

or

$$\frac{V^2}{V_c^2} = \frac{y}{y_o\sqrt{1 + (\frac{dy}{dx})^2}} \left\{1 + \frac{V_o^2 m}{\rho f}\frac{y_o}{y}\frac{d}{dx}\left[\frac{y}{y_o}\frac{d}{dx}(\frac{V}{V_o})^2\right]\right\} \quad . . . \quad (26)$$

To put this expression into dimensionless form let

$$\eta = \frac{y}{y_o} \qquad y_o d\eta = dy \quad . . . \quad (27)$$

$$\xi = \frac{x}{y_o} \qquad y_o d\xi = dx \quad . . . \quad (28)$$

and since

$$m = \frac{\rho y_o \pi^2}{32C^2}$$

then

$$\frac{V^2}{V_c^2} = \frac{\eta}{\sqrt{1 + (\frac{d\eta}{d\xi})^2}} \left\{1 + \frac{\pi^2}{32\eta}\frac{V_o^2}{V_c^2}\frac{d}{d\xi}\left[\eta\frac{d}{d\xi}(\frac{V}{V_o})^2\right]\right\} \quad . . . \quad (29)$$

The complicated nature of this expression would make it difficult to obtain a direct solution by integrating the differential equation which might be obtained by eliminating the ratio of $\frac{V}{V_o}$ between this equation and Equation (6). A trial method of solution will therefore be adopted instead. By this process a trial profile will be chosen and the ratio $(\frac{V}{V_o})^2$ will be computed for it from both the Drag Relation of Equation (6) and the Modified Chezy Relation of Equation (29). It will be considered that a satisfactory solution has been obtained when the two curves agree closely. An interesting feature of Equation (29) is that the quantity m does not occur in it. This indicates that the stable shape will fix a value for the ratio $(\frac{V_o}{V_c})$. This shape will also be independent of size.

For these purposes, if the relation between η and ξ can be assumed to be:

$$\eta = 1 + \alpha \xi^2 + \beta \xi^4 \quad \ldots \quad (30)$$

$$\frac{d\eta}{d\xi} = 2\alpha \xi + 4\beta \xi^3 \quad \ldots \quad (31)$$

or

$$\frac{y}{y_o} = 1 + \alpha\left(\frac{x}{y_o}\right)^2 + \beta\left(\frac{x}{y_o}\right)^4 \quad \ldots \quad (32)$$

$$\frac{dy}{dx} = 2\alpha\left(\frac{x}{y_o}\right) + 4\beta\left(\frac{x}{y_o}\right)^3 \quad \ldots \quad (33)$$

If

$$\frac{y}{y_o} = 0 \text{ and } \frac{dy}{dx} = -S \text{ when } x = a$$

then

$$0 = 1 + \alpha\left(\frac{a}{y_o}\right)^2 + \beta\left(\frac{a}{y_o}\right)^4 \quad \ldots \quad (34)$$

$$-S = 2\alpha\left(\frac{a}{y_o}\right) + 4\beta\left(\frac{a}{y_o}\right)^3 \quad \ldots \quad (35)$$

from which

$$\alpha = \frac{\left(\frac{S}{2}\frac{a}{y_o} - 2\right)}{\left(\frac{a}{y_o}\right)^2} \quad \ldots \quad (36)$$

$$\beta = \frac{\left(-\frac{S}{2}\frac{a}{y_o} + 1\right)}{\left(\frac{a}{y_o}\right)^4} \quad \ldots \quad (37)$$

DISCUSSION OF COMPILATION OF DATA AND CURVES

The computations in the tables at the end of this report are those used to obtain data for the curves shown on Figures 3 through 7. They consist of the development of the ratios of the mean velocity of the lamina at the distance x from the center of the channel to the mean velocity at the center for various top widths and transverse slopes. The procedure followed in making these calculations was to assume a value of $\frac{a}{y_o}$ for each value of S and to determine the ratio of $\frac{v}{v_o}$ from the Drag Relation and from the Modified Chezy Relation. When close agreement between the two curves was obtained, a satisfactory solution of Equation (29) was assumed to have been attained. In computing the values of $\frac{v}{v_o}$ from the Modified Chezy Relation, it was necessary to choose a value for the constant $\frac{v_o^2 m}{\rho f}$ which would make the curve agree with the one obtained from the Drag Relation. If agreement was not obtained, it was necessary to choose a new value of $\frac{a}{y_o}$ and make a new set of calculations.

It was found that the curves could be made to agree quite closely although $\frac{a}{y_o}$ was allowed to vary a great deal. Consequently, the greatest source of error was in choosing the value of $\frac{a}{y_o}$. It is believed that by careful adjustment of $\frac{a}{y_o}$ the curves could be made to coincide still more closely; however, the agreement shown in this set of curves is as close as is warranted by the approximations used in the development of these equations.

It may be noted that the solutions obtained by the modification for lateral transfer of drag agree very closely with the cosine solution. Since this is the case, the use of the cosine solution may give equally satisfactory results with considerably less work.

The range of transverse slope covered by this set of curves is from S = 0.3 to S = 1.0.

Graphical integration was used to evaluate $\frac{V_m}{V_o}$, $\frac{P}{y_o}$, $\frac{A}{y_o^2}$, and Q as follows:

$$\frac{V_m}{V_o} = \frac{2y_o^2}{A} \int_0^{\frac{a}{y_o}} \frac{V}{V_o} \eta \, d\xi \quad . . . \quad (38)$$

$$\frac{P}{y_o} = 2 \int_0^{\frac{a}{y_o}} \sqrt{1 + \left(\frac{d\eta}{d\xi}\right)^2} \, d\xi \quad . . . \quad (39)$$

$$\frac{A}{y_o^2} = 2 \int_0^{\frac{a}{y_o}} \eta \, d\xi \quad . . . \quad (40)$$

$$Q = 2V_o y_o^2 \int_0^{\frac{a}{y_o}} \frac{V}{V_o} \eta \, d\xi \quad . . . \quad (41)$$

It will be noted that the curve for $\frac{P}{y_o}$ is not used for solution of a problem by this method. This is because the concept of the wetted perimeter and the hydraulic radius is not in agreement with the drag distribution assumed in this solution. However, the curve is inserted to permit a comparison of results obtained by this method with similar results obtained from the ordinary formulas of hydraulics.

EXAMPLE

The procedure involved in the design of a stable channel can best be illustrated by the solution of an example.

For the particular type of soil through which the channel will run, the engineer will know or will be able to determine certain physical characteristics on which to base his design. It will be necessary to know the angle of repose of the soil and the maximum allowable drag force. An appropriate coefficient of roughness can be found in hydraulics tables, and the longitudinal slope on which the channel will run will be a predetermined factor. With these values specified, the quantity of water which the stable channel will carry will be a fixed value. Since it is desirable to specify the amount of water to be carried by the channel, it is necessary to have some method of modifying the stable channel shape to increase or decrease the flow. If the desired flow is greater than the stable channel shape would carry, it is proposed to insert a rectangular section at the center; and if the flow is to be less than the stable channel shape would carry, then a portion would be removed from the center. The portion to be removed in the latter case is easily determined by use of Figure 12.

The conditions assumed for the design of this particular channel are:

a. The tangent of the angle of repose of the soil is 0.6 ($S = 0.6$).

b. The material through which the channel is to run will stand a drag force of 0.1 pound per square foot ($\tau = 0.1$).

c. The longitudinal slope of the channel will be 0.0004 ($f = 0.0004$).

d. A reasonable value for the coefficient of roughness would be $n = 0.020$.

In a wide channel of uniform depth the drag force per unit of area of the bed would be $\tau = \rho fD$ and the velocity would be V_c. At the center of a stable channel, of equal depth, the lateral transfer of drag reduces the velocity from V_c to V_o and the drag relation there becomes

$$\tau = \rho f y_o \left(\frac{V_o}{V_c}\right)^2$$

An expression for the center depth can then be obtained from the above relation in the form

$$y_o = \frac{\tau}{\rho f \left(\frac{V_o}{V_c}\right)^2}$$

For the present case the upper curve of Figure 9 gives $\frac{V_o}{V_c} = 0.983$. Then

$$y_o = \frac{0.1}{(62.4)(0.0004)(0.983)^2} = 4.146 \text{ feet}$$

For an infinitely wide channel the following relation holds

$$V_c = C\sqrt{y_o f}$$

and for the conditions assumed above the value of C = 94.4 is obtained from hydraulic tables.

Then

$$V_c = 94.4\sqrt{(4.146)(0.0004)}$$

$$= 94.4\ (0.04072)$$

$$= 3.844 \text{ ft/sec}$$

From Figure 9

$$\frac{V_m}{V_o} = 0.869 \quad \text{and} \quad \frac{V_o}{V_c} = 0.983$$

Then

$$V_m = (0.869)(0.983)(3.844)$$

$$V_m = 3.28 \text{ ft/sec}$$

From Figure 8

$$\frac{A}{y_o^2} = 3.42$$

Or

$$A = (3.42)(4.146)^2$$

$$A = 58.79 \text{ ft}^2$$

Then since $Q = AV_m$

$$Q = (58.79)(3.28)$$

$$Q = 192.83 \text{ ft}^3/\text{sec}$$

From Figure 8 the value of $\frac{T}{y_o}$ is found to be 5.38, so that the top width of the stable channel is $5.38\ y_o$ or $(5.38)(4.146) = 22.30$. Having now obtained the top width and the center depth of the stable channel, it remains only to obtain the x and y coordinates of the channel bed. This is readily done by making use of Figure 11. For a transverse slope of $S = 0.6$ the following values of $\frac{x}{y_o}$ and $\frac{y}{y_o}$ are obtained and the values of x and y are calculated.

$\frac{x}{y_o}$	x	$\frac{y}{y_o}$	y
0	0	1.000	4.146
0.5	2.073	0.958	3.972
1.0	4.146	0.839	3.478
1.5	6.219	0.649	2.691
2.0	8.292	0.400	1.658
2.5	10.365	0.117	0.485
	11.15		0

If the channel is to carry less than the stable channel will carry, it is necessary to remove a portion of the channel. Suppose the channel is to carry 75 ft^3/sec. Then the ratio of Q to Q_o is $\frac{75}{192.83}$ or 0.389. From Figure 12 the value of $\frac{x}{a}$ is found to be 0.369, that is, 36.9 percent of the top width is to be removed from the center of the channel. The modified section with a top width of 14.07 feet is shown on Figure 13.

