Coastal Engineering Technical Note

DIRECTIONAL WAVE SPECTRA USING COSINE-SQUARED AND COSINE 2S SPREADING FUNCTIONS

PURPOSE: This Technical Note presents the basic concept of ocean wave directional spreading, why it is important in coastal engineering, and how it is characterized through parametric relationships. The COSINE-SQUARED and the COSINE 2S spreading functions are included.

INTRODUCTION: The wave energy at a point has an angular distribution as well as a distribution over a range of frequencies. This angular distribution of wave energy is termed "directional spreading." Spectral representations which include both the frequency distribution and the angular spreading of wave energy are known as "directional spectra."

Knowledge of directional spectra regarding their evolution in shallow water is important to coastal engineers. More accurate wave predictions can usually be achieved when directional spectra are considered. Indications are that spectral models without spreading can overpredict significant wave heights during refraction by as much as 20 percent when compared to directional spectral models (Collins, et al. 1981). This can have a significant effect on coastal structure design for nondepth-limited waves. Forristall, et al. (1978) concluded that a proper description of storm wave flow velocities is dependent on the correct representation of the directional spread of wave energy. Neglecting the spreading results in overprediction of flow velocities. The concept of directional spreading accounts for some important aspects of wave behavior which are known to occur. An example is given in Appendix C of ETL 1110-2-305 (Vincent and Lockhart 1983), where the analysis is based, in part, on directional spreading. Finally, more realistic models of coastal processes are being developed which require directional spectra as input.

The ability to represent directional spectra in parametric form permits simulation of real seas in numerical wave models, parametric wave models, and in CERC's new directional spectral wave basin.

NATURAL PROCESSES: In the initial stage of wave generation the wave crests are very short, and the waves can be seen propagating in many directions different from the wind direction. Waves on a small lake illustrate this initial growth stage. Over longer fetch distances the waves evolve into longer-crested waves. The wind generated waves develop so as to propagate generally in the direction of the wind; but there is still an angular spread of energy about the mean direction, and the representation of the wavefield with directional spectra is useful. Waves generated by a local storm illustrate this stage of wave growth. Finally, the waves leave the generation area and become swell waves. Ocean swell waves which have propagated long distances are long-crested, and the angular spreading of energy is very slight. However, representation of swell in terms of directional spectra is still a useful concept. Long-period swell arriving on the West Coast is a good example of long-crested waves.

When waves approach a coastal area, other factors begin to modify the directional nature of the wavefield. For example, refraction due to variations in depth can act to concentrate or disperse wave energy. Currents can also influence the directional spread of wave energy in shallow water as well as in deep water.

PARAMETRIC SPREADING FUNCTIONS: The basic premise of all parametric spreading functions is that the single-peaked directional spectrum is adequately described by the product of two functions:

$$E(f, \theta) = E(f) \cdot D(f, \theta)$$

where

$E(f, \theta)$ = directional spectral density function

$E(f)$ = one-dimensional energy spectral density function

$D(f, \theta)$ = angular spreading function

f = frequency in hertz

θ = direction in radians

The formulation of $D(f, \theta)$ requires that

$$\int_0^{\infty} \int_{-\pi}^{\pi} E(f)\, D(f, \theta)\, d\theta df = \int_0^{\infty} E(f)\, df$$

so that the total energy in the directional spectrum must be the same as the total energy in the corresponding one-dimensional spectrum. This parameterization can effectively represent the directional nature of a wavefield in the absence of complicating influences such as a large change in wind direction or the propagation of swell into the generation area, which results in a bimodal spectrum. Several idealized spreading functions are outlined below.

1. Cosine-Squared

$$D(\theta) = \begin{cases} \frac{2}{\pi} \cos^2(\theta-\theta_0) , & \text{for } (-\frac{\pi}{2} + \theta_0) < \theta < (\frac{\pi}{2} + \theta_0) \\ 0 , & \text{otherwise} \end{cases}$$

where θ_o = mean wave direction in radians.

The cosine-squared formulation is extremely simple because it is neither a function of frequency nor widespeed. It can be used to parameterize the directional spreading of wind seas. Similar formulations can be derived (cosine-fourth, for example) by changing the value of the exponent and adjusting the coefficient so that

$$\int_{-\frac{\pi}{2} + \theta_o}^{\frac{\pi}{2} + \theta_o} D(\theta)\, d\theta = 1$$

Most CERC spectral wave models, including the models used by the Wave Information Study (WIS) (CETN-I-19), use the cosine-fourth spreading function to represent locally generated waves (seas).

