CONTRACT REPORT S-71-6

THREE-DIMENSIONAL FINITE ELEMENT ANALYSES OF DAMS

by

G. Lefebvre, J. M. Duncan

May 1971

Sponsored by Office, Chief of Engineers, U. S. Army

Conducted for U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi

Under Contract No. DACW39-68-C-0078

By College of Engineering, Office of Research Services
University of California, Berkeley, California

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THREE-DIMENSIONAL FINITE ELEMENT ANALYSES OF DAMS

A Report of an Investigation

by

Guy Lefebvre

and

J. M. Duncan

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U. S. Army Engineers Waterways Experiment Station

CORPS OF ENGINEERS

Vicksburg, Mississippi

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FOREWORD

The work described in this report was performed under Contract No. DACW39-68-C-0078 "Behavior of Zoned Embankments and Embankments on Soft Foundations" between the U. S. Army Engineer Waterways Experiment Station and the University of California. This is the second report on investigations performed under this contract. The first report "Finite Element Analyses of Stresses and Movements in Embankments During Construction," by F. H. Kulhawy, J. M. Duncan, and H. Bolton Seed, was published in November, 1969. The research was sponsored by the Office, Chief of Engineers, under the Civil Works Investigations Engineering Studies 525, "Shear Characteristics of Undisturbed Weak Clays."

The general objective of this research, which was begun in June, 1968, is to develop methods for analysis of stresses and movements in embankments. Work on this project is conducted under the supervision of Professor J. M. Duncan, Associate Professor of Civil Engineering and Professor H. Bolton Seed, Professor of Civil Engineering. The project is administered by the Office of Research Services of the College of Engineering. The phase of the investigation described in this report was conducted, and the report was prepared, by Dr. Guy Lefebvre and J. M. Duncan.

The contract was monitored by Mr. D. C. Banks, Chief, Rock Mechanics Section, Soil and Rock Mechanics Branch, under the general supervision of Mr. J. P. Sale, Chief, Soils Division. Contracting Officer was COL Ernest D. Peixotto, CE, Director, U. S. Army Engineer Waterways Experiment Station.

SUMMARY

Three-dimensional analyses of dams were performed using a computer program developed recently by Wilson (1970). Three dams in V-shaped valleys were analyzed using incremental analysis procedures, in which construction of the dams was simulated in eight steps, and the results were compared with the results of two-dimensional incremental analyses.

The comparisons show that the stresses and displacements in the transverse section calculated by plane strain analyses agree closely with the results of three-dimensional analyses of dams in valleys having valley wall slopes as flat as 3:1 or flatter. The values of stress and displacement in the longitudinal section calculated by plane strain analyses agreed closely with the results of three-dimensional analyses for all three valley wall slopes analyzed (1:1, 3:1, and 6:1), but the results of plane stress analyses of the longitudinal section were found to differ significantly from the results of the three-dimensional analyses.

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LIST OF SYMBOLS

E	Young's modulus
u_v	vertical displacement
u_h	horizontal displacement
γ	unit weight
ν	Poisson's ratio
σ_1	major principal stress
σ_3	minor principal stress
τ_{max}	maximum shear stress

INTRODUCTION

Clough and Woodward (1967) developed procedures for performing finite element analyses of stresses and movements in earth dams during construction, and similar procedures have subsequently been employed in studies performed by Finn (1967); Kulhawy, Duncan and Seed (1969); and Kulhawy and Duncan (1970). The analyses performed by Clough and Woodward, as well as most of the subsequent analyses of dams, were two-dimensional plane strain analyses. Plane strain conditions are ideally representative of transverse sections in long dams of uniform cross-section, wherein there are no strains or movements normal to the plane analyzed. Plane strain analysis procedures have also been used by Covarrubias (1969) and by Casagrande and Covarrubias (1970) for analyses of the longitudinal sections of dams, to investigate tensile stresses which may lead to cracking.

The purpose of the study described in this report was to perform three-dimensional analyses of dams in V-shaped valleys, as shown in Fig. 1, and to compare the results of these analyses with the results obtained by performing two-dimensional analyses of the transverse and longitudinal sections. From these comparisons it has been possible to evaluate the accuracy of two-dimensional analyses.

COMPUTER PROGRAMS AND FINITE ELEMENT MESHES

The three-dimensional analyses conducted during this study were performed using a finite element computer program (SAP) which was developed by Wilson (1970). This computer program employs hexahedral or "brick" elements with six plane faces and eight nodal points at their vertices. Displacements are calculated at each nodal point and stresses are calculated at the centroid of each element and at the center of each of its faces. This computer program operates very efficiently; the last step of the incremental analyses, which involved a finite element mesh with 156 nodal points and a band width of 66, was performed in about 75 seconds of CDC 6400 time, or about 25 seconds of CDC 6600 time.

The plane strain analyses of the transverse section were also performed using SAP, but both the plane strain and the plane stress analyses of the longitudinal section were performed using a modified version of an incremental analysis computer program (LSBUILD) which was developed by Kulhawy, et. al. (1969) in previous studies conducted under this contract. See Appendix II for a comparison of the results calculated using these two computer programs.

Analyses were performed for dams in V-shaped valleys with three different valley wall slopes, 1:1 (1 horizontal: 1 vertical), 3:1, and 6:1

Valley Wall
Slope = 1:1

Valley Wall
Slope = 3:1

Valley Wall
Slope = 6:1

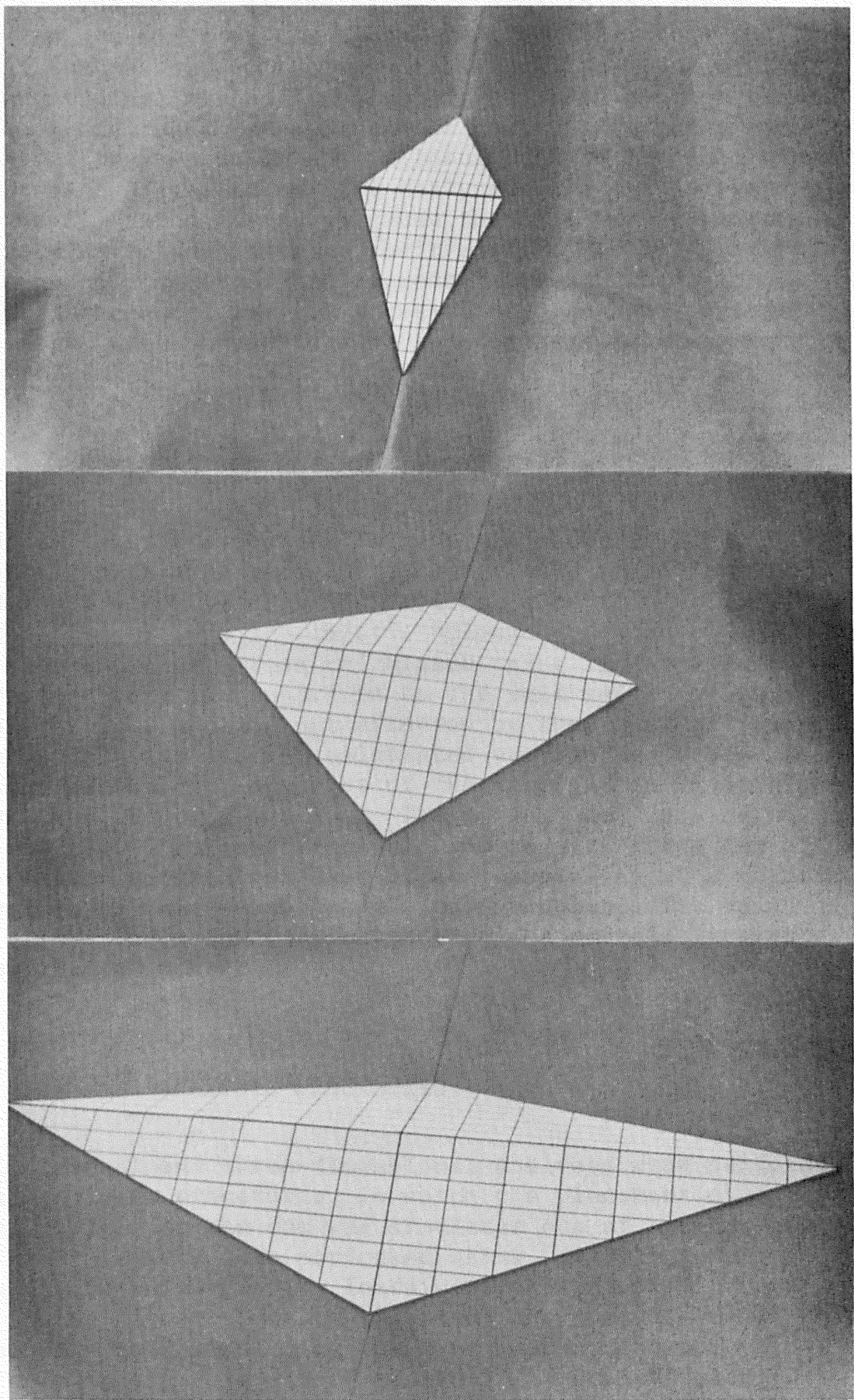

Fig. 1 PHOTOGRAPHS OF DAMS ANALYZED

as shown in Fig. 1. The shapes of the elements used in the three-dimensional analyses are shown by the lines drawn on the models in Fig. 1. The finite element meshes used in the two-dimensional analyses of the transverse and longitudinal sections, which had the same configurations in these sections as did the three-dimensional meshes, are shown in Figs. 2, 3, and 4. It may be noted that although the valley wall slopes were different for each of the three dams analyzed, the fill slopes were the same (2.5:1) in all three cases. Because the dams were symmetrical with respect to vertical planes through their maximum sections and through their crests, it was only necessary to represent one-fourth of the dam in the finite element meshes used for the three-dimensional analyses and one-half of the dam in the meshes used for the two-dimensional analyses.

DIMENSIONS AND MATERIAL PROPERTIES

Each of the dams analyzed was 160 ft high, as shown in Figs. 2, 3, and 4. Linear stress-strain characteristics were used in the analyses, with Young's modulus $E = 100$ tons/ft^2 and Poisson's ratio $\nu = 0.40$. The unit weight of the fill, γ, was 125 lb/ft^3. Clough and Woodward (1967) have shown that the values of stress calculated using linear stress-strain characteristics vary in proportion to the unit weight of the fill and the height of the dam, and are not affected by the value of Young's modulus. The values of displacement calculated vary in proportion to the unit weight and the square of the height of the dam, and in inverse proportion to the value of Young's modulus. These facts may be used to derive from the results of a linear analysis the results for a dam of similar shape, but with a different height, and consisting of a material with different values of unit weight and Young's modulus. The value of Poisson's ratio also affects the calculated stresses and displacements, but in a more complicated manner.

ANALYSIS PROCEDURES

Both two-dimensional and three-dimensional analyses were performed using incremental analysis procedures, in which the placement of successive layers of fill in the dam was simulated one at a time. Some two-dimensional analyses were also performed using "gravity turn-on" analysis procedures, in which gravity loads were applied simultaneously over the entire finite element mesh representing the complete dam. Both types of analyses were performed using the same meshes, shown in Figs. 2, 3, and 4, which contain eight layers of elements. The incremental analyses were performed using eight steps or increments, each one representing placement of one layer of elements. In the analyses performed using SAP, placement of a new layer was simulated by applying a pressure to the surface of the previous layer, the magnitude of the pressure being equal to the overburden pressure or the pressure which

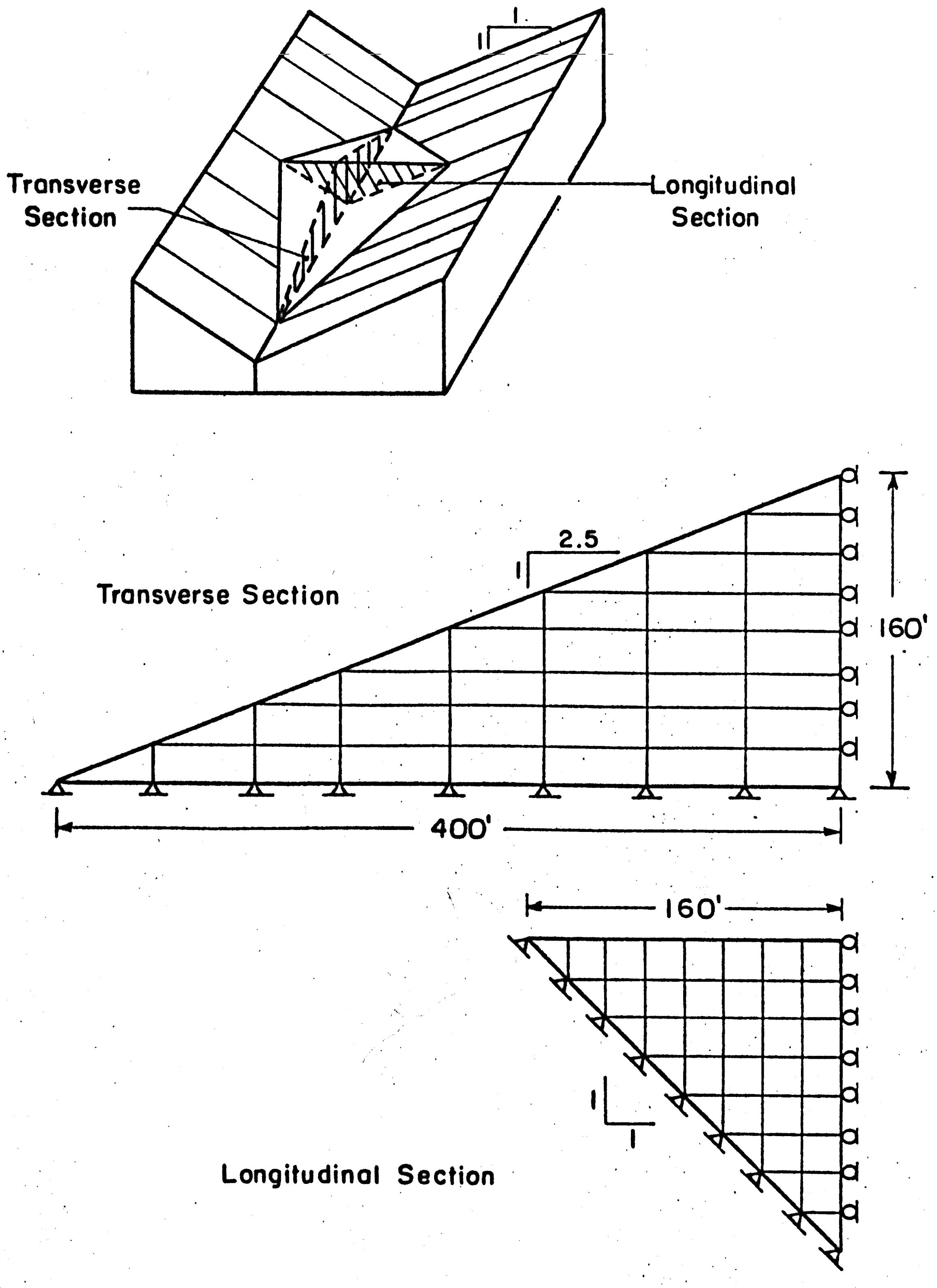

Fig. 2 FINITE ELEMENT MESH FOR DAM WITH 1:1 VALLEY WALL SLOPE

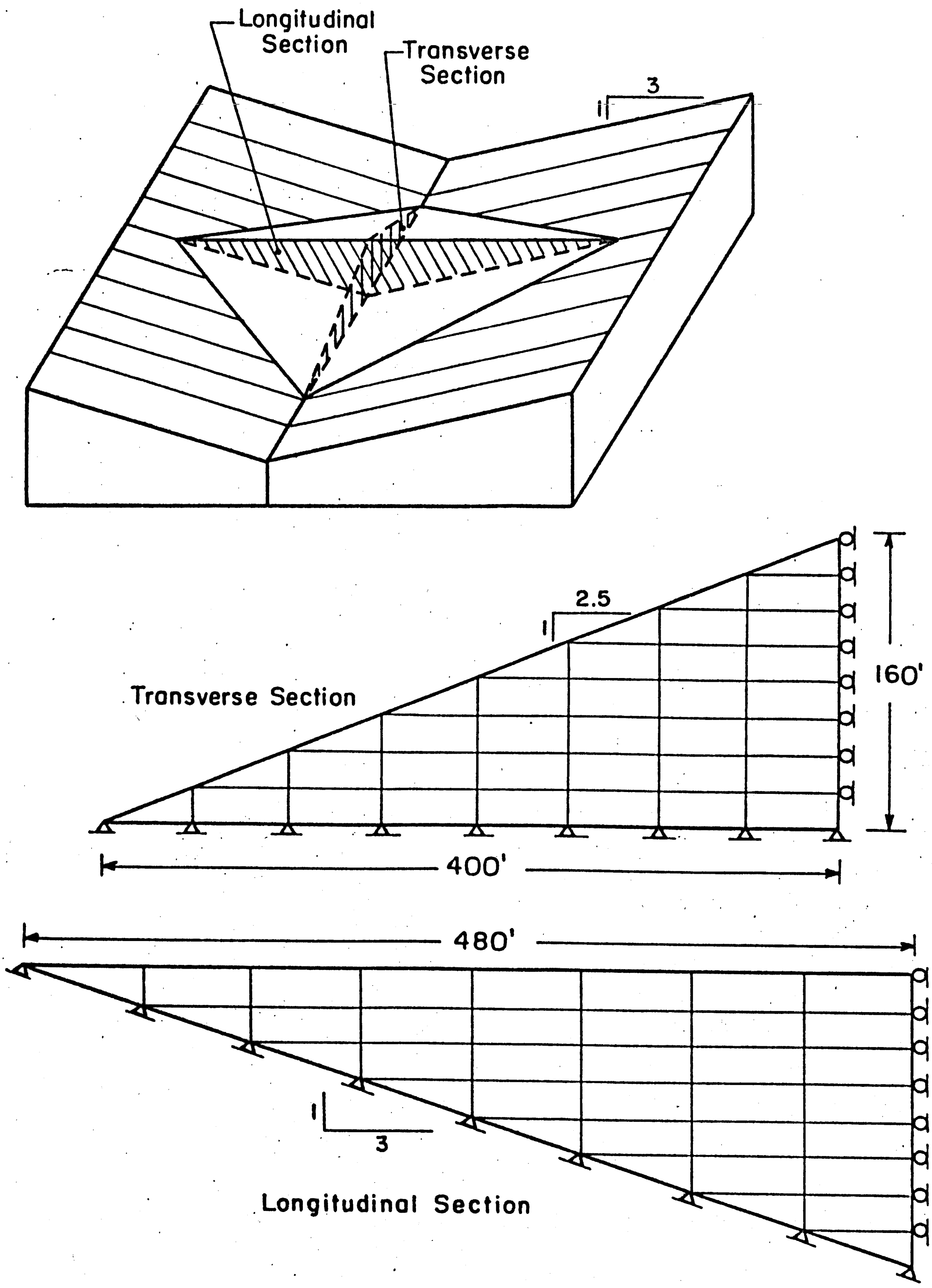

Fig. 3 FINITE ELEMENT MESH FOR DAM WITH 3:1 VALLEY WALL SLOPE

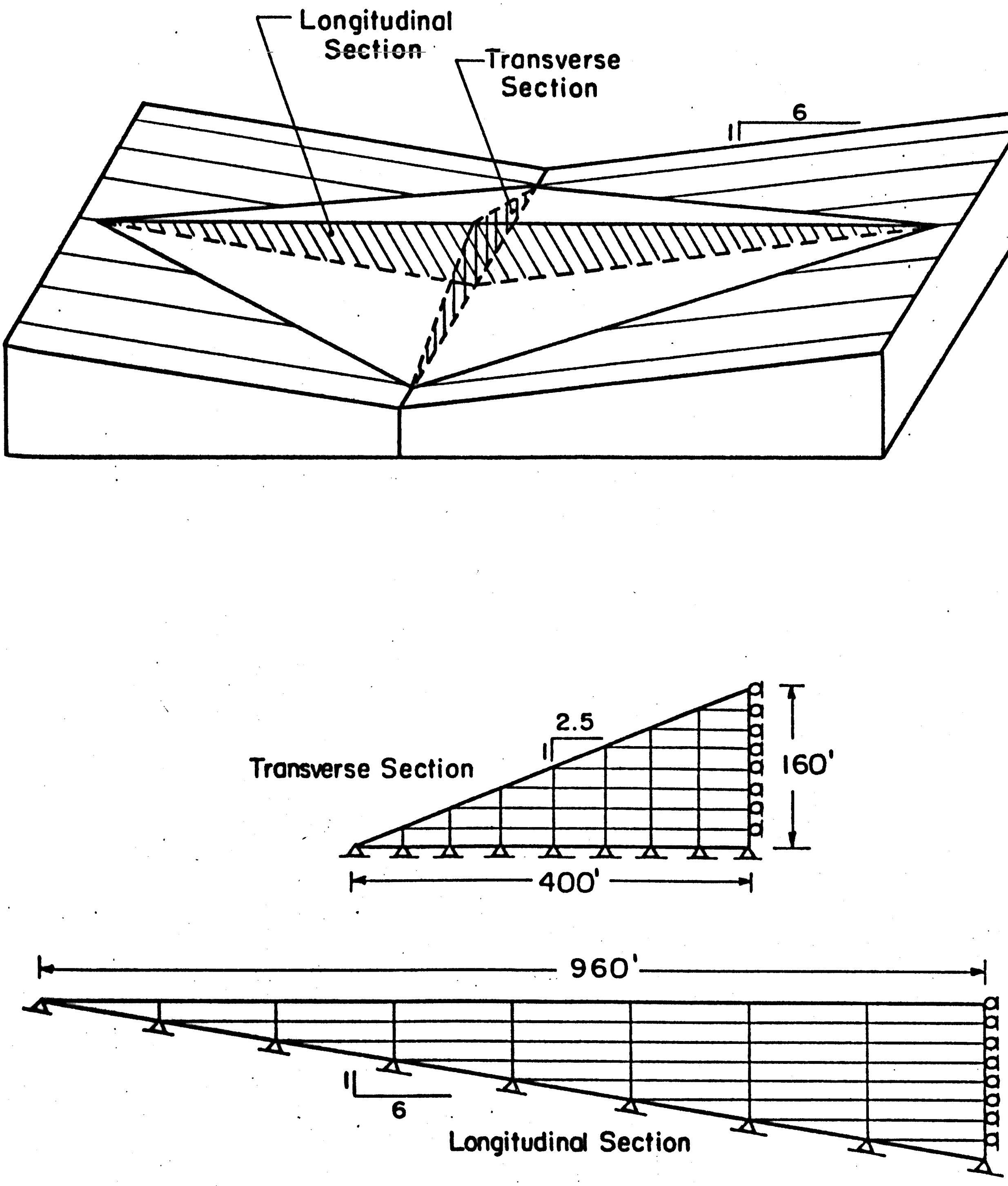

Fig. 4 FINITE ELEMENT MESH FOR DAM WITH 6:1 VALLEY WALL SLOPE

would have been exerted by a liquid having the same density. In the analyses performed using LSBUILD, placement of a new layer was simulated using the "dense liquid" technique described by Kulhawy, et. al. (1969). Different procedures were used in the two cases for reasons of convenience, but the two procedures are virtually identical with respect to results, and the fact that the procedures were different could not give rise to significant differences in calculated values of stress or displacement. In each case newly placed elements were assigned values of vertical stress commensurate with the overburden pressure at their centroids and values of horizontal stress equal to the overburden pressure multiplied by $\nu/(1-\nu)$. The displacements at the upper side of newly placed elements were set equal to zero, in accordance with procedures developed in previous studies (Kulhawy, et. al., 1969).

