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PROPERTIES OF EXPANSIVE CLAY SOILS

Report 2

A NUMERICAL PROCEDURE FOR PREDICTING HEAVE WITH TIME

by

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April 1975

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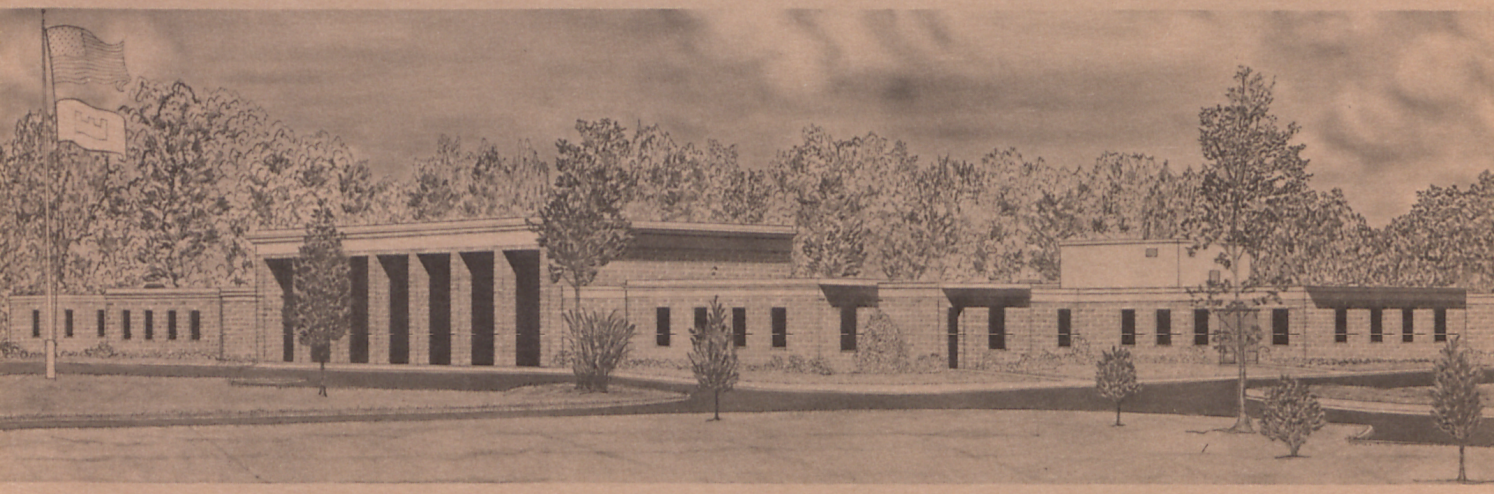

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A relatively economical and expedient numerical procedure and computer code based on the finite difference method were developed for predicting heave with time for heterogeneous expansive soils beneath structures idealized as two-dimensional. The computer code will also compute the potential heave assuming saturation of the foundation soils based on McDowell's swell data. The heave with time estimates should be useful for structures near which penetration of rainwater from the ground surface is permitted, provided that ponding of

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surface water and external water from leaking water and sewer lines does not occur. The potential heave should be useful if the foundation soils become saturated from excessive watering, poor surface drainage, and leaking water and sewer lines. The required input data include the structural load, specific gravity, liquid and plastic limits, initial dry density and water content, coefficient of vertical permeability, ratio of the horizontal to the vertical coefficient of permeability, and swell and suction relationships of the foundation soils. The swell and suction data in this report were adopted from published literature and were represented in the code by using a bicubic spline function. The remaining input data can be obtained from relatively simple laboratory tests. The procedure was applied to the prediction of heave with time for two field problems. In view of the approximate nature of the adopted swell and suction data, the heave predictions and extents of active zones influenced by heave show reasonable agreement with field observations. Field conditions that include the drainage pattern, amount and frequency of rainfall, and soil temperature were ignored. The suction computations suggest existence of perched water tables, which were confirmed by field piezometric data. The predicted heaves with time were substantially less than heaves determined by assuming completely saturated foundation soils and were relatively insensitive to the ratios of the horizontal to vertical coefficients of permeability. Further field data and results of swell and suction tests on undisturbed soils will be necessary to verify the generality of the formulation and code.

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PREFACE

The development of a numerical procedure for predicting heave is one phase of a continuing study under the work unit "Properties of Expansive Clay Soils." The work unit was started in 1967 under the sponsorship of the Office, Chief of Engineers, U. S. Army, Directorate of Military Construction. The initial studies were performed under the Operations and Maintenance, U. S. Army, program. The studies are now being performed under RDT&E Work Unit AT04 04 001.

The work reported herein was performed by Drs. L. D. Johnson and C. S. Desai, Research Group, Soil Mechanics Division, Soils and Pavements Laboratory (S&PL), U. S. Army Engineer Waterways Experiment Station (WES), Vicksburg, Mississippi. The report was reviewed by Messrs. W. C. Sherman, Jr., Research Group, and S. J. Johnson, Special Assistant to Chief, S&PL. Mr. J. P. Sale was Chief, S&PL, and Mr. C. L. McAnear was Chief, Soil Mechanics Division.

BG E. D. Peixotto, CE, and COL G. H. Hilt, CE, were Directors of the WES during the conduct of this study and the preparation of this report. Mr. F. R. Brown was Technical Director.

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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
inches	2.54	centimeters
feet	0.3048	meters
pounds per square inch	0.6894757	newtons per square centimeter
pounds per cubic foot	16.018489	kilograms per cubic meter

PROPERTIES OF EXPANSIVE CLAY SOILS

A NUMERICAL PROCEDURE FOR PREDICTING HEAVE WITH TIME

PART I: INTRODUCTION

Background

1. Expansive soils are found in many parts of the world and are responsible for damages to numerous types of military and civilian structures, with property losses exceeding two billion dollars annually within the United States alone.[1-3] The damages occur from differential heave or shrinkage of the foundation soils usually resulting from an increase or decrease in the water contents of the foundation soils. Placement of overlying structures often disturbs the moisture regime in the soil such that a new moisture profile tends to develop. The new moisture profile is usually one of greater water content because transpiration of moisture by vegetation and evaporation are inhibited beneath the structure and because sprinkling of plants and shrubs usually occurs adjacent to the structure. The water content of the foundation soil may decrease, however, if trees are planted adjacent to the structure or drought conditions occur.

2. Satisfactory and economical designs of foundations on expansive soils are difficult to achieve because techniques for accurate prediction of foundation heave are not available. Commonly used procedures for estimating swell potential are based on soil index tests,[4-9] but these do not consider the effects of field conditions on the rate and extent of vertical heave. Several two-dimensional (2D) numerical techniques based on finite element[10] and finite difference (FD)[11] methods have been developed that consider some field conditions, including nonhomogeneous soil strata, initial soil properties, and general weather patterns. These numerical techniques usually require a wide variety of input data that are difficult to obtain without elaborate laboratory equipment and costly testing techniques. Foundation designs for large structures such as multistory buildings have consequently been based on

a combination of experience, examination of subsoil and site conditions, and relatively simple laboratory swell tests. Soil investigations for designs of foundations for small structures such as one-story buildings and houses are usually much more limited to minimize costs.

Purpose and Scope

3. This report contains the results of the initial study of numerical methods for prediction of heave and rates of heave in expansive clay foundation soils as part of the work unit "Properties of Expansive Clay Soils." The purpose of this study is to develop a relatively economical and expedient procedure and computer program for predicting heave and rates of heave of expansive clay foundation soils. The computer program should require a minimum of input data which are relatively simple and inexpensive to obtain in the laboratory. The development of such a computer program capable of indicating a general trend of the magnitude and rate of heave is believed worthwhile in view of the high cost of most other methods for estimating heave and the general lack of confidence in the resulting heave estimations.

4. A computer code based on the FD method of numerical analysis prepared by Desai[12,13] for the transient flow problem was modified to predict heave with time beneath structures idealized as 2D. The procedures are based on the alternating direction explicit (ADE) scheme,[14,15] which has acceptable accuracy and can be more economical than the commonly used alternating direction implicit procedure.[12,14,15]

5. Experimental curves available in published literature[9,16] relating the swell and plasticity properties of similar cohesive remolded soils were adopted for the soils considered herein. Modifications to these experimental data may be necessary for soils in other areas. These curves are simulated by using a bicubic spline function[17,18] which permits accurate representation of the given parameters required in the FD scheme. Formulation and codes for the bicubic spline developed by Desai[18] were adopted herein. The remaining input data include the structural load, specific gravity of the soil, liquid limit, plasticity index,

initial dry density, water content, and coefficient of vertical permeability. The coefficient of horizontal permeability is input in terms of a fraction or multiple of the coefficient of vertical permeability. These properties can usually be estimated or easily obtained from standard tests, and additional tests for determination of swell properties are not necessary. Future studies will apply swell and suction data of the undisturbed expansive soils actually existing in the foundation materials, which should improve the accuracy of the input data and thereby lead to more accurate predictions of heave with time.

PART II: DETAILS OF THE PROCEDURE

Theory and Governing Equations

6. The solution of the time rate of change in heave of expansive clay foundation soils by the computer code (Appendix A) is based on the diffusion equation[19]*

$$\frac{\partial \tau}{\partial t} = \frac{\partial \tau}{\partial \theta} \frac{\partial}{\partial x_i} \left(k_{ij} \frac{\partial \tau}{\partial x_j} + k_{i3} \right) \tag{1a}$$

where

τ = total suction

t = time

θ = volumetric water content

x_i = coordinates ($i = 1,2,3$)

k_{ij} = permeability tensor--the proportion of velocity of flow in the ith direction contributed by a change in total head H in the jth direction

x_j = coordinate in the jth direction

k_{i3} = proportion of velocity of flow in the ith direction contributed by a change in total head H in the vertical direction

The total suction head is a function of the absolute temperature, surcharge pressure, pore water pressure, the concentration head, the absorptive head, and the gravity head.[19] Major assumptions in deriving Equation 1a are that the permeability tensor k_{ij} adequately describes the flow properties at a point, deformations in the soil mass are small, total suction head varies only with temperature and time, temperature remains constant, average density of water does not change with time or distance, and total head H equals the total suction head τ plus the gravity head.[19] The effect of these assumptions on heave is usually minor for most field conditions. The total suction pressure can be given by the sum of matrix and osmotic components (Table 1).[20] Matrix suction is associated with tensile stress in the pore water and sorptive forces

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* For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix D).

from the clay particles, while osmotic suction is associated with the concentration of soluble salts in the pore water.

7. Moisture flow is idealized as 2D, for which the governing Equation 1a reduces to

$$\frac{\partial \tau_m}{\partial t} = \frac{\partial \tau_m}{\partial \theta} \left(k_x \frac{\partial^2 \tau_m}{\partial x^2} + k_y \frac{\partial^2 \tau_m}{\partial y^2} \right) \frac{1}{\gamma_w} \tag{1b}$$

where

τ_m = matrix suction pressure

t = time

θ = volumetric water content

k_x = coefficient of permeability in the x direction

x = horizontal coordinate

k_y = coefficient of permeability in the y direction

y = vertical coordinate

γ_w = unit weight of water

Equation 1b assumes that the coefficient of permeability is independent of suction, which is not strictly true; however, the coefficient of permeability is essentially constant for the range of suctions of the Jackson, Mississippi, and Lackland, Texas, soils pertinent to this study.[21] Permeabilities of soils in other areas may vary significantly with suction and, if necessary, the computer code can be easily adjusted to consider the effect of suction on permeability. The osmotic suction is assumed not to change significantly with time. The actual effect of osmotic suction on heave is not well known.

8. The four main steps of the procedure for computation of heave as a function of time are (1) adoption of the boundary and initial conditions, (2) computation of the matrix suction at various time levels from solution of Equation 1b by the FD method, (3) conversion of the matrix suctions to corresponding water contents, and (4) computation of the heave. These steps are outlined below.

