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# Large Wave Tank Tests of Riprap Stability

by  
John P. Ahrens

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MAY 1975

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<p>Tests of riprap stability under wave attack were conducted at prototype scale in the large wave tank at the U.S. Army Coastal Engineering Research Center (CERC). Wave heights ranging from 1.4 to 6 feet and wave periods ranging from 2.8 to 11.3 seconds, were used. Three embankment slopes, 1 on 2.5, 1 on 3.5, and 1 on 5, were tested. The riprap stone was a diorite placed on the embankments by dumping. The median stone weights of the riprap ranged from 27 to 120 pounds.</p> <p>The study showed that wave period, because of its influence on breaker characteristics, has a significant effect on riprap stability. Wave conditions which produce the lowest riprap stability are predictable and are associated with a collapsing-type breaker. For the most dangerous wave conditions, the average stability coefficient (<math>K_{RR}</math>) was 2.70 for a 1 on 2.5 slope, 2.36 for a 1 on 3.5 slope, and 2.11 for a 1 on 5 slope.</p>		

## PREFACE

This report is published to assist coastal engineers in the planning, design, and construction of riprap structures. The work was carried out under the coastal construction research program of the U.S. Army Coastal Engineering Research Center (CERC).

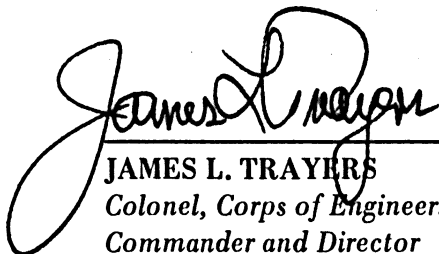
Tests of riprap stability were conducted to generalize and extend the work done at CERC by Thomsen, Wohlt, and Harrison (1972). Scale of the tests was large enough to alleviate scale effects. The many wave periods tested allows stable riprap structures to be designed for a variety of wave climates at slopes ranging from 1 on 2.5 to 1 on 5. The range of wave conditions used and the findings of Thomsen, Wohlt, and Harrison (1972), indicate that the results of this study can be extended to breakwater design. The 1972 study noted that gradation of riprap stone had little influence on stability to wave attack. However, since no wave overtopping occurred during this study and the steepest slope tested was 1 on 2.5, the application to breakwater design may be limited.

This report was prepared by John P. Ahrens, Oceanographer, Special Projects Branch, under the supervision of Dr. Robert M. Sorensen, Chief, Special Projects Branch, Research Division.

Mr. Ralph R. W. Beene, Soils Mechanics Branch, Office, Chief of Engineers (OCE), provided valuable insight on the problems of riprap design. Messrs. Arvid L. Thomsen, Paul E. Wohlt, and Alfred S. Harrison, authors of a CERC Technical Memorandum on riprap stability (TM-37; 1972), assisted this research by their knowledge of riprap design and by their familiarity with testing riprap stability in wave tanks. Extensive contributions were made to this study by Mr. Thorndike Saville, Jr., Mr. Rudolph P. Savage, Dr. Robert M. Sorensen, and Mr. George Simmons, CERC.

Comments on this publication are invited.

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**JAMES L. TRAYERS**  
*Colonel, Corps of Engineers*  
*Commander and Director*

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## SYMBOLS AND DEFINITIONS

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- D.N.F. . . . . Did not fail
- $D_{15}$  (riprap) : Equivalent diameter of stone; 15 percent of the total weight of the armor gradation is contributed by stones of lesser weight (feet)
- $D_{85}$  (filter) . . Equivalent diameter of stone; 85 percent of the total weight of the filter gradation is contributed by stones of lesser weight (feet)
- $D_{50}$  . . . . . Equivalent diameter of stone with median weight in feet =  $(W_{50}/0.658 \gamma^{1/3})$
- d. . . . . Water depth in feet; flat part of wave tank
- H . . . . . Wave height in feet
- $H_{td}$  . . . . . Wave height (feet) associated with limit of tolerable damage
- $H_o$  . . . . . Deepwater wave height (feet)
- $H_{zd}$  . . . . . Zero-damage wave height (feet)
- H (max) . . . . . Maximum wave height in a burst (feet)
- H (FAIL) . . . . . Wave height at which armor failed (feet)
- H (MODAL) . . . . . Modal wave height of a burst (feet)
- $K_{RR}$  . . . . . Stability coefficient for riprap =  $N_s^3 m$
- $K_{\Delta}$  . . . . . Stability coefficient for rubble-mound breakwaters
- L . . . . . Wavelength (feet)
- $L_o$  . . . . . Deepwater wavelength (feet)
- m . . . . . Tangent of slope angle
- $N_s$  . . . . . Stability number =  $H (W_{50} \gamma)^{1/3} (S - 1.0)$
- r . . . . . Average riprap layer thickness (feet)
- R . . . . . Runup (feet)
- S . . . . . Specific gravity of riprap stone; 2.71 for stone used in this study
- SWL . . . . . Stillwater line
- T . . . . . Wave period in seconds
- ~~W~~ . . . . . Individual armor-stone weight (pounds)
- $W_{50}$  . . . . . Median armor-stone weight (pounds); weight of stone where 50 percent of the total weight of the armor gradation is contributed by stones of lesser weight
- $\bar{W}$  . . . . . Average armor-stone weight (pounds)
- $\gamma$  . . . . . Unit weight of riprap stone; 169 pounds per cubic foot for stone used in this study
- $\theta$  . . . . . Angle formed between embankment slope and horizontal,  $\cot \theta = 1/m$

# LARGE WAVE TANK TESTS OF RIPRAP STABILITY

by  
*John P. Ahrens*

## I. INTRODUCTION

This report discusses tests of riprap stability conducted at prototype scale in the 635-foot-long wave tank at the U.S. Army Coastal Engineering Research Center (CERC). The tests were initiated in February 1969 and completed in December 1972.

There has been an increasing need for reliable information on the stability of riprap exposed to wave action. This need arises partly from an increase in the number and size of earth dams which must be protected from wave attack and partly from increased construction in coastal areas.

Dumped quarrystone usually is the cheapest material per ton which can be used for revetments exposed to waves. However, dumped stone has lower stability per pound to wave attack than most concrete armor units. Because of these characteristics, dumped quarrystone can serve as a reference by which the stability and cost of other revetments may be judged.

Factors specifically investigated during this study were the influence of riprap weight, embankment slope, and wave period on riprap stability. The influence of breaker characteristics, riprap layer thickness, and filter stone size on stability were also investigated. In addition to stability based on a "zero-damage" criterion, a greater level of damage which could, in some instances, be tolerated was defined and studied.

## II. BACKGROUND

A previous study conducted at CERC (Thomsen, Wohlt, and Harrison, 1972) indicated that riprap stability was strongly influenced by the wave period. Since riprap can be used to provide bank protection from waves in a variety of circumstances and locations, tests over a wide range of wave conditions were needed. The range of wave conditions used in this study permitted a comprehensive investigation of the influence of the wave period on riprap stability.

The effect of the wave period on riprap stability has been of some concern since the period is not considered in the equations for riprap stability given by Hudson (1958) and by the U.S. Army, Corps of Engineers (1971) in EM 1110-2-2300. Hudson's equation for rubble-mound stability is:

$$W_{50} = \frac{\gamma H^3}{K_{\Delta} (S-1.0)^3 \cot \theta}, \quad (1)$$

and the equation from EM 1110-2-2300 is:

$$\bar{W} = \frac{\gamma H^2}{1.82 (S-1.0)^3 \cot \theta}, \quad (2)$$

where  $H$  is a wave height (normally the zero-damage wave height),  $W_{50}$  is the median stone weight,  $\gamma$  is the unit weight of the stone,  $K_{\Delta}$  is the stability coefficient,  $S$  is the specific gravity of the stone,  $\cot \theta$  is the cotangent of the slope angle, and  $\bar{W}$  is the average weight of the riprap stone needed for stability.

Equations (1) and (2) are similar but differ on whether the square or the cube of the wave height should be used to compute a stable stone weight. One of the purposes of this study was to resolve how the wave height influences the stone stability.

Thomsen, Wohlt, and Harrison (1972) also found that the zero-damage wave heights of tests conducted in small wave tanks had to be increased from 20 to 70 percent to compare stability with similar large-scale tests; these scale effects were attributed to the influence of viscosity. The tests in this study were conducted in CERC's large wave tank primarily to eliminate problems due to scale effects.

### III. TEST SETUP AND PROCEDURES

The large wave tank was used for all of the tests described in this report. Profile views of the tank and test setup are shown in Figure 1. The tank is 635 feet long, 15 feet wide, with a depth of 20 feet. A stillwater depth of 15 feet was used for all tests. Details on the wave tank and generator are given in Coastal Engineering Research Center (1971). Sideboards were placed along a section of the top of the wave tank wall to allow the embankment to be built to a height of about 4 feet above the top of the tank. The sideboards permitted a freeboard for the embankment of about 9 feet above the stillwater level (SWL) which was enough to contain all of the wave runup. The distance between the toe of the embankment and the mean position of the wave generator blade varied from about 390 to 450 feet, depending on the slope of the embankment being tested.

The core of the embankment was compacted bank-run gravel. Enough fine material was available in the bank-run for a small bulldozer to easily grade and compact the embankment to the desired slope. The bank-run was essentially impermeable to wave activity. A layer of filter stone about one-half to two-thirds of a foot thick was placed between the core and the riprap. The size of the filter stone was such that the ratio of the 15 percent finer diameter of the riprap stone,  $D_{15}$  (riprap), to the 85 percent finer diameter of the filter stone,  $D_{85}$  (filter) was usually less than 4 and always less than 5. The size gradations of the core material and filter stone are shown in Figure 2. The relative position of the core, filter, and riprap are shown in Figure 1.

The riprap stone was a diorite with an average specific gravity of 2.71. This diorite generally has a blocky shape (Fig. 3.).

The riprap stone was dumped in the dry by skip onto the filter layer (Figs. 4, 5, and 6) and rearranged slightly by hand to achieve a reasonably uniform thickness. Detailed surveys made of the filter and riprap (discussed later) indicate that the method of placement did not allow the thickness of the riprap layer to be controlled closer than about  $\pm 0.25$  feet from



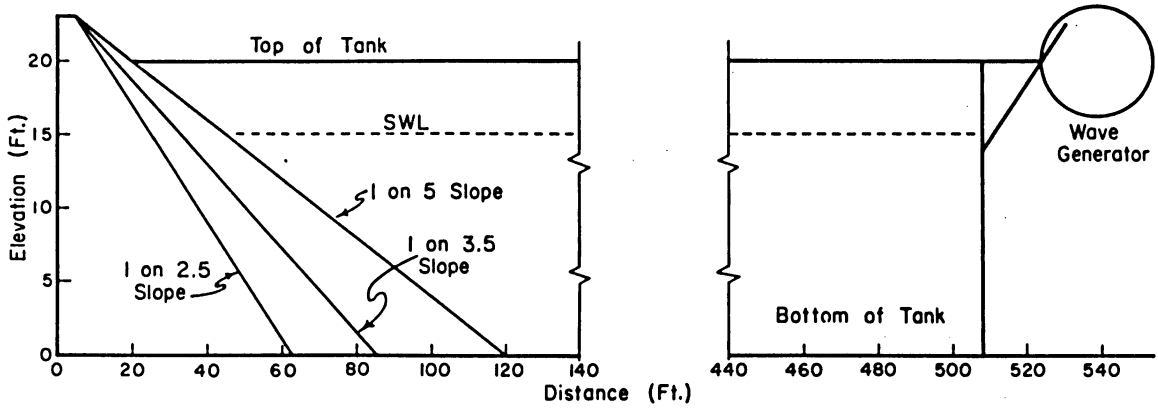
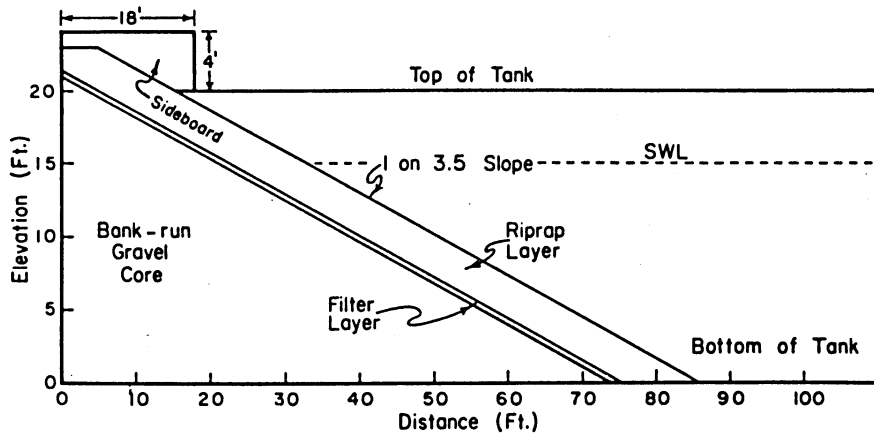


Figure 1. Profile view of wave tank and test setup.

