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U. S. ARMY
SNOW ICE AND PERMAFROST
RESEARCH ESTABLISHMENT
Corps of Engineers

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by T. R. Butkovich

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Corps of Engineers, U. S. Army

Wilmette, Illinois

PREFACE

This paper is an interim report on SIPRE project 22.1-10, Ultimate strength of ice and snow. The work was performed by Mr. Butkovich, physicist, under the direction of Dr. Henri Bader, Chief Scientist, former Chief of the Snow and Ice Basic Research Branch, and Mr. James A. Bender, Acting Chief, Snow and Ice Basic Research Branch. This paper reports results of two summers (1954-1955) of study in Northern Greenland on the strength properties of the upper layers of the Ice Cap. Results of tests on the work of disaggregation of snow, made by Mr. Bender, and on flexural strength of simple beams of snow, by Mr. S. Russell Stearns, are included in this report. The author was assisted by Pfc. Ronald D. Blotter during the summer of 1955.

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SUMMARY

Various strength properties of naturally compacted high-density snows, in the density range from 0.40 to 0.75 g/cm^3 are reported. Test results are given for: unconfined compression; unconfined and confined double shear; ring, flexural, and centrifugal tensile strength; torsional shear; and work of disaggregation. The expression

$$S = a(\gamma-\gamma_o)[1+b\,(\gamma-\gamma_o)^2]$$

relating the various strengths (S) to the density (γ) of snow fits all the results. The constants a and b are different for each test, while γ_o is constant for all tests. The work of disaggregation per unit volume was related to crushing, tensile, and shear strength at various lateral pressures, using the same empirical relationship. The results of the various tests measuring the tensile strength of the snow compare favorably with each other. An attempt was made to use the direct shear strength results in Coulomb's equation for the determination of Mohr's envelope of rupture for snow. These tests yield higher values than those obtained in unconfined compression tests. However, angles of internal friction obtained considering Mohr's envelope to be a straight line seem to agree with measurements taken on an unconfined compression specimen.

STRENGTH STUDIES OF HIGH-DENSITY SNOWS

INTRODUCTION

Several experimenters have devoted some time to the study of the shear, tensile, and crushing strength properties of snow. The data they report however, are rather meager, and a comparison of values obtained by the various authors shows quite a difference in their results. It is suspected that this is primarily due to the different conditions of temperature, techniques of measurement, and types of snow. Values found in existing publications are primarily for low-density snows, including new snow, old snow, and depth hoar (schwimmschnee). The work reported in this paper is for naturally compacted snows of higher densities, from 0.40 to 0.75 g/cm^3. It includes data, taken during two summers of study in Northern Greenland, on the strength properties of the upper layers of the Ice Cap.

The term "high-density snows" as used in this paper means snow of the type that lies below the surface in the upper layers of the Northern Greenland Ice Cap, in a no-melt zone. This snow has been naturally compacted, primarily by the weight of the overlying snow. Snow found in other regions of the earth where there is no continuous accumulation is usually of a lower density, varying from 0.1 to 0.4 g/cm^3.

During the 1954 field season a 100-ft deep pit was excavated and a density and stratigraphy profile was made. From these profiles, layers were chosen which would give a wide variation of densities. In the 100 ft depth a total of 53 layers were selected, with a density variation from 0.40 to 0.65 g/cm^3. The layers were sampled by drilling horizontally with a 3-in. coring auger. The snow in the horizontal cores was quite homogeneous with respect to density and grain size. The layers were selected so that no visible variations were included. In the 1955 field season, a single 3-in. vertical core hole was sunk an additional 55 ft down from the bottom of the 100-ft pit. The cores taken from this hole were also used in the tests to determine strengths for densities up to 0.75 g/cm^3. These cores were less homogeneous due to stratigraphic variation or layering of the snow.

In conjunction with the excavation of the 100-ft deep pit, a snow laboratory was made by excavating an area of about 17 x 9 ft to a depth of 8 ft. The excavation was roofed over, and snow platforms along its length were covered with wooden panels to provide table space.

The temperature in the deep pit below the 20 ft level was about -12°F, which is the mean annual temperature in this region. All of the cores taken for testing were left in the snow laboratory at least 12 hr before testing, in order to reach laboratory temperature.

The tests reported here were made on 3-in. diam cylindrical cores with a few exceptions. Specimens of the lower-density snows tested (below 0.45 g/cm^3) were taken with a standard snow tube, 2.28 in. in diameter.

Electric power was provided for lights and test equipment by a 110-volt, a-c generator.

TEMPERATURE CORRECTION FACTORS

Since the data reported in this paper were taken under field conditions, it was difficult to obtain any sort of temperature control. The snow laboratory's temperature varied from day to day and even from hour to hour, owing to the changing outside air temperature and to the heat radiation from the test personnel and the lighting system employed. During the 1954 field season the temperature of the snow laboratory varied between 14 and 23°F. Temperatures during the 1955 field season were considerably lower, because the roof of the laboratory was covered with a year's accumulation of snow. The maximum temperature then was 6°F, the minimum -1°F.

It is known that temperature affects the crushing strength of ice, and also torsional shear strength and tensile strength to a somewhat lesser extent. It was deemed necessary to correct all the values obtained to a common temperature, for which 14°F (-10°C) was chosen. The correction factors used were 1.4%/°F for crushing strength, and 0.5%/°F for ring tension and torsional shear values. The correction factor used for tensile strength was used for the double shear tests also, arbitrarily assuming a similar temperature relationship.

The temperature correction factors were taken from temperature effect curves for snow-ice given in SIPRE Research Paper 11 (Butkovich, 1954). They are not necessarily accurate for snow, but were the best values available. In any case, the maximum correction, about 15%, is less than the maximum scatter obtained for any set of tests, and certainly the corrections improve the values shown on the graphs for the temperature of 14°F.

CRUSHING STRENGTH

The Bibliography on snow, ice and permafrost (SIPRE Report 12, 1952-55) shows that only a very meager amount of data is reported on the ultimate unconfined compressive strength (crushing strength) of snow. This work contains nearly 12,000 abstracts covering the field of the title, and is a very extensive report on the existing literature on the subject.

A greater part of the tests on crushing strength of snow were made during the 1954 field season. The test specimens were 3-in. diam cylindrical cores taken horizontally from the walls of the 100-ft pit from each selected layer. The cores were trimmed to lengths of 6.7 ± 0.4 inches (17.0 ± 1.0 cm) in a miter box and the ends finished as nearly parallel as possible. A Carver hydraulic press with either a 2000-lb or a 5000-lb Baldwin load cell mounted in its head was used to apply and measure the load. The load cell is rated by its manufacturer to be accurate to ± 1% of the applied load. The loads were read on a Baldwin strain indicator, on which they were tracked to a maximum value. In many cases the load fell off to a lower value before actual rupture took place.

The cylindrical snow specimens were broken at the ambient temperature of the laboratory. Table I gives the crushing strength values and the mean values corrected to 14°F (-10°C). The rate of application of load was controlled manually and timed with a stop watch. The average rate was 7.4 psi/sec (0.5 kg/cm^2-sec) calculated by dividing the maximum load by the time required to attain it.

The specimens of snow often broke into characteristic shapes (Fig. 1). After the maximum load was reached and before the actual rupture took place the specimens visibly deformed, the top and bottom mushroomed, and after rupture the broken specimens consisted of either two cones, one at each end, or two partially conical pieces.

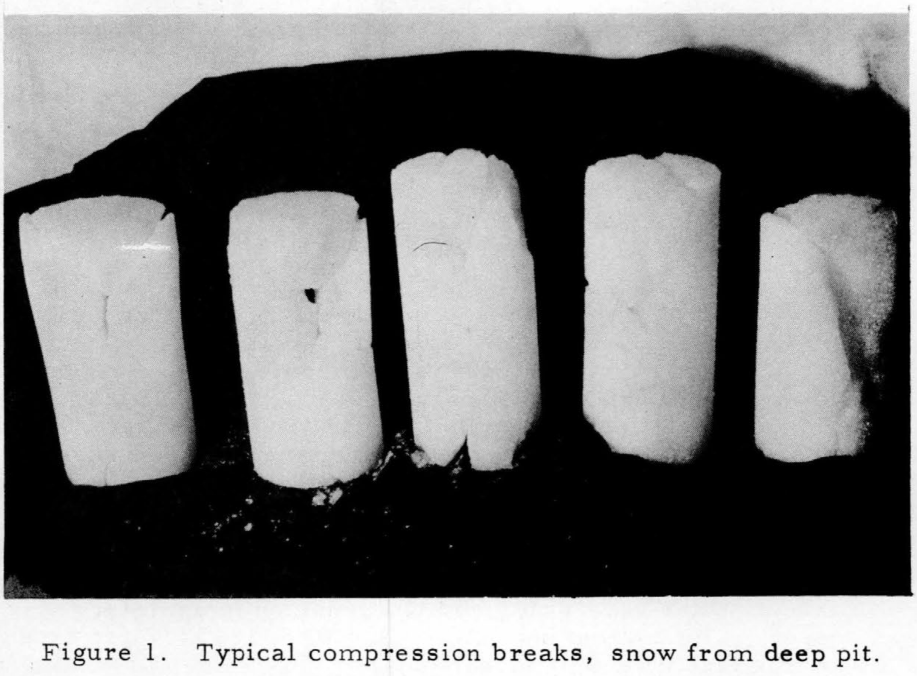

Figure 1. Typical compression breaks, snow from deep pit.