If the channel is to carry more than the stable channel will carry, it is necessary to add a rectangular section at the center. Assume that it is desired for the channel to carry 300 ft^3/sec, then the amount to be carried by the center section is 300 - 192.83 or 107.17 ft^3/sec. The mean velocity of the flow in an infinitely wide channel of depth y_o is V_c, which was determined to be 3.844 ft^3/sec in this case. For a center depth of 4.146 feet each foot of width would carry (4.146)(3.844) ft^3/sec or 15.94 ft^3/sec. The necessary width of the center section would then be $\frac{107.17}{15.94}$ or 6.72 feet.

It should be noted that there is a discrepancy between the velocity of the center section and the maximum velocity of the stable section. This discrepancy is relatively unimportant, however, since the difference is less than 2 percent of the maximum velocity for the center section; and, the lateral transfer of drag being small, the velocity at the edge of the center section would soon reach its maximum value at points nearer the center. The modified section with a top width of 29.02 feet is shown on Figure 13.

In conclusion, it may be noted that if water flowing in these channels carries sediment of the type coming from soil through which the channels run, it could be expected that the channels as designed would neither scour nor silt up. However, if clear water is to be carried, these channels would be stable for any slope less than 0.0004.

ACKNOWLEDGMENTS

The fundamental principles on which this work is based were laid down by Mr. E. W. Lane. This analysis has profited by the previous work of Mr. R. G. Conard who made many computations of the type shown in the tables and who succeeded in obtaining a close agreement between the Modified Chezy and Drag relations before he left the Bureau. The tables shown herein and the Figures 3 to 7, inclusive, represent a further refinement of Mr. Conard's work by Mr. Florey.

Table 1

Slope = 0.3

$(\frac{a}{y_o}) = 5.5$

α = - .038843

β = .00019124

$\frac{(12)}{1 + (11)}$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$(\frac{x}{y_o})$	$(\frac{x}{y_o})^2$	$(\frac{x}{y_o})^3$	$(\frac{x}{y_o})^4$	$\alpha(\frac{x}{y_o})^2$	$\beta(\frac{x}{y_o})^4$	$(\frac{y}{y_o})$	$2\alpha(\frac{x}{y_o})$	$4\beta(\frac{x}{y_o})^3$	$(\frac{dy}{dx})$	$(\frac{dy}{dx})^2$	$1 - \frac{1}{S^2}(\frac{dy}{dx})^2$	$(\frac{V}{V_o})^4$
0	0	0	0	0	0	1.00000	0	0	0	0	1.00000	1.00000
.4	.16	.064	.0256	-.00621	.000005	.99379	-.03107	.00005	-.03102	.00096	.98933	.98838
.8	.64	.512	.4096	-.02486	.00008	.97522	-.06215	.00039	-.06176	.00381	.95766	.95402
1.2	1.44	1.728	2.0736	-.05593	.00040	.94447	-.09322	.00132	-.09190	.00844	.90621	.89862
1.6	2.56	4.096	6.5536	-.09944	.00125	.90181	-.12429	.00313	-.12116	.01468	.83687	.82476
2.0	4.00	8.000	16.0000	-.15537	.00306	.84769	-.15537	.00612	-.14925	.02228	.75242	.73602
2.4	5.76	13.824	33.1776	-.22374	.00634	.78260	-.18644	.01057	-.17587	.03093	.65630	.63661
2.8	7.84	21.952	61.4656	-.30453	.01175	.70722	-.21751	.01679	-.20072	.04029	.55229	.53090
3.2	10.24	32.768	104.8576	-.39775	.02005	.62230	-.24858	.02507	-.22351	.04996	.44483	.42366
3.6	12.96	46.656	167.9616	-.50340	.03212	.52872	-.27966	.03569	-.24397	.05952	.33860	.31958
4.0	16.00	64.000	256.0000	-.62149	.04896	.42747	-.31073	.04896	-.26177	.06852	.23859	.22329
4.4	19.36	85.184	374.8096	-.75200	.07168	.31968	-.34180	.06516	-.27664	.07653	.14958	.13895
4.8	23.04	110.592	530.8416	-.89494	.10152	.20658	-.37288	.08460	-.28828	.08310	.07658	.07070
5.2	27.04	140.608	731.1616	-1.05030	.13983	.08953	-.40395	.10756	-.29639	.08785	.02379	.02187
5.5	30.25	166.375	915.0625	-1.17500	.17500	0	-.42726	.12727	-.29999	.08999	0	0

Slope = 0.3

$(\frac{a}{y_o}) = 5.5$

Table 2

$(\frac{V_o^2 m}{\rho f}) = .75$

(14) $(\frac{V}{V_o})^2$	(15) $\frac{d}{dx}(\frac{V}{V_o})^2$	(16) $(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2$	(17) $\frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	(18) $(\frac{y_o}{y})$	(19) $(\frac{V_o^2 m}{\rho f})(17)(18)$: $(\frac{V_o^2 m}{\rho f})(\frac{y_o}{y}) \cdot \frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	(20) 1+(19): 1 + last term	(21) $\sqrt{1 + (\frac{dy}{dx})^2}$	(22) $\frac{(7)}{(21)}$: $(\frac{V}{V_c})^2$	(23) (20)x(22): $(\frac{V}{V_c})^2$ Modified Chezy	(24) 1.05763 (23): $\frac{V^2}{V_o^2}$	(25) $\frac{V}{V_o}$	(26) $(\frac{V}{V_o})(\frac{y}{y_o})$
1.00000	.01458	.01453	-.07265	1.00000	-.05449	.94551	1.00000	1.00000	.94551	1.00000	1.00000	1.00000
.99417	.04358	.04290	-.07092	1.00625	-.05352	.94648	1.00048	.99331	.94015	.99433	.99716	.99097
.97674	.07195	.06906	-.06540	1.02541	-.05030	.94970	1.00190	.97337	.92441	.97768	.98878	.96428
.94796	.09950	.09185	-.05698	1.05879	-.04525	.95475	1.00421	.94051	.89795	.94970	.97452	.92040
.90816	.12560	.10987	-.04505	1.10888	-.03747	.96253	1.00731	.89526	.86171	.91137	.95466	.86092
.85792	.15010	.12235	-.03120	1.17968	-.02761	.97239	1.01108	.83840	.81525	.86223	.92856	.78713
.79788	.17312	.12896	-.01652	1.27779	-.01583	.98417	1.01535	.77077	.75857	.80229	.89571	.70098
.72863	.19435	.12920	-.00060	1.41399	-.00064	.99936	1.01995	.69339	.69295	.73288	.85608	.60544
.65089	.21395	.12313	.01518	1.60694	.01829	1.01829	1.02468	.60731	.61842	.65406	.80874	.50328
.56531	.23192	.11088	.03062	1.89136	.04343	1.04343	1.02933	.51365	.53596	.56685	.75289	.39807
.47254	.24945	.09319	.04422	2.33934	.07758	1.07758	1.03369	.41354	.44562	.47130	.68651	.29346
.37276	.26718	.07030	.05722	3.12813	.13424	1.13424	1.03756	.30811	.34947	.36961	.60796	.19435
.26589	.29502	.04368	.06655	4.84074	.24161	1.24161	1.04072	.19850	.24646	.26066	.51055	.10547
.14788	.49293	.02207	.07203	11.16944	.60340	1.60340	1.04300	.08584	.13763	.14556	.38152	.03416
0			-	∞	∞	∞	1.04402	0	0	0	0	0

Slope = 0.5 Table 3 $\alpha = -.1171875$

$\left(\frac{a}{y_o}\right) = 3.2$ $\beta = .00190735$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$\left(\frac{x}{y_o}\right)$	$\left(\frac{x}{y_o}\right)^2$	$\left(\frac{x}{y_o}\right)^3$	$\left(\frac{x}{y_o}\right)^4$	$\alpha\left(\frac{x}{y_o}\right)^2$	$\beta\left(\frac{x}{y_o}\right)^4$	$\left(\frac{y}{y_o}\right)$	$2\alpha\left(\frac{x}{y_o}\right)$	$4\beta\left(\frac{x}{y_o}\right)^3$	$\left(\frac{dy}{dx}\right)$	$\left(\frac{dy}{dx}\right)^2$	$1 - \frac{1}{S^2}\left(\frac{dy}{dx}\right)^2$	$\frac{(12)}{1 + (11)}$ $\left(\frac{V}{V_o}\right)^4$
0	0	0	0	0	0	1.00000	0	0	0	0	1.00000	1.00000
.2	.04	.008	.0016	-.00469	.00000	.99531	-.04688	.00006	-.04682	.00219	.99124	.98907
.4	.16	.064	.0256	-.01875	.00005	.98130	-.09375	.00049	-.09326	.00870	.96520	.95688
.6	.36	.216	.1296	-.04219	.00025	.95806	-.14063	.00165	-.13898	.01932	.92272	.90523
.8	.64	.512	.4096	-.07500	.00078	.92578	-.18750	.00391	-.18359	.03370	.86520	.83699
1.0	1.00	1.000	1.0000	-.11719	.00191	.88472	-.23438	.00763	-.22675	.05142	.79432	.75547
1.2	1.44	1.728	2.0736	-.16875	.00396	.83521	-.28125	.01318	-.26807	.07186	.71256	.66479
1.4	1.96	2.744	3.8416	-.22969	.00733	.77764	-.32813	.02094	-.30719	.09436	.62256	.56888
1.6	2.56	4.096	6.5536	-.30000	.01250	.71250	-.37500	.03125	-.34375	.11816	.52736	.47163
1.8	3.24	5.832	10.4976	-.37969	.02002	.64033	-.42188	.04449	-.37739	.14242	.43032	.37667
2.0	4.00	8.000	16.0000	-.46875	.03052	.56177	-.46875	.06104	-.40771	.16623	.33508	.28732
2.2	4.84	10.648	23.4256	-.56719	.04468	.47749	-.51563	.08124	-.43439	.18869	.24524	.20631
2.4	5.76	13.824	33.1776	-.67500	.06328	.38828	-.56250	.10547	-.45703	.20888	.16448	.13606
2.6	6.76	17.576	45.6976	-.79219	.08716	.29497	-.60938	.13409	-.47529	.22590	.09640	.07864
2.8	7.84	21.952	61.4656	-.91875	.11724	.19849	-.65625	.16748	-.48877	.23890	.04440	.03584
3.0	9.00	27.000	81.0000	-1.05469	.15450	.09981	-.70313	.20599	-.49714	.24715	.01140	.00914
3.2	10.24	32.768	104.8576	-1.20000	.20000	0	-.75000	.25000	-.50000	.25000	0	0