Boundary and Initial Conditions

9. The boundary conditions at the perimeter of the foundation

soils must be known and are given in this study as (Figure 1)

$$\frac{\partial \tau_m}{\partial x}(L,y) = \frac{\partial \tau_m}{\partial x}(0,y) = 0 \tag{2a}$$

$$\frac{\partial \tau_m}{\partial y}(x,0) = 0 \tag{2b}$$

$$\frac{\partial \tau_m}{\partial y}(x,D) = 0, \; 0 < x < BLEN \tag{2c}$$

where

L = length

D = depth

BLEN = total length of structure

The dimensions of the discretized mass should be sufficiently large in the x and y directions for the above boundary conditions to remain valid. The original suctions $\tau_m^o(x,0)$ along the baseline at $y = 0$ and $\tau_m^o(L,y)$ along the left-hand vertical boundary are assumed not to change with time. The suction of the surface soil $\tau_m(x,D)$ outside of the structure is assumed to be zero due to periodic watering. The right-hand vertical boundary is at the center line of the structure.

10. Both the initial matrix suction at zero surcharge pressure $\bar{\tau}_m^o$ and $\partial\tau_m^o/\partial w$ of the foundation soil required for solution of Equation 1b are determined from the initial water content w on the basis of known suction-water content relationships of appropriate undisturbed samples of the foundation clay soils. The suction-water content-plasticity index relationships reported by Black[16] for similar remolded British soils were adopted for this study (Appendix B). The plasticity properties of the Jackson[22] and Lackland[23] soils used in this study were similar to those of the remolded British soils. The suction-water content relationships for both undisturbed and remolded Lackland soil were also similar near the in situ natural water content.[24] Note that the behavior of undisturbed and remolded soils of identical composition and

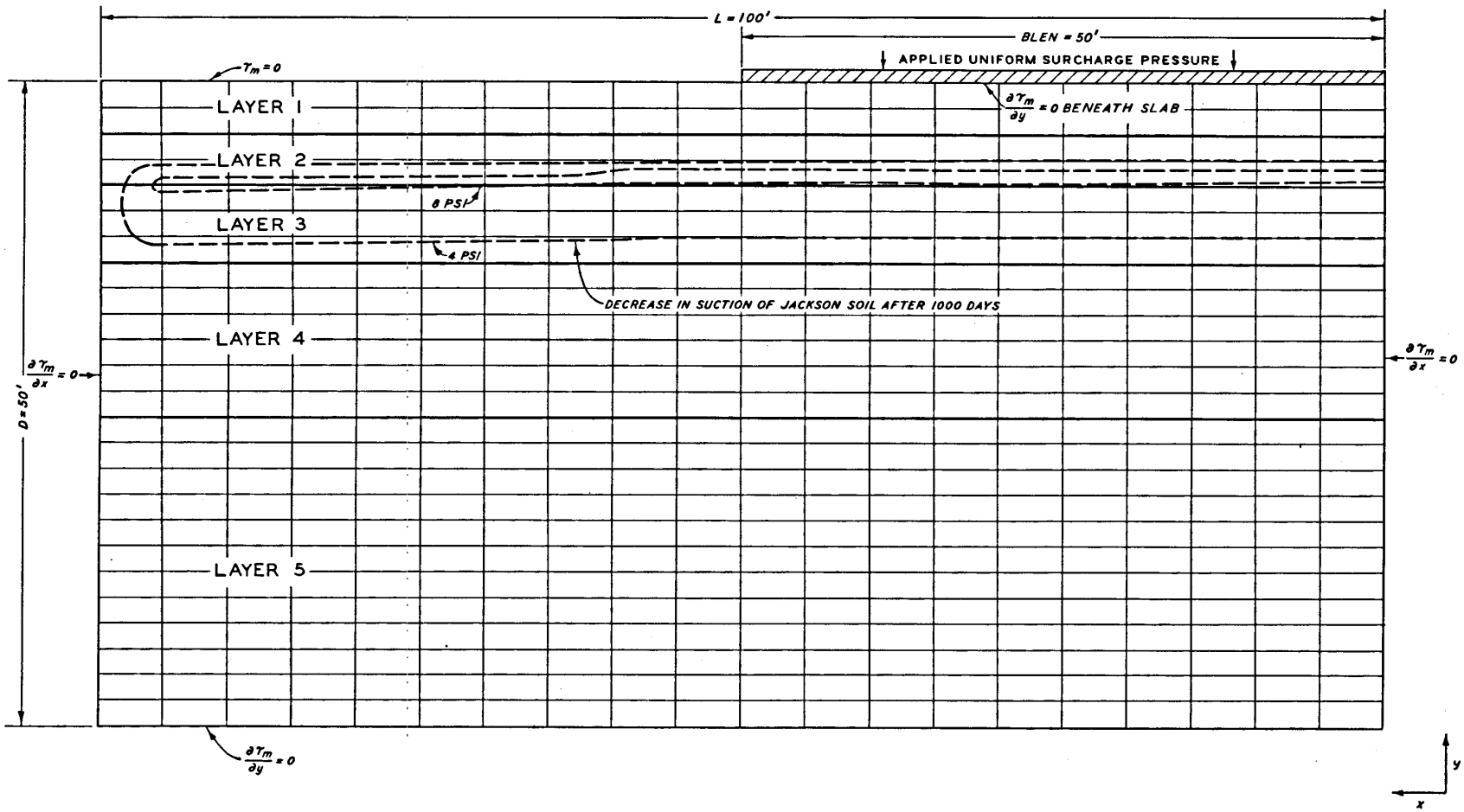

Figure 1. FD mesh and boundary conditions

water content can be different. However, data from remolded soils can be used for approximate analysis; it can be seen subsequently that computations based on data from remolded soils provide satisfactory solutions.

11. The initial suctions at zero surcharge pressure $\bar{\tau}_m^o$ must be converted to the initial in situ suctions τ_m^o to account for the soil overburden and surcharge pressure P as

$$\tau_m^o = \bar{\tau}_m^o - \alpha P \tag{3}$$

where α = compressibility factor. The compressibility factor is the fraction of applied pressure which is effective in changing the pore water pressure. The factor can be obtained by multiplying the unit weight of water in grams per cubic centimeter by the slope of a curve relating the reciprocal of the dry density in cubic centimeters per gram to water content in percent of dry weight.[25] The compressibility factor will be nearly equal to zero for incompressible soils, such as clean sands at low degrees of saturation, but will be equal to one for all fully saturated soils or for soils with a degree of saturation equal to unity. Heavy clay soils such as the Lackland and Jackson expansive clays are saturated to such a degree that a compressibility factor set equal to one is a reasonable approximation.[21]

12. The surcharge load applied to the foundation simulates the weight of the structure, and it is distributed to the foundation soil by the 2:1 slope method.[26] The overburden weight of the soil is added to the weight of the structure to determine the total surcharge pressure P at each point within the foundation soil. If $\bar{\tau}_m^o$ is exceeded by P, the in situ pore water pressures become positive, and the soil may undergo consolidation. This consolidation is also computed by the code. The pore water pressures at mesh points between regions of positive and negative (suction) pressures are set equal to zero.

Change of Matrix Suction with Time

13. The FD analogue[12-14] corresponding to Equation 1b is (Figure 2)

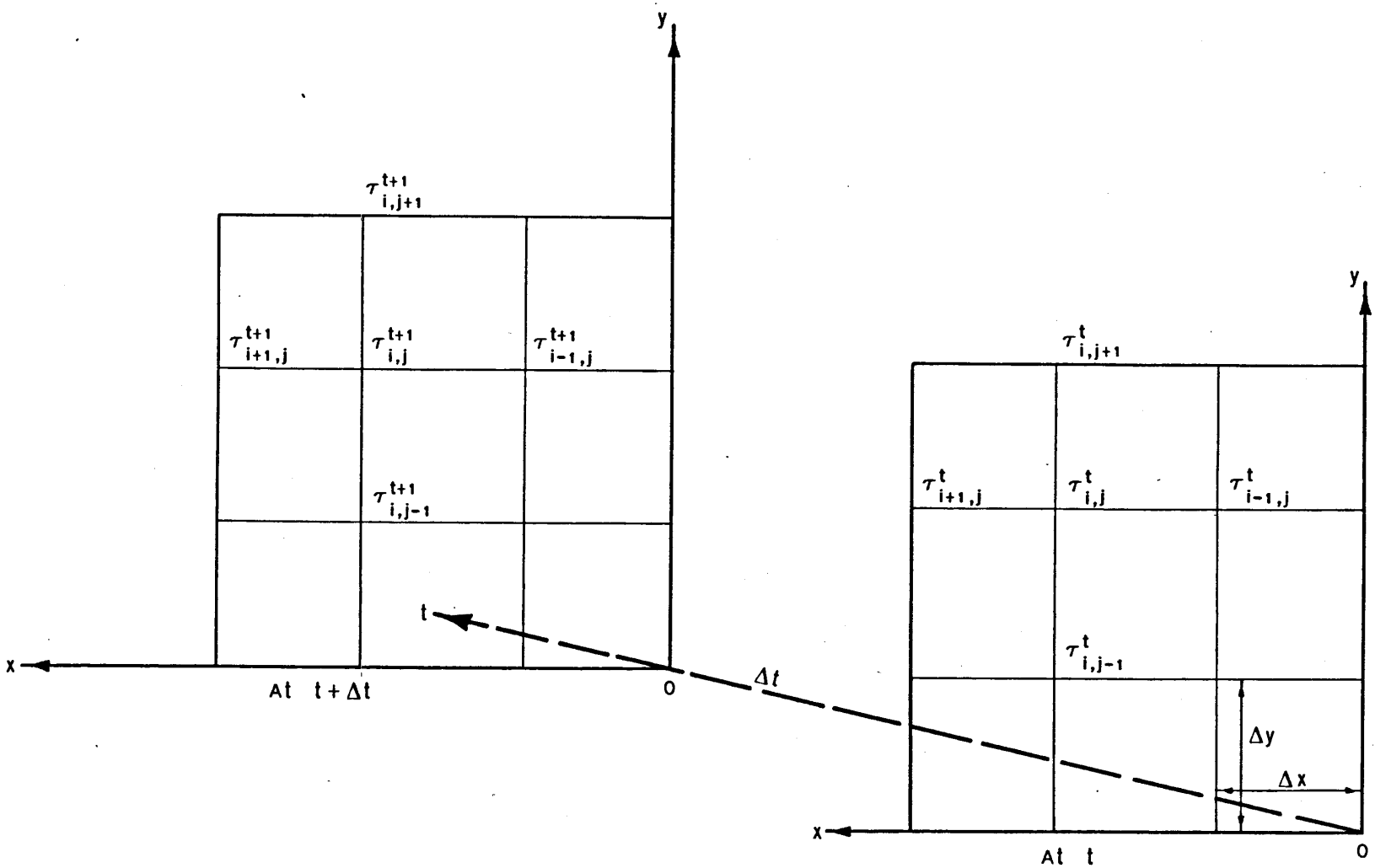

Figure 2. FD approximation

$$\frac{\tau_{m_{i,j}}^{t+1} - \tau_{m_{i,j}}^{t}}{\Delta t} = C \left[\frac{k_x \left(\frac{\tau_{m_{i-1,j}}^{t+1} - \tau_{m_{i,j}}^{t+1}}{\Delta x} - \frac{\tau_{m_{i,j}}^{t} - \tau_{m_{i+1,j}}^{t}}{\Delta x} \right)}{\Delta x} + \frac{k_y \left(\frac{\tau_{m_{i,j+1}}^{t+1} - \tau_{m_{i,j}}^{t+1}}{\Delta y} - \frac{\tau_{m_{i,j}}^{t} - \tau_{m_{i,j-1}}^{t}}{\Delta y} \right)}{\Delta y} \right] \tag{4}$$

where

$\tau_{m_{i,j}}$ = matrix suction at a point (x,y)

t = time

Δt = time increment

C = parameter

k_x, k_y = coefficient of permeability in x and y directions, respectively

$\Delta x, \Delta y$ = horizontal and vertical increments, respectively

As described previously, the values of τ_m are known at all times on the top, bottom, and left side (Figure 1); hence, the values of $\tau_{m_{i,j+1}}^{t+1}$ are available for the ADE procedure.

