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Energy of Snow Compaction and its Relation to Trafficability

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Snow Ice and Permafrost Research Establishment, Corps of Engineers, U. S. Army, Wilmette, Illinois

ENERGY OF SNOW COMPACTION AND ITS RELATION TO TRAFFICABILITY - J. K. Landauer and Frank Royse

Research Paper 14, October 56, 11pp-illus-tables. DA Proj 8-66-02-004, SIPRE Proj 22.1-9. Unclassified Report

Penetrometer tests were performed in the field on natural snow to determine the work of compaction. It was found that the work per unit area was independent of the penetrometer area and was not a strong function of velocity. The power used to compact snow is much less than that available from over-snow vehicles. Other effects must be responsible for the high observed energy losses.

UNCLASSIFIED

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2. Snow--Trafficability
3. Snow--Mechanical properties

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ENERGY OF SNOW COMPACTION AND ITS RELATION TO TRAFFICABILITY

by
J. K. Landauer and Frank Royse

ABSTRACT

Penetrometer tests were performed in the field on natural snow to determine the work of compaction. It was found that the work per unit area was independent of the penetrometer area and was not a strong function of velocity. The power used to compact snow is much less than that available from over-snow vehicles. Other effects must be responsible for the high observed energy losses.

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INTRODUCTION

In order to design vehicles for over-snow use, it is necessary to understand the mechanisms by which energy is lost in over-snow movement. For a tracked vehicle, energy will be expended in:

(1) Compacting snow under the track.
(2) Shearing snow at the sides of the track.
(3) Pushing aside and piling snow.
(4) Disaggregating snow.
(5) Slipping on snow.

These several mechanisms are not necessarily independent. A further loss of energy, not occurring directly in the snow, is due to an increase of internal friction in the suspension of the vehicle when riding on a yielding surface.

This paper deals primarily with (1), the energy of compaction.

A number of investigators have studied this problem. Bucher (1946) compacted snow at -2C to -3C in the laboratory and determined the density for a series of increasing pressures up to 35 psi. For simplicity he represented his data by the equation:

$$\gamma^2 = 0.03\sigma$$

where γ is the density of the snow in g/cm^3 and σ the pressure in psi. The fit of his data to this representation is poor. A more adequate representation is given by:

$$\gamma = \frac{\gamma_{ice}}{1+a\sigma^{-n}}$$

where _a_ and _n_ are constants. This equation expresses the boundary condition that the limiting density is that of ice.

Klein (1947) pushed a circular plate of 57.5 in^2 area vertically into the snow cover and determined the pressure-penetration curve up to a maximum of 3.5 psi. By numerically integrating this curve he arrived at the energy of compaction. The usefulness of his results is limited because he did not specify the snow conditions such as depth, density, and temperature, which ultimately determine the energy.

Barrett (1948) dropped weighted disks of 5, 10, and 20 in^2 area on snow and measured the penetration using pressures up to 2 psi. His results indicate that the work per unit area, _w_, is proportional to the square of the penetration, _h_: $w = ch^2$ (_h_ in in. and _w_ in ft-lb/in^2). Barrett finds a value of about 0.01 for _c_, but he does not state the snow temperature or density. The effect of plate size does not appear to be important.

Kragelski (Kondrat'eva _et al._, 1949) reports the results of experiments on snows at various temperatures from -1.5C to -21C under pressures up to 30 psi. For snow of initial density γ_0 (g/cm^3), at a temperature of _T_(C), and for a pressure σ (psi), the density becomes:

$$\gamma = \gamma_0 + \frac{K_1\sigma(K_2+T)}{\sigma+K_3}$$

where $K_1 = 3.8 \times 10^{-3}$, $K_2 = 96$, $K_3 = 0.08$.

Here, the effects of temperature and initial snow density are considered.

Yosida and Huzioka (1950) determined the load-penetration curve, finding:

$$\sigma = (5.9+5.3h+1.0h^2) \times 10^{-2}.$$

Scatter of experimental results limits the reliability of this equation. Snow parameters were unspecified in the available abstract.

Gold (1954) used a mechanical penetrometer and measured the penetration-pressure curve for penetration parallel to the surface. Although the snow conditions are defined, the horizontal measurement is of little value for trafficability purposes. Gold finds no deformation up to a critical pressure and then roughly a linear deformation up to his limit of about 8 psi. Plate sizes from 15 to 50 in^2 do not influence the results appreciably. No numerical analysis is made.