Table I. Crushing Strength of High-density Snow

Depth below June 54 surface (ft)	Mean * Density (g/cm^3)	Crushing Strength (psi)			Correct. Mean (to 14° F)	Amb. Temp. (°F)
		Max.	Min.	Mean		
8.7	0.405	49.7	21.3	31.2	34.5	21.3
9.7	0.354	28.4	9.9	21.3	23.0	17.6
11.6	0.444	58.2	45.4	52.5	51.7	13.1
14.1	0.428	52.5	44.0	48.3	47.4	13.1
16.1	0.420	42.6	28.4	35.5	34.5	13.1
17.7	0.454	102.2	75.3	86.6	84.8	13.1
19.8	0.450	83.8	59.6	71.0	70.5	13.1
22.0	0.483	136.3	98.0	112.2	110.7	13.1
22.5	0.465	117.9	89.5	100.8	99.2	13.1
23.3	0.498	200.2	73.2	190.3	190.3	—
23.9	0.480	154.8	86.6	127.8	130.8	15.8
25.6	0.520	228.6	170.4	200.2	196.9	13.1
26.9	0.490	150.5	130.6	140.6	143.8	15.8
27.9	0.543	245.7	217.3	232.9	232.9	14.0
29.2	0.457	85.2	52.5	68.2	67.5	14.0
29.7	0.509	140.6	119.3	125.0	125.0	14.0
30.8	0.472	112.2	75.3	96.6	97.8	14.9
31.3	0.516	174.7	157.6	164.7	166.8	14.9
32.8	0.466	133.5	105.1	117.9	119.4	14.9
33.6	0.530	207.3	164.7	183.2	185.6	14.9
34.9	0.517	207.3	193.1	201.6	201.6	14.0
37.1	0.564	288.3	274.1	276.9	276.9	14.0
38.4	0.532	191.7	164.7	174.7	174.7	14.0
40.0	0.530	235.7	193.1	220.1	220.1	14.0
41.5	0.528	207.3	146.3	183.2	183.2	14.0
44.8	0.558	249.9	207.3	235.7	235.7	14.0
45.4	0.527	180.3	133.5	166.1	166.1	14.0
46.9	0.540	203.1	139.2	178.9	178.9	14.0
47.9	0.571	299.6	217.3	248.5	251.6	14.9
49.5	0.562	265.5	207.3	238.5	241.6	14.9
50.8	0.587	333.7	296.8	303.9	307.7	14.9
51.5	0.564	252.8	224.4	234.3	237.2	14.9
54.0	0.580	340.8	305.3	328.0	332.1	14.9
54.8	0.591	349.3	248.5	303.9	307.7	14.9
57.6	0.578	275.5	188.9	240.0	243.0	14.9
58.4	0.596	305.3	234.3	274.1	277.5	14.9
59.7	0.589	305.3	267.0	288.3	291.9	14.9
62.0	0.569	261.3	187.4	222.9	225.7	14.9
66.3	0.614	377.7	319.5	347.9	352.2	14.9
69.5	0.595	292.5	230.2	259.9	266.0	15.8
72.7	0.613	349.3	296.8	298.2	306.3	15.8
75.4	0.608	336.5	278.3	299.6	307.7	15.8
79.9	0.626	349.3	291.1	322.3	330.7	15.8
85.0	0.632	363.5	298.2	339.4	347.9	15.8
86.4	0.623	347.9	306.7	328.0	336.5	15.8
96.8	0.641	382.0	319.5	352.2	369.5	17.6
97.7	0.642	328.0	255.6	303.9	319.1	17.6

* Mean of 6 tests.

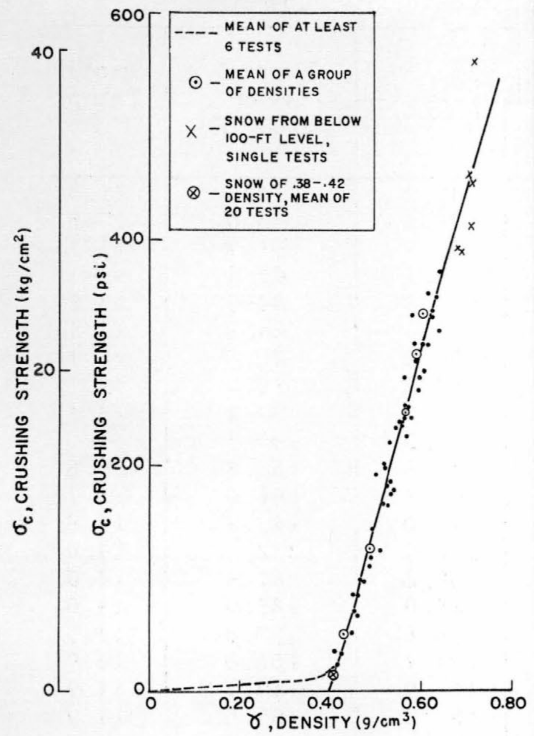

Figure 2. Crushing strength vs. density, snow from deep pit.

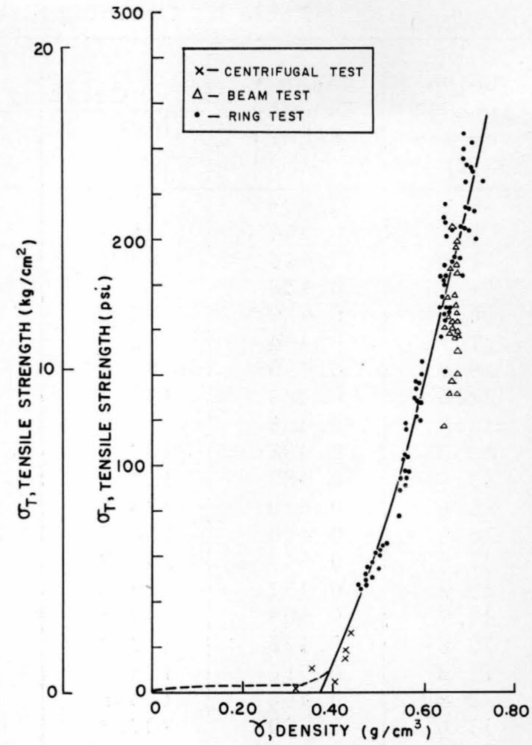

Figure 3. Tensile strength vs. density for high-density snow.

The graph of the crushing strength vs. density (Fig. 2) shows that the crushing strength of high density snow is a linear function of the density. Each point represents the mean value for at least six tests, and each circled point is the mean of a group of densities of the same approximate size. For example the mean value for all densities between 0.440 and 0.459 g/cm³ is shown as a circled point. A few tests were performed on vertical cores taken from below the 100-ft level. The values are represented by x's on the graph and are values for single tests.

An empirical relationship obtained from the data shown in Figure 2 above 0.39 g/cm³ density is:

$$\sigma_c = 1418\,(\gamma - 0.39) \qquad (1)$$

where σ_c is the crushing strength in psi, and γ is the density in g/cm³. This equation was determined by the method of least squares, and has a virtual intercept at 0.39 g/cm³ density. Equation (1) holds for densities from 0.39 to 0.75 g/cm³. The broken line is the manner in which it is assumed that strength actually goes to zero.

Insufficient data existed on the crushing strength for densities less than 0.40 g/cm³, so a number of tests were planned and conducted during the 1955 field season. It is impossible to obtain a 3-in. diam core sample with the auger when the snow is of low cohesion, as was the case here, so that the standard snow tube of 2.28-in. diam was used.

Tests were conducted successfully on snows of densities between 0.38 and 0.42 g/cm^3. The values obtained were again corrected to 14°F. Here the maximum temperature correction was 15% but for these low values it amounted to a maximum of less than 3 psi. The mean value for approximately 20 tests is shown in Figure 2. For these low-cohesion snows extreme care was exercised to prevent the samples from falling apart. Snow of density less than 0.35 g/cm^3 did fall apart when the samples were removed from the snow tube for testing. These snows have very low strengths.

A few tests on unconfined compressive strength of snow are reported in SIPRE Special Report 8 (Bird and Waterhouse, 1953), giving values obtained for 4-cm cubes. This snow was disaggregated, of uniform grain size, and artificially compacted. These values (Table II) are considerably higher than those obtained for Greenland snow. The most likely reason for this is the influence of the size of the test specimens. SIPRE Research Paper 15 (Butkovich, 1955) shows the influence of sample size on strength for ice. Smaller cross sections have higher strengths, and the ratio of length to width also influences the strength values obtained. End constraint increases with decreasing ratio of length to width, preventing the sample from failing under the lower loads.

Table II. Compressive Strength Values for 4-cm Cubes of Snow (from Bird and Waterhouse, 1953). Temp 14°F.

