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U.S. Army
Coastal Engineering
Research Center

A METHOD FOR CALCULATING AND PLOTTING SURFACE WAVE RAYS

TECHNICAL MEMORANDUM NO.17

DEPARTMENT OF THE ARMY
CORPS OF ENGINEERS

A METHOD FOR CALCULATING AND PLOTTING SURFACE WAVE RAYS

by

W. Stanley Wilson

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U. S. Army Coastal Engineering Research Center

February 1966

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COASTAL ENGINEERING RESEARCH CENTER

Addenda and Errata for Technical Memorandum No. 17

1. Since this Technical Memorandum was approved for publication, the Coastal Engineering Research Center has used the computer program in cooperation with the U. S. Army Engineer District, Wilmington, to plot rays in the vicinity of Oregon Inlet on the Outer Banks of North Carolina.

2. A card deck containing the program is available on loan from CERC for copying by the borrower. This deck is identical to that in Appendix C except for the following changes:

Page 35 Cards numbered RAYN 45 through RAYN 49 have been removed, and the following has been substituted:

```
396 GO TO (397,397,404), MIT
```

Page 36 The following has been inserted between MOVE 11 and MOVE 12:

```
    IF (D/DY - 0.005) 204,204,203
204 D = DY * 0.005
```

3. The sample data in figure 6 will also be provided to enable determination of satisfactory operation. If satisfactory, the resulting plot will be identical to that in figure 10; the printed output for the fifth ray will be identical to that in figure 8.

4. It is recommended that a Calcomp reference manual be used in conjunction with this report. The Calcomp subroutines are not listed in Appendix C, but are included in the card deck which can be borrowed from CERC. If an IBM 7094 computer and a Calcomp 670/564 plotting system are used, these subroutines will be the correct ones for use with this program. If another computer or plotter is used, other versions of the subroutines should be obtained.

5. Users may find little use for NUMCON and SHORE routines mentioned in Optional Computer Operations on page 11. If NSH = 0 and NCO = 0, no sounding card is needed, and these subroutines are not used.

ERRATA

Page ii, the LIST OF FIGURES should read:

Page 18, footnote 12 should read:
"... is given in figure 6 ..."

Page 26, the definition of COL = 0 should read:
"If COL ≠ 0 on a ... If COL = 0, the plotter ..."

Page 29, the definition of NXCMAT should read:
"If NXCMAT = 0, the ... if NXCMAT ≠ 0, the ..."

FOREWORD

An important aspect of any wave refraction analysis is the determination of wave-ray patterns for a coastal area of interest. Manual construction is both difficult and time consuming - especially when waves with many periods and directions of travel must be followed over an irregular bottom.

An alternative to manual construction is presented in this report. A digital computer and an incremental plotter are used to calculate and plot wave rays.

This study was begun at the Virginia Institute of Marine Science, Gloucester Point, Virginia, under Contract DA-49-055-CIV-ENG-64-5 with the Coastal Engineering Research Center, U. S. Army Corps of Engineers, Washington, D. C. The study was completed and the report prepared at the Johns Hopkins University, Baltimore, Maryland, under the same contract. The author, W. Stanley Wilson, is a graduate student in the Department of Oceanography at the University.

NOTE: Comments on this publication are invited. Discussion will be published in the next issue of the CERC Bulletin.

This report was prepared under authority of Public Law 166, 79th Congress, approved July 31, 1945, as supplemented by Public Law 172, 88th Congress, approved November 7, 1963.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS [1]

A	direction of ray travel
B	a given angle
C	wave speed
D	incremented distance between successive calculation points along a wave ray
g	acceleration due to gravity
H	wave height
h	water depth
h_o	maximum water depth at which refraction begins to be important for a group of sinusoids
K	ray curvature
L	wave length
L_d	deep-water wave length
T	wave period
W	conversion factor relating $\partial h/\partial n$ and $\partial C/\partial n$
α_1, α_2	arrows used in establishing grid boundaries
δ	a small distance used in initial positioning of grid boundaries
$\partial/\partial n$	partial with respect to the direction normal to a wave ray

[1] See APPENDIX B for definitions of symbols used in the computer program.

A METHOD FOR CALCULATING AND PLOTTING SURFACE WAVE RAYS

by

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The Johns Hopkins University
Baltimore, Maryland

ABSTRACT

A method using a digital computer and an incremental plotter for calculating and plotting wave rays is described. Given a grid of depth values, the initial position of a wave ray, and the direction of travel and the period of the wave, successive points along the ray path are calculated. For each point on the path, water depth and bottom slope are estimated from the depth grid by linear interpolation, wave speed and curvature are computed according to classic theory, and the location of the next successive point is approximated by an iteration procedure. The numerical results may be plotted automatically. An example of the results, obtained from an application of the method to Virginia Beach, Virginia, is presented.

Unless the bathymetry of an area is unusually smooth, this method calculates and plots wave rays faster than they can be manually constructed.

INTRODUCTION

As waves move toward a beach, their crests approach parallelism with the shoreline and their rays approach perpendicularity. Any wave-refraction analysis requires the determination of wave-ray patterns; however, if the bathymetry is irregular, the determination of the ray patterns--even for waves with a single period and direction of travel--can be both cumbersome and tedious.

Pierson, Neumann, and James (1955) explain how to estimate the effect of refraction on a continuous wave spectrum. They approximate the deep-water spectrum by a finite sum of discrete long-crested sinusoids, refracting each separately and recombining them to approximate the refracted spectrum. A cursory inspection of an application of their method (Pierson, Tuttell, and Woolley, 1953) reveals the extensive labor required of which by far the greater part is the construction of ray patterns for single sinusoids.

The method which they use to construct the rays is the manual method of Arthur, Munk, and Issacs (1952).

An alternative to manual construction is presented in this report. A digital computer and an incremental plotter are used to calculate and plot wave rays. Important references for the basic wave-refraction theory are Munk and Arthur (1952) and Dorrestein (1960). The computer program itself has evolved from Griswold and Nagle (1962), Mehr (1962), Griswold (1963), Harrison and Wilson (1964), and Wilson (1964).

METHOD

Initial Requirements. Let the coastal area of interest be specified and the group of sinusoids, characterized by their periods and directions of travel $\{T_i, A_i\}$, be given. It is necessary that a chart including the coastal area of interest and containing adequate bathymetric information be available.

Selection of Grid Boundaries. A rectangular X,Y-coordinate grid, whose boundaries also form a rectangle, is imposed on the chart of the region. The boundaries are identified by the lines $X = 0$, $X = AMM$, $Y = 0$, and $Y = ANN$. The position to be selected for these lines depends on the maximum water depth at which refraction begins to be important for the group of sinusoids, the given directions of travel for the group of sinusoids, and the bathymetry and coastal area of interest. Six examples, illustrated in sketches 1 through 6, will show how these lines are initially positioned.

The seaward extent of the region of analysis is approximated by drawing on the chart the contour whose value, h_c, represents the maximum water depth at which refraction begins to be important for the group of sinusoids. This depth, h_c, equals one-half the deep-water wave length, $1/2\ L_d = 2.56\ T^2$, of the longest-period sinusoid in the group. If an island or reef lies seaward from the h_c-contour (sketch 3), the seaward extent of the region of analysis must be extended to include it and the surrounding water whose depth is less than h_c.

The lateral extent of the region of analysis is approximated by considering the given directions of travel $\{A_i\}$ in conjunction with the h_c-contour. Arrows, α_1 and α_2, with the bounding directions for the set $\{A_i\}$ are drawn on the chart. Usually, α_1 and α_2 are pointed toward the area of interest. The exception (sketch 4) occurs when the area of interest lies in a bay, in which case α_1 and α_2 are directed toward the headlands. The lateral extent of the region of analysis must include the intersections of α_1 and α_2 with the h_c-contour; and, if an intersection lies on an island (sketch 3), the lateral extent must be extended to include it and the surrounding water whose depth is less than h_c.

The landward extent of the region of analysis must, of course, include the beach of the area of interest. When the area of interest lies on a small island (sketch 6), the region of analysis must include all water of depth less than h_c in the vicinity of the island.

When the extent of the region of analysis has been determined, the grid boundaries, X = 0, X = AMM, Y = 0, and Y = ANN, are positioned initially. Generally the Y-axis is established parallel with the direction of the h_c-contour; however, when the area of interest lies on a promontory (sketch 5), the Y-axis is parallel with a line connecting the intersections of α_1 and α_2 with the h_c-contour. <u>In each of the sketches</u>, the shoreline of the area of interest and that portion of the h_c-contour between α_1 and α_2 must lie a distance at least equal to δ inside the grid boundaries.[2] Note that the X-axis increases positively seaward from the area of interest and that the X,Y-coordinate system is right-handed; hence, all coordinate values are positive.

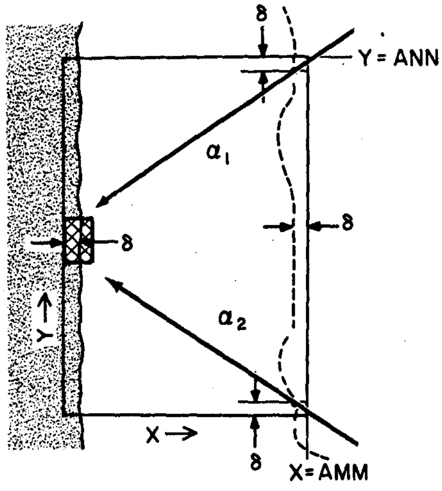
<u>Selection of Grid Interval.</u> Two opposing criteria govern the selection of the grid interval. The first requires that each grid cell be so small that its bottom topography can be approximated by a plane surface which need not necessarily be horizontal. The second requires that each grid cell be so large that the total number of grid points, which are the coordinate intersections, does not exceed 20,000.[3] Any cell size which meets both requirements can establish a grid interval satisfactory for the program. The number of feet per grid interval is needed for computations.

When both criteria are satisfied, the initially selected position of the grid boundaries is adjusted to make the distance

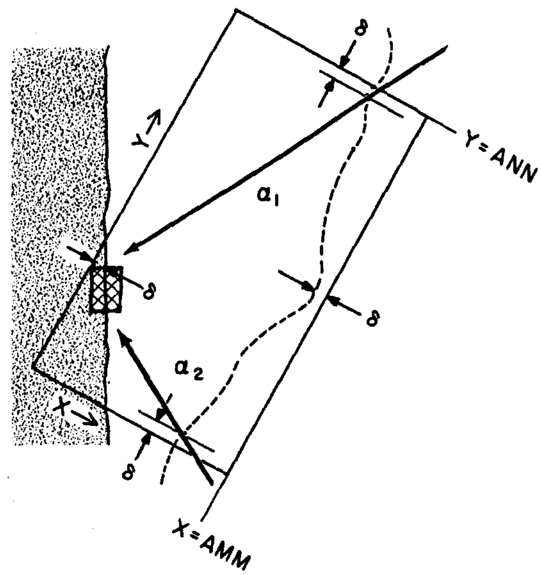
---

[2] δ, a small distance, serves to position the grid boundaries initially so that the grid interval can be selected. δ will then be set equal to two grid units.

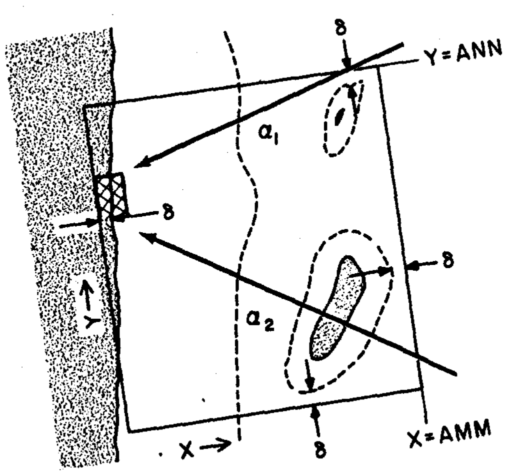
[3] This value depends on the storage capacity of the IBM 7094 computer. See the DISCUSSION section for storage requirements of the program when used with other computers.



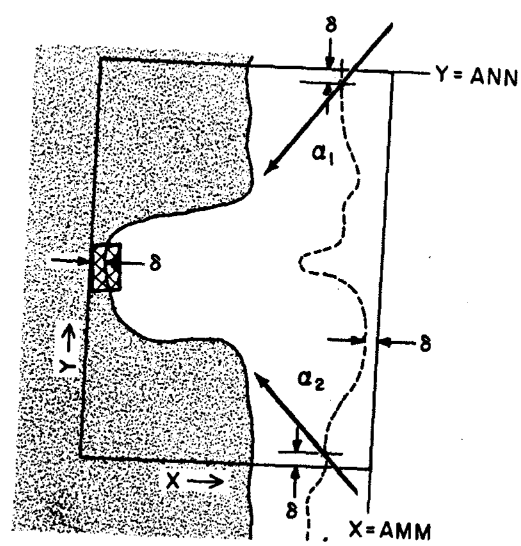
Sketch 1.



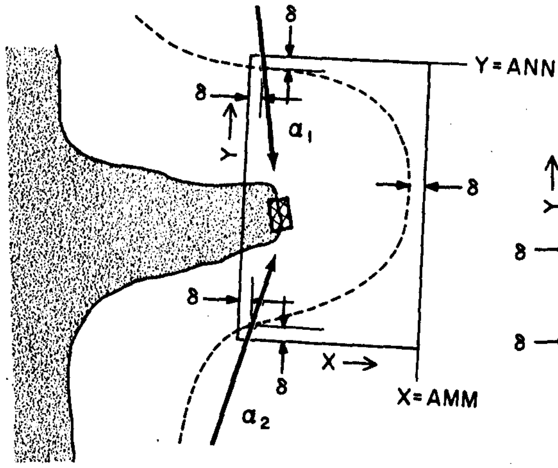
Sketch 2.



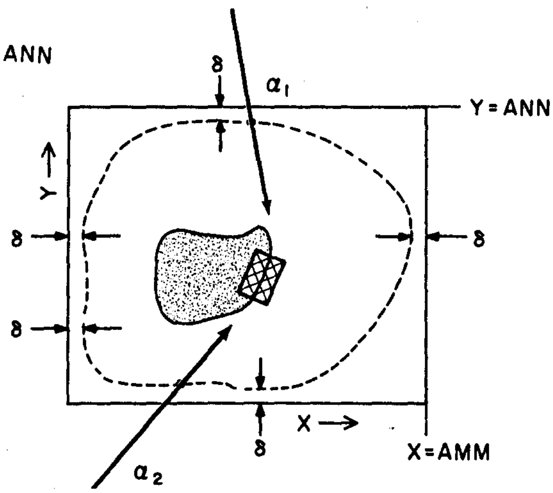
Sketch 3.



Sketch 4.

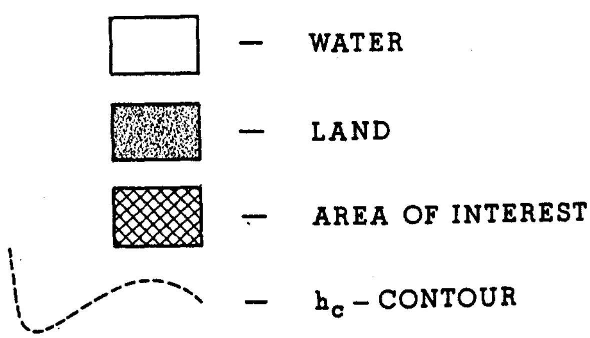


Sketch 5.

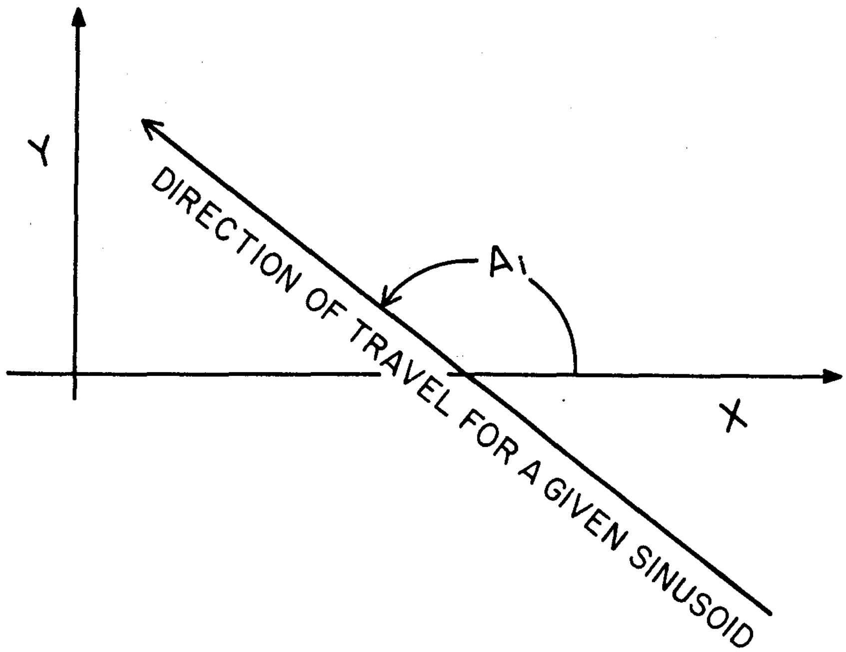


Sketch 6.

LEGEND



δ equal to two grid units and AMM and ANN integers.[4] Parallels and perpendiculars, whose intersections are the grid points, are ruled on the chart. The directions of travel $\{A_i\}$ for the group of sinusoids are expressed in degrees with respect to the direction of increasing X, as finally established (sketch 7).



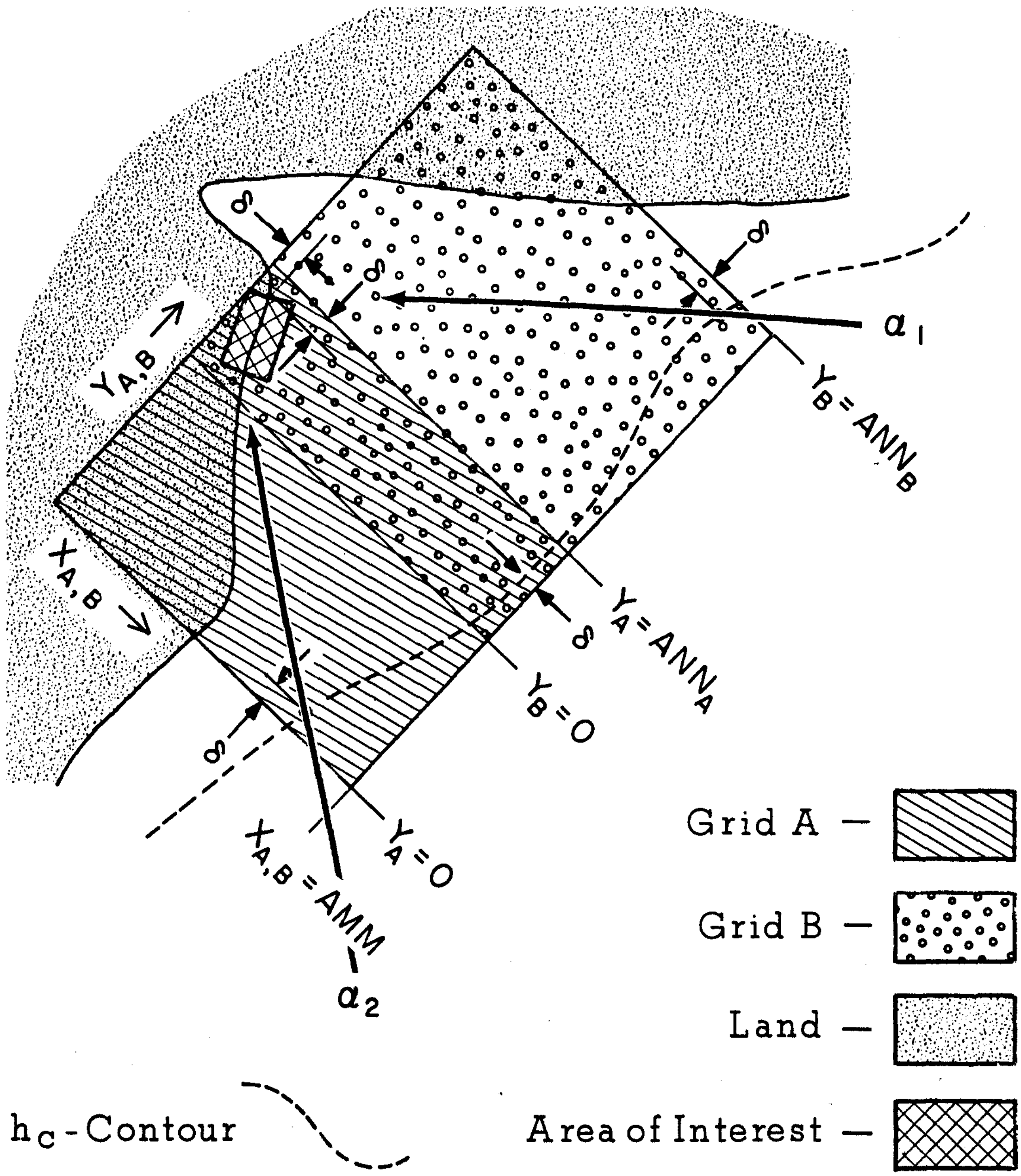
Sketch 7.