Slope = 0.5

$(\frac{a}{y_o}) = 3.2$

Table 4

$(\frac{V_o^2 m}{\rho f}) = 0.060$

(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
					$(\frac{V_o^2 m}{\rho f})$ (17)(18)	1+(19)		$\frac{(7)}{(21)}$	(20)x(22)	1.01667 (23)		
$(\frac{V}{V_o})^2$	$\frac{d}{dx}(\frac{V}{V_o})^2$	$(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2$	$\frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	$(\frac{y_o}{y})$	$(\frac{V_o^2 m}{\rho f})(\frac{y_o}{y}) \cdot \frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	1 + last term	$\sqrt{1 + (\frac{dy}{dx})^2}$	$(\frac{V}{V_c})^2$	$(\frac{V}{V_c})^2$ Modified Chezy	$\frac{V^2}{V_o^2}$	$\frac{V}{V_o}$	$(\frac{V}{V_o})(\frac{y}{y_o})$
1.00000	.02740	.02733	-.27330	1.00000	-.01640	.98360	1.00000	1.00000	.98360	1.00000	1.00000	1.00000
.99452	.08160	.08064	-.26655	1.00471	-.01607	.98393	1.00109	.99423	.97825	.99456	.99728	.99260
.97820	.13380	.12974	-.24550	1.01906	-.01501	.98499	1.00434	.97706	.96239	.97843	.98916	.97066
.95144	.18285	.17223	-.21245	1.04378	-.01330	.98670	1.00961	.94894	.93632	.95193	.97567	.93475
.91487	.22845	.20680	-.17285	1.08017	-.01120	.98880	1.01671	.91056	.90036	.91537	.95675	.88574
.86918	.26915	.23146	-.12330	1.13030	-.00836	.99164	1.02539	.86281	.85560	.86986	.93266	.82514
.81535	.30555	.24640	-.07470	1.19730	-.00537	.99463	1.03531	.80672	.80239	.81576	.90319	.75435
.75424	.33745	.25142	-.02510	1.28594	-.00194	.99806	1.04612	.74336	.74192	.75429	.86850	.67538
.68675	.36510	.24696	.02230	1.40351	.00188	1.00188	1.05743	.67380	.67507	.68632	.82844	.59026
.61373	.38855	.23354	.06710	1.56169	.00629	1.00629	1.06884	.59909	.60286	.61291	.78288	.50130
.53602	.40905	.21255	.10495	1.78009	.01121	1.01121	1.07992	.52020	.52603	.53480	.73130	.41082
.45421	.42675	.18473	.13910	2.09428	.01748	1.01748	1.09027	.43796	.44562	.45305	.67309	.32139
.36886	.44215	.15105	.16840	2.57546	.02602	1.02602	1.09949	.35314	.36233	.36837	.60693	.23566
.28043	.45560	.11241	.19320	3.39018	.03930	1.03930	1.10720	.26641	.27688	.28150	.53056	.15650
.18931	.46855	.06988	.21265	5.03804	.06428	1.06428	1.11306	.17833	.18979	.19295	.43926	.08719
.09560	.47800	.02385	.23015	10.01904	.13835	1.13835	1.11676	.08937	.10173	.10342	.32159	.03210
0			-	∞	∞	∞	1.11803	0	0	0	0	0

Table 5

Slope = 0.6

$\left(\frac{a}{y_o}\right) = 2.70$

$\alpha = -.1632373$

$\beta = .00357518$

$\frac{(12)}{1 + (11)}$

(1) $\left(\frac{x}{y_o}\right)$	(2) $\left(\frac{x}{y_o}\right)^2$	(3) $\left(\frac{x}{y_o}\right)^3$	(4) $\left(\frac{x}{y_o}\right)^4$	(5) $\alpha\left(\frac{x}{y_o}\right)^2$	(6) $\beta\left(\frac{x}{y_o}\right)^4$	(7) $\left(\frac{y}{y_o}\right)$	(8) $2\alpha\left(\frac{x}{y_o}\right)$	(9) $4\beta\left(\frac{x}{y_o}\right)^3$	(10) $\left(\frac{dy}{dx}\right)$	(11) $\left(\frac{dy}{dx}\right)^2$	(12) $1 - \frac{1}{S^2}\left(\frac{dy}{dx}\right)^2$	(13) $\left(\frac{V}{V_o}\right)^4$
0	0	0	0	0	0	1.00000	0	0	0	0	1.00000	1.00000
.2	.04	.008	.0016	-.00653	.00001	.99348	-.06529	.00011	-.06518	.00425	.98819	.98401
.4	.16	.064	.0256	-.02612	.00009	.97397	-.13059	.00092	-.12967	.01681	.95330	.93754
.6	.36	.216	.1296	-.05876	.00046	.94170	-.19588	.00309	-.19279	.03717	.89675	.86461
.8	.64	.512	.4096	-.10447	.00146	.89699	-.26118	.00732	-.25386	.06444	.82100	.77130
1.0	1.00	1.000	1.0000	-.16324	.00358	.84034	-.32647	.01430	-.31217	.09745	.72930	.66454
1.2	1.44	1.728	2.0736	-.23506	.00741	.77235	-.39177	.02471	-.36706	.13473	.62575	.55145
1.4	1.96	2.744	3.8416	-.31994	.01373	.69379	-.45706	.03924	-.41782	.17457	.51508	.43853
1.6	2.56	4.096	6.5536	-.41789	.02343	.60554	-.52236	.05858	-.46378	.21509	.40253	.33128
1.8	3.24	5.832	10.4976	-.52889	.03753	.50864	-.58765	.08340	-.50425	.25427	.29369	.23415
2.0	4.00	8.000	16.0000	-.65295	.05720	.40425	-.65295	.11440	-.53855	.29004	.19433	.15064
2.2	4.84	10.648	23.4256	-.79007	.08375	.29368	-.71824	.15227	-.56597	.32032	.11022	.08348
2.4	5.76	13.824	33.1776	-.94025	.11862	.17837	-.78354	.19769	-.58585	.34322	.04661	.03470
2.6	6.76	17.576	45.6976	-1.10348	.16338	.05990	-.84883	.25135	-.59748	.35698	.00839	.00618
2.7	7.29	19.683	53.1441	-1.19000	.19000	0	-.88148	.28148	-.60000	.36000	0	0

Slope = 0.6

$(\frac{a}{y_o}) = 2.7$

Table 6

$(\frac{V_o^2 m}{\rho f}) = 0.075$

(14) $(\frac{V}{V_o})^2$	(15) $\frac{d}{dx}(\frac{V}{V_o})^2$	(16) $(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2$	(17) $\frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	(18) $(\frac{y_o}{y})$	$(\frac{V_o^2 m}{\rho f})$ (17)(18) (19) $(\frac{V_o^2 m}{\rho f})(\frac{y_o}{y}) \cdot \frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	1+(19) (20) 1 + last term	(21) $\sqrt{1 + (\frac{dy}{dx})^2}$	$\frac{(7)}{(21)}$ (22) $(\frac{V}{V_c})^2$	(20)x(22) (23) $(\frac{V}{V_c})^2$ Modified Chezy	1.03095 (23) (24) $\frac{V^2}{V_o^2}$	(25) $\frac{V}{V_o}$	(26) $(\frac{V}{V_o})(\frac{y}{y_o})$
1.00000	.04015	.04002	-.40020	1.00000	-.03002	.96998	1.00000	1.00000	.96998	1.00000	1.00000	1.00000
.99197	.11850	.11657	-.38275	1.00656	-.02889	.97111	1.00212	.99138	.96274	.99254	.99626	.98976
.96827	.19215	.18236	-.32895	1.02672	-.02533	.97467	1.00837	.96588	.94141	.97055	.98516	.95952
.92984	.25800	.23719	-.27415	1.06191	-.02183	.97817	1.01842	.92467	.90448	.93247	.96564	.90934
.87824	.31525	.27384	-.18325	1.11484	-.01532	.98468	1.03172	.86941	.85609	.88258	.93946	.84269
.81519	.36295	.29266	-.09410	1.18999	-.00840	.99160	1.04759	.80216	.79542	.82004	.90556	.76098
.74260	.40190	.29462	-.00980	1.29475	-.00095	.99905	1.06524	.72505	.72436	.74678	.86416	.66743
.66222	.43325	.28146	.06580	1.44136	.00711	1.00711	1.08378	.64016	.64471	.66466	.81527	.56563
.57557	.45840	.25537	.13045	1.65142	.01616	1.01616	1.10231	.54934	.55822	.57550	.75862	.45937
.48389	.47885	.21857	.18400	1.96603	.02713	1.02713	1.11994	.45417	.46649	.48093	.69349	.35274
.38812	.49595	.17307	.22750	2.47372	.04221	1.04221	1.13580	.35592	.37094	.38242	.61840	.24999
.28893	.51325	.12114	.25965	3.40507	.06631	1.06631	1.14905	.25558	.27253	.28096	.53006	.15567
.18628	.53835	.06414	.28500	5.60632	.11984	1.11958	1.15897	.15390	.17230	.17763	.42146	.07518
.07861	.78610	.02354	.40600	16.69449	.50835	1.50835	1.16489	.05142	.07756	.07996	.28277	.01694
0			-	∞	∞	∞	1.16619	0	0	0	0	0