14. The parameter C at each mesh point is given by

$$C = \frac{\partial \tau_m^t}{\partial \theta} = \frac{\partial \tau_m^t}{\partial w} \times \frac{\partial w^t}{\partial \theta} \tag{5a}$$

For small volume changes,

$$\frac{\partial w^t}{\partial \theta} = \frac{1 + e^t}{G_s} = V_T^t \tag{5b}$$

and

$$C = \frac{\partial \tau_m^t}{\partial w} \times V_T^t \tag{5c}$$

where

e^t = void ratio at each mesh point and at time t

G_s = specific gravity

V_T^t = total specific volume at each mesh point and at time t

Known relationships between matrix suction and water content are necessary to evaluate $\partial\tau_m^t/\partial w$; evaluation of V_T is discussed later.

15. Equation 4 is also useful for computing decreases in positive pore water pressure, which lead to consolidation, by setting the parameter C for each nodal point of the mesh (Figure 1) as

$$C = \frac{1}{\gamma_w m_v} \tag{6a}$$

where γ_w = unit weight of water. The coefficient of volume change m_v is computed by

$$m_v = \frac{\Delta e}{(1 + e_o)\Delta\sigma} = \frac{PVS}{100\Delta\sigma} \tag{6b}$$

where

Δe = ultimate change in void ratio from the initial void ratio e_o

PVS = potential volumetric shrinkage (Appendix C)

$\Delta\sigma$ = ultimate change in effective stress or the initial positive pore water pressure

16. For the horizontal layering in this study, the FD equations were modified to allow for continuity of flow across the interfaces.

$$k_{y_n}\left(\frac{\partial\tau_m}{\partial y}\right)_n = k_{y_{n+1}}\left(\frac{\partial\tau_m}{\partial y}\right)_{n+1} \tag{7}$$

where k_y denotes the vertical coefficient of permeability and n refers to the soil layer.

Conversion of Matrix Suction to Water Content

17. The computed values of matrix suction τ_m of each mesh point

at time level $t + 1$ are corrected to refer to suction at zero surcharge pressure by using Equation 3. The water content w of each mesh point is obtained from the matrix suction at zero surcharge pressure by reference to the known suction-water content relationship (Appendix B), which is coded into the computer program through the spline function.[18]

Computation of Heave

18. The volumetric heave at various time levels for each mesh point (Figure 1) is computed by the formula

$$\frac{\Delta V}{V} = \frac{V_T^t - V_{TI}}{V_{TI}} \tag{8}$$

where

ΔV = change in volume V

V_T^t = total specific volume at time t

V_{TI} = initial specific volume

The total specific volume is evaluated from the water content w by[11]

$$V_T^t = V_{TI} + \left(V_{TP} - V_{TI}\right)\left(\frac{w - w_o}{w_{max} - w_o}\right)^Q \tag{9}$$

where

$V_{TP} = V_{TI}(1 + PVS/100)$, maximum total specific volume

w = water content computed from FD procedure

w_o = initial water content

$w_{max} = \left[\left(V_{TP}G_s - 1\right)/G_s\right] \times 100$

$Q = \left[\left(w_{max} - w_o\right)/\left(V_{TP} - V_{TI}\right) \times 100\right]$

The initial specific volume is evaluated from the initial void ratio e_o by

$$V_{TI} = \frac{1 + e_o}{G_s} \tag{10}$$

19. The percent potential volumetric swell PVS or the percent

swell after saturation at each mesh point must be found from known swell-surcharge pressure relationships, which are coded into the computer program through the bicubic spline function.[18] The swell-or shrinkage-surcharge pressure relationships for remolded Texas clays and the procedure for predicting potential heave given by McDowell[9] were adopted for this study (Appendix C). The McDowell method is a popular procedure and is used widely;[27] however, the swell data of these remolded Texas soils may not be applicable for all expansive soils and modifications to these data or additional data may be necessary. The amount of swell accumulating in a soil mass after a practical period of time, i.e., five or ten years or the useful life of the structure, may be much less than the PVS , depending on the soil permeability, exposure to surface water, magnitude and distribution of the soil suction, and other field conditions.

20. The amount of heave that occurs in the vertical direction is often assumed to be equal to one-third of the volumetric heave. The actual amount of volumetric swell that occurs in the vertical direction depends on the nature of the soil and confining conditions. A tight soil with high lateral restraint may heave entirely in the vertical direction (maximum level of heave), while a heavily cracked soil may exhibit approximately one-third of the volumetric swell in the vertical direction (minimum level of heave).[28] Heave may be between these limits for many cases.

Outline of the Procedure

21. The formulation and computer code (Appendix A) may be used to compute (a) the total potential heave or the potential vertical rise PVR and (b) the minimum level of heave with time along the ground surface and base of the structure for a representative section of the 2D profile. The PVR is calculated to be approximately one-third of the sum of the potential volumetric swells PVS of the discretized masses between adjacent coordinates in a vertical column of soil.[9] The PVS is expected to develop from adding water until the degree of saturation reaches 100 percent and the soil is completely saturated. The maximum potential heave and maximum level of heave with time are found by

multiplying by three the PVR and the minimum level of heave with time, respectively. The computer code computes the distribution of heave along the vertical column of soil beneath the center of the structure and the column of soil in the discretized mass most distant from the structure. Suction, derivative of suction with respect to water content, and the change in void ratio are also printed for each coordinate of the discretized mass at the desired time levels.

22. The input data (Appendix A) include (a) identification of the problem (one card), (b) space/time parameters (3 or more cards), and (c) soil properties (one card for each soil). The input data on each soil property card consist of the specific gravity G_s, initial dry density γ_{do}, liquid limit LL, plasticity index PI, the initial water content w_o, and the coefficient of vertical permeability VPERM. The ratio of the horizontal to the vertical coefficient of permeability PFACT is included in the space/time parameter card 2B. The coordinates of each soil must be described.

23. The PVR or vertical swell expected from complete saturation of the soil in inches* along the ground surface and base of the structure will be given whether or not increments of output NOUT are specified in the "time of output" card 2C (Appendix A). NTIME can be made zero and a blank card substituted for card 2C. Values for the PFACT and VPERM are not necessary. The PVR may be quite useful if the foundation soil is expected to reach a degree of saturation of 100 percent during the life of the structure. Factors that contribute toward complete saturation of foundation soils include leaky underground water and sewer pipes, excessive watering of the ground surface, dripping faucets, and poor drainage of surface water.

24. The heave with time computations will be provided if NTIME is not set equal to zero and increments for output NOUT are provided. The heave with time computations may be useful for structures located where most field conditions have not been disturbed by construction or

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* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 4.

occupancy and water is permitted to infiltrate vertically from the ground surface. Examples may include houses and buildings near which surface water is not permitted to pond and underground water and sewer pipes are made watertight with flexible joints.

PART III: APPLICATIONS

25. The procedure and computer code were applied to the solution of the heave behavior of the Jackson and Lackland field test sections. A view of the Jackson site is shown in Figure 3. The covered 100-ft-square area contains a layer of sand about 0.8 ft thick between two impermeable membranes, which exerts a pressure of about 1 psi to the underlying soil. The soil profiles of both test sites were divided into five soil layers (Table 2). Because the ratios of the horizontal to vertical coefficients of permeability of the Jackson and Lackland field problems were not known, values of various ratios from 0.1 to 10 were input to examine the effect on heave behavior. For both these problems, the horizontal increment $\Delta x = 5$ ft, the vertical increment $\Delta y = 2$ ft, and the time increment $\Delta t = 10$ days were adopted.

26. The output was obtained at various time levels of 50, 100, 150, 200, 400, 600, 800, 1000, 3000, and 5000 days. The total cost for such a run on the GE-600 computer was about $37.00 with 9 min of processor time.

Figure 3. A view of Jackson test site

Jackson Site

27. The potential vertical rise PVR is 2.4 in. at the center of the Jackson test section, but Figure 4 shows that only about 0.8 in. had accumulated beneath the center area within 1620 days (4.4 yr). The matrix suctions of the Jackson soil are small (15 psi),[20,21] and these coupled with the small coefficient of permeability may have caused the slow rate of heave. In view of the approximations involved in adopting swell and suction data for other soils,[9,16] the correlation between the computed and actual heave seems satisfactory. The computations based on the assumption (marked minimum in Figure 4) that the vertical heave is one-third of the maximum or total volumetric change are closer to the observed data during the initial time levels, but the trend of actual behavior shows that the heave may lie somewhere between one-third and the total volume change during later time levels (Figure 4). The computed amounts of heave with time appear to level off after 2000-3000 days

