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LONGSHORE CURRENT VELOCITY: A REVIEW OF THEORY AND DATA

by

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Longshore Current Velocity: A Review of Theory and Data

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Abstract. A proven prediction of longshore current velocity is not available, and reliable data on longshore currents are lacking over a significant range of possible flows. Theoretical studies have been based on over-simplified models, and empirical predictions have been hampered by lack of data. The empirically modified, momentum-flux theory now accepted as the best prediction is based on an untenable assumption and supported by inappropriate data. Regardless of their validity, however, all six of the testable equations agree fairly well with at least one of six sets of published data, and two agree with both of the better sets of data. These two equations may be used as empirical guides for velocity prediction in the absence of a proven theory. The best prospect for a generally valid velocity prediction appears to be an empirical correlation based on reliable data.

INTRODUCTION

Longshore currents flow parallel to the shoreline and reach their highest velocities between the point of wave breaking and the shoreline. These currents are assumed to be generated by the longshore component of motion in waves that approach the shoreline obliquely.

The effect of longshore currents on landing craft during World War II led to the comprehensive field, laboratory, and theoretical study of Putnam *et al.* [1949]. That study, as amended by the work of Inman and Quinn [1951], remains the accepted treatment of longshore currents. More recently, the ability of longshore currents to erode, transport, and deposit beach sediment has prompted further studies by engineers, who must predict the future effect of these currents on coastal structures, and by geologists, who wish to interpret the past importance of these currents in distributing sediment [Bruun, 1963; Brebner and Kamphuis, 1963; Inman and Bagnold, 1963; Harrison and Krumbein, 1964; Galvin and Eagleson, 1965].

The principal aim of published longshore current studies has been to develop a method for predicting the longshore current velocity. As a result of these studies, there are now at least 12 different equations to predict velocity and at least 352 published field and laboratory measurements of longshore currents. This paper reviews published field and laboratory observations that permit a description of longshore current flow, and it evaluates the theories proposed to predict longshore current velocity. The review covers papers appearing in North American sources; it is selective rather than exhaustive, emphasizing recent re-

sults and omitting data known to exist but unpublished [Galvin and Nelson, 1967, p. 3].

DATA

The variables. A review of eight published sets of field and laboratory data indicates that only five variables (Figure 1) have been measured in all studies:

Fig. 1. Definition of variables ($\beta = d_b/H_b$, $\sigma = a_t/H_b$). In field studies, slope is approximated from bottom profiles. MWL is mean water level.

a mean longshore velocity (V), a wave height (H), direction (θ), period (T), and a beach profile or slope (m). In these studies, other variables such as breaker depth, breaker position, and wave length were measured, but none of these additional variables was measured in more than two of the eight studies. Although it would be useful to relate longshore current velocity to deep-water conditions, as Brebner and Kamphuis [1963] point out, it is better to attempt to relate the current to the nearest condition of the breaking wave. The choice of the breaker position eliminates the effects of shoaling and refraction over the usually unknown offshore hydrography, the field problem of obtaining simultaneous deep-water and surf zone measurements, and the necessity in laboratory studies of computing hypothetical deep-water directions and wave heights. Since most investigators have measured the height (H_b) and direction (θ_b) at the breaker point, data from the studies that include only deep-water height (H_0) and direction (θ_0) [Brebner and Kamphuis, 1963; Saville, 1950] have been recomputed for the breaker position [Galvin and Nelson, 1967].

Laboratory data. The term 'observation' denotes all the measurements made during given wave conditions. The four laboratory studies under discussion contain 225 observations [Putnam et al., 1949; Saville, 1950; Brebner and Kamphuis, 1963; Galvin and Eagleson, 1965]. (The last-named study will be referred to hereafter as *TM 10*.) Seven other observations have been made by Eagleson [1965] on the growth of the current immediately downstream of an obstacle. Twelve of the 38 observations in *TM 10* do not include information on all five variables.

The laboratory basins used in these studies had test beaches that measured, between training walls, from 6 meters (20 feet, *TM 10*) to 18 meters long (60 feet)

[Saville, 1950]. Beach slopes in most tests were about 0.1, but they varied from 0.05 to 0.26. Most beach surfaces were of smooth concrete, but anything from sheet metal to bonded gravel has been used [Putnam *et al.*, 1949]. Saville's beach consisted of loose sand in the surf zone and a plane concrete surface off shore. In the four studies, off-beach water depth ranged between 0.3 and 0.6 meter.

There has been no systematic test of the relation between basin geometry and longshore currents, but it is known that wave heights vary irregularly with time and position in wave basins [Savage, 1962]. Basin geometry also affects the return of the fluid in the surf zone to the offshore part of the basin and, thus, the development of the currents known to be non-uniform in the longshore direction. Finally, basin geometry may affect the refraction pattern of waves on the beach, unless the training walls are curved to follow the wave orthogonals.