Mean Density (g/cm^3)	Average Unconfined Compressive Strength (psi)	
	Loaded Normal to Compaction Plane	Loaded Parallel to Compaction Plane
0.59	263	207
0.61	391	356
0.68	604	569

TENSILE STRENGTH

Various techniques are available for measuring the tensile strength of snow. One method developed for use with low density snows (below 0.45 g/cm^3) is the centrifugal tensile strength apparatus. This device consists of a cylindrical container into which snows from a standard tube can be inserted. The snow is notched slightly at the axis of rotation and is driven by an electric motor controlled by a rheostat. A tachometer gives the rpm directly. The theory and a nomograph for this apparatus are given in SIPRE Report 7 (1951). A number of tests employing this apparatus were conducted during the 1954 field season. It was possible to test snows to a depth of 8.7 ft (265 cm) where the density reached 0.44 g/cm^3. Testing snows of higher densities and consequently higher strengths would require a greater rpm than was available. Table III shows the results of these tests. A number of tests using a centrifugal apparatus with the lower density snows are reported by Bucher (1948). Twenty-three single values between 0.372 and 0.427 g/cm^3 density fall in the same range as those shown here with deviations of the same order. The mean density for these tests was 0.411 g/cm^3, with a mean tensile strength of 9.2 psi (0.65 kg/cm^2), SIPRE Report 4 (1951) shows some values for tensile strength obtained at the University of Minnesota, again for the lower-density snows, which agree with results obtained with the centrifugal tensile strength apparatus.

Mr. R. R. Philippe of the Office, Chief of Engineers, while visiting the test site in 1954 brought to the attention of the author a method used successfully for the determination of the tensile strength of rock, concrete, and other brittle material. A ring of the material is caused to fail by applying a compressive load normal to its axis. The failure plane then is always parallel to the direction of the application of the load. The theory for these tests is discussed in an article by Ripperger and Davids (1947), with a discussion by Philippe and Mellinger.

A few tests employing high-density snows near 0.60 g/cm^3 were conducted during the 1954 field season. The results of these tests proved to be quite reproducible, and it was decided that this method should be further investigated. In the 1955 field season a large number of ring tests were prepared. The specimens were cut from the 3-in. diam cores, to lengths of approximately $2\frac{1}{2}$ in. A 1-in. diam hole was drilled coaxially with the core cylinder. The tensile strength values shown in Table IV were calculated from the equation

$$\sigma_T = \sigma_\theta = \frac{PK}{\pi r_o} \qquad (2)$$

where σ_T is the tensile stress at failure, which is equal to σ_θ the tangential stress at failure, $\underline{P}$ is the measured load at failure per unit of ring length, r_o is the outer radius of the ring, and $\underline{K}$ is the so-called concentration factor. For these calculations, K = -10.917 was obtained from a table in the previously cited reference. The derivation of the above equation depends on the assumption that the tested material is truly elastic. Within the limitations of this assumption, a ring test will yield the tensile strength of a brittle material. Snow is primarily a visco-plastic material, with insignificant elastic deformation at the lower densities. However, the elastic deformation increases appreciably with increasing density. If the loading is performed fast enough, the plastic effects decrease appreciably in magnitude. The ring specimens were loaded at a high rate (average of 9.5 psi/sec) at which the deformation is essentially elastic.

Figure 3 is the graph of ring tensile strength vs. density. All values shown on this curve were corrected to 14°F. Each point represents single values. The curve was drawn from the means of these values (Table IV). An empirical equation relating ring tensile strength to density was determined from the solid line portion of the curve:

$$\sigma_T = 503(\gamma - 0.37)\left[1 + 2.88(\gamma - 0.37)^2\right] \qquad (3)$$

where σ_T is the ring tensile strength in psi, and γ the density in g/cm³. The x's on the graph are values obtained for the lower density snow with the centrifugal tensile strength apparatus.

A number of center loading beam tests were conducted by S. R. Stearns during the 1955 field season on snow from the 100-ft level in the deep pit.

A total of 31 tests were made and the individual values are plotted in Figure 3. The mean of these values corrected to +14°F is 162 psi at a mean density of 0.673 g/cm³. This value is slightly lower than that obtained in ring tests. However, a scatter can be observed.

Table III. Centrifugal Tensile Strength

Depth (ft)	Mean Density (g/cm³)	Density Range (g/cm³)	Tensile Strength (psi)			Corr. Mean* (to 14°F)
			Max.	Min.	Mean*	
0.4	0.317	0.309-0.327	1.92	0.68	1.43	1.44
1.6	0.404	0.395-0.412	7.31	2.73	4.78	4.82
2.4	0.352	0.339-0.370	13.63	8.09	10.17	10.41
5.2	0.427	0.422-0.436	19.02	10.93	15.07	15.49
7.2	0.428	0.408-0.433	19.88	15.62	17.56	18.23
8.7	0.442	0.427-0.454	29.54	20.02	24.85	26.05

* Mean of 8 tests.

Table IV. Ring Tensile Strength

Mean Density (g/cm³)	Density Range (g/cm³)	Ring Tensile Strength (psi)			Corr. Mean* (to 14° F)
		Max.	Min.	Mean*	
0.481	0.454-0.501	64.0	49.0	55.5	52.6
0.541	0.502-0.560	110.0	65.0	89.4	84.4
0.575	0.560-0.590	145.0	99.0	124.4	117.8
0.613	0.513-0.639	260.0	126.0	160.2	151.7
0.648	0.644-0.652	227.0	179.0	199.2	188.6
0.661	0.655-0.666	272.0	149.0	201.2	190.6
0.681	0.670-0.690	268.0	194.0	223.0	211.2
0.695	0.691-0.699	272.0	216.0	238.7	226.1
0.716	0.706-0.727	292.0	210.0	250.5	237.2

* Mean of 10 tests.

SHEAR STRENGTH

During the 1954 field season, a large number of double shear tests were made on snows taken from the same layers of the 100-ft pit as those for the crushing strength tests. A double shear apparatus designed by the Arctic Construction and Frost Effects Laboratory was used. The test results and a shear strength-density curve are shown in SIPRE Report 20 (1955). It was felt at that time that this apparatus for determining the shear strength of snow did not yield good results and caused failures primarily in tension, such as occur in the bending of beams. However, there was no other apparatus available at that time. A similar apparatus was modified to take a longer test specimen, and the dimensions changed to improve the fit of the specimens and to reduce the play of the moving parts. This improved apparatus (Fig. 4) was used for the tests conducted during the 1955 field season.

The original 6-in. tube length was changed to 12 in., divided into three equal segments. The center segment was attached to a mechanical loading jack, through either a 500 or a 2000 lb proving ring, and the two outer segments were attached rigidly to the base of the apparatus (Fig. 4). The tube was equipped so that confined double shear tests could also be made. A tapered pin retained a plug at one end of the tube, while a 200-kg Dillon dynamometer attached to a manually loaded piston was used to apply and measure a lateral load. With this device it was possible to obtain a maximum of 60 psi lateral pressure on the 3-in. diam snow samples.

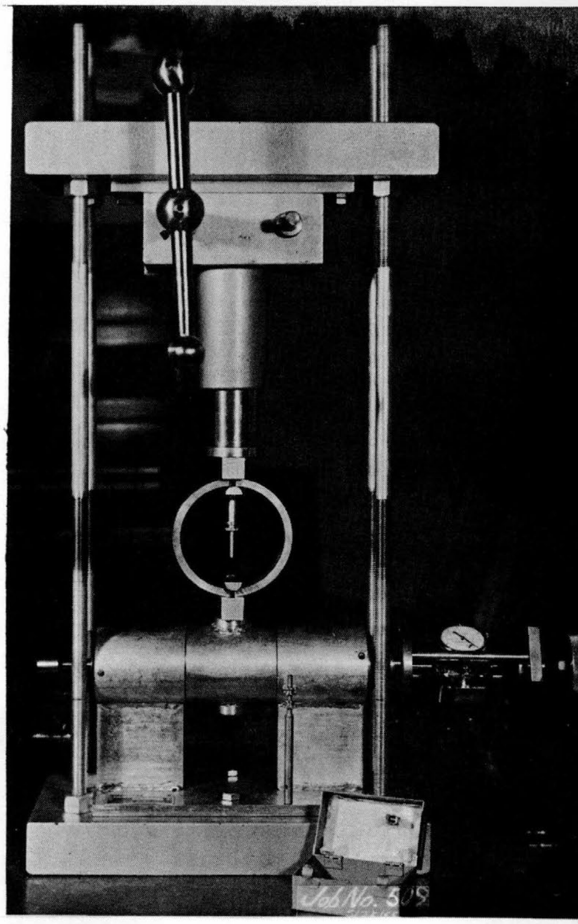

Figure 4. Double shear apparatus used for 1955 tests.