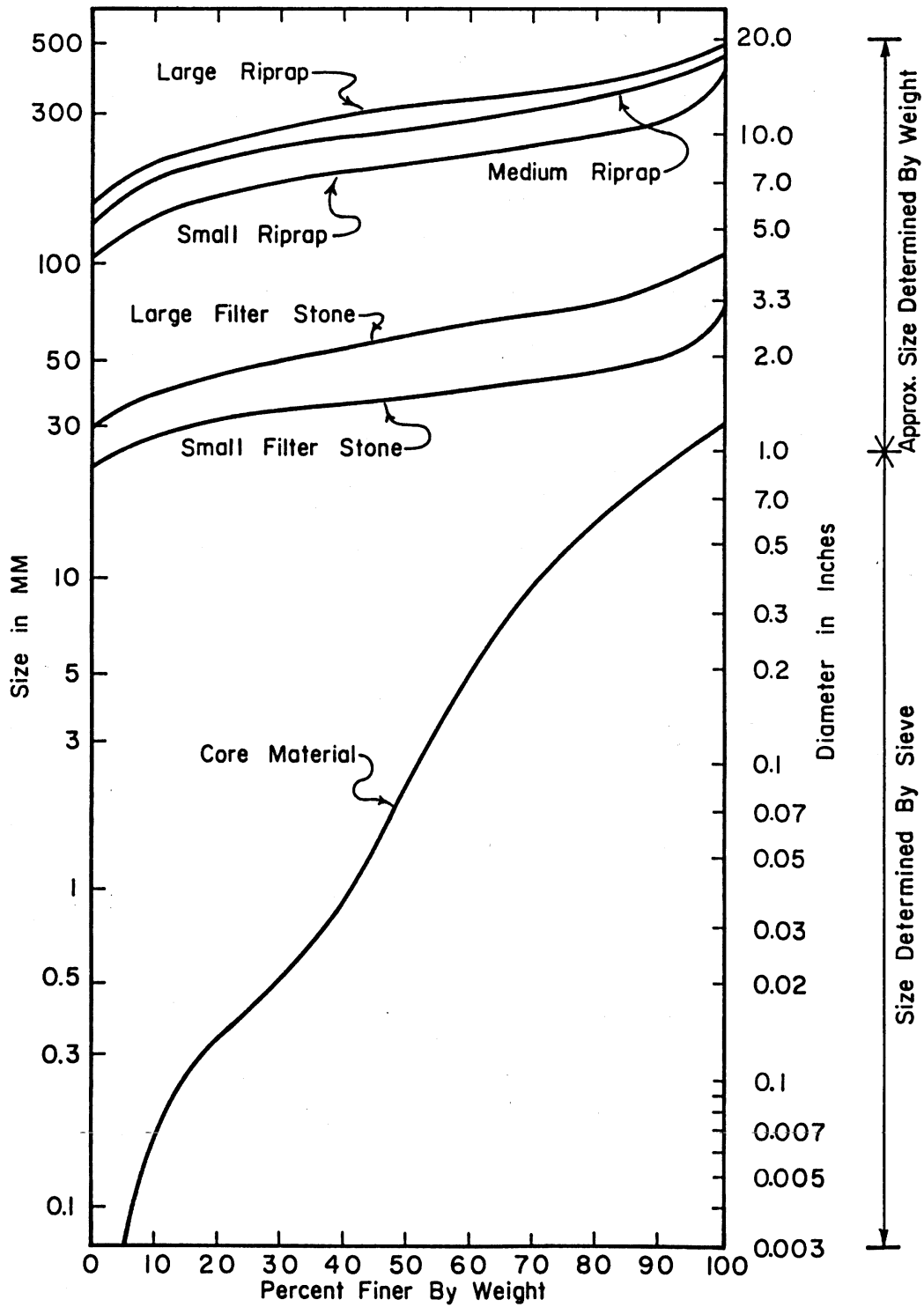


Figure 2. Size gradation of riprap, filter, and core.



Figure 3. Riprap-protected embankment in large wave tank.

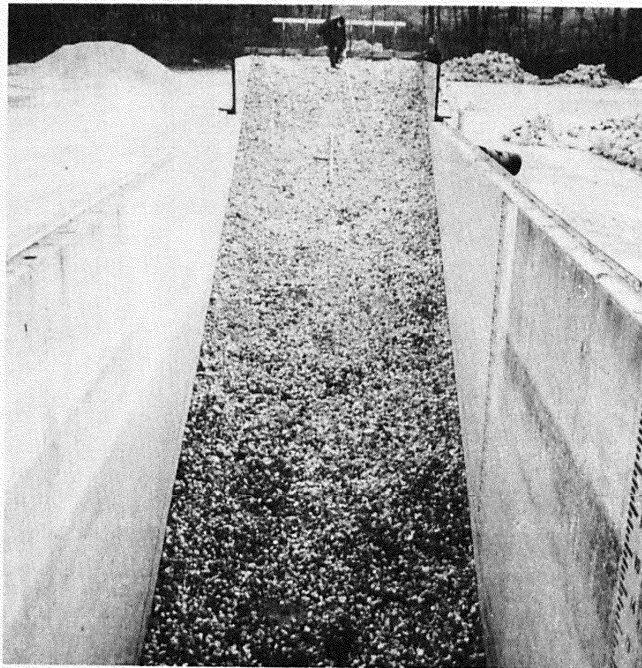


Figure 4. Embankment with filter layer in place.



**Figure 5.** End loader filling skip with riprap.



**Figure 6.** Dumping riprap from skip onto filter layer.

desired thickness. No attempt was made to achieve placement of riprap better than would be expected under good construction practice. Riprap in place before testing is shown in Figure 3.

The porosity or amount of void space in the "undisturbed dumped-state" of the riprap layer was found to range from 33 to 47 percent with an average value of about 40 percent. Porosity was estimated by surveying the riprap layer to determine the volume of a known weight of stone. Since the riprap layer was usually only 1.5- to 2-median-stone-diameters thick, the porosity values are an estimate rather than precise values. The porosity is important because it is needed to estimate the coverage on an embankment that would be expected from a given weight of riprap. In addition, porosity affects the stability of the riprap by its influence on the wave runoff and return flow.

The independent parameters which were varied systematically in this study were: wave height, embankment slope, riprap weight, and wave period. Wave heights ranged from about 1.4 to 6 feet; five wave periods ranging from 2.8 to 11.3 seconds were used. Three embankment slopes, 1 on 2.5, 1 on 3.5, and 1 on 5 were used in the tests, and three stockpiles of riprap with median weights ranging from 27 to 120 pounds were tested.

To determine the influence of riprap weight on stability, the stone was selected from one of three stockpiles or "batches" of riprap. Stockpiles of riprap were sorted by different median weights but with the same relative size gradation. This gradation corresponded closely with that specified on Plate 11 in EM 1110-2-2300 (U.S. Army, Corps of Engineers, 1971). In this gradation the specified maximum stone weight is 4 times the median weight and the minimum weight is one-eighth the median. A typical gradation of a sample from each of the three (small, medium and large) riprap stockpiles is shown in Figure 2. Stone was occasionally added to the stockpiles to replace losses due to breakage that occurred during construction and because of wave action or to obtain enough stone to cover a flatter slope. Breakage and additions of stone to the stockpiles caused some fluctuation in the stockpile median weight. The median weight of one stockpile ranged from 27 to 36 pounds, another from 73 to 78 pounds, while the third remained constant at 120 pounds (no appreciable breakage). The stockpile with the smallest stone suffered the most breakage, probably because the small stones were easily moved about by wave action. Most breakage for this stone occurred on the 1 on 5 slope. When dislodged by waves on the 1-on-5 slope, the small stones would be abraded back and forth over the surface of the riprap by wave action. However, on the 1 on 2.5 and 1 on 3.5 slopes, the dislodged stones rolled down the slope and were removed permanently from the zone of major wave action.

A group of tests with the same slope and riprap from the same stockpile would usually include one test for each of the five wave periods (2.8, 4.2, 5.7, 8.5, and 11.3 seconds). At times, however, no tests were conducted at a wave period of 2.8 seconds if the maximum wave height for this period, about 4 feet, was judged too small to do extensive damage to the riprap. Combinations of slopes and riprap weights tested are shown in Table 1.

Table 1. Test conditions.

Embankment slope	Range of median weights of riprap (stone) stockpiles		
	Small 27 to 36 lbs.	Intermediate 73 to 78 lbs.	Large 120 lbs.
1 on 2.5	tested	tested	untested
1 on 3.5	tested	tested	tested
1 on 5	tested	tested	untested

Replicate tests using the same embankment slope, wave period and riprap weight as in earlier tests, were conducted to obtain an estimate of the reproducibility of the tests. Test conditions repeated were generally those showing low riprap stability so that the worst conditions were given the closest scrutiny.

Wave heights used in this study were determined before construction of the riprap-protected embankment, by collecting wave height data with a wave-absorber beach in the tank. Thus, the wave heights generated at specific periods and stroke settings were free of the influence of reflected waves. The wave heights obtained in this manner were assigned the same wave period and stroke setting during the stability tests.

The slope of the test embankment was steep enough so that an appreciable part of the wave energy striking it was reflected. Wave energy reflected from the slope traveled back and was reflected from the generator blade; if the generator were run continuously, wave energy would travel back to the slope along with the generated waves. To prevent the unpredictable superposition of waves traveling toward the embankment, the generator was run in short bursts with interludes between bursts to allow the wave energy in the tank to dampen out. The burst duration was equal to the time it takes the wave group to travel from the generator to the embankment and back to the generator blade. Burst duration ranged from 103 seconds for the 2.8-second wave period to a duration of 43 seconds for the 11.3-second wave period. The quiet interlude lasted from about 2 minutes for short-period waves to about 4 minutes for long-period waves.

Depending on the depth-to-wavelength ratio, i.e.,  $d/L$ , of the waves in a burst, there are one to three waves noticeably higher than the modal height of the group. This nonuniformity in the wave heights of a burst is commonly called the *last wave effect*, since the largest wave usually occurs near the end of the burst (Madsen, 1970). However, there is another high wave near the beginning of a burst so that the highest waves bracket smaller waves of almost uniform height, considered the modal waves (Fig. 11, in Thomsen, Wohlt, and Harrison, 1972). When the wave height distribution of the bursts was investigated, it was found that the ratio of the largest wave,  $H$  (maximum), divided by the modal wave,  $H$  (modal), was practically constant for a given period. The average value of  $H/H$ , decreased monotonically with period from 1.27 for the 2.8-second wave to 1.04 for the 11.3-second wave.