In the laboratory, the wave period (T) can be measured accurately to 0.01 sec, usually sufficient for 1% accuracy. Beach slopes (m) for typical plane concrete laboratory beaches are also accurately measurable, but the slopes of sand beaches involve some approximation. Typical laboratory waves about 5 cm high can be measured to within 5% by resistance gages or ordinary point gages [TM 10; Ursell *et al.*, 1958]. This accuracy is all that is warranted, since wave height can vary by greater amounts along the beach, probably owing to the effects of basin geometry (see Table 1).

The breaker angle (θ_b) is both the most difficult of the variables to measure accurately and probably the most important for determining the strength of the current. Putnam *et al.* [1949] measured θ_b from vertical photographs, and Galvin and Eagleson [1965] used a tedious method of visual observation with a specially designed protractor. The protractor gave results agreeing well with the theoretical values obtained from standard small amplitude refraction theory (see Table 7 in TM 10).

Longshore current velocity on laboratory beaches has been measured by tim-

TABLE 1. Wave Height Variation on a 6-Meter Laboratory Beach
Breaker height, in centimeters, at distance x from upstream training wall.*

Test Number	T , sec	x , meters		
		0.9	3.1	5.2
II 3	1.13	5.9	7.0	6.0
II 4	1.25	4.0	5.2	4.0
II 5	1.38	5.9	6.6	4.7
II 7	1.25	0.7	1.4	1.0
II 8	1.25	2.3	4.8	3.1
II 9	1.25	4.3	6.4	5.7
II 10	1.25	4.7	6.0	5.4
II 11	1.25	5.2	6.8	5.4
IV 4	1.25	5.5	5.6	4.9
IV 9	1.25	4.7	4.6	4.0

* Data from Galvin and Eagleson [1965]. Each height is average of twenty consecutive waves measured with wire resistance gage.

ing the travel of dye [Putnam *et al.*, 1949; Saville, 1950], immiscible fluids [Brebner and Kamphuis, 1963] or wooden floats [TM 10]. The per cent difference between float velocity and the velocity of the midpoint of a dye patch ranged from less than 1 to 10% in five comparisons (Table 5 in TM 10), the float velocities tending to be higher than the dye velocities.

Field data. If measurements are made of a uniform train of waves in the ocean, it is possible to measure repeatedly an average wave period to within about 5% of a mean value, either with a stop watch or from an oscillograph of the water surface (Table 4 in Galvin and Savage [1966]). Under favorable conditions, wave height can be measured with similar accuracy from the oscillographs, but accuracy can be significantly poorer.

Wave direction is less accurately measurable by ordinary field techniques. Single measurements of wave direction are probably only rarely accurate to within about 2° , and such an error is relatively important for the small values of θ_b that commonly occur. Often θ_b is measured only to the nearest 5° [Moore and Scholl, 1961; Putnam *et al.*, 1949]. Longshore current velocity has been measured by timing the travel of dye [Putnam *et al.*, 1949; Moore and Scholl, 1961], seaweed, a weighted soccer ball [Inman and Quinn, 1951], and water-filled balloons [Galvin and Savage, 1966]. With care and repetition, most such measurements can be within 0.25 ft/sec of the mean of the data. Beach slopes are obtained from profiles through the surf zones and seem to be subject to less variation than may be initially suspected. Thirteen carefully controlled profiles obtained at Nags Head, North Carolina, during five consecutive days of changing wave conditions had nearshore slopes ranging only from 0.025 to 0.030 (profiles supplied by Louisiana State University Coastal Studies Institute, March 1964).

Although measurements can produce numbers that are repeatable within reasonable limits, field data are uncertain because of the uncontrolled variables that influence the longshore current velocity. The two principal unknowns in field work are the effect of variations in nearshore hydrography [Sonu *et al.*, 1967] and the effect of substituting a mean value for the distribution of values of each measured variable. Another field problem is the difficulty of measuring all variables simultaneously, so that they refer to the same physical condition.

Flow changes. When non-uniformity in the alongshore direction has been measured (only in the laboratory [Brebner and Kamphuis, 1963; Eagleson, 1965; and TM 10]), the current typically had a low velocity near the upstream wall, increased to a maximum on the downstream half of the beach, and then fell off as the downstream training wall was neared. By using miniature current meters calibrated against the travel of floats, it was possible to measure the one- and two-dimensional velocity distributions in the surf zone. Figure 2 shows the one-dimensional longshore current growth for three tests which differed only in the direction of wave approach. These results are typical of much data (Figure 27 and Table A7 in TM 10) indicating that the current requires more distance to reach its maximum velocity at low θ_b than at high θ_b .