If both criteria cannot be satisfied with a single grid, two overlapping grids--each including the area of interest--can be used (sketch 8). Each must satisfy both criteria.[5]

Selection of Depth Values. A Z-axis is established vertically with Z = 0 at sea level and increasing positively downward so that the X,Y,Z-coordinate system is left-handed. Any unit may be used to measure water depth; however, the program needs the conversion factor to feet (DCON). For example, if depths are given in meters, DCON = 3.28 (ft/m). Should the depths be given in feet, DCON = 1 (ft/ft).

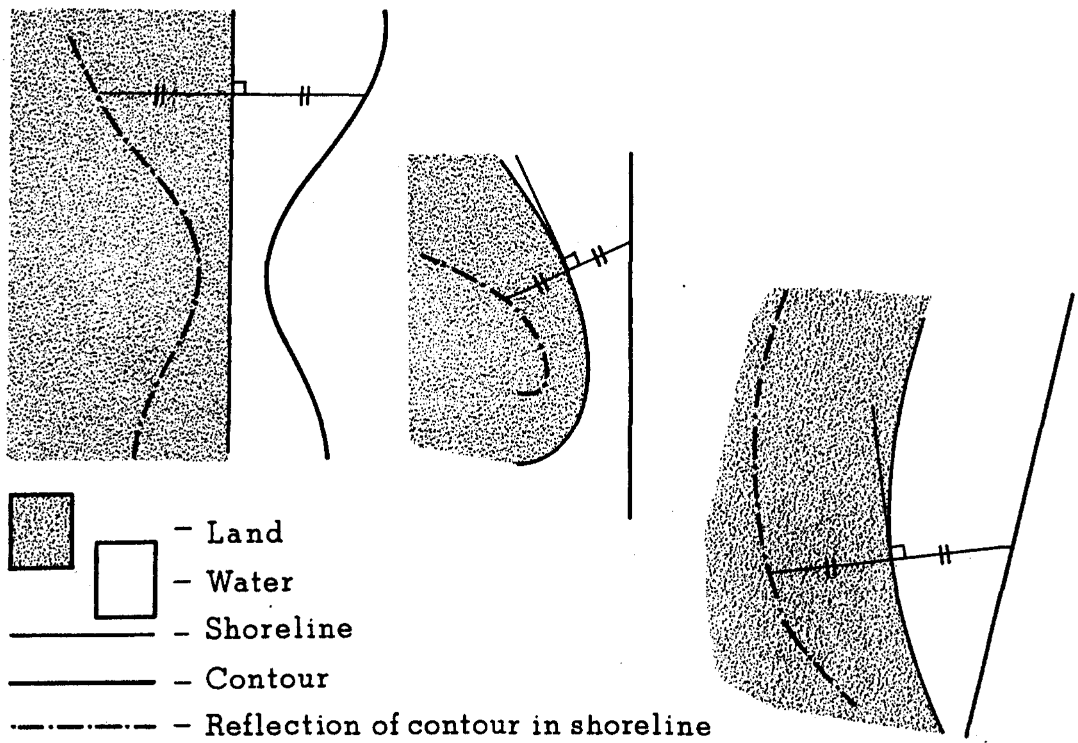
[4] The computer program requires that MM = AMM + 1 be an integral multiple of 10. This requirement can be changed by altering the controlling input format statement.

[5] If more than two grids are used, the rays can be transferred from one grid to the other. There is no provision for this operation in the present computer program.



Sketch 8.

Estimate from the chart the water depths at every grid intersection where the depth is non-negative. Record these depth values. Negative depth values are associated with grid points which lie on land. They are assigned as discussed below. Draw depth contours in a strip extending at least three grid units seaward from the shoreline. On the land draw the reflections of these contours in the shoreline (sketch 9). The "depth" assigned to each reflected contour is the negative of the water depth associated with the contour being reflected. From these land contours, negative depth values are estimated and recorded for every grid point lying two grid units or less from the shoreline. Depth values need not be considered for grid points further inland.



Sketch 9.

Selection of Ray Origins. To determine the ray pattern for a given sinusoid, origin points must be specified for individual rays. A segment of a single wave crest of the sinusoid (T_1, A_1)--represented by a straight line--is drawn on the chart in deep water $(h > 1/2\ L_{d1})$. Origin points for the rays are selected along this crest, and the coordinate values for each point are recorded (sketch 10).[6] This method of choosing wave-ray origins is used if the ray pattern for a single sinusoid having a fixed direction of travel and approaching the area of interest from the sea is wanted.

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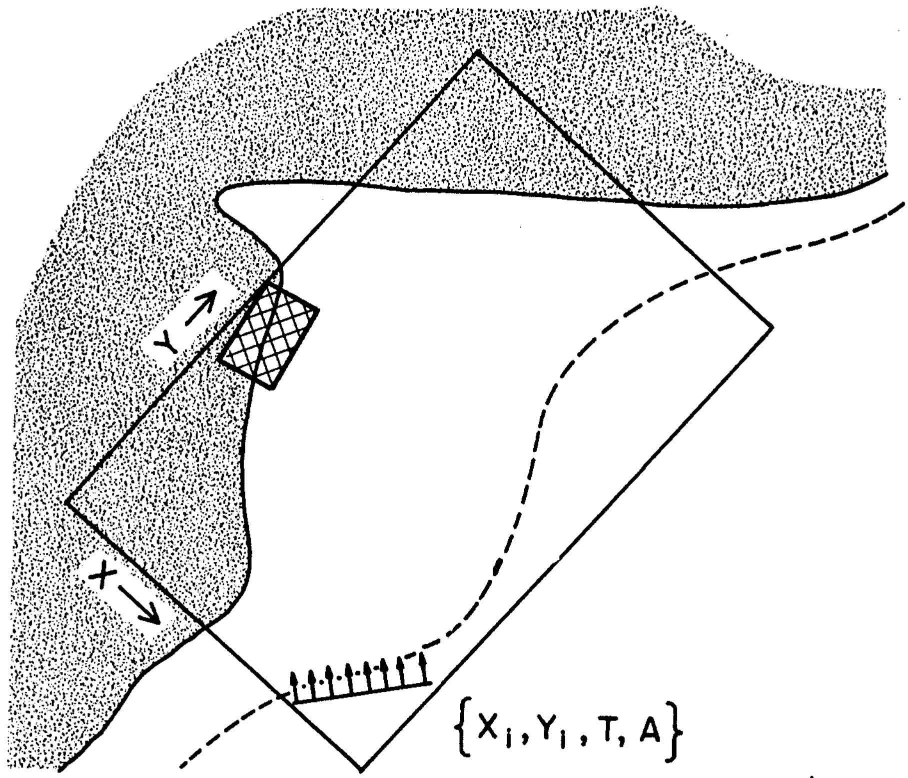
[6] No ray origin may be placed closer than one-half grid unit to a grid boundary.

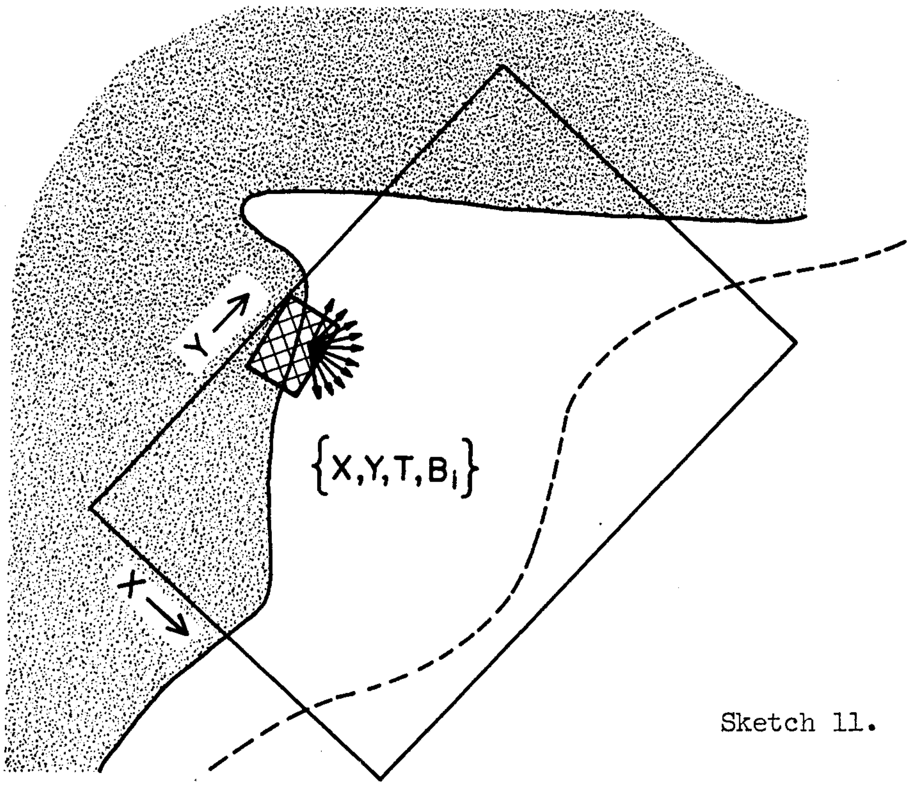


Sketch 10.

If a group of rays converging upon a single shallow-water point from many directions is desired, then the reverse tracing procedure (Dorrestein, 1960) is used. For the point of interest, a given T, and a set of angles $\{B_i\}$ at the point, the ray paths radiating from the point are traced (sketch 11). When the rays reach deep water, their directions give the directions of travel $\{A_i\}$ corresponding to the selected angles $\{B_i\}$.

Computer Operations.[7] The computer starts with a ray origin and approximates the path by calculating successive points. For this calculation the computer needs the array of depth values, the wave period (T), the direction of travel (A), and the coordinates of the initial position (X,Y). At each point a plane is fitted by

[7] All operations described in this section are performed by the computer.



Sketch 11.

least-squares to the four closest depth values. Water depth (h) and the gradients $\partial h/\partial X$ and $\partial h/\partial Y$ are obtained from the plane. The change in depth normal to the ray ($\partial h/\partial n$) is found from

$$\frac{\partial h}{\partial n} = - \frac{\partial h}{\partial X} \sin A + \frac{\partial h}{\partial Y} \cos A .$$

Wave speed (C) and $\partial C/\partial n$ are calculated with

$$C = \frac{gT}{2\pi} \tanh \left(\frac{2\pi h}{CT} \right)$$

and

$$\frac{\partial C}{\partial n} = \frac{\partial h}{\partial n} \cdot W$$

where[8]

$$W = \frac{1}{k'} \cdot \left[\frac{1}{\frac{Ck''}{1+k''C} + \frac{Ck''}{1-k''C} + \ln (1+k''C) - \ln (1-k''C)} \right] .$$

[8] See Harrison and Wilson (1964) Appendix F for the derivation of W.

In this expression $k' = T/4\pi$ and $k'' = 2\pi/gT$. Ray curvature (K) is computed with

$$K = \frac{1}{C} \left(\frac{-\partial C}{\partial n}\right) .$$

Denoting the current point (P_n) and the next succeeding point (P_{n+1}), P_{n+1} is reached from P_n by iterating with

$$\Delta A = (K_n + K_{n+1}) D_n/2 ,$$

$$A_{n+1} = A_n + \Delta A ,$$

$$\bar{A} = (A_n + A_{n+1}) /2,$$

$$X_{n+1} = X_n + D_n \cos \bar{A} ,$$

and

$$Y_{n+1} = Y_n + D_n \sin \bar{A}$$

where D_n, the incremented distance between points, is given by the ratio h_n/L_d. (See Griswold and Nagle, 1962; Griswold, 1963.)

Computations stop when the ray reaches the shore or a border of the grid. The coordinates of the points defining the ray path just completed are recorded on a magnetic tape. The process is repeated for each ray origin specified. Later, the information contained on this tape is used by the plotter to draw the rays.

Optional Computer Operations. The computer may be made to perform any or all of three optional operations. The first calculates the coordinates along a ray of the positions occupied by a wave crest at equal time intervals (CIN). If the value of CIN is chosen so that CIN/T = M where M is an integer, the result of the calculation can be interpreted as the positions of every Mth crest in a sinusoidal wave train.

The second operation obtains coordinate values of points on the depth grid where the linearly-interpolated depth equals zero. This option, if exercised, provides the plotter with data from which it can draw an approximate position of the shoreline.[9]

The third operation enables the plotter to enter selected soundings on the ray diagram. If a judicious selection of water

[9] The approximation becomes poorer as $\partial h/\partial X|_{h=0}$ approaches zero positively.

depths is made, an idea of the bathymetry of the region of analysis can be formed directly from the ray plot without referring to the chart of the region.

The results of these optional operations are recorded on the same magnetic tape used for the ray calculations.

Plotting Operations. The plotter transforms the calculations recorded on the magnetic tape into a series of plots. Each plot shows ray paths and is bordered and labeled. If the options have been exercised, the plot will also show travel-time marks, the shoreline, and soundings. (See figures 10-15 in APPENDIX D.)

The maximum dimensions of the plotting surface are 120 feet by 29.5 inches. The position of the border and label of each plot is controlled by AMM, ANN, and HT, as shown in figure 1.[10] AMM and ANN have already been determined by the depth grid. HT must be chosen and may be set equal to its maximum value of 28 inches—1.5 inches less than the respective plotting dimension. On the other hand, it may be chosen to make the scale of the plot (SCL) equal to a specified value. In the latter case, HT = GRID • SCL • ANN • 12, where GRID equals the number of feet per grid interval.

Before beginning a series of plots, the plotter pen is set (15.0 - HT/2) inches from the bottom of the roll of paper where HT is the height selected for the first plot. This establishes the origin, as shown in figure 1.

DISCUSSION

Computers Compatible with the Program. Although the program for calculating wave rays was written in Fortran II for the IBM 7094 computer, any computer operating with the Fortran monitor system and satisfying the memory requirement can be used. Approximately 11,000 positions are required for the data (exclusive of the depth grid) and the program. In addition, one memory position is required for each coordinate intersection on the depth grid.

[10] These are the dimensions for the Calcomp 564 plotter. See the DISCUSSION section for the dimensions if other plotters are used.

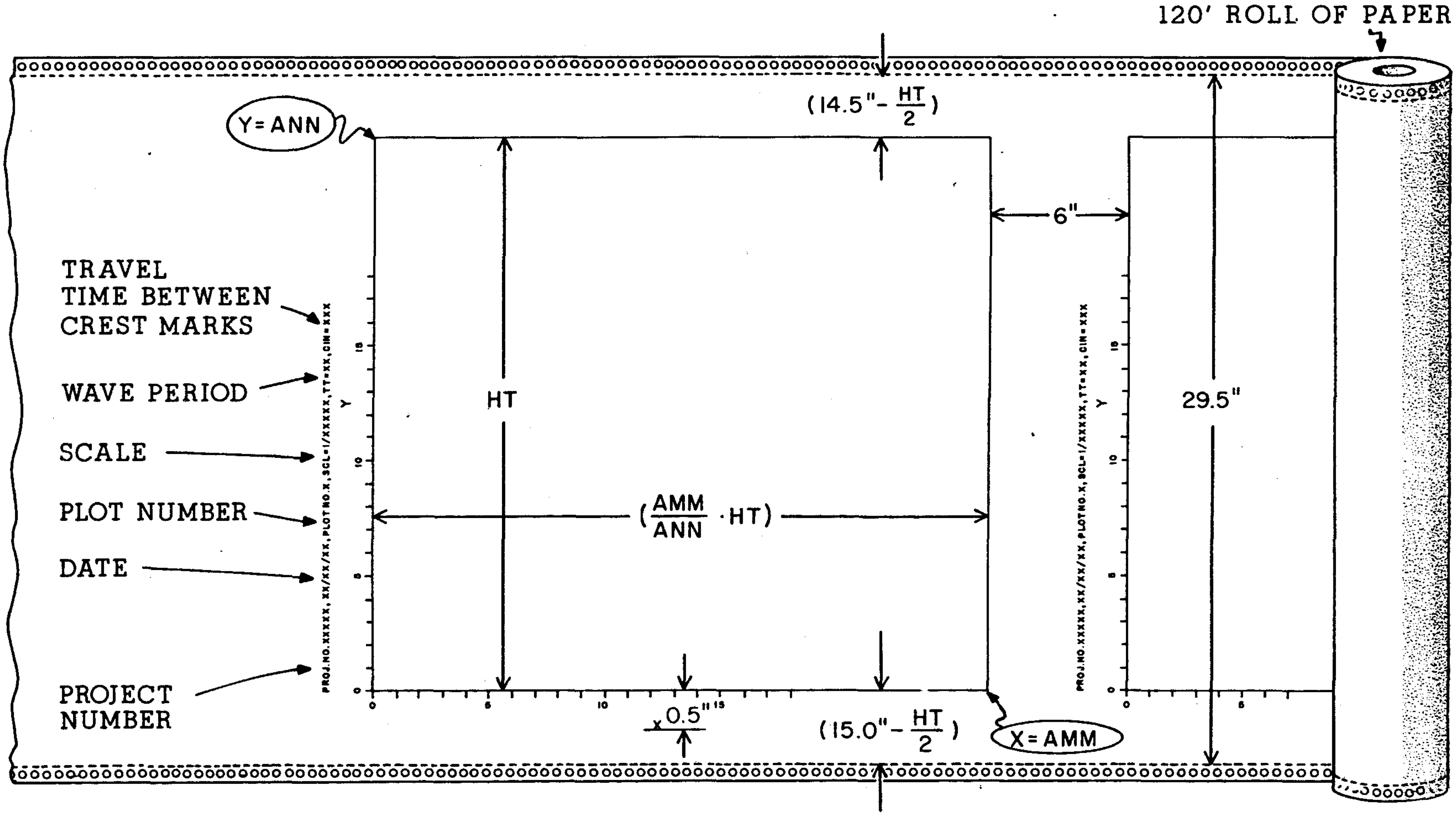


Figure 1. Orientation of Plot Border and Label.

Plotting Systems Compatible with the Program. The Calcomp 670/564 plotting system was used. This plotting system, produced by California Computer Products, Inc.[11], consists of a Model 670 magnetic-tape unit and a Model 564 plotter. (See figure 2.) The tape unit simultaneously reads information from the magnetic tape and instructs the plotter in graphing the information. The drum-type plotter translates information on the tape into deflections parallel with the X-axis by rotating a drum and deflections parallel with the Y-axis by moving a pen carriage along a track parallel with the axis of the drum. A given line is approximated with this system by movements of the pen in any of eight directions in .005-inch steps.

Calcomp makes other tape units and plotters which are compatible with the system. Among the tape units are Model Numbers 570, 670, 750, 760, and 770; among the plotters are 502, 563, 564, and 763. With the exception of the 502 plotter, whose plotting dimensions are 31 inches by 34 inches, the plotting dimensions of all these plotters are 120 feet by 29.5 inches. These models differ in plotting speed, step size, number of basic directions, and ability to read various densities of magnetic tape.

Recommendation for a Field Test. A practical field test of the method described in this report could be performed if it were possible to locate a coastal area having a well-defined swell. A hydrographic survey would be necessary if adequate bathymetric information were not available. Aerial photography could provide the deep-water wave length and the crest pattern for the swell. It would then be possible to compute the period of the swell and to draw a ray pattern. A depth grid would be prepared, and coordinate values obtained for initial points of rays. Ray paths would be computed, plotted, and compared with those obtained from the photography.