Frequently, the highest wave or two in the burst caused stone movement. Using the modal wave height to characterize the height of a wave burst would result in an invalid comparison of riprap stability for tests with different wave periods because the modal height is not representative of the effective height for the different wave periods. The effective height is higher than the modal height and less than the maximum wave height of a burst. For this study, the effective burst wave height (effective wave height) was taken as the average of the one-third highest waves in the burst. To designate this effective wave height as the significant wave height would be misleading due to the great difference in height distribution of a burst of waves in the wave tank and the height distribution of waves in nature. The ratio of the effective-to-modal wave height of a burst decreased monotonically from 1.11 for a period of 2.8 seconds to 1.04 for a wave period of 11.3 seconds.

Frequent surveys were taken of the slope surface to document and follow the progress of damage to the riprap. One slope observer was always present to record riprap performance during the tests. Photographs and movies were taken of the riprap during testing to provide documentation and insight into the processes involved in the interaction between the riprap and the waves. Movies included real-time, slow-motion, and time-lapse photography.

After a test was completed, a crane removed the riprap layer exposing the underlying filter layer. Removal of the riprap usually disturbed the filter layer. The disturbed filter layer would be prepared for the next test by raking and, if necessary, adding stone to produce a smooth surface. The filter layer was then surveyed and the riprap replaced. After the initial riprap survey was taken, the first waves were generated and testing started.

The filter and riprap were surveyed with a sounding rod with a 6-inch-diameter circular foot. The foot was attached to the rod by a ball and socket joint. The survey pattern for both filter and riprap was a square grid with grid points 2- by 2-feet apart in the horizontal plane.

The first wave height run on the newly placed riprap was chosen at about 30 percent lower than the height expected to dislodge stones. A standard for riprap stability at a particular height was established, specifying the minimum number of waves required to strike the riprap. The minimum number ranged from 400 waves for the 11.3-second period up to 1,900 waves for the 2.8-second period. At times more waves were required than the minimum to give stability at a particular wave height. When the riprap was judged stable at a wave height, the riprap was resurveyed and the wave generator adjusted to generate waves approximately 10 percent greater than the previous height. These steps were followed during testing:

- (a) Remove riprap from embankment,
- (b) smooth filter stone,
- (c) survey filter layer,
- (d) replace riprap on filter bed,
- (e) initial survey of riprap,

- (f) run first wave height until no net stone movement,
- (g) resurvey riprap,
- (h) increase wave height about 10 percent,
- (i) run waves until no net stone movement,
- (j) repeat steps g, h and i until riprap fails,
- (k) take final riprap survey, and
- (l) return to first step for next test.

#### IV. DEFINITION AND DISCUSSION OF IMPORTANT CONCEPTS AND TERMS.

As the wave height is increased during testing, a height is reached that starts to dislodge stones from the slope face. The dislodged stones will usually roll or be carried down the slope by the return flow of the runup. Figure 7 shows survey profiles from a typical test where wave action has removed a considerable amount of stone from around the stillwater line (erosion zone) and deposited the stone in a pile (accretion zone) starting about 3 feet below the stillwater line. Stones moved by wave action tend to find stable positions that reduce the void space within the riprap layer. The reduction in void space in the accretion zone can be seen in Figure 7 by noting that the area of the erosion zone is greater than the area of the accretion zone.

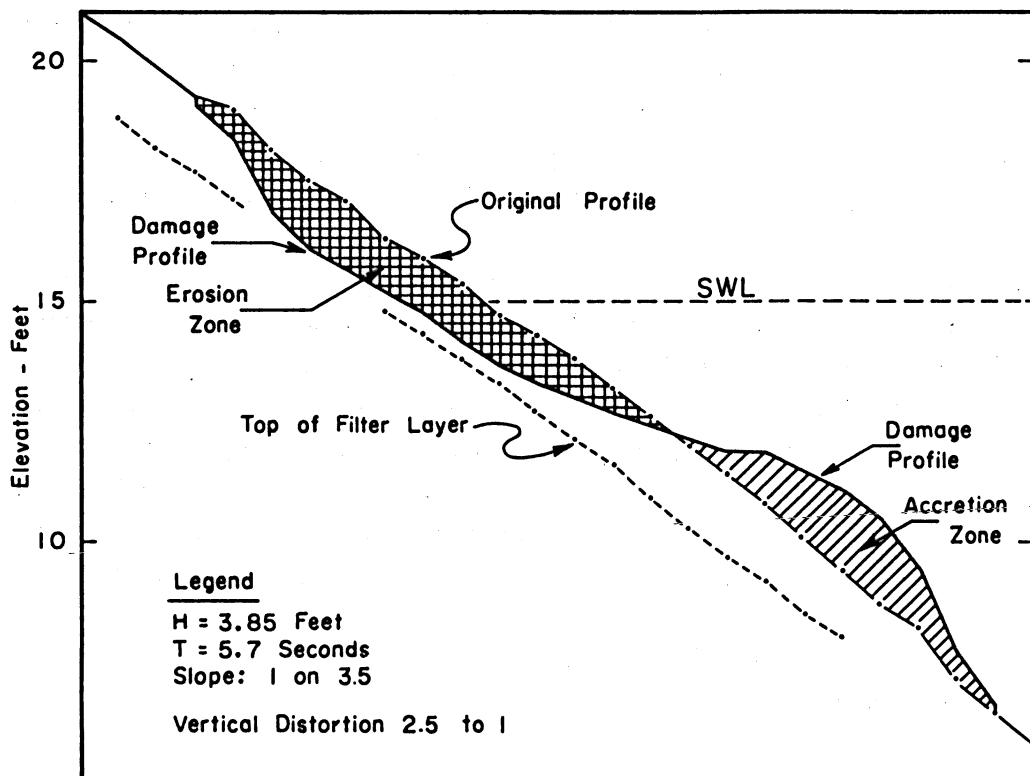


Figure 7. Typical damage profile.



The zero-damage wave height ( $H_{zd}$ ) is the primary measure of riprap stability used in this report. A simple definition is that the zero-damage wave height is the highest wave height for which the riprap is stable. This depends upon a clear definition of *stable* which in the ultimate sense means no movement. In practice some rock movement was accepted below the zero-damage level, since dumping the riprap leaves some stones in unstable positions.

After consideration of visual observations, movies, photographs and surveys of the riprap, it was decided that a reasonable zero-damage wave height could be set at the height that caused a loss to the riprap layer erosion zone of 1.5 cubic feet per foot of tank width. The damage profile shown in Figure 7 is the survey profile of test 23 at failure; the loss of 20 cubic feet per foot from the erosion zone far exceeds the zero-damage level of 1.5 cubic feet per foot. In practice, the zero-damage wave height was estimated by interpolating between the two wave heights which caused volumetric damage bracketing the zero-damage value of 1.5 cubic feet per foot. This loss of 1.5 cubic feet per foot from the erosion zone includes loose stones, some settlement or packing and the normal void space of the riprap layer.

In judging primary stability, two other parameters are used in this report; they are essentially dimensionless versions of the zero-damage wave height. The two parameters are the stability number,  $N_s$ , and the stability coefficient,  $K_{RR}$ . The stability coefficient,  $K_{RR}$ , is similar to the stability coefficient,  $K_{\Delta}$ , shown in equation (1), which is frequently used in evaluating the stability of armor units in rubble-mound breakwaters (Hudson, 1958). The stability number,  $N_s$ , is given by:

$$N_s = \frac{H}{(W_{50}/\gamma)^{1/3} (S-1.0)}, \quad (3)$$

where  $H$ , the wave height, is normally the zero-damage wave height. The relationship between  $K_{RR}$  and  $N_s$  is:

$$K_{RR} = \frac{(N_s)^3}{\cot \theta} \quad (4)$$

The zero-damage wave height, or its dimensionless versions, is used as the measure of primary stability. However, since riprap has the ability to protect the embankment from waves higher than the zero-damage level, other measures of stability (i.e., secondary stability) are useful. The wave height at which the riprap fails is one measure of secondary stability. Failure was "defined" and the test terminated for the 1 on 2.5 and 1 on 3.5 slopes when enough riprap stones were displaced so that the core material could be removed through the filter layer by wave action. For the 1 on 5 slope tests, it was necessary to allow the wave action to do considerably more damage to the riprap than was required on the steeper slopes before core material could be pulled through the filter layer. Some tests for

the 1 on 5 slope had substantially all of the damage occurring below the stillwater line so that visual observations of the erosion area were either difficult or impossible. For these reasons, the decision to stop a test because the riprap "failed" was subjective and consequently, it was necessary to define another measure of secondary stability. Photos of the embankment at failure are shown in Figures 8 and 9.

A more quantitative measure of secondary stability than failure was developed by estimating the wave height,  $H_{td}$ , associated with the "greatest tolerable damage," a condition considered to be less severe than the damage at failure.  $H_{td}$  is the wave height at which failure is imminent because wave action has removed or displaced enough riprap stones in the erosion zone to cause some exposure of filter stone; however, enough shelter is provided by adjacent riprap so that no filter stone is being removed. Waves slightly higher than  $H_{td}$  would be expected to start removing filter stone in places where the riprap protection is thin.  $H_{td}$  was computed from the survey data by considering both the volume of riprap removed from the erosion zone and the severity of penetration of damage into the riprap layer. The Appendix gives details of this computation. The formal method of calculating  $H_{td}$  yielded results consistent with the description of greatest tolerable damage. Conceptually,  $H_{td}$  is similar to the "limited-damage wave height" discussed by Thomsen, Wohlt, and Harrison (1972). An example of the appearance of riprap at a level of damage corresponding approximately with the greatest tolerable damage is shown in Figure 10.

The ratio  $H_{td}/H_{zd}$  is defined as the reserve stability. It is a measure of how much the wave height can exceed the zero-damage wave height and still have the riprap provide adequate protection to the embankment. The percent reserve stability is given by  $(H_{td}/H_{zd} - 1.0) \times 100$ .

## V. EXPERIMENTAL RESULTS

A tabulation of some of the most important parameters and data from this study is given in Table 2. In the table tests are numbered chronologically in the first column and test conditions are given in the next four columns, i.e., embankment slope, median riprap weight, wave period, and the average riprap layer thickness. The sixth through eighth columns give data related to the primary stability of the riprap, i.e.,  $H_{zd}$ ,  $N_s$ , and  $K_{RR}$ . The 9th and 10th columns are measures of secondary stability, i.e., the reserve stability and the failure wave height. The last column gives the runup ratio,  $R/H$ , where  $R$ , the runup, is the average vertical excursion of the wave uprush above the stillwater line, and  $H$  is the observed height of the wave causing the runup. The runup ratio is an average value for the waves having heights equal to or less than the zero-damage wave height. The small range of wave heights between the start of a test and zero-damage was such that the runup ratio varied only slightly. Runup data for waves greater than the zero-damage wave height were collected but not included in the average since the riprap was damaged at wave heights above  $H_{zd}$  and the runup would not be representative of a plane slope.



Figure 8. Riprap at failure, tank dry.



**Figure 9.** Closeup of failure shown in Figure 8.



**Figure 10.** Riprap at a level of damage corresponding to the "greatest tolerable damage."

Table 2. Basic data.

