Research Paper 16

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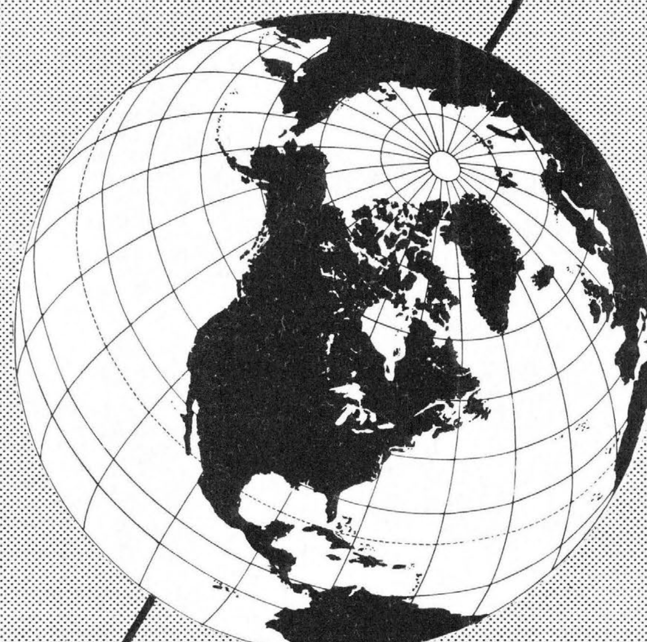
Measurements on Anisotropy of Thermal Conductivity of Ice



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Snow Ice and Permafrost Research Establishment, Corps of Engineers, U. S. Army, Wilmette, Illinois
MEASUREMENTS ON ANISOTROPY OF THERMAL CONDUCTIVITY OF ICE-
J. K. Landauer and H. Plumb

Research Paper 16, April 55, 4 pp-illus-tables.
DA Proj 8-66-02-004, SIPRE Proj 22.1-14.
Unclassified Report

A comparison technique is used to measure the anisotropy in the thermal conductivity of ice. Samples of laboratory-grown monocrystals, glacial monocrystals and polycrystalline commercial ice were studied. No effects due to the grain boundaries are observed. The experiments indicate that the conductivity in the direction of the c-axis may be about 5% greater than normal to it. The probable error in the results is about 2%. Analysis of the data leads to the conclusion that, if a difference in conductivity exists, it is less than 8%. More accurate experimentation is necessary to specify the anisotropy with greater precision.

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II. Plumb, H.
III. Snow Ice and Permafrost Research Establishment

GPO 829387-1

MEASUREMENTS ON ANISOTROPY OF THERMAL CONDUCTIVITY OF ICE

by
J. K. Landauer and H. Plumb

ABSTRACT

A comparison technique is used to measure the anisotropy in the thermal conductivity of ice. Samples of laboratory grown monocrystals, glacial monocrystals and polycrystalline commercial ice were studied. No effects due to the grain boundaries are observed. The experiments indicate that the conductivity in the direction of the c-axis may be about 5% greater than normal to it. The probable error in the results is about 2%. Analysis of the data leads to the conclusion that, if a difference in conductivity exists, it is less than 8%. More accurate experimentation is necessary to specify the anisotropy with greater precision.

Introduction

A number of experiments to determine the thermal conductivity of ice have been reported in the literature (Artsybashev and Parfianovich, 1929; Jackob and Erk, 1929). Some observers have claimed that there is a greater conductivity in the direction of the crystallographic c-axis than in a direction normal to it (Forbes, 1874). This anisotropy has not been found by others (Straneo, 1898). It has been suggested recently that preferred growth of ice with the c-axis in the direction of freezing could be accounted for by anisotropic conductivity (Lliboutry, 1955).* To test this hypothesis it is necessary to know the degree of anisotropy. Since Forbes method involves determining the rate of growth of new ice in a known temperature gradient, it is unlikely that an accuracy of greater than 10% could be expected. His observed anisotropy of about 6% could be in error by this amount. To resolve this problem it was decided to measure the conductivity with a more precise method.

Method

The difference in thermal conductivity normal and parallel to the c-axis of ice is measured using a comparison technique. Two samples, similar except for orientation, are mounted in identical measuring assemblies (Fig. 1) and tested simultaneously. A temperature gradient $T_B - T_C$ is maintained across the ice sample A by electrically heating a copper block B to temperature T_B and cooling copper block C by flow of a constant-temperature antifreeze solution at temperature T_C. The electrical power is adjusted so that the gradient across both samples is the same. If there is a difference in conductivity of ice samples A and A' then the applied power P and P' must be different to maintain a constant temperature difference. T_C is made equal to T_C' by circulating the antifreeze equally through the two assemblies by means of a "Y" junction. The temperatures in both cases are then the same and heat losses due to conduction and radiation should be similar. Because these losses are not known, this method does not provide an accurate method for determining the absolute conductivity, but is only useful for measuring differences.