Several series of tests were made to obtain a relationship between density and shear strength for both the unconfined case and the confined double shear at various lateral pressures. The 12-in. long, 3-in. diam samples used were taken from horizontal layers in the 100-ft pit, selected to give a wide range of densities. A number of vertical cores taken from below the 100-ft level were included. All of the shear tests were performed at a rate of approximately 5 psi/sec. The loading was accomplished manually and the loading rate was kept as uniform as possible throughout the test. Three types of failures were observed. First, at a very low load, the sample could be heard to fail, although the load did not drop off considerably. If the tests were stopped at this point, and the sample removed from the shear device for examination, it was always noted that the failure was somewhere near the center of the specimen. This failure was undoubtedly a tension failure similar to that which occurs in bending beams of other brittle materials.

During the 1954 field season, using the ACFEL double shear apparatus, the tests were stopped at the sound of the first failure. The results of these tests (SIPRE Report 20, 1955) were considerably lower than those obtained during the 1955 field season, and were of the same order as the values obtained for the tension failures with the improved

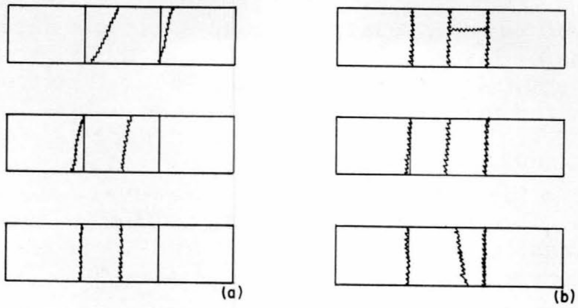

Figure 5. Sketch of typical breaks in double shear (solid lines denote apparatus; wavy lines denote breaks). a) Single shear with tension break as generally occurred for densities <0.50 g/cm³. b) Double shear with tension break as generally occurred for densities >0.50 g/cm³.

apparatus. Therefore it is quite logical to assume that the earlier test results are erroneous because of the manner in which they were made. Loads were measured at the first failure only. Because of the shorter length (6 in.) of these test specimens, it was impossible to determine whether the failure was due to shearing or bending.

In 1955, the test was continued after the tension failure, and the load increased until a failure occurred where the load dropped off to zero. Two types of failures were noted: When the snow density was less than 0.50 g/cm³, the sample generally failed on only one shear plane (Fig. 5a); when the density was greater than 0.50 g/cm³, the samples failed on both shear planes (Fig. 5b). However, all the data reported here were treated as though a double shear failure occurred in all cases. Figure 6 is a graph of the unconfined shear strength vs. density values shown in Table V. The points represent values obtained for single tests of horizontal cores, while the x's represent single values obtained for vertical cores. The solid line in the graph was drawn from the mean values of all the tests. It is noted that a majority of the values for the vertical cores are to the right of this line, which indicates that the shear strength values may be too low for the reported densities of the samples. The densities for which these values are reported are mean densities for the 12-inch vertical specimen. It is known that there is a considerable variation of density with depth. For example: a vertical core of mean density 0.65 g/cm³ may have layers with a density variation from 0.63 to 0.67 g/cm³. Since the lower densities have lower shear strength, it is reasonable to assume that the failure would occur in the 0.63 g/cm³ layer if possible. Therefore the point would correctly be plotted at 0.63 rather than 0.65 g/cm³. However it would be quite difficult to determine the density for the layers where the failures occurred. The broken line represents the assumed extrapolation of the curve to zero.

Table V. Shear Strength of High-Density Snow
Unconfined Double Shear

Mean Density (g/cm³)	Density Range (g/cm³)	Shear Strength(psi) Max.	Min.	Mean	Corr. Mean* (to 14°F)
0.420	0.407-0.431	27.4	13.4	20.7	19.4
0.435	0.432-0.443	32.8	16.2	25.4	23.9
0.445	0.443-0.447	32.0	16.2	25.5	24.0
0.454	0.448-0.457	42.1	16.3	29.4	27.7
0.462	0.458-0.466	44.0	30.4	37.8	35.6
0.476	0.468-0.482	55.1	22.8	39.1	36.9
0.494	0.482-0.507	75.6	29.4	47.9	45.2
0.566	0.547-0.590	108.5	45.5	79.9	75.4
0.621	0.592-0.649	171.3	65.5	132.4	124.8
0.664†	0.650-0.677	222.0	122.2	152.5	143.8
0.692†	0.680-0.702	212.0	114.0	153.8	145.0
0.716†	0.703-0.731	247.0	157.0	215.6	203.3

* Mean of 10 tests; † Vertical samples.

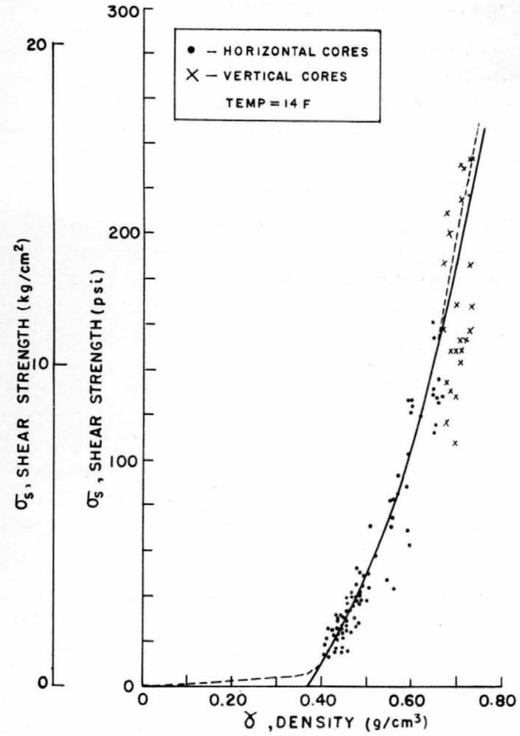

Figure 6. Unconfined double shear vs. density for high-density snow.

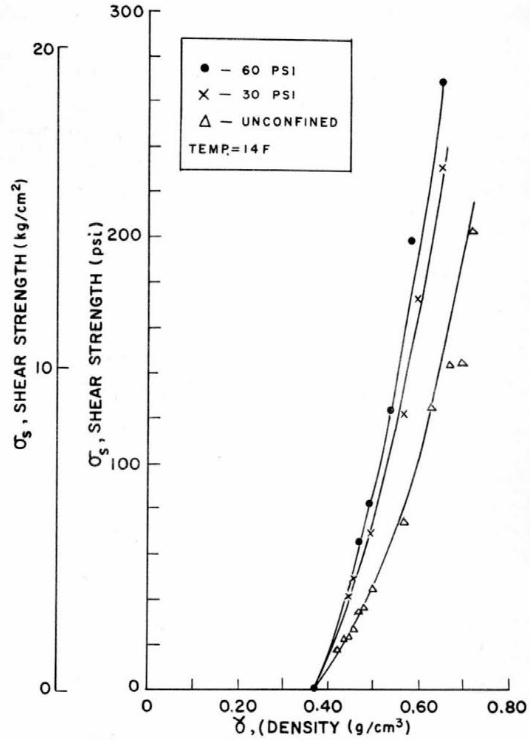

Figure 7. Shear strength vs. density for 0, 30, and 60 psi lateral pressures

Several investigators have reported values for unconfined shear strength of snow. SIPRE Report 7 (1951) gives a few values obtained by the University of Minnesota for the lower density snows, and zero normal load. The means of these tests are 0.50 kg/cm² (7.1 psi) for 0.420 g/cm³ density. Fuchs (1949) reports single shear values for disaggregated, artificially compacted snow as a function of temperature. Table VI gives the values taken from his curves at -10°C (14°F).

Table VI. Shear Strength of Disaggregated, Artificially Compacted Snow (from Fuchs, 1949).

Density (g/cm³)	Shear Strength (kg/cm²)	(psi)
0.487	1.5	21.3
0.615	2.7	38.3
0.631	4.9	69.6
0.686	8.1	115.0
0.862 (snow-ice)	12.2	173.2

These values are somewhat lower than those found for the Greenland snow, but there are several factors which may explain the difference. The shear specimens were all loaded at an extremely fast rate. The loading to failure was accomplished in 1 to 2 sec.

Another and probably more important factor was that the specimens were loaded to failure only seconds after they were compacted, not allowing age hardening.

Another series of tests were accomplished using various lateral pressures. For these tests a sample of 10.5 in. length was used in the 12-in. tube. The remaining portion of the tube was occupied by the plug at one end to retain the specimen and a piston device with the Dillon dynamometer at the other. After the snow core was loaded, the desired lateral pressure was applied. Immediately after this was attained, the shearing force was applied, keeping the lateral pressure constant.

Table VII gives the results of these tests. These values were also corrected to 14°F by using the previously mentioned correction factor. Figure 7 is the graph of the shear strength vs. density for 0, 30, and 60 psi lateral pressures. An empirical equation was obtained relating the shear strength to density for the three curves which is:

$$\sigma_s = a(\gamma - 0.37)\left[1 + b(\gamma - 0.37)^2\right] \tag{4}$$

where σ_s is the shear strength in psi, γ is the density in g/cm³, and a and b are the constants for the various lateral pressures. The values for a and b at 14°F are 333 and 7.04 respectively for zero lateral pressure; 558 and 5.44 for 30 psi lateral pressure; and 643 and 6.13 for 60 psi lateral pressure.