SUMMARY

Unless the bathymetry of an area is unusually smooth, the method described in this report calculates and plots wave rays faster than they can be constructed manually. The method requires the availability of a compatible computer and plotting system and the preparation of a grid of depth values for each region of analysis.

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[11] 305 Muller Avenue, Anaheim, California, 92801.

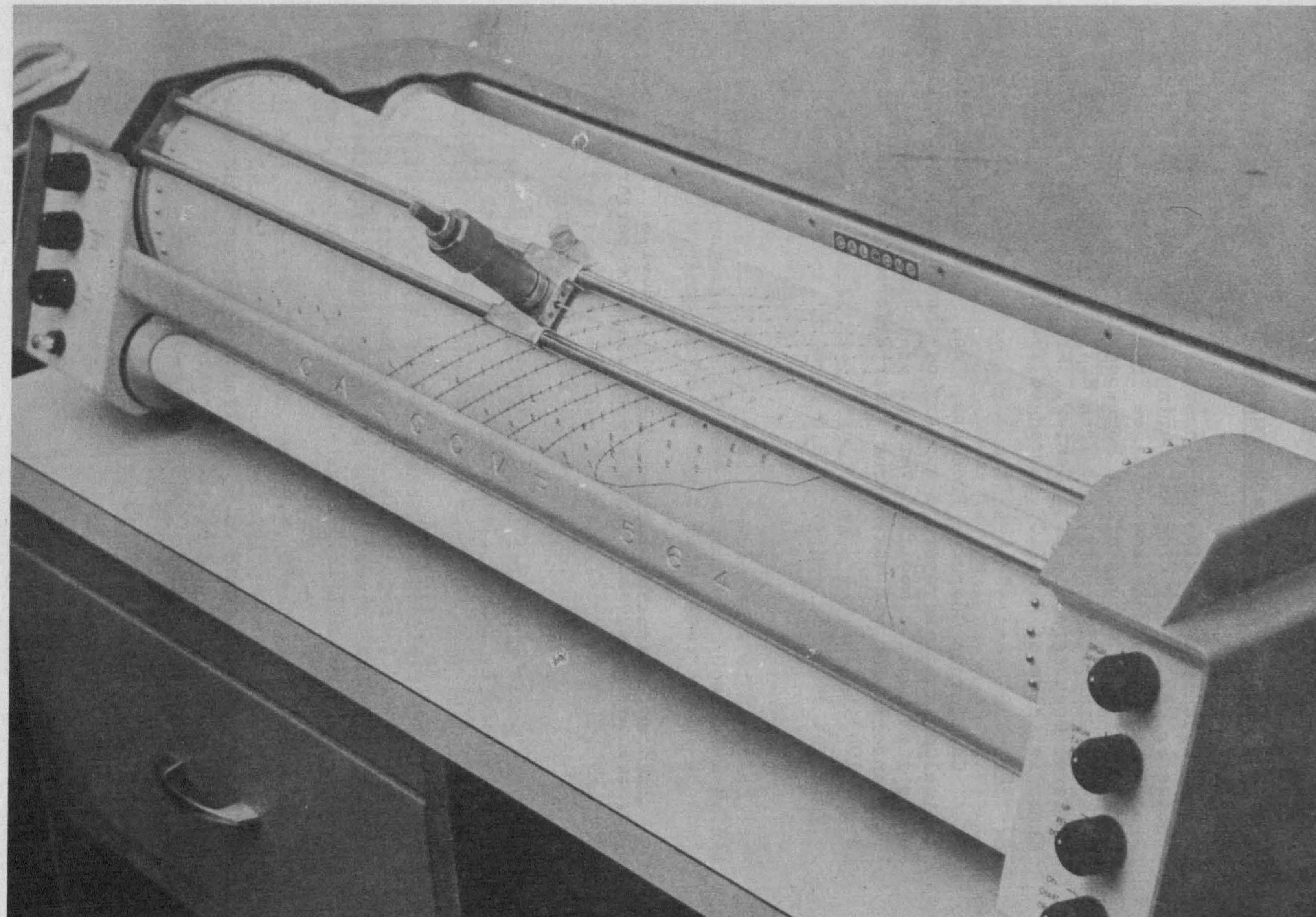

Figure 2. The Calcomp 564 Plotter.

ACKNOWLEDGEMENTS

Guidance and advice were provided by the author's major professor, Dr. Blair Kinsman, during the course of this study. Programing assistance was obtained from Miss Patricia M. Powers and Mr. Willard Graves of the Homewood Computing Center, the Johns Hopkins University, and from Mr. Rudi Saenger of the National Oceanographic Data Center. Mr. Saenger and Mr. John Chakalis, also of N.O.D.C., arranged free use of the plotting system.

California Computer Products, Inc., granted permission to include in this report a summary of the Calcomp subroutines and a listing of a modified version of their AXIS subroutine. This information was taken from their Reference Manual, copyright 1963.

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APPENDIX A.

Description of Computer Program

The computer program for calculating and plotting wave rays consists of a main program and sixteen subroutines. The main program and twelve of the subroutines are included in the generalized flow chart of the program as shown in figure 3. Definitions of variables used in this section are found in APPENDIX B.

1. MAIN PROGRAM.

MAIN controls the operation of the computer program and receives all input. Any given operation of the computer program entails the preparation of a magnetic tape containing instructions for the plotter to produce one or more plots—each plot to contain, for a specified wave period and depth grid, a group of refracted rays whose origins and initial angles were given. The input, when punched on cards and arranged for a given operation of the computer program, appears physically as is shown in figure 4.[12]

For the first plot, MAIN calls TITLE, NUMCON, and SHORE subroutines to prepare the general features of the plot. For each ray, MAIN calls RAYN to compute the path. When all paths have been computed for the first plot, the cycle is repeated for each additional plot.

2. TITLE.

TITLE produces the information necessary for the plotter to write the label and draw straight-line borders for each plot. The label includes a project number, date, plot number, and wave period. Depending on the value of NAX, TITLE may call AXIS in order to prepare instructions for drawing and calibrating the X- and Y-axes.

3. AXIS.[13]

AXIS prepares instructions so that the plotter can draw, calibrate, and label the X- and Y-axes. Calibrations are located

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[12] The input listing for an example operation of the computer program is given in figure 7 in APPENDIX D.

[13] This subroutine was modified from a Calcomp subroutine of the same name.

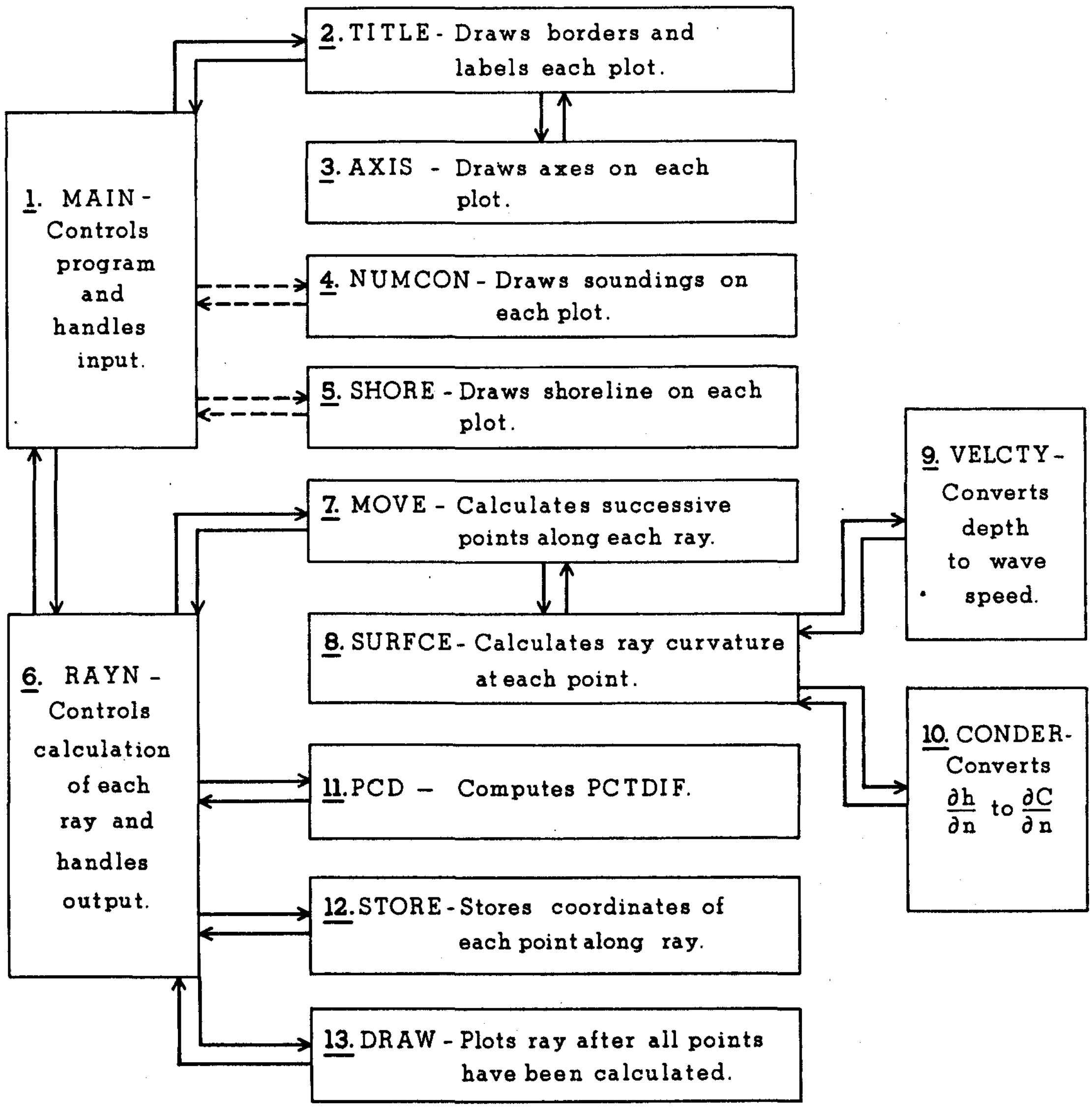


Figure 3. Generalized Flow Chart of Computer Program. SYMBOL, NUMBER, PLT670, and BCDFL not included. (– – – – – – optional subroutines)

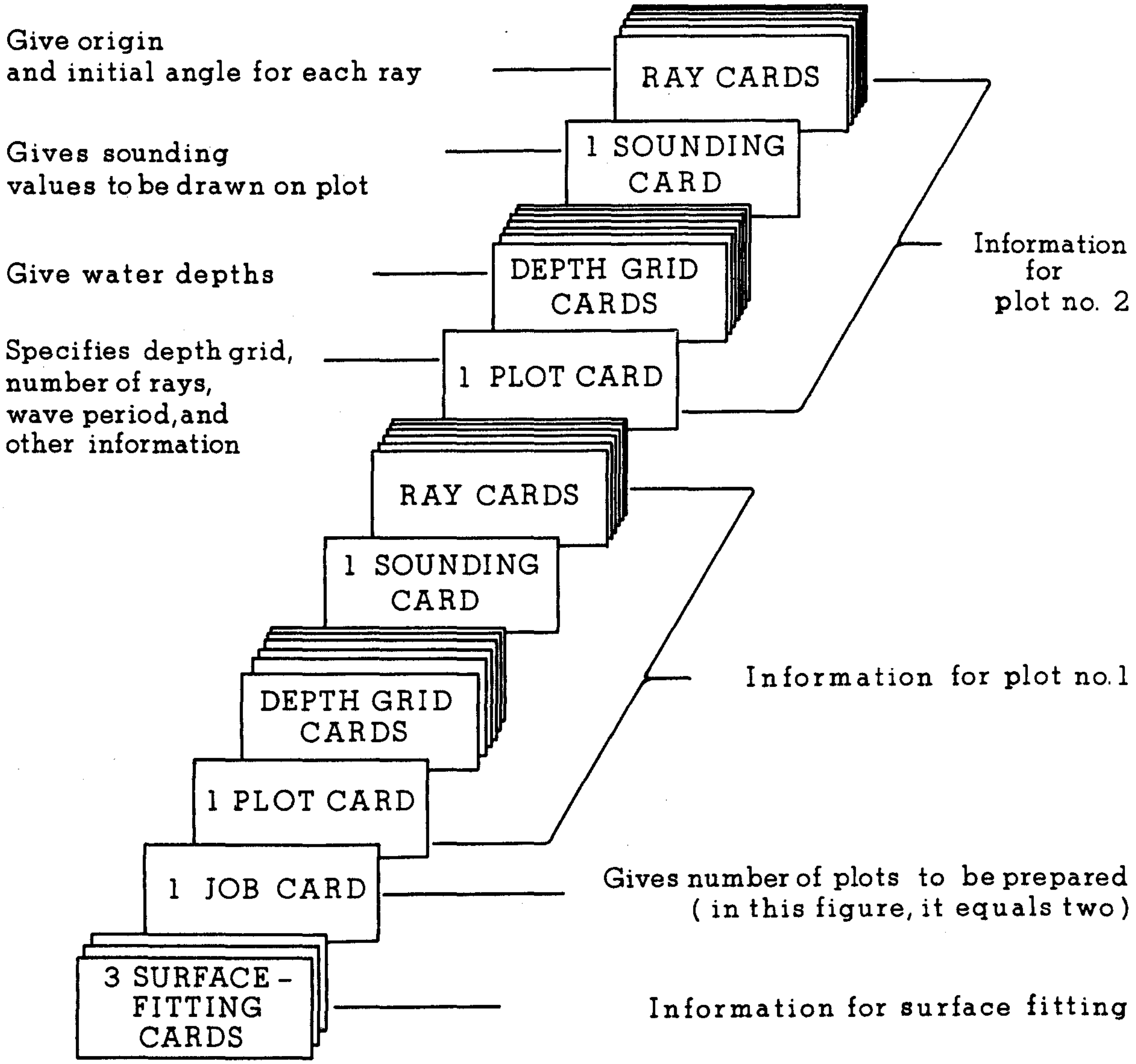


Figure 4. Generalized Diagram of Input for Computer Program.

at integral values along the axes. In this modification of AXIS, which is listed in APPENDIX C, the product of DY and SIZE must be an integer. For information concerning Calcomp's version of AXIS, see their Reference Manual (California Computer Products, Inc., 1963).

4. NUMCON.

NUMCON is called by MAIN to draw specified sounding values on a particular plot. NCO gives the number of sounding values, and each value is represented by an element of the CTOUR array. For each integral value of Y where $1 \leq Y \leq ANN - 1$ and for each CTOUR value, NUMCON prepares the necessary instructions for the plotter so that the value of CTOUR will be drawn at those positions on the plot where the linearly-interpolated value of CMAT equals the value of CTOUR. This subroutine cannot be used unless $\frac{\partial h}{\partial X}\Big|_{h=0} \geq 0$ for the entire depth grid. When NCO $\leq$ 0, NUMCON is not called, and a sounding card is not used in the input.

5. SHORE.

If NSH $\neq$ 0, SHORE is called by MAIN to prepare the necessary plotting instructions so that the shoreline can be drawn. Coordinates of points, where the linearly-interpolated value of CMAT equals zero, are calculated. The shoreline, when plotted, consists of a line connecting these points. This subroutine cannot be used unless $\frac{\partial h}{\partial X}\Big|_{h=0} \geq 0$ for the entire depth grid. If NSH = 0, SHORE is not called.

6. RAYN.

RAYN is called by MAIN to determine the path of each wave ray. RAYN calls MOVE to obtain the coordinates of each point along a ray. After each point is located, RAYN first calls PCD to obtain PCTDIF and then calls STORE to store the coordinates of the point. After all points have been computed, RAYN calls DRAW so that the coordinates of the points can be transformed into plotting instructions.

Computations of new points along a ray terminate when one of the following five conditions is encountered. Listed with each is the message that is produced for the printed output.

(1) MIT = 3, CURVATURE APPROXIMATIONS NOT CONVERGING.

(2) NGO = 2, RAY REACHED GRID BOUNDARY.

(3) NDP = 2, RAY REACHED SHORE.

(4) (D/DY) ≤ 0.005, RAY REACHED SHALLOW WATER.

(5) MAX + KCIN ≥ MMAX, DIMENSION OF OUTPUT-ARRAYS EXCEEDED.

RAYN outputs information in either of two formats, depending on the value of NPT. If NPT ≠ 0, MAX, X, Y, ANGLE, TIME, PCTDIF, DEP, and D are output for each point along a ray; if NPT = 0, X, Y, ANGLE, and TIME are output only for the origin and terminal points of a ray. An example of each format is given in figures 8 and 9 in APPENDIX D.

7. MOVE.

MOVE is called by RAYN to calculate the coordinates of the next point along a wave ray. D is computed, and the curvature used in getting the present point is used to approximate the location of the next point. MOVE then calls SURFCE to obtain the curvature at the approximated position of the next point. The average of the curvatures at the present and new points is taken and is used to obtain a second approximation of the next point. This procedure continues for a maximum of 20 times or until two successive curvature averages differ by a factor less than 0.00009/D. If this convergence occurs, the new point is accepted and MIT = 1. If the average curvatures used on the 18th and 20th trials have converged to less than 0.00009/D, the curvature used to obtain the new point is the average of the curvatures used on the 19th and 20th trials. This is done because the curvature approximations have converged to two values. MIT = 2 in this situation and, if NPT ≠ 0, causes a message CURVATURE AVERAGED to appear with the printed output. If neither convergence condition is satisfied, MIT = 3 and no new point is accepted.

Before returning to RAYN, the coordinates of the new point are tested to see if the point lies one-half grid unit from the edge of the grid. If this is true, NGO = 2; if this is not true, NGO = 1.

8. SURFCE.

SURFCE is called by MOVE to calculate curvature for a specific point along a wave ray. The four closest CMAT values are stored in C(4), and the coefficients, E(3), of the plane fitted to C(4) are calculated. DEP is obtained by interpolating on this plane. NDP = 1 if DEP > 0; NDP = 2 if DEP ≦ 0. If NDP = 2, control is transferred back to MOVE. Otherwise, VELCTY is called to obtain CXY. If (DEP/WL) > 0.5, NFK = 1 and curvature, FK, = 0; if (DEP/WL) ≦ 0.5, NFK = 2 and FK is computed after calling CONDER to obtain the partial of wave speed normal to the ray.

9. VELCTY.

VELCTY is called by SURFCE each time a wave speed, CXY, is to be obtained. If NFK = 1, CXY = CXXO. If NFK = 2, the expression on page 10 is used to obtain CXY.

10. CONDER.

CONDER is called by SURFCE to convert the partial of water depth with respect to the direction normal to a ray into the partial of wave speed with respect to the normal. The expression on page 10 is used.

11. PCD.

PCD is called by RAYN to compute PCTDIF for a given point along a ray. For each of the four depth values, C, closest to the point, PCD obtains the percent difference between C and the corresponding point on the plane fit to the four C's. PCTDIF represents the maximum of these four differences.

12. STORE.

STORE is called by RAYN after each point along a ray has been calculated. The X,Y-coordinates are stored in the AX and AY arrays, respectively. If CIN > 0, the X,Y-coordinates representing the position of a wave crest at equal time intervals along a ray are calculated and similarly stored in AX and AY. If CIN ≦ 0, these crest-position coordinates are not calculated.

13. DRAW.

DRAW is called by RAYN after all points have been calculated for a given ray so that the coordinates of these points can be transformed into plotting instructions. In order to minimize plotting time, odd-numbered rays are plotted beginning with the initial point; even-numbered rays are plotted beginning with the terminal point. Note the description of FAN in APPENDIX B to see how rays are numbered. If CIN > 0, marks are placed along a ray to designate crest positions; if CIN ≤ 0, these marks are not entered.

14. SYMBOL.[14] CALL SYMBOL (X,Y,HEIGHT,BCD,THETA,N)

This subroutine is used to prepare plotter instructions for drawing characters where:

(1) X,Y are the coordinates of the lower, left-hand corner of the first character to be drawn,

(2) HEIGHT specifies the character height and spacing in inches where the spacing between the lower left-hand corners of two successive characters equals 6/7 HEIGHT,

(3) BCD is the string of characters to be drawn and is written either as Hollerith information or as a variable containing alphanumeric information,

(4) THETA is the line angle where THETA = 0 for characters to be drawn from left to right and THETA = 90 for those to be drawn from bottom to top, and

(5) N is the number of characters to be drawn.

