Coastal Engineering Technical Note

ONE-DIMENSIONAL WAVE-CURRENT INTERACTION

by Jane McKee Smith

PURPOSE

To describe the theory and a computer program for calculating the wave height, wavelength, and wave steepness for monochromatic waves interacting with a uniform current for a one-dimensional channel.

BACKGROUND

Ocean waves entering an inlet against an ebb current steepen (wave height increases and wavelength decreases) and can pose a hazard to navigation and dredging operations. On a flood current, wave steepness decreases (wave height decreases and wavelength increases). Design or modification of inlet channels requires consideration of wave-current interaction. Information presented in this Coastal Engineering Technical Note (CETN) may assist engineers in understanding and quantifying navigable conditions at existing inlets, and in making modifications to channels and jetties that might alter the current and waves in the entrance channel.

ASSUMPTIONS

This CETN treats the situation of one-dimensional wave-current interaction, which is a reasonable approximation for narrow inlets. Figure 1 shows a schematic of an idealized inlet configuration. The waves are propagating parallel to the channel and the current is either flowing in the same direction as the waves (+U) or opposing the waves (-U). In the ocean region offshore of the inlet, the current is assumed

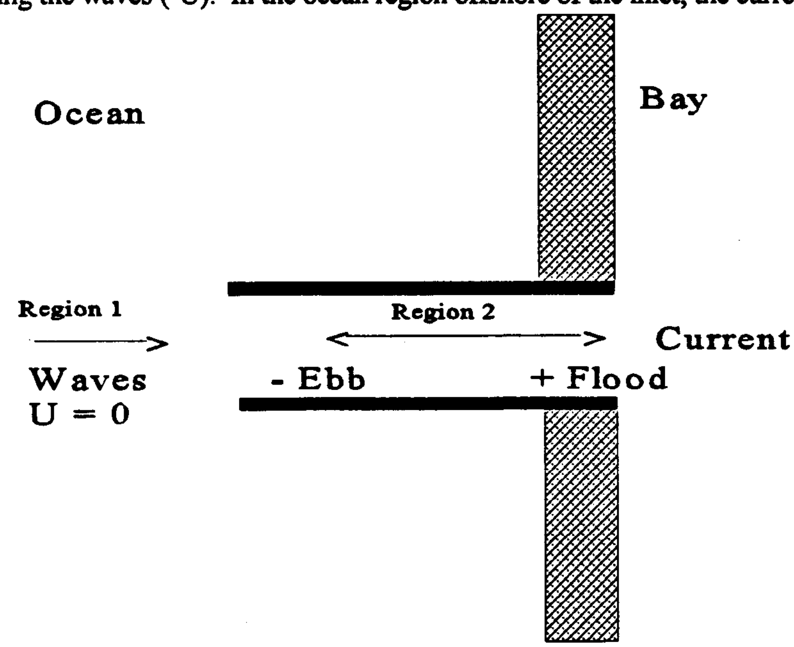

Figure 1. Schematic for wave-current interaction calculation

to be negligible. Wave shoaling is included in the approach given in this CETN, but refraction is neglected. Wave breaking due to depth and wave steepness are estimated using Miche's (1951) criterion.

Assumptions are summarized below:

a. Monochromatic waves.
b. Depth-uniform current.
c. Linear wave theory.
d. One-dimensional wave, current, and bathymetry variations (parameters vary only in the cross-shore direction).
e. Depth and current vary gradually through the channel.
f. Breaking is represented by Miche's (1951) criterion.
g. The current is not altered by the waves.
h. The current is negligible in the offshore (input) region.
i. Refraction is neglected.

In field situations, wave refraction, irregular waves, and nonuniform currents will influence the results.

METHODOLOGY

The dispersion relationship for waves and currents traveling in the same direction or directly opposing each other is (Jonsson 1990, and others):

$$\omega - kU = \sqrt{gk \tanh kh} \tag{1}$$

where ω is angular frequency, k is wave number, U is current velocity, g is gravitational acceleration, and h is water depth. The wave period ($T = 2\pi/\omega$) is assumed to remain constant as the wave propagates from still water onto the current. For the situation where $U = 0$, Equation 1 reverts to the standard dispersion equation. Wave blocking (stopping of waves by an opposing current) occurs for relatively strong ebb currents for which there is no solution for Equation 1 ($|U| > (gT)/(8\pi)$ in deep water and $|U| > (gh)^{1/2}$ *in shallow water*). The strong current prevents the wave from propagating through the channel, and wave energy is dissipated by breaking.