COMPARISON OF INCREMENTAL AND GRAVITY TURN-ON TWO-DIMENSIONAL ANALYSES OF THE TRANSVERSE SECTION

The results of two-dimensional gravity turn-on and incremental analyses of the transverse section are compared in Figs. 5 and 6. The values of major principal stress (σ_1) calculated by these two procedures may be seen to be very nearly the same, whereas the values of minor principal stress (σ_3) calculated using the incremental analysis procedures are somewhat smaller than those calculated using gravity turn-on procedures. The values of horizontal displacement (u_h) shown in the lower part of Fig. 6, are very similar for both analyses, but the values of vertical displacement (u_v) shown in the upper part of the same figure, are considerably different. The values of vertical displacement calculated by means of the gravity turn-on analysis increase continually from the bottom to the top of the cross-section, whereas those calculated by means of the incremental analysis are largest near mid-height, as are the during-construction settlements measured in actual dams. Because of the closer correspondence between the results of incremental analyses and settlements measured in dams, the comparisons of the results of two-dimensional and three-dimensional analyses were made using the results of incremental analyses.

RESULTS FOR THE TRANSVERSE SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES

Values of stress and displacement in the transverse section calculated using three-dimensional incremental analyses are shown in Figs. 7 through 12. In each figure, results are shown for the three valley wall slopes analyzed, with the results for the 1:1 valley wall slope at the top, the 3:1 slope in the center and the 6:1 slope at the bottom.

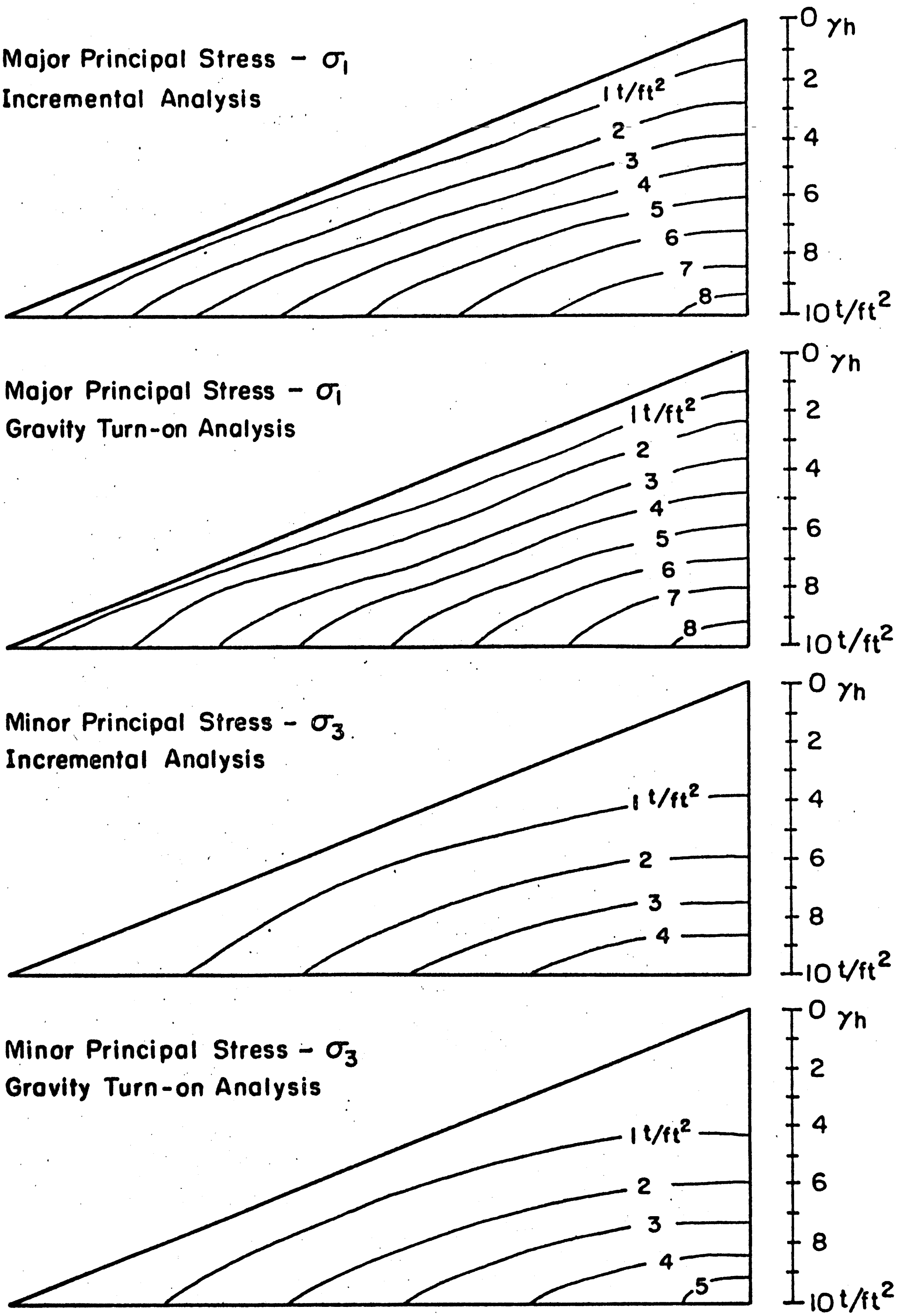

Fig. 5 COMPARISON OF STRESSES IN THE TRANSVERSE SECTION CALCULATED USING INCREMENTAL AND GRAVITY TURN-ON TWO-DIMENSIONAL PLANE STRAIN ANALYSES

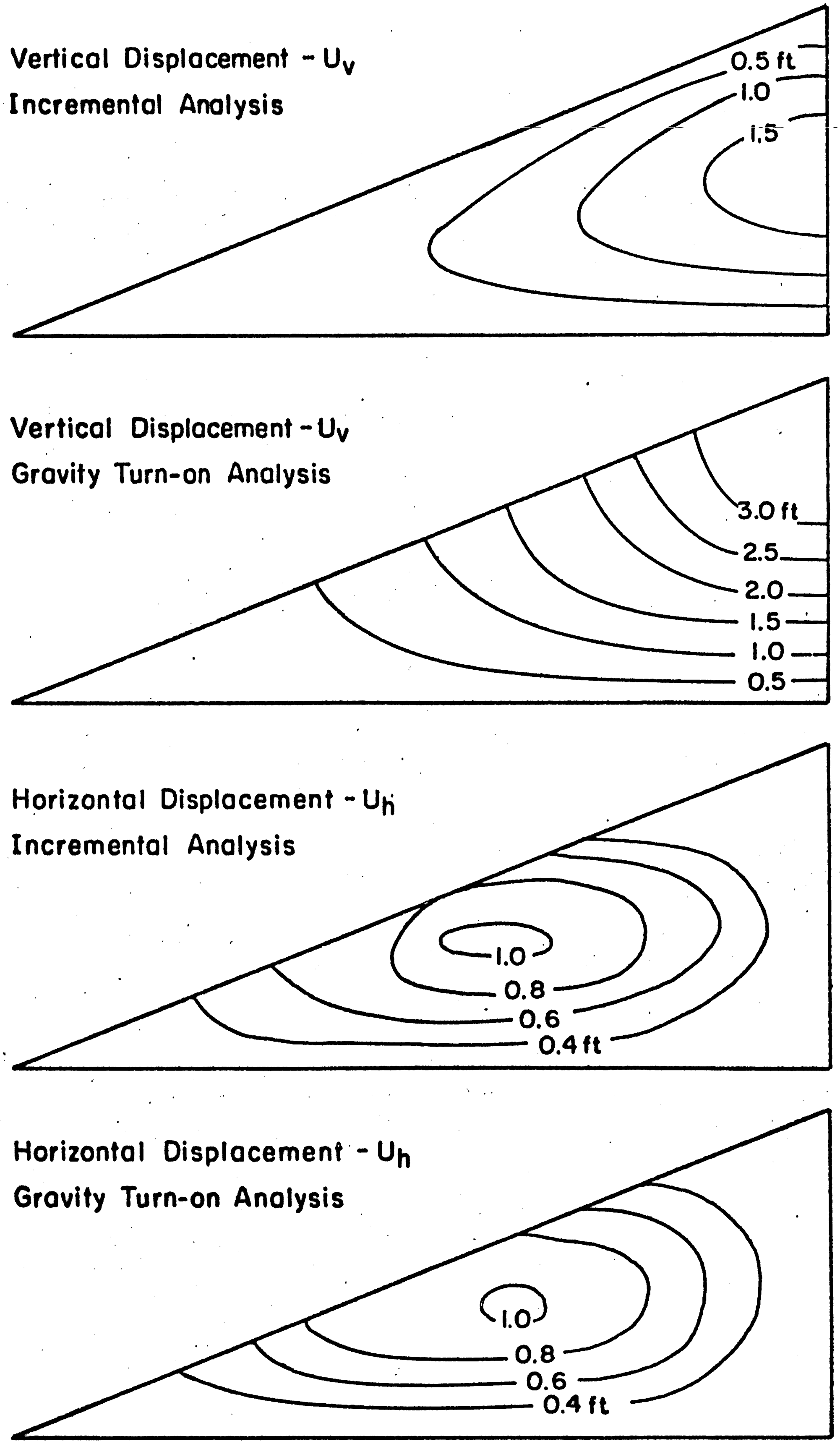

Fig. 6 COMPARISON OF DISPLACEMENTS IN THE TRANSVERSE SECTION CALCULATED USING INCREMENTAL AND GRAVITY TURN-ON TWO-DIMENSIONAL PLANE STRAIN ANALYSES

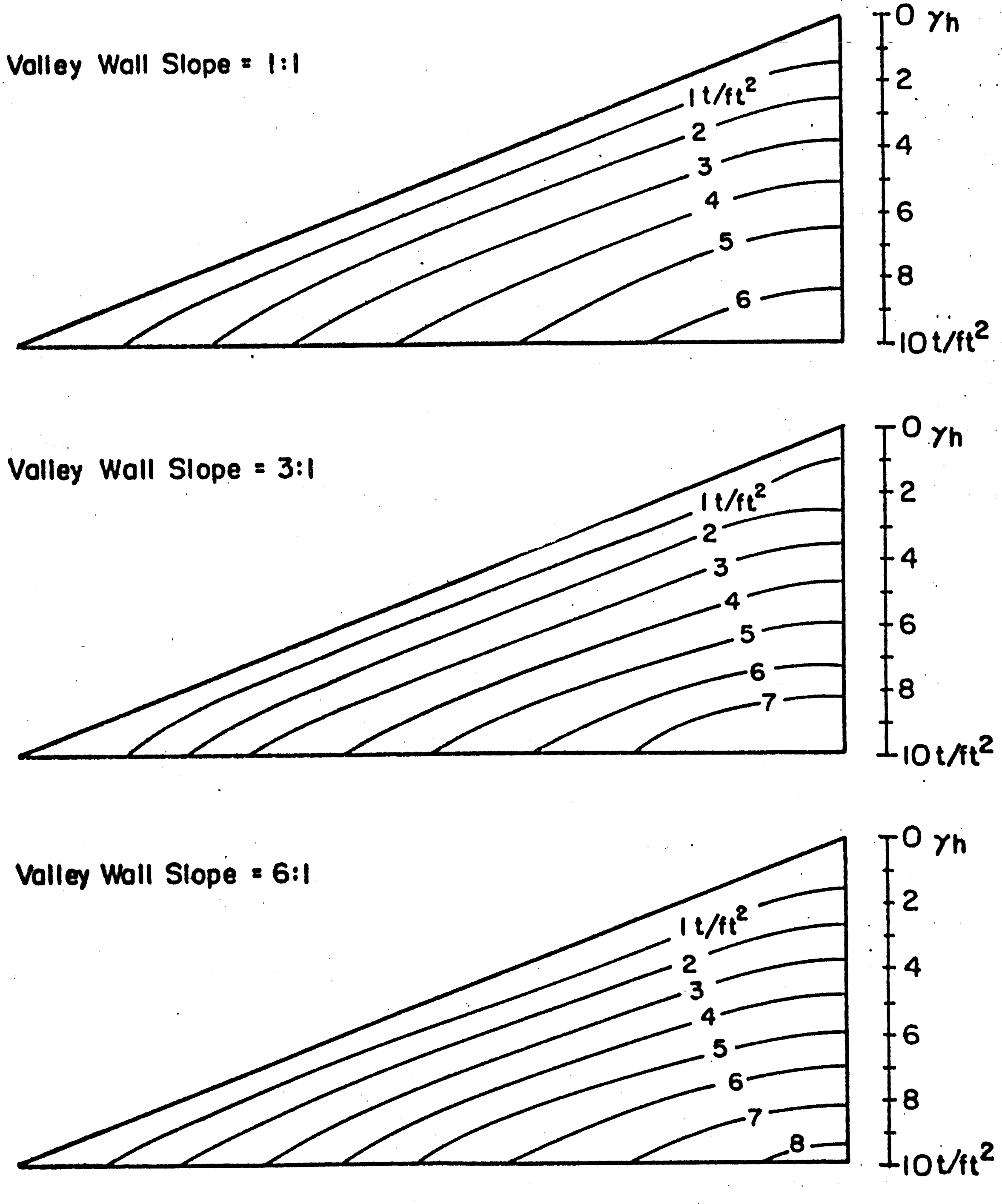

Fig. 7 CONTOURS OF VALUES OF MAJOR PRINCIPAL STRESS (σ_1) IN THE TRANSVERSE SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

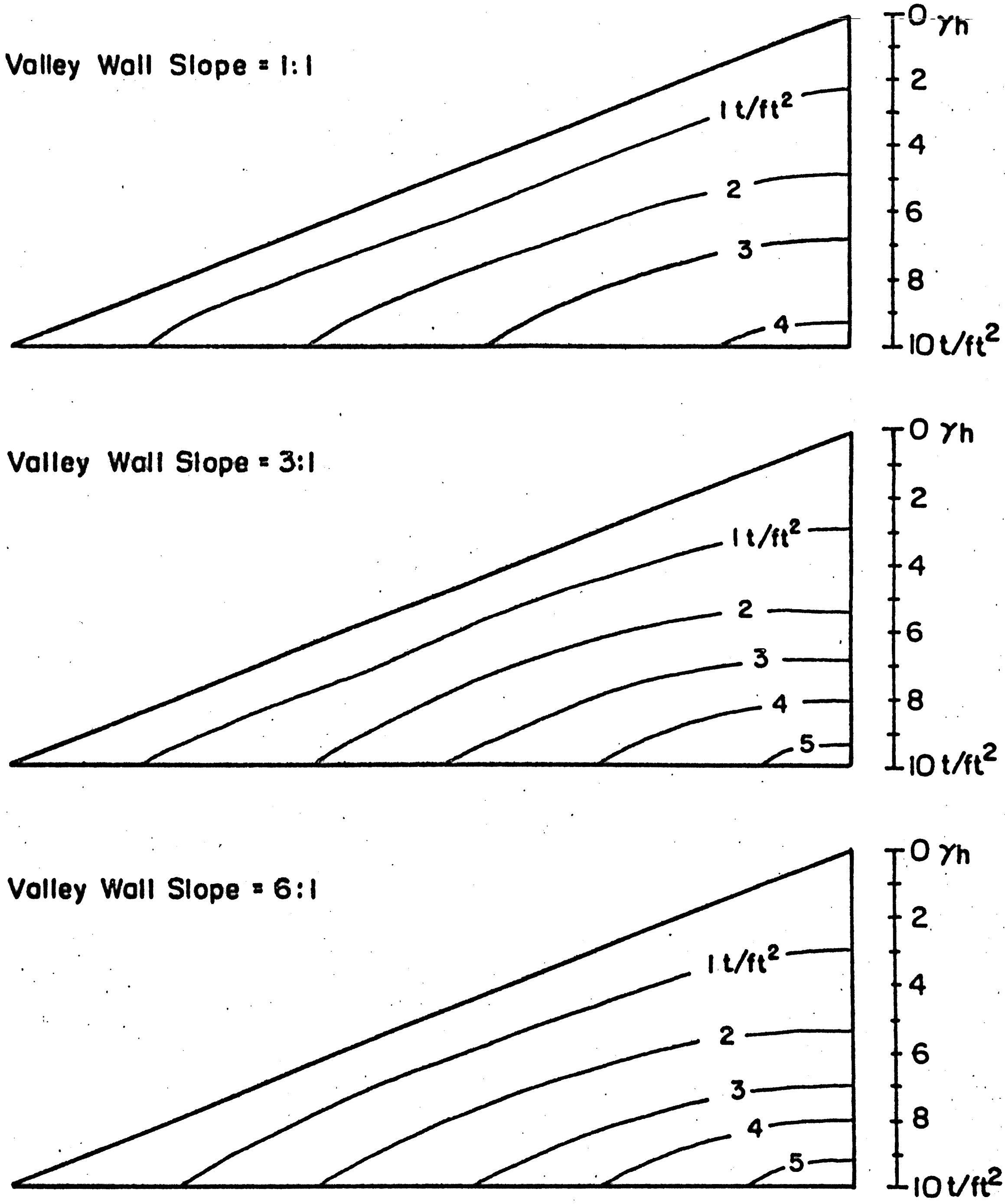

Fig. 8 CONTOURS OF VALUES OF MINOR PRINCIPAL STRESS (σ_3) IN THE TRANSVERSE SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