Theory

The quantity of heat per unit time, Q, flowing through the sample is:

$$Q = ks \frac{\Delta T}{\ell} \tag{1}$$

where k is the conductivity, s is the cross-sectional area and $\Delta T = T_B - T_C$ is the

* There is now evidence which indicates that lake ice does not have a strong preferred direction of growth.

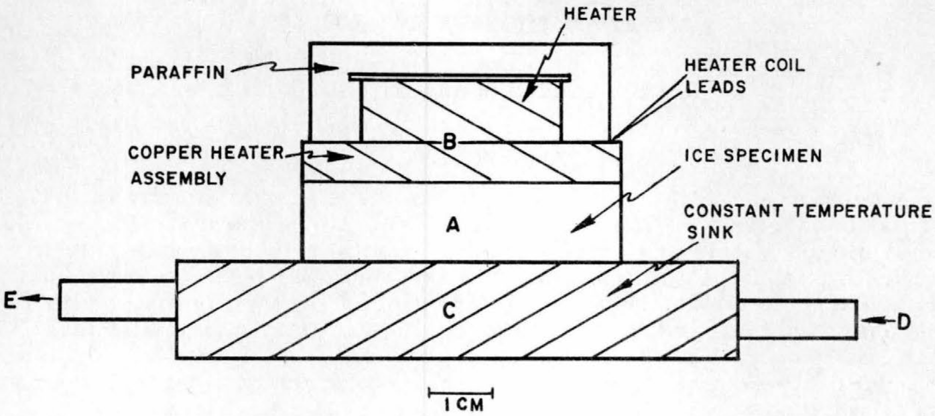

Figure 1. One of the two identical conductivity apparatuses. A is the sample; B is the heating assembly; C is the constant temperature sink cooled by antifreeze flow D and E. The thermocouple connections are not shown.

temperature difference across the sample of thickness ℓ. The applied electrical power, P, must be the sum of Q and the heat losses, L. The experimental conditions are such that s, ℓ, ΔT and L are the same for both assemblies, so:

$$\left.\begin{aligned} P &= Q+L = ks\,\frac{\Delta T}{\ell} + L \\ P' &= Q'+L = k's\,\frac{\Delta T}{\ell} + L \end{aligned}\right\} \qquad (2)$$

Therefore:

$$k-k' = \frac{\ell}{s\,\Delta T}\,(P-P'). \qquad (3)$$

Experimental

The two assemblies are placed a few centimeters apart in a cardboard box in a room at temperature -5.0 ± 0.5°C. The antifreeze temperature is -5.0 ± 0.1°C. The copper block C has interior baffles to ensure good heat transfer. Both copper blocks B and C have plane ground surfaces. The heating coils for B are covered with paraffin to reduce heat losses. Power is supplied to the heating coils by storage batteries and is measured by a potentiometer. Generally, the temperature difference between B and C is about 1°C.

Copper-constantan thermocouples are inserted in small holes in B and C and measurements of $T_B - T_B'$ and $T_B' - T_C'$ can be made. The powers P and P' are adjusted until $T_B = T_B'$. A waiting time of a few hours is necessary to ensure temperature equilibrium.

Samples

Measurements have been made on laboratory-grown monocrystalline ice, glacial monocrystalline ice, commercial ice, and plate glass (see Table I). The laboratory-grown ice has a marbled appearance when viewed under polarized light. This is probably due to small angle boundaries (less than 1 deg) between the c-axis of neighboring finger-sized crystals. These boundaries extend through the crystals in directions near to the c-axis. The glacier ice is well annealed and monocrystalline. The commercial ice consists of nearly parallel pencil-shaped crystals extending through the sample. The crystallographic orientation of these pencils is random. Plate-glass samples were used as a check on the apparatus.

Table I. Specimens tested.
Samples are placed in the apparatus with their smallest dimension in the direction of the heat flow.