15. NUMBER. CALL NUMBER (X,Y,HEIGHT,FLOAT,THETA,N)

This subroutine is used to prepare plotter instructions for drawing floating-point numbers where:

---

[14] This and the following three subroutines are not listed in APPENDIX C, but they are available from California Computer Products, Inc. The versions of these four subroutines may differ depending on the computer and plotter systems being used, but their calling statements will be the same. Only a description of their calling statements will be given here; more information, including typical listings, is contained in the Calcomp Reference Manual (California Computer Products, Inc., 1963).

(1) X,Y,HEIGHT, and THETA are the same as for SYMBOL,

(2) FLOAT is the floating-point number to be drawn, and

(3) $-1 \leq N \leq 11$ where N gives the number of decimal places to be drawn. For -1 and 0, no decimal places will be drawn; however, -1 suppresses the decimal point and 0 does not.

16. PLT670.

This subroutine has two entry points used by the computer program:

```
CALL PLOTS (BUFFER(N),N)
CALL PLOT (X,Y,IPEN)
```

The first is used to initialize plotting operations by reserving an output buffer region for plotter information. The limits on the dimension of this region are $120 \leq N \leq 180{,}000$, where it is recommended that N be at least 2000.

The second entry point is used to issue instructions to move the pen to a new location (X,Y) specified in inches, where the pen will be up if IPEN = 3 and down if IPEN = 2. If IPEN < 0, the call is an end-of-plot entrance. This call is used to issue instructions to establish a new origin at the point to which the pen is to move. During the plotting operation, the only time that plotting can be stopped is after the pen has moved to a newly established origin. If IPEN = 999, a terminating mark is written on the tape.

17. BCDFL.

This subroutine is used internally by the Calcomp subroutines to convert fixed- and floating-point numbers into BCD.

APPENDIX B.

Principal Variables Used in Program

A	In the input: the initial ray angle measured in degrees relative to the direction of increasing X; internally in the program: the ray angle in radians for a specific calculation point along a ray. Note sketch 7 on page 6 to see how A is measured.
AMM,ANN	Maximum values of X and Y, respectively, for a particular depth grid.
ANGLE	The ray angle in degrees for a specific calculation point along a ray.
AX,AY	Two arrays used for temporary storage of output information. The dimension of these arrays is specified by MMAX.
BUFFER	An array used for temporary storage of plotter output information. Limits on the dimensioning of this array are given in the description of PLT670 in APPENDIX A.
C(4)	The four values of the depth grid which are closest to a specific calculation point along a ray.
CIN	If CIN > 0: in the input and output, the travel time in seconds between successive crest marks along a ray; internally in program, the same time as above but measured in hours. If CIN = 0: no crest marks will be placed along rays.
CMAT	The array representing the grid of depth values. The dimension of this array is given by MM,NN.
COL	If COL = 0 on a particular ray card, the plotter—when plotting this information—will pause before that ray is plotted. If COL ≠ 0, the plotter will not pause. This pause is called an "end-of-plot entrance" and is discussed in the PLT670 section of APPENDIX A.
CTOUR	An array specifying the sounding values in feet which are to be drawn on a particular plot. NCO gives the number of these values.

CXXO	Deep-water wave speed in feet per second.
CXY	Wave speed in feet per second at a specific calculation point along a ray.
D	The incremented distance in grid units between successive calculation points along a ray.
DATE1,DATE2	The date given in the form xx/yy/zz, where xx is the day, yy the month, and zz the year.
DCON	The conversion factor necessary for the product of DCON and CMAT to be a depth measured in feet.
DEP	The water depth in feet at a specific calculation point along a ray.
DN	Before CONDER is called: the partial of depth with respect to the direction normal to a ray (feet per grid unit); after CONDER is called: the partial of wave speed with respect to the direction normal to a ray (feet/second per grid unit).
DY	The number of grid units per inch for a specific plot.
E(3)	The coefficients of the equation of the plane fitted to the four closest depth values around a specific calculation point along a ray.
EM(4,3)	This and the S(3,3) array are used in obtaining E(3). (See Harrison and Wilson, 1964, Appendix C for the derivation of these arrays.)
FAN	FAN $\neq$ 0 (for rays originating from a point) causes rays to be numbered at their terminal points; FAN = 0 (for rays originating from points spaced along a crest) causes their origin points to be numbered.
FK	Ray curvature (grid units $^{-1}$).
GRID	The number of feet per grid unit for a particular depth grid.
HT	The height in inches for a specific plot.
I,J	Indices for CMAT; I = X + 1, J = Y + 1.

KCIN	The number of crest marks calculated along a ray which do not coincide with calculation points used for plotting the ray path.
KREST	The number of crest marks calculated along a particular ray.
LI	LI + 5 = the number of lines printed per page. The value of LI depends on the page height and printer being used by the computing center running the program.
MAX	The serial number of a specific calculation point along a ray.
MIT	MIT = 1 if the curvature approximations in MOVE have converged to one value; MIT = 2 if they have converged to two values; MIT = 3 if they have not converged.
MM,NN	The dimensions for a particular depth grid.
MMAX	The dimension of the AX and AY arrays.
MXPLOT	The number of plots to be prepared for a given operation of the computer program.
N	The ray number.
NAX	If NAX = 0, the borders of a given plot will be uncalibrated; if NAX $\neq$ 0, the borders will be calibrated with integral values of grid units.
NCO	If NCO is an integer where $1 \leq$ NCO ≤ 5, it specifies the number of CTOUR values to be input; if NCO ≤ 0, no CTOUR values will be input and no sounding card is needed.
NDP	If DEP > 0, NDP = 1; if DEP $\leq$ 0, NDP = 2.
NFK	If (DEP/WL) > 0.5, NFK = 1; if (DEP/WL) $\leq$ 0.5, NFK = 2.
NOR	The number of rays to be calculated for a given plot.
NPLOT	The plot number.
NPT	This determines the format of the printed output; see the discussion of RAYN in APPENDIX A.
NSH	If NSH $\neq$ 0, the shoreline will be drawn on a particular plot; if NSH = 0, it will not be drawn.

NXCMAT	If NXCMAT $\neq 0$, the depth grid will be input for a particular plot; if NXCMAT = 0, the depth grid used for the previous plot will be used again.
PCTDIF	An estimate of how well the linear-interpolation surface fits the four depth values which are closest to a specific calculation point along a ray. See the description of PCD in APPENDIX A.
PROJCT	Six digits of alphanumeric information used to identify each operation of the computer program.
RT	The width in inches of the plot.
S(3,3)	See EM.
SCL	The scale of the plot.
SCLI	1/SCL.
TIME	The total time in hours necessary for a wave crest to travel from a ray origin to a specific calculation point of the same ray.
TT	The wave period in seconds.
WL	The deep-water wave length in feet.
X,Y	The coordinates of a specific calculation point along a ray.

APPENDIX C.