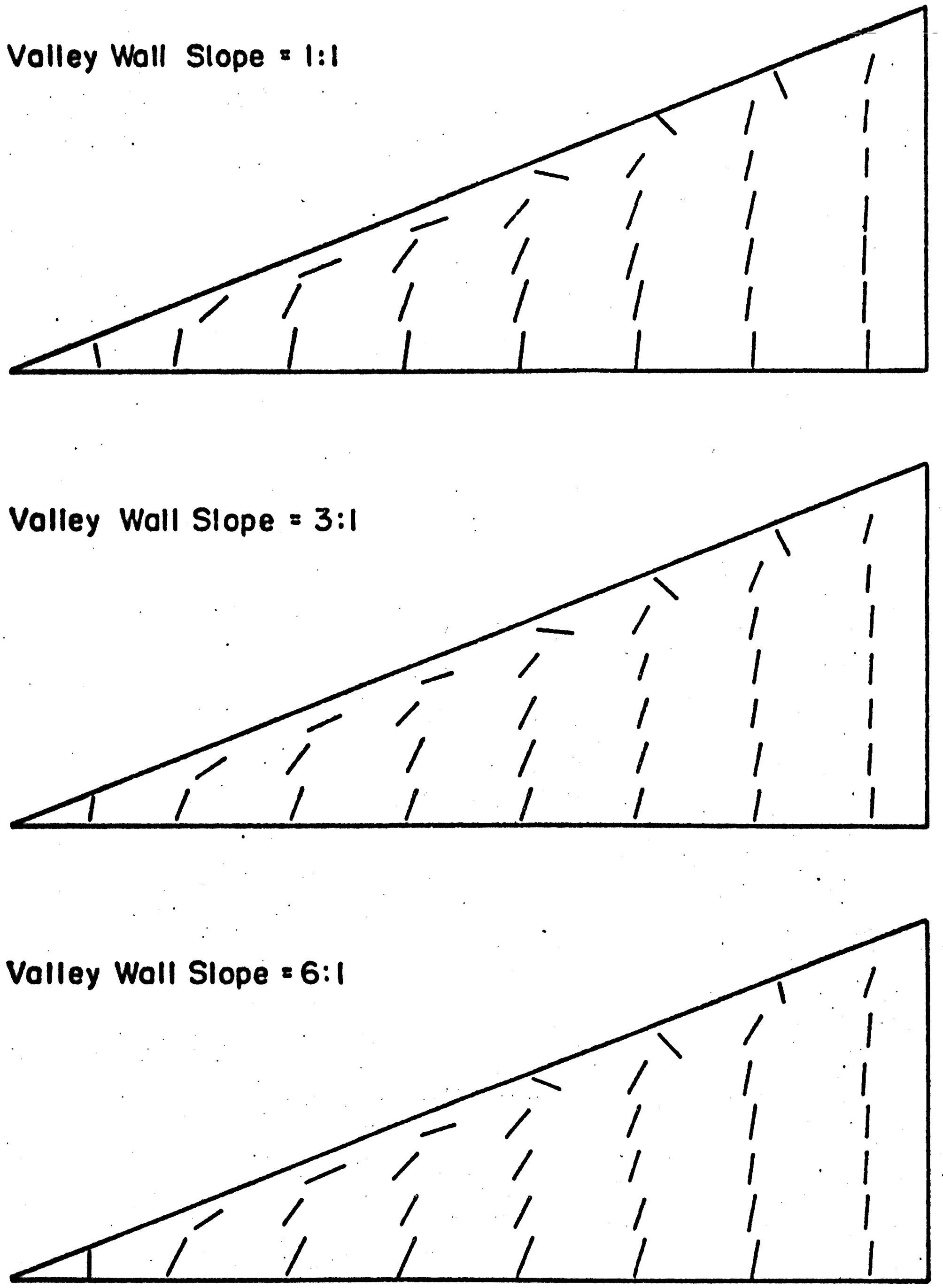

Fig.9 MAJOR PRINCIPAL STRESS ORIENTATIONS IN THE TRANSVERSE SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

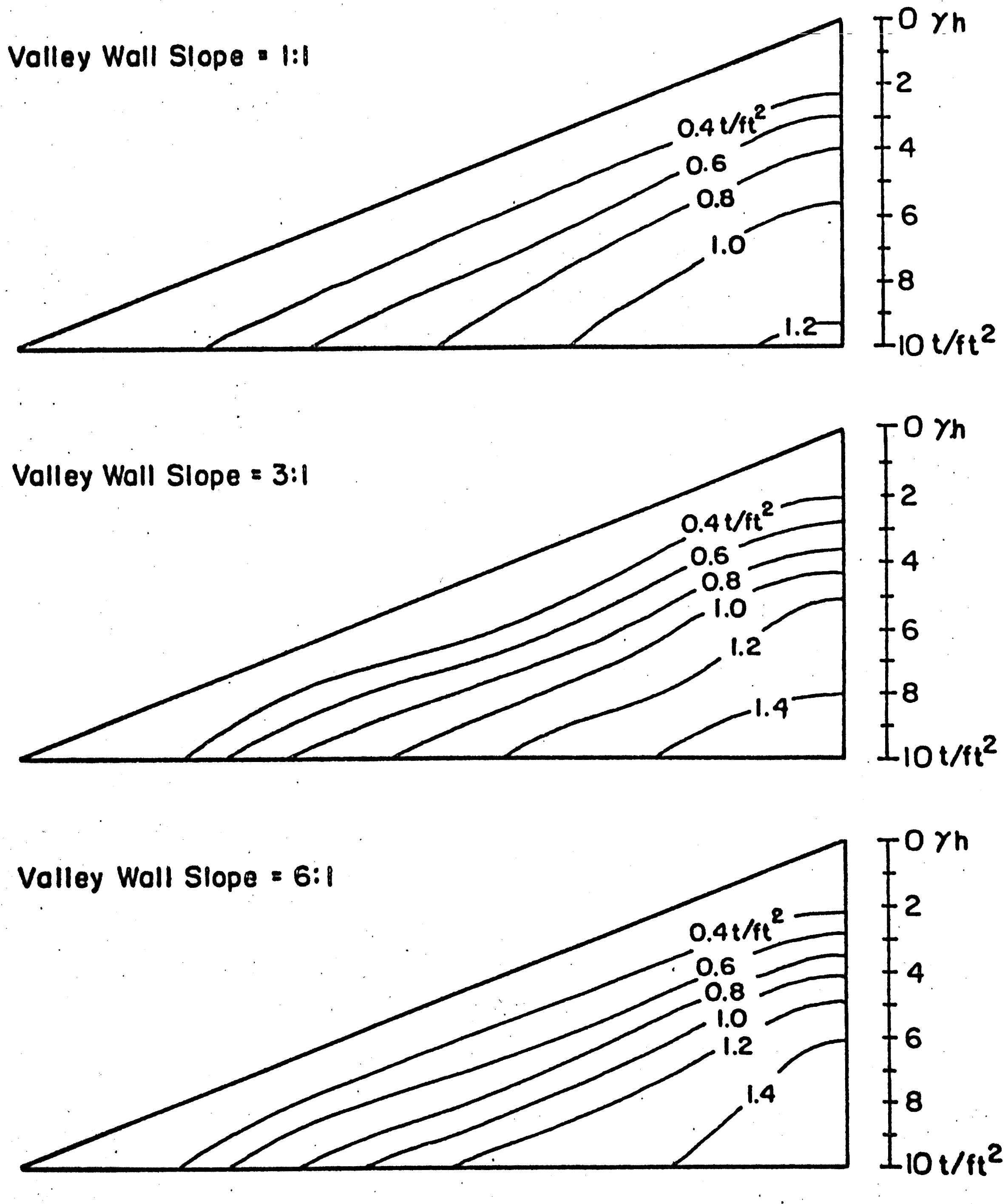

Fig. 10 CONTOURS OF MAXIMUM SHEAR STRESS (τ_{max}) IN THE TRANSVERSE SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

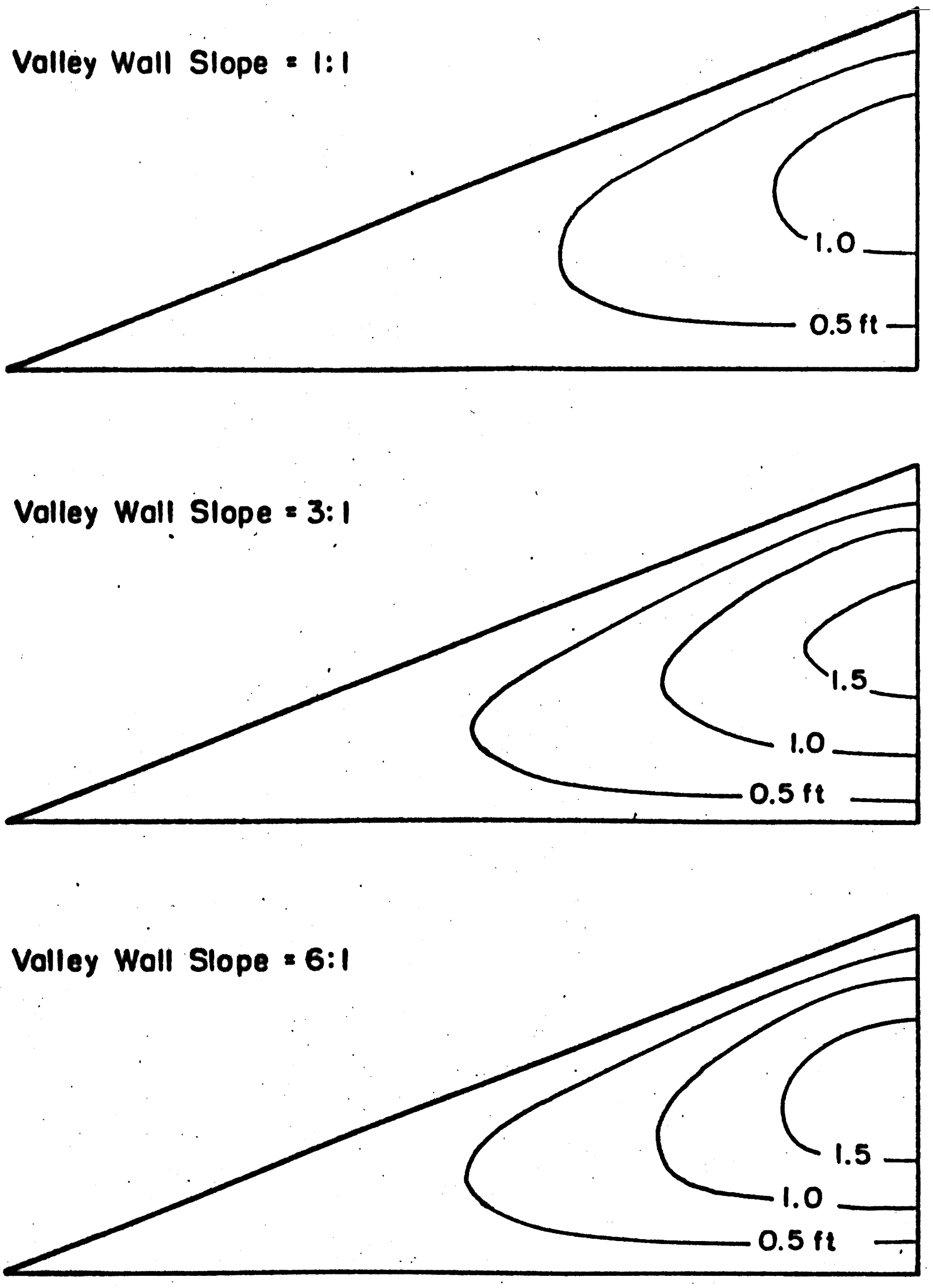

Fig. 11 CONTOURS OF VALUES OF VERTICAL DISPLACEMENT IN THE TRANSVERSE SECTION (u_v) CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

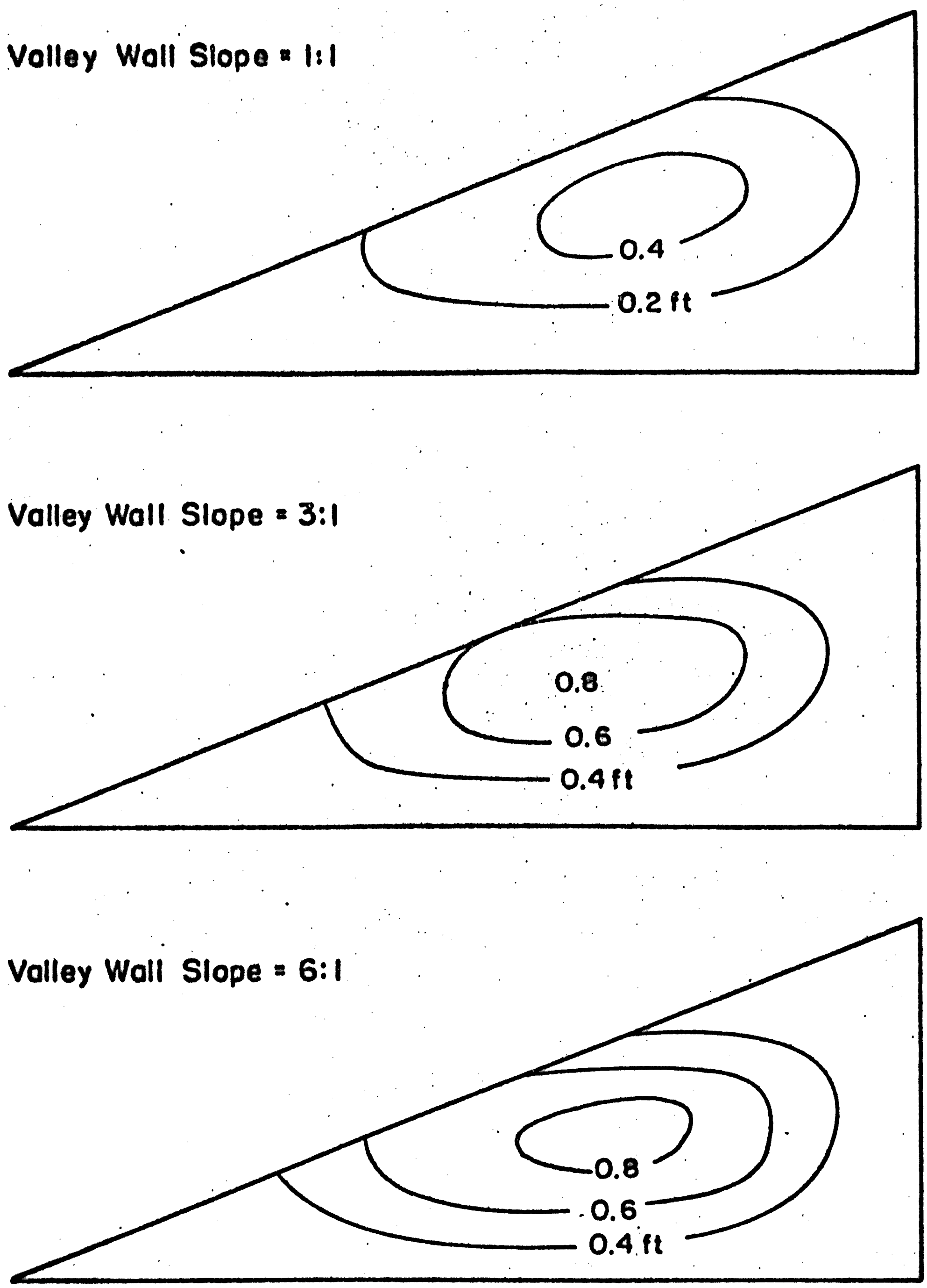

Fig. 12 CONTOURS OF VALUES OF HORIZONTAL DISPLACEMENT IN THE TRANSVERSE SECTION (u_h) CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

The effect of the valley wall slope on the major principal stresses (σ_1) in the dam may be seen in Fig. 7. Although the values of σ_1 in the upper part of the dam are very nearly the same in all three cases, the values near the base of the dam are smaller for the steeper valley wall slopes, indicating a significant degree of cross-valley arching within the dams in the steeper valleys. The values of minor principal stress (σ_3) shown in Fig. 8 indicate a similar influence of valley wall slope, with the values near the base of the dam decreasing with increasing steepness of the valley wall. The orientations of the major principal stresses, which are depicted by the orientations of the lines shown in Fig. 9, may be seen to be virtually the same for all three cases analyzed. The values of maximum shear stress (τ_{max}) shown in Fig. 10 may be seen to follow essentially the same pattern of variation as the major and minor principal stresses, with the values of the base of the dam being smaller for the steeper valley wall slopes.

The settlements or vertical displacements (u_h) within the dams are shown by the contours in Fig. 11. It may be seen that the magnitudes of the settlements decrease with increasing steepness of the valley wall, indicating that the steeper valley walls tend to restrict and reduce the vertical displacements within the dam. The contours shown in Fig. 12 indicate that the steeper valley walls also restrict the horizontal movements. These affects are consistent with the cross-valley arching in the case of the steeper valley walls noted previously with respect to the stresses in the dams.

COMPARISONS OF RESULTS OF PLANE STRAIN AND THREE-DIMENSIONAL ANALYSES OF THE TRANSVERSE SECTION

Values of the major and minor principal stresses in the transverse section calculated using three-dimensional and plane strain analyses are compared in Figs. 13 and 14. The contours shown in these figures represent the values of stress calculated by the plane strain analyses, expressed in percentage of the values calculated by the three-dimensional analyses. It may be seen that the values of σ_1 calculated by plane strain analyses are somewhat larger than the values calculated by three-dimensional analyses. This difference, which results from cross-valley arching in the three-dimensional dams, is greatest for the steepest valley wall slope. For the 1:1 valley wall slope the values of σ_1 near the base of the dam calculated by the plane strain analysis are about 20% larger than those from the three-dimensional analyses. For the 3:1 valley wall slope the difference is only about 5% throughout most of the dam, indicating that conditions in dams with 3:1 or flatter valley wall slopes correspond fairly closely to plane strain. The larger differences near the slopes in the lower portions of the dams are in regions where the stresses are quite small, and would not be of primary interest for most purposes.

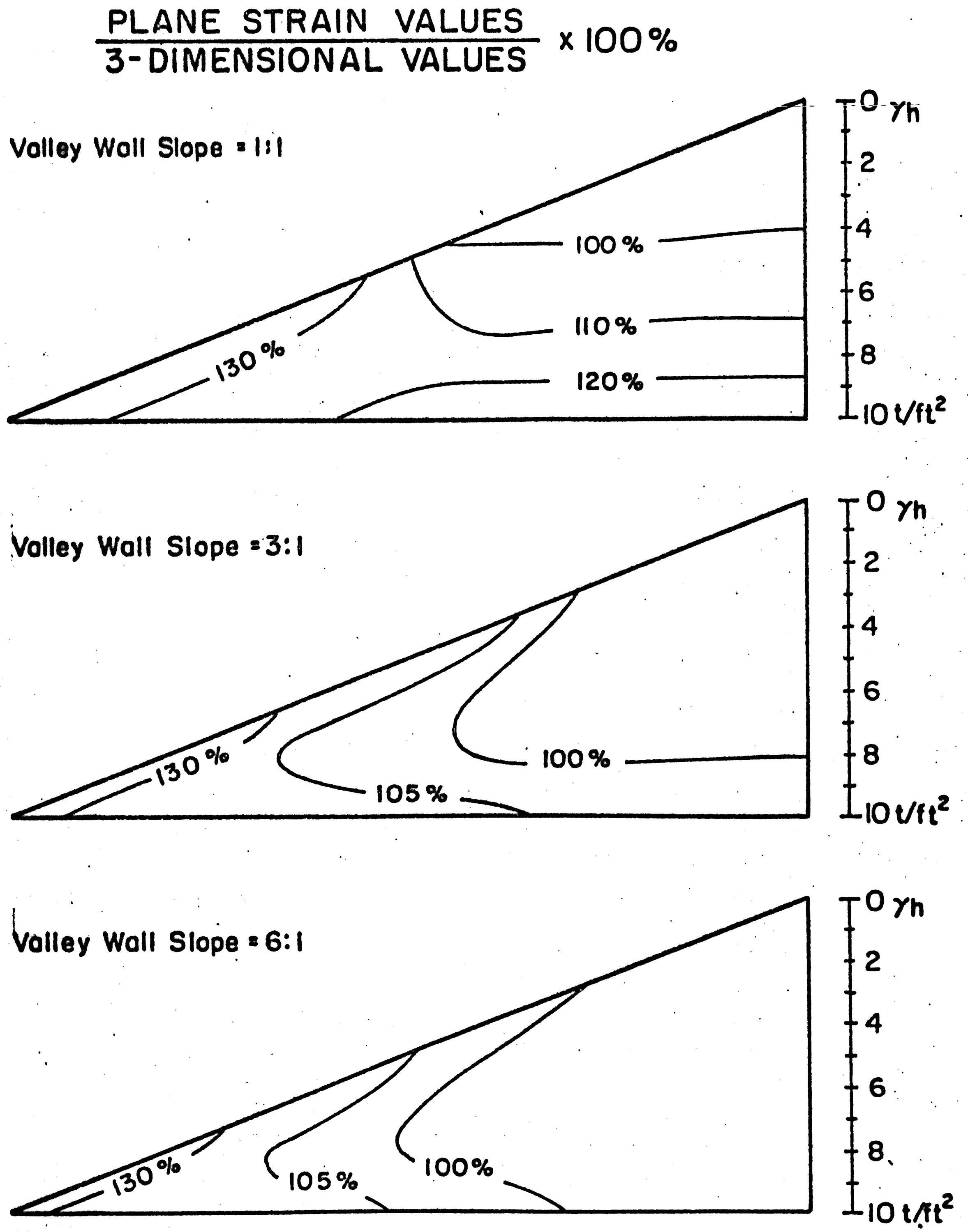

Fig. 13 CONTOURS OF VALUES OF MAJOR PRINCIPAL STRESS (σ_1) IN THE TRANSVERSE SECTION CALCULATED BY PLANE STRAIN ANALYSIS, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

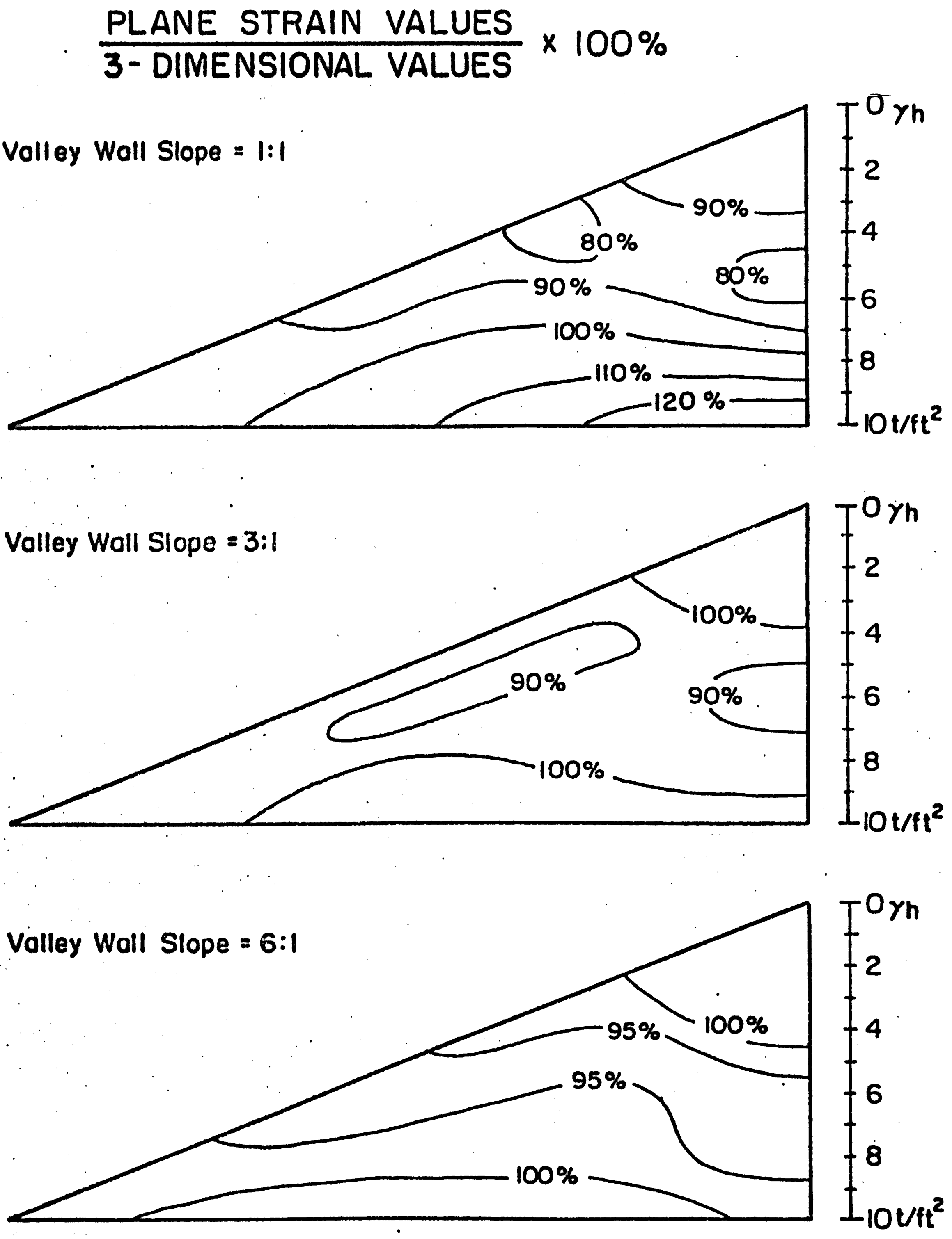

Fig.14 CONTOURS OF VALUES OF MINOR PRINCIPAL STRESS (σ_3) IN THE TRANSVERSE SECTION CALCULATED BY PLANE STRAIN ANALYSIS, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

The contours in Fig. 14 show that the values of σ_3 calculated by plane strain analyses are smaller than those calculated by three-dimensional analyses. The difference arises from the restraint imposed by the valley walls and is greatest for the steepest valley walls. Near the center of the dam in the valley with 1:1 walls, the values of σ_3 calculated by plane strain analysis are about 80% of those calculated by three-dimensional analysis; for the 3:1 valley wall slope, the plane strain values are about 90% as large as those from the three-dimensional analysis.