Test No.	Embankment slope	Median riprap weight (lbs.)	Wave period (sec.)	Average riprap layer thickness (ft.)	Zero-damage wave height (ft.)	Stability number $N_s$	Stability coefficient $K_{RR}$	Stability reserve	Failure wave height (ft.)	Average runup ratio, R/H
1	1 on 2.5	28	2.8	1.82	2.58	2.75	8.30	1.36	3.81	0.98
2		28	5.7	1.37	2.12	2.26	4.60	1.10	2.41	1.44
3		28	4.2	1.54	2.05	2.18	4.16	1.22	2.68	1.21
4		28	8.5	1.46	2.42	2.58	6.85	1.20	3.20	1.61
5		28	11.3	1.58	2.66	2.83	9.09	1.24	3.26	1.81
6		28	8.5	0.95	2.17	2.31	4.94	1.06	2.34	1.62
7		28	5.7	1.21	1.98	2.11	3.75	1.13	2.41	1.51
8		28	4.2	1.03	1.82	1.94	2.91	-----	2.03	1.22
9		28	2.8	1.10	2.48	2.64	7.37	1.17	3.00	1.01
10		28	4.2	1.06	1.77	1.88	2.68	1.08	2.03	1.22
11		78	5.7	1.40	2.44	1.85	2.52	1.16	2.97	1.36
12		78	8.5	1.58	3.06	2.32	4.97	1.22	3.84	1.61
13		78	2.8	1.41	3.29	2.49	6.18	1.21	D.N.F.	1.01
14		78	4.2	1.45	2.60	1.97	3.05	1.14	3.16	1.23
15		78	11.3	1.35	3.35	2.54	6.52	1.12	3.97	1.69
16		78	5.7	1.38	2.50	1.89	2.71	1.20	3.13	1.32
17	1 on 3.5	75	2.8	1.37	3.79	2.91	7.01	-----	D.N.F.	0.73
18		75	4.2	1.29	3.32	2.55	4.71	1.31	4.38	1.06
19		75	5.7	1.52	2.62	2.01	2.32	1.26	3.85	1.15
20		75	8.5	1.44	3.02	2.32	3.55	1.12	3.52	1.45
21 <sup>1</sup>		75	11.3	-----	-----	-----	-----	-----	-----	-----
22		75	11.3	1.55	3.31	2.54	4.67	1.19	4.27	1.56
23		75	5.7	1.47	2.76	2.12	2.71	1.23	3.85	1.11
24		75	8.5	1.53	2.94	2.25	3.27	1.21	3.68	1.44
25		75	4.2	1.72	3.11	2.38	3.88	1.30	4.75	0.91
26		75	5.7	1.37	2.48	1.90	1.97	1.14	3.13	1.17
27		27	2.8	0.96	2.82	3.04	8.03	1.18	3.81	0.75
28		27	4.2	1.00	2.10	2.26	3.31	1.25	2.88	1.06
29		27	5.7	0.98	1.88	2.03	2.38	1.16	2.41	1.21
30		27	8.5	0.85	2.10	2.26	3.31	1.09	2.34	1.55
31		27	11.3	0.96	2.52	2.72	5.73	1.10	2.98	1.78
32		120	5.7	1.39	3.39	2.22	3.14	1.27	4.58	1.05
33		120	8.5	1.50	3.18	2.08	2.59	1.24	4.16	1.36
34		120	11.3	1.59	3.90	2.56	4.78	1.19	5.22	1.50
35		120	4.2	1.82	3.76	2.47	4.28	1.34	5.76	0.91
36		120	5.7	1.38	2.96	1.94	2.09	1.28	4.58	1.08
37	120	8.5	1.69	3.14	2.06	2.49	1.26	4.16	1.42	
38	1 on 5	36	5.7	1.42	2.63	2.58	3.42	1.46	4.34	0.98
39		36	8.5	1.33	2.23	2.18	2.08	1.26	3.36	1.33
40		34	2.8	1.25	3.30	3.29	7.15	-----	4.04	0.54
41		33	4.2	1.01	2.68	2.70	3.95	1.31	4.02	0.83
42		32	8.5	1.23	2.24	2.28	2.38	1.21	3.68	1.35
43		31	5.7	1.00	2.60	2.68	3.83	1.20	3.37	1.05
44		31	11.3	1.06	2.54	2.61	3.58	1.42	3.97	1.49
45		73	5.7	1.43	2.86	2.21	2.17	1.60	5.27	0.90
46		73	8.5	1.35	2.72	2.10	1.86	1.27	4.16	1.23
47		73	11.3	1.16	3.04	2.35	2.60	1.36	4.27	1.36
48		73	8.5	1.31	2.86	2.21	2.17	1.30	4.77	1.25
49		73	4.2	1.32	3.44	2.66	3.77	1.39	5.39	0.81
50		73	5.7	1.52	2.94	2.27	2.35	1.56	5.27	0.96

1. Test terminated before zero-damage wave height.

D.N.F. = Did Not Fail

The tests which were repeated to check reproducibility are identified in Table 2; e.g., test 6 is a repeat of test 4, test 7 is a repeat of test 2, etc. Test 42 is considered a replicate of test 39, in spite of the small difference in the median riprap weights, because both used stone from the same riprap batch.

## VI. DISCUSSION OF RESULTS

### 1. Influence of the Riprap Layer Thickness on the Zero-Damage Wave Height.

The reproducibility tests provide a way to evaluate the influence of riprap layer thickness on the primary stability. Since there was a relatively high degree of variability in the riprap layer thickness due to the method of placement, this variability is also present in the original and repeated tests. The method used to determine the influence of the riprap layer thickness on the primary stability is shown in Table 3. The original test, and any repeat tests, form the "reproducibility group" in Table 3. Table 2 shows that tests 1 and 9 form such a reproducibility group, as do tests 2 and 7, etc. For illustration, only 3 of 14 groups are listed in Table 3. Tables 2 and 3 show that the average zero-damage wave height for test 1 is about 2 percent higher than the group average and test 9 is about 2 percent lower than average. Test 1 had a riprap layer about 25 percent greater than the group average; test 9 was about 25 percent less. A scatter plot of the percent deviations of the zero-damage wave heights versus the riprap layer thicknesses for the 14 reproducibility groups is shown in Figure 11. The figure was plotted by using data for all reproducibility groups as shown in the third and fourth columns in Table 3.

There is some positive correlation between thicker riprap layers and higher zero-damage wave heights, but the correlation is not high (Fig. 11). Since the linear correlation for the scatter plot in Figure 11 is about 0.4, it is concluded that the normal variations in the thickness of the riprap layer experienced during these tests had little influence on the zero-damage stability. Thomsen, Wohlt, and Harrison (1972) also concluded that there was little or no influence of the riprap thickness on the zero-damage stability.

The percent deviations in the zero-damage wave heights, shown in the third column of Table 3, can also be used to estimate the reproducibility of the zero-damage wave heights obtained in this study. The standard deviation of the percent deviations of the zero-damage wave height from the reproducibility group average for all groups is 3.64 percent. Assuming the percent deviations in  $H_{zd}$  from the group averages have a normal distribution, a standard deviation of 3.64 percent indicates that the 95 percent confidence limits on the reproducibility of  $H_{zd}$ , for a test in this study, are about  $\pm 7.3$  percent. Confidence limits of  $\pm 7.3$  percent on  $H_{zd}$  imply that the 95 percent confidence limits on the stability coefficient from a single test are about  $\pm 23$  percent because the stability coefficient is a function of the wave height cubed.

### 2. Influence of the Median Riprap Weight and Embankment Slope on Stability.

Figures 12, 13, and 14 show the zero-damage wave height plotted versus the median riprap weight for embankment slopes of 1 on 2.5, 1 on 3.5, and 1 on 5, with the wave

Table 3. Deviation in the zero-damage wave heights and riprap layer thickness for tests conducted to check the reproducibility of testing.

Group	Number	Percentage of deviation from group average	
		$H_{zd}$	Riprap layer thickness
I	1	+1.98	+24.8
I	9	-1.98	-24.8
II	2	+2.05	+ 6.5
II	7	-2.05	- 6.5
III	3	+9.04	+27.1
III	8	-3.19	-14.9
III	10	-5.85	-12.2

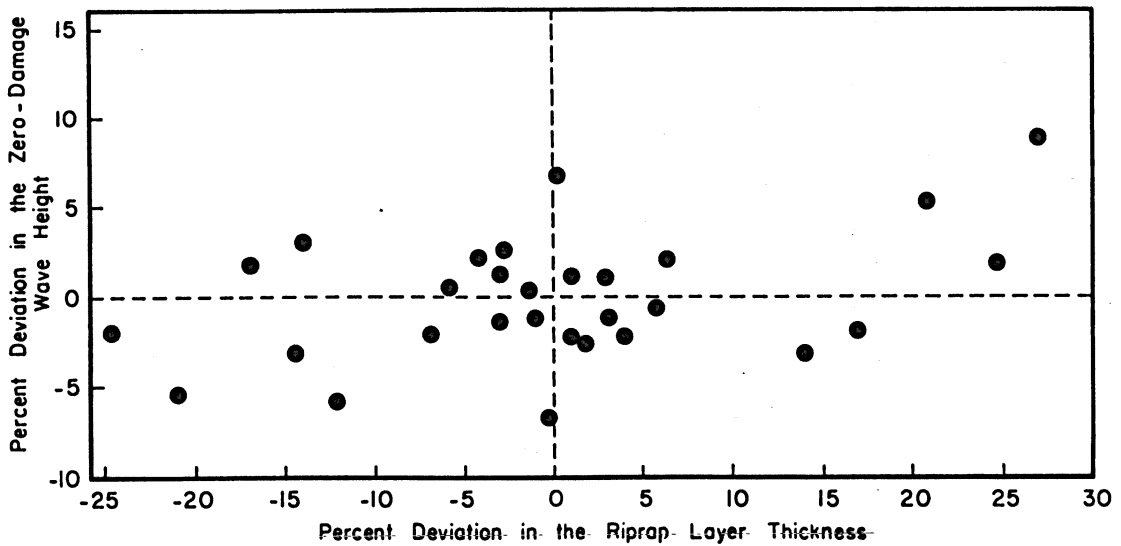


Figure 11. Scatter plot of percent deviations of the zero-damage wave height and the riprap layer thickness.

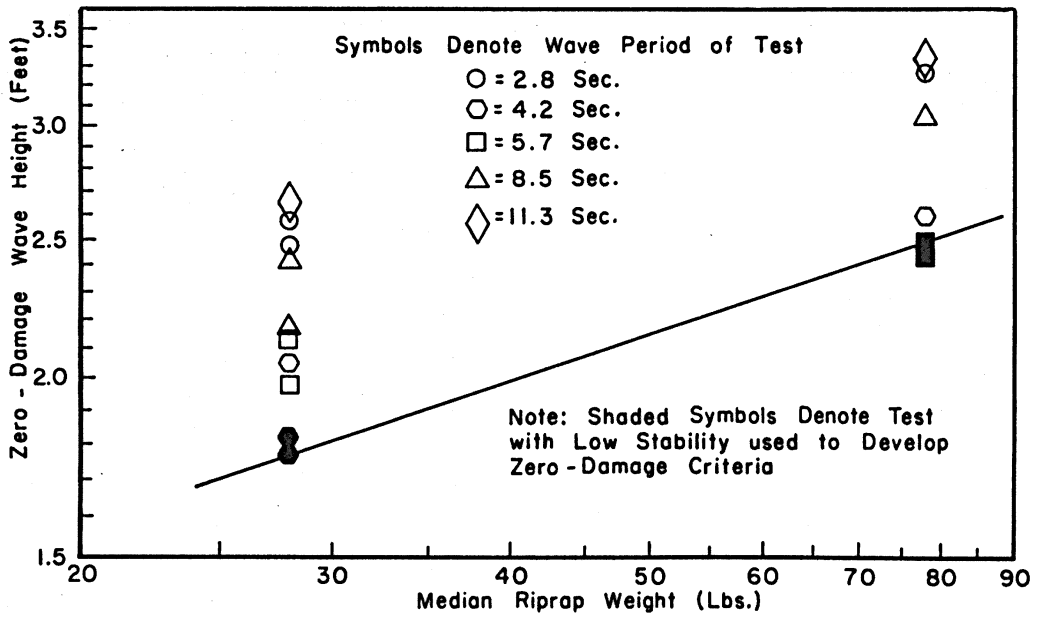


Figure 12. Zero-damage wave height versus median riprap weight for a 1 on 2.5 slope.