T. R. Butkovich and M. Diamond, SIPRE (personal communication) measured the penetration of dropped weights with pressures up to 2.9 psi. They found

$$\sigma = .46h$$

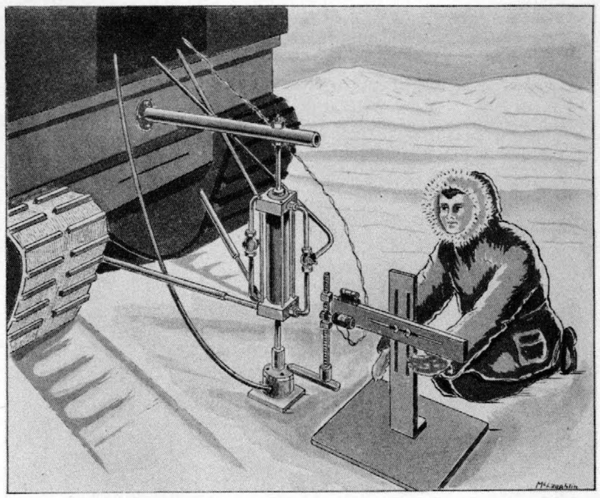

Figure 1. Penetrometer mounted at rear of an Otter. Recorder and hydraulic pump are in the vehicle.

for snow of density near 0.3 g/cm^3 at a temperature of -6C.

Although these measurements give an indication of the magnitude of the work necessary for compaction, the conditions are not sufficiently well defined for a reliable value. For the loaded plate tests, it was necessary to drop the plate from above the surface in order to obtain any penetration. This introduced an error due to the added kinetic energy. It is also apparent that methods involving measurement of density changes are difficult to analyze because of initial snow inhomogeneity and the difficulty of determining the depth to which the density changes occur. In none of the tests was the velocity of penetration considered as a parameter. It was thought that tests more nearly duplicating the conditions of an over-snow vehicle would be worth performing. To this end penetrometer tests at variable speed and with specification of snow parameters were carried out.

METHOD

Field experiments were performed on natural snow at Keweenaw Field Station, Houghton, Michigan, on 9 January 1956 and again on 7 March 1956. Penetrometer tests were made in the following manner. An electrically driven hydraulic ram was mounted on an Otter vehicle (Fig. 1). A 2000-lb capacity load cell was mounted on the lower end of the ram and transmitted force to aluminum plates of 4 x 4 in., 6 x 6 in., and 10 x 10 in. The load cell rested in a depression at the center of the plate but the plate was not constrained to remain normal to the ram. Although the ram penetrated normal to the snow surface, the plate could tilt to some extent. Results of runs for which the tilt exceeded about 10 deg were discarded. The penetration measured was the distance traveled by the center of the plate relative to the snow surface and was limited to 10 cm by the measuring apparatus. Both penetration and load were continuously recorded on a double-pen Brown "Electronik" recorder.

An experimental site with an apparently level surface and uniform snow characteristics was chosen. All tests on a given day were performed in the same general area. Air and snow temperatures and snow depths were measured frequently. The snow profiles on the two days are shown in Figure 2.

Tests were run for rates of penetration from 0.056 to 1.59 ft/min and for the three plate sizes. The recorder data were transferred to stress-strain curves and integrated by planimetry. Because of the collapse characteristics (see Fig. 3) the

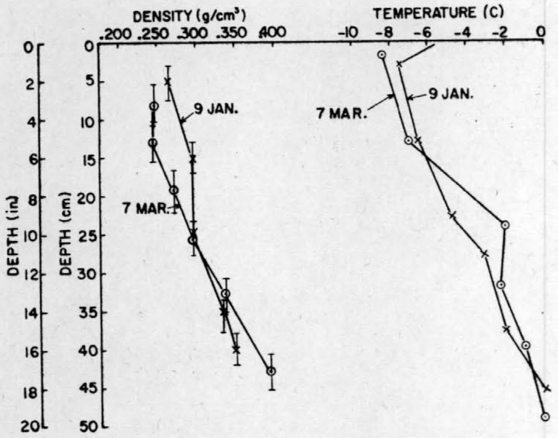

Figure 2. Snow observations at snow compaction test sites, Keweenaw Field Station, Houghton, Michigan, 9 January and 7 March, 1956. Total snow depth = 48 cm on 9 Jan.; 56 cm on 7 March