The stress orientations calculated for both analyses are shown in Fig. 15. Except for the orientations calculated for the odd-shaped elements along the slope, the orientations may be seen to be virtually identical for both analyses.

As shown in Fig. 16, the values of maximum shear stress (τ_{max}) calculated using plane strain analysis are considerably larger than those calculated using the three-dimensional analysis procedures for the 1:1 valley wall slope. In the central portion of the dam the difference amounts to about 40% for the 1:1 valley wall slope, about 10% for the 3:1 valley wall slope, and less than 10% for the 6:1 valley wall slope. The larger percentage differences near the toes of the slopes occur in regions where the values of τ_{max} are comparatively small, and would not be expected to be of great importance for most purposes.

The contours in Fig. 17 show that the vertical displacements near the center of the dam calculated by means of plane strain analysis are about 50% greater than those calculated by three-dimensional analysis of the dam in the valley with a 1:1 valley wall slope. For the flatter valley wall slopes the difference is much smaller. The larger percentages near the surface of the slope correspond to calculated values which are relatively small.

For the dam in the valley with the steepest valley wall slope, the values of horizontal displacement calculated by plane strain analysis are considerably greater than those calculated by three-dimensional analysis, as shown in Fig. 18. The difference is greatest near the base of the dam where the displacements are small and where the effect of restraint from the valley walls is greatest. Even within the zone of maximum displacement near midheight of the dam, the values calculated by plane strain analysis are about twice as large as those calculated by three-dimensional analysis. For the dams in valleys with less steep valley walls, the differences are considerably smaller. The fact that the values calculated by the plane strain analysis in the upper portions of these dams are somewhat smaller than those calculated by the three-dimensional analysis is believed to be due to a tendency for increased outward movement near the base of the dam to be accompanied by a tendency for inward movement near the top of the dam. This type of effect is more pronounced in materials with high values of Poisson's ratio and less pronounced in materials with low values of Poisson's ratio.

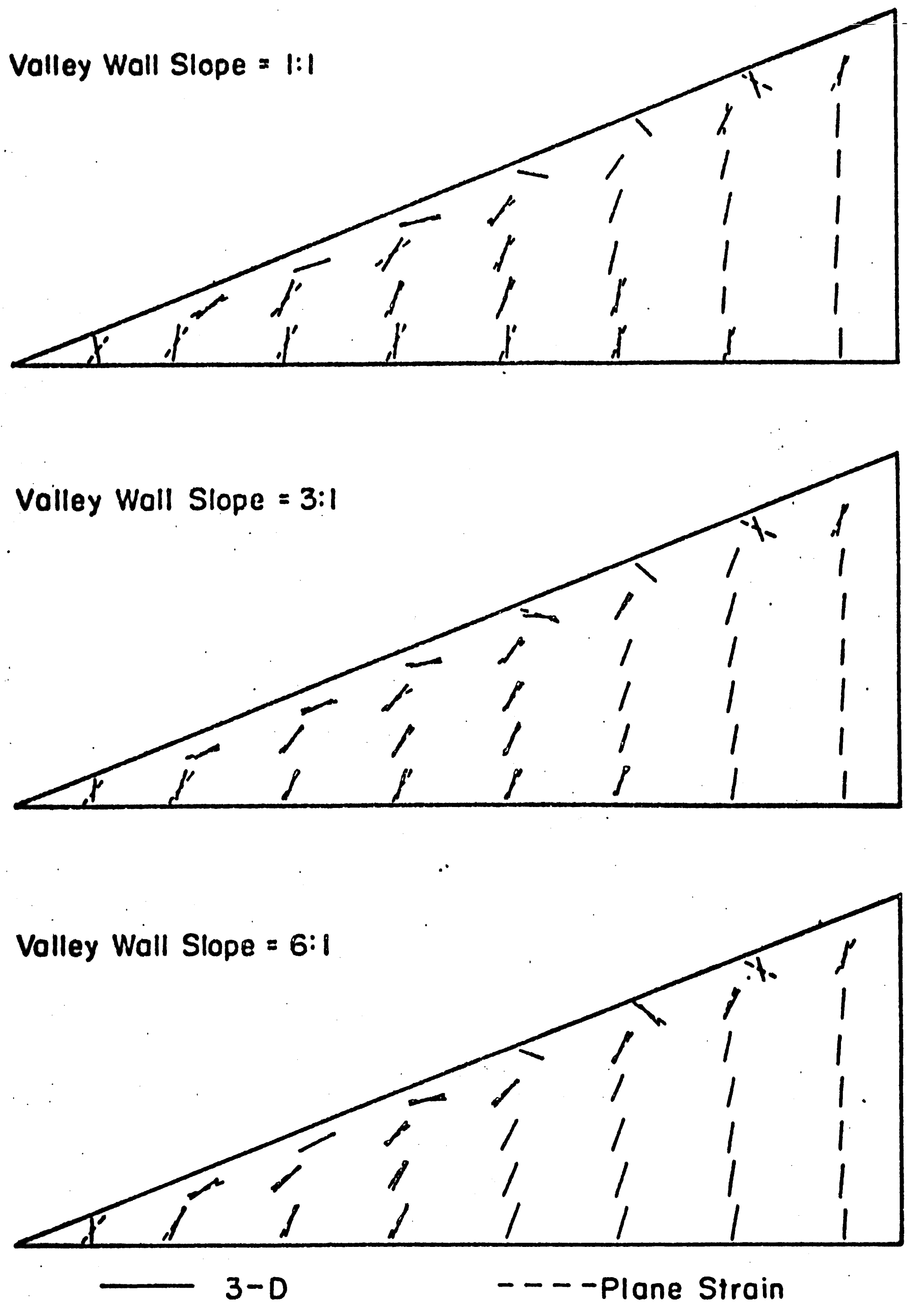

Fig. 15 COMPARISONS OF MAJOR PRINCIPAL STRESS ORIENTATIONS IN THE TRANSVERSE SECTION CALCULATED BY PLANE STRAIN ANALYSIS WITH ORIENTATIONS CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES.

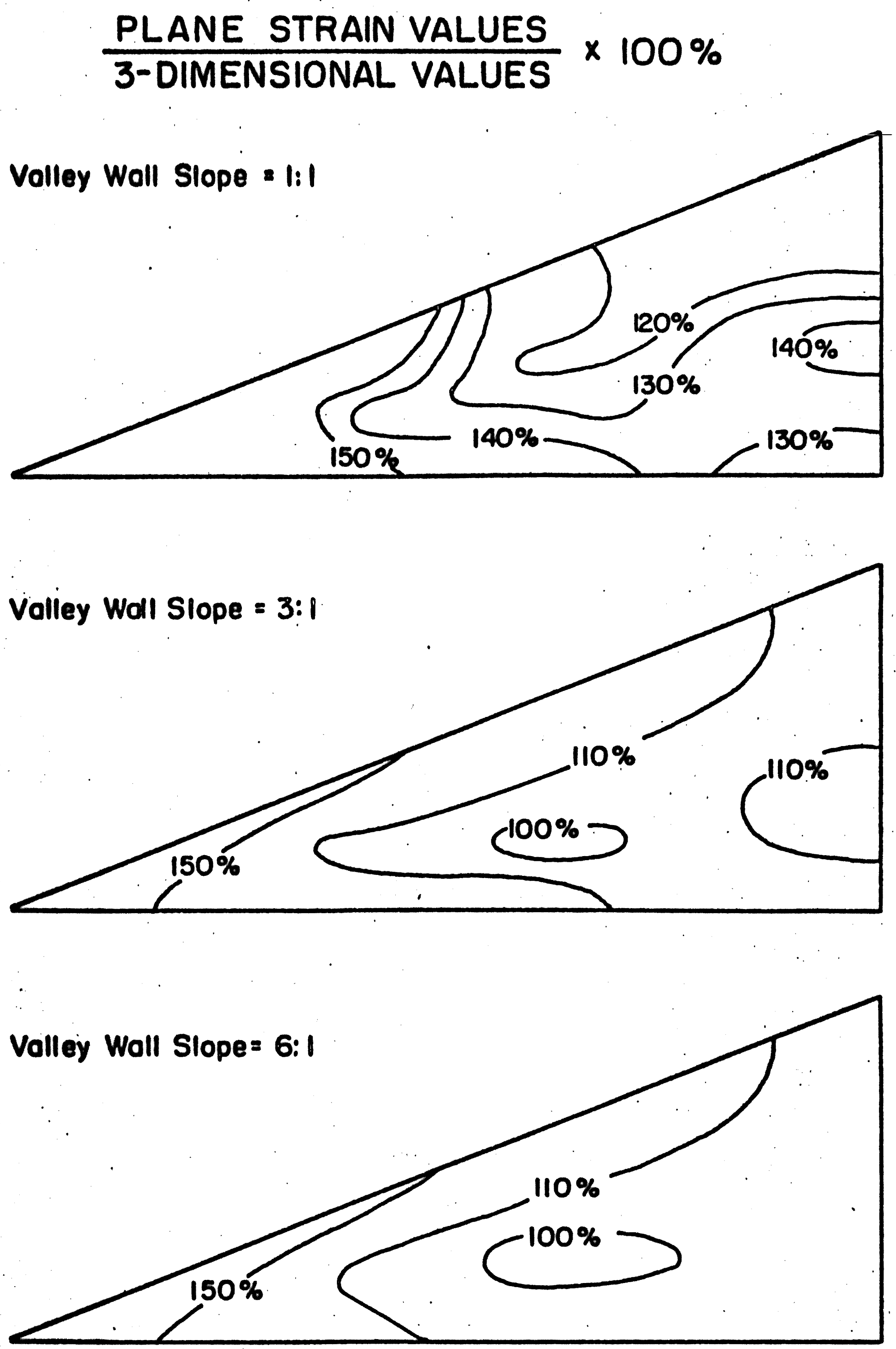

Fig. 16 CONTOURS OF VALUES OF MAXIMUM SHEAR STRESS (τ_{max}) IN THE TRANSVERSE SECTION CALCULATED BY PLANE STRAIN ANALYSIS, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES FOR THREE DIFFERENT VALLEY WALL SLOPES

$$\frac{\text{PLANE STRAIN VALUES}}{\text{3-DIMENSIONAL VALUES}} \times 100\%$$

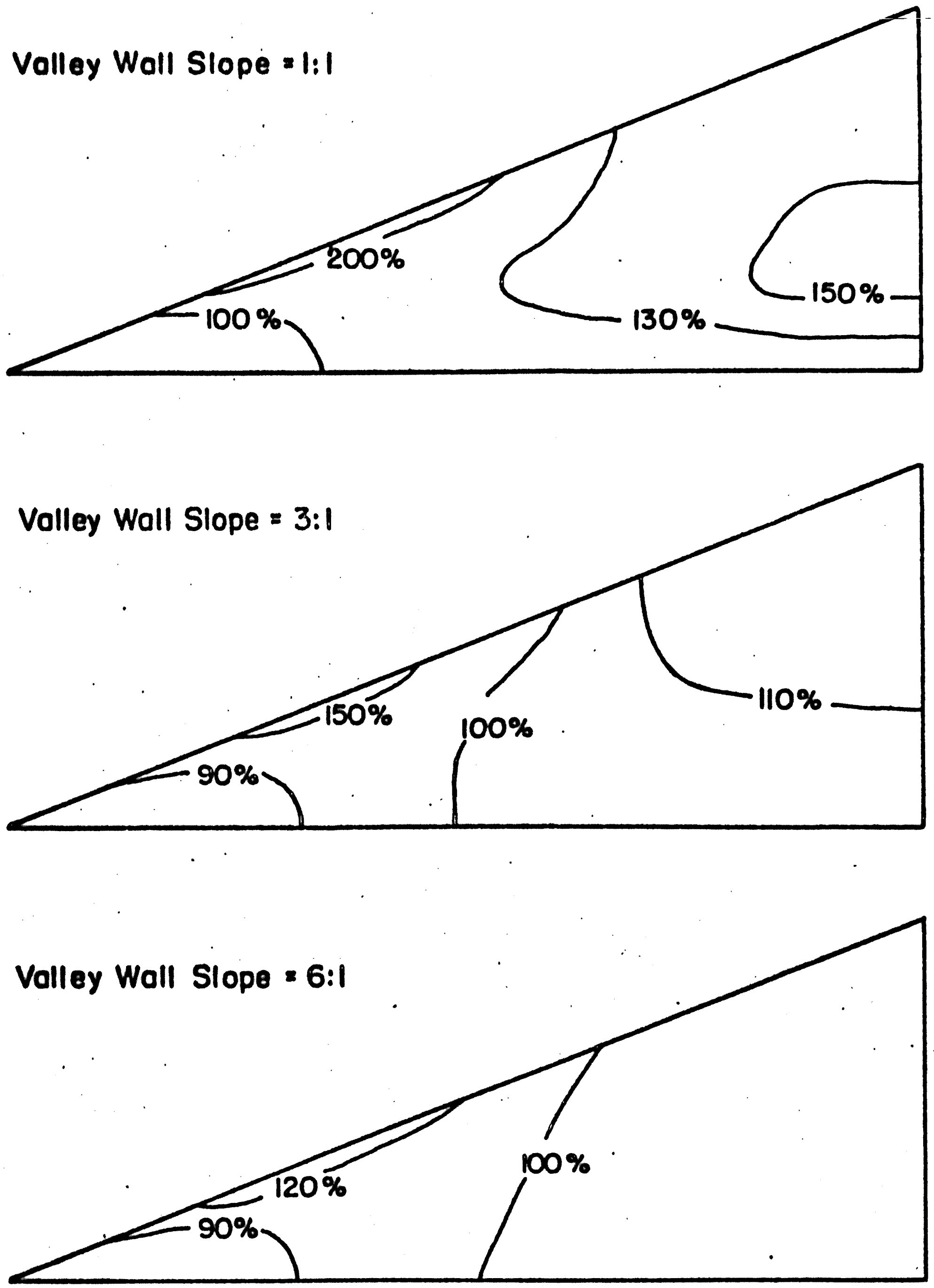

Fig.17 CONTOURS OF VALUES OF VERTICAL DISPLACEMENT (u_v) IN THE TRANSVERSE SECTION CALCULATED BY PLANE STRAIN ANALYSIS, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES FOR THREE DIFFERENT VALLEY WALL SLOPES

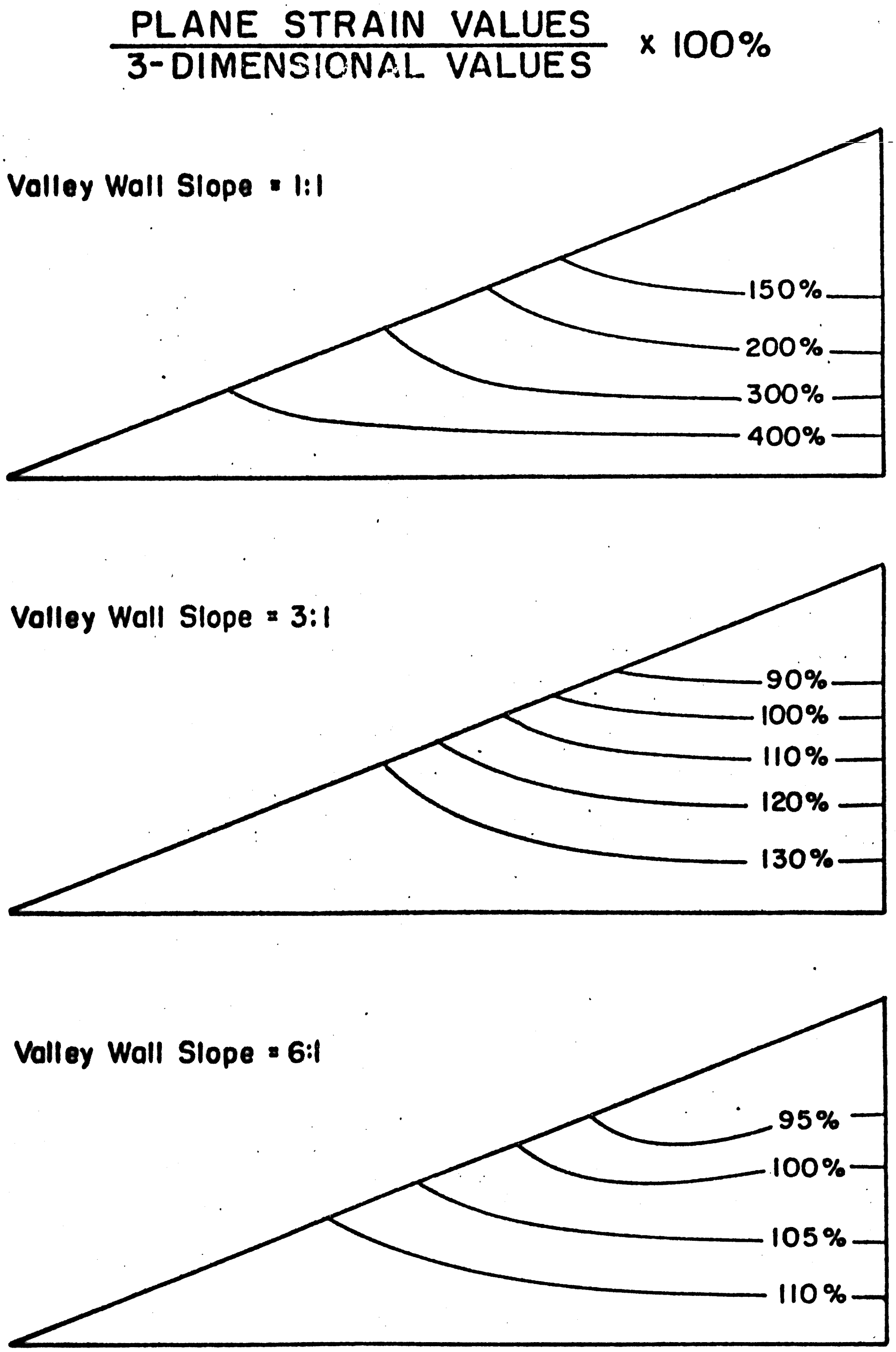

Fig. 18 CONTOURS OF VALUES OF HORIZONTAL DISPLACEMENT (u_h) IN THE TRANSVERSE SECTION CALCULATED BY PLANE STRAIN ANALYSIS, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

COMPARISON OF INCREMENTAL AND GRAVITY TURN-ON TWO-DIMENSIONAL ANALYSES OF THE LONGITUDINAL SECTION

Plane strain analyses of the longitudinal section of the dam in the valley with a 3:1 valley wall slope were performed using both incremental and gravity turn-on analysis procedures. The values of major and minor principal stress, σ_1 and σ_3, which are shown in Fig. 19, may be seen to be essentially the same for both analyses. At the base of the dam, near the center of the valley, the values calculated by the incremental analysis are about 5% greater than those calculated by the gravity turn-on analysis.

The calculated displacements, shown in Fig. 20, may be seen to be quite different for the two types of analysis. The vertical displacements (u_v) calculated by incremental analysis are zero at the top and bottom of the section and are largest near midheight, whereas the values calculated by the gravity turn-on analysis increase continually with height above the bottom, reaching a maximum at the top of the dam. Similarly, the values of horizontal displacement (u_h) calculated by the incremental analysis are larger near midheight, while those calculated by the gravity turn-on analysis are largest at the crest of the dam. Covarrubias (1969) has pointed out that the movements calculated by incremental analyses are similar to those which occur during construction, and that those calculated by gravity turn-on analyses correspond better to the deformations which occur after construction. The analytical results described in subsequent sections of this report were determined using incremental analysis procedures.