Program Listing

```
*       LIST8
*       LABEL
CMAIN     PROGRAM
C       PROGRAM FOR THE CALCULATION AND PLOTTING OF SURFACE WAVE RAYS.
C       WRITTEN IN FORTRAN II FOR THE IBM 7094 COMPUTER AND
C       THE CALCOMP 670/564 PLOTTING SYSTEM.
C       PHYSICAL TAPE UNIT A6 IS USED FOR PLOTTER OUTPUT.
C       THIS PROGRAM NEEDS THE FOLLOWING SUBROUTINES...TITLE,AXIS,
C       NUMCON,SHORE,RAYN,MOVE,SURFCE,VELCTY,CONDER,PCD,STORE,DRAW,
C       SYMBOL,NUMBER,PLT670,BCDFL.
C
C       THIS PROGRAM WAS PREPARED BY W.S.WILSON, DEPT.OF OCEANOGRAPHY,
C       JOHNS HOPKINS UNIVERSITY, IN PURSUANCE OF CONTRACT DA-49-055-
C       CIV-ENG-64-5 WITH THE COASTAL ENGINEERING RESEARCH CENTER,
C       U.S.ARMY CORPS OF ENGINEERS.      JULY 21,1965.
C
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200
     10),AX(1000),AY(1000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GR
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
      CALL PLOTS (BUFFER(2000),2000)
      MMAX = 1000
      LI = 57
      LII = (LI-4)/3
      READ INPUT TAPE 5,5,((S(I,J),J=1,3),I=1,3)
    5 FORMAT(6F12.8)
      READ INPUT TAPE 5,7,((EM(L,I),L=1,4),I=1,3)
    7 FORMAT(12F6.2)
      READ INPUT TAPE 5,500,MXPLOT,PROJCT,DATE1,DATE2
  500 FORMAT (I3,A6,2A4)
      DO 399 NPLOT=1,MXPLOT
      READ INPUT TAPE 5,401,TT,NOR,MM,NN,GRID,DCON,NSH,NCO,NXCMAT,NPT,
     1 NAX,CIN,HT
  401 FORMAT (F5.1,3I4,2F7.0,5I4,F7.0,F9.3)
      CIN = CIN / 3600.
      WL = 32.2*TT**2./6.2831854
      AMM = MM-1
      ANN = NN-1
      DY = ANN/HT
      SCLI = GRID*DY*12.
      CALL TITLE (NPLOT,NAX,SCLI,HT)
      IF (NXCMAT) 3939,3938,3939
 3938 READ INPUT TAPE 5,11,((CMAT(I,J),I=1,MM),J=1,NN)
   11 FORMAT (10(F3.0,1X))
 3939 IF (NCO) 493,493,494
  494 READ INPUT TAPE 5,495,(CTOUR(KC),KC=1,NCO)
  495 FORMAT (5F8.2)
      CALL NUMCON (MM,NN,NCO)
  493 IF (NSH) 3936,3937,3936
 3936 CALL SHORE (MM,NN)
 3937 DO 15 N=1,NOR
      MAX = 1
      READ INPUT TAPE 5,6,A,X,Y,FAN,COL
    6 FORMAT (F7.2,2F6.2,2F3.0)
      IF (COL) 4322,4321,4322
 4322 CALL PLOT (0.0,0.0,-3)
 4321 A = A * .0174532925
      CALL RAYN (X,Y,A,NPLOT,N,MMAX,LI,NPT,LII)
   15 CONTINUE
```

```
  399 CONTINUE
      CALL PLOT (0.0,0.0,-3)
      CALL PLOT (0,0,999)
      WRITE OUTPUT TAPE 6,9999
 9999 FORMAT (17H1THIS IS THE END.)
      CALL EXIT
      END
*      LIST8
*      LABEL
      SUBROUTINE TITLE (NPLOT,NAX,SCLI,HT)
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200
     10),AX(1000),AY(1000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GR
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
      IF (NPLOT - 1) 701,701,700
  700 CALL PLOT (RT+6.0,(YHT-HT)/2.0,-3)
  701 RT = AMM/DY
      XNPLOT = NPLOT
      CALL SYMBOL (-1.5,0.0,0.21,81HPROJ.NO.                    , PLOT NO.
     1 , SCL = 1/          , TT =        , CIN =         ,90.,81)
      CALL NUMBER (-1.5,13.50,0.21,CIN*3600.,90.,-1)
      CALL NUMBER (-1.5,11.16,0.21,TT ,90.,1)
      CALL NUMBER (-1.5,08.46,0.21,SCLI,90.,-1)
      CALL NUMBER (-1.5,06.12,0.21,XNPLOT,90.,-1)
      CALL SYMBOL (-1.5,02.88,0.21,DATE1,90.,4)
      CALL SYMBOL (-1.5,03.60,0.21,DATE2,90.,4)
      CALL SYMBOL (-1.5,01.44,0.21,PROJCT,90.,6)
      IF (NAX) 705,704,705
  704 CALL PLOT (0.0,0.0,3)
      CALL PLOT (0.0,HT,2)
      GO TO 706
  705 CALL AXIS (0.,0.,1HY,1,HT ,90.,0.,DY)
      CALL AXIS (0.,0.,1HX,-1,RT,0.,0.,DY)
      CALL PLOT (0.0,HT,3)
  706 CALL PLOT (RT,HT,2)
      CALL PLOT (RT,0.0,2)
      IF (NAX) 707,708,707
  708 CALL PLOT (0.0,0.0,2)
  707 CALL PLOT (0.0,0.0,-3)
      YHT = HT
      RETURN
      END
*      LIST8
*      LABEL
      SUBROUTINE AXIS (X,Y,BCD,NC,SIZE,THETA,YMIN,DY)
C     MODIFIED FROM A CALCOMP SUBROUTINE OF THE SAME NAME.
C     REPRODUCED WITH PERMISSION FROM
C     CALIFORNIA COMPUTER PRODUCTS,INC.,ANAHEIM,CALIF.
      SIGN = 1.0
      IF(NC) 1,2,2
    1 SIGN = -1.0
    2 NAC =XABSF (NC)
      TH=THETA*0.017453294
      N = DY * SIZE + 0.5
      CTH = COSF (TH)
      STH = SINF (TH)
      TN = N
      XB = X
      YB = Y
      XA = X - 0.1 * SIGN * STH
```

```
      YA = Y + 0.1 * SIGN * CTH                                         AXIS  18
      CALL PLOT (XA,YA,3)                                               AXIS  19
      DO 20  I =1,N                                                     AXIS  20
      CALL PLOT (XB,YB,2)                                               AXIS  21
      XC = XB + CTH/DY                                                  AXIS  22
      YC = YB + STH/DY                                                  AXIS  23
      CALL PLOT (XC,YC,2)                                               AXIS  24
      XA = XA + CTH/DY                                                  AXIS  25
      YA = YA + STH/DY                                                  AXIS  26
      CALL PLOT (XA,YA,2)                                               AXIS  27
      XB = XC                                                           AXIS  28
   20 YB = YC                                                           AXIS  29
      ABSV = YMIN + TN                                                  AXIS  30
      XA = XB - (.20 * SIGN - .05) *STH - .02857* CTH                   AXIS  31
      YA = YB + (.20 * SIGN - .05) * CTH - .02857* STH                  AXIS  32
      N = N + 1                                                         AXIS  33
      DO 30 I = 1,N                                                     AXIS  34
      IF (MODF(ABSV,5.)) 100,101,100                                    AXIS  35
  101 CALL NUMBER (XA,YA,0.1,ABSV,THETA,-1)                             AXIS  36
  100 ABSV = ABSV - 1.                                                  AXIS  37
      XA = XA - CTH/DY                                                  AXIS  38
   30 YA = YA - STH/DY                                                  AXIS  39
      TNC = NAC + 7                                                     AXIS  40
      XA = X + (SIZE / 2.0 -.06 * TNC)*CTH - (-.07 + SIGN*.36)* STH     AXIS  41
      YA = Y + (SIZE / 2.0 -.06 * TNC)*STH + (-.07 + SIGN*.36)* CTH     AXIS  42
      CALL SYMBOL (XA,YA,0.14,BCD,THETA,NAC)                            AXIS  43
      RETURN                                                            AXIS  44
      END                                                               AXIS  45
*       LIST8                                                           NUMCON00
*       LABEL                                                           NUMCON01
      SUBROUTINE NUMCON (MM,NN,NCO)                                     NUMCON02
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200NUMCON03
     10),AX(1000),AY(1000),CTOUR(5)                                     NUMCON04
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,IT,CXY,MAX,GRNUMCON05
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                     NUMCON06
      NOD = NN-1                                                        NUMCON07
      MOD = MM-1                                                        NUMCON08
      DO 5000 J=2,NOD                                                   NUMCON09
      YJ = J-1                                                          NUMCON10
      KKK = 1                                                           NUMCON11
      DO 8000 KC=1,NCO                                                  NUMCON12
      KWIT = 0                                                          NUMCON13
      NDIF = 3                                                          NUMCON14
      I = MM-1                                                          NUMCON15
      DO 1010 II=1,MOD                                                  NUMCON16
      XI = I-1                                                          NUMCON17
      IL = I+1                                                          NUMCON18
      XL = IL-1                                                         NUMCON19
      IF (KWIT) 21,21,8000                                              NUMCON20
   21 IF (CMAT(I,J)) 10,10,20                                           NUMCON21
   10 KWIT = 1                                                          NUMCON22
   20 IF (CMAT(I,J)*DCON-CTOUR(KC)) 12,11,13                            NUMCON23
   11 AX(KKK) = XI                                                      NUMCON24
      AY(KKK) = CTOUR(KC)                                               NUMCON25
      KKK = KKK+1                                                       NUMCON26
      NDIF = 3                                                          NUMCON27
      GO TO 1010                                                        NUMCON28
   12 GO TO (14,77,14),NDIF                                             NUMCON29
   14 NDIF = 1                                                          NUMCON30
      GO TO 1010                                                        NUMCON31
```

```
   13 GO TO (77,15,15),NDIF                                              NUMCON32
   15 NDIF = 2                                                           NUMCON33
      GO TO 1010                                                         NUMCON34
   77 SLPX = (DCON*(CMAT(IL,J)-CMAT(I,J)))/(XL-XI)                       NUMCON35
      XP = (CTOUR(KC)-DCON*CMAT(I,J))/SLPX + XI                          NUMCON36
      AX(KKK) = XP                                                       NUMCON37
      AY(KKK) = CTOUR(KC)                                                NUMCON38
      KKK = KKK+1                                                        NUMCON39
      GO TO (81,82),NDIF                                                 NUMCON40
   81 NDIF = 2                                                           NUMCON41
      GO TO 1010                                                         NUMCON42
   82 NDIF = 1                                                           NUMCON43
 1010 I = I-1                                                            NUMCON44
 8000 CONTINUE                                                           NUMCON45
      KKK = KKK-1                                                        NUMCON46
      IF (KKK-1) 5000,668,670                                            NUMCON47
  670 KKL = KKK-1                                                        NUMCON48
      DO 997 IA=1,KKL                                                    NUMCON49
      IAD = IA+1                                                         NUMCON50
      DO 997 IB=IAD,KKK                                                  NUMCON51
      IF (AX(IA)-AX(IB)) 997,997,996                                     NUMCON52
  996 XMIN = AX(IA)                                                      NUMCON53
      AX(IA) = AX(IB)                                                    NUMCON54
      AX(IB) = XMIN                                                      NUMCON55
      XMIN = AY(IA)                                                      NUMCON56
      AY(IA) = AY(IB)                                                    NUMCON57
      AY(IB) = XMIN                                                      NUMCON58
  997 CONTINUE                                                           NUMCON59
  668 IF (XMODF(J,2))104,103,104                                         NUMCON60
  103 KONE = KKK                                                         NUMCON61
      KADD = -1                                                          NUMCON62
      LAST = +1                                                          NUMCON63
      GO TO 105                                                          NUMCON64
  104 KONE = +1                                                          NUMCON65
      KADD = +1                                                          NUMCON66
      LAST = KKK                                                         NUMCON67
  105 CALL NUMBER (AX(KONE)/DY,YJ/DY,0.10,AY(KONE),0.0,-1)               NUMCON68
      IF (KONE-LAST) 109,5000,109                                        NUMCON69
  109 KONE = KONE + KADD                                                 NUMCON70
      GO TO 105                                                          NUMCON71
 5000 CONTINUE                                                           NUMCON72
      CALL PLOT (0.,0.,-3)                                               NUMCON73
      RETURN                                                             NUMCON74
      END                                                                NUMCON75
*      LIST8                                                             SHORE 00
*      LABEL                                                             SHORE 01
      SUBROUTINE SHORE (MM,NN)                                           SHORE 02
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200SHORE 03
     1O),AX(1000),AY(1000),CTOUR(5)                                      SHORE 04
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GRSHORE 05
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                      SHORE 06
      PONTF(X1,X2,D1,D2) = X1 - D1*((X1-X2)/(D1-D2))                     SHORE 07
      IC = 3                                                             SHORE 08
      DO 1 J=1,NN                                                        SHORE 09
      YJ = J-1                                                           SHORE 10
      JL = J-1                                                           SHORE 11
      YL = JL-1                                                          SHORE 12
      I  = MM                                                            SHORE 13
      DO 2 II=1,MM                                                       SHORE 14
      XI = I-1                                                           SHORE 15
```

```
      IL = I+1                                                          SHORE 16
      XL = IL-1                                                         SHORE 17
      IF (CMAT (I,J)) 100,200,300                                       SHORE 18
  100 IF (IC-2) 101,101,102                                             SHORE 19
  101 XP = PONTF(XI,XL,CMAT(I,J),CMAT(IL,J))                            SHORE 20
      CALL PLOT (XP/DY,YJ/DY,IC)                                        SHORE 21
      IC = 2                                                            SHORE 22
      GO TO 1                                                           SHORE 23
  102 IF (J-1) 101,101,103                                              SHORE 24
  103 YP = PONTF(YJ,YL,CMAT(1,J),CMAT(1,JL))                            SHORE 25
      CALL PLOT (0.,YP/DY,IC)                                           SHORE 26
      IC = 2                                                            SHORE 27
      XP = PONTF(XI,XL,CMAT(I,J),CMAT(IL,J))                            SHORE 28
      CALL PLOT (XP/DY,YJ/DY,IC)                                        SHORE 29
      GO TO 1                                                           SHORE 30
  200 IF (II-MM) 201,202,201                                            SHORE 31
  202 CALL PLOT (XI/DY,YJ/DY,IC)                                        SHORE 32
      IF (IC-2) 203,203,204                                             SHORE 33
  203 IC = 3                                                            SHORE 34
      GO TO 1                                                           SHORE 35
  204 IC = 2                                                            SHORE 36
      GO TO 1                                                           SHORE 37
  201 IF (IC-2) 207,207,206                                             SHORE 38
  206 IF (J-1) 207,207,209                                              SHORE 39
  209 YP = PONTF(YJ,YL,CMAT(1,J),CMAT(1,JL))                            SHORE 40
      CALL PLOT (0.,YP/DY,IC)                                           SHORE 41
      IC = 2                                                            SHORE 42
  207 CALL PLOT (XI/DY,YJ/DY,IC)                                        SHORE 43
      IC = 2                                                            SHORE 44
      GO TO 1                                                           SHORE 45
  300 IF (II-MM) 2,302,2                                                SHORE 46
  302 IF (IC-2) 303,303,1                                               SHORE 47
  303 YP = PONTF(YJ,YL,CMAT(1,J),CMAT(1,JL))                            SHORE 48
      CALL PLOT (0.,YP/DY,IC)                                           SHORE 49
      IC = 3                                                            SHORE 50
      GO TO 1                                                           SHORE 51
    2 I = I-1                                                           SHORE 52
    1 CONTINUE                                                          SHORE 53
      CALL PLOT (0.0,0.0,-3)                                            SHORE 54
      RETURN                                                            SHORE 55
      END                                                               SHORE 56
*      LIST8                                                            RAYN  00
*      LABEL                                                            RAYN  01
      SUBROUTINE RAYN (X,Y,A,NPLOT,N,MMAX,LI,NPT,LII)                   RAYN  02
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200RAYN  03
     10),AX(1000),AY(1000),CTOUR(5)                                     RAYN  04
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GRRAYN  05
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                     RAYN  06
      NDP = 1                                                           RAYN  07
      NFK = 1                                                           RAYN  08
      NGO = 1                                                           RAYN  09
      KREST = 0                                                         RAYN  10
      KCIN = 0                                                          RAYN  11
      CALL SURFCE (X,Y,A,FK,NFK,NDP)                                    RAYN  12
      CALL MOVE (X,Y,A,FK,NGO,MIT,NFK,NDP)                              RAYN  13
      TIME = 0.0                                                        RAYN  14
      ANGLE=A*57.29577951                                               RAYN  15
      IF (NPT) 100,101,100                                              RAYN  16
  100 WRITE OUTPUT TAPE 6,7,PROJCT,DATE1,DATE2,NPLOT,TT,N               RAYN  17
    7 FORMAT (1H1,11HPROJECT NO.,A6,2H, ,2A4,10H, PLOT NO.,I3,10H, PERIORAYN  18
```

```
   1D =,F5.1,14H SEC., RAY NO.,I3,1H.//)
     WRITE OUTPUT TAPE 6,150
 150 FORMAT (1H ,3X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,6X,4HTIME,4X,6HPCTDI
    1F,5X,5HDEPTH,6X,1HD//)
     GO TO 19
 101 IF (N-1) 800,800,801
 801 IF (XMODF(N,LII)) 803,800,803
 800 WRITE OUTPUT TAPE 6,850,PROJCT,DATE1,DATE2,NPLOT,TT
 850 FORMAT (12H1PROJECT NO.,A6,2H, ,2A4,10H, PLOT NO.,I3,10H, PERIOD =
    1,F5.1,5H SEC.///)
     WRITE OUTPUT TAPE 6,851
 851 FORMAT (8H RAY NO.,4X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,6X,4HTIME//)
 803 WRITE OUTPUT TAPE 6,853,N,MAX,X,Y,ANGLE,TIME
 853 FORMAT (1H ,I6,1X,I7,2F9.2,F11.2,F10.3)
     GO TO 19
   3 MAX=1+MAX
     IF (MAX+KCIN-MMAX) 399,400,400
 400 WRITE OUTPUT TAPE 6,401
 401 FORMAT (80X,36HDIMENSION OF OUTPUT-ARRAYS EXCEEDED.)
     GO TO 15
 399 ZCXY = CXY
     CALL MOVE (X,Y,A,FK,NGO,MIT,NFK,NDP)
     GO TO (396,402),NDP
 402 WRITE OUTPUT TAPE 6,403
 403 FORMAT (80X,18HRAY REACHED SHORE.)
     GO TO 15
 396 IF (D/DY - .005) 700,700,702
 700 WRITE OUTPUT TAPE 6,701
 701 FORMAT (80X,26HRAY REACHED SHALLOW WATER.)
     GO TO 15
 702 GO TO (397,397,404), MIT
 404 WRITE OUTPUT TAPE 6,405
 405 FORMAT (80X,40HCURVATURE APPROXIMATIONS NOT CONVERGING.)
     GO TO 15
 397 IF (NPT) 180,20,180
 180 IF (XMODF(MAX,LI)) 20,5,20
   5 WRITE OUTPUT TAPE 6,7,PROJCT,DATE1,DATE2,NPLOT,TT,N
     WRITE OUTPUT TAPE 6,150
  20 TIME = TIME + (D*GRID/(1800.*(CXY+ZCXY)))
     ANGLE=A*57.29577951
  19 IF (NPT) 160,161,160
 160 CALL PCD (C,E,PCTDIF)
     WRITE OUTPUT TAPE 6,12,MAX,X,Y,ANGLE,TIME,PCTDIF,DEP,D
  12 FORMAT (I7,2F9.2,F11.2,F10.3,F10.1,F10.2,F10.3)
 161 KMAX = MAX
     PX = X
     PY = Y
     CALL STORE (X,Y,A,KMAX,TIME,KCIN,KREST)
     GO TO (10,11), MIT
  11 IF (NPT) 170,10,170
 170 WRITE OUTPUT TAPE 6,9
   9 FORMAT (1H+,80X,19HCURVATURE AVERAGED.)
  10 IF (MAX-1) 4,4,13
   4 GO TO (3,402),NDP
  13 GO TO (3,406),NGO
 406 WRITE OUTPUT TAPE 6,407
 407 FORMAT (80X,26HRAY REACHED GRID BOUNDARY.)
  15 IF (NPT) 190,191,190
 191 WRITE OUTPUT TAPE 6,1233,N,KMAX,PX,PY,ANGLE,TIME
1233 FORMAT (1H+,I6,1X,I7,2F9.2,F11.2,F10.3,//)
```

```
190 CALL DRAW (N,KMAX,KCIN,KREST)
    RETURN
    END
*     LIST8
*     LABEL
    SUBROUTINE MOVE (X,Y,A,FK,NGO,MIT,NFK,NDP)
    DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200
   10),AX(1000),AY(1000),CTOUR(5)
    COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GR
   1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
    MIT = 1
    GO TO (201,202),NFK
201 D = 0.5
    GO TO 203
202 D = DEP / WL
203 IF (MAX-2) 38,102,104
102 FKBAR=FK
104 DO 20 IT=1,20
 39 DELA=FKBAR*D
    AA=A+DELA
    ABAR=A+.5*DELA
    DELX = D * COSF(ABAR)
    DELY = D * SINF(ABAR)
    XX=X+DELX
    YY=Y+DELY
    CALL SURFCE (XX,YY,AA,FKK,NFK,NDP)
    GO TO (101,6), MIT
101 GO TO (10,38),NDP
 10 FKBAR = 0.5 * (FK + FKK)
    IF (IT - 18) 5,37,9
 37 FKKPP = FKBAR
  5 IF (MAX - 2) 7,7,9
  7 IF (IT - 1) 20,20,9
  9 IF (ABSF(FKKP-FKBAR) - (0.00009/D)) 6,6,20
 20 FKKP = FKBAR
    IF (ABSF(FKKPP - FKBAR) - (0.00009/D)) 18,18,17
 17 MIT = 3
    GO TO 38
 18 FKBAR = 0.5 * (FKBAR + FKKP)
    MIT = 2
    GO TO 39
  6 IF ((XX-0.5)*((AMM-0.5)-XX))2,2,3
  3 IF ((YY-0.5)*((ANN-0.5)-YY))2,2,8
  2 NGO = 2
  8 X = XX
    Y = YY
    A = AA
    FK = FKK
 38 RETURN
    END
*     LIST8
*     LABEL
    SUBROUTINE SURFCE (X,Y,A,FK,NFK,NDP)
    DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200
   10),AX(1000),AY(1000),CTOUR(5)
    COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GR
   1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
    I=X+1.
    J=Y+1.
    FI=I
```

```
      FJ=J
      XL=X+1.-FI
      YL=Y+1.-FJ
      IF (MAX-1) 1,1,4
    4 IF (ZI-FI) 1,2,1
    2 IF (ZJ-FJ) 1,3,1
    1 ZI = FI
      ZJ = FJ
      C(1)=CMAT(I,J)
      C(2)=CMAT(I+1,J)
      C(3)=CMAT(I+1,J+1)
      C(4)=CMAT(I,J+1)
      DO 318 II=1,3
      YVW(II) = 0.
      DO 318 L=1,4
  318 YVW(II) = YVW(II)+C(L)*EM(L,II)
      DO 319 II=1,3
      E(II) = 0.
      DO 319 JJ=1,3
  319 E(II) = E(II)+S(II,JJ)*YVW(JJ)
    3 DEP = (E(1) + E(2)*XL + E(3)*YL) * DCON
      IF (DEP) 320,320,324
  320 NDP = 2
      GO TO 403
  324 IF ((DEP/WL)-0.5) 321,321,322
  321 NFK = 2
      GO TO 323
  322 NFK = 1
  323 CALL VELCTY (CXY,TT,MAX,DEP,NFK)
      PCX = E(2) * DCON
      PCY = E(3) * DCON
      DN = -PCX*SINF(A) + PCY*COSF(A)
      CALL CONDER (DN,TT,CXY,MAX,NFK)
      GO TO (401,402),NFK
  401 FK = 0.0
      GO TO 403
  402 FK = -DN/CXY
  403 RETURN
      END
*      LIST8
*      LABEL
      SUBROUTINE VELCTY (CXY,TT,MAX,DEP,NFK)
      IF (MAX - 1) 101,101,102
  101 BAR = 6.2831854/TT
      CXXO = TT*32.2/6.2831854
      CCC = CXXO
      GO TO 103
  102 CCC = XCXY
  103 GO TO (104,105),NFK
  104 CXY = CXXO
      GO TO 106
  105 DO 1000 M=1,90
      CXY = CXXO*TANHF(BAR*DEP/CCC)
      IF (ABSF(CXY-CCC)-.00005) 106,1000,1000
 1000 CCC = (CXY+CCC)/2.
  106 XCXY = CXY
      RETURN
      END
*      LIST8
*      LABEL
```

```
      SUBROUTINE CUNDER (DN,TT,CXY,MAX,NFK)                                     CONDER02
      IF (MAX - 1) 101,101,102                                                  CONDER03
  101 C1 = TT/12.5663708                                                        CONDER04
      C2 = 6.2831854/(32.2*TT)                                                  CONDER05
  102 GU TO (105,104),NFK                                                       CONDER06
  104 C3 = C2*CXY                                                               CONDER07
      A1 = C3/(1.+C3)                                                           CONDER08
      A2 = C3/(1.-C3)                                                           CONDER09
      A3 = LOGF(1.+C3)                                                          CONDER10
      A4 = LOGF(1.-C3)                                                          CONDER11
      DN = (DN/C1)*(1./(A1+A2+A3-A4))                                           CONDER12
  105 RETURN                                                                    CONDER13
      END                                                                       CONDER14
*       LIST8                                                                   PCD   00
*       LABEL                                                                   PCD   01
      SUBROUTINE PCD (C,E,PCTDIF)                                               PCD   02
      DIMENSION C(4),E(3)                                                       PCD   03
      IF(C(1)*C(2)*C(3)*C(4)) 901,900,901                                       PCD   04
  900 PCTDIF = 999.                                                             PCD   05
      GO TO 902                                                                 PCD   06
  901 P1 = ABSF((C(1)-E(1))/C(1))                                               PCD   07
      P2 = ABSF((C(2)-E(1)-E(2))/C(2))                                          PCD   08
      P3 = ABSF((C(3)-E(1)-E(2)-E(3))/C(3))                                     PCD   09
      P4 = ABSF((C(4)-E(1)-E(3))/C(4))                                          PCD   10
      PCTDIF = 100. * MAX1F(P1,P2,P3,P4)                                        PCD   11
  902 RETURN                                                                    PCD   12
      END                                                                       PCD   13
*       LIST8                                                                   STORE 00
*       LABEL                                                                   STORE 01
      SUBROUTINE STORE (X,Y,A,KMAX,TIME,KCIN,KREST)                             STORE 02
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200STORE 03
     10),AX(1000),AY(1000),CTOUR(5)                                             STORE 04
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GRSTORE 05
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN                             STORE 06
      IF (CIN) 403,403,410                                                      STORE 07
  410 IF (KMAX-1) 400,400,401                                                   STORE 08
  400 AT = 0.0                                                                  STORE 09
  403 K = KMAX + KCIN                                                           STORE 10
      AX(K) = X                                                                 STORE 11
      AY(K) = Y                                                                 STORE 12
      IF (CIN) 205,205,402                                                      STORE 13
  402 ZA = A                                                                    STORE 14
      ZCXY = CXY                                                                STORE 15
      GO TO 205                                                                 STORE 16
  401 ET = TIME - AT                                                            STORE 17
      IF (CIN - ET) 405,404,403                                                 STORE 18
  404 K = KMAX + KCIN                                                           STORE 19
      AX(K) =-X                                                                 STORE 20
      AY(K) = Y                                                                 STORE 21
      KREST = KREST + 1                                                         STORE 22
      AT = AT + CIN                                                             STORE 23
      GO TO 402                                                                 STORE 24
  405 DSC = (ET-CIN)*(CXY+ZCXY)*3600./(GRID*2.)                                 STORE 25
      AA = (A+ZA)/2.                                                            STORE 26
      XM = DSC*COSF(AA)                                                         STORE 27
      YM = DSC*SINF(AA)                                                         STORE 28
      K = KMAX + KCIN                                                           STORE 29
      AX(K) =-X+XM                                                              STORE 30
      AY(K) = Y-YM                                                              STORE 31
      KREST = KREST + 1                                                         STORE 32
```

```
      KCIN = KCIN+1
      AT = AT + CIN
      GO TO 401
  205 RETURN
      END
*      LIST8
*      LABEL
      SUBROUTINE DRAW (N,KMAX,KCIN,KREST)
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,100),C(4),BUFFER(200
     10),AX(1000),AY(1000),CTOUR(5)
      COMMON S,EM,E,YVW,CMAT,C,BUFFER,AX,AY,CTOUR,PROJCT,D,TT,CXY,MAX,GR
     1ID,DCON,DEP,WL,AMM,ANN,DY,FAN,DATE1,DATE2,CIN
      XN = N
      KMAX = KMAX + KCIN
      IF (AX(KMAX)) 600,601,601
  600 AX(KMAX) = -AX(KMAX)
      KREST = KREST - 1
  601 IF (XMODF(N,2)) 104,103,104
  103 KTWO = KMAX-1
      KADD = -1
      LAST = +1
      MC = KREST + 1
      IF (FAN) 200,201,200
  200 CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.35/DY,XN,0.0,-1)
  201 CALL PLOT (AX(KMAX)/DY,AY(KMAX)/DY,3)
      IF (KMAX - 1) 106,106,105
  104 KTWO = +2
      KADD = +1
      LAST = KMAX
      MC = 0
      IF (FAN) 111,110,111
  110 CALL NUMBER (AX(1)/DY,AY(1)/DY,0.35/DY,XN,0.0,-1)
  111 CALL PLOT (AX(1)/DY,AY(1)/DY,3)
      IF (KMAX - 1) 106,106,105
  105 IF (CIN) 300,300,301
  301 IF (AX(KTWO)) 302,300,300
  300 CALL PLOT (AX(KTWO)/DY,AY(KTWO)/DY,2)
      GO TO 303
  302 AX(KTWO) = -AX(KTWO)
      WI = 0.05
      MC = MC + KADD
      IF (XMODF(MC,10)) 500,501,500
  501 WI = 0.10
  500 XPN = AX(KTWO)/DY
      YPN = AY(KTWO)/DY
      K = KTWO-KADD
      XPL = AX(K)/DY
      YPL = AY(K)/DY
      DSC = SQRTF((XPN-XPL)**2.+(YPN-YPL)**2.)
      CALL PLOT (XPN,YPN,2)
      XB = +WI*(YPN-YPL)/DSC
      YB = -WI*(XPN-XPL)/DSC
      CALL PLOT (XPN+XB,YPN+YB,2)
      CALL PLOT (XPN-XB,YPN-YB,2)
      CALL PLOT (XPN,YPN,2)
  303 IF (KTWO-LAST) 109,106,109
  109 KTWO = KTWO + KADD
      GO TO 105
  106 IF (KADD) 208,108,108
  208 IF (FAN) 205,107,205
```

```
107 CALL NUMBER (AX(1)/DY,AY(1)/DY,0.35/DY,XN,0.0,-1)                    DRAW  55
    GO TO 205                                                             DRAW  56
108 IF (FAN) 207,205,207                                                  DRAW  57
207 CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.35/DY,XN,0.0,-1)              DRAW  58
205 RETURN                                                                DRAW  59
    END                                                                   DRAW  60
```

APPENDIX D.

Example of Computing and Plotting Operations

To illustrate the computing and plotting operations, a portion of the coast south of Cape Henry and including Virginia Beach was selected as an area of interest. Two depth grids, one large and one small, were established. (See figure 5.) The grid interval for each equals 3038 feet, and each origin is located at 76°1.9'W, 36°39.5'N. U.S. Coast and Geodetic Survey boat sheets 5988, 5990, 5991, 5992, 5993, and 6595 and charts 1222 and 1227 were used to obtain depth values at grid intersections. Figure 7 shows the smoothed contours of the depth grids. It is apparent from the figure that this portion of the continental shelf includes many irregularities.

An annotated listing of input to the computer program is given in figure 6. (This input was used to produce the plot shown in figure 10.)[15] Two listings of printed output from the computer are given in figures 8 and 9. (Figure 8—for NPT = 1—gives the printed output for ray number 5 of figure 10; figure 9—for NPT = 0—gives the printed output for ray numbers 18 through 31 of figure 14.)

Six plots are presented in figures 10 through 15. Figure 10 shows a ray pattern for T = 4 sec and A = 120° on the small grid. On this figure the option to enter soundings has been exercised for 10, 20, 30, and 40 feet. (Figure 2 shows this plot being drawn.) Figure 11 shows ray patterns for T = 4 sec and A = 120°, 180°, 210°, and 240° on the large grid. Figure 12 illustrates how rays for T = 6 sec can be refracted away from a point in shallow water. Figures 11 and 12 show how the 20- and 40-foot soundings have been entered.

Figures 13, 14, and 15 show ray patterns for T = 6 sec and A = 180°, 195°, and 210°, respectively. The contours of the depth grid in figure 7 have been superimposed on these three figures in order to show the relation between the bathymetry and the ray patterns.

The computing time required to produce the information for these six plots was 0.14 hours. In contrast, the plotting time was much longer—3.5 hours.

---

[15] If the computer program has been modified or if the program is not being used with a Calcomp 670/564 plotting system, the input listing of figure 6 and the plot in figure 10 provide the necessary information to conduct a test.