Date	Depth (cm)	Grain Size (mm)
9 Jan.	0-37	~ 1
9 Jan.	37-48	~0.5
7 Mar.	0-1	Wind crust
7 Mar.	1-22	~ 3
7 Mar.	22	Wind crust
7 Mar.	22-56	~ 1

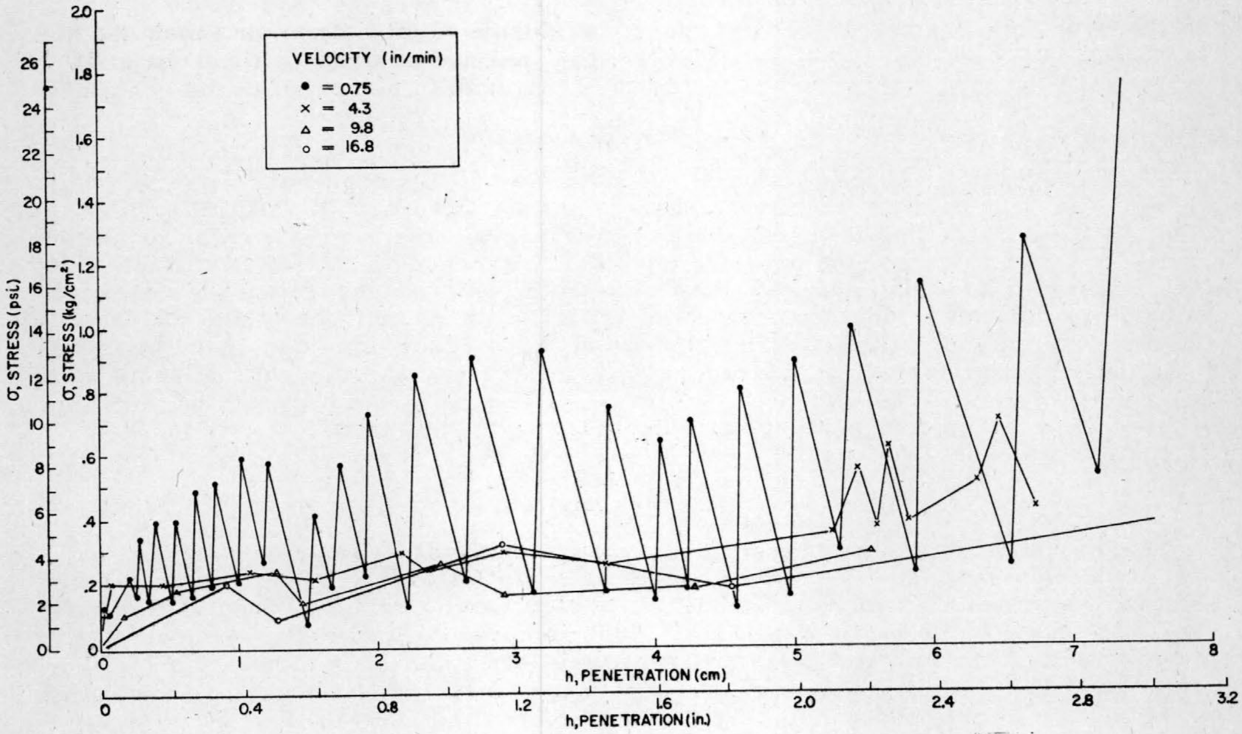

Figure 3. Stress-penetration curves for four velocities, 7 March 1956. Note the major collapse features on the low velocity curves. Small amplitude oscillations on higher velocity curves have been smoothed out.

integration was obtained to various limits of penetration, which proved more consistent than integration to limits of load. The work-penetration data for various plate sizes obtained 9 January are shown in Figure 4. The average curves obtained 7 March for various velocities with the 6 x 6 in. plate are shown in Figure 5. The indicated errors are the mean deviations of the results for experiments of a particular plate size and velocity. Each average represents four or five runs within the velocity range (for example, see Fig. 6). Logarithmic plots of work per unit area vs. penetration are presented in Figures 7 and 8. The characteristics of the data obtained during the two experimental periods are somewhat different. It is believed that difference in slope may characterize the nature of the snow.