RESULTS FOR THE LONGITUDINAL SECTIONS CALCULATED USING THREE-DIMENSIONAL ANALYSES

Contours of the major principal stress (σ_1) calculated using three-dimensional analyses are shown in Fig. 21. It may be seen that the values of σ_1 increase uniformly with depth except near the valley walls. In the dams in the valleys with steeper valley walls, the values of σ_1 at the bottom of the dam in the center are smaller than in the dam in the valley with the flattest valley wall slope, indicating some degree of cross-valley arching. In addition, the values of σ_1 in elements adjacent to the valley walls are increased somewhat in the upper portions of the dams in the steeper valleys, and are reduced somewhat in the lower portion, as compared to the values of σ_1 calculated for the dam in the flattest valley. Similar effects of the valley wall slope may be seen in Fig. 22, which shows contours of the minor principal stress, σ_3. The values near the base of the dam may be seen to be somewhat smaller for the dam in the steepest valley.

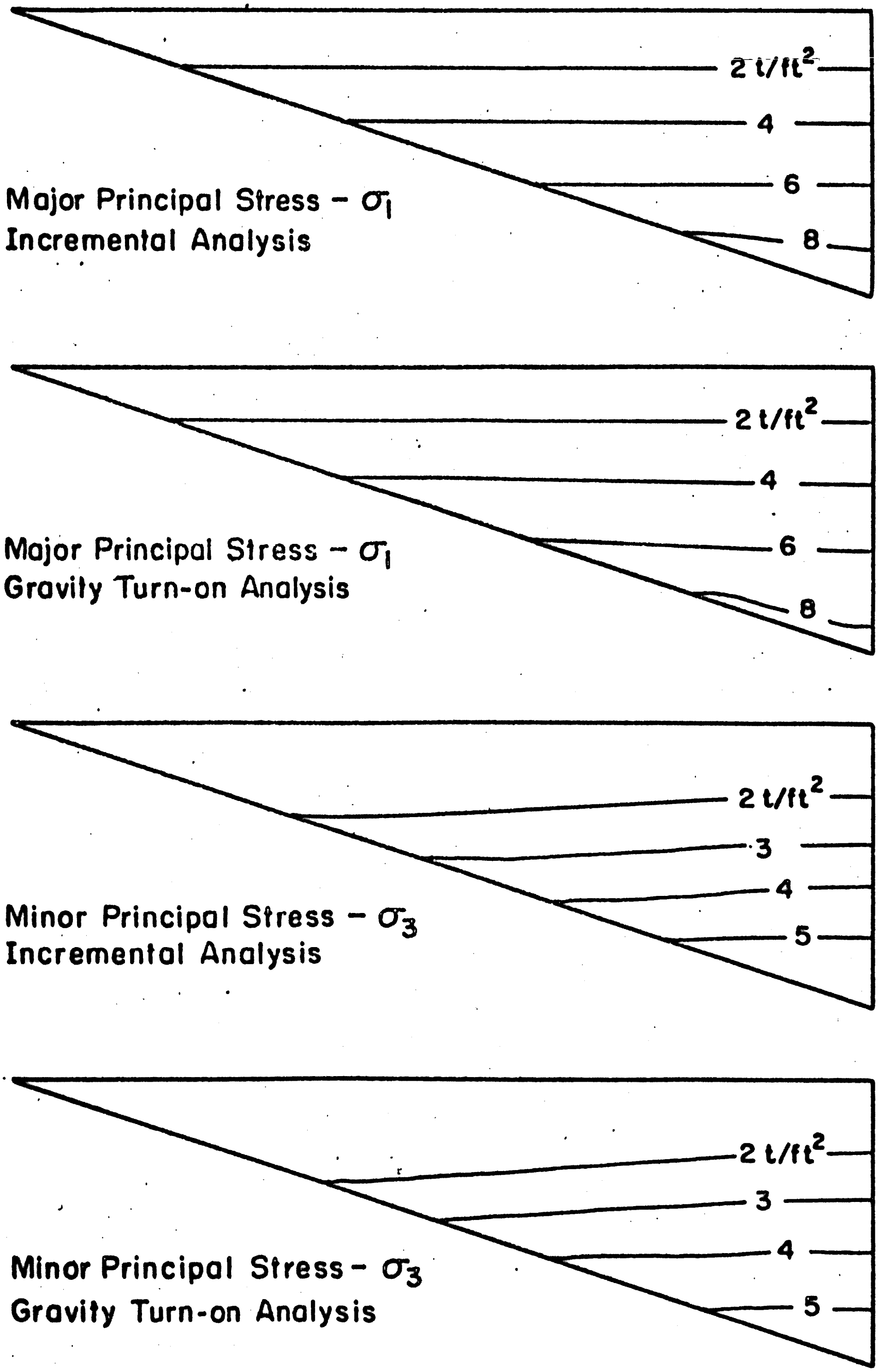

Fig. 19 COMPARISON OF STRESSES IN THE LONGITUDINAL SECTION CALCULATED USING INCREMENTAL AND GRAVITY TURN-ON PLANE STRAIN ANALYSES FOR 3:1 VALLEY WALL SLOPE

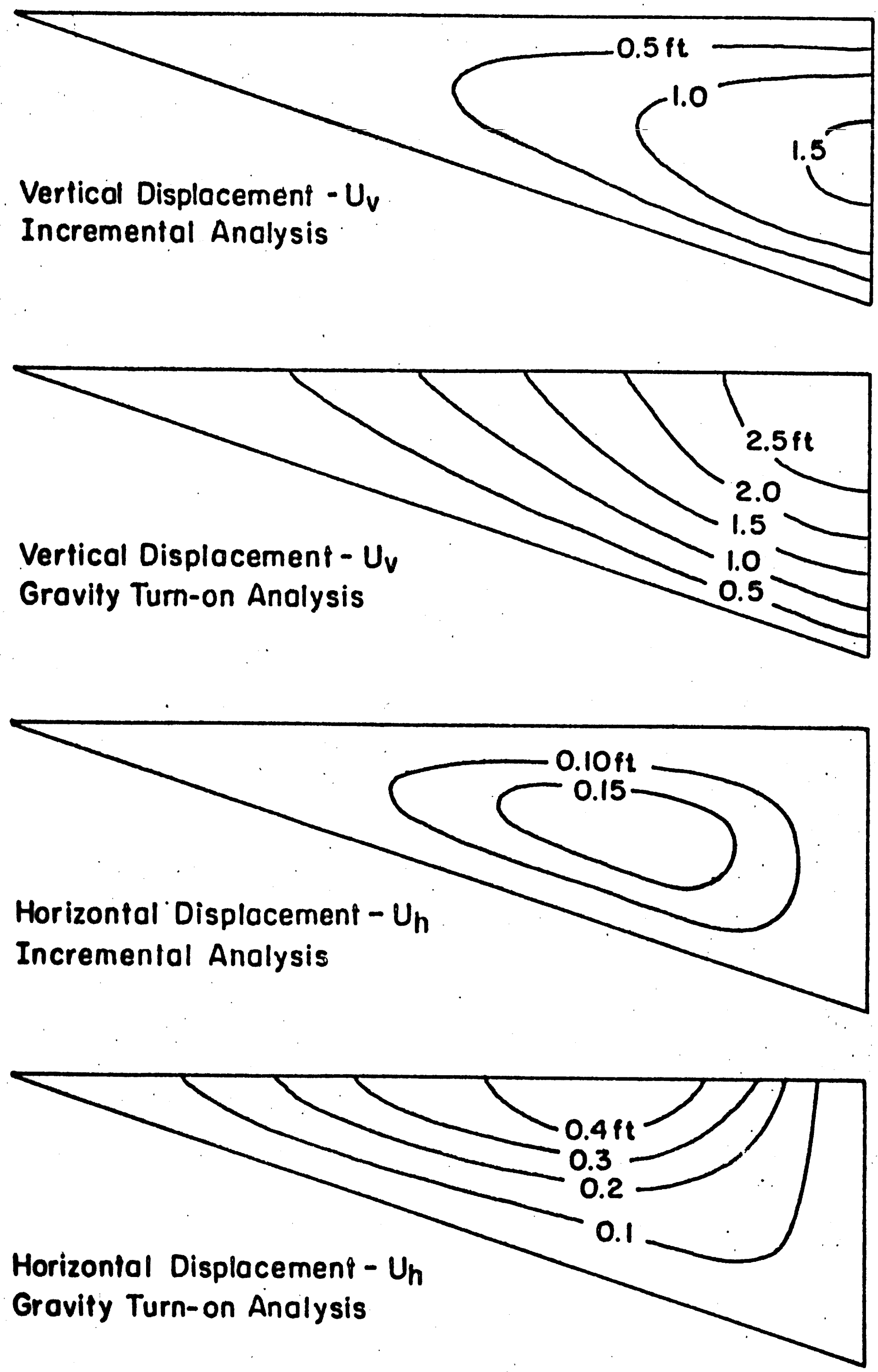

Fig. 20 COMPARISON OF DISPLACEMENTS IN THE LONGITUDINAL SECTION CALCULATED USING INCREMENTAL AND GRAVITY TURN-ON PLANE STRAIN ANALYSES FOR 3:1 VALLEY WALL SLOPE

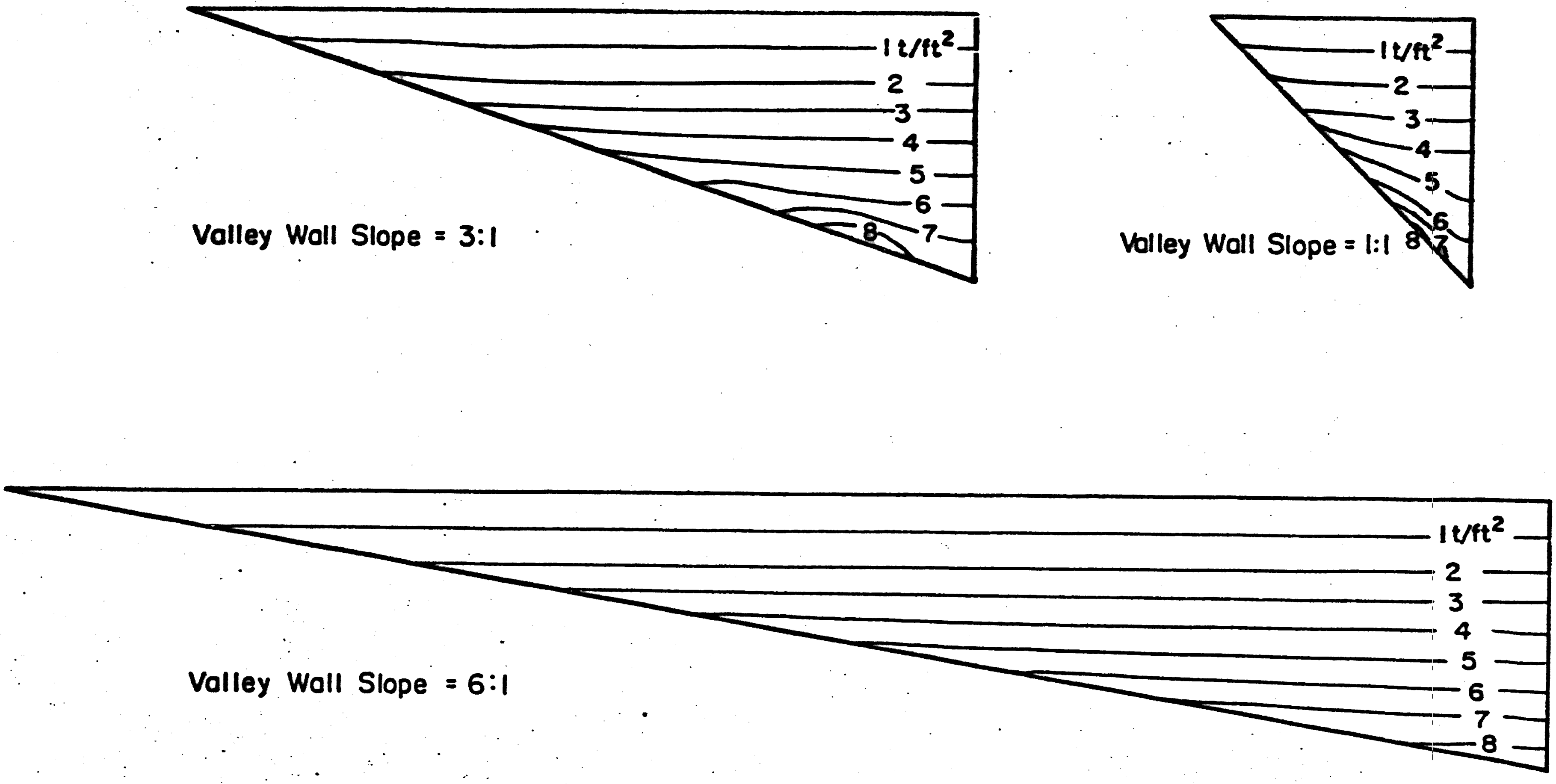

Fig. 21 CONTOURS OF VALUES OF MAJOR PRINCIPAL STRESS (σ_1) IN THE LONGITUDINAL SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

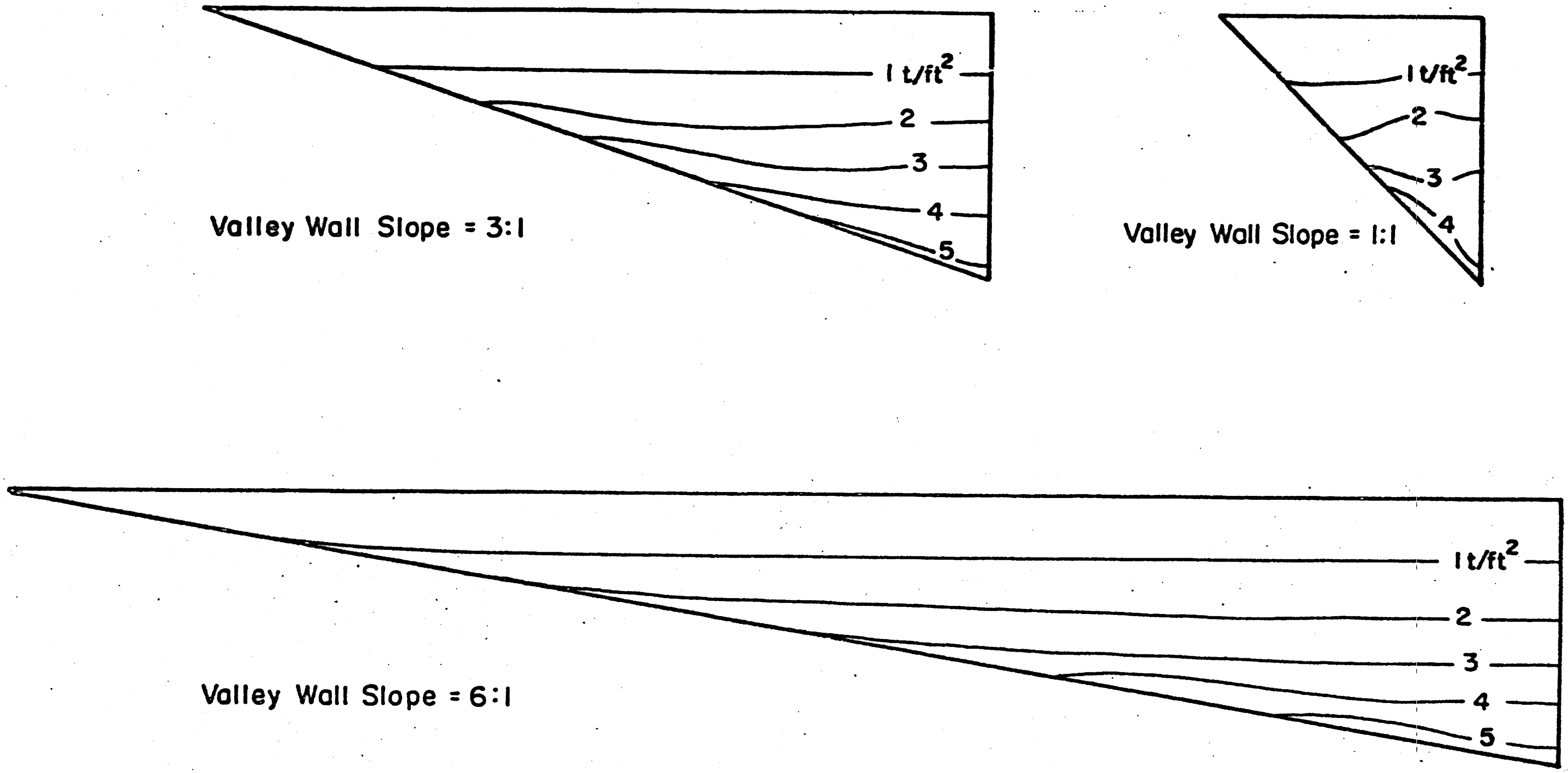

Fig. 22 CONTOURS OF VALUES OF MINOR PRINCIPAL STRESS (σ_3) IN THE LONGITUDINAL SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

The calculated stress orientations are shown in Fig. 23 by lines indicating the orientations of the major principal stress. It may be noted that these orientations are very nearly vertical near the top of all three dams and that they change gradually with depth, approaching directions normal to the valley walls in the lower parts of the cross-sections.

Values of the maximum shear stress (τ_{max}) are shown in Fig. 24. The values increase with depth and reach slightly larger values for the dam in the steepest valley than for the dams in the flatter valleys.

Contours of vertical displacement (u_v) are shown in Fig. 25. In each of the three cases illustrated the variations are similar, but the maximum value, near the center of the section, is smaller for the dam in the steeper valley as a result of cross-valley arching. The maximum values for the dams in the valleys with 3:1 and 6:1 valley wall slopes are about the same, indicating that the effects of the valley walls are small in these cases.

Contours of horizontal displacement (u_h) are shown in Fig. 26. It may be seen that the maximum value of u_h in the dam in the valley with a 3:1 valley wall slope is larger than that for either the flatter or the steeper valley wall slope. It is believed that this occurs because there are two counteracting effects of increasing valley wall slope: Firstly, there is a greater tendency for horizontal movement. (Note that if the base of the dam was horizontal there would be no horizontal displacement in the longitudinal direction during construction.) Secondly, as the valley walls become steeper, they offer greater restraint against movement. (Note that if the dam was a very thin wedge in a valley with very steep walls approaching vertical, there would be almost no horizontal displacement in the longitudinal direction during construction.) As a result of these two counteracting influences, the magnitude of the horizontal displacement first increases and then decreases as the valley wall slope becomes steeper.