A number of tests were conducted to obtain shear strength vs. lateral pressure data, in order to describe snow in terms of Coulomb's law of friction. Snows of densities near 0.45 g/cm³ were chosen. The test results (Table VIII) are discussed in the section on angle of internal friction and apparent cohesion.

Table VII. Shear Strength of High-Density Snow, Confined Double Shear

Mean Density (g/cm³)	Density Range (g/cm³)	Shear Strength (psi) Max.	Shear Strength (psi) Min.	Shear Strength (psi) Mean	Corr. Mean* (to +14°F)	Lateral Pressure (psi)
0.444	0.438-0.447	49.9	36.3	44.6	42.1	30
0.452	0.447-0.461	54.3	45.5	51.5	48.6	30
0.491	0.470-0.510	86.9	52.9	72.7	68.5	30
0.567	0.562-0.572	143.4	95.2	129.2	122.0	30
0.595	0.586-0.602	198.0	168.0	183.2	172.8	30
0.649	0.646-0.652	296.8	206.0	247.7	233.6	30
0.442	0.430-0.454	54.8	23.6	44.1	41.9	60
0.467	0.462-0.478	86.6	58.6	69.0	65.4	60
0.489	0.482-0.499	100.7	77.1	87.2	82.2	60
0.540	0.507-0.564	155.3	95.1	130.7	123.3	60
0.581	0.564-0.597	244.0	161.6	209.4	197.5	60
0.633	0.600-0.655	342.3	231.3	284.5	268.6	60

* Mean of 8 tests.

Table VIII. Shear Strength of Snow, Density 0.45 g/cm³, at Various Lateral Pressures.

Mean Density (g/cm³)	No. tests	Density Range (g/cm³)	Shear Strength (psi) Max.	Shear Strength (psi) Min.	Shear Strength (psi) Mean	Corr. Mean (to +14°F)	Lateral Pressure (psi)
0.449	24	0.44-0.457	35.4	16.3	24.4	23.0	0
0.447	8	0.433-0.460	44.0	29.6	35.0	33.0	5
0.453	10	0.445-0.459	54.6	34.7	45.0	42.4	15
0.450	14	0.446-0.457	53.8	47.2	50.5	47.6	30
0.450	12	0.440-0.457	58.1	46.4	52.9	49.9	45
0.449	5	0.436-0.463	68.4	34.2	52.8	49.8	60

TORSIONAL SHEAR STRENGTH

A method of measuring the shear strength by applying a torque to 3-in. diam cores was tried on a few specimens during the 1955 field season. Tests were made on the higher density layers only, because the handling necessary to prepare the specimens would not be possible for the snows of lower density and, consequently, lower cohesion. Horizontal cores were taken for these tests from 0.50, 0.55, 0.60, and 0.65 g/cm^3 density layers in the deep pit.

The apparatus used for these tests (Fig. 8) consists of a device which holds one end of the specimen stationary, while the other end is free to rotate. A spring balance is attached to a lever arm on the rotating end for the determination of the torque. It can be seen that this apparatus was designed to accept square-end cross sections. It was necessary to put the ends of the core into metal boxes, pack slush around it, and wait until it froze solid. This required a great deal of time, 3 to 4 hours for each end at the ambient temperature of the snow laboratory.

The rate of loading was controlled and timed manually with a stop watch. The average loading rate for all the tests reported in Table IX was 9 psi/sec. This fast rate was used to reduce the plastic effects described in the tensile strength section of this paper.

The type of breaks that occurred in the tests were the typical 45° helical torsion breaks that usually occur with brittle materials (Fig. 9). This is not the kind of break that would be characteristic of a pure shear test.

Table IX. Torsional Shear Strength of Snow.
(corrected to +14°F.)

Mean Density (g/cm^3)	Density Range (g/cm^3)	Torsional Shear Strength (psi)			
		Max.	Min.	Mean*	Corr. Mean*
0.496	0.477-0.503	37.7	27.5	34.2	33.2
0.535	0.510-0.559	62.3	34.9	47.2	45.8
0.571	0.560-0.584	63.2	55.6	59.2	57.4
0.594	0.592-0.598	74.6	63.2	69.4	67.3
0.640	0.599-0.652	109.5	66.0	88.4	85.8

*Mean of 5 tests.

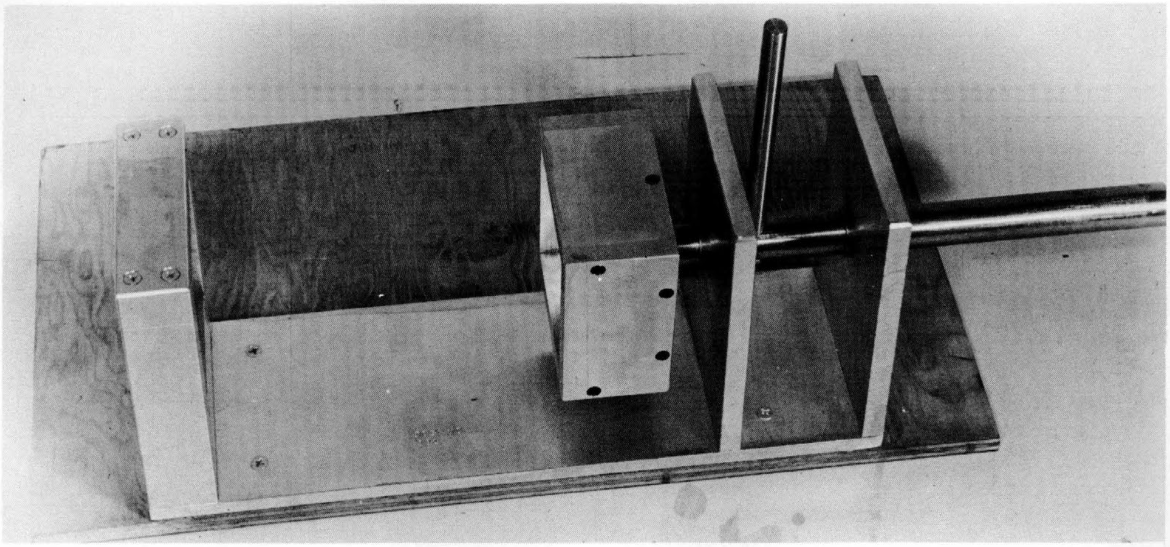

Figure 8. Apparatus used for torsional shear strength tests.

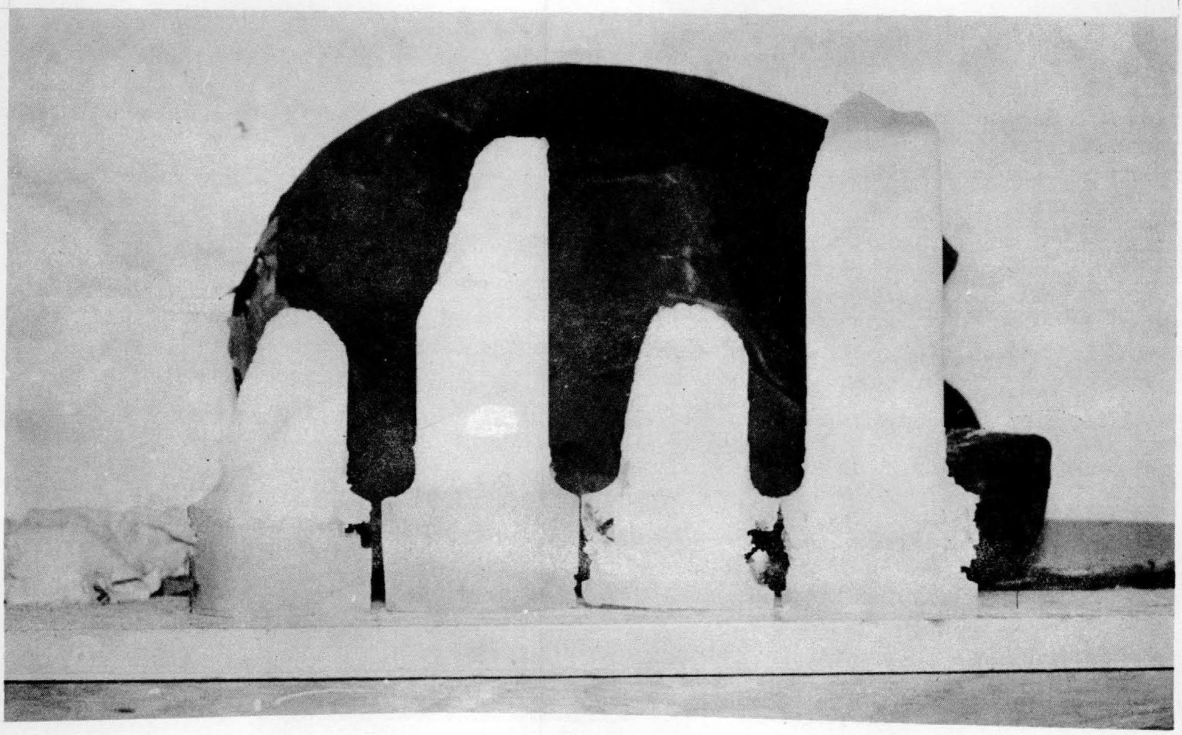

Figure 9. Typical torsion breaks, high-density snow from deep pit.