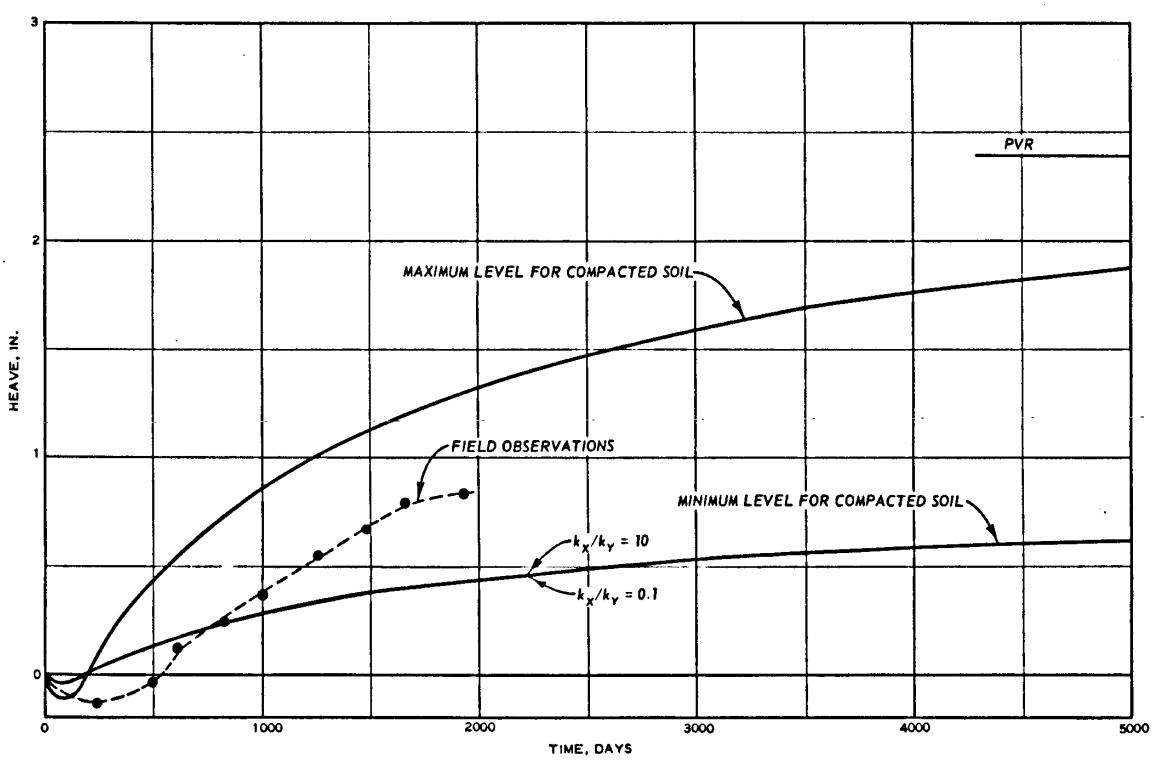

Figure 4. Comparisons for heave at Jackson test section

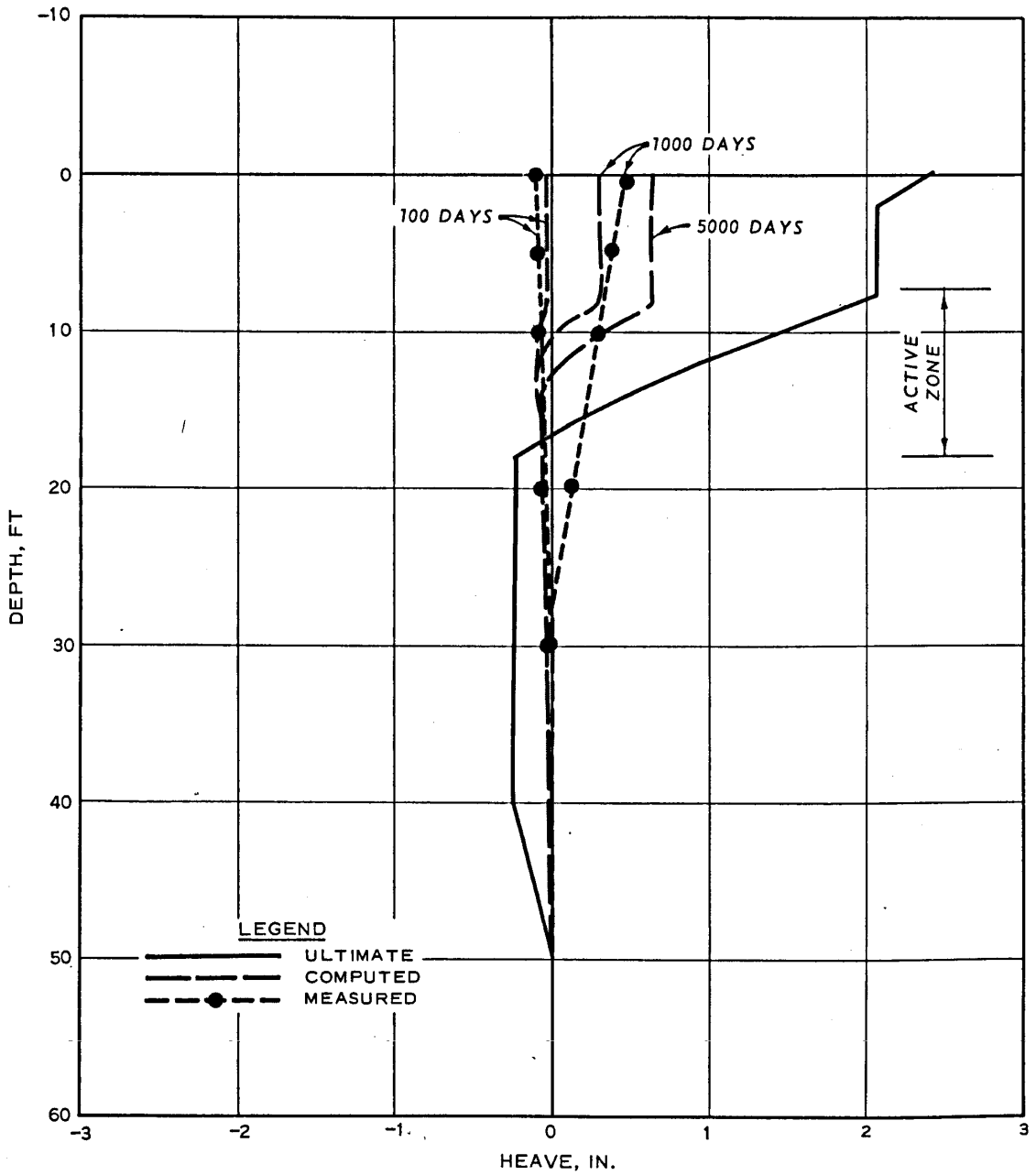

Figure 5. Comparisons for heave profiles at Jackson test section

(5 to 8 yr). Most soil heave is found to accumulate within from 0 to 5 to 8 yr after construction.[29-31] The magnitude of the ratio of the horizontal to vertical coefficient of permeability (0.1 or 10) was found to have a negligible effect on the heave behavior of the Jackson soil. The computed heave profile beneath the center of the test section given in Figure 5 is consistent with the field data and shows that the active zone lies approximately from 8 to 18 ft in depth. The topsoil above 8 ft of depth contains a lean clay which is not expansive.[21] As can be seen in Figure 5, the foundation soil experiences consolidation instead of swell below about 18 ft of depth. This may be due to the initial distribution of suction in the foundation soils. If the relative magnitude of applied load is sufficiently large, part of the pore water medium may pass from suction to positive pressures causing consolidation instead of heave.

28. Field piezometric data indicate a perched water table from 5 to 20 ft or more below ground surface and a deep water table at about 125 ft of depth.[21] The computed soil suctions suggest a perched water table in the vicinity of 2-6 ft of depth, which appears reasonable in view of the rough estimation of the suction properties of the foundation soil as taken from the literature. The change in the suction pressure profile of the Jackson soil in pounds per square inch after 1000 days is also given in Figure 1.

Lackland Site

29. The PVR calculated for the Lackland soil is 3.0 in. beneath the center of the slab foundation. The amount of heave is expected to accumulate much more rapidly than for the Jackson soil, with more than 1 in. expected within the first 1000 days (Figure 6). The rate of heave again appears to level off after 2000-3000 days. Variations in the ratios of the horizontal to vertical coefficients of permeability indicated that a higher ratio of 10 showed a slight change in heave of about 0.2 in. more beneath the section than did the lower ratio of 0.1. The

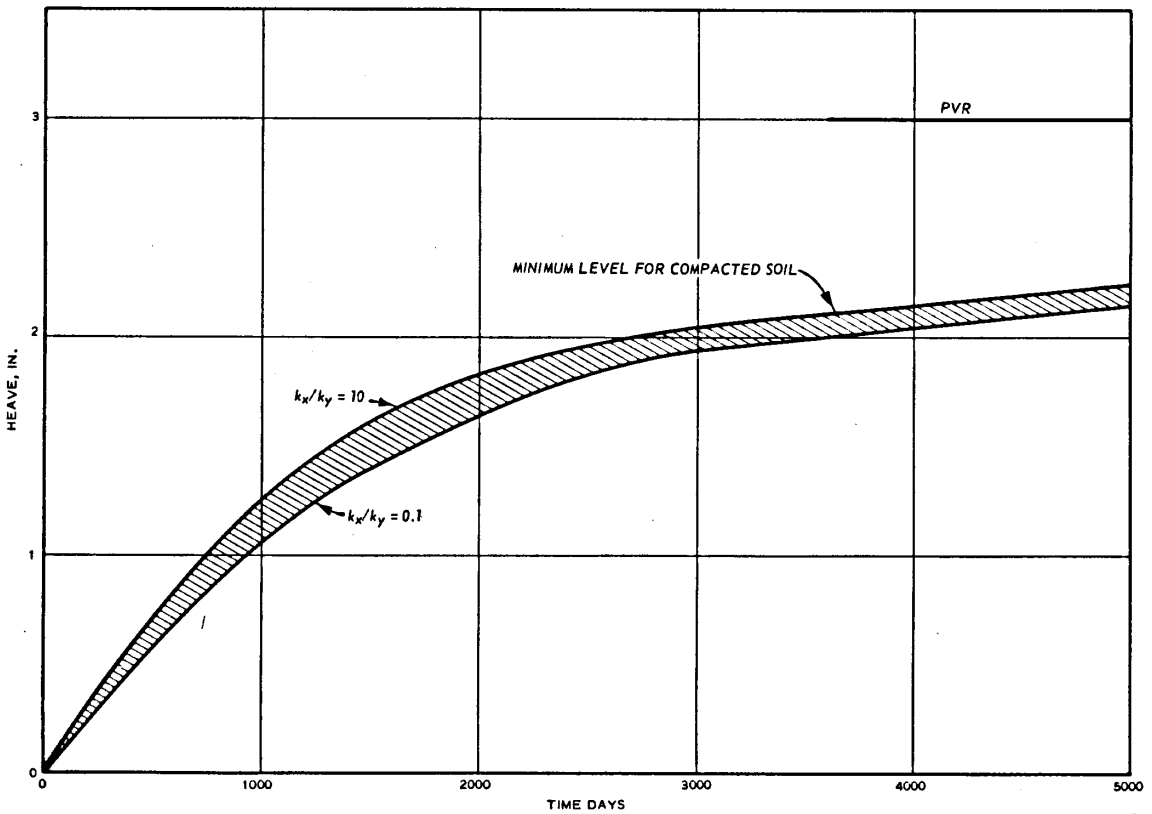

Figure 6. Predicted heave at Lackland test section

predicted heave profile in Figure 7 shows an active zone down to 30 ft of depth, with most heave occurring in the top 10 ft. The absence of a change in heave is evident from 10 to 14 ft in the low-plasticity area. Field experience[32,33] shows that most heave occurs in the top 8 or 10 ft of soil. This is in agreement with the computed results.

30. Field piezometric data indicate a perched water table from 8 to 35 ft below ground surface and a deep water table below 45 ft of depth. The computed soil suctions indicate a perched water table from 10 to 14 ft of depth and a deep water table below 42 ft of depth. These comparisons again appear reasonable in view of the adoption of the suction and swell properties for the foundation soils as taken from the literature.