Measurements of typical two-dimensional distributions of velocity in laboratory longshore currents have also been made (Figure 24 and Table A7 in TM 10). The maximum velocities appeared between the still water line on the beach and

Fig. 2. Velocity distributions along a laboratory beach as a function of mean breaker angle θ_b . All other conditions constant: wave period, 1.25 sec.; wave height, 4.25 cm at 35-cm depth; beach, concrete with 0.11 slope.

the breaker position, usually initially closer to the still water line than to the breaker position. As the current flowed down the beach, the local maxima had a tendency to migrate toward the breaker position. Under certain laboratory conditions, longshore currents may turn seaward into a rip-like current, but such laboratory rip currents are not known to have been observed during any of the laboratory studies discussed here.

On natural beaches, rip currents may cause unsteadiness in longshore current flow [Putnam *et al.*, 1949], but such changes may not always be present [Galvin and Savage, 1966]. The non-uniformity of longshore currents on natural beaches has not been studied quantitatively, although the results of Inman and Quinn [1951] indicate non-uniform flow may have been present during their observations.

EVALUATION OF DATA

Overlapping laboratory data. There is only one laboratory condition that is nearly repeated in three studies and only about four conditions that appear in two laboratory studies. The common laboratory conditions must be drawn from data taken on smooth, plane beaches of approximately 1 on 10 slope. Observations numbered 15, 138, and 213 satisfy these restrictions, are drawn from three different laboratory studies, and have approximately the same wave conditions (Table 2).

TABLE 2. Comparison of Overlapping Laboratory Data

Observation Number*	H_b , cm	T , sec	θ_b , deg	m	V , m/sec	References
15	7.3	0.99	28.0	0.10	0.51	Putnam <i>et al.</i>
138	5.5	1.00	26.0	0.10	0.39	Brebner and Kamphuis
213	5.5	1.00	28.0	0.11	0.66	Galvin and Eagleson
17	4.9	1.63	18.8	0.10	0.29	Putnam <i>et al.</i>
205	4.9	1.50	6.9	0.11	0.37	Galvin and Eagleson
48	3.0	1.00	7.5	0.10	0.14	Brebner and Kamphuis
199	2.7	1.00	2.9	0.11	0.14	Galvin and Eagleson
72	4.9	1.00	14.0	0.10	0.34	Brebner and Kamphuis
201	5.2	1.00	14.1	0.11	0.54	Galvin and Eagleson
18	4.9	1.98	18.4	0.10	0.23	Putnam <i>et al.</i>
88	4.6	1.00	18.0	0.10	0.36	Brebner and Kamphuis

* Observation number refers to the numbers assigned to the data in the compilation of Galvin and Nelson [1967]. All data are for smooth cement laboratory beaches.

The only important difference in wave variables is that observation 15 has a wave height of 7.3 cm and the other two observations have heights of 5.5 cm. Despite the similar values of wave variables, the observed velocities differ greatly. The velocity in observation 213 (*TM 10*) is 70% larger than the velocity in observation 138 (Brebner and Kamphuis) and 28% larger than the velocity in observation 15 (Putnam et al.). Assuming velocity increases with wave height, the discrepancy is even greater because of the difference in heights. On the other hand, the difference in velocity between observations 15 and 138 is in the direction to be expected from the lower value of wave height and angle in 138. The four pairs of observations, representing somewhat similar wave conditions, also show the same trend (Table 2). The velocities in *TM 10* are significantly greater than those measured by Putnam et al. [1949] and by Brebner and Kamphuis [1963].

Causes of velocity differences. The size of the differences is considerably larger than is to be expected from differences between float and dye methods of velocity measurement. The consistent sign of the differences makes it unlikely that they are due to random error. The velocities were measured at approximately the same position on the beach, thus ruling out large errors due to non-uniformity of the current. (Putnam et al., Brebner and Kamphuis, and Galvin and Eagleson measured velocities at, respectively, 6.1, 4.6–6.1, and 4.9 meters, from the start of the current.) In the order just listed, lengths of the test beaches in these studies were 12 meters, about 11 meters, and 6–9 meters. Since backwater effects are less on the longer beaches, the higher velocities on the shorter beach (*TM 10*) remain unexplained.

A possible explanation of the difference in measured velocities is that the wave heights in the three studies are not comparable because of different types of breaking or different definitions of the breaking point. Although it does not seem to have been remarked before, the data of Putnam et al. (Figure 3) agree as well with Miche's theory for the maximum stable wave as do the experiments usually quoted as evidence in favor of this theory [Danel, 1952]. Miche's theory appears to be valid under conditions producing spilling breakers, implying that the currents measured by Putnam et al. were produced by spilling waves. Since the data

Fig. 3. Comparison of laboratory breaker heights with the theory of Miche [1944]. L_b was computed from $T \sqrt{gd_b}$. Data of Galvin and Eagleson [1965] are the average heights of 20 consecutive waves.

of Putnam et al. followed Miche's theory, their agreement with the data of Brebner and Kamphuis, in the few cases where it is possible to compare (Table 2), is to be expected because the wave heights in Brebner and Kamphuis' data were computed from a graph derived from Miche's theory [*Le Méhauté*, 1961]. The data in *TM 10*, which do not agree with Miche's theory (Figure 3), were for currents produced by plunging breakers.