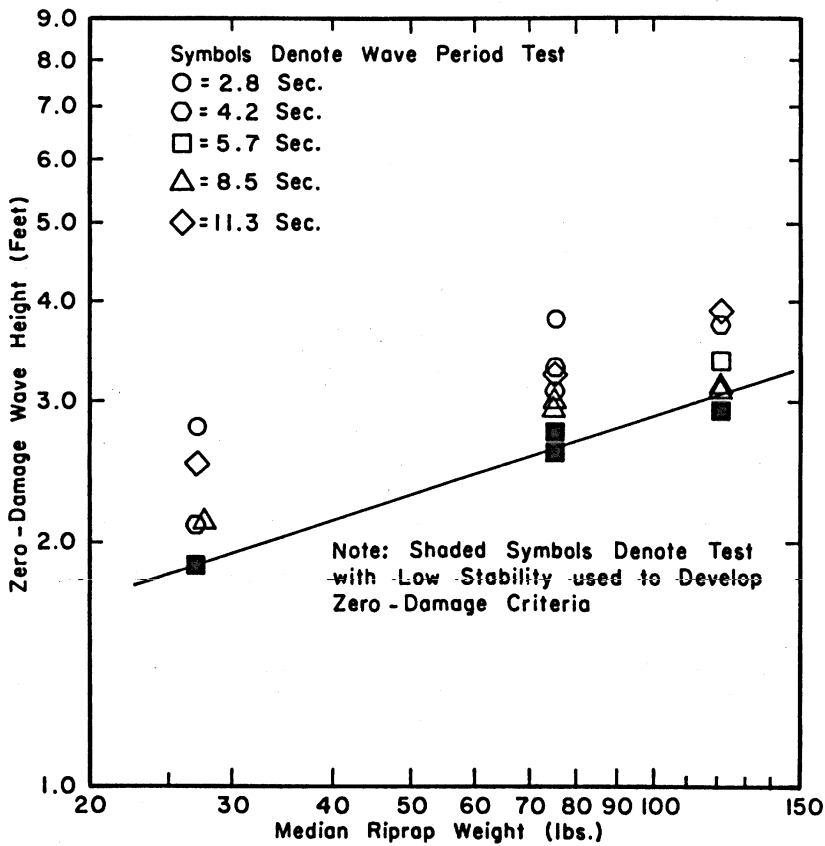


Figure 13. Zero-damage wave height versus median riprap weight for a 1 on 3.5 slope.



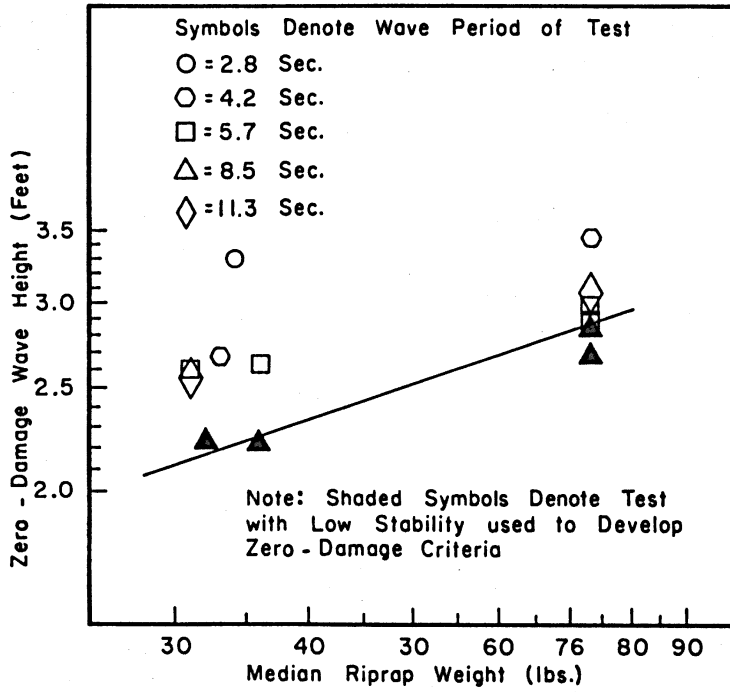


Figure 14. Zero-damage wave height versus median riprap weight for a 1 on 5 slope.

period of each test identified. Tests having the lowest stability within a group of tests using the same riprap stockpile batch and embankment slope are shown by shaded symbols in the figures. In some groups the two lowest stability tests are shaded; the purpose of choosing two tests from some batch-slope groups was to obtain a more balanced sample of data. Each figure shows four shaded symbols which are representative of the tests having low stability on each of the three embankment slopes. Low stability tests are appropriate for use in determining design stability criteria.

A regression line is fitted to the shaded points in each figure (Figs. 12, 13, and 14). The equation of these lines has the form:

$$W_{50} = aH^b, \quad (5)$$

The coefficient  $b$ , the slope of the lines, was chosen to be 3 in order to be consistent with Hudson's (1958) equation. Since the lines follow the trend of the data well for the three embankment slopes, letting  $b = 3$  is reasonable.

Equation (5) can thus be written:

$$W_{50} = aH^3 = \frac{\gamma H^3}{K_{RR} (S-1.0)^3 \cot \theta}, \quad (6)$$

where  $K_{RR}$  is the stability coefficient for riprap. Solving for  $K_{RR}$  in equation (6) yields,

$$K_{RR} = \frac{\gamma}{a (S-1.0)^3 \cot \theta}. \quad (7)$$

Using the values of  $a$ , determined from the regression lines in Figures 12, 13 and 14, a stability coefficient can be associated with each embankment slope; these are 2.70, 2.36, and 2.11 for slopes of 1 on 2.5, 1 on 3.5, and 1 on 5, respectively. As shown in the figures, regression lines are a conservative measure of the data point field. Consequently, the desired stability coefficients are conservative.

This low stability analysis shows that the stability coefficient decreases as the embankment slope decreases. To further investigate the influence of the slope on stability, a plot was prepared. The plot (Fig. 15) shows the stability number,  $N_s$ , of the 12 tests with low stability, i.e., the 4 tests for each slope which are shaded in Figures 12, 13, and 14, plotted versus the cotangent of the embankment slope. These 12 points are also shaded in Figure 15. A regression line fitted to these 12 points has the equation,

$$N_s = 1.55 (\cot \theta)^{0.214} \quad (8)$$

The line in Figure 15 passing through the shaded points has the equation,

$$N_s = 1.54 (\cot \theta)^{2/9} \quad (9)$$

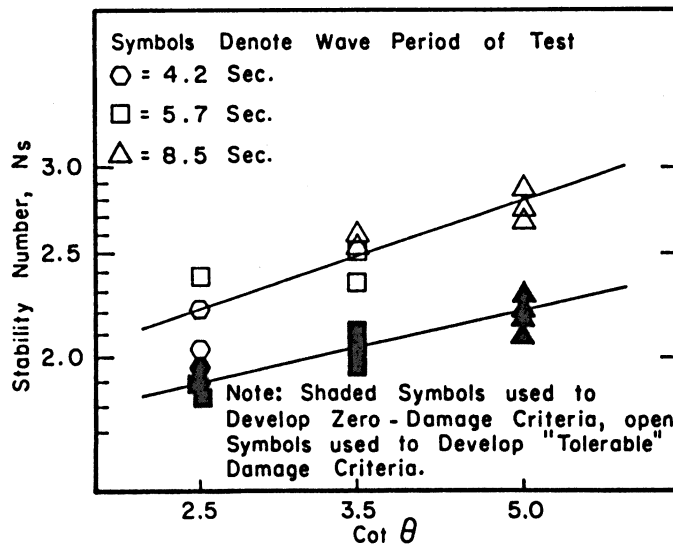


Figure 15. Stability number versus the cotangent of the slope angle.

which is almost identical to the regression line (equation 8) but has a simpler form. The tests having low stability, as measured by their zero-damage wave height, suggest a modification of Hudson's (1958) equation by using equation (9). Cubing equation (9) and rearranging terms yields:

$$W_{50} = \frac{H_{zd}^3}{(3.62) (S-1.0)^3 (\cot \theta)^{2/3}} \quad (10)$$

Equation (10) shows that the slope has less influence on the stability, at least for the worst wave conditions, than would be expected from Hudson's equation (1).

Thomsen, Wohlt, and Harrison (1972) noted an opposite affect of the slope on stability, finding that the stability of the riprap tended to increase more than would be expected from Hudson's formula as the slope decreased. The differing results from this study and that of Thomsen, Wohlt, and Harrison, may be due to the influence of the breaker type on stability (discussed below).

### 3. Influence of Wave Period, Shoaling, Breaker Height, and Breaker Type on Stability:

The tests indicated that breaker type had a strong influence on the riprap stability. The wave conditions that were transitional between plunging and surging yielded the lowest stability coefficients. These waves have been called *collapsing breakers* by Galvin (1968). The conditions which produce collapsing breakers can be predicted using Galvin's "offshore breaker parameter,"  $H_o/L_o m^2$ , where  $H_o$  is the unrefracted deepwater wave height,  $L_o$  is the deepwater wavelength, and  $m$  is the tangent of the angle formed by the embankment face and the horizontal.

The stability coefficient calculated from the zero-damage wave height for each test plotted versus the corresponding value of the offshore breaker parameter is shown in Figure 16. Generally, the tests with the lowest stability fall between values of 0.095 and 0.250 for the offshore breaker parameter, which roughly corresponds with the collapsing breaker condition on the riprapped slope. For values of the offshore breaker parameter greater than 0.30 the wave would plunge cleanly on the riprapped slope, and for values less than about 0.080 the wave would surge up the embankment without breaking. Values of the offshore breaker parameter—given here to provide boundaries between the various breaker types—are not the same as those by Galvin (1968), but were determined to be consistent with the conditions observed in this study. Galvin considered a value of 0.09 of the offshore breaker parameter as the transition value between collapsing and plunging breakers; for this study the transition value for the two breaker types was about 0.30. The large variation in transition values is probably due to differences in the roughness and porosity of the slopes; Galvin used a smooth, impermeable slope. He also used flatter slopes and calculated the wave heights for the breaker parameter from linear-wave generator theory.

A large variability in riprap stability for tests in the plunging wave region is indicated by Figure 16. As shown by the symbol shape in the figure, the variability is related to wave period with shorter-period waves having higher stability coefficients. The plot in Figure 16 suggests that shoaling of the waves greatly influences the riprap stability for plunging waves. Whether shoaling affects the stability for breaker conditions other than plunging is unclear.

The dependence of riprap stability on shoaling is demonstrated in Figure 17. This figure shows the stability coefficient (calculated using observed zero-damage breaker heights) plotted versus the offshore breaker parameter. Since breaker heights were measured frequently for plunging wave conditions, estimates of the zero-damage breaker height are judged reliable. Stability coefficients obtained by using the zero-damage breaker height range from 6.5 to 11.4, and average 9.2 for the 16 tests with plunging breakers. The method of analysis used in Figure 17 has reduced the scatter related to stability to plunging breakers as shown in Figure 16. Evidently part of the variability in stability for tests with plunging waves shown in Figure 16 is due to differences in shoaling characteristics for waves of different periods. Average values of the shoaling ratio of plunging waves during these tests is shown in Table 4. The shoaling ratio is the ratio of the breaker height divided by the wave height just offshore of the toe of the embankment in the flat part of the wave tank.