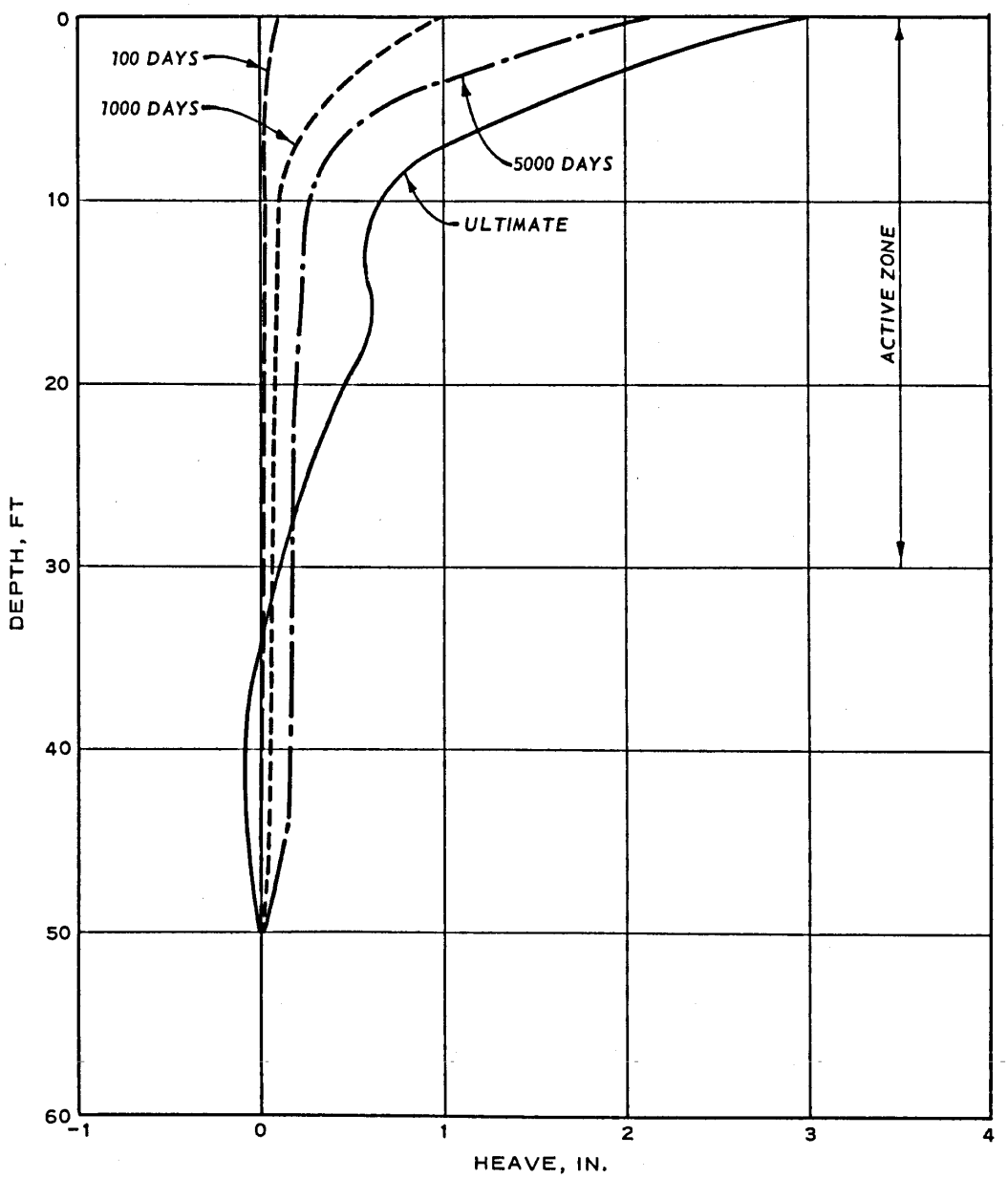

Figure 7. Predicted heave profiles at Lackland test section

PART IV: CONCLUSIONS

31. A numerical procedure based on the ADE scheme of the FD method was developed to evaluate amount and rate of heave of foundations in expansive soils. Use of the bicubic spline permits accurate representation of known values of swell and suction parameters for incorporation in the numerical scheme. The swell and suction data available from other published works are used in the procedure. The amount of other input data required for the proposed procedure is relatively small and easily obtained from standard tests. These data include specific gravity, liquid limit, plasticity index, initial dry density and water content, coefficient of vertical permeability, and the ratio of the horizontal to vertical coefficient of permeability.

32. A factor of 100 change in the ratio of the horizontal to vertical coefficient of permeability has a minimal effect on the heave and rate of heave of the tested soils beneath the center of the test sections.

33. Much of the predicted heave from both test sites occurs within 10-20 ft of the ground surface. The rate at which heave occurs levels off after 2000-3000 days (5-8 yr) with relatively small amounts accumulating after 3000 days.

34. The computer code predicts a perched water table and a deeper water table at the test sections. Piezometric data confirm the existence of these water tables in roughly comparable locations.

35. Differences between field and computed behavior are attributed to the adoption of swell and suction properties of remolded soils taken from the literature. Field conditions that include drainage pattern, amount and frequency of rainfall, and soil temperature are not considered. Much additional field data and results of swell and suction tests on undisturbed soils will be necessary to verify the program.

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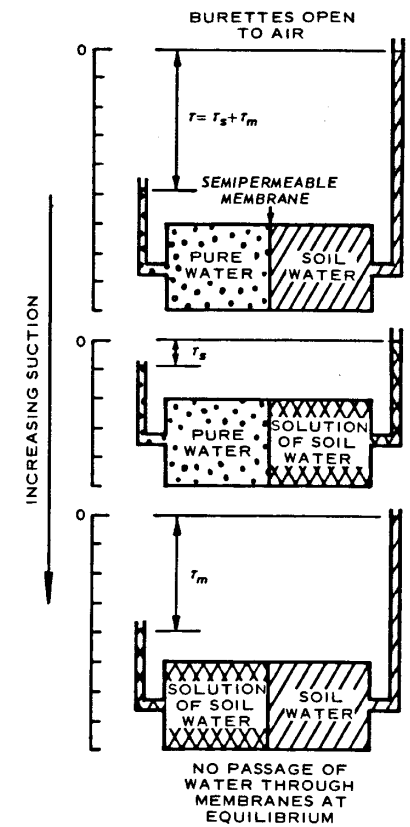
Table 1

Definitions of Suction

Term	Symbol	Definition*	Illustration
Total suction	τ	The negative gage pressure, relative to the external gas pressure** on the soil water, to which a pool of pure water must be subjected in order to be in equilibrium through a semipermeable (permeable to water molecules only) membrane with the soil water	
Osmotic (solute) suction	τ_s	The negative gage pressure to which a pool of pure water must be subjected in order to be in equilibrium through a semipermeable membrane with a pool containing a solution identical in composition with the soil water	
Matrix (soil water) suction	τ_m	The negative gage pressure, relative to the external gas pressure** on the soil water, to which a solution identical in composition with the soil water must be subjected in order to be in equilibrium through a porous permeable wall with the soil water	



* From Reference 20 of text.

** The magnitude of the matrix suction is reduced by the magnitude of the external gas pressure. The osmotic suction is determined by the concentration of soluble salts in the pore water and can be given by $\tau_s = \frac{RT}{v_w} \log_e \frac{P}{P_o}$ where R is the universal gas constant, T is absolute temperature, v_w is volume of a mole of liquid water, P is vapor pressure of the pore water extract, and P_o is vapor pressure of free pure water.

Table 2

Profile of the Example Problems

Depth ft	Coordinates x	Coordinates y	Specific Gravity	Dry Density lb/ft^3	Liquid Limit	Plasticity Index	Water Content %	Coefficient of Permeability, k_y ft/day
				A. Jackson Soil				
0-4	25-26	1-21	2.68	97	55	30	25	4.4×10^{-2}
4-8	23-24	1-21	2.70	104	45	25	25	1.02×10^{-2}
8-14	20-22	1-21	2.74	78	100	75	40	1.26×10^{-3}
14-26	14-19	1-21	2.73	75	110	80	45	1.02×10^{-3}
26-50	1-13	1-21	2.71	82.5	95	65	40	4.4×10^{-4}
				B. Lackland Soil				
0-10	22-26	1-21	2.70	95	60	40	22	7.6×10^{-3}
10-14	20-21	1-21	2.72	108	27	14	20	3.4×10^{-2}
14-22	16-19	1-21	2.73	92	78	53	30	2.4×10^{-4}
22-38	8-15	1-21	2.74	92	82	57	29	1.7×10^{-4}
38-50	1-7	1-21	2.73	96	85	60	28	1.0×10^{-4}

APPENDIX A: THE COMPUTER PROGRAM

Identification of Letter Symbols

NPROB	Number of problem
NX	Number of horizontal coordinates
NBX	Number of horizontal coordinates beneath structure
NY	Number of vertical coordinates
NBY	Number of vertical coordinates contained in structure
NMAT	Number of soils
NTIME	Number of times output desired
NOUT	Number of increments at which output is desired
DT	Time increment, days
PFACT	Ratio of horizontal to vertical coefficient of permeability
DY	Vertical distance between coordinates, ft
DX	Horizontal distance between coordinates, ft
QQ	Pressure exerted by structure at ground surface, psi
BLEN	Total length of structure = 2DX(NBX - 1), ft
B	Total width of structure, ft
Y_i	i^{th} vertical coordinate
Y_{i+1}	i^{th} + 1 vertical coordinate
X_i	i^{th} horizontal coordinate
X_{i+1}	i^{th} + 1 horizontal coordinate
GS	Specific gravity
γ_D	Dry density, pcf
LL	Liquid limit, %
PI	Plasticity index, %
W_o	Initial water content, %
VPERM	Coefficient of vertical permeability, in./day

Input Data

1. Identification of Problem (One Card)

NPROB	DESCRIPTION OF PROBLEM
1 5 9	80

2. Space/Time Parameters

A. Integers (one card)

NX	NBX	NY	NBY	NMAT	NTIME			
1	10	20	30	40	50	60	70	80

B. Decimals (one card)

DT	PFACT	DX	DY	QQ	BLEN	B		
1	10	20	30	40	50	60	70	80

C. Time of output (integer values). Continue on additional cards until NOUT (NTIME).

NOUT(1)	NOUT(2)	NOUT(3)						NOUT(8)
1	10	20	30	40	50	60	70	80

3. Soil Properties (One Card for Each Material)

Y_i	Y_{i+1}	X_i	X_{i+1}	GS	γ_D	LL	PI	W_o	VPERM
1 5	10	15	20	30	40	50	60	70	80

4. Input Listing Example

```
 1     Jackson Test Section - 50 ft depth

      26       1      21       11       5       10
      10.      .1     5.0      2.0      1.      100.      100.
       5      10     20       40      60      80      100       150
     300     500

 1    13    1    21    2.71    82.5    95.    65.    40.    4.4E-02
 1    19    1    21    2.73    75.    110.    80.    45.    10.2E-02
20    22    1    21    2.74    78.    100.    75.    40.    12.6E-04
23    24    1    21    2.70   104.     45.    25.    25.    10.2E-03
25    26    1    21    2.68    97.     55.    30.    25.     4.4E-02
```

5. Programmed Input

The remaining cards contain the information to set up the bicubic spline functions of Figures B1 and C1. These data cards may be changed to obtain other spline functions which may be more representative of actual soil swell and suction properties. Data for this study are as follows:

```
1         10        20        30        40        50        60        70        80
 30        20         5         6
  1         0.        0.
  5        30.        0.
  6         0.       10.
 10        30.       10.
 11         0.       20.
 15        30.       20.
 16         0.       30.
 20        30.       30.
 21         0.       40.
 25        30.       40.
 26         0.       50.
 30        30.       50.
       1         1         2         7         6
       4         4         5        10         9
       5         6         7        12        11
       8         9        10        15        14
       9        11        12        17        16
      12        14        15        20        19
      13        16        17        22        21
      16        19        20        25        24
      17        21        22        27        26
      20        24        25        30        29
      0.        7.       14.       22.       30.        -.4        .8        4.
      9.      13.8       -.8       -.5       1.2        4.8       8.7      -1.2
    -1.1        0.       2.3       5.2      -1.6       -1.6      -1.0        .8
     3.0      -2.0      -2.0      -2.0       -.4        1.5
    5.15      5.15      5.15      5.15      5.15       -2.0       -.4       1.7
    2.25      2.55      -2.0     -1.63        0.        .90      1.38      -2.0
    -2.0     -1.25      -.1        .55      -2.0       -2.0      -2.0      -.66
    -.11      -2.0      -2.0      -2.0     -1.65       -.72
    17.5      44.0      76.0     111.0     150.0       14.6      34.0      56.0
    83.0     110.0      11.0      25.0      40.0       57.5      76.0       7.7
    17.5      28.0      38.0      48.0       5.0       11.0      17.5      23.0
    29.0       2.5       5.2       8.0      11.0       13.5
```

6. Output Listing Example

A. Problem number and description

B. Material properties

Specific Gravity	Dry Density lb/ft^3	Limits LL	Limits PI	Initial Water Content, %	Coefficient of Vertical Permeability in./day

C. Potential heave data

Nodal Point Coordinate		Suction psi	Derivative Suction psi	Water Content, %		Surcharge Pressure psi	Void Ratio	
Y	X			Initial	Final		Initial	Final

Horizontal Nodal Point	Ultimate Heave in.