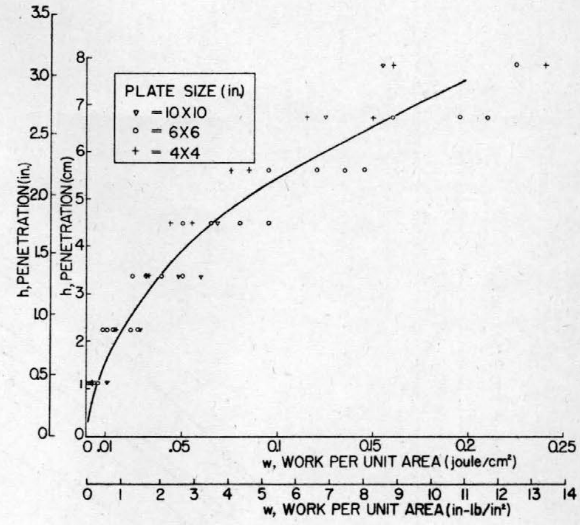

Figure 4. Depth of penetration vs. work per unit area for three plate sizes, 9 January 1956. Mean curve indicated. Compactive velocity = 0. 87 - 0. 98 in/min.

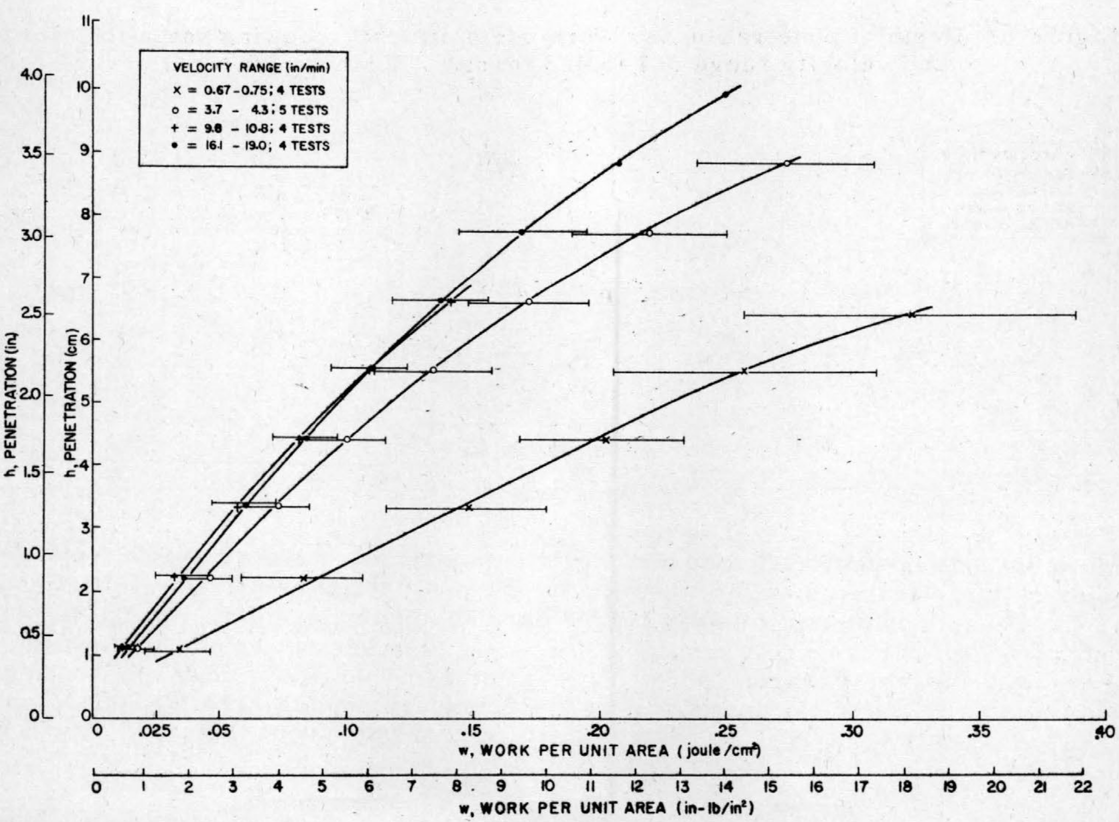

Figure 5. Depth of penetration vs. work per unit area, for four velocity ranges, 7 March 1956. Indicated errors are the mean deviations.