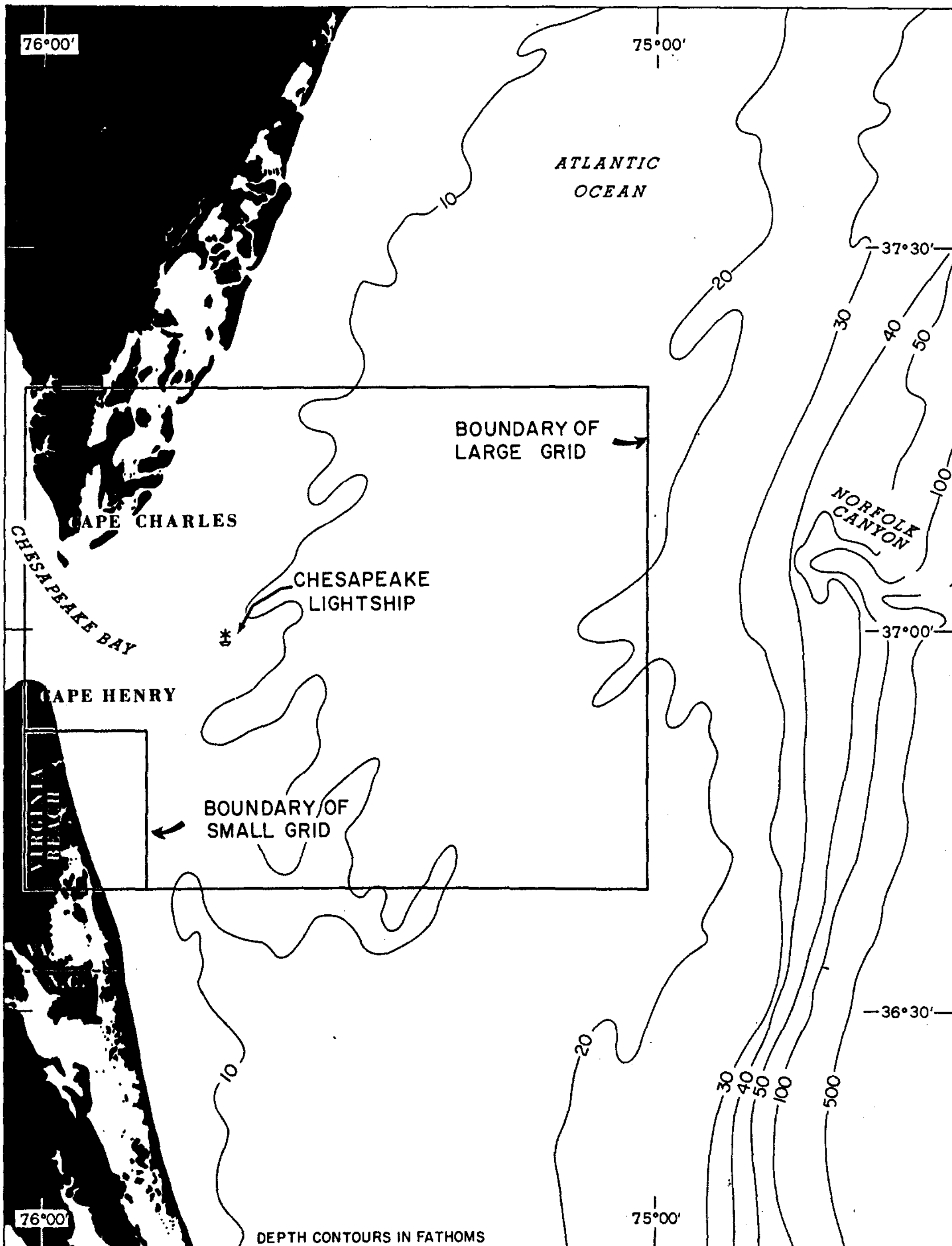


Figure 5. Location of Depth Grids.

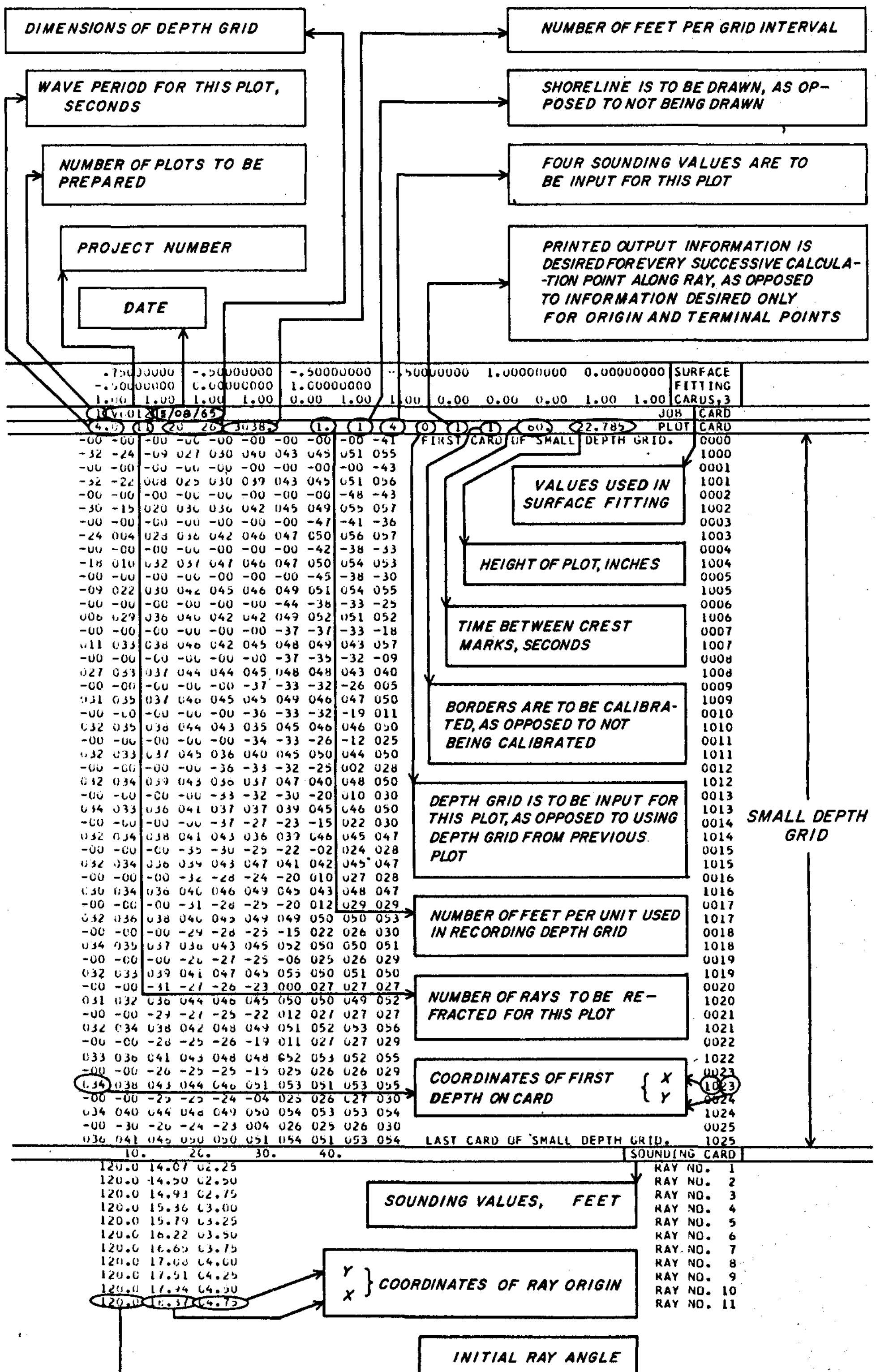


Figure 6. Example of Input for Computer Program.

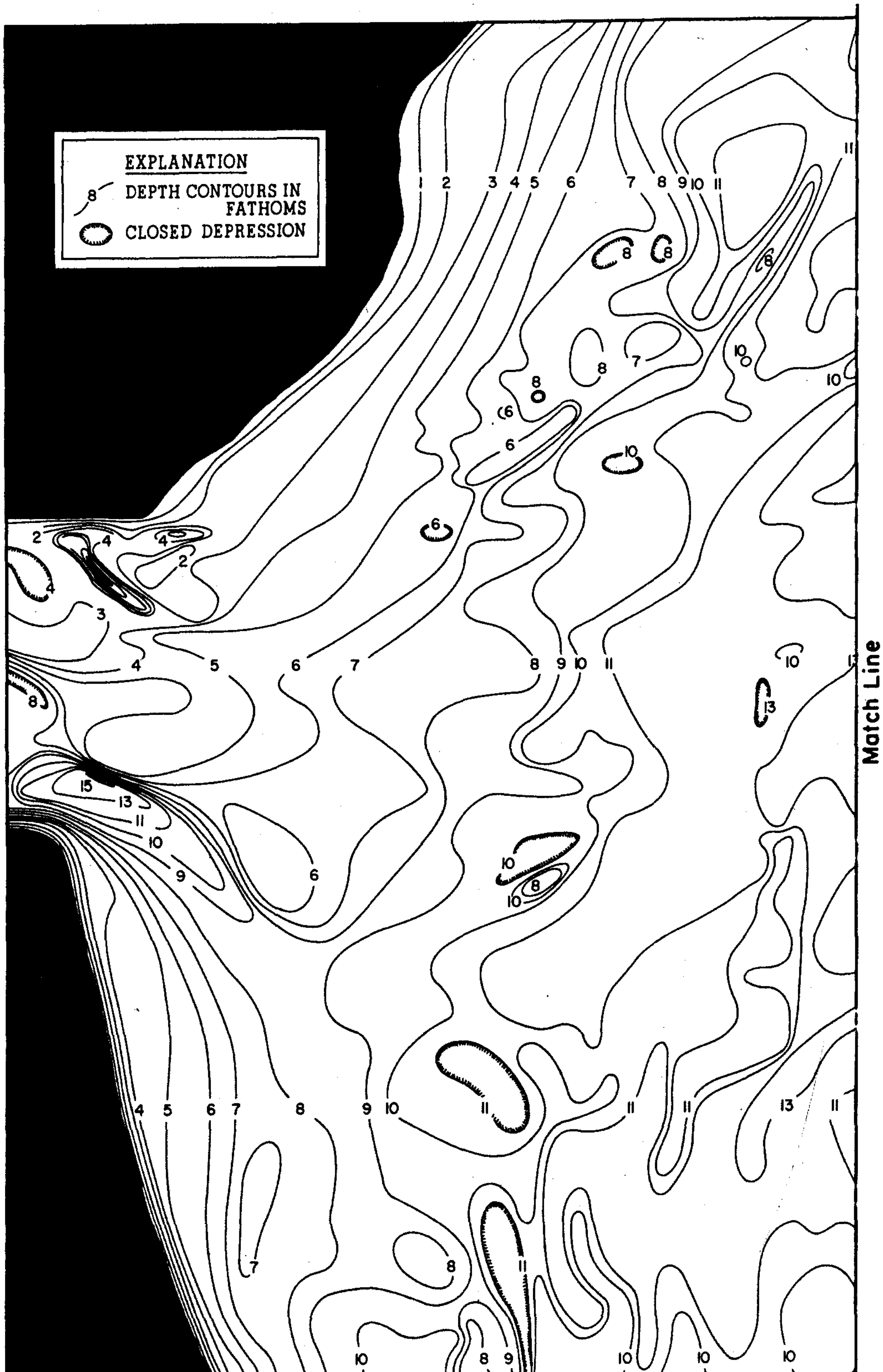
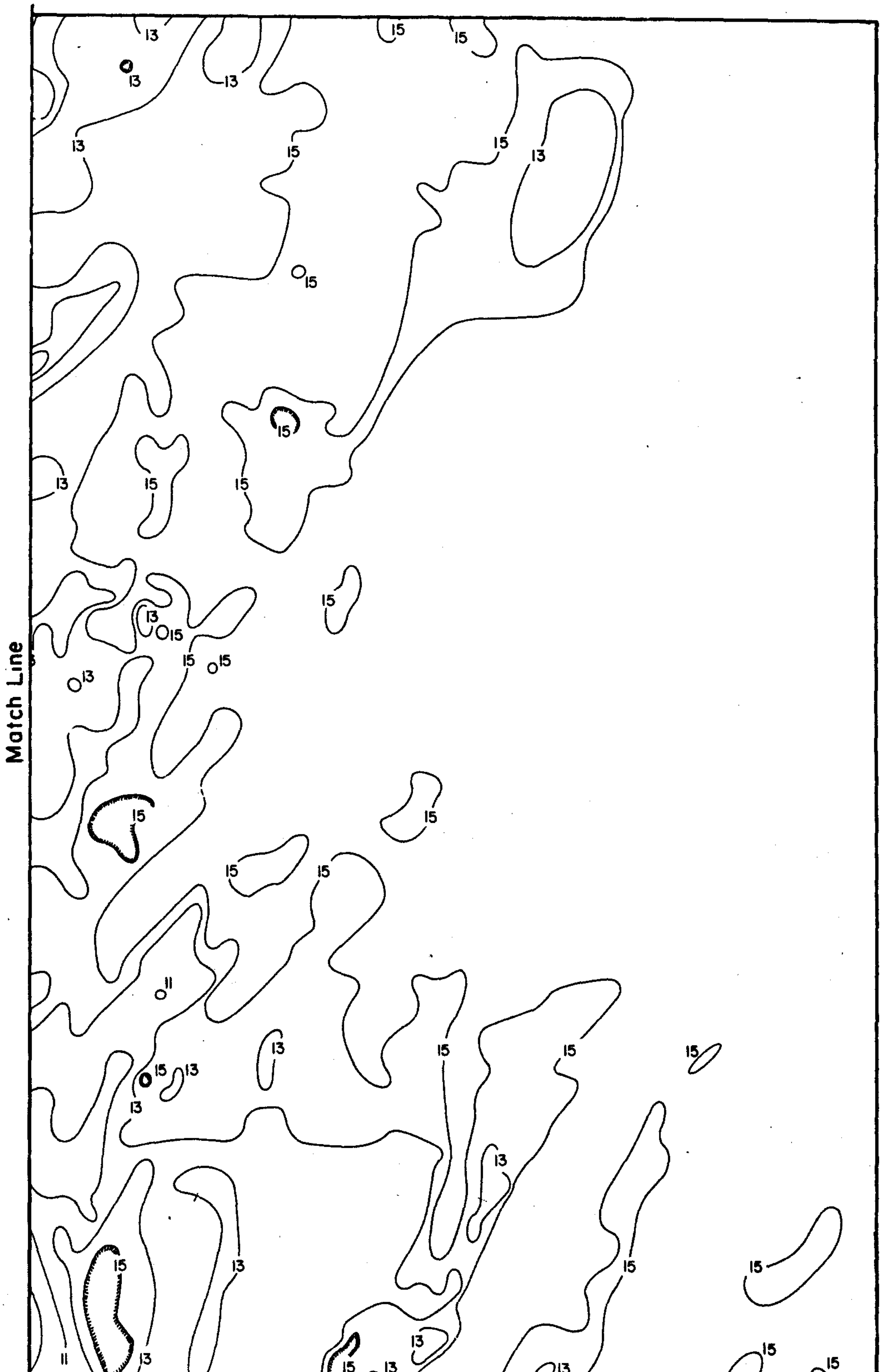


Figure 7. Bathymetry of Depth Grids



(after Harrison and Wilson, 1964).

PROJECT NO. VB012, 08/15/65, PLOT NO. 1, PERIOD = 4.0 SEC., RAY NO. 5.

MAX	X	Y	ANGLE	TIME	PCTDIF	DEPTH	D
1	15.79	3.25	120.00	0.	0.	46.79	0.500
2	15.54	3.68	120.00	0.021	0.	46.54	0.500
3	15.29	4.12	120.00	0.041	1.1	46.20	0.500
4	15.04	4.55	120.00	0.062	1.1	46.13	0.500
5	14.79	4.98	120.00	0.082	1.1	45.52	0.500
6	14.54	5.42	120.00	0.103	0.6	44.07	0.500
7	14.29	5.85	120.00	0.124	0.6	42.43	0.500
8	14.04	6.28	120.00	0.144	1.8	41.73	0.500
9	13.79	6.71	120.00	0.165	3.7	42.85	0.500
10	13.54	7.15	120.00	0.185	2.4	43.92	0.500
11	13.29	7.58	120.00	0.206	2.4	44.42	0.500
12	13.04	8.01	120.00	0.226	0.6	44.25	0.500
13	12.79	8.45	120.00	0.247	1.4	43.27	0.500
14	12.54	8.88	120.00	0.268	1.4	41.70	0.500
15	12.29	9.31	120.06	0.288	2.0	39.77	0.500
16	12.05	9.73	120.19	0.308	2.0	37.73	0.485
17	11.81	10.13	120.29	0.327	0.8	37.41	0.460
18	11.58	10.52	120.35	0.346	0.8	36.01	0.456
19	11.36	10.90	120.43	0.364	0.8	34.66	0.439
20	11.15	11.27	120.55	0.382	0.8	33.81	0.423
21	10.94	11.62	120.67	0.399	0.8	33.47	0.412
22	10.73	11.97	120.73	0.416	0.8	33.33	0.408
23	10.52	12.32	120.77	0.433	2.3	33.17	0.406
24	10.31	12.67	120.80	0.450	2.3	33.24	0.405
25	10.11	13.02	120.81	0.467	2.3	33.29	0.405
26	9.90	13.37	120.86	0.484	1.7	32.83	0.406
27	9.69	13.71	120.95	0.500	1.7	31.87	0.400
28	9.49	14.04	121.05	0.517	1.8	30.94	0.389
29	9.30	14.37	121.16	0.533	1.8	30.03	0.377
30	9.11	14.68	121.28	0.548	1.8	29.15	0.366
31	8.92	14.98	121.51	0.563	4.5	28.54	0.355
32	8.74	15.28	121.80	0.578	3.1	27.02	0.348
33	8.57	15.56	122.01	0.592	3.1	27.00	0.329
34	8.39	15.84	122.23	0.605	3.1	26.98	0.329
35	8.22	16.11	122.39	0.619	0.9	27.53	0.329
36	8.04	16.40	122.47	0.634	0.9	27.86	0.336
37	7.85	16.68	123.21	0.648	0.	25.83	0.340
38	7.67	16.94	124.73	0.661	0.	23.36	0.315
39	7.51	17.18	126.25	0.674	27.1	21.22	0.285
40	7.35	17.38	127.63	0.685	27.1	20.31	0.259
41	7.20	17.58	129.11	0.696	27.1	19.37	0.248
42	7.05	17.76	130.68	0.707	27.1	18.42	0.236
43	6.90	17.92	134.34	0.717	10.4	16.62	0.225
44	6.75	18.06	140.85	0.727	25.0	12.27	0.203
45	6.63	18.15	148.05	0.735	25.0	8.65	0.150
46	6.53	18.20	155.38	0.741	25.0	5.79	0.106
47	6.47	18.22	162.22	0.747	25.0	3.71	0.071
48	6.42	18.24	168.16	0.751	25.0	2.29	0.045
49	6.40	18.24	173.08	0.754	25.0	1.38	0.028
50	6.38	18.24	177.01	0.756	25.0	0.82	0.017
51	6.37	18.24	180.09	0.758	25.0	0.48	0.010
52	6.36	18.24	182.47	0.759	25.0	0.28	0.006

RAY REACHED SHALLOW WATER.

Figure 8. Example of Output from Computer Program (NPT = 1).

PROJECT NO. VB012, 08/15/65, PLOT NO. 5, PERIOD = 6.0 SEC.