D. Periodic data

CYCLES TIME, days

Nodal Point Coordinate		Suction psi	Derivative Suction, psi	Void Ratio	Change in Void Ratio
Y	X				

Horizontal Nodal Points	Heave in.

```
      C     MOISTURE FLOW SPECIAL EXPLICIT AND HEAVE PREDICTION
      C     DEVELOPED BY L. D. JOHNSON AND C. S. DESAI
            COMMON MM,NRP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
           1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),SS,
           2XX(30),XY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
           3RPI(80,30),RERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
            DIMENSION TITLE(9)
       2500 READ 10,NPROB,(TITLE(I),I=1,9)
            IF(NPROB.LE.0)GO TO 999
            PRINT 15,NPROB,(TITLE(I),I=1,9)
            CALL DATAIN
            GAW=62.43
            READ 60,(U(I),I=1,NNP)
            CALL SPLINE
            CALL UHEAVE
      C     SUCTION AND DERIVATIVE WRP TO WATER CONTENT
            READ 60,(U(I),I=1,NNP)
            CALL SUCT
            READ 60,(U(I),I=1,NNP)
            CALL SPLINE
            NC=0
            TIME=0.
       1500 NC=NC+1
            TIME=TIME+DT
            CALL TSTEP
            DO 1000 I=1,NTIME
            J=NOUT(I)
            IF(NC.EQ.J)GO TO 980
       1000 CONTINUE
            IF(NC.GT.NOUT(NTIME))GO TO 2000
            GO TO 1500
        980 CONTINUE
            PRINT 75,NC,TIME
            CALL HEAVE
            PRINT 80
            DO 1200 J=1,NX
            DELH=0.
            DO 1250 I=2,NY
            IF(G(I,J).LE..01)GO TO 1255
            EI=((G(I,J)*GAW)/GAM(I,J))-1.
            ER=E(I,J)-EI
            GO TO 1240
       1255 ER=0.
            EI=0.
       1240 IF(I.EQ.2)ER=ER/2.
            DELH=DELH+(DY/3.)*ER/(1.+EI)
            IF(J.EQ.1.OR.J.EQ.NX)GO TO 1260
            GO TO 1250
       1260 PRINT 72,I,DELH
       1250 CONTINUE
            PRINT 70,J,DELH
       1200 CONTINUE
```

```
      GO TO 1500
 2000 CONTINUE
      GO TO 2500
  999 CALL EXIT
   10 FORMAT(I5,3X,9A8)
   15 FORMAT(/ 8H PROBLEM,I5,3H.. ,9A8)
   60 FORMAT(8F10.3)
   70 FORMAT(5X,I5,10X,F15.5)
   72 FORMAT(10X,I5,10X,F15.5)
   75 FORMAT(7H CYCLES,I5,5H TIME,F10.4/)
   80 FORMAT(85H  HORIZONTAL COOR      HEAVE          /)
      END
*****W  1286 (STOP) STATEMENT IS MISSING-SIMULATED
```

```
R0734 01   07-08-74    08,820

      SUBROUTINE DATAIN
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),PERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      READ 20,NX,NBX,NY,NBY,NMAT,NTIME
      READ 25,DT,PFACT,DX,DY,QQ,BLEN,B
      READ 20,(NOUT(I),I=1,NTIME)
      DX=12.*DX
      DY=12.*DY
      B=12.*B
      BLEN=12.*BLEN
      PRINT 27
      DO 50 I=1,NMAT
      READ 30,  I1,I2,J1,J2,U(I),UX(I),UY(I),UXY(I),XX(I),YY(I)
      PRINT 32,U(I),UX(I),UY(I),UXY(I),XX(I),YY(I)
      DO 55 II=I1,I2
      DO 55 JJ=J1,J2
      G(II,JJ)=U(I)
      GAM(II,JJ)=UX(I)
      E(II,JJ)=UY(I)
      PPI(II,JJ)=UXY(I)
      WC(II,JJ)=XX(I)
      PERX(II,JJ)=YY(I)
      ALOAD(II,JJ)=0.
      SU(II,JJ)=0.
   55 CONTINUE
   50 CONTINUE
C     SOIL LOADING
      NYY=NY-1
      NXY=NY-NBY+1
      DO 100 I=1,NYY
      IL=NY-I
      DO 100 J=1,NX
      IF(IL.GE.NXY.AND.J.LT.NBX)GO TO 100
      GAMM=GAM(IL,J)*(1.+WC(IL,J)/100.)
      GAMM=GAMM/1728.
      ALOAD(IL,J)=ALOAD(IL+1,J)+DY*GAMM
  100 CONTINUE
      NYY=NY-NBY+1
      DO 205 J=1,NBX
  205 ALOAD(NYY,J)=QQ
      DYY=0.
      ICOUNT=0
      DO 207 II=1,NYY
      DYY=DYY+DY
      I=NYY-II
      NXX=NBX+ICOUNT
      NOX=NXX-NBX
      RNOX=NOX
      DXX=RNOX*DX
      DISYY=0.5*DYY
```

```
      DO 206 J=1,NXX
      ALOAD(I,J)=(CQ*B*BLEN)/((BLEN+DYY)*(B+DYY))+ALOAD(I,J)
  206 CONTINUE
      IF(DYY.LE.DISTY)ICOUNT=ICOUNT+1
  207 CONTINUE
      READ 20,NNP,NEL,NCONF,NSTR
      B=1
  110 READ 35,M,XX(M),YY(M)
      IF(M-N)120,130,140
  120 PRINT 40,M
      GO TO 110
  140 DF=M+1-N
      RX=(XX(M)-XX(N-1))/DF
      RY=(YY(M)-YY(N-1))/DF
  150 XX(N)=XX(N-1)+RX
      YY(N)=YY(N-1)+RY
  130 CONTINUE
      N=N+1
      IF(M-N)160,130,150
  160 IF(N.LE.NNP)GO TO 110
  170 L=0
  175 READ 20,M,(IE(M,I),I=1,4)
  180 L=L+1
      IF(M-L)185,185,190
  190 IE(L,1)=IE(L-1,1)+1
      IE(L,2)=IE(L-1,2)+1
      IE(L,3)=IE(L-1,3)+1
      IE(L,4)=IE(L-1,4)+1
  185 CONTINUE
      IF(M-L)195,195,180
  195 IF(NEL-L)200,200,175
  200 CONTINUE
   20 FORMAT(8I10)
   25 FORMAT(8F10.3)
   27 FORMAT(///60H  SP GR    DRY DENSITY    LL      PI      INITIAL WC    V
     1ER PERM  /)
   30 FORMAT(4I5,5F10.2,E10.3)
   32 FORMAT(5F10.2,E10.3)
   35 FORMAT(I5,2F10.3)
   40 FORMAT(5X,26H ERROR IN ELEMENT CARD NO.,I5/)
      RETURN
      END
```

```
R0734 01   07-08-74    08.827

      SUBROUTINE UWEAVE
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),PERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      GAW=62.43
      DO 300 II=1,NY
      DO 300 JJ=1,NX
      IF(G(II,JJ).LE..01)GO TO 490
      WCH=.47*E(II,JJ)+2.
      WCL=.2*E(II,JJ)+9.
      VOLMIN=.223*PPI(II,JJ)-3.1
      VOLMAX=.34*PPI(II,JJ)-2.4
      IF(WC(II,JJ).GE.WCH)GO TO 310
      IF(WC(II,JJ).LE.WCL)GO TO 320
      S3=((VOLMAX-VOLMIN)*(WCH-WC(II,JJ))/(WCH-WCL))+VOLMIN
      GO TO 330
  310 S3=VOLMIN
      GO TO 330
  320 S3=VOLMAX
  330 CONTINUE
      EC=ALOAD(II,JJ)
      MM=0
      CALL BICUBE
      EI=((G(II,JJ)*GAW)/GAM(II,JJ))-1.
      E(II,JJ)=EI+((1.+EI)*UC)/100.
      GO TO 380
  490 E(II,JJ)=0.
  300 CONTINUE
      RETURN
      END
```

```
R0734 01  07-08-74   09.044
      SUBROUTINE SUCY
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),BERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      GAW=62.43
      PRINT 84
      DO 600 I=1,NNP
      XX(I)=(0.73)*XX(I)
  600 YY(I)=2.*YY(I)
      CALL SPLINE
      DO 650 II=1,NY
      DO 650 JJ=1,NX
      IF(II.EQ.NY.AND.JJ.GE.NBX)GO TO 640
      S3=PPI(II,JJ)
      EC=WC(II,JJ)
      IF(EC.LE.1.)GO TO 640
      MM=1
      CALL BIGUBE
      UC=2.30258*UC
      SU(II,JJ)=EXP(UC)-ALOAD(II,JJ)
      EEE=(1.+E(II,JJ))/G(II,JJ)
      EEE=EEE*1728./GAW
      DSW(II,JJ)=-ET*2.30258*SU(II,JJ)*EEE*100.
      EI=(G(II,JJ)*GAW)/GAM(II,JJ)-1.
      EE=(EI-E(II,JJ))/(1.+EI)
      IF(SU(II,JJ))630,630,645
  630 IF(EE)640,640,635
  635 DMV=-EE/SU(II,JJ)
      DSW(II,JJ)=1728./(GAW*DMV)
      GO TO 650
  640 SU(II,JJ)=0.
      DSW(II,JJ)=0.
      E(II,JJ)=EI
      GO TO 650
  645 IF(EE)650,640,640
  650 CONTINUE
      DO 660 II=1,NY
      DO 660 JJ=1,NX
      EI=((G(II,JJ)*GAW)/GAM(II,JJ))-1.
      WCF=(E(II,JJ)/G(II,JJ))*100.
      IF(E(II,JJ).EQ.EI)WCF=WC(II,JJ)
      IF(SU(II,JJ).LT.0..AND.SU(II,JJ+1).GT.0.)GO TO 648
      IF(SU(II,JJ).GT.0..AND.SU(II,JJ+1).LT.0.)GO TO 648
      GO TO 649
  648 SU(II,JJ)=0.
      DSW(II,JJ)=0.
      WCF=WC(II,JJ)
      E(II,JJ)=EI
  649 IF(SU(II,JJ).LT.0..AND.SU(II+1,JJ).GT.0.)GO TO 655
      IF(SU(II,JJ).GT.0..AND.SU(II+1,JJ).LT.0.)GO TO 655
      GO TO 651
```

```
  655 SU(II,JJ)=0.
      DSW(II,JJ)=0.
      WCF=WC(II,JJ)
      E(II,JJ)=EI
  651 WRITE(2,88)WC(II,JJ),WCF
      WCI=WC(II,JJ)
      PRINT 87,II,JJ,SU(II,JJ),DSW(II,JJ),WCI,WCF,ALOAD(II,JJ),EI,E(II,
     1JJ)
  660 CONTINUE
      REWIND 2
      PRINT 65
      DO 510 J=1,NX
      DELH=0.
      DO 520 I=2,NY
      IF(G(I,J).LE..01)GO TO 515
      EI=((G(I,J)*GAW)/GAM(I,J))-1.
      ER=E(I,J)-EI
      GO TO 516
  515 ER=0.
      EI=0.
  516 IF(I.EQ.2)ER=ER/2.
      DELH=DELH+(DY*ER)/((1.+EI)*3.)
      IF(J.EQ.1.OR.J.EQ.NX)GO TO 525
      GO TO 520
  525 PRINT 86,I,DELH
  520 CONTINUE
      PRINT 85,J,DELH
  510 CONTINUE