COMPARISONS OF RESULTS OF PLANE STRAIN AND THREE-DIMENSIONAL ANALYSES OF THE LONGITUDINAL SECTION

Contours of values of the major principal stress calculated by plane strain analyses, expressed in percentage of the values calculated by three-dimensional analyses, are shown in Fig. 27. For all three dams the values calculated by plane strain analyses are about 15% larger than those calculated by three-dimensional analyses. The plane strain values are larger because the thickness of the dam, measured normal to the plane of the section shown in Fig. 27, is considered to be uniform in plane strain analyses. Consequently the weight of the overlying material above any horizon in the dam is somewhat greater for the plane strain analyses

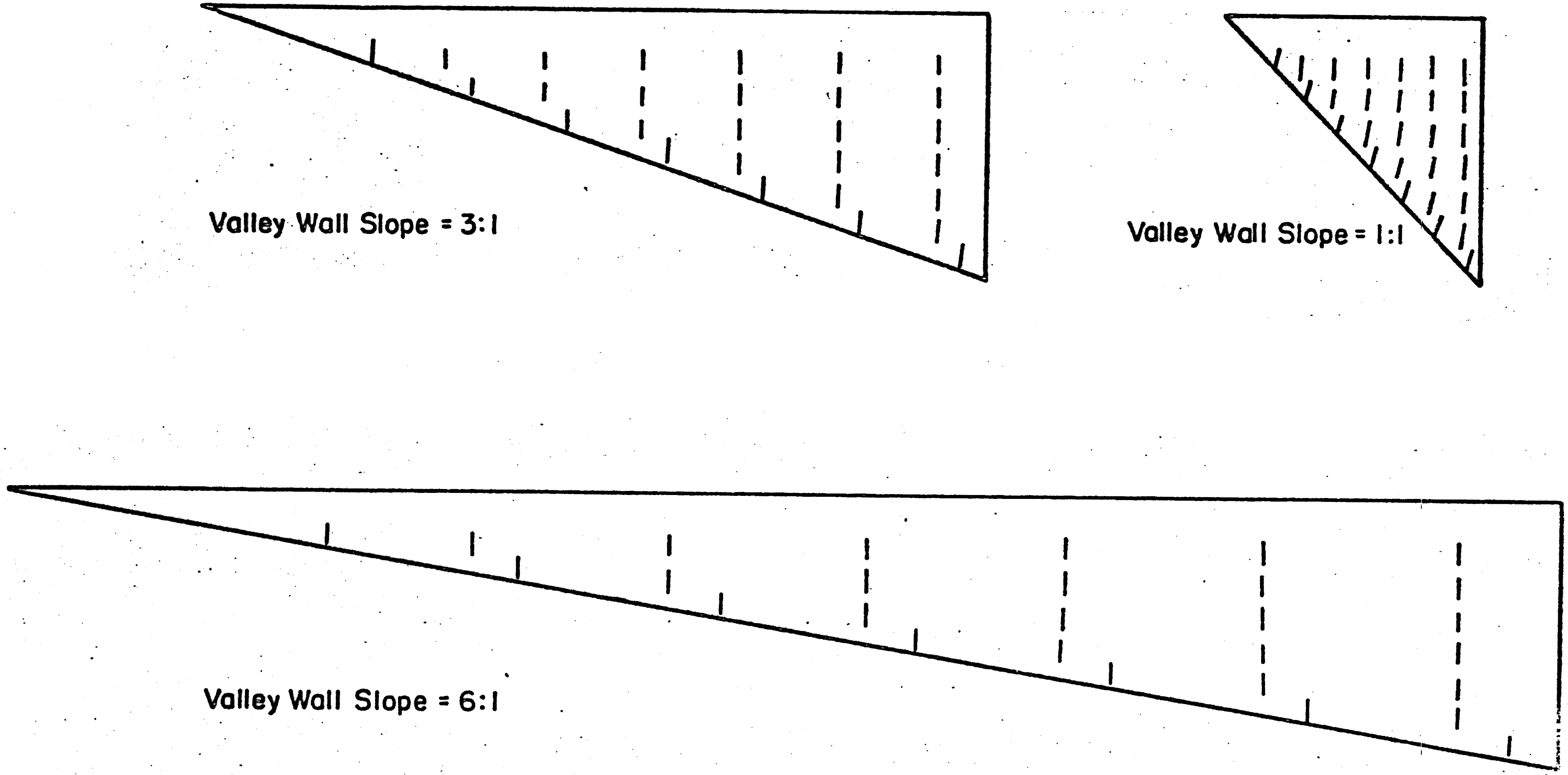

Fig. 23 MAJOR PRINCIPAL STRESS ORIENTATIONS IN THE LONGITUDINAL SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

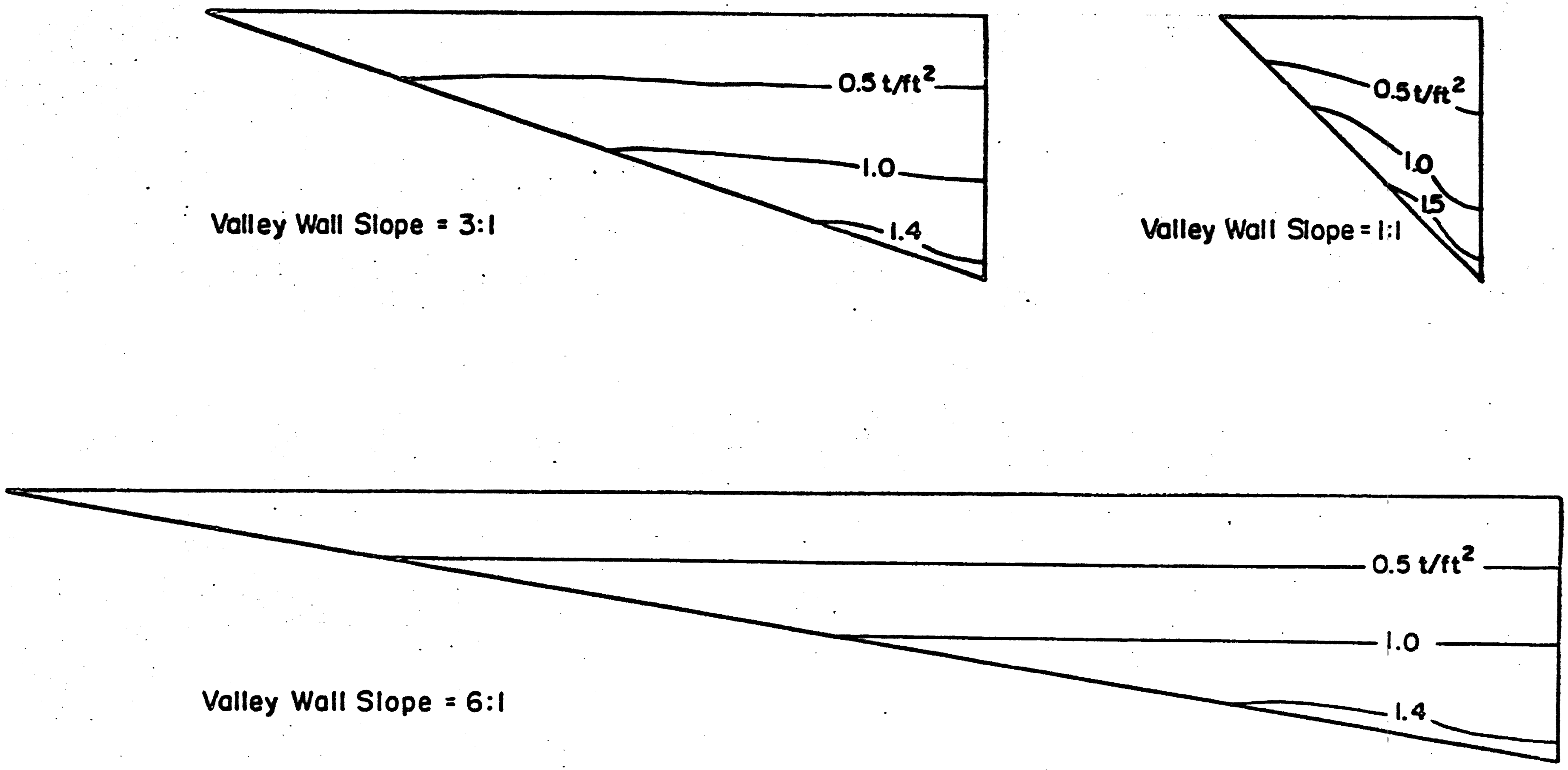

Fig. 24 CONTOURS OF VALUES OF MAXIMUM SHEAR STRESS (τ_{max}) IN THE LONGITUDINAL SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

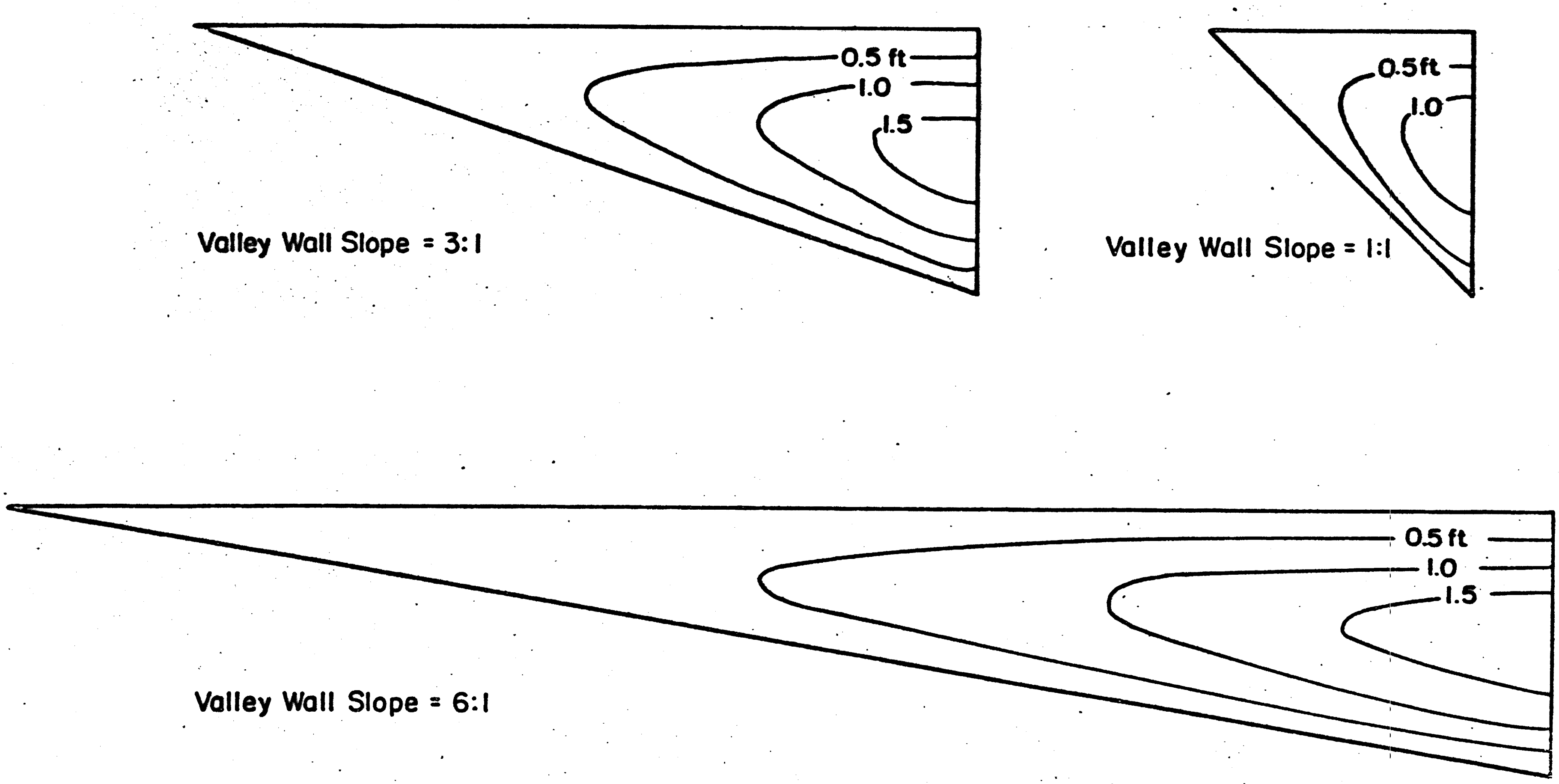

Fig. 25 CONTOURS OF VALUES OF VERTICAL DISPLACEMENT (u_v) IN THE LONGITUDINAL SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

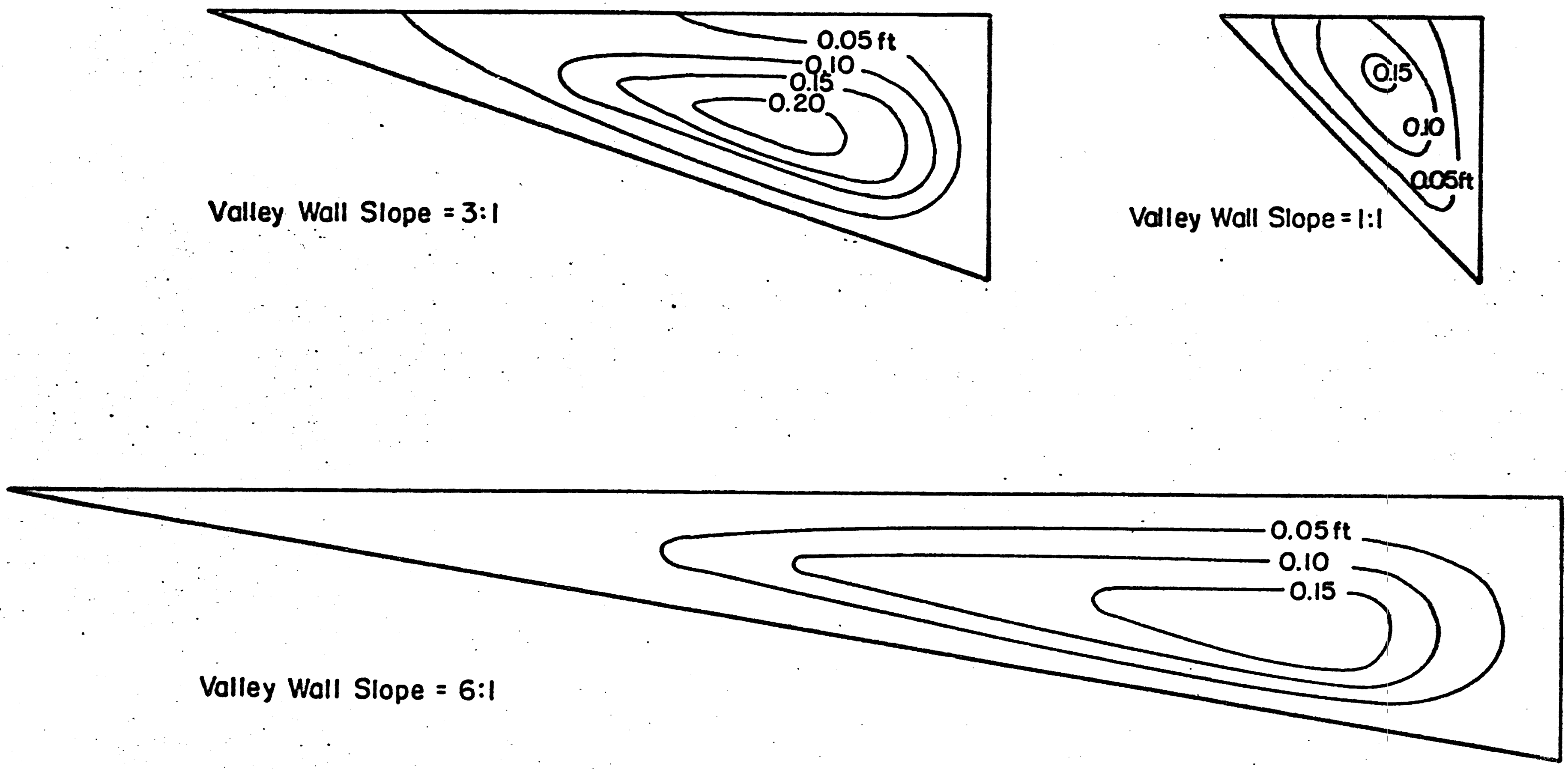

Fig. 26 CONTOURS OF VALUES OF HORIZONTAL DISPLACEMENT (u_h) IN THE LONGITUDINAL SECTION CALCULATED USING THREE-DIMENSIONAL ANALYSES WITH THREE DIFFERENT VALLEY WALL SLOPES

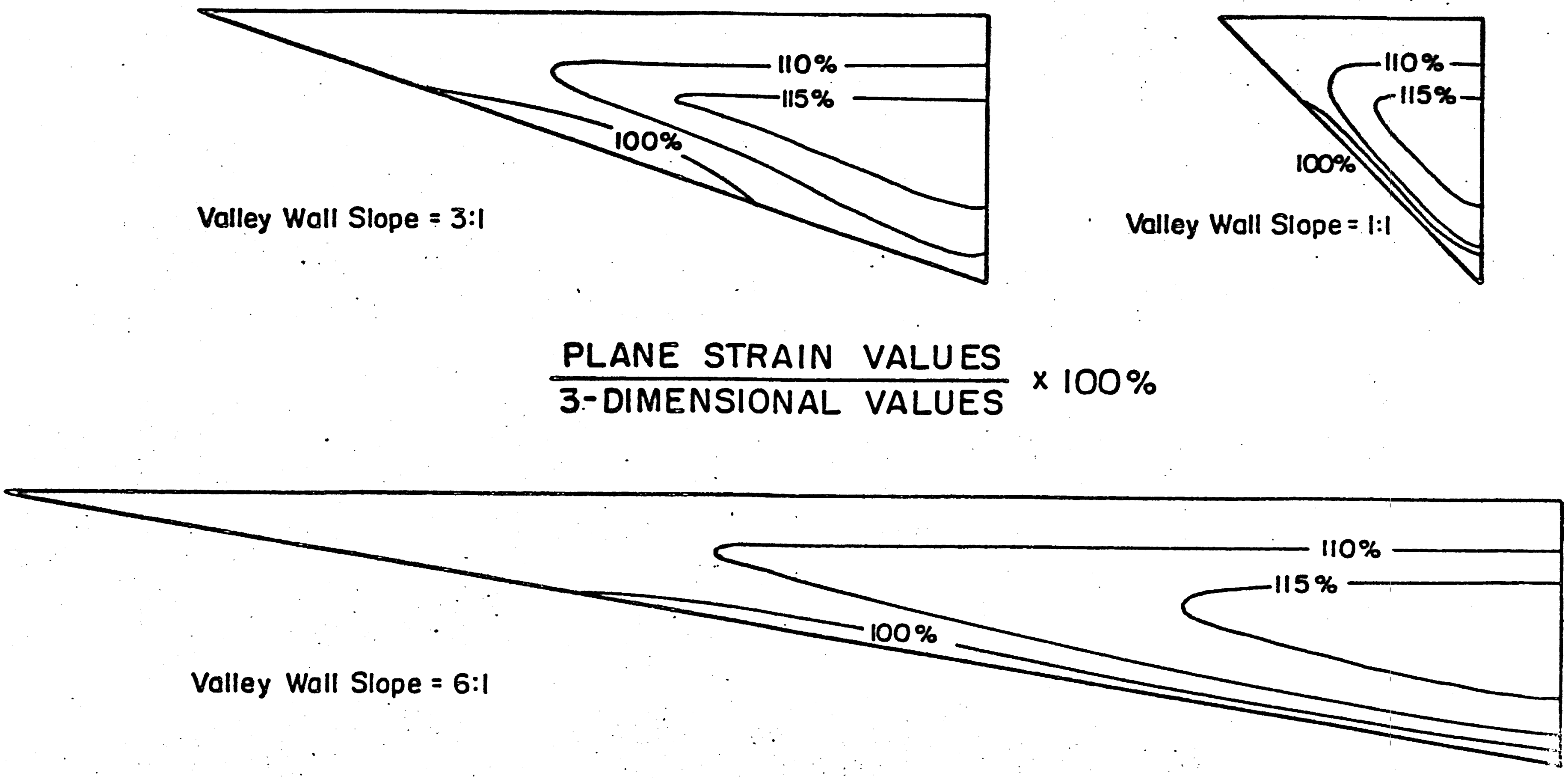

Fig.27 CONTOURS OF VALUES OF MAJOR PRINCIPAL STRESS (σ_1) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRAIN ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

than for the three-dimensional analyses, and as a result the values of σ_1 are also greater.

The values of minor principal stress (σ_3) calculated by the two analyses are compared in Fig. 28. The values of σ_3 calculated by the plane strain analysis are greater than those calculated by the three-dimensional analysis, by as much as 30% in each of the three cases analyzed. These differences are believed to result from the fact that both the overburden and the restraint are larger in the plane strain analyses. The amount by which the values of σ_3 calculated by plane strain analyses exceed those calculated by three-dimensional analyses would be expected to increase as the value of Poisson's ratio for the dam material increased.

The orientations of the major principal stresses, shown in Fig. 29, may be seen to be virtually the same for the plane strain and the three-dimensional analyses for all three valley wall slopes.

The contours in Fig. 30 show that the values of maximum shear stress (τ_{max}) calculated by plane strain analyses are somewhat smaller than those calculated by three-dimensional analyses. The difference increases with increasing steepness of the valley wall from about 10% for the 6:1 valley wall slope to about 20% for the 3:1 and 1:1 valley wall slopes.

As shown in Fig. 31, the values of vertical displacement calculated by the plane strain analyses are 10% to 20% larger than those calculated by the three-dimensional analyses near the centerline at the crest and are about 10% smaller near the abutments. The fact that the values of u_v calculated by plane strain analyses are larger than those calculated by three-dimensional analyses in one part of the dam and smaller in other parts is believed to be due to the counteracting influences of greater overburden and greater restraint in the plane strain analyses, which were discussed previously. These factors also appear to affect the values of horizontal displacement (u_h). As shown in Fig. 32, the values of u_h from plane strain analyses are larger near the top and bottom of the dam and smaller near midheight than the values calculated by three-dimensional analyses. It may be noted that the largest percentage differences shown in both Figs. 31 and 32 are near the tops and bottoms of the sections, where the magnitudes of the calculated displacements were smallest.