Using k calculated from Equation 1, the wavelength is given by:

$$L = \frac{2\pi}{k} \tag{2}$$

In shallow water, Equation 1 reduces to:

$$k = \frac{\omega}{U + \sqrt{gh}} \tag{3}$$

and Equation 2 becomes:

$$L = \left(U + \sqrt{gh}\right) T \tag{4}$$

Wave height is determined from the conservation of wave action (Jonsson 1990, and others):

$$\frac{\partial}{\partial x}\left(\frac{E(C_{g_r} + U)}{\omega_r}\right) = 0 \tag{5}$$

where E is wave energy, C_{gr} is relative group velocity of the waves, x is wave propagation direction, and ω_r is relative angular frequency. The subscript r represents variables measured relative to the current, i.e., variables in a coordinate system moving with the current. Wave energy is determined from linear theory as:

$$E = \frac{1}{8} \rho g H^2 \tag{6}$$

where H is wave height and ρ is water density. Relative angular frequency is given by:

$$\omega_r = \sqrt{gk \tanh kh} \tag{7}$$

Equation 7 is similar to Equation 1 for the situation of U=0, but its application is different. Equation 7 is used to solve directly for ω_r with the value of k determined from Equation 1. The relative group velocity is given by:

$$C_{g_r} = \frac{1}{2} \frac{\omega_r}{k} \left(1 + \frac{2kh}{\sinh 2kh}\right) \tag{8}$$

Applying Equation 5 between an offshore Region 1 where the current is negligible and a Region 2 in the channel (which may have a different depth and current) gives:

$$\left(\frac{E\ C_g}{\omega}\right)_1 = \left(\frac{E\ (C_{g_r} + U)}{\omega_r}\right)_2 \tag{9}$$

Solving for the wave height in Region 2 gives:

$$H_2 = H_1 \sqrt{\left(\frac{C_g}{\omega}\right)_1 \left(\frac{\omega_r}{C_{g_r} + U}\right)_2} = H_1 \sqrt{\left(\frac{C_{g_1}}{C_{g_{r_2}} + U_2}\right)\left(\frac{1}{1 + \frac{U_2}{C_{r_2}}}\right)} \qquad (10)$$

where $C = \omega/k$ is the wave celerity. The second expression on the right-hand side of Equation 10 for H_2 is obtained by substituting $\omega_1 = \omega_2 = \omega_{r2} + k_2 U_2$. If Regions 1 and 2 are both in shallow water, Equation 10 reduces to:

$$H_2 = \frac{H_1}{\left(1 + \frac{U_2}{\sqrt{gh}}\right)} \qquad (11)$$

COMPUTER PROGRAM

The interactive FORTRAN program wc1d32 solves Equation 1 by Newton-Raphson iteration to calculate the wave number. The wave height is calculated from Equation 10. The maximum wave height is limited by (Miche 1951):

$$H_{max} = 0.14\ L\ \tanh kh \qquad (12)$$

In deep water, Equation 12 reduces to a maximum wave steepness $H_{max}/L = 0.14$, and in shallow water, it reduces to $H_{max} = 0.88h$. Breaking in the program output is denoted with a "B." Research to improve the criterion for wave breaking on a current is in progress in the Coastal Inlets Research Program.

Example program results are shown in Figure 2. The figure illustrates the increase in wave steepness caused by an increasing ebb current and a decreasing depth. The initial wave conditions are H =2.0 m and T = 8 s at an offshore depth of 15 m. As the ebb current increases, the wave height increases and wavelength decreases, resulting in increased wave steepness. Wave blocking occurs between water depths of 4 and 5 m for an ebb current of -3.0 m/s and T = 8 s.

An executable version of the wave-current interaction program is available for 32-bit PCs (running Windows 95 or Windows NT). The file name is wc1d32.exe. An example application of the program is shown below.