      LA=1
      YY(1)=-2.
      DO 700 I=2,NNP
      NN=(LA*NNP)/NSTR +1
      IF(I.EQ.NN)GO TO 705
      YY(I)=YY(I-1)
      GO TO 710
  705 YY(I)=YY(I-1)+1.
      LA=LA+1
  710 CONTINUE
  700 CONTINUE
   65 FORMAT(35H HORIZONTAL COOR      ULTIMATE HEAVE  /)
   84 FORMAT(100H II    JJ       SUCTION       DSUCTION    INITIAL WC
     1FINAL WC      LOAD          EI            EF       /)
   85 FORMAT(5X,I5,10X,F15.5)
   86 FORMAT(10X,I5,10X,F15.5)
   87 FORMAT(2I5,7F13.5)
   88 FORMAT(2F15.5)
      RETURN
      END
*****W  1470 EQUALITY OR NON-EQUALITY COMPARISON MAY NOT BE MEANINGFUL IN LOGICAL
            IF EXPRESSIONS
```

```
      SUBROUTINE TSTEP
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),PERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      DO 800 II=2,NY
      DO 800 JN=1,NX
      NBY1=NY-NBY+1
      NX1=NX+1
      JJ=NX1-JN
      IF(JJ.EQ.NX)GO TO 810
      IF(JJ.EQ.1)GO TO 860
      IF(JJ.LT.NBX.AND.II.EQ.NBY1)GO TO 870
      IF(JJ.EQ.NBX.AND.II.GT.NBY1)GO TO 860
      IF(JJ.LT.NBX.AND.II.GT.NBY1)GO TO 800
      IF(JJ.GE.NBX.AND.II.EQ.NY)GO TO 800
      RY=PERX(II,JJ)/PERX(II-1,JJ)
      RX=PERX(II,JJ)/PERX(II,JJ-1)
      BY=DT*DSW(II,JJ)*PERX(II,JJ)/DY
      BX=DT*DSW(II,JJ)*PERX(II,JJ)*PFACT/DX
      BX1=BX/DX
      BY1=BY/DY
      B1=1.+BX1+BY1
      B2=1.-BX1-BY1
      F1=(1.-RX)/(1.+RX)
      F2=(1.-RY)/(1.+RY)
      SY=SU(II+1,JJ)*(SU(II+1,JJ)-SU(II-1,JJ))*F2
      SX=SU(II,JJ-1)*(SU(II,JJ+1)-SU(II,JJ-1))*F1
      C1=B2*SU(II,JJ)
      C2=BX1*(SU(II-1,JJ)+SY)
      C3=BX1*(SU(II,JJ+1)+SX)
      SU(II,JJ)=(C1+C2+C3)/B1
      GO TO 810
  860 SU(II,JJ)=SU(II,JJ+1)
      GO TO 810
  870 SU(II,JJ)=SU(II-1,JJ)
  810 CONTINUE
      IF(SU(II,JJ))800,800,830
  830 CONTINUE
C     DERIVATIVE WRP TO WATER CONTENT
      S3=PPI(II,JJ)
      EC=SU(II,JJ)+ALOAD(II,JJ)
      IF(EC.LT..01)GO TO 770
      GO TO 775
  770 EC=.01
  775 EC=ALOG10(EC)
      MM=2
      CALL BICUBE
      EEE=(1.+E(II,JJ))/G(II,JJ)
      EEE=EEE**2.679
      DSW(II,JJ)=(1./ET)*SU(II,JJ)*100.*EEE**2.30258
  800 CONTINUE
```

```
      SUBROUTINE HEAVE
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),BERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      PRINT 93
      GAW=62.43
      DO 1100 II=1,NY
      DO 1100 JJ=1,NX
      IF(G(II,JJ).LE..01)GO TO 1130
      READ(2,91)WCI,WCF
      EI=((G(II,JJ)*GAW)/GAM(II,JJ))-1.
      EF=WCF*G(II,JJ)/100.
      ER=EF-EI
      IF(SU(II,JJ))1115,1117,1118
 1118 CONTINUE
      S3=PPI(II,JJ)
      EC=SU(II,JJ)+ALOAD(II,JJ)
      IF(EC.LE..0)GO TO 1110
      GO TO 1120
 1110 EC=.01
 1120 EC=ALOG10(EC)
      MM=0
      CALL BICUBE
      WR=(WCF-WCI)/100.
      QQ=G(II,JJ)*(WR/ER)
      AB=ER/G(II,JJ)
      IF(UC-WCI)1116,1117,1119
 1116 WS=(WCI-UC)/100.
      WW=WS/WR
      V=((1.+EI)/G(II,JJ))-AB*(WW**QQ)
      E(II,JJ)=V*G(II,JJ)-1.
      IF(WS.GE.WR)GO TO 1117
      GO TO 1130
 1119 WS=(UC-WCI)/100.
      WW=WS/WR
      V=((1.+EI)/G(II,JJ))+AB*(WW**QQ)
      E(II,JJ)=V*G(II,JJ)-1.
      IF(UC.GE.WCF)E(II,JJ)=EF
      GO TO 1130
 1117 E(II,JJ)=EI
      GO TO 1130
 1115 DMV=1728./(DSW(II,JJ)*GAW)
      EE=ER/(1.+EI)
      E(II,JJ)= (1.+EI)*DMV*((EE/DMV)-SU(II,JJ))+EI
 1130 DE=E(II,JJ)-EI
      PRINT 94,II,JJ,SU(II,JJ),DSW(II,JJ),E(II,JJ),DE
 1100 CONTINUE
      REWIND 2
   91 FORMAT(2F15.5)
   93 FORMAT(70H   II   JJ          SUCTION      DSUCTION     VOID RATIO
     1    DELTA E      /)
```

```
      SUBROUTINE SPLINE
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),SS,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),RERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      NCT=1
      N=NCONF
  210 ID=NCONF*NCT
      II=ID-NCONF+1
      JJ=NCONF-1
      DO 220 I=II,ID
      J=NCONF-JJ
      X(J)=XX(I)
      Y(J)=U(I)
  220 JJ=JJ-1
      CALL SOLV(N)
      IC=1
      DO 225 I=II,ID
      UX(I)=S(IC)
  225 IC=IC+1
      NCT=NCT+1
      IF(NCT.LE.NSTR)GO TO 210
      NCT=1
      N=NSTR
  230 IT=NCONF*(NSTR-1)
      ID=IT+NCT
      II=ID-IT
      JJ=NSTR-1
      DO 235 I=II,ID,NCONF
      J=NSTR-JJ
      X(J)=YY(I)
      Y(J)=U(I)
  235 JJ=JJ-1
      CALL SOLV(N)
      IC=1
      DO 240 I=II,ID,NCONF
      UY(I)=S(IC)
  240 IC=IC+1
      NCT=NCT+1
      IF(NCT.LE.NCONF)GO TO 230
      NCT=1
      N=NCONF
  243 ID=NCONF*NCT
      II=ID-NCONF+1
      JJ=NCONF-1
      DO 245 I=II,ID
      J=NCONF-JJ
      X(J)=XX(I)
      Y(J)=UY(I)
  245 JJ=JJ-1
      CALL SOLV(N)
      IC=1
```

```
      DO 250 I=II,ID
      UXY(I)=S(IC)
  250 IC=IC+1
      NCT=NCT+1
      IF(NCT.LE.NSTR)GO TO 243
      RETURN
      END
```

```
      SUBROUTINE SCLV(N)
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),XY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),PERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      DIMENSION A(20),B(20),C(20),D(20),P(20),Q(20)
      N1=N-1
      DO 2010 I=2,N
 2010 H(I)=X(I)-X(I-1)
      DO 2020 I=2,N
 2020 A(I)=1./H(I)
      A(1)=0.
      DO 2030 I=2,N1
      T1=1./H(I)+1./H(I+1)
 2030 B(I)=2.*T1
      B(1)=2.*(1./H(2))
      B(N)=2.*(1./H(N))
      DO 2040 I=1,N1
 2040 C(I)=1./H(I+1)
      C(N)=0.
      DO 2050 I=2,N1
      T1=(Y(I)-Y(I-1))/(H(I)*H(I))
      T2=(Y(I+1)-Y(I))/(H(I+1)*H(I+1))
 2050 D(I)=3.*(T1+T2)
      T1=(Y(2)-Y(1))/(H(2)*H(2))
      B(1)=3.*T1
      T2=(Y(N)-Y(N-1))/(H(N)*H(N))
      D(N)=3.*T2
C     FORWARD PASS
      Q(1)=C(1)/B(1)
      DO 2100 I=2,N1
      T1=B(I)-A(I)*Q(I-1)
 2100 Q(I)=C(I)/T1
      P(1)=D(1)/B(1)
      DO 2110 I=2,N
      TEM=D(I)-A(I)*P(I-1)
      T1=B(I)-A(I)*Q(I-1)
 2110 P(I)=TEM/T1
C     BACK SOLUTION
      S(N)=P(N)
      I=N-1
 2120 S(I)=P(I)-Q(I)*S(I+1)
      IF(I.EQ.1)GO TO 2150
      I=I-1
      GO TO 2120
 2150 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BICUBE
      COMMON MM,NNP,NEL,NCONF,NSTR,NX,NBX,NY,NBY,NTIME,DT,PFACT,DX,DY,
     1NOUT(10),X(20),Y(20),S(20),H(20),U(30),UX(30),UY(30),UXY(30),S3,
     2XX(30),YY(30),DSW(30,30),E(30,30),GAM(30,30),G(30,30),WC(30,30),
     3PPI(30,30),PERX(30,30),ALOAD(30,30),SU(30,30),IE(25,4),EC,ET,UC
      DO 400 M=1,NEL
      I=IE(M,1)
      J=IE(M,2)
      K=IE(M,3)
      L=IE(M,4)
      IF(S3.GE.XX(I).AND.S3.LE.XX(J))GO TO 410
      GO TO 400
  410 IF(EC.GE.YY(I).AND.EC.LE.YY(L))GO TO 420
  400 CONTINUE
  420 CONTINUE
      AA=XX(J)-XX(I)
      BB=YY(L)-YY(I)
      S3N=S3-XX(I)
      ECN=EC-YY(I)
      SL=S3N/AA
      T=ECN/BB
      S2=SL*SL
      S3=SL*SL*SL
      T2=T*T
      T3=T2*T
      F1=1.-3.*S2+2.*S3
      F2=S2*(3.-2.*SL)
      F3=AA*SL*(1.-SL)*(1.-SL)
      F4=AA*S2*(SL-1.)
      DO 430 KJ=1,2
      IF(MM.EQ.0)GO TO 440
      IF(KJ.EQ.2)GO TO 440
      G1=(-6.*T+6.*T2)/BB
      G2=(6.*T-6.*T2)/BB
      G3=3.*T2-4.*T+1.
      G4=3.*T2-2.*T
      GO TO 450
  440 G1=1.-3.*T2+2.*T3
      G2=T2*(3.-2.*T)
      G3=BB*T*(1.-T)*(1.-T)
      G4=BB*T2*(T-1.)
  450 H(1)=F1*G1*U(I)
      H(2)=F2*G1*U(J)
      H(3)=F2*G2*U(K)
      H(4)=F1*G2*U(L)
      H(5)=F3*G1*UX(I)
      H(6)=F4*G1*UX(J)
      H(7)=F4*G2*UX(K)
      H(8)=F3*G2*UX(L)
      H(9)=F1*G3*UY(I)
      H(10)=F2*G3*UY(J)
      H(11)=F2*G4*UY(K)
```

```
      H(12)=F1*G4*LY(L)
      H(13)=F3*G3*UXY(I)
      H(14)=F4*G3*UXY(J)
      H(15)=F4*G4*UXY(K)
      H(16)=F3*G4*UXY(L)
      IF(MM.EQ.0)GO TO 460
      IF(KJ.EQ.2)GO TO 460
      ET=0.
      DO 470 KK=1,16
  470 ET=ET+H(KK)
      IF(MM.EQ.2)GO TO 485
      GO TO 430
  460 UC=0.
      DO 480 KK=1,16
  480 UC=UC+H(KK)
      IF(MM.EQ.0)GO TO 485
  430 CONTINUE
  485 CONTINUE
      RETURN
      END
```