2. Cosine-2S

$$D(f, \theta) = \left(\frac{2^{(2s-1)}}{\pi}\right)\left(\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}\right)\cos^{2s}\left(\frac{\theta-\theta_o}{2}\right)$$

where

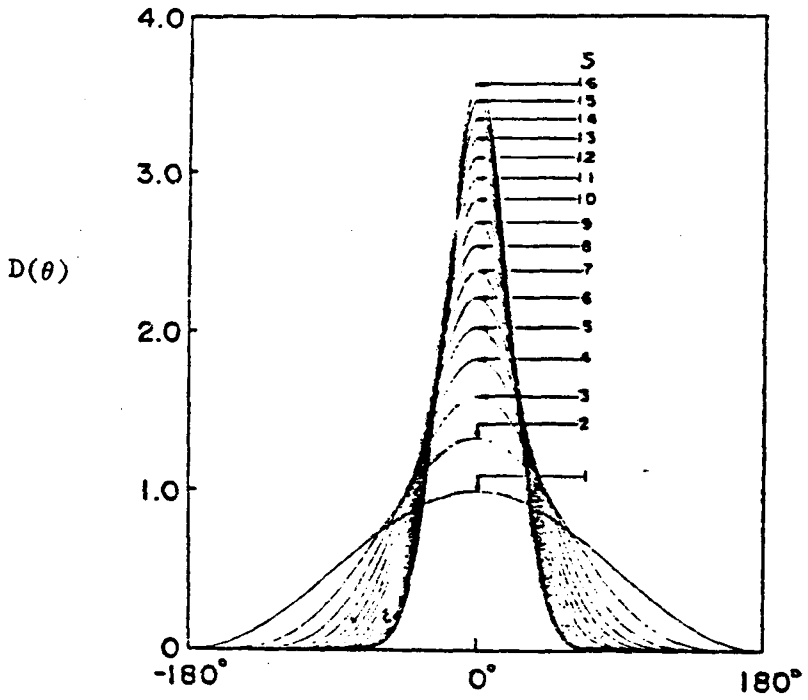
Γ = the Gamma function

θ_o = the mean wind direction

s = the spreading parameter which is a function of frequency and windspeed

This spreading function was proposed by Longuet-Higgins, et al. (1963). Three of the most commonly used and referenced formulations for the parameter s are as follows: (a) s = constant; (b) Mitsuyasu, et al. (1975); and (c) Hasselmann, et al. (1980).

The first parameter, (a), is comparable to consine-nth, which is a generalization of the cosine-squared spreading function presented earlier. The change in directional spread as a function of s is illustrated in Figure 1. Increasing the power of the cosine causes a narrowing of the directional spread. Swell is more typically represented by the narrow spreads



$$\left(\frac{\theta - \theta_o}{2}\right)$$

Figure 1. Idealized angular distribution

$$D(\theta) = G(s)\cos^{2s}\left(\frac{\theta - \theta_o}{2}\right)$$

in Figure 1. Most numerical and physical modeling of directional spectra make use of this formulation. A simple approximation to this formulation is discussed in CETN-I-6(1985).

The last two formulations, (b) and (c), are based on field data in deep water. Both include windspeed and peak spectral frequency in the parameterization, although windspeed is not a critical part of Hasselmann's, and it could be left out. While these latter two parameterized spreading functions more accurately depict nature, they generally are not used in numerical models because they are inefficient numerically; and their increased accuracy is lost in the coarse angular resolution of the models.

APPLICATION: Practical applications of parametric directional spreading functions include numerical and physical modeling of waves and analysis of directional field data. Field measurement analysis often involves selecting a value for n or s in the parameterized spreading function which optimizes the fit to measured directional attributes. When measurements of directional spectra are unavailable or impractical, reasonable approximations for numerical or physical modeling can be made by combining a spreading function with either measured one-dimensional spectra or with empirical one-dimensional spectral formulations, such as the JONSWAP spectrum in deep water or the TMA spectrum in shallow water (Hughes 1984). The composite directional spectra then provide useful input for shallow-water spectral transformation numerical or physical models. Present shallow-water numerical models allow calculation of directional spectra in the nearshore region; and they account for effects such as refraction, bottom friction, nonlinear interactions, wave breaking, and currents.

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