Further evidence that the experiments of Putnam et al. involved spilling breakers is provided by the breaker depth-to-height ratio, β . Data on breaker depth are given by Putnam et al. and in *TM 10*, and the β ratios are summarized in Table 3. In general, wave steepness and beach slope determine both β [Iversen,

TABLE 3. Laboratory Breaking Depth-to-Height Ratios

Slope	Number of Observations	Average β^*	Reference
0.10 smooth slope	4	1.34	Putnam et al. [1949]
All slopes (0.07 to 0.26)	37	1.39	Putnam et al. [1949]
0.11 smooth slope	54	0.85	Galvin and Eagleson [1965] (observations of 12 wave conditions)
0.10 smooth slope	163	0.86	CERC (unpublished data from films of 12 wave conditions)

* $\beta = d_b/H_b$, where d_b is the mean water depth at the breaker point. $\beta = 1.28$ in solitary wave theory.

1953] and type of breaking [Patrick and Wiegel, 1955]. By comparing the laboratory data with the curves of Iversen and of Patrick and Wiegel (see Wiegel [1964], p. 175), it appears that the data of Putnam et al. are in the spilling range and that the data of *TM 10*, obtained with plunging breakers, are in the plunging range of the graph of Patrick and Wiegel. But even though the wave data of Putnam et al. imply spilling, spilling rarely occurs on the steep slopes used in their tests, and photographs accompanying the paper of Putnam et al. clearly indicate breaking by plunging. This suggests that breaker position may have been differently defined, but information on this point is not available.

Evaluation of laboratory studies. It is necessary to determine if any of the four diverse sets of laboratory data are useful for evaluating theories of longshore current flow. Because of the uncertainties brought out in the paragraph above (see also discussion of Figure 4 in later section), it is assumed that the laboratory data of Putnam et al. are not useful for the purpose of evaluating the theories. Since the breaker data were not measured by Saville or by Brebner and Kamphuis, only the laboratory data of *TM 10* remain. In this review these data will be assumed to supply the most useful test of theory for laboratory conditions, although they have recognized experimental uncertainties (Figure 2 and Table 1).

Evaluation of field studies. The available data do not permit comparisons between field studies, even to the limited extent possible for the laboratory studies. The data of Inman and Quinn [1951] are already averages of measurements

from 15 positions spaced at 90-meter intervals along the beach. They demonstrate convincingly that local conditions are not necessarily representative of the average conditions along a stretch of beach, since the standard deviation of the average velocity in this study usually exceeds the mean. However, since velocity predictions developed from physical models are all based on local conditions of the breaker, the data of Inman and Quinn, being the average of differing local conditions, are not appropriate for evaluating these models.

The data of *Moore and Scholl* [1961] are measurements of generally low velocities accompanying low waves on steep beaches, a condition not represented in any other study. However, the precision of measurement was low: H_b averaged about 0.4 meter and was measured to the nearest 0.1 meter; θ_b was measured to the nearest 5° , with the result that 36 to 71 observations were either 0° or 5° . This study includes many observations of low flows, several of which flowed against the direction of the breaking wave. It is possible that, for such low velocity conditions, other physical factors exert relatively more influence on the velocity than does the wave direction.

The five observations of *Galvin and Savage* [1966] include useful supplementary information on variability encountered under field conditions but are themselves too few to be useful. Of the four sets of field data available, only the 18 observations of Putnam et al. appear to be useful for evaluating predictions of longshore current velocity.

VELOCITY PREDICTION

Equations to predict longshore current velocity can be grouped into three classes, according as they are based predominately on: (1) conservation of energy of momentum [*Putnam et al.*, 1949; equation 28 of *TM 10*; *Eagleson*, 1965]; (2) conservation of mass [*Bruun*, 1963; *Inman and Bagnold*, 1963; equation 35 of *TM 10*]; (3) empirical correlation of data [*Inman and Quinn*, 1951; *Brebnner and Kamphuis*, 1963; *Harrison and Krumbein*, 1964].

Energy and momentum equations. The first application of physical principles to derive equations for predicting longshore current velocity was made by *Putnam et al.* [1949]. They (and most subsequent investigators) used the solitary wave description of the breaker [*Munk*, 1949] to obtain relations for the energy and momentum flux into a unit length of the surf zone, and they assumed a simple partition of the energy and momentum flux in the surf zone. This led to an energy equation for the mean longshore current velocity:

$$V = [6.97g(s/f)(mH_b^2/T) \sin 2\theta_b]^{1/3} \quad (1)$$

For details of typical derivations, see the original analysis of *Putnam et al.* [1949] or the more elaborate treatment of *Eagleson* [1965].