APPENDIX B: ADOPTED SUCTION-WATER CONTENT RELATIONSHIPS

1. Suction-water content data (Figure B1) provided for the mesh points are those of some remolded British soils with plastic properties approximated by[16*]

$$PI = 0.838LL - 14.2 \qquad \text{(B1)}$$

where LL = liquid limit

If the plastic properties of the soil are significantly different from those represented by Equation B1, an effective water content can be calculated by assuming that for any given plasticity index, soils with an equal consistency index ($I_c = \frac{LL - w}{PI}$) will have the same suction.[16] The effective water content can then be used to determine the suction properties or the converse.

---

* References are listed on pages 27-29.

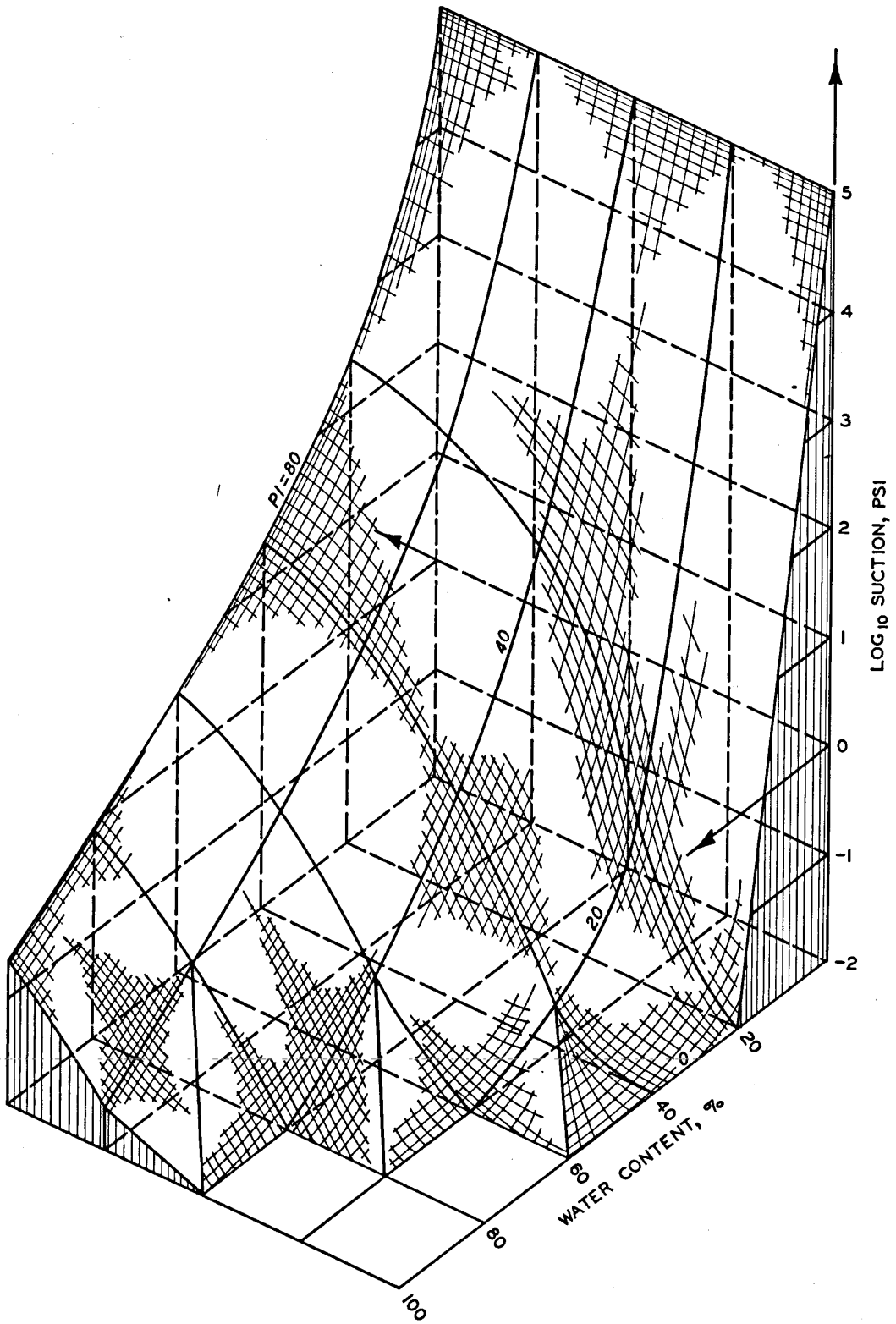


Figure B1. Representation of suction relationships by bicubic spline function (from Reference 16)

APPENDIX C: ADOPTED POTENTIAL VOLUMETRIC SWELL-SURCHARGE PRESSURE RELATIONSHIPS

1. The percent potential volumetric swell or shrinkage in situ PVS of foundation clay soils such as remolded Texas clays can be found by McDowell's procedure.[9*] For known values of surcharge pressure P and potential volumetric change at 1-psi surcharge pressure PVC, the PVS can be determined from a family of curves (Figure C1) represented by the bicubic spline function.[18]

2. The percent potential volumetric change at 1-psi surcharge pressure is represented by

$$PVC = \frac{100(PVMAX - PVMIN)\ (w_H - w)}{(w_H - w_L)} + PVMIN \qquad (C1)$$

where

PVMAX = .34PI - 2.4

PVMIN = .223PI - 3.1

PI = plasticity index

w = percent initial water content

The foregoing expressions for percent maximum volume swell PVMAX and percent minimum volume swell PVMIN at 1-psi pressure are found to approximately fit the available data.[9] The initial water content w must be within the range of the high (optimum) water content w_H and the low water content w_L, where

$$w_H = 0.47LL + 2 \qquad (C1a)$$

$$w_L = 0.2LL + 9 \qquad (C1b)$$

The percent high water content w_H corresponds to the maximum capillary sorption obtained by laboratory tests on specimens molded at

---

* References are listed on pages 27-29.

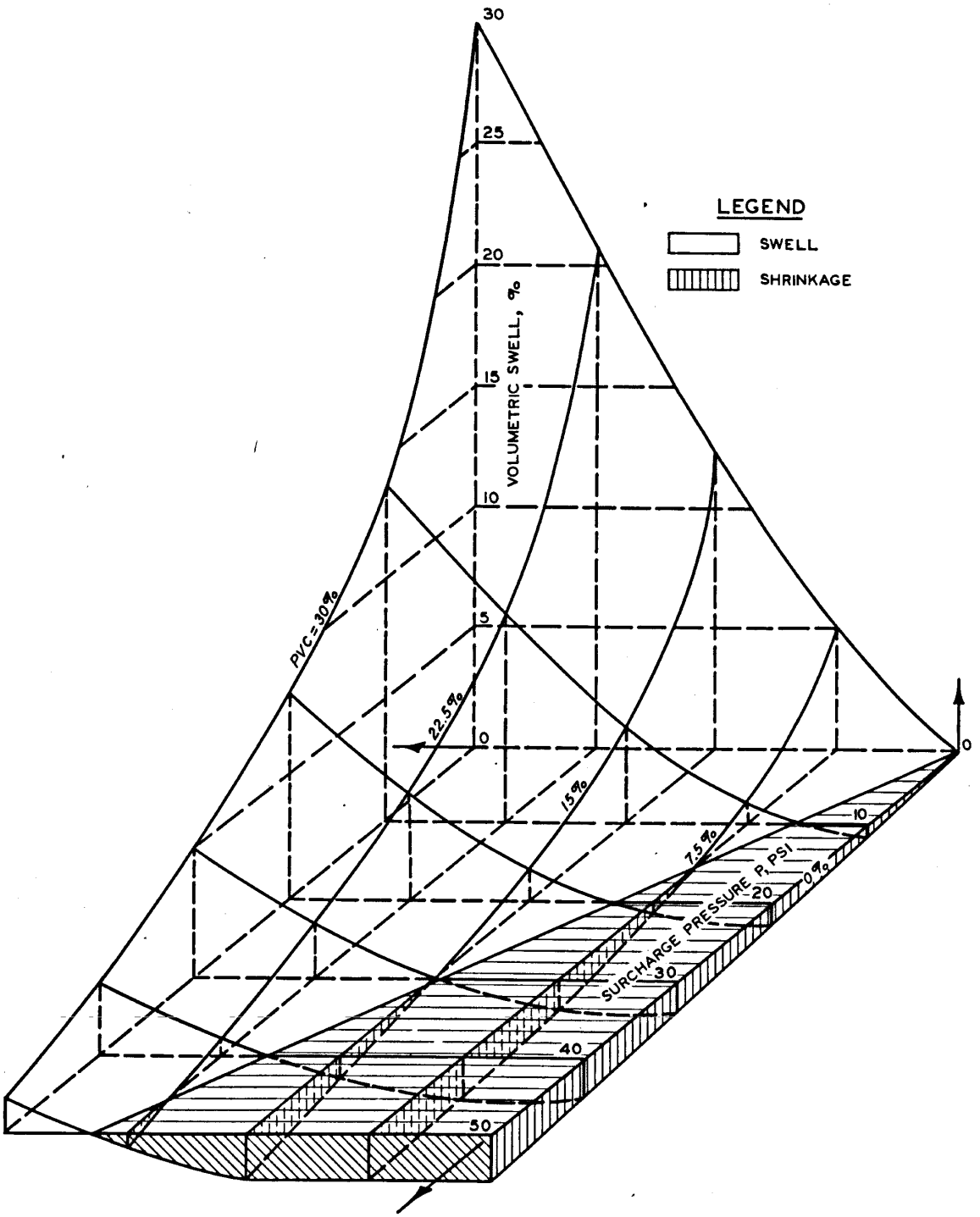


Figure C1. Representation of swell relationships by bicubic spline function (from Reference 9)

optimum. The percent low water content w_L represents the minimum moisture contents from which swelling clays usually expand.[9] Moisture content had generally shown that soils will approach the higher value w_H or lower value w_L below a thin, impermeable, top stratum.[9]

3. The accumulative heave at the ground surface is denoted as the potential vertical rise PVR and computed by the code as

$$PVR = \frac{1}{3} \sum_{i=0}^{n} (\Delta y_i)\ (PVS_i) \qquad (C2)$$

where

n = number of vertical increments in the mesh

Δy = dimension of vertical increment i , inches

APPENDIX D: NOTATION

ADE	Alternating direction explicit
BLEN	Total length of structure
C	A parameter
D	Depth
e	Void ratio
e_o	Initial void ratio
e^t	Void ratio at each mesh point and at time t
FD	Finite difference
G_s	Specific gravity
H	Total head
I_c	Consistency index
k_{ij}	Permeability tensor--the proportion of velocity of flow in the ith direction contributed by a change in total head in the jth direction
k_{i3}	The proportion of velocity of flow in the ith direction contributed by a change in total head H in the vertical direction
k_x	Coefficient of horizontal permeability
k_y	Coefficient of vertical permeability
L	Length
LL	Liquid limit
m_v	Coefficient of volume change
n	Number of soil layer
P	Surcharge pressure; also vapor pressure of the pore water extract
P_o	Vapor pressure of free pure water
PI	Plasticity index
PVC	Percent volume change at 1 psi
PVMAX	Percent maximum volume swell at 1 psi
PVMIN	Percent minimum volume swell at 1 psi
PVR	Potential vertical rise
PVS	Percent in situ potential volumetric swell or shrinkage
Q	A constant = $\left[(w_{max} - w_o)/(V_{TP} - V_{TI}) \times 100\right]$
R	Universal gas constant

t	Time
T	Absolute temperature
v_w	Volume of a mole of liquid water
V	Volume
V_T^t	Total specific volume at each mesh point and at time t
V_{TI}	Total initial specific volume
V_{TP}	Maximum total specific volume
w	Water content, percent dry weight
w_H	High (optimum) water content, percent dry weight
w_L	Low water content, percent dry weight
w_{max}	$\left[(V_{TP}G_s - 1)/G_s\right] \times 100$
w_o	Initial water content, percent dry weight
x	Horizontal coordinate
x_i	Coordinate in the ith direction
x_j	Coordinate in the jth direction
y	Vertical coordinate
α	Compressibility factor
γ_{do}	Initial dry density
γ_w	Unit weight of water
Δe	Ultimate change in void ratio from the initial void ratio
Δt	Time increment
ΔV	Change in volume
Δx	Horizontal increment
Δy	Vertical increment
$\Delta\sigma$	Ultimate change in effective stress or the initial positive pore water pressure
Δ_{yi}	Dimension of vertical increment i, inches
θ	Volumetric water content, percent
σ	Effective stress or initial positive pore water pressure
τ	Total suction
τ_m	Matrix suction
τ_m^o	Initial in situ matrix suction
$\bar{\tau}_m^o$	Initial matrix suction at zero surcharge pressure

$\tau_{m_{i,j}}$ Matrix suction at point (x,y)

τ_s Osmotic suction