RAY NO.	MAX	X	Y	ANGLE	TIME	
18	1	89.42	45.26	195.00	0.	
18	261	4.78	25.91	189.39	2.480	RAY REACHED SHALLOW WATER.
19	1	89.16	46.23	195.00	0.	
19	261	4.51	26.90	191.07	2.480	RAY REACHED SHALLOW WATER.
20	1	88.90	47.20	195.00	0.	
20	263	4.42	27.44	188.74	2.480	RAY REACHED SHALLOW WATER.
21	1	88.64	48.17	195.00	0.	
21	264	4.41	27.49	189.27	2.480	RAY REACHED SHALLOW WATER.
22	1	88.38	49.14	195.00	0.	
22	265	3.60	31.01	201.93	2.483	RAY REACHED SHALLOW WATER.
23	1	88.12	50.11	195.00	0.	
23	274	3.93	29.37	191.21	2.491	RAY REACHED SHALLOW WATER.
24	1	87.86	51.08	195.00	0.	
24	272	4.21	28.33	191.49	2.487	RAY REACHED SHALLOW WATER.
25	1	87.60	52.05	195.00	0.	
25	264	3.44	31.39	202.65	2.475	RAY REACHED SHALLOW WATER.
26	1	87.34	53.02	195.00	0.	
26	274	3.93	29.36	194.25	2.489	RAY REACHED SHALLOW WATER.
27	1	87.08	53.99	195.00	0.	
27	254	0.47	34.64	184.08	2.516	RAY REACHED GRID BOUNDARY.
28	1	86.82	54.96	195.00	0.	
28	273	4.13	28.83	194.60	2.489	RAY REACHED SHALLOW WATER.
29	1	86.56	55.93	195.00	0.	
29	290	0.32	38.31	155.14	2.563	RAY REACHED GRID BOUNDARY.
30	1	86.30	56.90	195.00	0.	
30	296	0.38	39.40	155.94	2.578	RAY REACHED GRID BOUNDARY.
31	1	86.04	57.87	195.00	0.	
31	285	0.31	37.43	194.57	2.541	RAY REACHED GRID BOUNDARY.

Figure 9. Example of Output from Computer Program (NPT = 0).

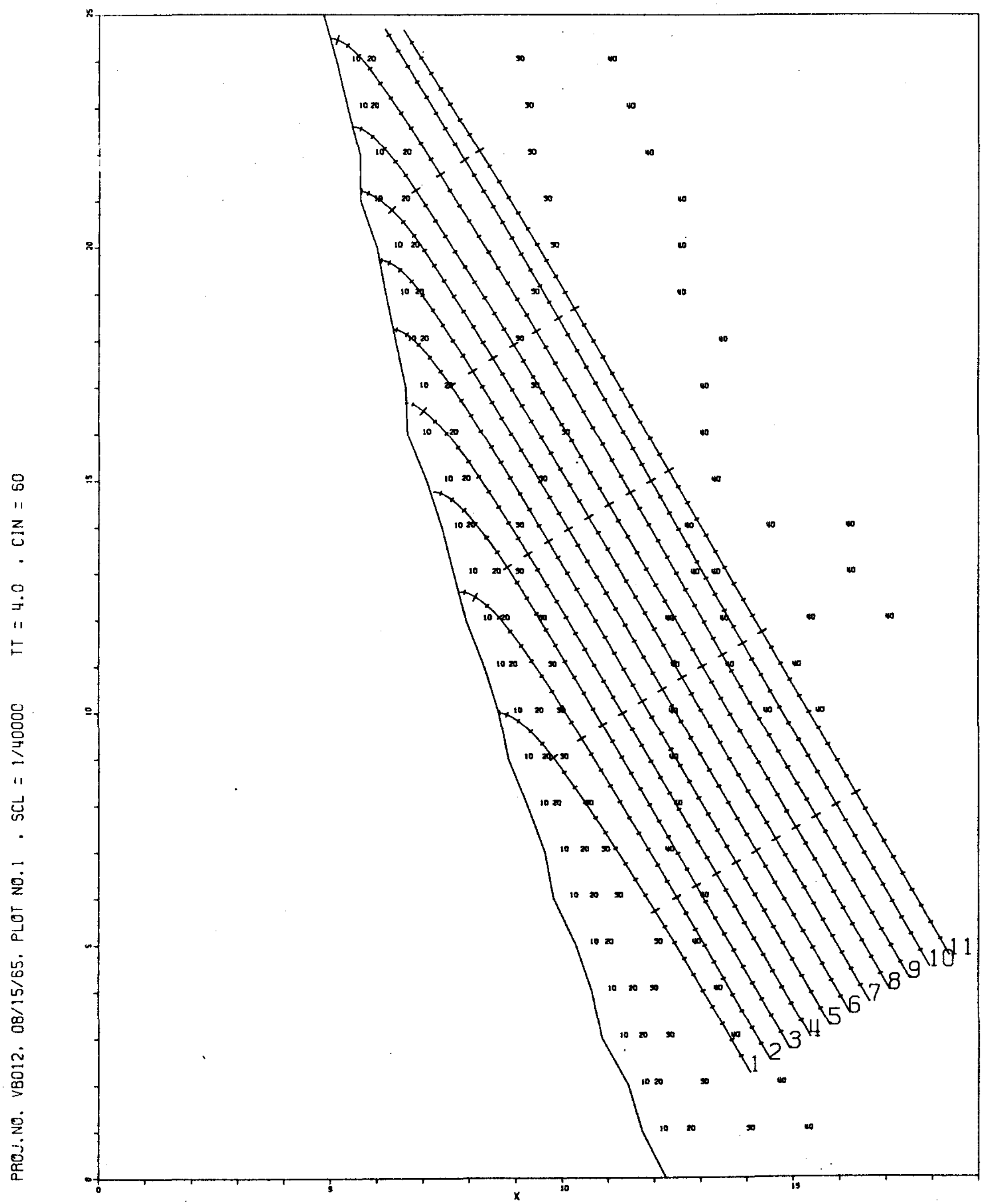


Figure 10. Ray Pattern ($T = 4$ sec, $A = 120^{\circ}$) on Small Grid.

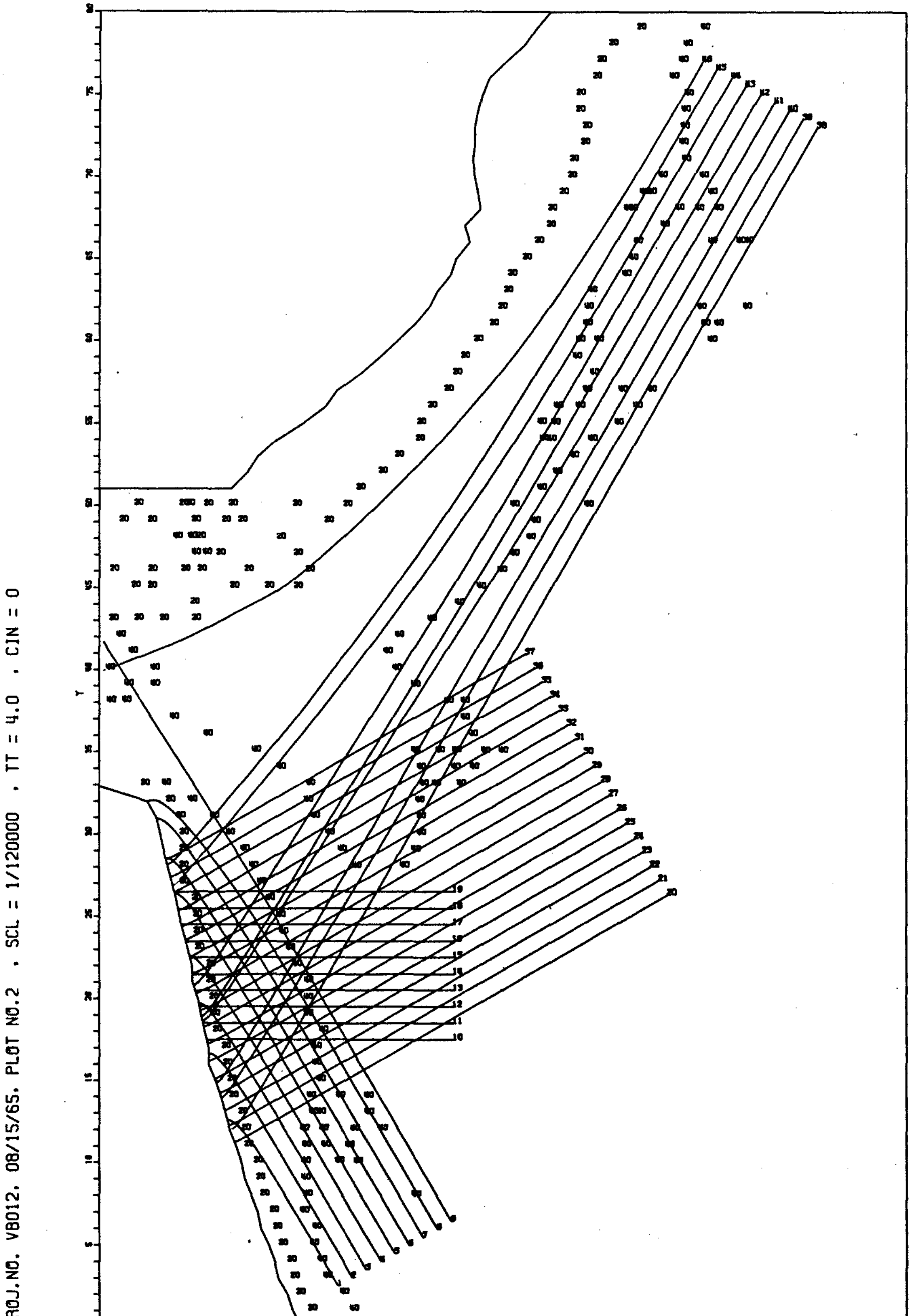


Figure 11. Ray Patterns (T = 4 sec; A = 120°, 180°, 210°, 240°) on Large Grid.

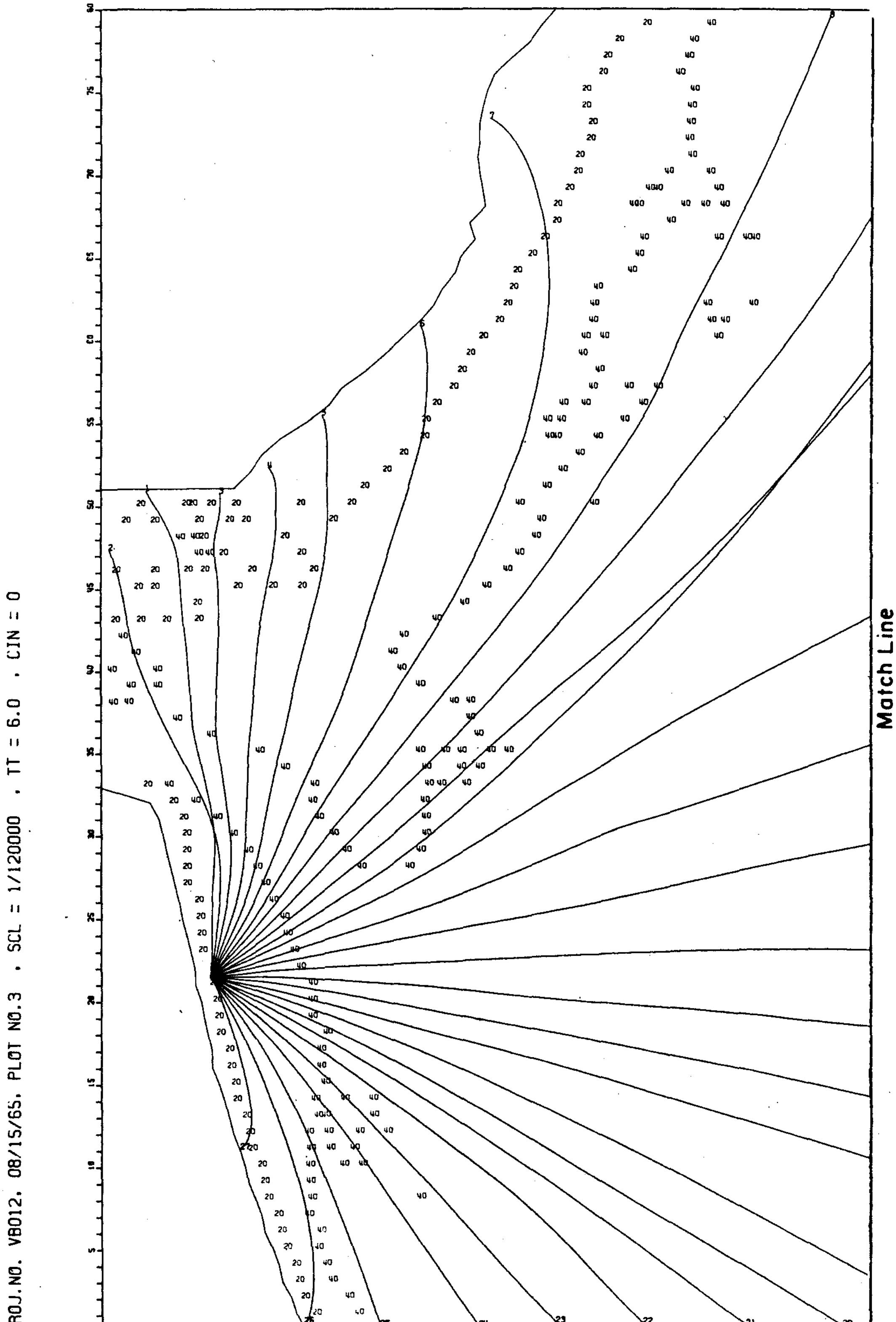
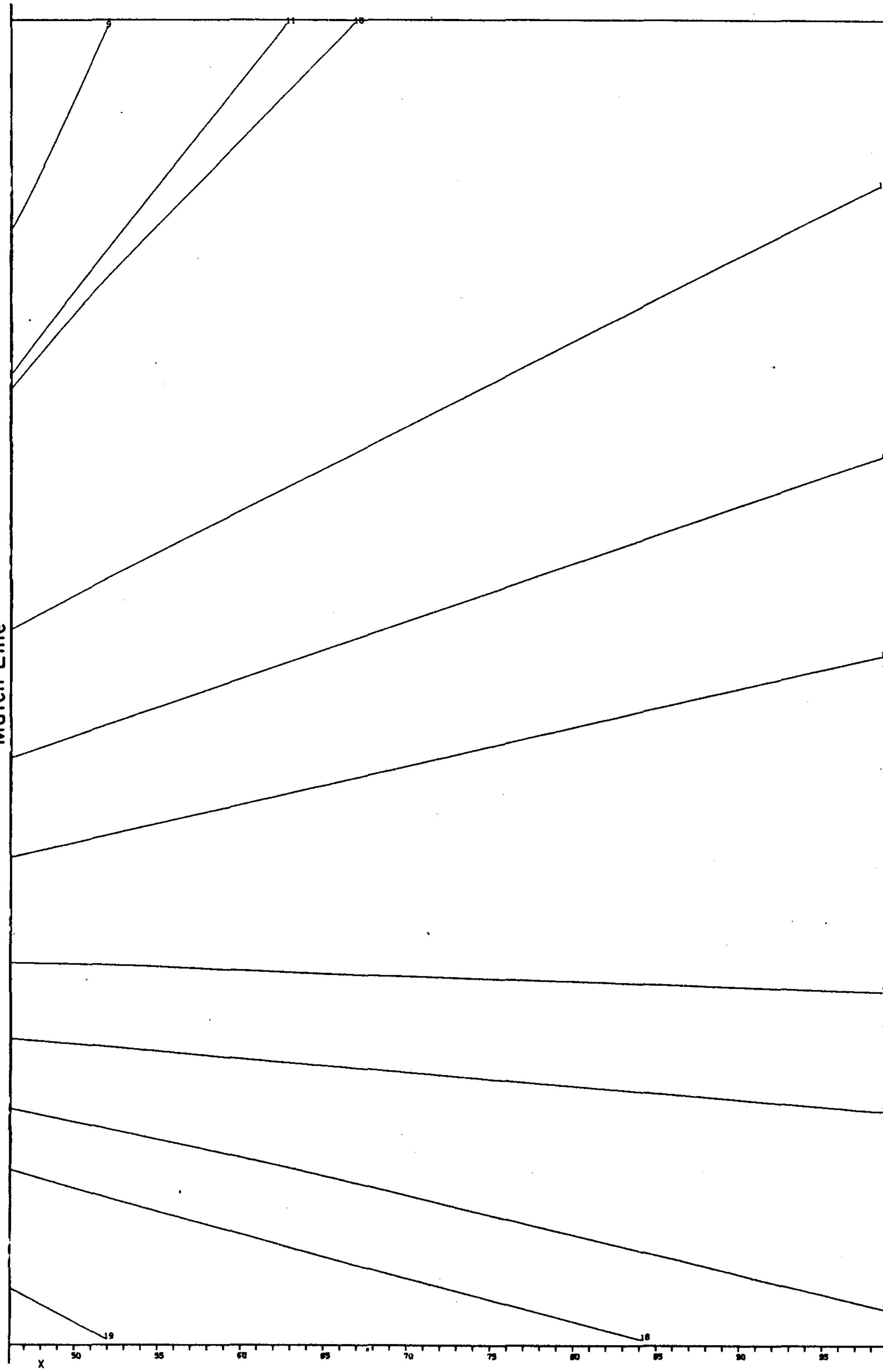


Figure 12. Rays (T = 6 sec) Refracted from a Point on Large Grid.