The torsional shear strength was calculated from an equation whose derivation can be found in Timoshenko and MacCullough (1949). The equation is

$$S = \frac{16M_t}{\pi d^3} \qquad (5)$$

where S is the shearing stress up to the elastic limit, M_t is the applied torque, and d is the diameter of the circular cross section. This equation is said to hold only within the elastic limit. In case of failure, Timoshenko speaks of a fictitious shearing stress and the value obtained is called the torsional modulus of rupture. These are the values reported as the torsional shear strengths at the various densities.

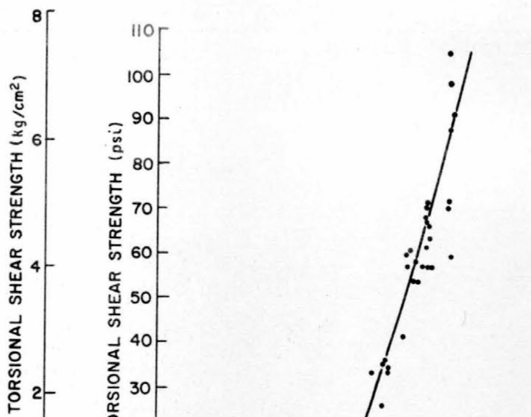

Figure 10. Torsional shear strength vs. density for high-density snow.

Figure 10 is a plot of the torsional shear strength vs. density. The values obtained are somewhat lower than those obtained in the double shear tests. Since direct shear tests tend to give higher values of shear strength than triaxial tests these results are not unreasonable. However the helical type of break suggests a tension failure. It is tentatively suggested that Timoshenko's nomenclature be adopted, and results obtained in this manner be called the torsional modulus of rupture of the material.

WORK OF DISAGGREGATION OF SNOW

An investigation of the possibility of using the work of disaggregation of snow as a parameter of snow hardness was begun by SIPRE staff members several years ago. Because it would be useful to know the relationship between hardness and strength of snow, this analysis was made. It is also useful to know the work of disaggregation in snow removal and snow compaction problems.

An instrument to measure the work of disaggregation of snow was developed by H. Bader and B. L. Hansen (SIPRE Report 7). It consists of a variable speed motor driving an 8-in. diam wooden wheel 3 in. wide, up to several revolutions per second. (Figure 11). The wooden wheel has a number of steel pins on its periphery against which snow from a standard tube can be fed slowly at a constant rate while the wheel is rotating. As the steel pins disaggregate the snow, a torque is exerted on the spiked wheel, which appears on a record. This instrument was later modified by J. A. Bender for field use. It has a drive through a planetary gear system which is counterbalanced by a coiled spring, and any additional torque required to rotate the spiked wheel is reflected on a drum, and thereby recorded.

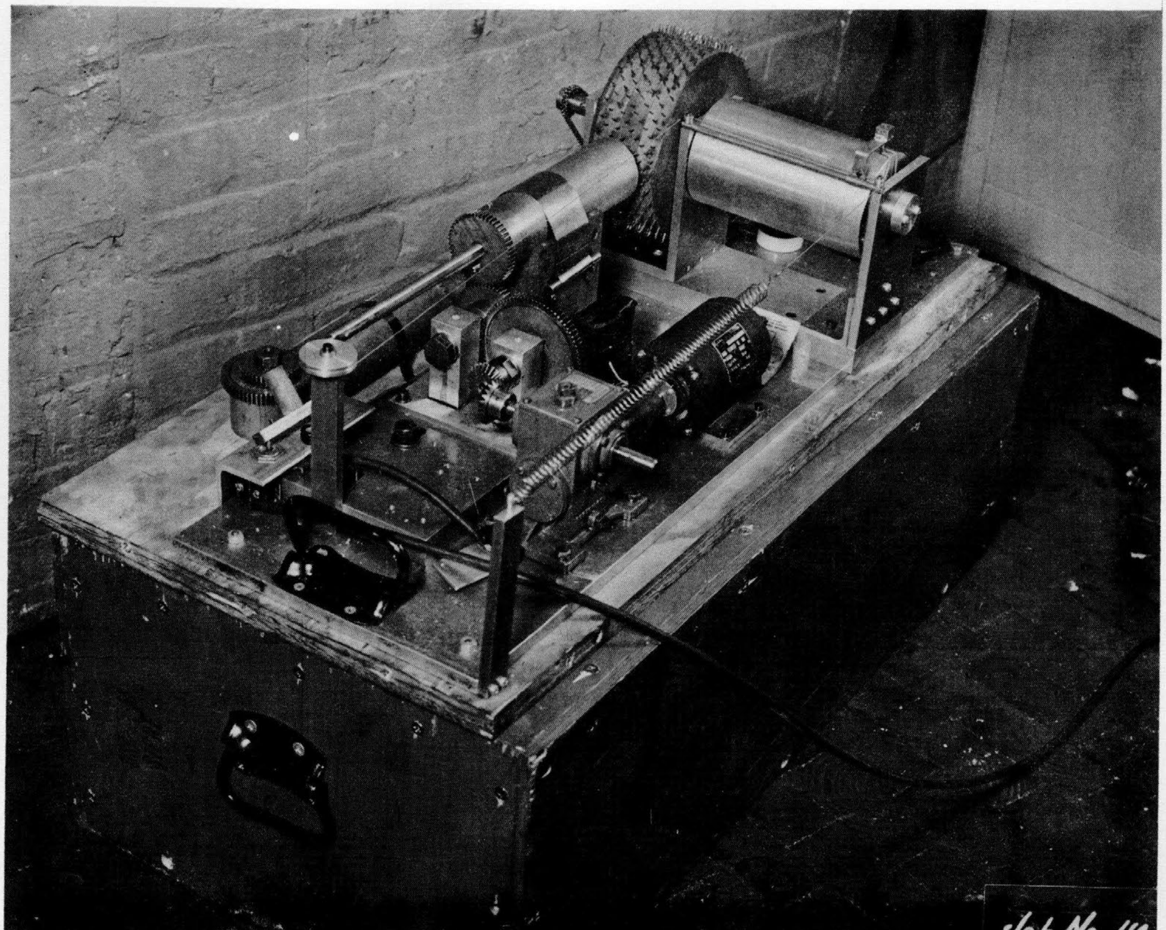

Figure 11. Apparatus for measuring work of disaggregation of snow.

The tests that are reported here were made by Mr. Bender during the 1955 field season. He used the lower-density broken double shear specimens and density profile samples taken in the upper 15 ft of the snow cover. Snows of densities greater than

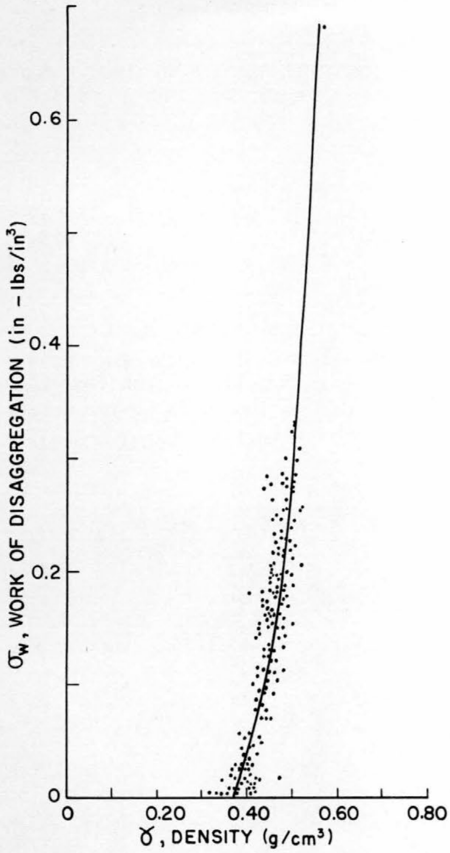

Figure 12. Work of disaggregation per unit volume vs. density for high density snows. $\sigma_w = 1.195(\gamma - 0.37)[1+65.12(\gamma-0.37)^2]$.

Table X. Work of Disaggregation of Snow.

Mean* Density (g/cm^3)	Density Range (g/cm^3)	Work to Disaggregate 1 in^3 of Snow (psi)
0.343	0.314-0.367	0.0044
0.379	0.369-0.390	0.0240
0.398	0.391-0.404	0.0318
0.409	0.406-0.415	0.0397
0.420	0.415-0.427	0.0627
0.432	0.427-0.435	0.1148
0.440	0.437-0.443	0.1510
0.447	0.445-0.449	0.1428
0.452	0.449-0.454	0.1804
0.458	0.456-0.460	0.2078
0.464	0.461-0.469	0.1788
0.476	0.471-0.479	0.2418
0.488	0.479-0.500	0.2414
0.527	0.504-0.569	0.4024

* Mean of 10 tests.

0.55 g/cm^3 were too hard for use with the present design of the disaggregator.