The wave variables were defined in Figure 1. Equation 1 involves two unknown constants: s , the fraction of incoming energy flux available to the longshore current; and f , the Darcy-Weisbach friction factor. (The original coefficient k has been replaced by the more commonly used f .) The acceleration due to gravity is represented by g . Momentum conservation was used to obtain a second equation for the mean longshore current velocity:

$$V = \frac{a}{2} \left[\left(1 + \frac{4}{a} \sqrt{2.28gH_b} \sin \theta_b \right)^{1/2} - 1 \right] \quad (2)$$

where

$$a = 20.88(m H_b / fT) \cos \theta_b \quad (3)$$

(*Castanho* [1964, equation 10] has recently rederived (2) in a slightly different form.) There are then two equations, (1) and (2), involving three unknowns, V , s , and f . No solution is possible for V unless one of the constants, s or f , is known. Equation 2 was used together with the field and laboratory data to obtain an f value for the field and each of the laboratory beaches. This value of f was then put back into (2) to predict the velocity [*Putnam et al.*, 1949].

This procedure may be criticized because the measurements are used to obtain f and then f is used to predict the measurements; thus it is clear that no independent evaluation of the equation has been made.

There is, however, a more basic objection to the derivation of the equation itself. In deriving (1) and (2), it was assumed that the longshore current is driven by the alongshore component of motion in the breaking wave and that this component is simply the alongshore component of the velocity of the breaker crest given by $C_b \sin \theta_b$. C_b is the speed of the breaking wave crest according to solitary wave theory. Since energy losses accompany the current generation, the assumption implies the inequality (equation 14 of *Putnam et al.* [1949]):

$$C_b \sin \theta_b > V \quad (4)$$

It is a well observed fact, however, that water in the breaking wave is largely withdrawn from the surf zone just prior to breaking. If a longshore current is flowing in the surf zone, then the water particles in the breaking wave will have an alongshore component of motion in addition to $C_b \sin \theta_b$. Furthermore, Figure 4 shows that this inequality (4) does not hold for many observations. For the data from *TM 10* (the bottom histogram of Figure 4), (4) does not hold in 25 of 26 cases; for the data of *Brebner and Kamphuis* [1963] (one-third of which are plotted on the middle histogram), (4) does not hold in 65 of 94 cases on the 1:10 slope.

Fig. 4. Laboratory tests of the hypothesis of *Putnam et al.* [1949] that the ratio $V/(C_b \sin \theta_b)$ is less than 1.0.

In constructing Figure 4, the assumptions of Putnam et al. were used. C_b was obtained from the solitary wave equation

$$C_b = \sqrt{gh} \quad (5)$$

where h is the elevation of the wave crest above the bottom, given in terms of wave height as

$$h = H_b(1 + \beta - \sigma) \quad (6)$$

β and σ are defined in Figure 1. Putnam et al. assume $\beta = 1.28$ and $\sigma = 0$. If the values $\beta = 0.85$ and $\sigma = 0.3$ are used instead (*TM 10*), C_b would be 18% smaller, thus increasing the disagreement with the hypothesis (4).

There are at least three reasons why Putnam et al. did not discover that the inequality (4) could be violated: (1) their data were for high values of θ_b (only 3 of 55 observations had $\theta_b < 10^\circ$, and 12 had $\theta_b > 30^\circ$; rough refraction calculations indicate that in some laboratory experiments the wave generator must have made about 90° angle with the beach, a condition that introduces serious reflection problems); (2) the derivation, in effect, sets $\sigma = 0$ by equating trough elevation with mean water level; and (3) their experimental conditions were such that β was much higher than has been found for plunging waves in other studies (Table 3 and accompanying discussion). All three of these factors artificially increase the left side of (4), so that (4) was not violated by their data, but at the time their data were the only available.

More space than would otherwise be appropriate has been devoted to this discussion because equation 2, as modified by *Inman and Quinn* [1951], is currently the accepted description of longshore current velocity [*Wiegel*, 1964; *Bruun*, 1963]; and the reasons for rejecting it should be made clear. Inman and Quinn's subsequent modification is discussed below (see equation 17).

The fact that longshore velocity in the surf zone affects the alongshore component of fluid motion in the breaker was used in another momentum equation (p. 52, *TM 10*) in which the mean alongshore velocity in the breaker, $V_b(x)$, was distinguished from the mean velocity in the surf zone, $V(x)$. The equation agrees qualitatively with the dependence of velocity profile on angle (Figure 2), but it is not useful for quantitative prediction because both V and V_b are unknown.