COMPARISONS OF RESULTS OF PLANE STRESS AND THREE-DIMENSIONAL ANALYSES OF THE LONGITUDINAL SECTION

Because the upstream and downstream faces of dams are stress-free, it was considered possible that plane stress analyses of the longitudinal section might provide results corresponding closely to the results of three-dimensional analyses. To examine this possibility in detail,

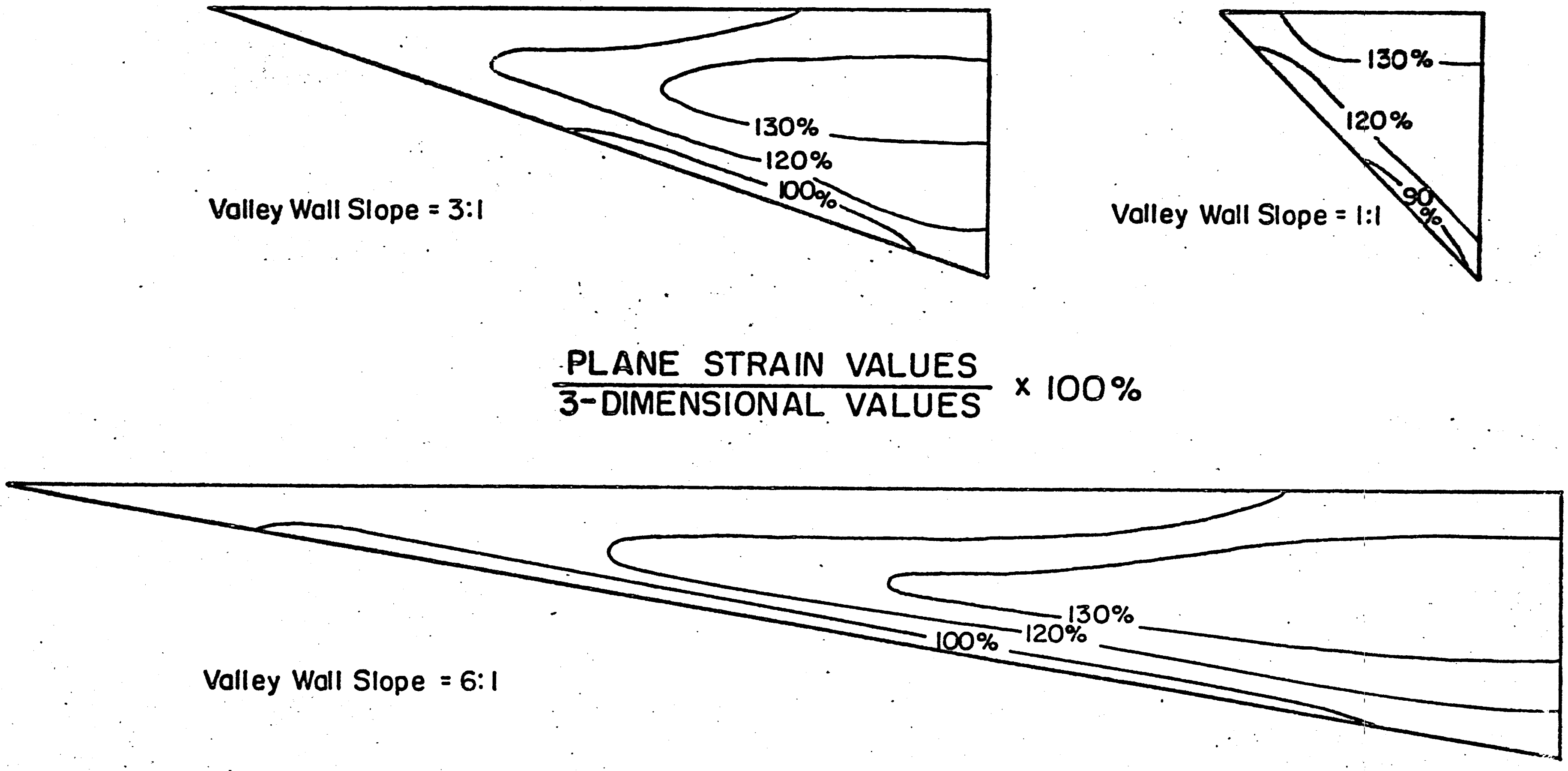

Fig. 28 CONTOURS OF VALUES OF MINOR PRINCIPAL STRESS (σ_3) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRAIN ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

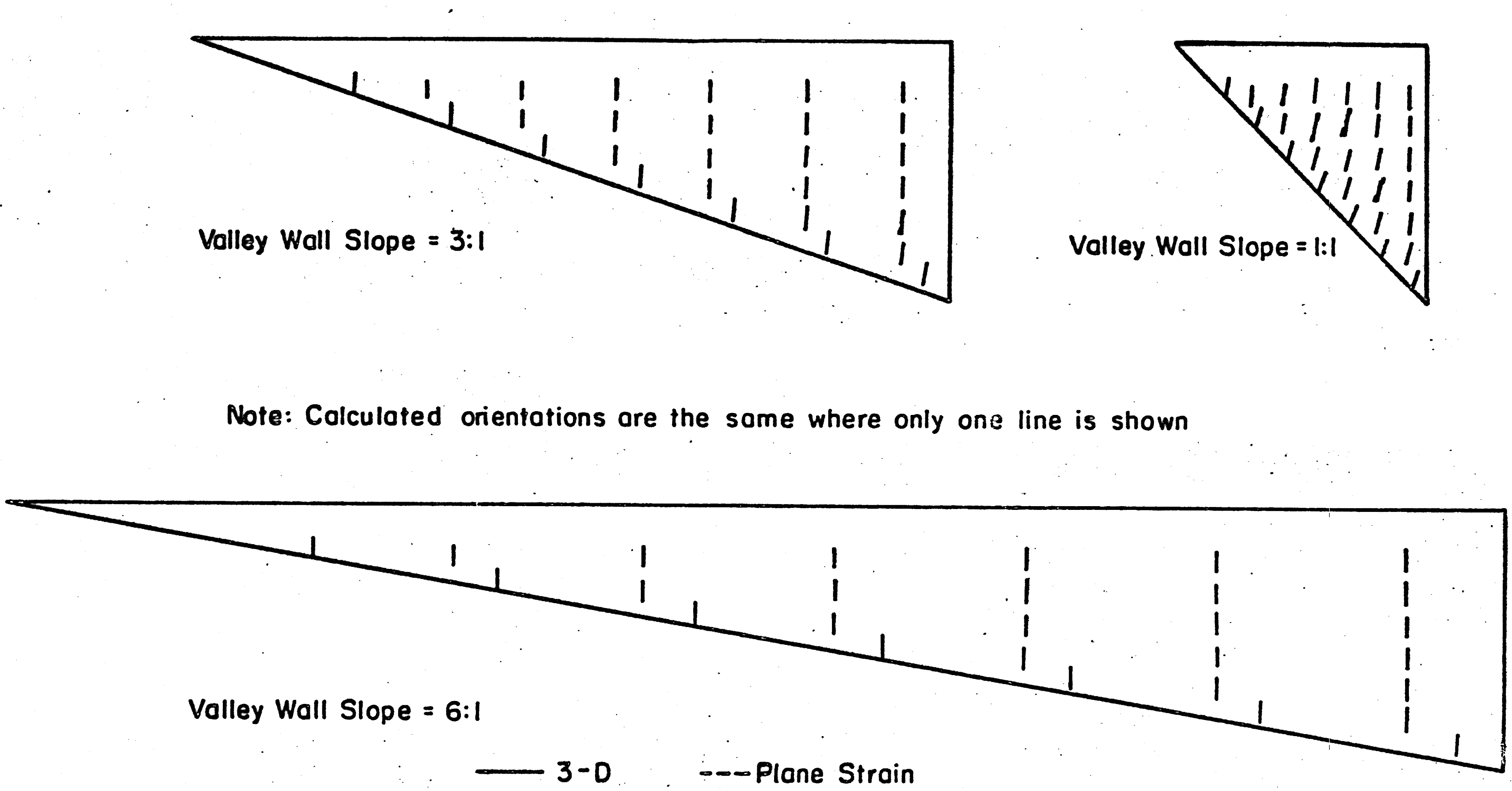

Fig. 29 COMPARISONS OF MAJOR PRINCIPAL STRESS ORIENTATIONS IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRAIN ANALYSES WITH ORIENTATIONS CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

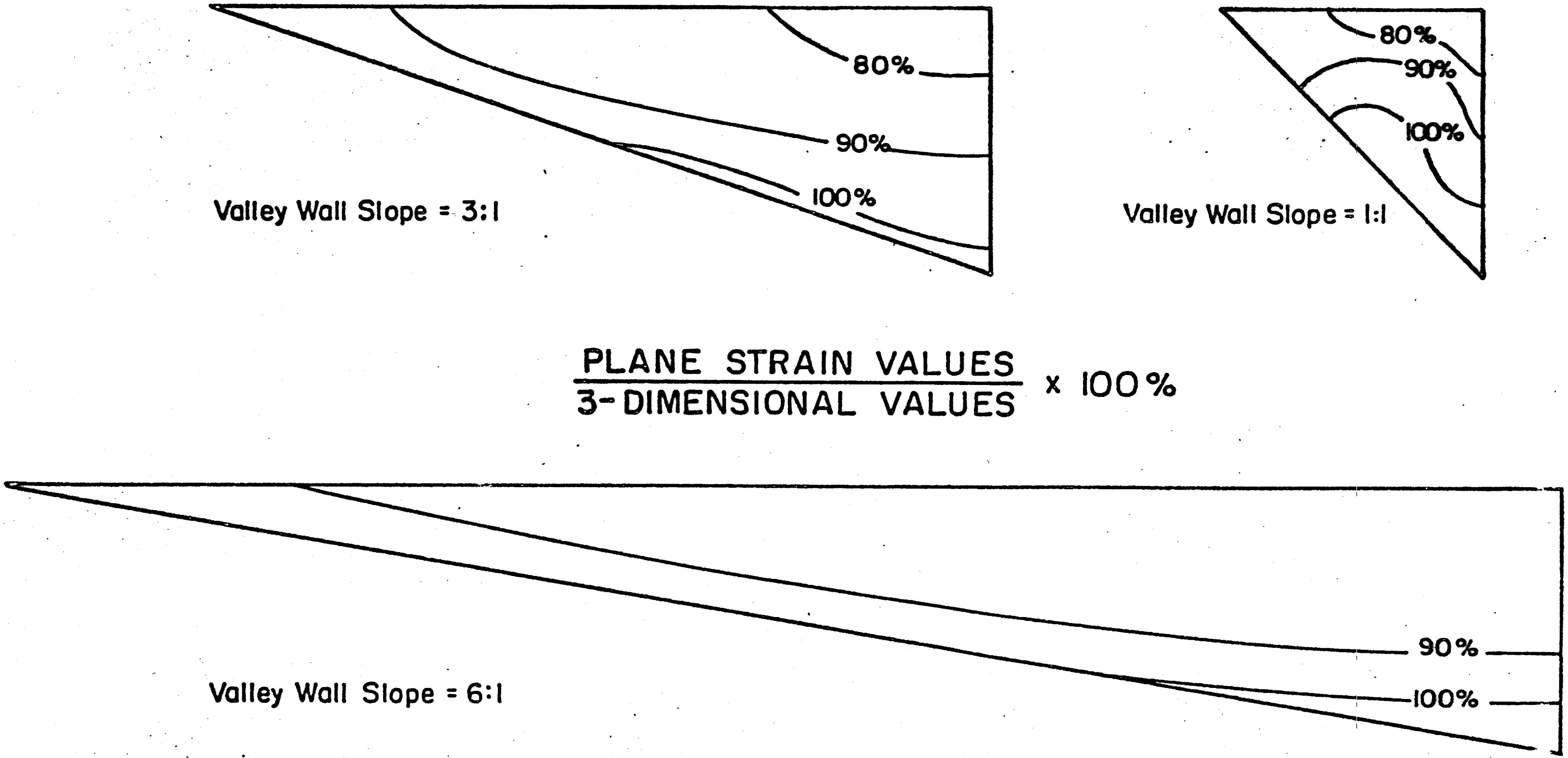

Fig. 30 CONTOURS OF VALUES OF MAXIMUM SHEAR STRESS (τ_{max}) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRAIN ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

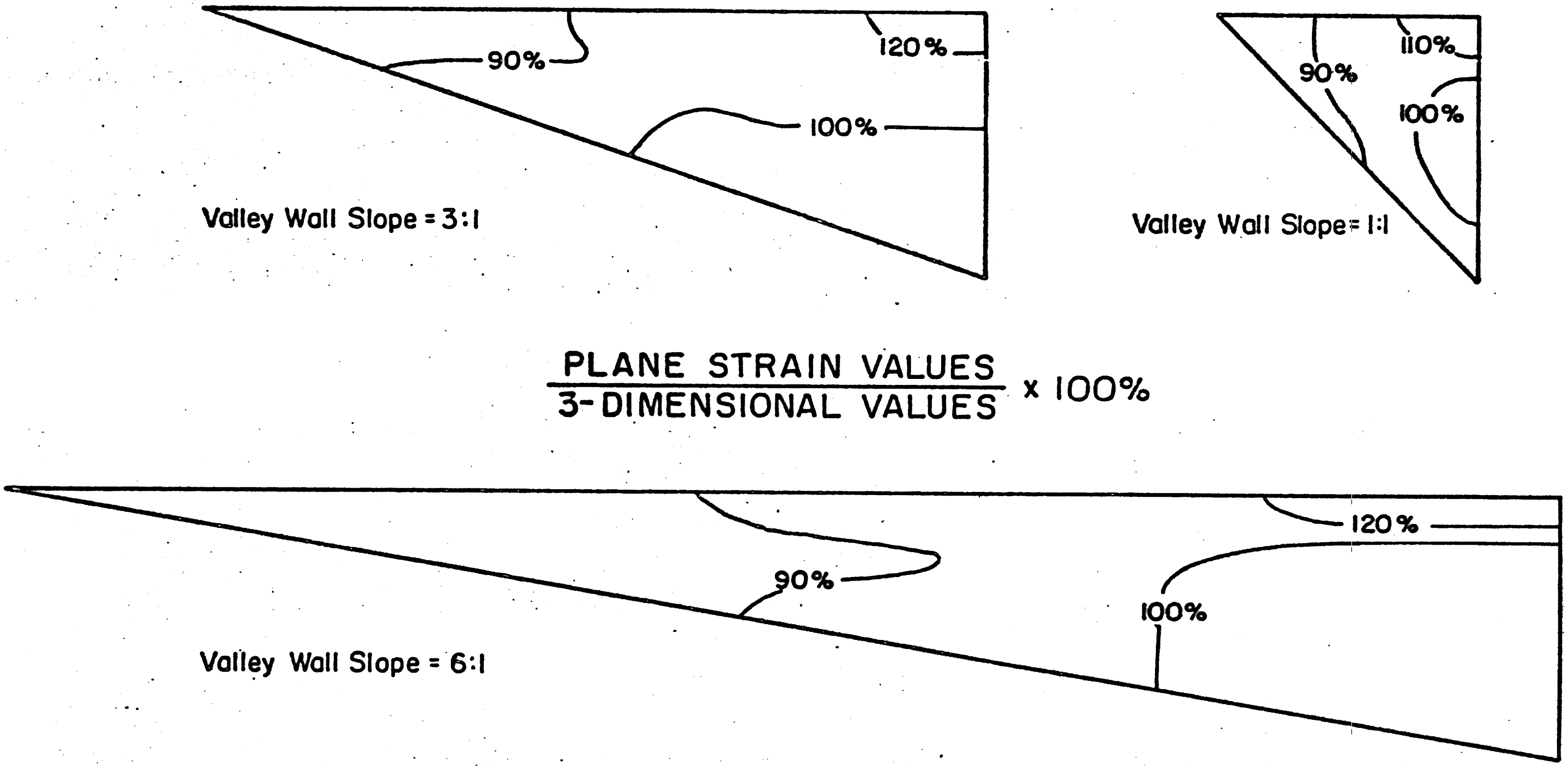

Fig. 31 CONTOURS OF VALUES OF VERTICAL DISPLACEMENT (u_v) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRAIN ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

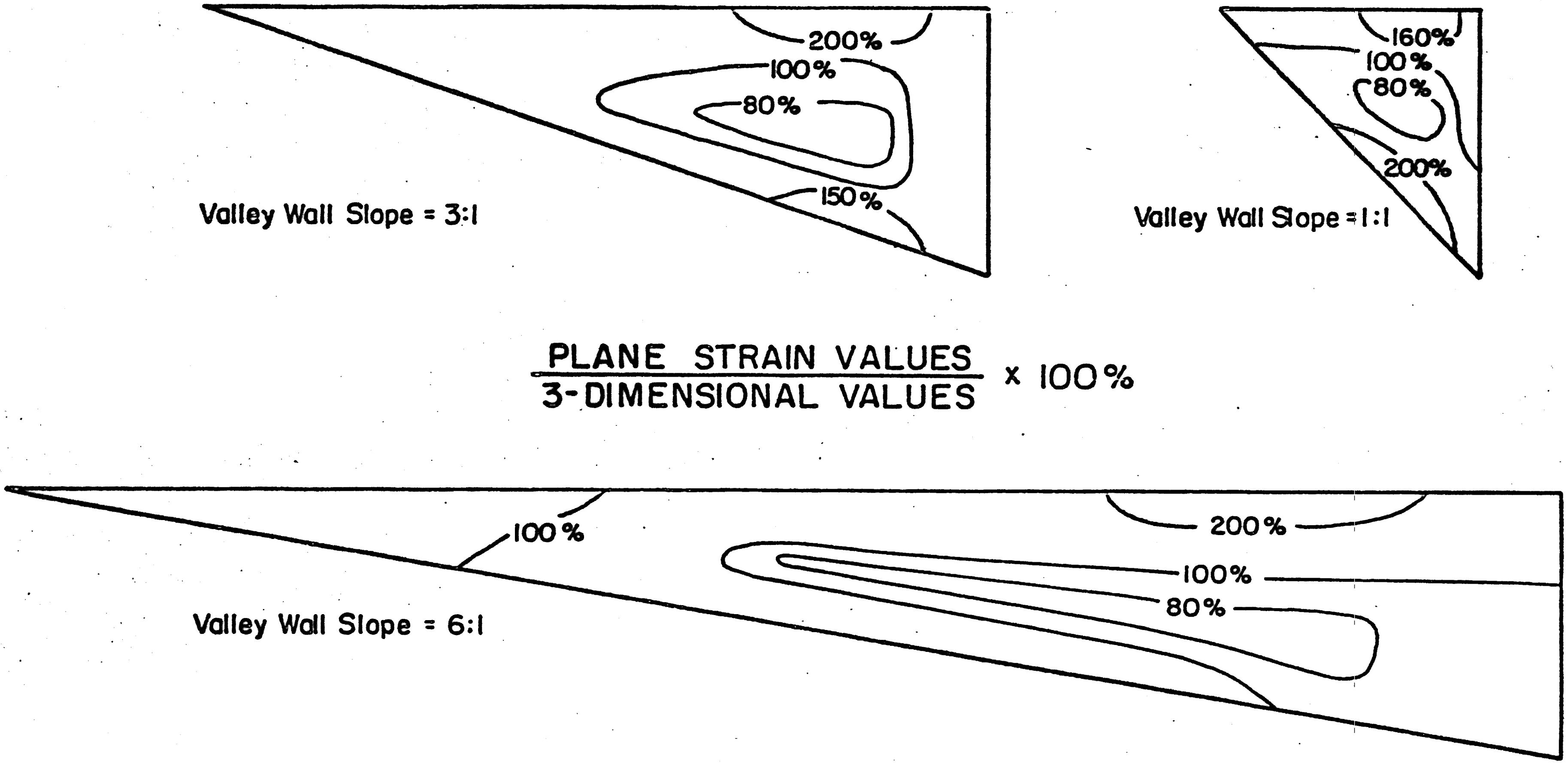

Fig. 32 CONTOURS OF VALUES OF HORIZONTAL DISPLACEMENT (u_h) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRAIN ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

analyses of the longitudinal sections were performed for the longitudinal section and the results were compared with the results of the three-dimensional analyses.

Contours of the values of the major principal stress (σ_1) calculated by the plane stress analyses, expressed in percent of the values calculated by the three-dimensional analyses, are shown in Fig. 33. It may be noted that the values calculated by the plane stress analysis are larger in all three cases, the largest difference being about 20% for the steepest valley wall slope. The fact that the values calculated by the plane stress analysis are greater is believed to be due to the fact that the thickness of the section is constant throughout its height and as a result the overburden at any level is greater.

The values of minor principal stress calculated by the two analyses are shown in Fig. 34. It may be noted that throughout the major part of all three dams the plane stress values are lower than the values from the three-dimensional analyses, the difference amounting to as much as 60% in the case of the dam in the steepest valley.

The calculated stress orientations are shown in Fig. 35. Throughout the cross-sections of all three dams, including the triangular elements adjacent to the valley wall slopes, the orientations calculated by the plane stress analyses are almost exactly the same as those calculated by the three-dimensional analyses.

The contours in Fig. 36 show that the values of maximum shear stress (τ_{max}) calculated by the plane stress analysis are considerably greater than the values calculated by the three-dimensional analyses. It may be seen that the difference amounts to 70% to 80% in all three cases shown. The larger percentages of difference, furthermore, occur in the lower parts of the cross-sections, where the magnitudes of the shear stresses are largest.

As shown in Fig. 37, the vertical displacements (u_v) calculated by the plane stress analyses exceed those calculated by the three-dimensional analyses by as much as 100%. The contours shown in Fig. 38 indicate that the horizontal displacements are also overestimated by the plane stress analyses, by even greater percentages. The larger displacements in the plane stress analyses are attributable to the fact that the degree of restraint in plane stress is considerably smaller than in the three-dimensional analyses.