Thomsen, Wohlt, and Harrison (1972) conducted most of their riprap stability tests in the 635-foot-long wave tank with a wave period of 3.67 seconds and a stillwater depth of 15 feet. A 3.67-second period should produce collapsing breakers on a 1 on 2 slope. Possibly, Thomsen, Wohlt, and Harrison encountered the most damaging wave conditions on the 1 on 2 slope, while their tests on the flatter slopes of 1 on 3 and 1 on 5 resulted in plunging breakers which are inherently less damaging to the riprap. The transition in breaker

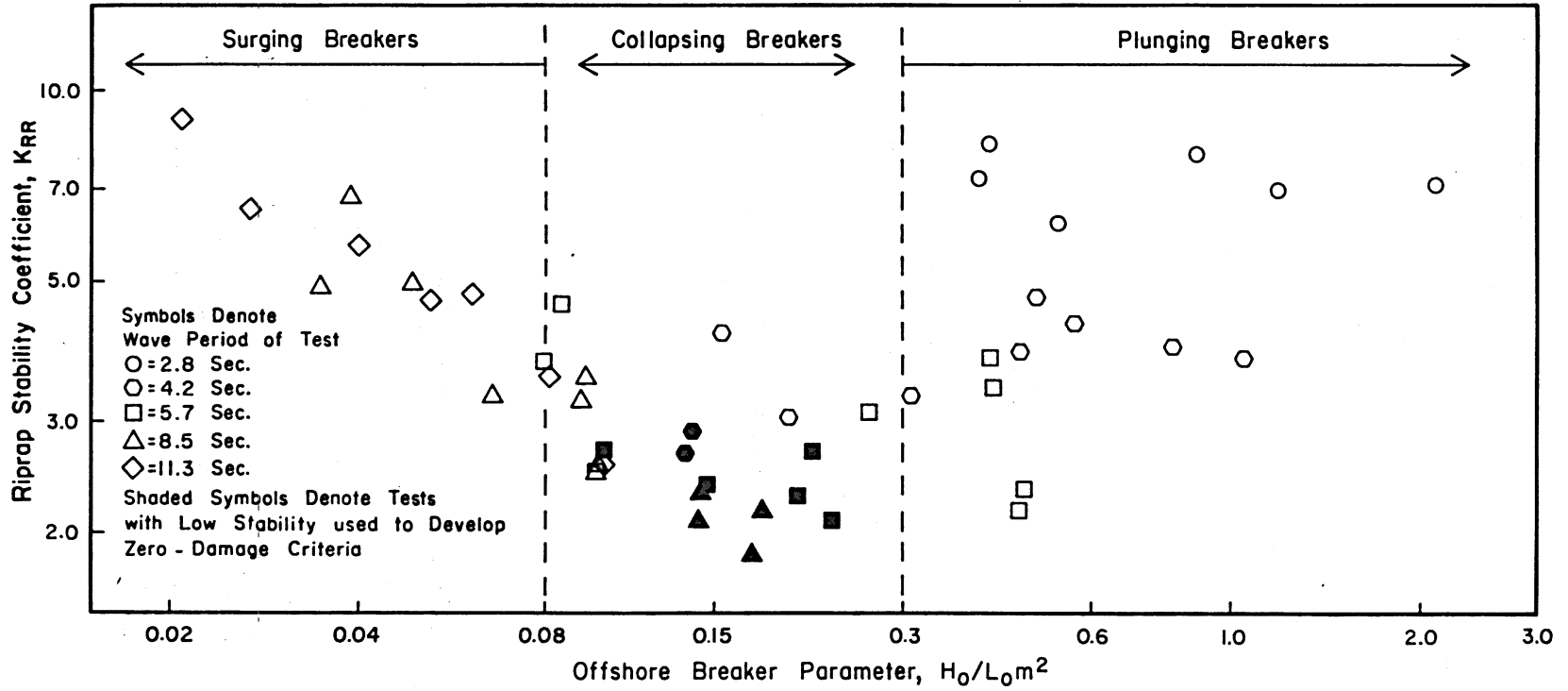


Figure 16. Stability coefficient versus the offshore breaker parameter.

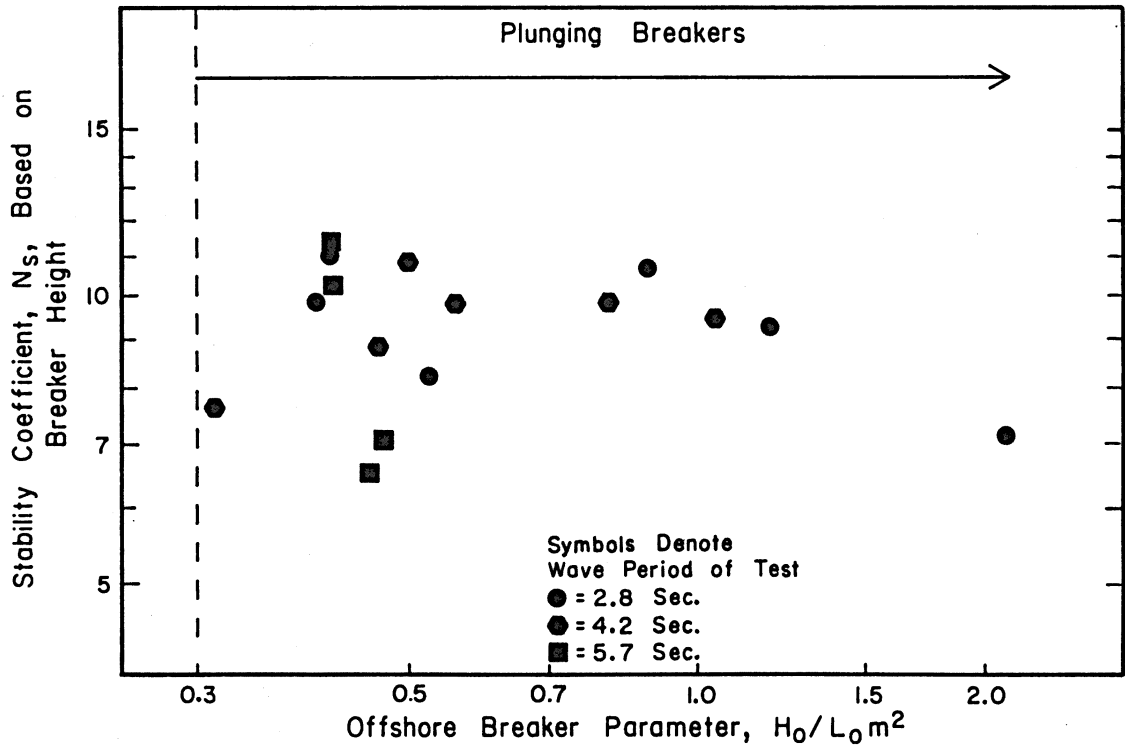


Figure 17. Stability coefficient, based on breaker height, versus offshore breaker parameter for tests with plunging breakers.

Table 4. Shoaling ratio for plunging breakers.

Wave period (seconds)	Slopes 1 on 2.5	Slopes 1 on 3.5	Slopes 1 on 5
2.8	1.10	1.10	1.00
4.2	none observed	1.32	1.36
5.7	none observed	none observed	1.44

Note: Plunging breakers were not observed for wave periods of 8.5 and 11.3 seconds.

characteristics that occurred because of changes in slope for a fixed wave period may account for the unexpectedly large increase in stability with flatter slopes noted by Thomsen, Wohlt, and Harrison.

The embankment slope influences the stability of riprap in two ways: (a) the flatter the slope the greater the inherent stability of the riprap due to the decrease in the component of gravitational attraction along the slope, and (b), the slope, along with wave height and period, water depth and roughness and porosity of the riprap, affects the breaker characteristics which in turn affects the riprap stability.

In tests with long-period waves (8.5 and 11.3 seconds) on the 1 on 5 slope, the waves built a berm with riprap. Figure 7 shows a typical damage profile and Figure 18 shows a berm-type damage profile (test 42 at failure). Tests 39, 44, 46, 47, and 48 also had berm-type damage profiles. Hedar (1960) shows a damage profile with a berm, but this is the first known profile of this type observed at prototype scale.

#### 4. Secondary Stability of Dumped Riprap.

Dumped-stone riprap can provide protection to an embankment beyond the zero-damage level. Because it is impractical to design structures that will never be damaged, the economic importance of reserve stability is considerable. Two major factors affecting the reserve

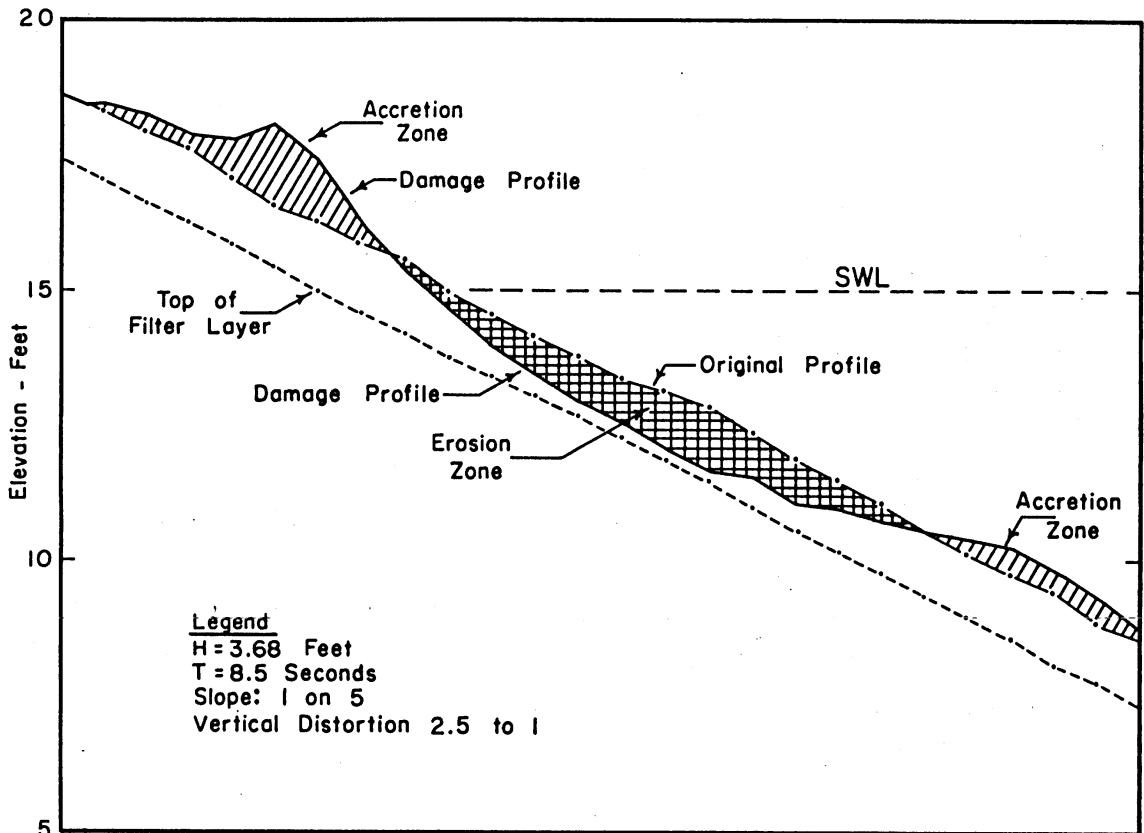


Figure 18. Typical berm-type damage profile.

stability of dumped-stone riprap are the embankment slope and the thickness of the riprap layer. This effect would be expected since the amount of stone available to protect the embankment from wave attack is proportional to the slope and the riprap thickness. Figure 19 shows the percent reserve stability plotted versus the product of the riprap layer thickness and the cotangent of the slope angle, i.e.,  $r \cot \theta$ . The figure shows that the reserve stability generally increases as the riprap layer gets thicker and the slope flatter ( $r \cot \theta$  increases); however, there is considerable scatter in this trend.