Slope = 0.80 Table 7 $\alpha = -.30000$

$(\frac{a}{y_o}) = 2.0$ $\beta = .0125$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
$(\frac{x}{y_o})$	$(\frac{x}{y_o})^2$	$(\frac{x}{y_o})^3$	$(\frac{x}{y_o})^4$	$\alpha(\frac{x}{y_o})^2$	$\beta(\frac{x}{y_o})^4$	$(\frac{y}{y_o})$	$2\alpha(\frac{x}{y_o})$	$4\beta(\frac{x}{y_o})^3$	$(\frac{dy}{dx})$	$(\frac{dy}{dx})^2$	$1 - \frac{1}{S^2}(\frac{dy}{dx})^2$	$\frac{(12)}{1 + (11)}$ $(\frac{V}{V_o})^4$
0	0	0	0	0	0	1.00000	0	0	0	0	1.00000	1.00000
.1	.01	.001	.0001	-.003	0	.99700	-.06	.00005	-.05995	.00359	.99439	.99083
.2	.04	.008	.0016	-.012	.00002	.98802	-.12	.00040	-.11960	.01430	.97766	.96388
.3	.09	.027	.0081	-.027	.00010	.97310	-.18	.00135	-.17865	.03192	.95012	.92073
.4	.16	.064	.0256	-.048	.00032	.95232	-.24	.00320	-.23680	.05607	.91239	.86395
.5	.25	.125	.0625	-.075	.00078	.92578	-.30	.00625	-.29375	.08629	.86517	.79644
.6	.36	.216	.1296	-.108	.00162	.89362	-.36	.01080	-.34920	.12194	.80947	.72149
.7	.49	.343	.2401	-.147	.00300	.85600	-.42	.01715	-.40285	.16229	.74642	.64220
.8	.64	.512	.4096	-.192	.00512	.81312	-.48	.02560	-.45440	.20648	.67738	.56145
.9	.81	.729	.6561	-.243	.00820	.76520	-.54	.03645	-.50355	.25356	.60381	.48168
1.0	1.00	1.000	1.0000	-.30	.01250	.71250	-.60	.05000	-.55000	.30250	.52734	.40487
1.1	1.21	1.331	1.4641	-.363	.01830	.65530	-.66	.06655	-.59345	.35218	.44972	.33259
1.2	1.44	1.728	2.0736	-.432	.02592	.59392	-.72	.08640	-.63360	.40145	.37273	.26596
1.3	1.69	2.197	2.8561	-.507	.03570	.52870	-.78	.10985	-.67015	.44910	.29828	.20584
1.4	1.96	2.744	3.8416	-.588	.04802	.46002	-.84	.13720	-.70280	.49393	.22823	.15277
1.5	2.25	3.375	5.0625	-.675	.06328	.38828	-.90	.16875	-.73125	.53473	.16448	.10717
1.6	2.56	4.096	6.5536	-.768	.08192	.31392	-.96	.20480	-.75520	.57033	.10886	.06932
1.7	2.89	4.913	8.3521	-.867	.10440	.23740	-1.02	.24565	-.77435	.59962	.06309	.03944
1.8	3.24	5.832	10.4976	-.972	.13122	.15922	-1.08	.29160	-.78840	.62157	.02880	.01776
1.9	3.61	6.859	13.0321	-1.083	.16290	.07990	-1.14	.34295	-.79705	.63529	.00736	.00450
2.0	4.00	8.000	16.0000	-1.200	.20000	0	-1.20	.40000	-.80000	.64000	0	0

10726

Slope = 0.80

$(\frac{a}{y_o}) = 2.0$

Table 8

$(\frac{V_o^2 m}{\rho f}) = 0.020$

					$(\frac{V_o^2 m}{\rho f})$ (17)(18)	1+(19)		$\frac{(7)}{(21)}$	(20)x(22)	1.01871 (23)		
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
$(\frac{V}{V_o})^2$	$\frac{d}{dx}(\frac{V}{V_o})^2$	$(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2$	$\frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	$(\frac{y_o}{y})$	$(\frac{V_o^2 m}{\rho f})(\frac{y_o}{y}) \cdot \frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	1 + last term	$\sqrt{1 + (\frac{dy}{dx})^2}$	$(\frac{V}{V_c})^2$	$(\frac{V}{V_c})^2$ Modified Chezy	$\frac{V^2}{V_o^2}$	$\frac{V}{V_o}$	$(\frac{V}{V_o})(\frac{y}{y_o})$
1.00000			-.91860	1.00000	-.01837	.98163	1.00000	1.00000	.98163	1.00000	1.00000	1.00000
.99540	.04600	.04593	-.89350	1.00301	-.01792	.98208	1.00179	.99522	.97738	.99567	.99783	.99484
.98177	.13630	.13528	-.82600	1.01212	-.01672	.98328	1.00712	.98104	.96464	.98269	.99131	.97943
.95955	.22220	.21788	-.71510	1.02764	-.01470	.98530	1.01583	.95794	.94386	.96152	.98057	.95419
.92949	.30060	.28939	-.58630	1.05007	-.01231	.98769	1.02765	.92670	.91529	.93242	.96562	.91958
.89243	.37060	.34801	-.43430	1.08017	-.00938	.99062	1.04225	.88825	.87992	.89638	.94677	.87650
.84940	.43030	.39144	-.28730	1.11904	-.00643	.99357	1.05922	.84366	.83824	.85392	.92408	.82578
.80137	.48030	.42017	-.14390	1.16822	-.00336	.99664	1.07810	.79399	.79132	.80612	.89784	.76855
.74930	.52070	.43456	-.01610	1.22983	-.00040	.99960	1.09839	.74028	.73998	.75382	.86823	.70598
.69403	.55270	.43617	.09560	1.30685	.00250	1.00250	1.11962	.68345	.68516	.69798	.83545	.63929
.63629	.57740	.42661	.19140	1.40351	.00537	1.00537	1.14127	.62430	.62765	.63939	.79962	.56973
.57671	.59580	.40747	.26460	1.52602	.00808	1.00808	1.16283	.56354	.56809	.57872	.76074	.49851
.51571	.61000	.38101	.32940	1.68373	.01109	1.01109	1.18383	.50169	.50725	.51674	.71885	.42694
.45370	.62010	.34807	.37410	1.89143	.01415	1.01415	1.20378	.43920	.44541	.45374	.67360	.35613
.39086	.62840	.31066	.41370	2.17382	.01799	1.01799	1.22226	.37637	.38314	.39031	.62475	.28740
.32737	.63490	.26929	.44310	2.57546	.02282	1.02282	1.23884	.31342	.32057	.32657	.57146	.22189
.26329	.64080	.22498	.46660	3.18552	.02973	1.02973	1.25313	.25051	.25796	.26279	.51263	.16092
.19860	.64690	.17832	.48760	4.21230	.04108	1.04108	1.26476	.18770	.19541	.19907	.44617	.10592
.13327	.65330	.12956	.50420	6.28062	.06333	1.06333	1.27341	.12503	.13295	.13544	.36802	.05860
.06708	.66190	.07914	.52340	12.51564	.13101	1.13101	1.27878	.06248	.07066	.07198	.26829	.02144
0	.67080	.02680	-	∞	∞	∞	1.28062	0	0	0	0	0

Table 9

Slope = 1.0

$(\frac{a}{y_o}) = 1.6$

$\alpha = -.46875$

$\beta = .03051758$

$\frac{(12)}{1 + (11)}$

(1) $(\frac{x}{y_o})$	(2) $(\frac{x}{y_o})^2$	(3) $(\frac{x}{y_o})^3$	(4) $(\frac{x}{y_o})^4$	(5) $\alpha(\frac{x}{y_o})^2$	(6) $\beta(\frac{x}{y_o})^4$	(7) $(\frac{y}{y_o})$	(8) $2\alpha(\frac{x}{y_o})$	(9) $4\beta(\frac{x}{y_o})^3$	(10) $(\frac{dy}{dx})$	(11) $(\frac{dy}{dx})^2$	(12) $1 - \frac{1}{S^2}(\frac{dy}{dx})^2$	(13) $(\frac{V}{V_o})^4$
0	0	0	0	0	0	1.00000	0	0	0	0	1.00000	1.00000
.1	.01	.001	.0001	-.00469	0	.99531	-.09375	.00012	-.09363	.00877	.99123	.98261
.2	.04	.008	.0016	-.01875	.00005	.98130	-.18750	.00098	-.18652	.03479	.96521	.93276
.3	.09	.027	.0081	-.04219	.00025	.95806	-.28125	.00330	-.27795	.07726	.92275	.85657
.4	.16	.064	.0256	-.07500	.00078	.92578	-.37500	.00781	-.36719	.13483	.86517	.76238
.5	.25	.125	.0625	-.11719	.00191	.88472	-.46875	.01526	-.45349	.20565	.79435	.65886
.6	.36	.216	.1296	-.16875	.00396	.83521	-.56250	.02637	-.53613	.28744	.71256	.55347
.7	.49	.343	.2401	-.22969	.00733	.77764	-.65625	.04187	-.61438	.37746	.62254	.45195
.8	.64	.512	.4096	-.30000	.01250	.71250	-.75000	.06250	-.68750	.47266	.52734	.35809
.9	.81	.729	.6561	-.37969	.02002	.64033	-.84375	.08899	-.75476	.56966	.43034	.27416
1.0	1.00	1.000	1.0000	-.46875	.03052	.56177	-.93750	.12207	-.81543	.66493	.33507	.20125
1.1	1.21	1.331	1.4641	-.56719	.04468	.47749	-1.03125	.16248	-.86877	.75476	.24524	.13976
1.2	1.44	1.728	2.0736	-.67500	.06328	.38828	-1.12500	.21094	-.91406	.83550	.16450	.08962
1.3	1.69	2.197	2.8561	-.79219	.08716	.29497	-1.21875	.26819	-.95056	.90356	.09644	.05066
1.4	1.96	2.744	3.8416	-.91875	.11724	.19849	-1.31250	.33496	-.97754	.95558	.04442	.02271
1.5	2.25	3.375	5.0625	-1.05469	.15450	.09981	-1.40625	.41199	-.99426	.98855	.01145	.00576
1.6	2.56	4.096	6.5536	-1.20000	.20000	0	-1.50000	.50000	-1.00000	1.00000	0	0