Match Line



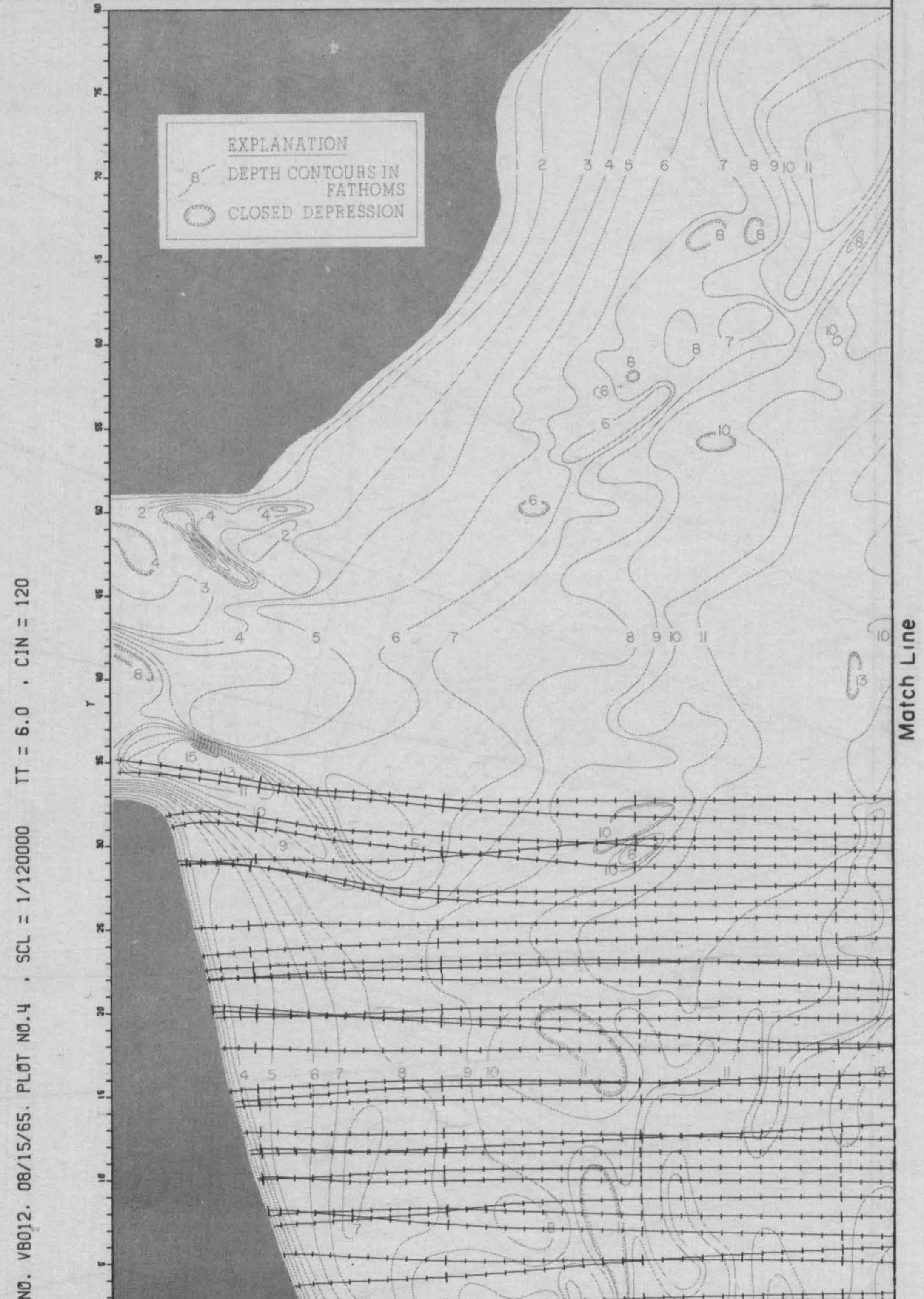
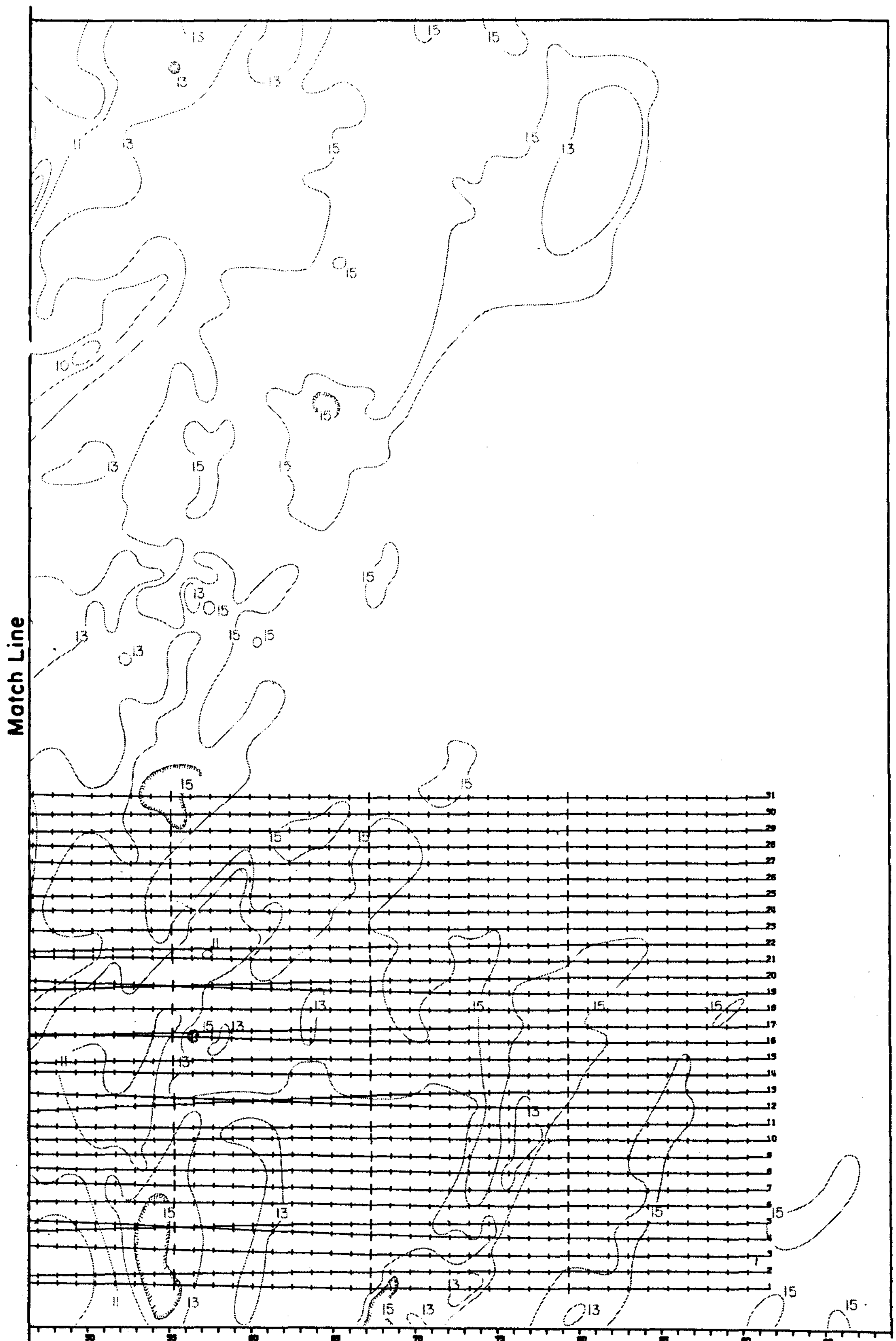


Figure 13. Ray Pattern (T = 6 sec, A = 180°) on Large Grid.


Match Line

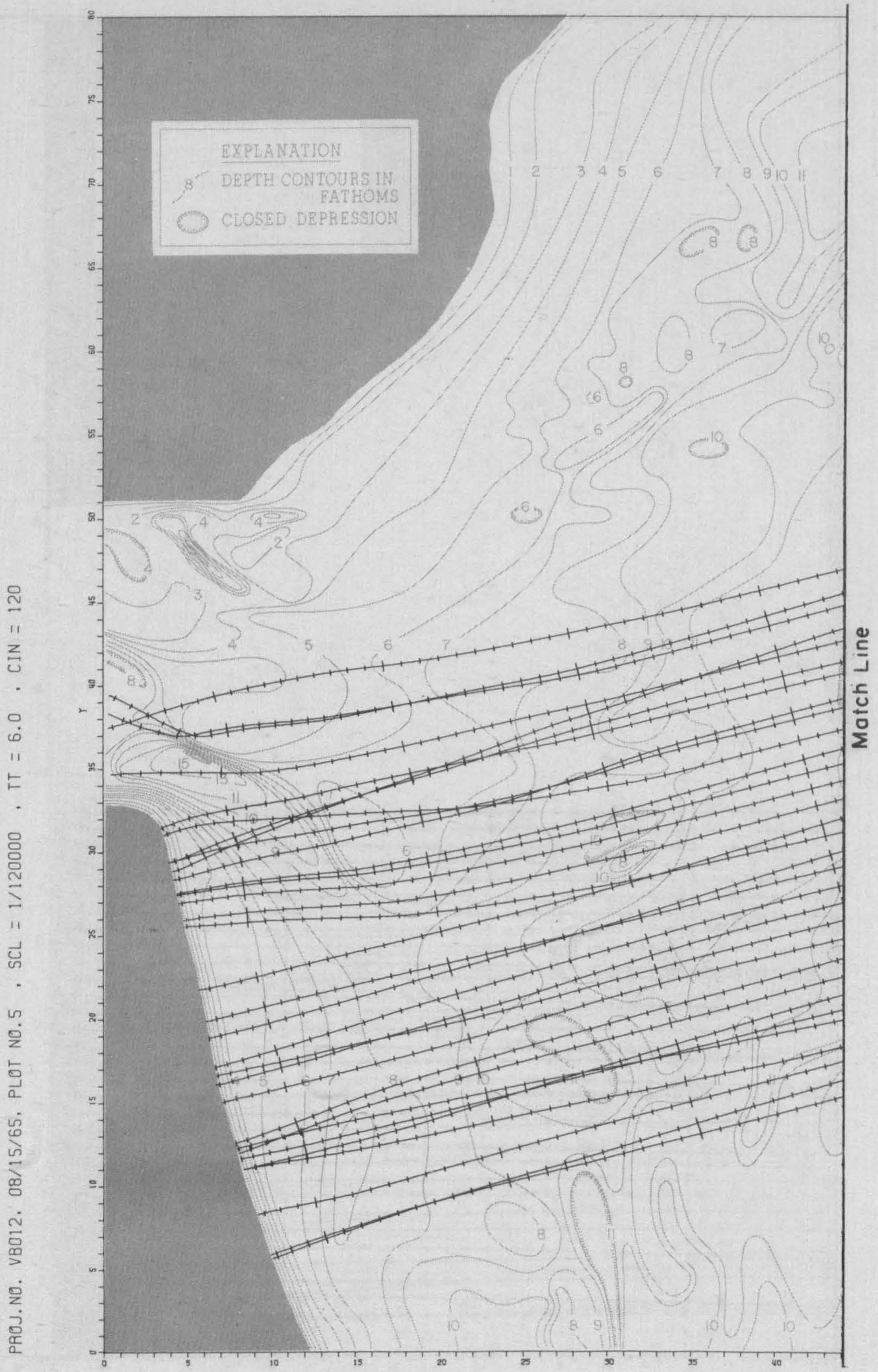
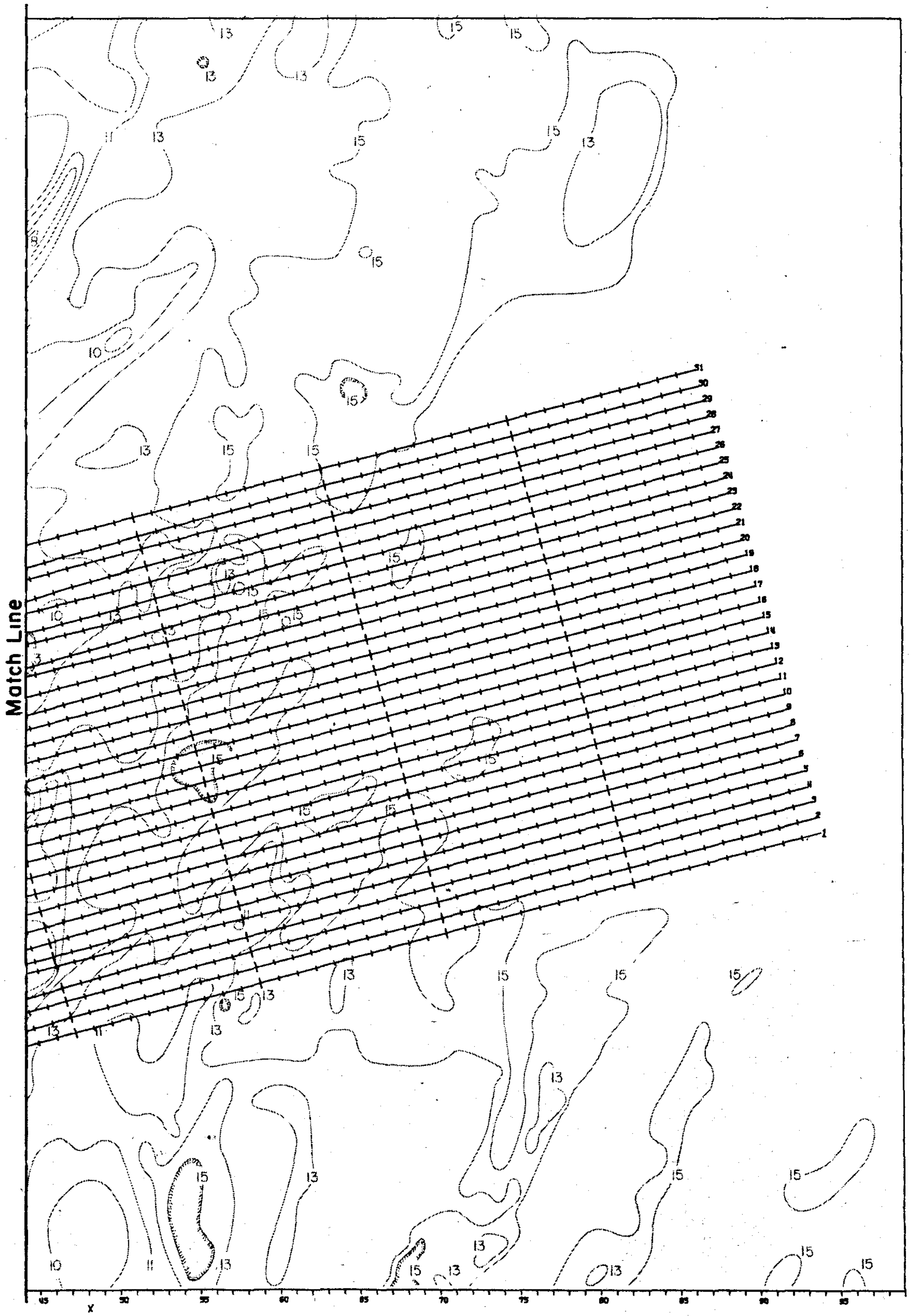


Figure 14. Ray Pattern (T = 6 sec, A = 195°) on Large Grid.


Match Line
45
X
50
55
60
65
70
75
80
85
90
95

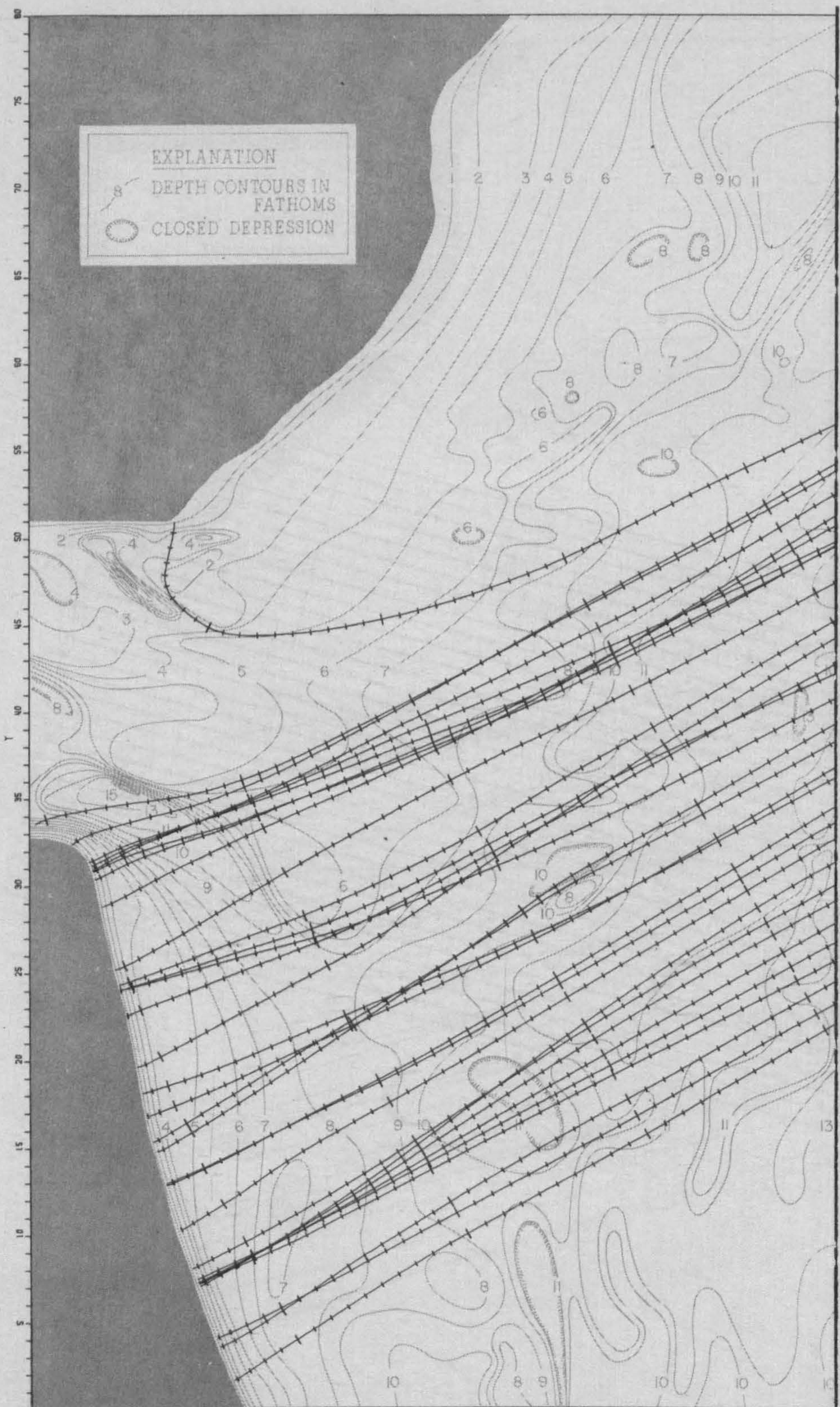
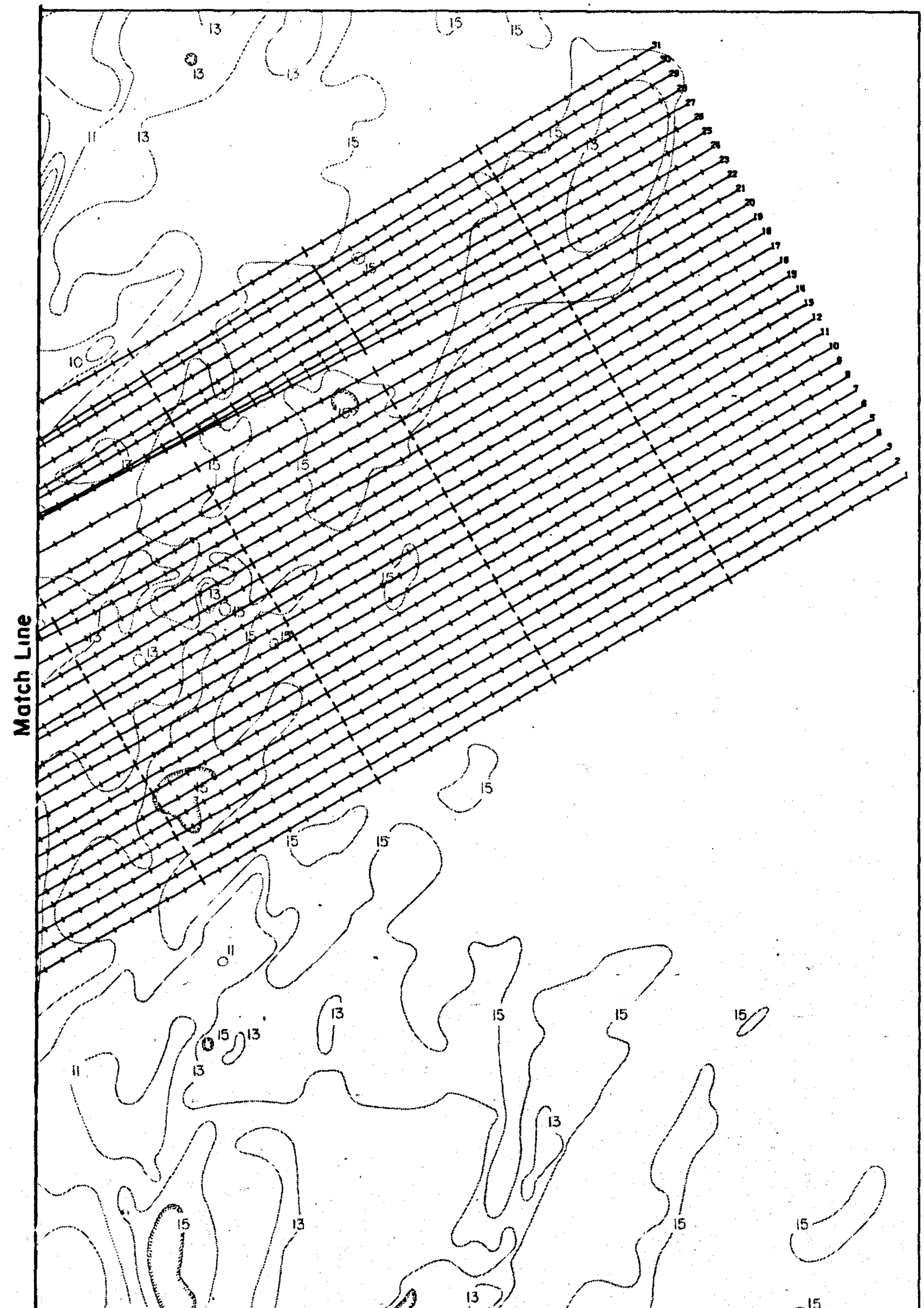


Figure 15. Ray Pattern (T = 6 sec, A = 210°) on Large Grid.


Match Line

U.S. ARMY COASTAL ENGRG. RES. CENTER, CE., WASH. D.C.

A METHOD FOR CALCULATING AND PLOTTING SURFACE WAVE RAYS by W. Stanley Wilson, February 1966, 57 pp., including 15 illustrations

TECHNICAL MEMORANDUM No. 17 UNCLASSIFIED

1. Wave Refraction
2. Virginia Beach, Virginia
3. Computer Programs
4. Plotter Systems

I Wilson, W.S.
II Title

A method using a digital computer and incremental plotter for calculating and plotting wave rays (orthogonals) is described. Given grid of depth values, initial position of wave ray, and direction of travel and period of wave, successive points along ray path are calculated. For each point on path, water depth and bottom slope are estimated from depth grid by linear interpolation; wave speed and curvature computed according to classic theory; and location of next successive point approximated by iteration procedure. Numerical results may be plotted automatically. Example of results, obtained by application of method at Virginia Beach, Virginia, is presented. Unless the bathymetry of area is unusually smooth, this method is faster than manual construction. The computer program is included.

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