The work per unit volume was computed from

$$\sigma_w = \frac{\tau\theta}{V} \tag{6}$$

where σ_w is in psi, τ the measured torque in inch-pounds, θ in radians, and $\underline{V}$ the volume in cubic inches. A total of 140 tests were made, for which densities were known. All values were corrected to 14°F. Table X was assembled from the mean values of groups of 10 tests each. Figure 12 is a plot of all the data.

An empirical equation was formulated relating the work of disaggregation per unit volume to density:

$$\sigma_w = 1.195(\gamma-0.37)\left[1+65.12(\gamma-0.37)^2\right] \tag{7}$$

where σ_w is the work of disaggregation per unit volume in psi, and γ the density in g/cm^3. Further analysis using this equation is contained in the discussion of results.

ANGLE OF INTERNAL FRICTION AND APPARENT UNIT COHESION

Coulomb's equation assumes that the shear strength of a material subjected to a lateral pressure is determined by the relationship

$$S = c + p \tan \phi \tag{7}$$

in which p is the lateral pressure (normal stress), c is a constant called the apparent unit cohesion, and ϕ is the angle of internal friction. Furthermore, it is assumed that the values of c and ϕ are independent of the states of stress which preceded the failure. These conditions are approximately true for cemented sand and moist sand. Equation (7) is a straight line, often called a line of rupture or Mohr's envelope of rupture.

Figure 13 is a Mohr's envelope of rupture curve constructed from the shear strength vs. lateral pressure data shown in Table VIII. This shows that Coulomb's equation does not hold without corrections for high-density snows. The envelope of rupture, instead of being a straight line, has a pronounced curvature. Similar relationships were found for the rupture of marble and limestone (Robertson, 1955). It should also be emphasized that the shear failures that occurred were not under true triaxial conditions. These snow samples broken in double shear were merely loaded with axial pressures, but were not under a uniform pressure distribution. This may be a cause for the discrepancies between the results and Equation (7).

After the shear stress was applied, it was noted, for the specimens subjected to the low lateral pressures of 5 and 15 psi, that it was necessary to move the piston out to maintain a constant load, indicating that the snow specimen was expanding. For the higher lateral pressures of 45 and 60 psi, it was necessary to move the piston in to maintain a constant load, indicating a decrease in volume of the specimen. This was not due to simple consolidation of the snow because no decrease in volume occurred in the relatively long period of time before application of shear stress.

The shear strength data was plotted on a Mohr's envelope of rupture diagram (Fig. 14) using the unconfined compression and ring tension values for 0.45 g/cm^3 density. If the tensile strength and the unconfined compressive strength are known, then these values serve to define the origin for the determination of Mohr's envelope of rupture for an ideal material. The values for shear strength (represented by x's) fall considerably higher than those obtained from the straight line relationship. The fault of this analysis probably lies in the type of test performed. Taylor (1948) states that larger relative values of the shear strength are obtained in direct shear than in cylindrical compression tests. Failure must occur on defined planes in direct shear tests and, therefore, such tests on isotropic materials tend to give somewhat larger strength values than compression tests, in which failure occurs in the weakest plane.

Values of apparent unit cohesion, c, and angle of internal friction, ϕ, can be determined in terms of tensile strength σ_T and unconfined compressive strength σ_c from the straight line relationship shown in Figure 14:

$$\phi = \sin^{-1} \frac{\sigma_c - \sigma_T}{\sigma_c + \sigma_T} \tag{8}$$

and

$$c = \frac{\sigma_T}{2 \cos \phi}(1 + \sin \phi). \tag{9}$$

Table XI gives the values obtained from equations 8 and 9. The angle of internal friction ranges between 19° and 23°.

The angle of rupture cone obtained with unconfined compressive strength tests is another means of determining the angle of internal friction. The angle of rupture, θ, bears the following relationship to the angle of internal friction:

$$\phi = 2(\theta - 45). \tag{10}$$

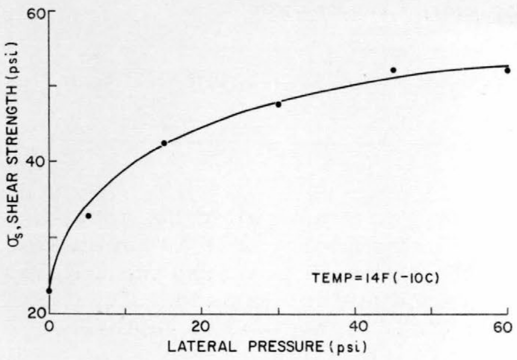

Figure 13. Shear strength vs. lateral pressure for snows of mean density of 0.45 g/cm³. Each point represents a mean of 10 tests.

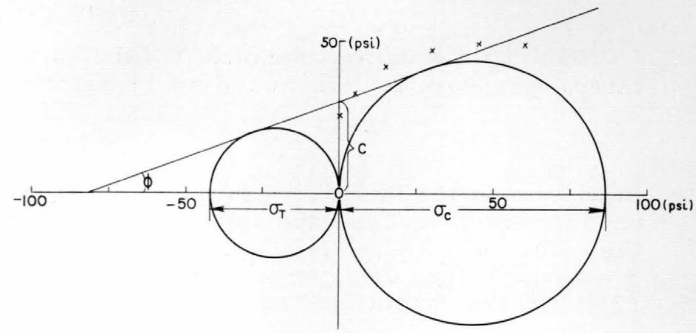

Figure 14. Shear strength data plotted on a Mohr's envelope of rupture diagram.

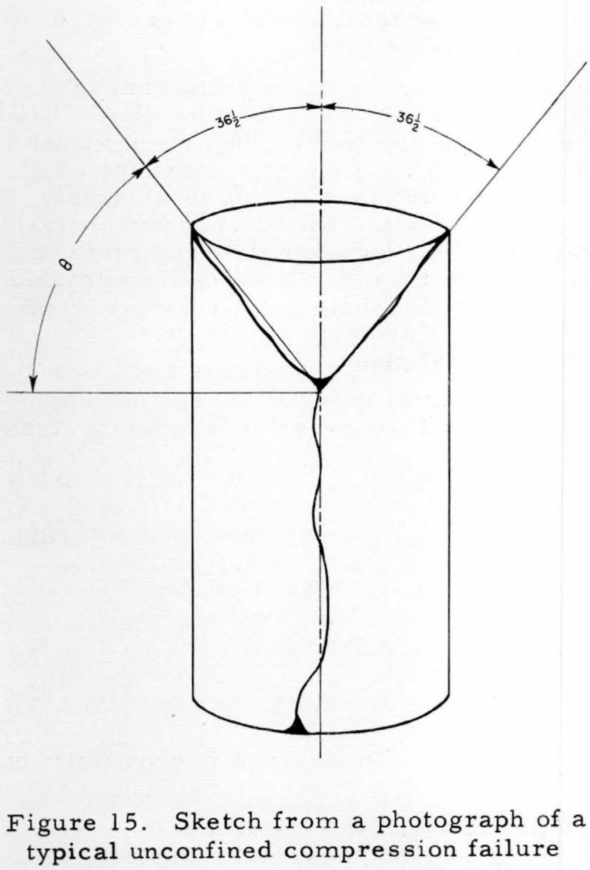

Figure 15. Sketch from a photograph of a typical unconfined compression failure of a high-density snow specimen.

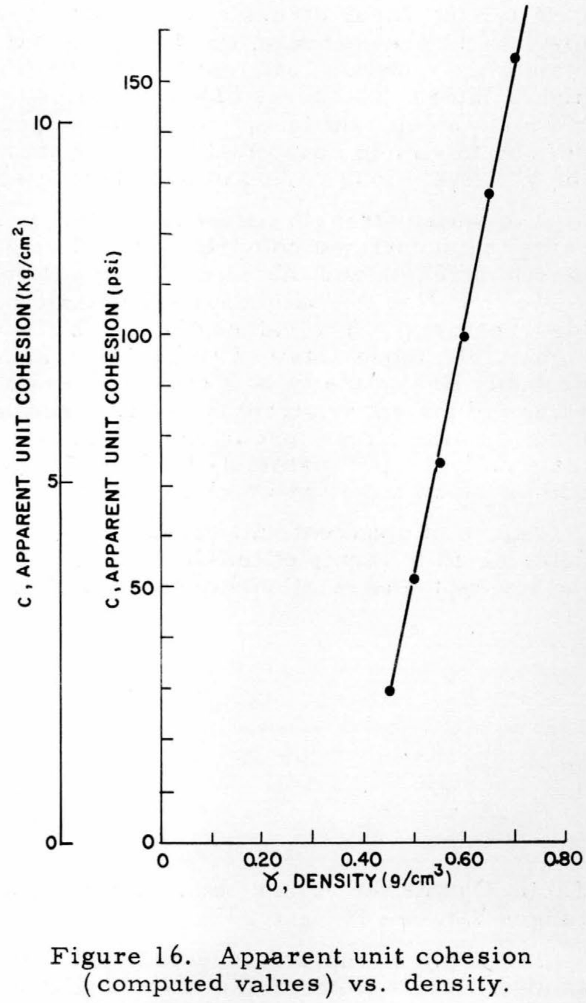

Figure 16. Apparent unit cohesion (computed values) vs. density.