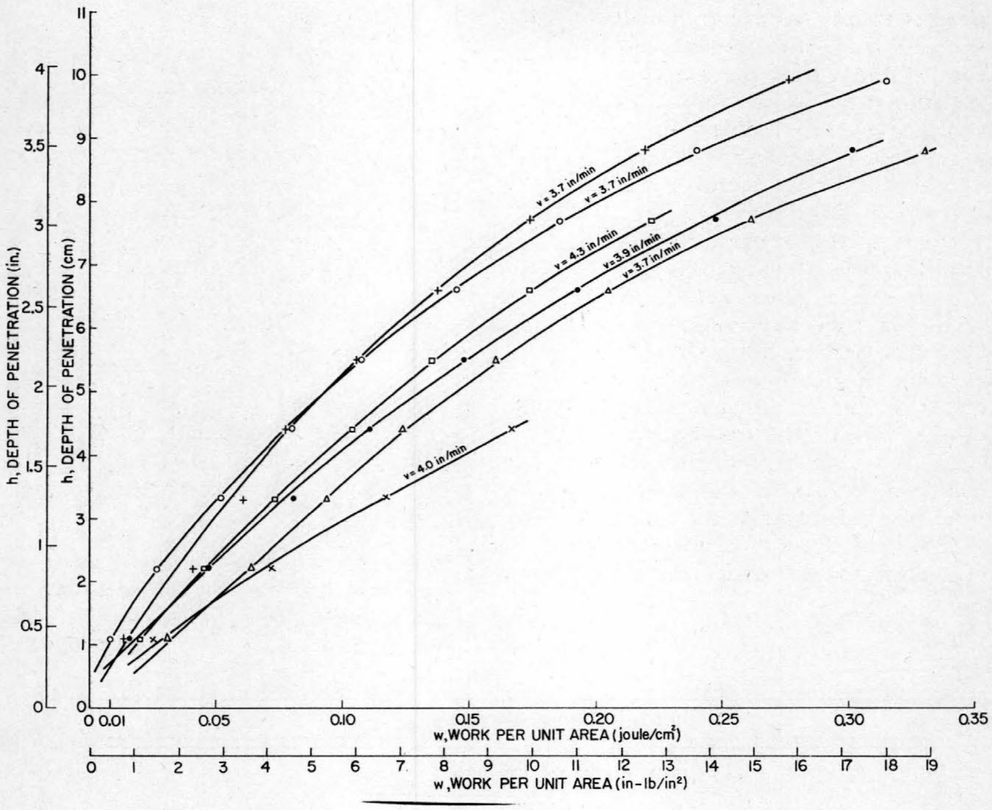

Figure 6. Depth of penetration vs. work per unit area, showing variations for the velocity range 3.7 to 4.3 in/min. 7 March 1956.

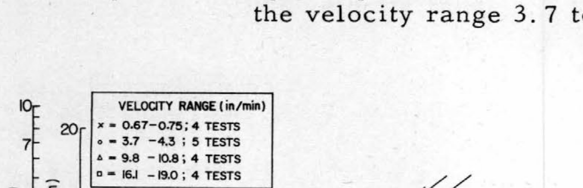

Figure 7. Logarithmic plot of depth of penetration vs. work per unit area for 4 velocity ranges, 7 March 1956. Mean deviation indicated for several curves.

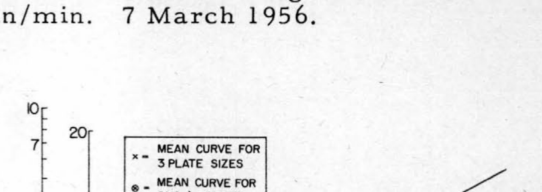

Figure 8. Logarithmic plot of depth of penetration vs. work per unit area for same velocity range on two days, 9 January and 7 March, 1956. Mean deviation indicated.