Recently, *Eagleson* [1965] developed an elegant model of longshore current growth in which longshore current velocity is the only unknown:

$$\frac{V^2(x)}{A} = 1 - \left[1 - \frac{V^2(0)}{A} \right] e^{-Bx} \quad (7)$$

where, for usual beach slopes

$$A = \frac{3}{8} \frac{gH_b^2 n_b}{d_b f} m \sin \theta_b \sin 2\theta_b \quad (8)$$

$$B = \frac{2}{5} \left[\frac{f}{md_b \sin \theta_b} \right] \quad (9)$$

The f in (8) and (9) is evaluated from the Karman-Prandtl resistance equation for steady, uniform, turbulent flow in pipes of given roughness. According to (7),

the predicted velocity profiles are always concave downward, and steady state is always approached faster for lower values of θ_b . These two predictions do not agree with data now available (e.g. Figure 2).

Conservation of mass equations. Bruun [1963] suggests two theories of longshore current flow based mostly on conservation of mass in the surf zone: a rip current theory and a continuity theory using the Chezy equation. Basic to both is a factor Q_b , the cross-sectional area of the significant wave (the average of the highest third of the waves). Bruun assumes that only those waves whose heights equal or exceed the significant wave height, i.e., the highest one-sixth of all waves, break on the bar and contribute to the current. Q_b is given as an integral [Bruun, 1963, p. 1070], which Brebner and Kamphuis [1963, p. 10] integrate. This integrated result can be written, using Bruun's assumptions, as

$$Q_b = 0.38H_b^2 \quad (10)$$

H_b refers to the breaking height of the significant wave. Assuming that the laboratory case represents a spectrum so narrow that all the waves are nearly the significant wave height, the right side of (10) will be multiplied by 6 in the laboratory case.

In essence, the continuity theory [Bruun, 1963, p. 1074] is derived from the following assumptions. A wave breaks at an angle to the beach contributing mass to the surf zone and locally raising the mean water level in the surf zone. This gives a slope to the water surface which creates a longshore current whose velocity is controlled by the frictional head loss in the current (thus introducing energy considerations). A value for the slope S can be derived by dividing the rate of rise of the mean water level at a point on the beach ($Q_b \cos \theta_b / bT$) by the speed at which the plunge point of the breaker travels down the beach ($C_b / \sin \theta_b$) so that

$$S = (Q_b \sin 2\theta_b) / (2bC_bT) \quad (11)$$

where b is the distance between the breaker point and the shoreline. This slope is then used in the Chezy equation for open channel flow.

In order to test it here by available data, the equation has been modified by the assumption that $b = m/d_b$ and, further, by the assumption that the hydraulic radius in the Chezy equation is expressible as kd_b , where $k = 0.8$ in the field and 0.5 in the laboratory. This results in

$$V = C_f' [H_b^{3/2} m (\sin 2\theta_b) / T]^{1/2} \quad (12)$$

where C_f' is a constant (0.25 in laboratory, 0.13 in field) times the 'Chezy' coefficient C_f .

The rip current theory supposes that the breaking waves dump their water across the longshore bar into the trough and that this water runs along the trough until reaching a low point in a bar, where it flows seaward as a rip current. This leads to an equation for non-uniform longshore current which Bruun [1963, p. 1072] averages and formulates in terms of shear stress. However, using the same assumptions (including $\beta = 1.0$), the rip current theory can be arranged in a simpler form without including shear:

$$V = 2.31 \lambda m (\cos \theta_b) / T \quad (13)$$

Equations 12 and 13 are expressions of the ideas presented in Bruun's paper but not stated in this form.

Another rip current theory, developed independently by *Inman and Bagnold* [1963], can be written

$$V = 2.31 \lambda m (\cos \theta_b \sin \theta_b) / T \quad (14)$$

again employing the assumption that $\beta = 1.0$. Both rip current theories have the same form except for the addition of $\sin \theta_b$ to (14), arising from the assumption of Inman and Bagnold that only the alongshore motion in the breaking wave contributes to the current. Since no data on λ collected simultaneously with longshore current data have been published, neither equation 13 nor 14 can be tested, but for reasonable choices of λ , T , and m , it appears that velocity predicted by (13) is unreasonably high and that velocity given by Inman and Bagnold (14) is not impossible. The difficulty with (13) comes from the assumption that all return flow is by rip currents.

The final equation suggested by conservation of mass considerations (*TM 10*) equates the flow rate of the longshore current to an alongshore flow through the wave area, $A_w C_b \sin \theta_b$, so that after solving for V

$$V = K g m T \sin 2\theta_b \quad (15)$$

where

$$K = (1 + \beta - \sigma) / 2\beta^2 \quad (16)$$

K can range from about 0.6 to about 1.1 for appropriate choices of β and σ . Because of the approximation involved in (15), $K = 1$ is used throughout this discussion. The form of (15) is unusual in that V is independent of H_b . This independence of H_b , also found in (13) and (14), seems intuitively implausible at first sight, but the mass rate of flow will still depend strongly on H_b , since H_b will determine the cross-sectional area of the current.