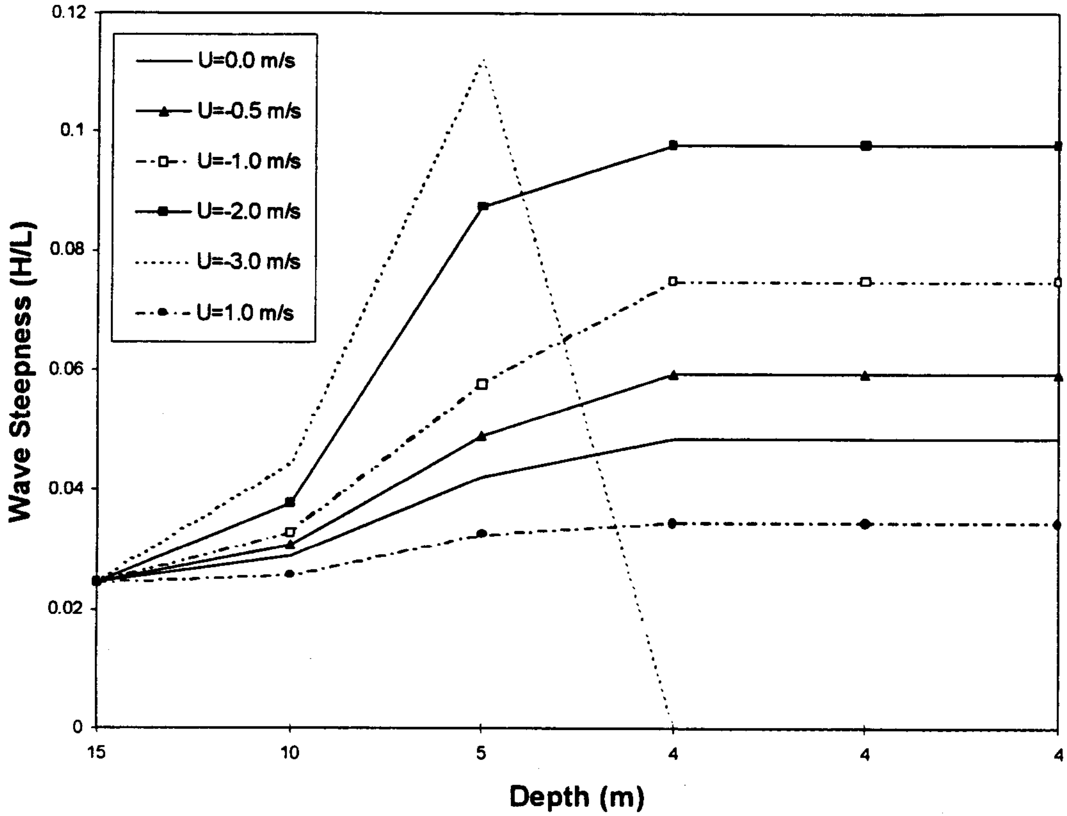

Figure 2. Wave steepness for a 2-m, 8-s wave approaching and entering a 4-m-deep channel as a function of current velocity in the channel

EXAMPLE APPLICATION

Inlet "A" is the site of numerous accidents, causing vessel damage and potential loss of life, because of steep and breaking waves in the inlet entrance. Inlet A is long and narrow. The maximum operational navigation condition for this Pacific Coast entrance is $H = 5$ m and $T = 15$ s in a depth of 15 m. The program wc1d32 is used to evaluate some initial alternatives for improving navigation at the entrance. The existing conditions in the entrance are a depth of 6 m and maximum ebb current of -1.0 m/s. Improvement options include increasing the channel depth to 10 m or 13 m. Widening of the channel could also reduce the maximum ebb flow to approximately 0.5 m/s.

PROGRAM wc.f v. 2.0, 25 Feb 1997

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wc.f calculates the height, length, and steepness for waves propagating from
the ocean (negligible current) into a channel with an ebb or flood current

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input wave height (H), period (T), and depth (h) in the ocean
free format: H in meters, T in seconds, and h in meters
5.0, 15.0, 15.0
input depth (h) and current velocity (U) in the channel
free format: h in meters and U in meters/second (-ebb, +flood)
6.0, -1.0

 T (s)  h (m)  U (m/s)  H (m)  L (m)   H/L   C (m/s)  Cr (m/s)
 15.0  15.00    .00    5.00  173.8  .0288  11.59   11.59
 15.0   6.00  -1.00    5.03   97.3  .0517   6.49    7.49 B

enter y to run another case or enter q to quit
y
```

The results are tabulated on the screen and written to an output file wc.out. The first line is the input conditions and the second is the output. Results for project alternatives are given below:

Existing conditions:

T (s)	h (m)	U (m/s)	H (m)	L (m)	H/L	C (m/s)	Cr (m/s)	
15.0	15.00	.00	5.00	173.8	.0288	11.59	11.59	
15.0	6.00	-1.00	5.03	97.3	.0517	6.49	7.49	B

Option 1 (channel depth of 10 m):

T (s)	h (m)	U (m/s)	H (m)	L (m)	H/L	C (m/s)	Cr (m/s)
15.0	15.00	.00	5.00	173.8	.0288	11.59	11.59
15.0	10.00	-1.00	6.14	128.0	.0480	8.53	9.53

Option 2 (channel depth of 13 m):

T (s)	h (m)	U (m/s)	H (m)	L (m)	H/L	C (m/s)	Cr (m/s)
15.0	15.00	.00	5.00	173.8	.0288	11.59	11.59
15.0	13.00	-1.00	5.76	146.4	.0394	9.76	10.76

Option 3 (channel depth of 13 m and reduction of ebb current to -0.5 m/s):

T (s)	h (m)	U (m/s)	H (m)	L (m)	H/L	C (m/s)	Cr (m/s)
15.0	15.00	.00	5.00	173.8	.0288	11.59	11.59
15.0	13.00	-.50	5.43	154.7	.0351	10.31	10.81

The initial conditions include high wave steepness ($H/L = 0.052$) and breaking waves in the entrance. Deepening the channel decreases the wave steepness (H/L decreases by 24 percent for deepening the channel to 13 m) and eliminates breaking. Reducing the ebb current further reduces the wave steepness. This simple example only examines one component of the potential project (wave steepness and breaking). In a detailed study, the impacts of increased wave energy in the harbor, increased wave heights in the inlet, and changes in scour/deposition in the channel would also be investigated. More advanced numerical or physical modeling may be used to optimize the plan, accounting for three-dimensional bathymetry and irregular, directional waves.

ADDITIONAL INFORMATION

For further information, contact Jane McKee Smith, U.S. Army Engineer Waterways Experiment Station, Coastal and Hydraulics Laboratory, at (601)634-2079, FAX (601)634-4314, or jm.smith@cerc.wes.army.mil.

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