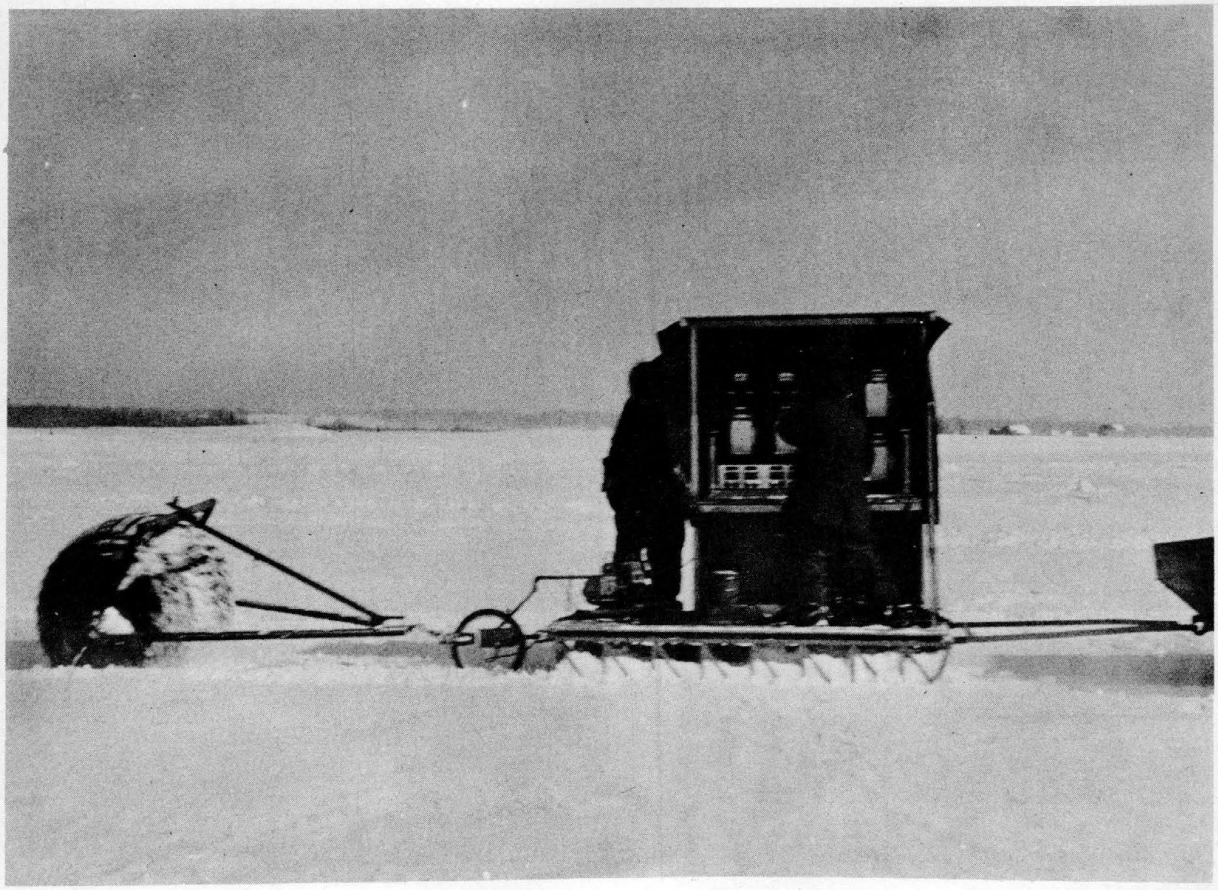

Figure 9. Rolligon bag being pulled by Otter. The drawbar pull is measured by load cell, which can be seen through the fifth wheel. Velocity and load are recorded on the instrument sled.

On 8 March another series of experiments was performed to determine the drawbar force required to pull a Rolligon bag over the snow surface at several velocities. The rubber bag, weighing about 1000 lb, was pulled behind an instrument sled and compacted the snow in the strip of undisturbed snow between the tracks left by the sled runners. Velocity and drawbar force were electrically recorded on the instrument sled. Tow speeds of 3.3 to 22 ft/min were provided by drawing the instrument sled by winch; to obtain higher speeds of 58 to 360 ft/min, the sled was towed behind an Otter (Fig. 9). The drawbar force was averaged over a fifty to several hundred foot distance after a steady velocity was reached. The data are shown in Figure 10. Because the drawbar force may also depend on velocity when pulled over a hard surface, no quantitative conclusions should be drawn from the Rolligon data.

DISCUSSION

From Figures 7 and 8, it can be seen that the energy per unit area $\underline{w}$, involved in compacting snow is a function of the penetration, $\underline{h}$, and can be represented by:

$$w = ah^r \qquad (1)$$

where $\underline{a}$ and $\underline{r}$ are constants.