Since tests with a riprap layer thickness of between 1.5 and 2 median-stone diameters are probably the most typical of normal revetment design, the reserve stability of these tests was given special attention. The median-stone diameter is given by:

$$D_{50} = \left( \frac{W_{50}}{0.65\gamma} \right)^{1/3} = 1.15 \left( \frac{W_{50}}{\gamma} \right)^{1/3} \quad (11)$$

$W_{50}/\gamma$  is the volume of a stone of median weight, the cube root of which gives a length that is converted approximately to the sieve diameter by the factor 1.15. This relationship was developed empirically by Thomsen, Wohlt, and Harrison (1972). Tests with a riprap layer thickness between 1.5 and 2.0 median-stone diameters, i.e.,  $1.5 \leq r/D_{50} \leq 2.0$ , are referred to as "qualifying" tests. The data related to reserve stability for the qualifying tests is summarized in Table 5 and shows that the reserve stability tends to increase as the embankment slope decreases for riprap with approximately the same thickness.

**Table 5. Summary of data related to reserve stability for tests qualifying as typical with a riprap layer thickness between 1.5 and 2 median stone diameters.**

Slope	No. of tests	Average, $r/D_{50}$	Average reserve stability, $H_{td}/H_{zd}$
1 on 2.5	9	1.66	1.16
1 on 3.5	13	1.67	1.22
1 on 5.0	10	1.64	1.35

Another method of demonstrating the secondary stability is shown in Figure 15 where stability numbers calculated using the wave height associated with the limit of tolerable damage (unshaded symbols) are plotted versus the cotangent of the slope angle. The secondary stability is dependent on the riprap thickness; however, for the data shown in Figure 15 this influence has been mitigated by only considering data from the qualifying tests. Twelve tests were used in the analysis of reserve stability: tests 7, 10, 14, 16, 19, 20, 29, 37, 39, 42, 46, and 48. These tests were chosen because they qualified on the basis of riprap layer thickness, had low reserve stability, and represented a balanced sample of tests. Seven basic conditions of slope and riprap size were tested (Table 1) and at least one test from each condition was used to give a total of four tests for each slope.



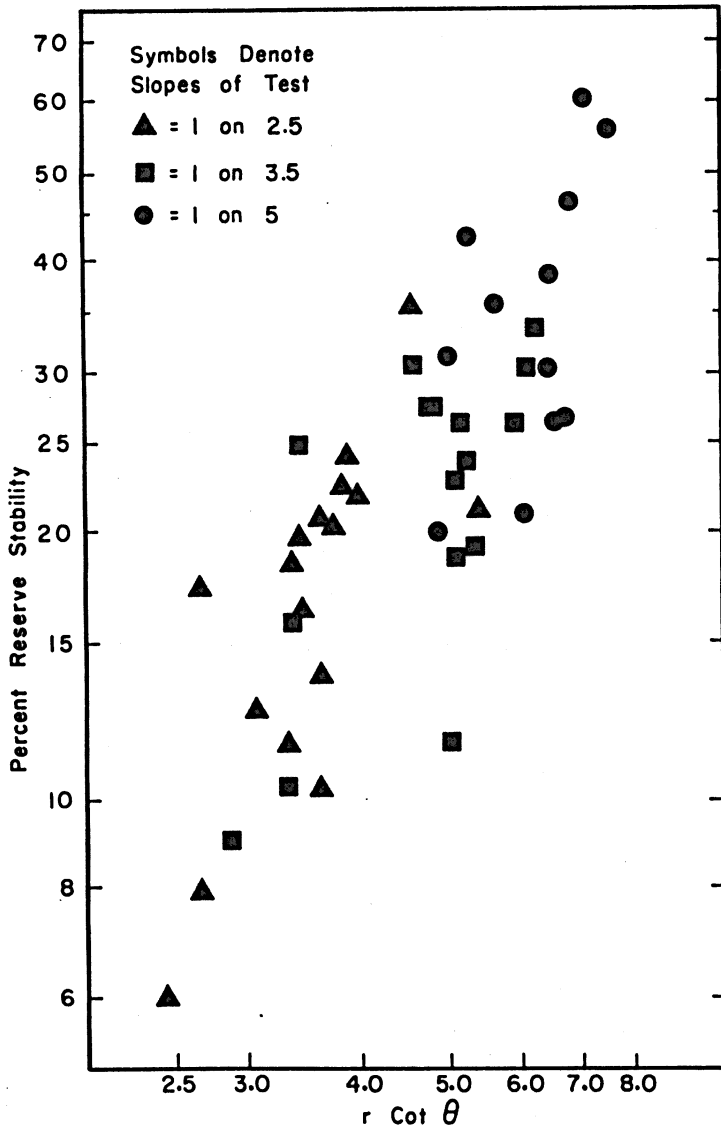


Figure 19. Percent reserve stability versus the product of the riprap layer thickness and the cotangent of the slope angle.

A regression line fitted to the 12 unshaded points representing low secondary stability (Fig. 15) has the equation,

$$N_s = 1.67 (\cot \theta)^{0.32} . \quad (12)$$

The equation of the line through the unshaded points in Figure 15 is

$$N_s = 1.64 (\cot \theta)^{1/3} , \quad (13)$$

which is almost identical to equation (12), but has a simpler form. The two lines in Figure 15 provide a reasonably conservative guide to the primary and secondary stability of the riprap. Equation (13), of the line demonstrating secondary stability, is consistent with Hudson's (1958) equation for a stability coefficient of 4.37. Hudson's equation can be used for riprap design using a stability coefficient of 4.37 for situations where some wave damage can be tolerated. It also indicates in Figure 15 that the zero-damage stability is less dependent on the embankment slope than the stability associated with the limit of tolerable damage.

#### 5. Influence of the Filter Layer on Stability.

The size of the filter stone used was such that the ratio of the 15 percent finer size of the riprap to the 85 percent finer size of the filter, i.e.,  $D_{15}$  (riprap)/ $D_{85}$  (filter), was always less than 5 and except for four tests it was always less than 4. In the four tests where the ratio  $D_{15}/D_{85}$  fell between 4 and 5, the filter size was considered to be only marginally satisfactory. The four tests were 11, 12, 13, and 26.

The impression from observing tests 11, 12, and 26 was that the smaller filter stone did reduce the stability slightly. For example, test 11 has a lower zero-damage wave height than the repeated version, test 16, and test 26 has a lower zero-damage wave height than similar tests 19 and 23. However, the differences between these zero-damage wave heights may not be significant.

During tests 11, 12, and 26 some filter stone was removed through the riprap before wave action had done substantial damage to the riprap layer. These were the only tests where filter stone was removed through an intact riprap layer. This loss of filter reduced the stability of the riprap stones. No filter loss was noted during test 13 and there appeared to be no loss of stability due to the small filter stone used for this test. The short-wave period of 2.8 seconds used for test 13 may account for the better performance of the filter for this test, as compared to tests 11, 12, and 26 which had wave periods of 5.7, 8.5, and 5.7 seconds, respectively.

In summary, it appears that the riprap filter ratio,  $D_{15}/D_{85}$ , of between 4 and 5 is only marginally satisfactory. Possibly the smaller filter is satisfactory for the shorter-period waves but the longer-period waves tend to pull filter stone out through the riprap.

## 6. Comparison of Riprap Stability from Various Sources.

There are several formulas or procedures used in the design of riprapped embankments exposed to waves. For comparison, this study and the following sources are considered: Hudson and Jackson (1962), EM 1110-2-2300 in U.S. Army, Corps of Engineers (1971), Harrison (1974), and U.S. Army, Corps of Engineers, Coastal Engineering Research Center (1973). All of these sources have adopted a stability equation having the general form,

$$W_{50} = \frac{\gamma H^a}{K (S-1.0)^3 (\cot \theta)^b} \quad (14)$$

which originated with the work of Hudson (1958) on rubble-mound breakwaters. In equation (14), a, b, and K are empirically determined coefficients. Variations in equation (14) used by the above sources are summarized in Table 6. The average riprap weight normally used in the EM 1110-2-2300 formula (U.S. Army, Corps of Engineers, 1971) was converted to median weight by multiplying by the factor 4/3 for consistency with the other stability formulas which use the median weight rather than the average weight. The approximate relationship,

$$W_{50} \cong \frac{4 \bar{W}}{3},$$

was found empirically from numerous stone samples selected from the riprap batches during this study and is considered representative of the gradation in EM 1110-2-2300 (U.S. Army, Corps of Engineers, 1971), commonly referred to as EM.

Table 6. Variations in the general form of the stability equation.

Source	Coefficients			Remarks
	a	b	K	
Hudson and Jackson (1962)	3	1	2.2	K = 2.2 for breaking waves (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1973).
	3	1	2.5	K = 2.5 for nonbreaking waves (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1973).
Thomsen, Wohlt, and Harrison (1972)	3	0	14	K = 14 for a slope of 1 on 2
	3	0	18	K = 18 for a slope of 1 on 2.5
	3	0	27	K = 27 for a slope of 1 on 3
	3	0	51	K = 51 for a slope of 1 on 5
EM 1110-2-2300 U.S. Army Corps of Engineers (1971)	2	1	1.36	
This study	3	2/3	3.62	Average zero-damage level for the worst wave conditions (Equation 10).
This study	3	1	4.37	Average limit of tolerable damage for the worst wave conditions.

A graphical comparison of riprap stability as given by the sources in Table 6 is shown in Figures 20 and 21. The figures show the median riprap weight versus the embankment slope for a given significant wave height of 5 and 10 feet.

To plot Figures 20 and 21 the unit weight of the stone,  $\gamma$  in equation (14), was assumed to be 169 pounds per cubic foot and the specific gravity,  $S$ , was 2.71; these values are characteristic of the stone used in this study.

For the stability equation of Thomsen, Wohlt, and Harrison (1972) in Figures 20 and 21 the wave height,  $H$ , in equation (14) has been set equal to 1.4 times the design significant wave height. Normally the significant wave height would be used for the design of a rubble structure (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1973); however, this procedure is inappropriate for Thomsen, Wohlt, and Harrison, since they used the maximum height of a wave burst to judge stability. Increasing the design significant wave by a factor of 1.4 is suggested by Harrison (1974) to adapt the results given by Thomsen, Wohlt, and Harrison for design purposes.

In Figures 20 and 21 the curves given by Hudson and Jackson (1962) and the EM curve in EM 1110-2-2300 (U.S. Army, Corps of Engineers, 1971) are intended for design use. The curves of Thomsen, Wohlt, and Harrison (1972) and of this study should possibly be more conservative to correct for uncertainties in relating the wave height to windspeed and fetch to be used for design; however, they are close to being design curves. Taking into account the considerations discussed, Figures 20 and 21 show there is good agreement among Hudson and Jackson (1962), Thomsen, Wohlt, and Harrison, and this study in predicting stable riprap weights. The divergence of stability between Thomsen, Wohlt, and Harrison and this study for flatter slopes is probably due to the influence of breaker characteristics discussed earlier.