Slope = 1.0

$(\frac{a}{y_o}) = 1.6$

Table 10

$(\frac{V_o^2 m}{\rho f}) = .009$

(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
					$(\frac{V_o^2 m}{\rho f})$ (17)(18)	1+(19)		$\frac{(7)}{(21)}$	(20)x(22)	1.11752 (23)		
$(\frac{V}{V_o})^2$	$\frac{d}{dx}(\frac{V}{V_o})^2$	$(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2$	$\frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	$(\frac{y_o}{y})$	$(\frac{V_o^2 m}{\rho f})(\frac{y_o}{y})\cdot\frac{d}{dx}\left[(\frac{y}{y_o})\frac{d}{dx}(\frac{V}{V_o})^2\right]$	1 + last term	$\sqrt{1+(\frac{dy}{dx})^2}$	$(\frac{V}{V_c})^2$	$(\frac{V}{V_c})^2$ Modified Chezy	$\frac{V^2}{V_o^2}$	$\frac{V}{V_o}$	$(\frac{V}{V_o})(\frac{y}{y_o})$
1.00000	.08730	.08710	-1.74200	1.00000	-.01568	.98432	1.00000	1.00000	.98432	1.00000	1.00000	1.00000
.99127	.25470	.25172	-1.64620	1.00471	-.01488	.98512	1.00438	.99097	.97622	.99177	.99588	.99121
.96580	.40290	.39068	-1.38960	1.01906	-.01274	.98726	1.01725	.96466	.95237	.96754	.98364	.96524
.92551	.52370	.49328	-1.02600	1.04378	-.00964	.99036	1.03791	.92307	.91417	.92873	.96371	.92329
.87314	.61440	.55618	-.62900	1.08017	-.00611	.99389	1.06528	.86905	.86374	.87750	.93675	.86722
.81170	.67740	.58254	-.26360	1.13030	-.00268	.99732	1.09802	.80574	.80358	.81638	.90354	.79938
.74396	.71690	.57812	.04420	1.19730	.00048	1.00048	1.13465	.73609	.73644	.74817	.86497	.72243
.67227	.73860	.55031	.27810	1.28594	.00322	1.00322	1.17365	.66258	.66471	.67530	.82177	.63904
.59841	.74810	.50603	.44280	1.40351	.00559	1.00559	1.21353	.58713	.59041	.59982	.77448	.55182
.52360	.74990	.45073	.55300	1.56169	.00777	1.00777	1.25286	.51109	.51506	.52326	.72337	.46320
.44861	.74770	.38853	.62200	1.78009	.00996	1.00996	1.29032	.43537	.43971	.44671	.66836	.37546
.37384	.74480	.32241	.66120	2.09428	.01246	1.01246	1.32467	.36046	.36495	.37076	.60890	.29074
.29936	.74280	.25376	.68650	2.57546	.01591	1.01591	1.35481	.28659	.29115	.29579	.54386	.21117
.22508	.74380	.18352	.70240	3.39018	.02143	1.02143	1.37970	.21379	.21837	.22185	.47101	.13893
.15070	.74810	.11158	.71940	5.03804	.03262	1.03262	1.39842	.14194	.14657	.14890	.38588	.07659
.07589	.75890	.03787	.73710	10.01904	.06646	1.06646	1.41016	.07078	.07548	.07668	.27691	.02764
0			-	∞	∞	∞	1.41421	0	0	0	0	0