Empirical correlation. The equation of *Inman and Quinn* [1951], a modification of (2) obtained by introducing an empirical expression for f , is given as

$$V = \{[(1/4R^2) + \sqrt{2.28gH_b \sin \theta_b}]^{1/2} - (1/2R)\}^2 \quad (17)$$

where $R = 108.3 m H_b \cos \theta_b / T$. The expression for f was obtained from a least-squares fit of three sets of data: the field data of Inman and Quinn, and the field data and laboratory data of Putnam et al. For reasons discussed in the previous section, only the field data of Putnam et al. seem appropriate to the correlation (see discussion of data).

Brebner and Kamphuis [1963] correctly question the assumptions necessary to obtain equations for velocity from conservation laws, and they give good reasons in favor of developing empirical equations. It is unfortunate that the two equations they developed require data for deep-water conditions, which are available in only two studies [*Saville*, 1950; *Brebner and Kamphuis*, 1963] and in practice have been computed rather than measured.

In another study involving offshore data, *Harrison and Krumbein* [1964]

performed a linear multiregression analysis on unpublished data from Virginia Beach, Virginia, and found, for the 11 variables of their study, that the combination of 6 variables that causes the greatest reduction in variability is T , H_0 , m , onshore wind speed, offshore wind speed, and wave direction, in that order. Other combinations of variables do very nearly as well, however, and, surprisingly, only 1 or 2% of the variability is due to wave direction. This may be explained by the fact that wave direction was measured at a point 1000–1300 feet offshore, where the relative depth varied from deep water to shallow water depending on wave period. Under such conditions the relation between measured velocity and measured direction is complicated by the unevaluated refraction effects which will vary with deep-water wave direction, wave period, and bottom topographic irregularities.

EVALUATION

Theoretical evaluation. From the previous section it is apparent that, in attempting to arrive at some solution, theoretical treatments of longshore currents have considered models that are drastic simplifications of longshore currents. It has been necessary to assume relations for wave speed, wave shape, water particle velocity, cross-sectional area of the current, velocity distribution and energy distribution within the current, and alongshore variation of the waves and current. Any one of the approximations might be plausible under proper conditions, but the necessity to use them all has produced theories that only occasionally resemble longshore currents as they actually flow.

Basic descriptive data necessary for an adequate explanation of longshore currents are still lacking. Waves on beaches may break by spilling, plunging, or surging, but the prediction of breaker types is not possible except in general terms, and the control volume approach of the theories does not distinguish among them. The magnitude and velocity distribution of the current should be affected by breaker type, since the relative amount and location of energy dissipation depends on breaker type and since the energy available for the current is determined by how much is lost in breaking.

Data are also lacking on how longshore currents fit into the nearshore current system. Most theories assume rip currents are absent, but some assume them present at a known spacing [Bruun, 1963; Inman and Bagnold, 1963]; yet the existence of rip currents and their spacing are dependent variables of the system. It has already been pointed out that the neglect of the longshore current already present in the breaking wave makes the original theory of Putnam *et al.* [1949] internally inconsistent and that an attempt to account for this velocity [TM 10] merely adds another dependent variable to the theory. Sonu *et al.* [1967] have recently pointed out that the whole nearshore current system, and particularly the longshore current, depends sensitively on variations in nearshore hydrography.

A theory based on conservation of energy (or momentum) flux appears to be impractical because (1) the flux assignable to the longshore current is such a small fraction of the flux in the incident waves [TM 10] (usually less than a tenth, often less than a hundredth); (2) energy flux in the current is not a constant fraction of the incident flux; and (3) computed flux in the unsteady, non-

uniform current depends on coefficients analogous to the empirical coefficients measured in steady, uniform, open-channel flow [Putnam *et al.*, 1949; Bruun, 1963; *TM 10*; Eagleson, 1965].

A continuity theory might be more practical than a momentum or energy theory, since the mass flow in the current is the dominant mass flow in the surf zone. This is particularly true for low velocity flows, where observation indicates that the current velocity is merely the averaged alongshore component of the surge-induced water particle velocity. However the approximations necessary in the theories developed so far [Inman and Bagnold, 1963; Bruun, 1963; Galvin, 1963] are extreme.

Because of the inadequacies of physical theory, empirical correlation offers the most promise [Brebner and Kamphuis, 1963], but as yet such correlations have not been developed from useful data. The acquisition of reliable data is a limiting factor, and there is increasing evidence that current depends on the off-shore and alongshore boundaries to such an extent that each set of data is applicable only to the peculiar conditions producing it (Sonu *et al.* [1961]; also experimental work in progress at the Coastal Engineering Research Center).

Empirical evaluation. Most theories have been put forward with experimental evidence supporting their validity, yet this review indicates that no available theory is adequate and that most available data are inappropriate tests of existing theories.