In all the stability equations considered, except for the equation from EM 1110-2-2300 (U.S. Army, Corps of Engineers, 1971), the riprap weight is proportional to the cube of the wave height. In the EM equation riprap weight is proportional to the square of the wave height and the relative position of the EM curve changes with respect to the other curves for different design wave heights (compare Figs. 20 and 21). These figures also show that the EM formula predicts rock weights considerably smaller than the other three sources with the differences between this formula and other sources increasing as the wave height increases. The EM curve even predicts riprap weights smaller than predicted by the curve for the limit of tolerable damage (Figs. 20 and 21). Because of the large discrepancies between the EM curve and the three sources based on extensive wave tank tests, continued use of the EM formula cannot be justified. The origins of the EM equation are obscure and the source of information on which it is based appears to be a few field observations of riprap stability.

## VII. CONCLUSIONS

The major conclusions from this study are:

- a. The primary measure of riprap stability was the zero-damage wave height. It was found that for the range of riprap layer thicknesses studied the thickness had very little influence on the zero-damage wave height.

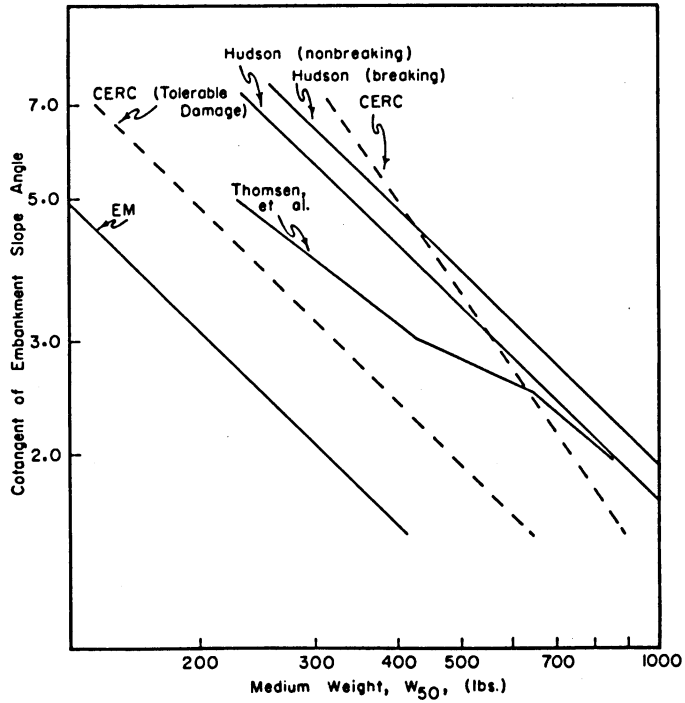


Figure 20. Median riprap weight versus cotangent of slope angle as given by various sources of a wave height of 5 feet.

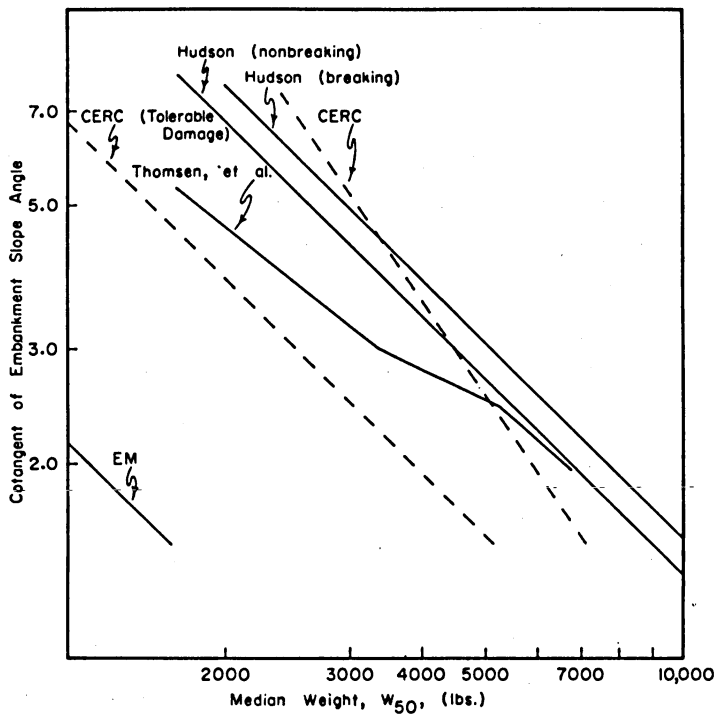


Figure 21. Median riprap weight versus cotangent of slope angle as given by various sources for a wave height of 10 feet.

b. For wave conditions that produced the lowest zero-damage wave heights the average stability coefficient,  $K_{RR}$ , was 2.70 for a 1 on 2.5 slope, 2.36 for a 1 on 3.5 slope, and 2.11 for a 1 on 5 slope. It was found that the stability number,  $N_S$ , for these wave conditions can be predicted by the formula,

$$N_S = 1.54 (\cot \theta)^{2/9} ,$$

where  $\theta$  is the angle between the embankment slope and the horizontal.

c. This study showed that wave period has a strong affect on riprap stability. Wave period effect is caused by the influence of period on the breaker type and shoaling characteristics of the waves. The breaker type referred to by Galvin (1968) as a *collapsing breaker* consistently yielded the lowest riprap stability. Breaker types observed during this study proved to be predictable through the use of Galvin's "offshore breaker parameter." Since the inherent stability of the rock, and the breaker and shoaling characteristics of the waves are all strongly dependent on the embankment slope, the influence of the slope on stability is more complex than generally acknowledged.

d. Dumped-quarrystone riprap can provide protection for wave heights above the zero-damage level. A "tolerable" level of damage was defined where a considerable amount of riprap displacement was noted but no filter stone was being lost and no soil was being pulled through the filter and riprap layers. Tolerable was applied to this level of damage because at times, damage of this type can be tolerated and is usually easy to repair. For riprap designs accepting tolerable damage a stability coefficient as high as 4.37 can be used. However, the stability coefficient and maximum wave height for tolerable damage are dependent on the thickness of the riprap. The stability coefficient of 4.37 is associated with a riprap layer between 1.5 and 2.0 median-stone-diameter thick.

e. It is assumed that if the critical riprap-filter size ratio, i.e., the ratio of the 15 percent diameter of the riprap to the 85 percent diameter of the filter stone, is less than 5, no filter stone will be washed through the riprap by wave action. In four tests this critical ratio was between 4 and 5 and in three of these four tests filter stone was observed being removed through the riprap by wave action. For riprap exposed to wave attack, the critical riprap-filter ratio should be 4 or less.

f. Riprap stability obtained in this study for the most dangerous wave conditions (collapsing breakers) was found to be consistent with results from other wave tank studies. In making this comparison the following sources were considered: Hudson and Jackson (1962), Thomsen, Wohlt, and Harrison (1972), U.S. Army, Corps of Engineers, Coastal Engineering Research Center (1973), and Harrison (1974). However, the EM 1110-2-2300 (U.S. Army, Corps of Engineers, 1971) predicts stable riprap weights considerably smaller than can be justified by this study.

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## APPENDIX

### COMPUTATION OF THE WAVE HEIGHT ASSOCIATED WITH THE "GREATEST TOLERABLE DAMAGE"

The wave height associated with the "greatest tolerable damage" is estimated by use of the failure parameter, F. This parameter was developed empirically to measure the progress towards failure accompanying the increases in wave height during a test. Generally a test was terminated when the waves were high enough to remove substantial amounts of filter stone through openings in the damaged riprap layer. In some tests before a substantial amount of filter stone was removed, core material would be pulled out through the filter and damaged riprap layer; this condition was considered severe enough to stop a test and declare it a "failure." Thus, failure was meant to imply that the riprap was sufficiently damaged so that it could no longer protect the embankment from wave attack, rather than failure of the entire embankment.

Observations of numerous tests showed that failure occurred in two general ways. One type of failure was associated with the removal of a great deal of riprap over a wide area. The other type of failure involved removal of a relatively small amount of riprap from a confined area. Long-period waves (8.5- and 11.3-second waves) tended to produce damage over a wide area and short-period waves ( $T = 2.8$  seconds) tended to produce more concentrated areas of damage.

Many failures were a combination of the two general types. The embankment was judged to be equally threatened by both types of damage. The different characteristics of the damage distribution made it impossible to judge how close to failure a test was solely on the basis of the volume of stone removed from the erosion zone. Thus, the failure parameter was developed to measure progress towards failure regardless of the distribution of damage and to weigh the two general types about equally.

The failure parameter is given by:

$$F = \left[ \left( \frac{\% P_s \text{ (rms)}}{0.80} \right)^2 + \left( \frac{\% VD}{0.19} \right)^2 \right]^{1/2}, \quad (A-1)$$

where  $\% P_s \text{ (rms)}$  is the percent, significant root mean square penetration of damage into the riprap layer;  $\% VD$  is the percent volumetric damage, and 0.80 and 0.19 were empirically determined factors to give approximately equal weight to the two terms in the equation. The parameter  $\% P_s \text{ (rms)}$  in equation (A-1) is given by:

$$\% P_s \text{ (rms)} = \left( \frac{\sum_{i=1}^M P_i^2}{M} \right)^{1/2} (\cos \ell/r), \quad (A-2)$$

where the  $P_i$ 's are the differences (penetrations into the riprap layer) between the survey values associated with some wave height compared to survey values at the same points of the reference survey. The reference survey is that of the riprap made before any waves have



been run. The differences at a point between surveys is caused by the removal of riprap by wave action. In equation (A-2),  $\theta$  is the angle between the slope and the horizontal,  $r$  is the average riprap layer thickness, and  $M$  is an integer equal to one-third the number of survey points in the damage or erosion zone. The erosion zone (Figs. 7 and 18) was determined from the average profile obtained from six complete profiles of the riprap; therefore, the number of survey points in the erosion zone is always divisible by three, and  $M$  is always an interger. The  $P_i$ 's chosen are the one-third representing the greatest penetration into the riprap layer in the erosion zone. The cosine term in equation (A-2) projects the  $P_i$ 's into the plane of the riprap layer from the vertical, where they were obtained from the surveys. By dividing the penetration by  $r$ , the thickness of the riprap layer, equation (A-2) can be expressed as a percentage.

In equation (A-1) the parameter % VD is given by:

$$\% VD = \frac{VD \sin \theta}{4H_{zd} r}, \quad (A-3)$$

where VD is the volume of riprap (including void space) removed by wave action from the erosion zone (Figs. 7 and 18) per foot of tank width. Equation (A-3) normalizes VD by the volume of the riprap layer per foot of tank width within a band of two zero-damage wave heights,  $H_{zd}$ , on either side of the stillwater line. A band with vertical limits of  $\pm 2 \times H_{zd}$  about the stillwater line is large enough so that it is unlikely that riprap would be disturbed by wave action above or below these limits. The slant length along the slope equivalent to four zero-damage wave heights is given in equation (A-3) by  $4 \times H_{zd} \div \sin \theta$ .

The factors 0.80 and 0.19 in equation (A-1) were adjusted not only to give about equal weight to the two terms in the equation but also to give a value of  $F = 1$  at the limit of tolerable damage. How the failure parameter increased during a typical test is shown below.

From this table, logarithmic interpolation gives a wave height of 3.39 feet corresponding to a value of  $F = 1$ ; this wave height is considered the limit of tolerable damage. Other methods of determining the limits of tolerable damage were tried but this approach worked the best. The failure parameter gave values of  $H_{td}$  which were consistent with visual and photographic observations of the tests.

Test 23, T = 5.7 seconds,  $W_{50} = 75$  pounds, slope = 1 on 3.5.

Run number	Wave height (ft.)	Failure parameter, F
1	2.01	0.296
2	2.21	0.300
3	2.41	0.311
4	2.61	0.339
5	2.81	0.427
6	2.97	0.626
7	3.13	0.737
8	3.37	0.978
9	3.61	1.328
10	3.85	2.987