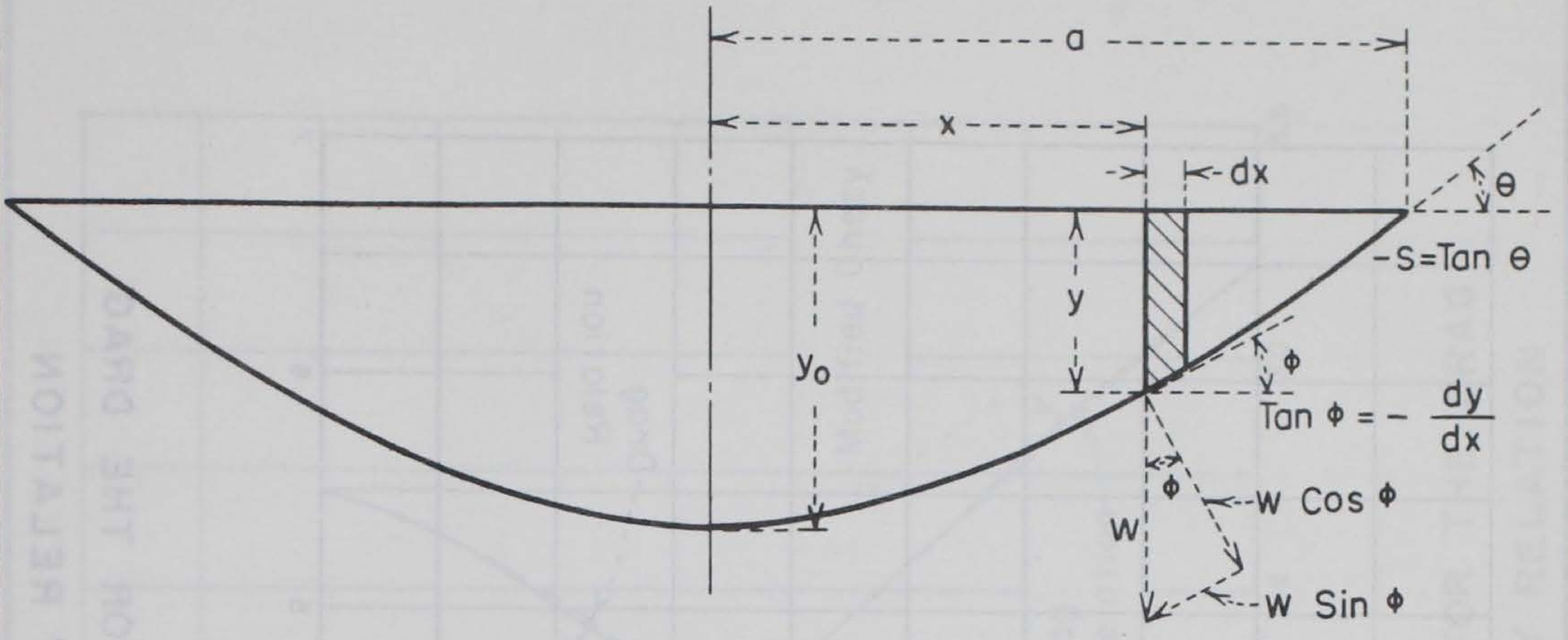

FIGURE 1

FIGURE 2

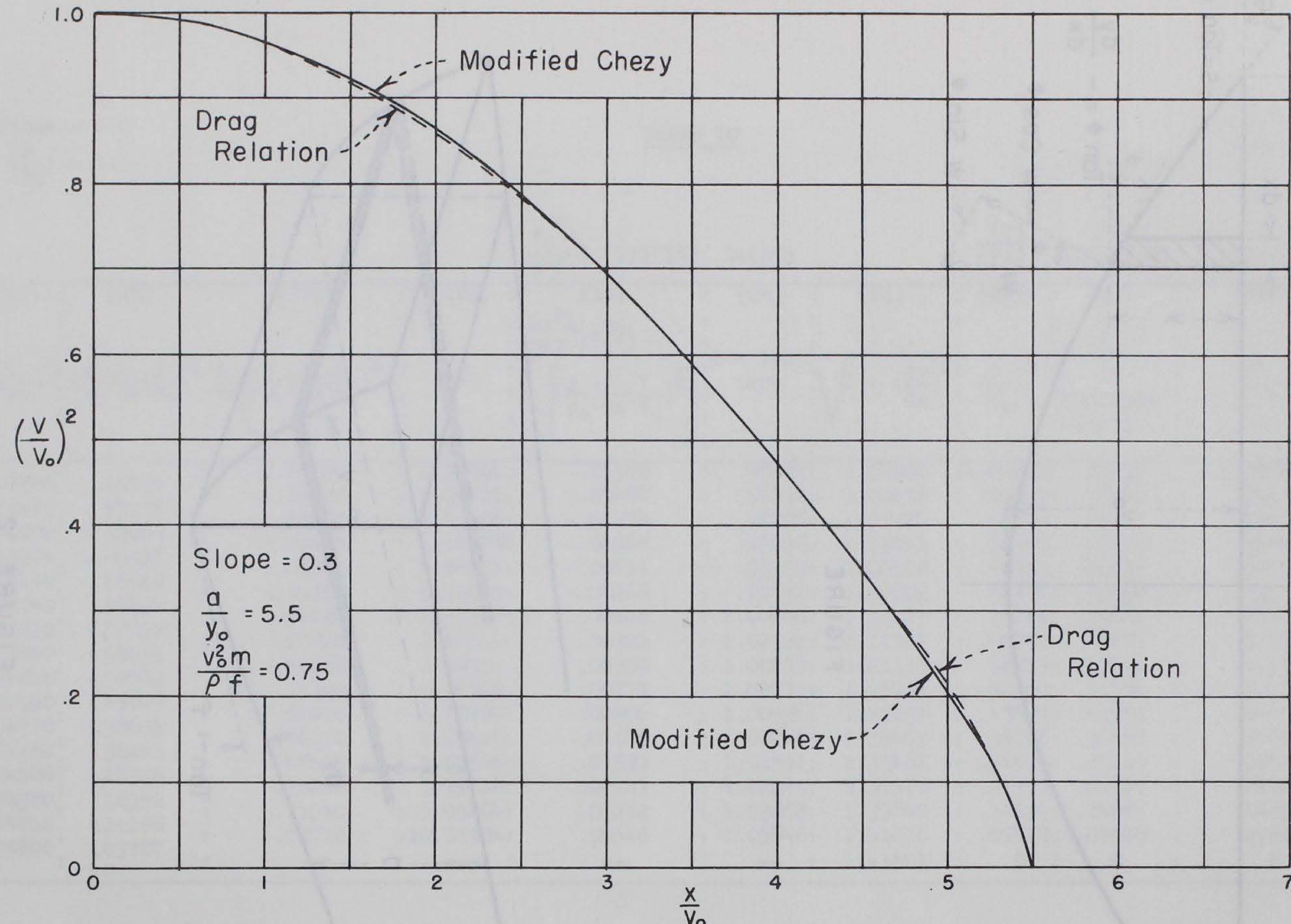

FIGURE 3

COMPARISON OF THE VALUES OF $\left(\frac{V}{V_o}\right)^2$ FOR THE DRAG RELATION AND THE MODIFIED CHEZY RELATION

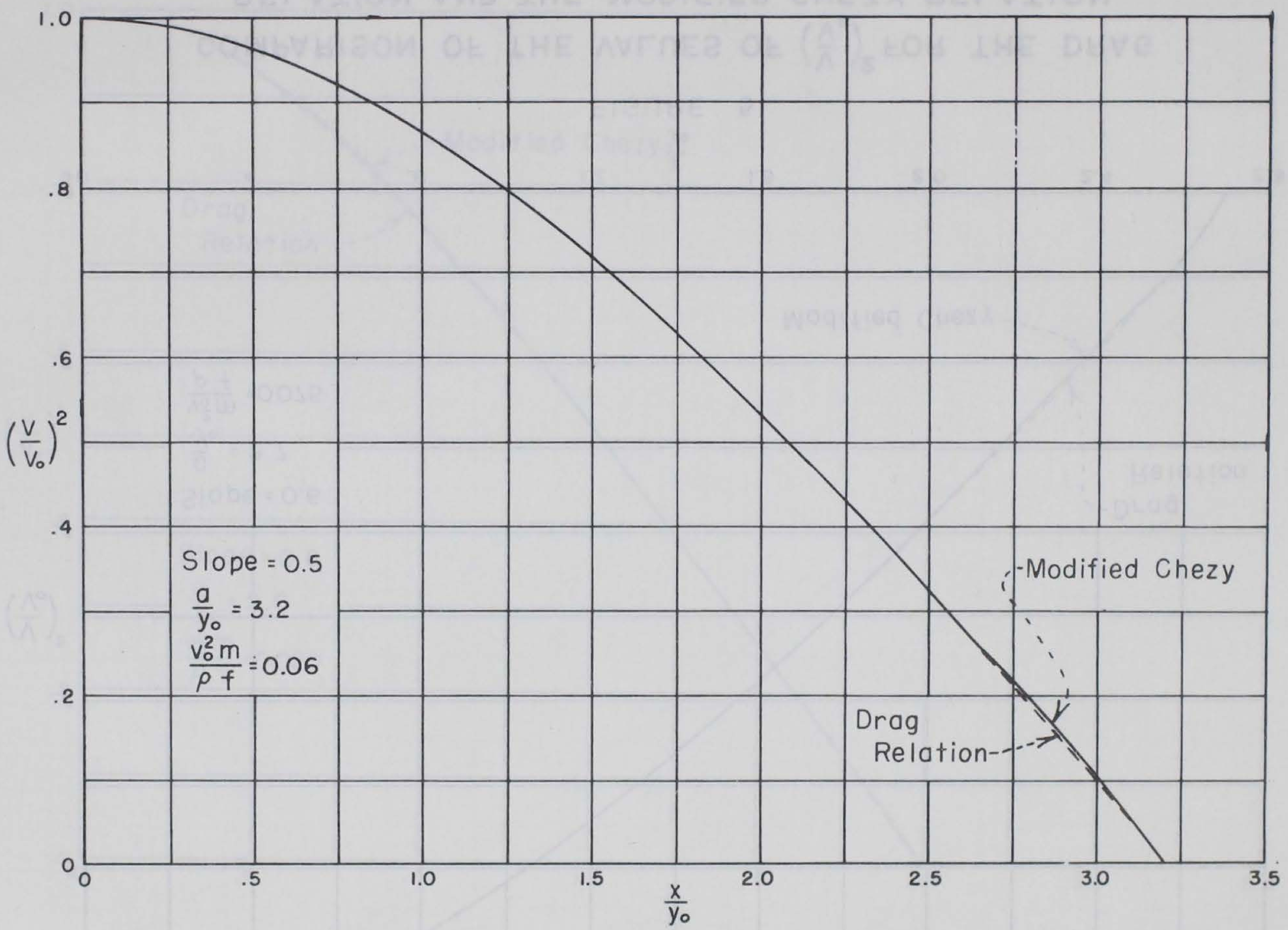

FIGURE 4

COMPARISON OF THE VALUES OF $\left(\frac{V}{V_o}\right)^2$ FOR THE DRAG RELATION AND THE MODIFIED CHEZY RELATION