To examine this paradox, all the testable equations were evaluated with each of the eight sets of data. The six testable equations, functions only of the variables that have been measured, include the energy (1) and momentum (2) equations of Putnam *et al.*, the asymptotic version (7) of Eagleson's equation, the modified version (12) of Bruun's 'Chezy' equation, the continuity equation (15) of *TM 10*, and the correlation equation (17) of Inman and Quinn. Velocities predicted by these equations were calculated on the CDC 1604 at the Computer Center of the University of Wisconsin. Output was obtained both in numerical form and as Calcomp plotter graphs of predicted versus measured velocity for each combination of equation and data set. Agreement between observed and predicted velocity was judged qualitatively by how closely the plotted data clustered around the 45° line between the axes.

It was necessary to supply constants for the two equations of Putnam *et al.* and the equation of Bruun. Ten values of the constants were chosen over a range from unreasonably low to unreasonably high values, based on the open channel analogy. The computation of these three equations was then iterated for the ten values of the constants, and best values of the constants were identified as those producing the best correlation on the plots.

It was found that every equation gave at least fair agreement with some set of data, and, conversely, except for the field data of Moore and Scholl and of Inman and Quinn, each set of data was predicted at least fairly well by some one of the equations. Best predictions come from the continuity theories of Bruun (equation 12, Figure 5) and *TM 10* (equation 15, Figure 6). These are the only theories to produce fair correlations with both the better sets of data (the field data of Putnam *et al.* [1949] and the laboratory data of *TM 10*). The laboratory

data for $\theta_d = 51^\circ$ on Figure 6 closely parallels the predicted curve but is displaced from it.

Each of the four other equations was able to correlate some set of data, although equation 17, derived empirically from three sets of data, agrees fairly well with only one of them, in the qualitative sense of this discussion. Figure 7 shows some of the better correlations that can be obtained for the other four equations. Good correlation of the Brebner-Kamphuis data can be produced by most of the equations, probably because the range of experiments was limited and θ_b and H_b were computed.

Figures 5, 6, and 7 are empirical evidence that apparent correlation between

Fig. 5. Best observed agreement between a testable theory and the two usable sets of data. Theory is equation 12 with $C_f = 110 \text{ m}^{1/2}/\text{sec}$.

Fig. 6. Comparison between measured velocities and velocities predicted by equation 15. θ_d is the angle between the wave generator and the beach.

theory and data is no guarantee that the theory accurately describes the phenomenon. Such a correlation for some sets of data is possible because (1) there are eight sets of data each covering a limited range of possible conditions, (2) laboratory data are internally correlated, owing to systematic changes in the experimental variables, and (3) the free constant in many equations gives an added possibility of adjustment.

CONCLUSIONS

1. In spite of many studies since the pioneer paper of Putnam *et al.* [1949], there is still no adequate theoretical prediction of longshore current velocity. Previous studies include nine papers containing twelve velocity equations, of which only six can be evaluated with available data.

2. Most available data are not useful. Reliability of experimental techniques and statistics of variability are usually unknown. Available published data

Fig. 7. Examples of misleading correlation between theory and data. *Left:* equation 12 with $C_t = 110 \text{ m}^{1/2}/\text{sec}$. *Center:* equation 2 with $f = 3.0$. *Right:* equation 1 with $s/f = 25$.

include four sets of laboratory measurements and four sets of field measurements totaling 352 observations. The better sets of data are the field data of *Putnam et al.* [1949] and the laboratory data of *TM 10*.

3. Longshore currents are considerably more complicated than the models on which theoretical studies have been based. Important factors not accounted for in the theories include breaker type, nearshore hydrography, and the nearshore current system.

4. The presently accepted theory of longshore current motion is an empirical modification [*Inman and Quinn*, 1951] of the original momentum flux equation of *Putnam et al.* [1949]. The momentum equation is shown, both conceptually and experimentally (Figure 4), to be based on an erroneous assumption, and the empirical modification, derived from three sets of data, is a fair predictor for only one of these three sets of data.

5. Demonstration of agreement between a proposed theory and a given set of data is no proof that the theory is valid. Such agreement could be found between each of the six testable equations and some one of the four laboratory sets of data, independently of the validity of the theory or the reliability of the data. Two theories based on conservation of mass (equations 12 and 15) were the only ones to produce fair agreement with both of the two better sets of data. These two equations may be taken as empirical guides to velocity prediction in lieu of a general theory.

6. On the basis of energy or momentum flux, longshore currents are a secondary feature of the surf zone, but on the basis of net mass flow they are a primary feature. This suggests that conservation of mass may be better than conservation of momentum or energy as a basis for longshore current theories.

7. At present the best approach to a meaningful prediction of longshore current velocity is through empirical correlation of reliable data.

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