The stress-work per unit area relation can be derived from the above equation since:

$$w = \int_0^h \sigma dh.$$

Taking the derivative with respect to $\underline{h}$, the following relation is found:

$$\sigma = arh^{r-1}$$

the penetration being:

$$h = \left(\frac{\sigma}{ar}\right)^{\frac{1}{r-1}}.$$

The energy per unit area is then:

$$w = \left(\frac{\sigma a^{-\frac{1}{r}}}{r}\right)^{\frac{r}{r-1}} \qquad (2)$$

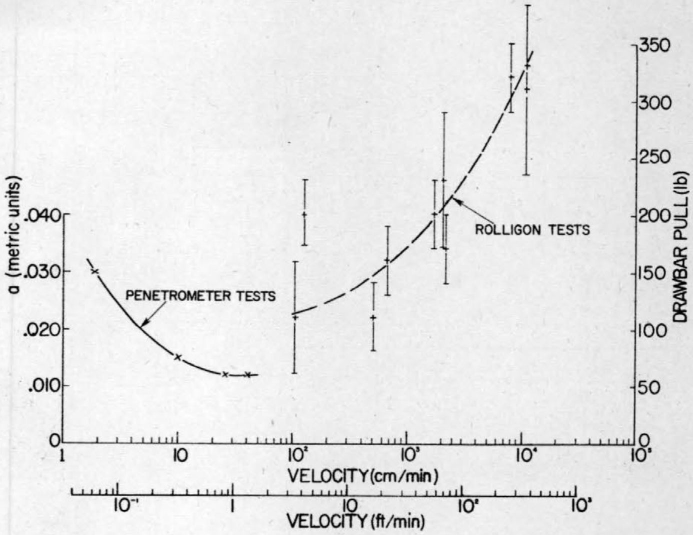

Figure 10. Dependence of work on velocity, 7 March 1956.

Ordinate for the penetrometer data is constant $\underline{a}$, in $w = ah^r$.

Ordinate for the Rolligon data is drawbar pull.

The data of 9 January give $\underline{a} = 1.2$ (in the English system, see Table I) and $\underline{r} = 1.85$. For a pressure of 1.25 psi, the work done is 0.3 in-lb/in². Compared with estimates made from the results of previous workers (Table II), this value is very low. However, it must be noted that in no case are the experiments really comparable. The experiments using dropped weights include a potential energy term. For instance, in the tests by Butkovich and Diamond, the weights were dropped from about 1 inch. For a pressure of 1.25 psi, this contributes an energy of 1.25 in-lb/in². When this value is subtracted from their calculated energy, the result agrees adequately with ours. In most of the other cases, the snow parameters have not been specified sufficiently to make a reliable comparison. Gold's tests were made in a direction normal to ours and the snow layering might account for some of the difference.

Before the results obtained here can be used to calculate the energy going into compaction for an over-snow vehicle, it is necessary to know the dependence of energy on velocity of penetration; its variation with compacting area; and whether the manner of compaction is similar in both cases.

The results of the penetrometer tests for various velocities, as well as the Rolligon bag results, indicate that there may be a dependence of work on velocity. That is, a given pressure will penetrate to different depths depending on the velocity of application of the pressure. However, the work done is not a strong function of velocity.

An increase by a factor of about 200 will bring the maximum penetrometer velocity to the neighborhood of vehicle velocities. Examination of Figure 10 shows that the work will probably change by much less than a factor of ten in this region.

The results for various plate sizes show, in agreement with Gold (1954) and Barrett (1948), that the work per unit area is independent of the size. This result probably holds while the plate dimensions are less than or much greater than the snow depth. The stress distribution under the plate will be altered when the depth of the snow becomes comparable to the plate dimensions. The stress distribution under a long, narrow plate (i.e. a track) extends to a depth only slightly larger than that for a square plate having the same width. The energy analysis therefore should be

Table I. Work Per Unit Area, w, Calculated From Equation 2.
(a, r = constants from Equation 1; σ = pressure)

Date	v, Velocity (in/min)	r	a		w (in-lb/in²) for	
			Metric System *	English System *	σ = 1.25 psi	σ = 2.0 psi
9 Jan	0.87 - 0.98	1.85	0.0043	1.2	0.3	1.0
7 Mar	0.67 - 0.75	1.32	0.029	5.4	0.004	0.03
7 Mar	3.7 - 4.3	1.32	0.014	2.8	0.03	0.2
7 Mar	9.8 - 10.8	1.32	0.011	2.1	0.08	0.6
7 Mar	16.1 - 19.0	1.32	0.011	2.1	0.08	0.6

*	Metric	English
h	cm	in.
σ	joule/cm³	psi
w	joule/cm²	in-lb/in²

Table II. Sample Values for Energy of Compaction, for σ= 1.25 psi.