Table XI. Angle of Internal Friction and Apparent Unit Cohesion Using Crushing Strength and Ring Tensile Strength Data, Corrected to 14°F.

γ (g/cm³)	σ_c (psi)	σ_T (psi)	ϕ (deg)	c (psi)	sin ϕ
0.45	85	41.2	20	29.6	0.347
0.50	156	68.6	23	51.7	0.389
0.55	227	99.0	23	74.8	0.393
0.60	298	133.3	23	99.8	0.382
0.65	369	177.4	21	128.0	0.351
0.70	440	218.0	19	155.0	0.337

$\sigma_c = 1418\,(\gamma - 0.39)$.

$\sigma_T = 503\,(\gamma - 0.37)\,[1 + 2.88(\gamma - 0.37)^2]$.

$$c = \frac{T}{2\cos\phi}\,(1 + \sin\phi).$$

$$\phi = \sin^{-1}\frac{\sigma_c - \sigma_T}{\sigma_c + \sigma_T}.$$

Figure 15 was made from a photograph of a typical unconfined compression failure of a high-density snow specimen, $\theta = 90° - 36.5° = 53.5°$, and $\phi = 17°$, which falls near the range obtained by the other method. Diamond and Hansen (1956) report values of angle of internal friction obtained with a shear vane on lower density snows (0.20 - 0.32 g/cm³) in the range of 20° to 30°.

A graph of the apparent unit cohesion data vs. density data shown in Table XI (Fig. 16) shows a linear relationship. Comparison with the ring tensile strength vs. density curve (Fig. 3) shows that the two curves are nearly parallel, with the apparent unit cohesion values approximately 25% below the corresponding tensile strength values. The similarity of slopes is quite interesting. However, Terzaghi (1943) states that, for cohesive materials such as clays, the values of c and ϕ in this equation merely represent two empirical coefficients, and that there is no relation between apparent and true cohesion other than the name.

The analysis that was performed and is presented here is an attempt to present as comprehensive a picture as possible of the strength analysis of snow. Coulomb's law fails for the shear strength data. As already suggested, this probably is due to the direct shear type of test performed.

DISCUSSION OF RESULTS

For all the tests made — unconfined compression; unconfined and confined double shear; ring, flexural, and centrifugal tensile strength; torsional shear tests; and the work of disaggregation per unit volume, the curves of strength vs. density continued through $\sigma = 0$ would fall between $\gamma_o = 0.35$ and $\gamma_o = 0.40$ g/cm³. An attempt was made to express all the strength data in a common equation, involving only a difference in constants. It was found that the expression

$$\sigma = a\Delta\gamma(1 + b\Delta\gamma^2) \tag{11}$$

could be made to fit all of the curves, where $\Delta\gamma = \gamma - \gamma_o$ and γ_o is the density at $\sigma = 0$. For the double shear tests the most likely intercept for the 0, 30, and 60 psi lateral pressure curves was chosen as 0.37 g/cm³. A straight line was drawn through the intercept tangent to the curves; a in equation (11) is the slope of the straight line. The value of b, which is an indication of the curvature, was determined by selecting a point on the curve at 0.67 of the maximum strength value and substituting the value of σ and $\Delta\gamma$ from here. The values of a and b that best represent the empirical data are:

	a	b
Unconfined compression	1418	0
Tension	503	2.88
Double shear — unconfined	333	7.04
Double shear — 30 psi lateral pressure	558	5.44
Double shear — 60 psi lateral pressure	643	6.13
Torsional shear	230	5.73
Work of disaggregation per unit volume	1.195	65.1

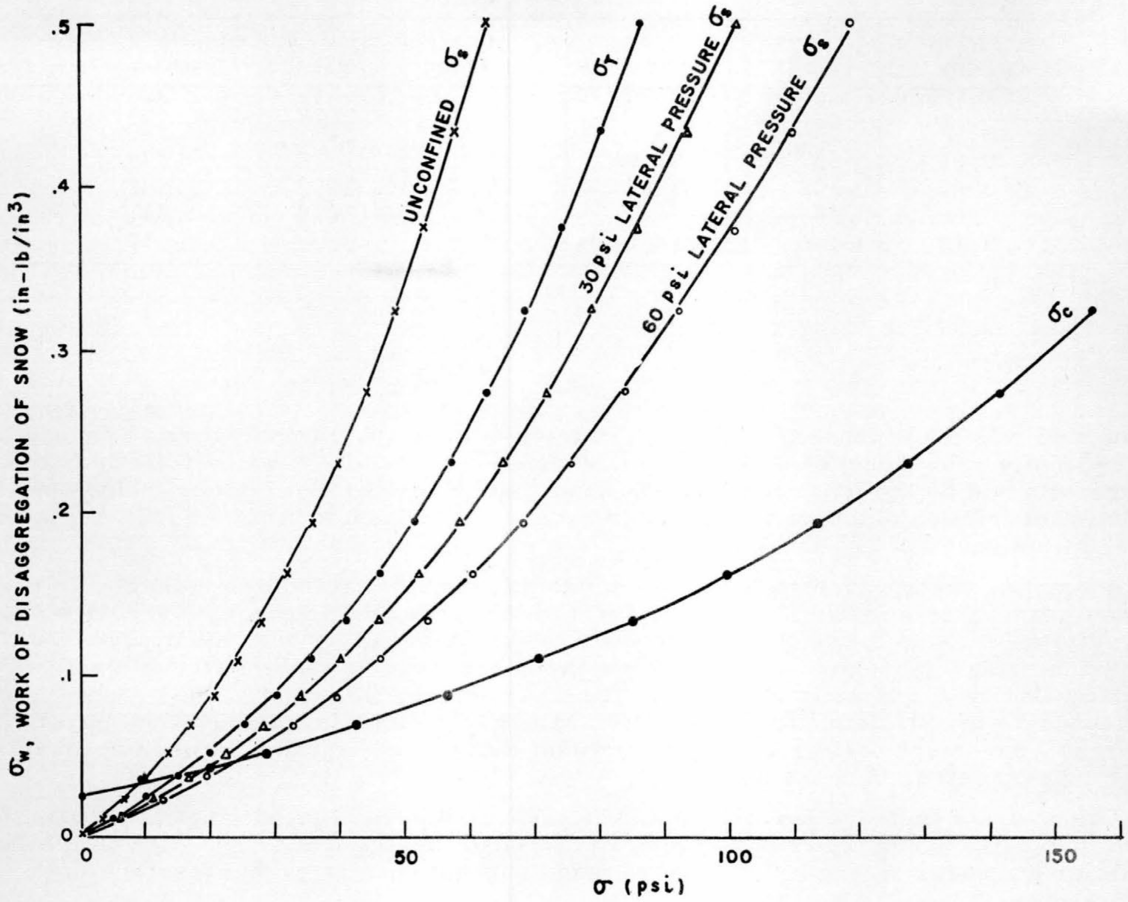

Figure 17. Work of disaggregation of snow vs. crushing and tensile strength, and shear strength at various lateral pressures.

For the data on work of disaggregation per unit volume, least square determinations were made for various values of the intercept γ_o and the use of $\gamma_o = 0.37$ g/cm^3 produced the least deviation from the empirical values.

This interpretation is not meant to indicate that snow of lower densities than 0.37 g/cm^3 has absolutely no strength, since strengths have been measured for these snows. Kuroda (1929) measured 63 g/cm^2 (0.9 psi) tensile strength and 3 g/cm^2 (0.04 psi) shear strength for loose powder snows. A different type of structure exists below this critical density. It has been noticed that snow disaggregated and heaped always assumes an uncompacted density between 0.35 and 0.42 g/cm^3. It is known that strength is some function of the contact area of the grains. At the lower densities, the number of contacts and the total contact area between grains decrease very rapidly, and also the strength.

Work of disaggregation was related to crushing and tensile strength, and to shear strength at various lateral pressures, using the empirical relationships previously determined (Fig. 17). The value of γ_o for all the equations is 0.37 g/cm^3, except for the crushing strength for which $\gamma_o = 0.39$ g/cm^3. Figure 17 also shows the the relationship between the various strengths.

The evaluation of the ring test for the determination of the tensile strength of snow shows that this test yields results comparable with those obtained by other means. The flexural strengths of simple beams obtained by Professor Stearns agree very well with the ring values, although the beams show more scatter. The strength values obtained with the centrifugal tensile strength apparatus also fall in line, although done on lower density snows. It is recommended that ring tests be used for the determination of tensile strength of materials that can be cored easily, because of the excellent reproducibility of results and the relative ease of preparation of the test specimen.

The tests for the determination of the shear strength were performed in direct shear. Although the results by themselves are quite reproducible, and can be formulated, the usefulness of direct shear tests is questionable. It has been stated that direct shear tests yield higher values than those obtained in unconfined compression tests, therefore Coulomb's equation cannot be used with these results for the determination of Mohr's envelope of rupture. However the angles of internal friction obtained considering Mohr's envelope to be a straight line do seem to agree by two methods, although not enough data are available to draw any definite conclusions. The entire problem of ultimate strength of snow will be investigated further, using triaxial and other test methods.

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