Samples	Type	Orientation	Size (cm)
$A_{\parallel}$	Lab. grown	heat flow $\parallel$ $\underline{c}$-axis	4.55 x 4.55 x 1.31
$A_{\perp}$	Lab. grown	heat flow $\perp$ $\underline{c}$-axis	4.55 x 4.55 x 1.31
B_1	Lab. grown	heat flow $\perp$ $\underline{c}$-axis	4.73 x 4.73 x .521
B_2	Lab. grown	heat flow $\perp$ $\underline{c}$-axis	4.73 x 4.73 x .521
C_1	glass	—	.317 thick
C_2	glass	—	.317 thick
$D_{\parallel}$	glacier	heat flow $\parallel$ $\underline{c}$-axis	4.0 x 4.0 x 1.4
$D_{\perp}$	glacier	heat flow $\perp$ $\underline{c}$-axis	4.0 x 4.0 x 1.4
$E_{\parallel}$	commercial	heat flow $\parallel$ pencils	4.72 x 4.72 x 1.17
$E_{\perp}$	commercial	heat flow $\perp$ pencils	4.72 x 4.72 x 1.17

Table II. Experimental results.

Date	Sample in Assembly 1	Sample in Assembly 2	Power Applied (w) P_1	P_2	$\frac{P_1-P_2}{P_{av}}$ (%)	$\frac{k_{\parallel}-k_{\perp}}{k_{av}}$ (%)
27 May	$A_{\parallel}$	$A_{\perp}$	0.368	0.326	+12	+12
31 May	$A_{\perp}$	$A_{\parallel}$	.341	.348	- 2	2
1 June	$A_{\parallel}$	$A_{\perp}$	.361	.337	+ 7	7
1 June	$A_{\perp}$	$A_{\parallel}$	.342	.349	- 2	2
2 June	$A_{\parallel}$	$A_{\perp}$	.353	.324	+ 8	8
3 June	$A_{\perp}$	$A_{\parallel}$	.319	.337	- 5	5
8 June	B_1	B_2	.325	.328	- 1	
8 June	B_2	B_1	.329	.326	+ 1	
9 June	B_1	B_2	.325	.326	0	
9 June	B_2	B_1	.329	.326	+ 1	
10 June	B_1	B_2	.321	.329	- 2	
13 June	B_2	B_1	.322	.332	- 3	
15 June	B_1	B_2	.339	.332	+ 2	
16 June	$A_{\perp}$	$A_{\parallel}$	.307	.340	-10	10
17 June	$A_{\parallel}$	$A_{\perp}$	.340	.313	+ 8	8
20 June	$A_{\perp}$	$A_{\parallel}$	.304	.338	- 9	9
21 June	$A_{\parallel}$	$A_{\perp}$	.338	.315	+ 7	7
22 June	C_1	C_2	.773	.796	- 3	
22 June	C_2	C_1	.795	.792	0	
23 June	C_1	C_2	.795	.789	+ 1	
24 June	C_2	C_1	.769	.791	- 3	
24 June	$D_{\parallel}$	$D_{\perp}$	.154	.151	+ 2	2
24 June	$D_{\perp}$	$D_{\parallel}$	.150	.153	- 2	2
12 July	$E_{\parallel}$	$E_{\perp}$	.320	.308	+ 4	
13 July	$A_{\perp}$	$A_{\parallel}$	.294	.300	- 2	2
13 July	$A_{\parallel}$	$A_{\perp}$	.310	.297	+ 4	4
15 July	$E_{\perp}$	$E_{\parallel}$	.303	.297	+ 2	
18 July	$A_{\parallel}$	$A_{\perp}$	.279	.289	- 4	- 4

All ice surfaces are milled and lapped. Thermal contact between the ice and the copper blocks was obtained by using kerosine (later a light silicone grease) on the contact surfaces. From time to time samples are switched from one assembly to the other in order to reduce systematic errors.

Results

Experiments between 27 May and 24 June 1955 were performed by H. Plumb and the later ones by J. Landauer. The results of the experiments are shown in Table II.

Absolute values of the conductivity agree roughly with the average of values obtained by others (i.e., $k = 0.020$ w/cm-°C). In some cases the observed conductivity is 10 to 20% less than the accepted value. In any case the loss term in Equation 2 must be fairly small, so that the percentage difference in applied powers is approximately equal to the percentage difference in conductivity. This approximation is used for the calculation in Table II.

The average of all results indicates that the conductivity is 5.0% greater parallel to the c-axis than normal to it. The mean deviation from this value is 3.4% and the probable error is 2.1%. In analyzing the data it is found that the accuracy as indicated in the tests with glass and identical ice samples is also about 3%. The indicated errors may be misleading, however, for it should be noted that high values occurred predominantly for the early experiments while low values occurred for the later ones. The conclusion to be drawn is that if a difference in conductivity exists it is less than 8%.

No significant differences are apparent between the laboratory-grown, glacial, and commercial ice samples. Thus, grain boundaries do not affect the conductivity to within the experimental error.

The thesis that easy growth in the direction of the c-axis is due to anisotropy in the thermal conductivity is probably not valid. An 8% difference in conductivity could account for only a small statistical probability in favor of growth along the c-axis.

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