Investigator	Method	γ_o Initial Density (g/cm³)	T, Temp. (C)	Total Snow Depth (in)	w, work (in-lbs/in²)	Remarks
Bucher	Lab	.25	-2	5	.45	Used modified representation
Klein	Pene-trometer				2.4	No snow parameters known
Barrett	Weights	~.2		35	1.9	
Kragelski	Weights	.15	-6	5	1.9	Integrated density change unknown
Yosida	Weights				4.5	Snow parameters unknown
Gold	Pene-trometer	.2	-3		6.6	Compacted along surface
Butkovich and Diamond	Weights	.3	-6	∞	1.7	
Landauer and Royse	Pene-trometer	.25	-6	20	0.3	

applicable when the snow depth is greater than the track width. For shallower snows an undetermined correction must be made.

The question as to whether the penetrometer compacts snow in a manner similar to a vehicle is less easily settled. For the penetrometer, the pressure applied to the snow surface builds up until a collapse occurs. During the build-up, the penetration increases only slightly while the load (Otter) is lifted. At collapse, the load and penetrometer drop downward until stopped by the snow. The penetration, therefore, increases stepwise but with a linear envelope determined by the rate of oil flow into the ram. These steps are only appreciable for the lowest velocities, amounting to 0.5 in. at the maximum. At higher velocities the steps are negligible. A vehicle moving over the snow applies its load in an analogous manner. The track comes into contact with the snow at a rate equal to its forward velocity and produces a series of collapses until the snow supports the vehicle. In reality, however, vibratory motion and shear may produce greater penetration than a smoothly applied pressure. Thus the work done by a penetrometer may be less than that expended by a vehicle.

Assuming that work per unit area is independent of velocity of penetration and compacting area, and that the compacting mechanisms are similar, the energy of compaction is computed as follows:

The power, P, expended by a vehicle in compacting snow is the work for the area compacted divided by the time, t, required:

$$P = \frac{wbvt}{t}$$

Here b is the total width of track and vt is the distance traveled at velocity v. As before, w is the work per unit area. Therefore:

$$P = wbv.$$

Example:

For a weasel,

$b = 40$ in.

If pressure = 1.25 psi,

$w = 0.3$ in-lb/in^2 (Table I)

and, for velocity

$v = 100$ in/sec (5.7 mph),

$P = 1200$ in-lb/sec = 0.2 hp.

This should be compared with the available horsepower, which is certainly much less than the rated value of 65 hp. Assuming the available power is between 10 and 20 hp, the amount going into compaction is less than a few percent of the total expended.

SUMMARY AND CONCLUSIONS

As the various physical properties of the snow (temperature, density, grain size) could not be controlled, the error produced by variation of these properties was minimized by multiple testing in a restricted area during the shortest possible time interval. The experimental site of 7 March was, of necessity, slightly removed from the site used on 9 January. Both sites were situated, however, in open fields with an undisturbed snow cover of similar depth and density (Fig. 2). A pronounced but variable wind crust and lower surface temperatures were observed at the site of March 7. These conditions help to explain the higher initial stresses encountered at this site, and may account for the difference in the form of the integrated curves. For low stresses, the work done should be less when the snow is crusted and the penetration small. At higher stresses the effect of the crust should be relatively unimportant.

As long as the plate size is small compared to the snow depth, the work of compaction per unit area is independent of the area of the plate. This implies that side shearing contributes a negligible energy relative to the compactive energy.

Although the dependence of work on velocity of compaction has not been quantitatively established, some trends seem to be apparent. The work appears greater at low and high velocities than at intermediate rates. Low velocity penetration proceeds with a major collapse feature (Fig. 3). Hardening of the snow may occur between collapses at low velocities. At higher velocities less hardening takes place in the shorter time between collapses. At still higher velocities, a general trend of increasing work with velocity (perhaps a velocity-dependent friction) becomes dominant. The results indicate, however, that there is no strong dependence of work on velocity in the region between 10^{-3}ft/sec and 10 ft/sec.

Since the power going into compacting the snow is much less than is available in over-snow vehicles, the question arises as to where the power is going. An understanding of this power loss might make possible the designing of a very much more efficient vehicle.

The energetics of simple plate compaction tests do not yield absolute data for trafficability use. Yet, since vehicles obviously labor when they sink deeply in snow, measurement of penetration of loaded plates may be useful in predicting trafficability. The explanation of the high energy losses, however, must be sought elsewhere than in the work of compaction.

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