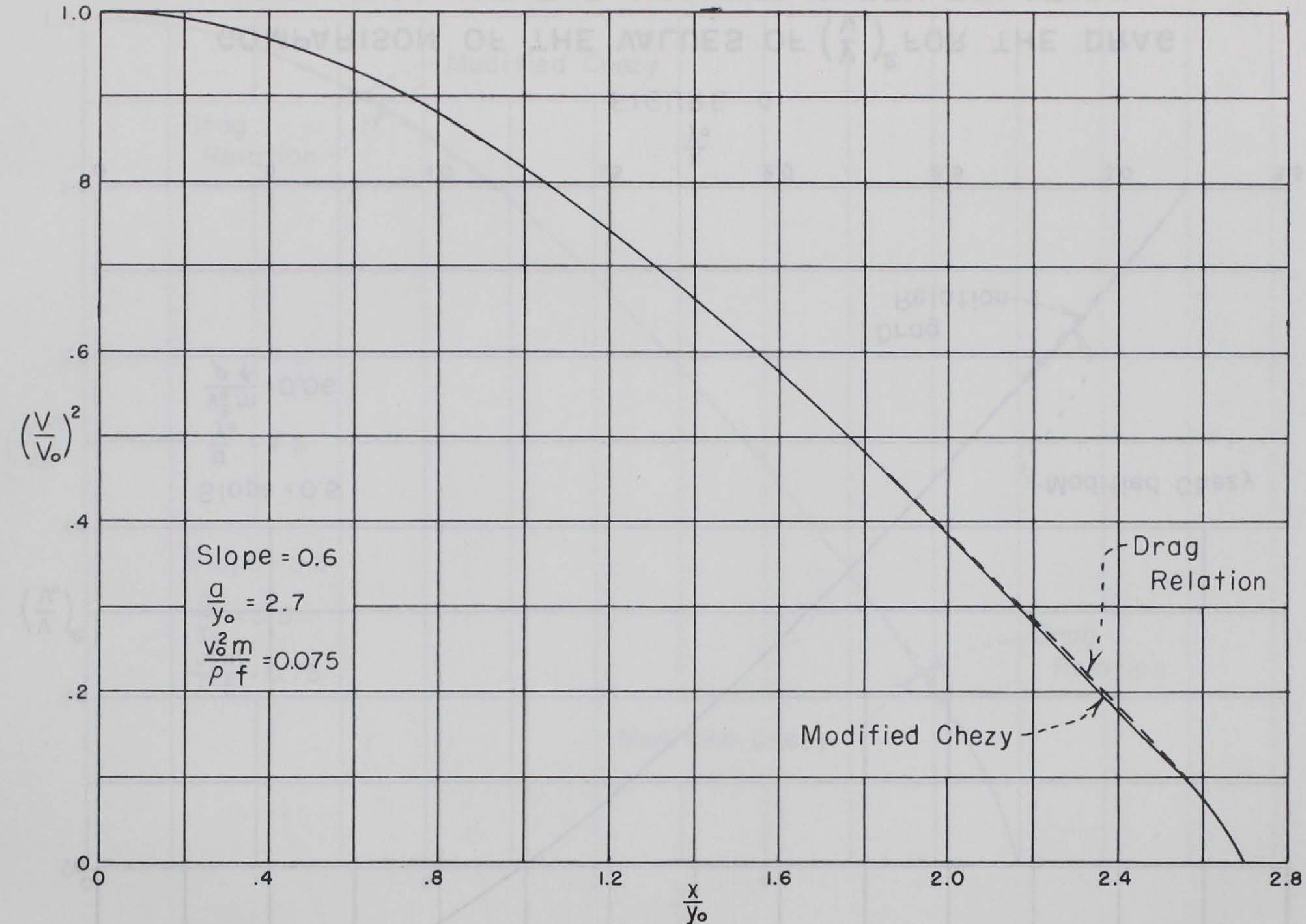

FIGURE 5

COMPARISON OF THE VALUES OF $\left(\frac{V}{V_o}\right)^2$ FOR THE DRAG RELATION AND THE MODIFIED CHEZY RELATION

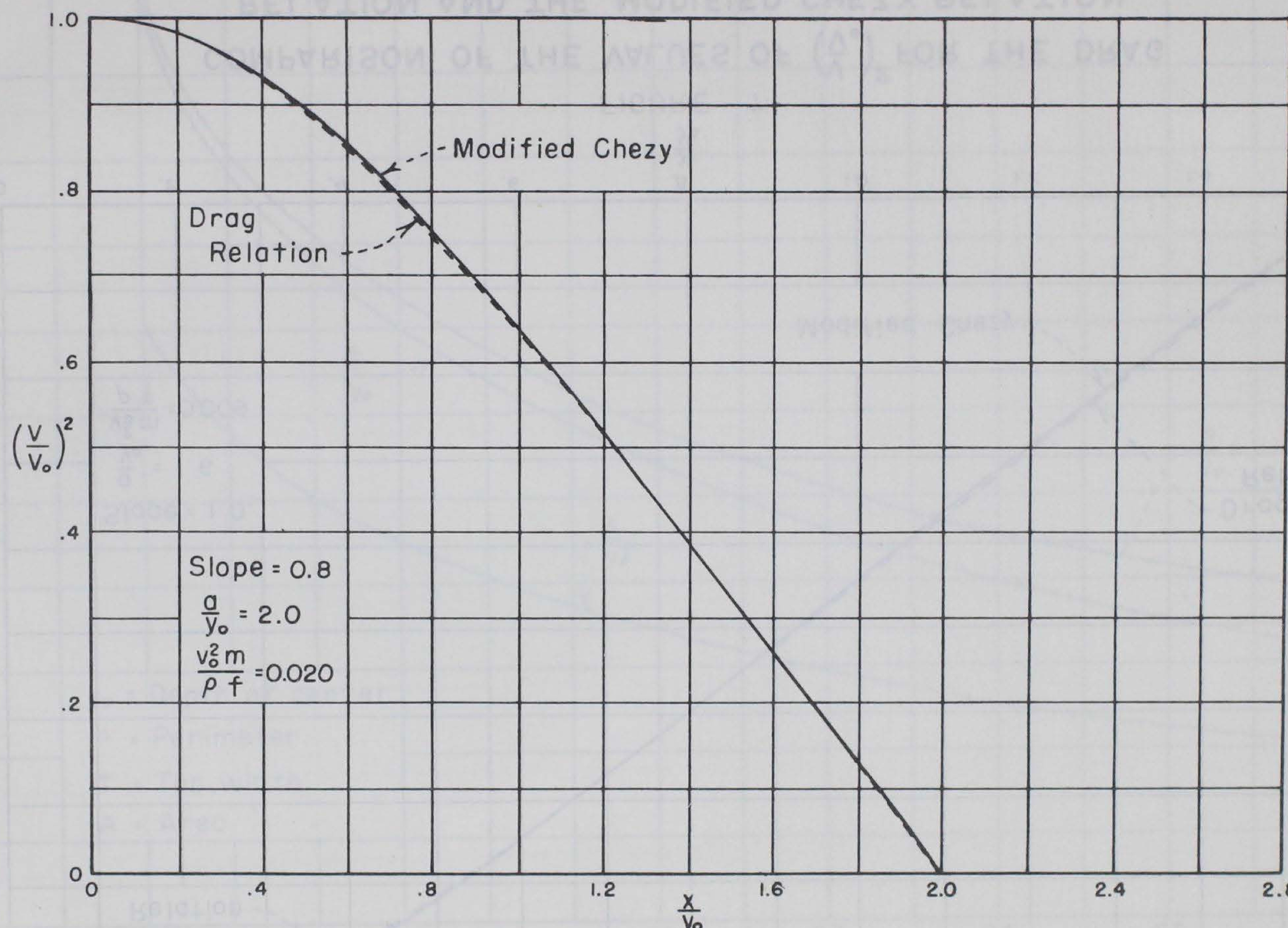

FIGURE 6

COMPARISON OF THE VALUES OF $\left(\frac{V}{V_o}\right)^2$ FOR THE DRAG RELATION AND THE MODIFIED CHEZY RELATION

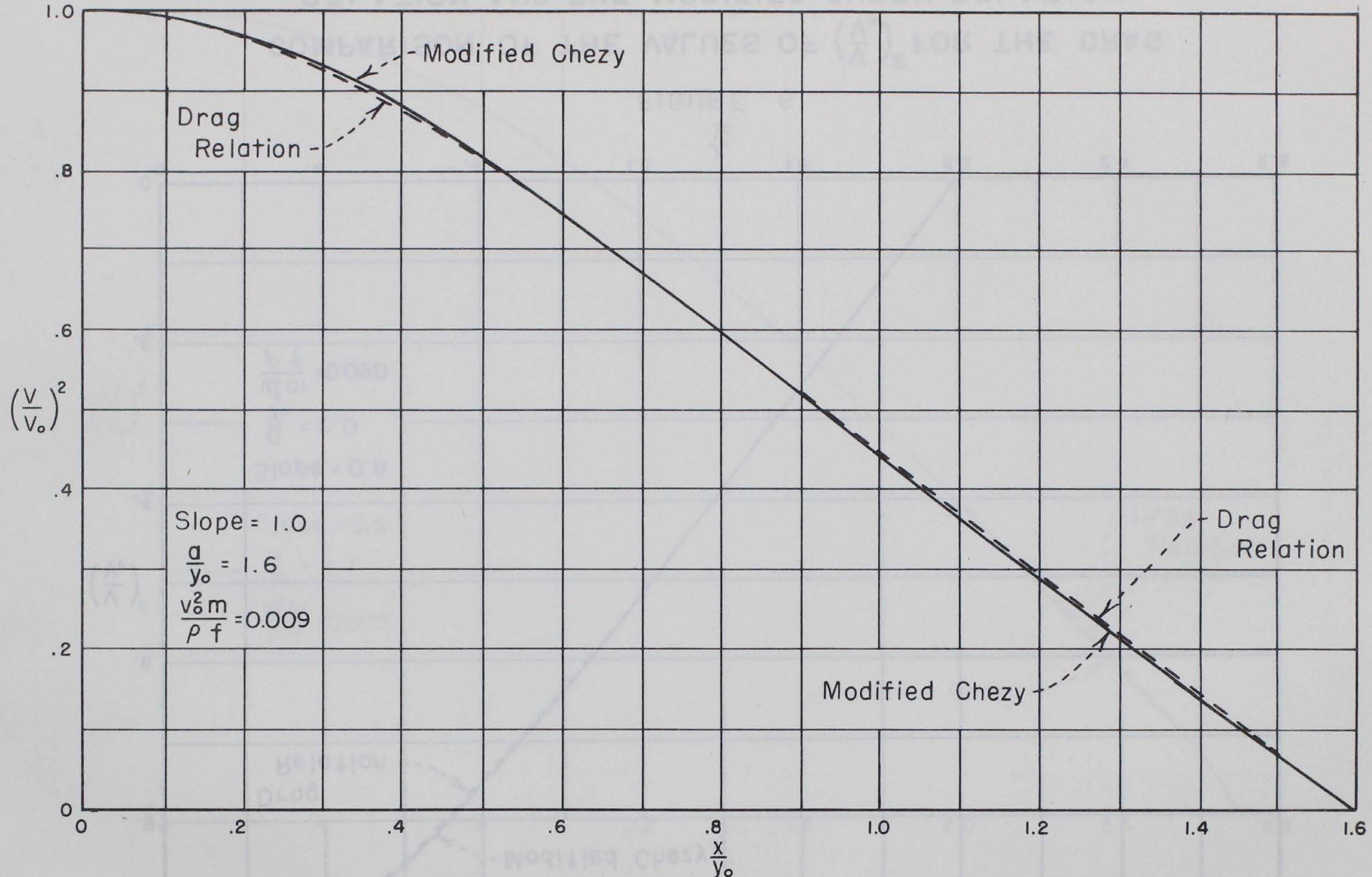

FIGURE 7

COMPARISON OF THE VALUES OF $\left(\frac{V}{V_o}\right)^2$ FOR THE DRAG RELATION AND THE MODIFIED CHEZY RELATION

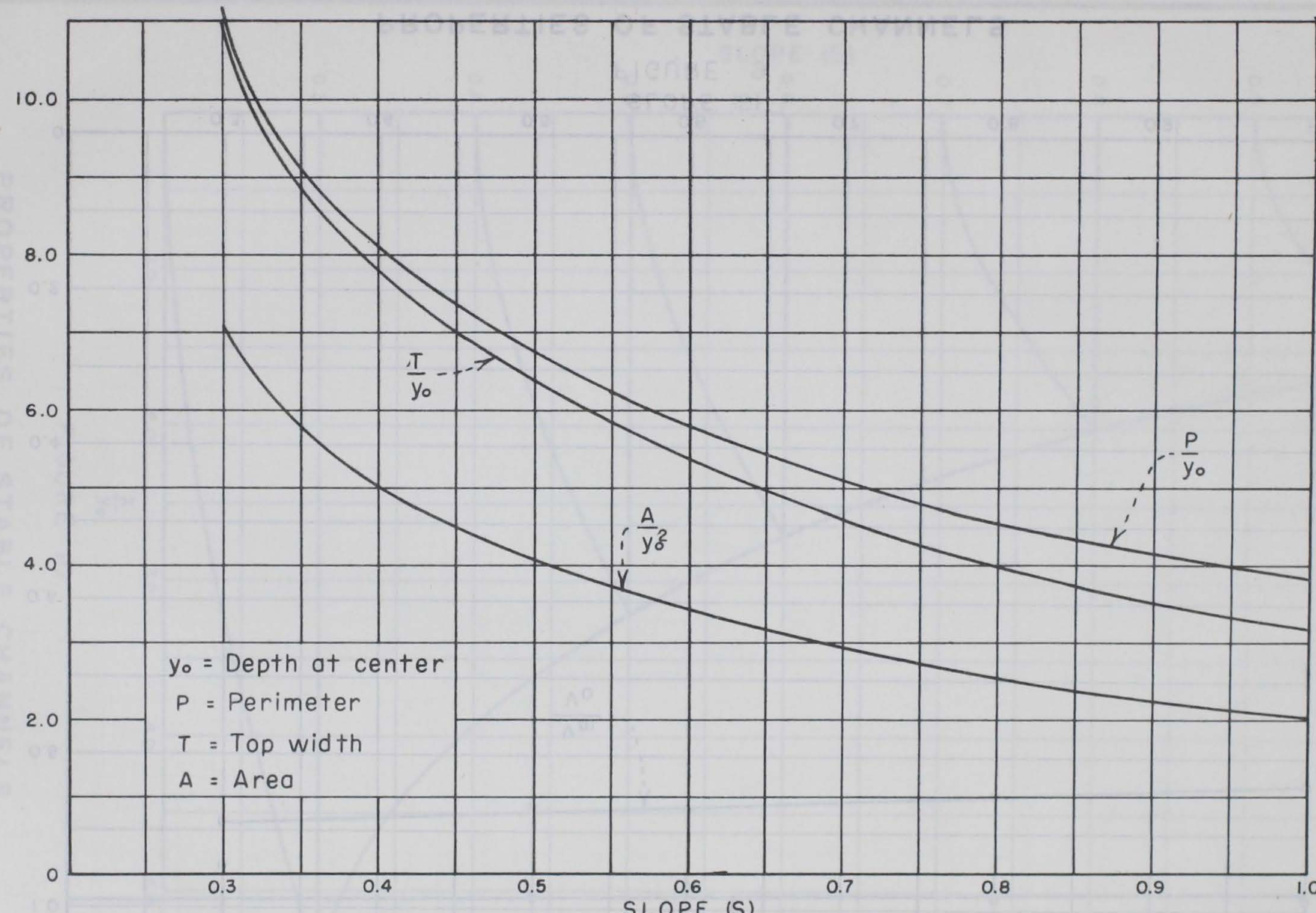

FIGURE 8

PROPERTIES OF STABLE CHANNELS

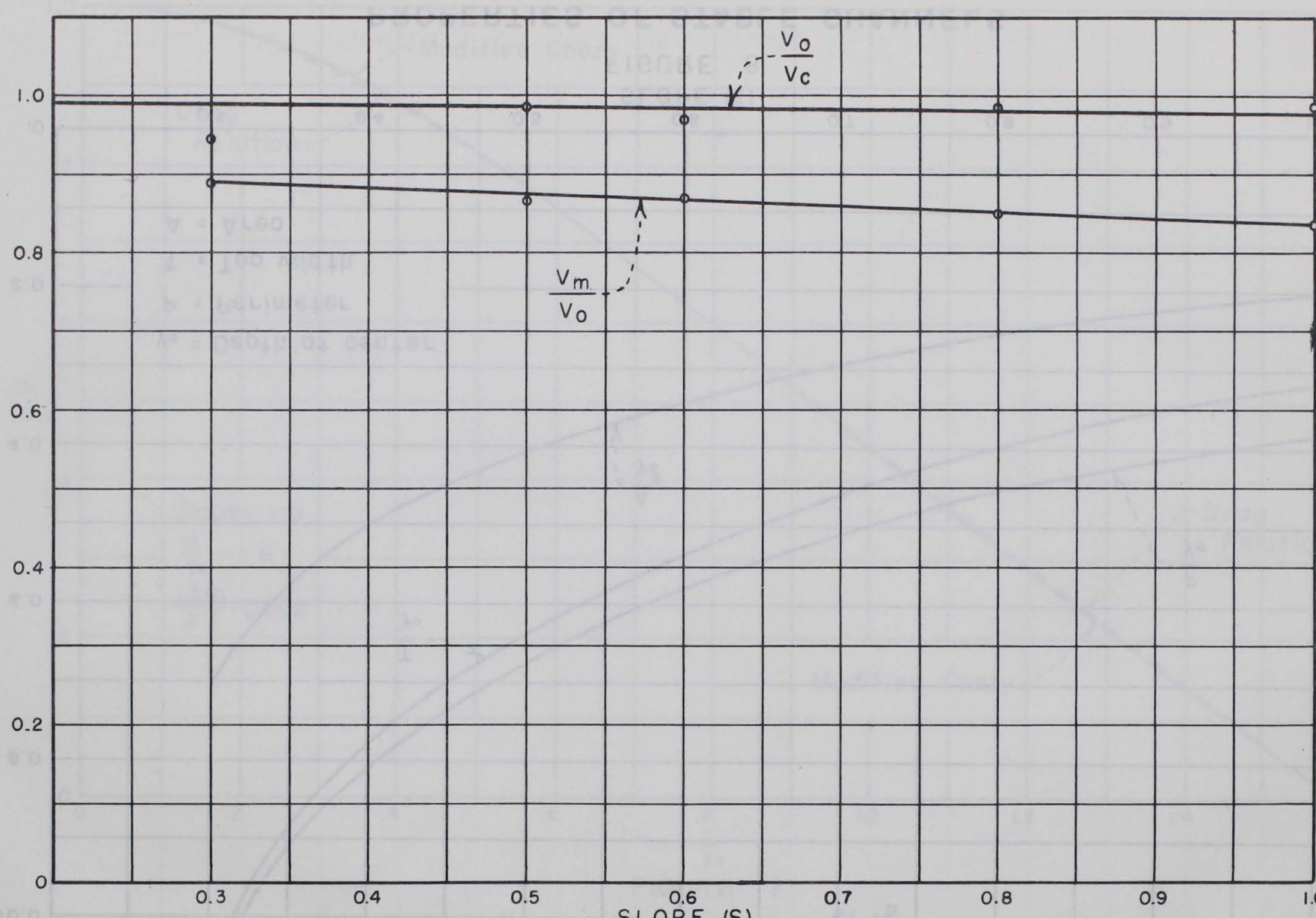

FIGURE 9

PROPERTIES OF STABLE CHANNELS

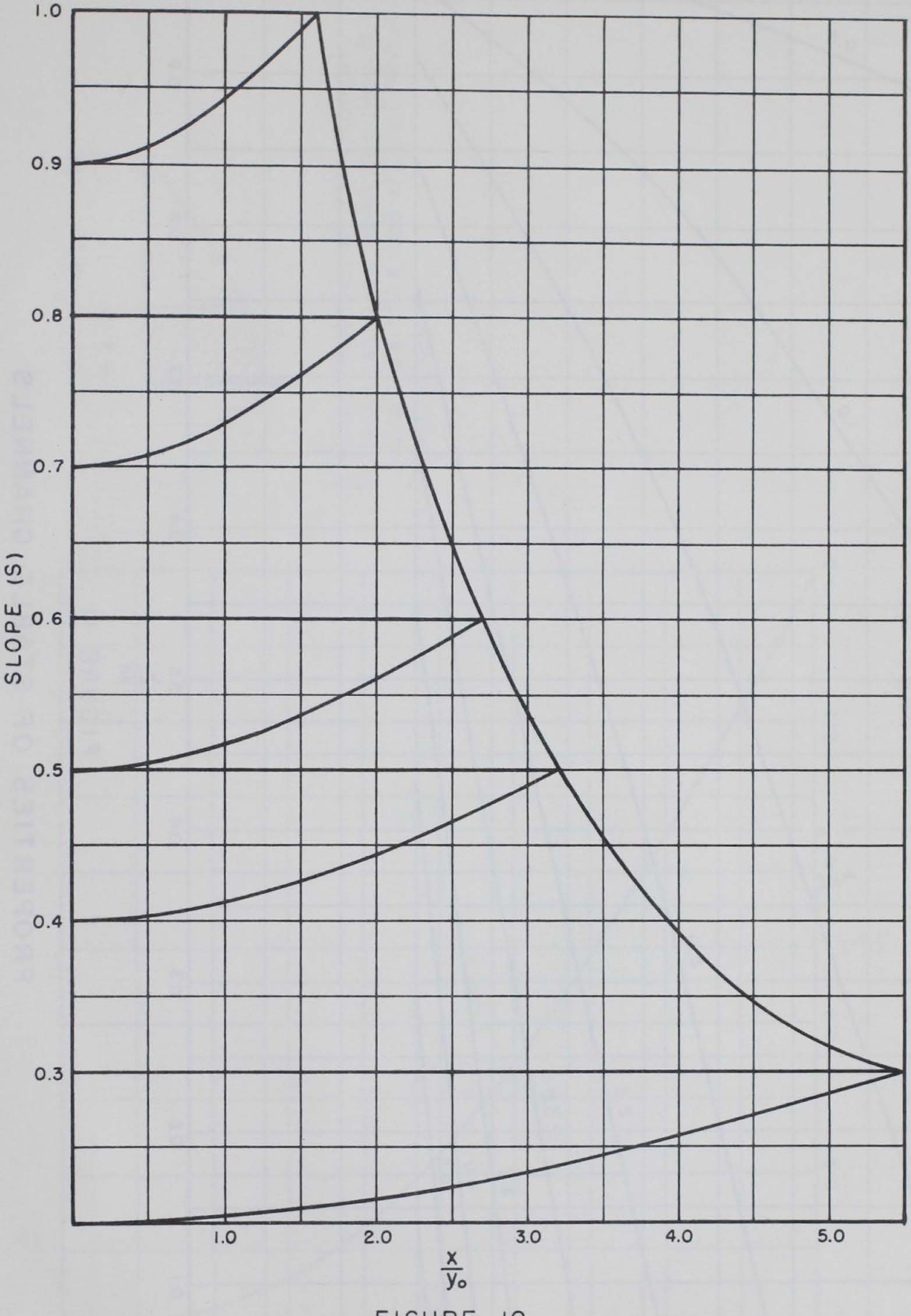

FIGURE 10

PROPERTIES OF STABLE CHANNELS

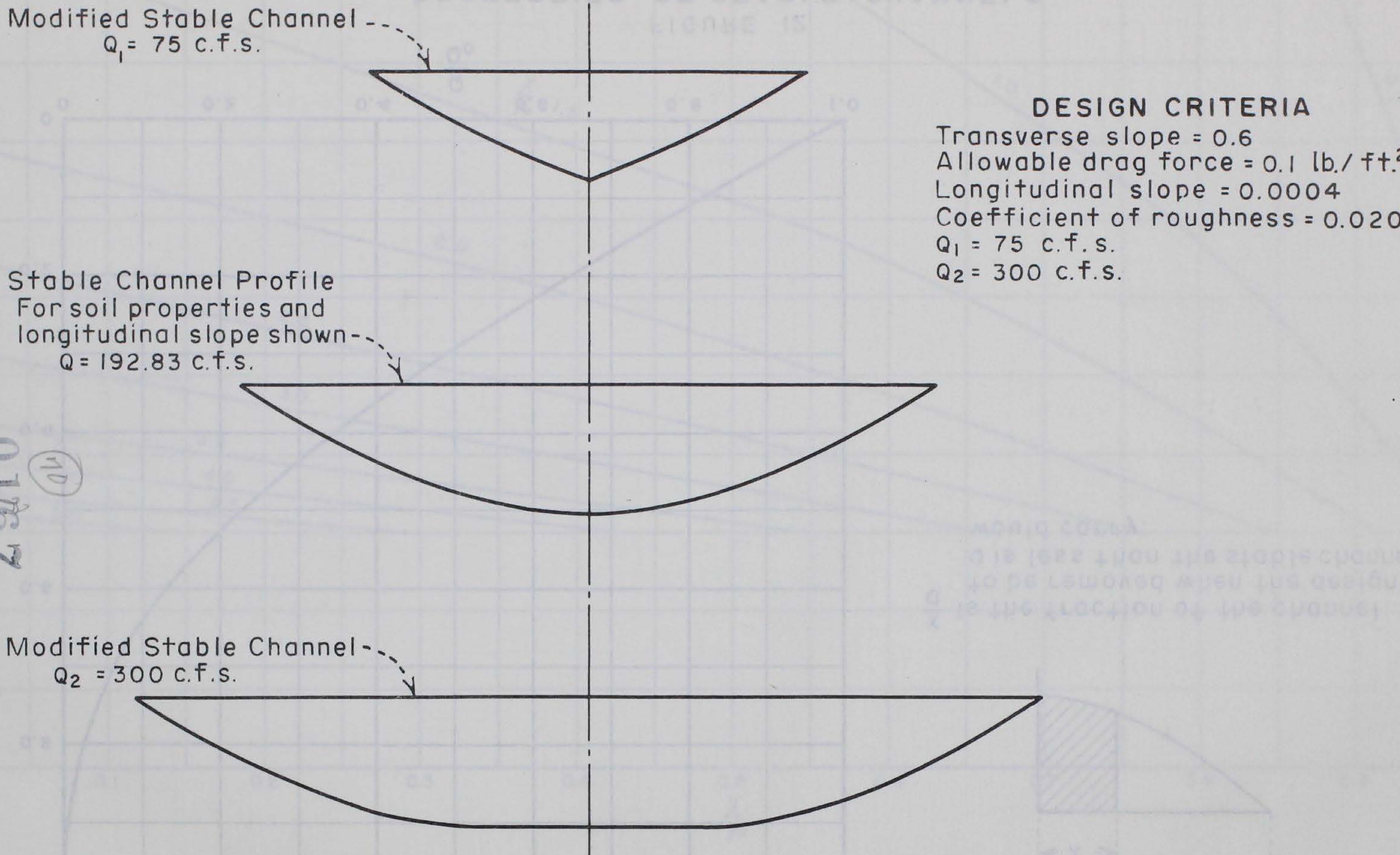

FIGURE 13

STABLE CHANNEL PROFILES