SUMMARY AND CONCLUSIONS

Three-dimensional incremental finite element analyses were performed for three dams in V-shaped valleys with three different valley wall slopes - 1:1, 3:1, and 6:1. The results of these analyses were compared with the results of plane strain analyses of the maximum transverse

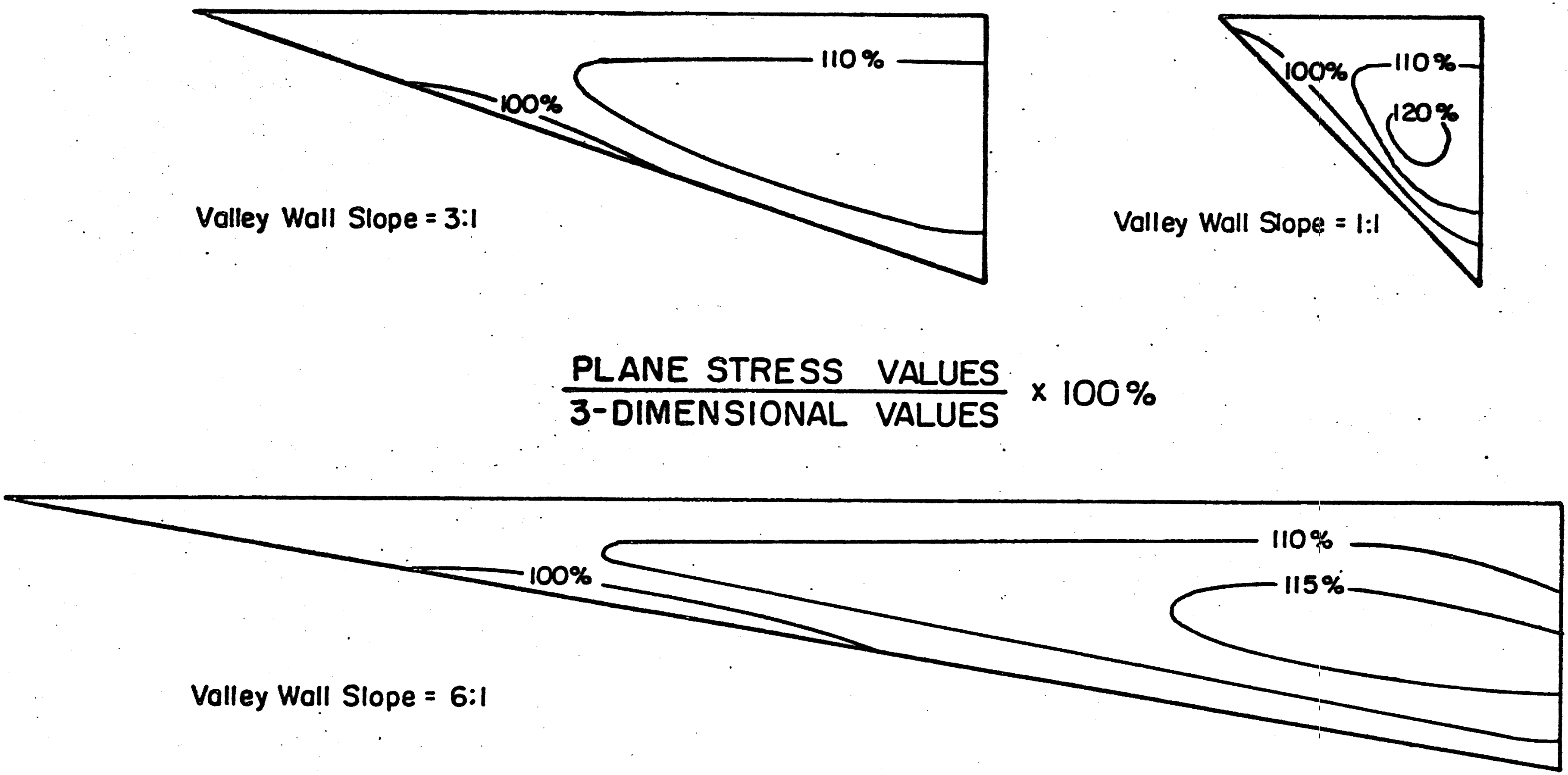

Fig. 33 CONTOURS OF VALUES OF MAJOR PRINCIPAL STRESS (σ_1) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRESS ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

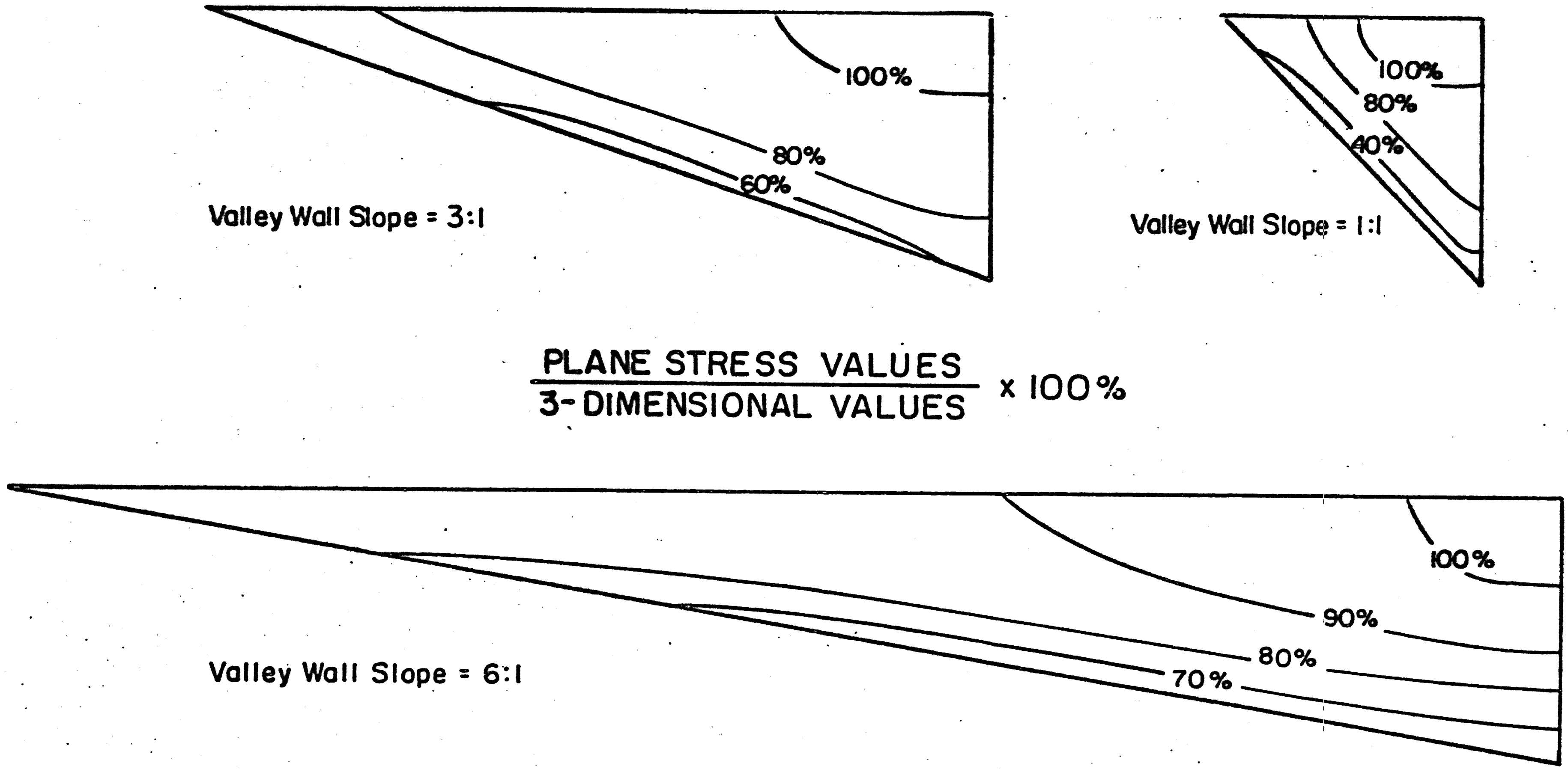

Fig. 34 CONTOURS OF VALUES OF MINOR PRINCIPAL STRESS (σ_3) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRESS ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

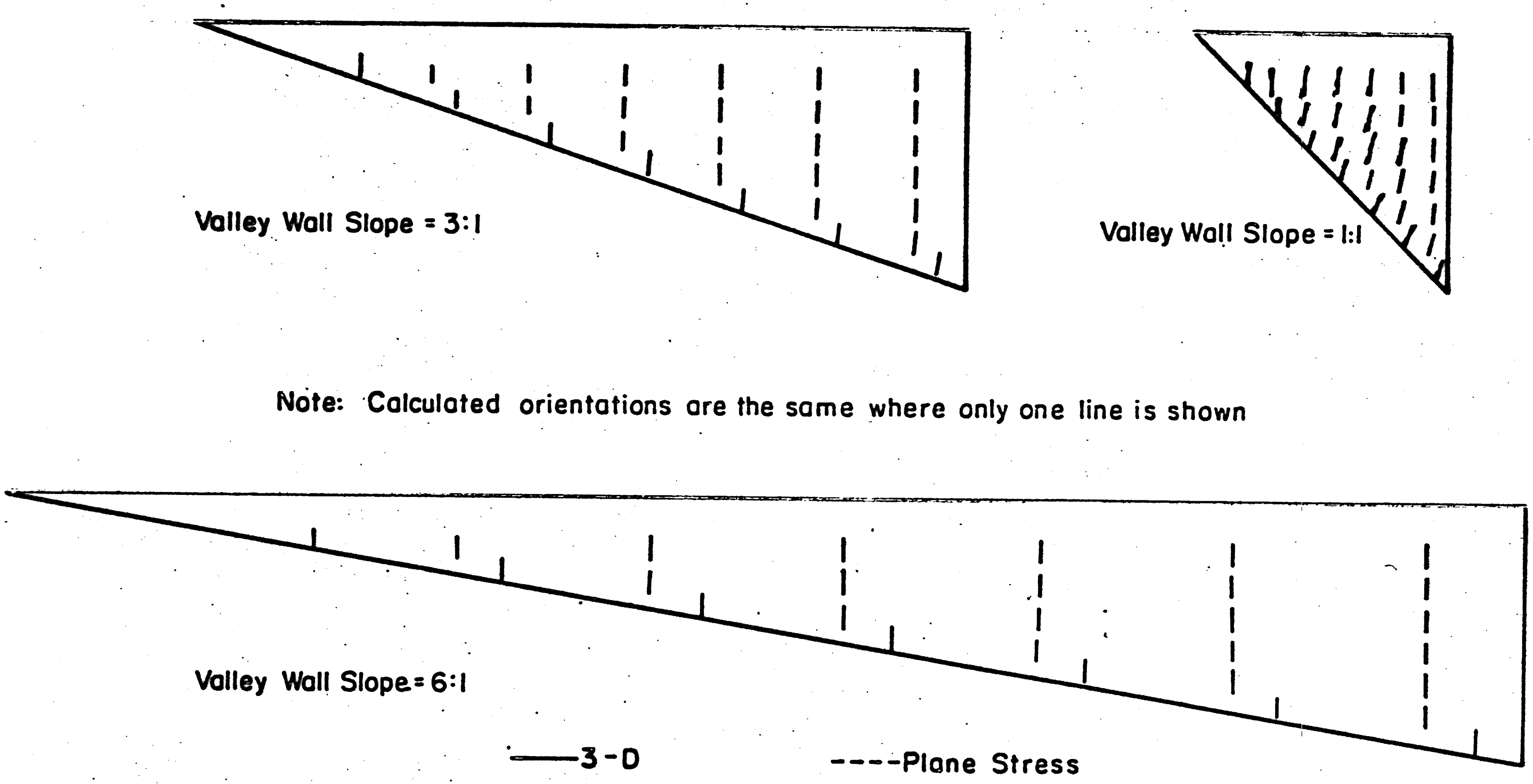

Fig. 35 COMPARISONS OF MAJOR PRINCIPAL STRESS ORIENTATIONS IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRESS ANALYSES WITH ORIENTATIONS CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

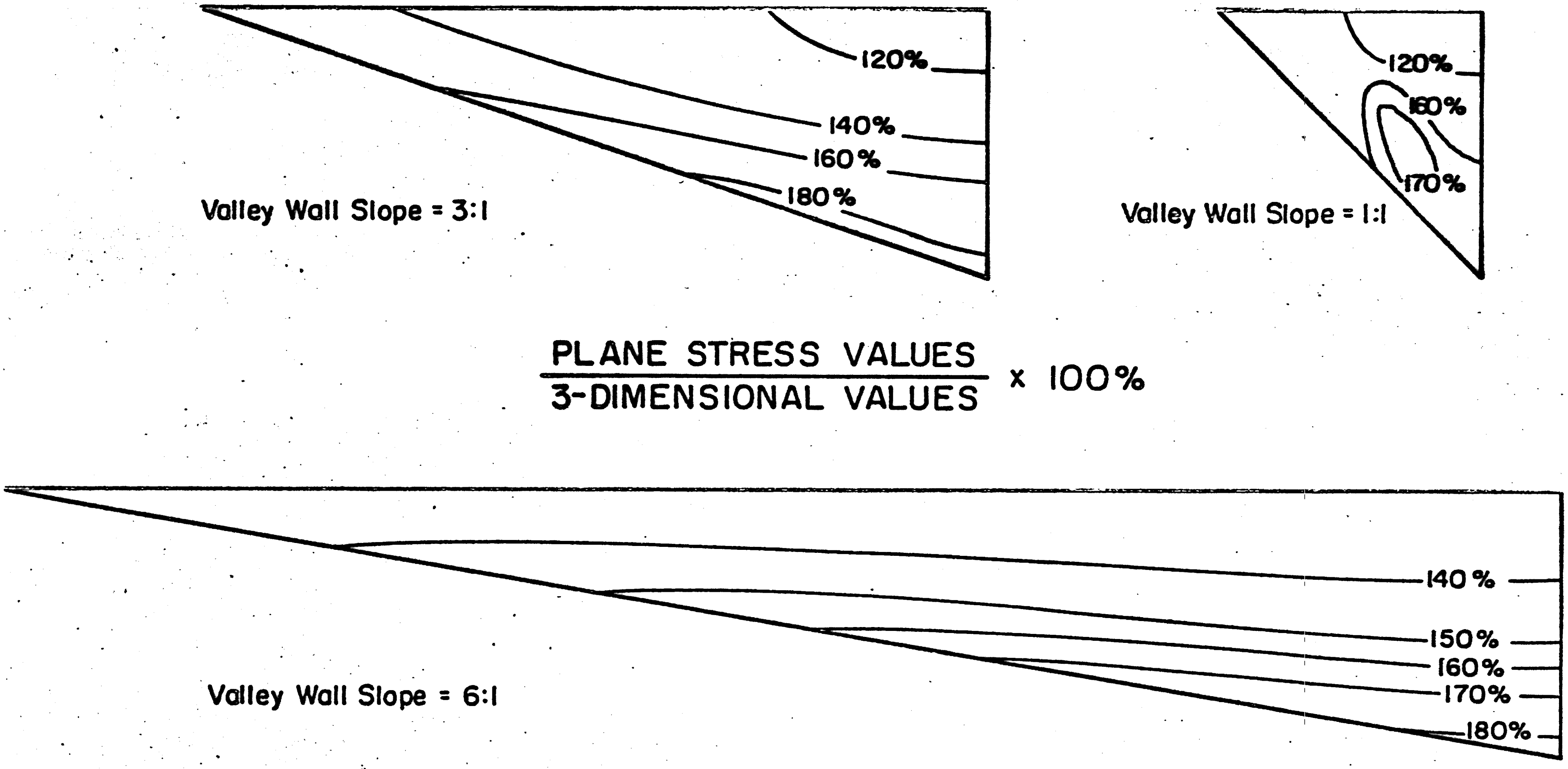

Fig.36 CONTOURS OF VALUES OF MAXIMUM SHEAR STRESS (τ_{max}) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRESS ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

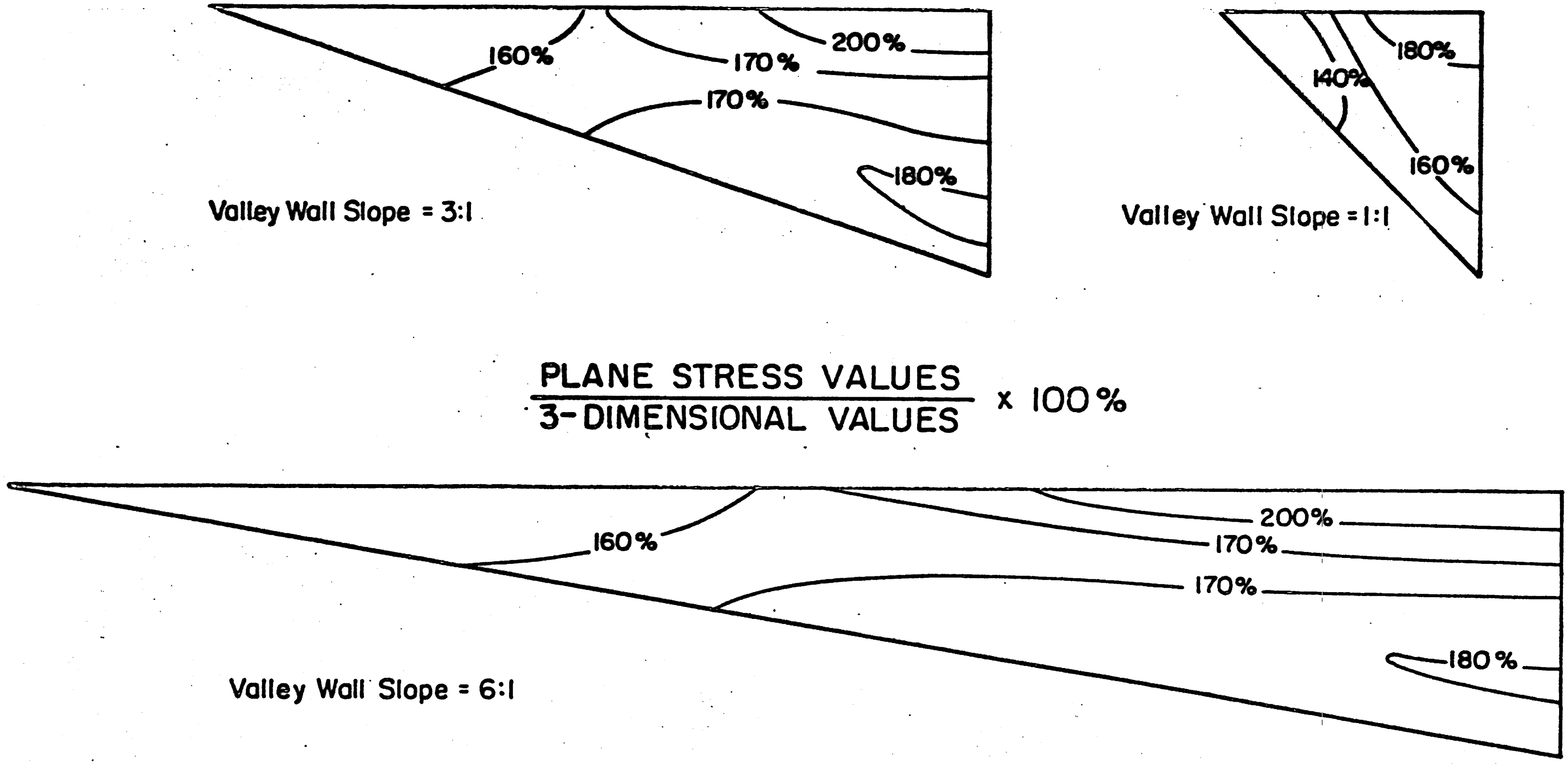

Fig.37 CONTOURS OF VALUES OF VERTICAL DISPLACEMENT (u_v) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRESS ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

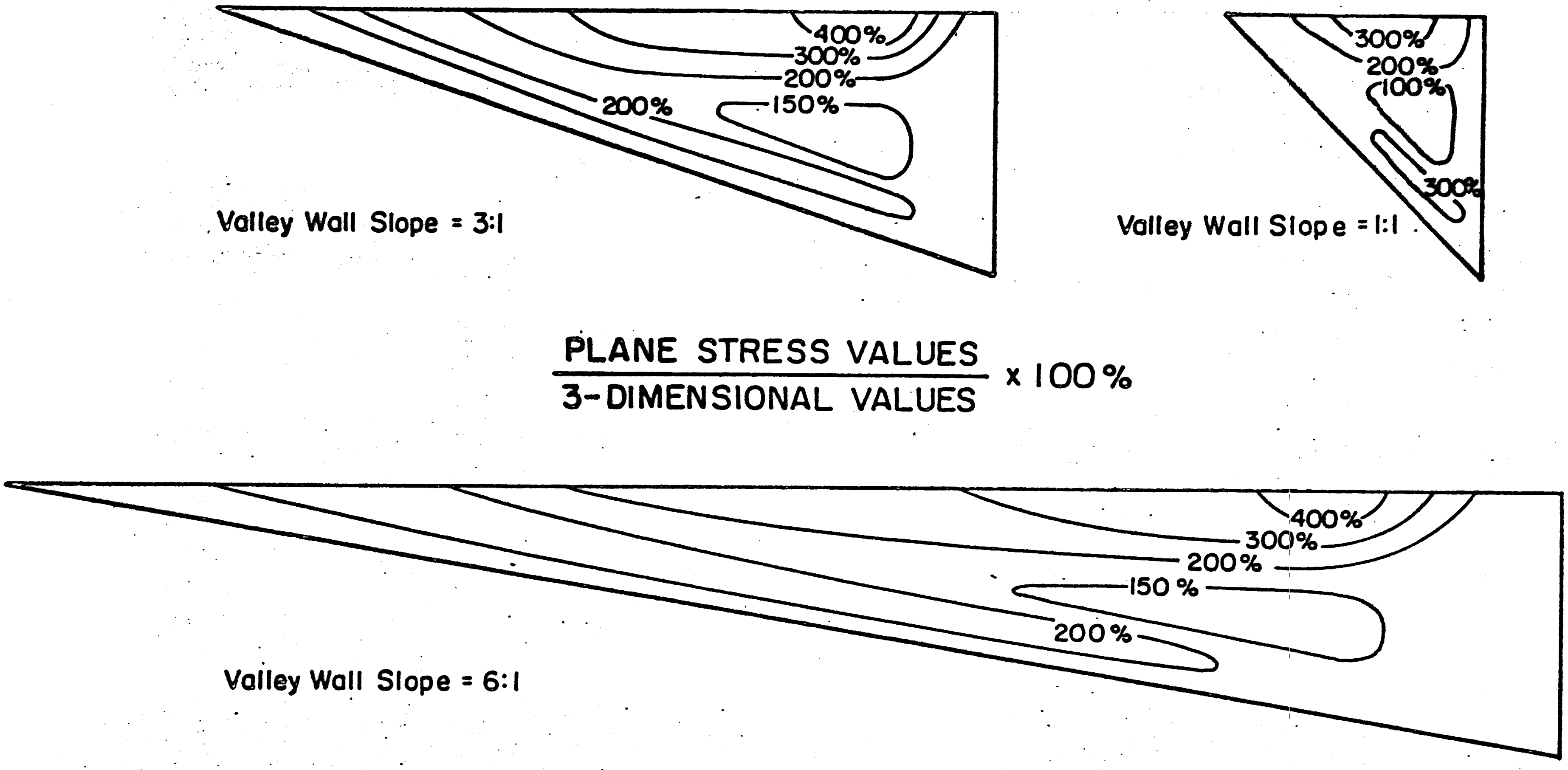

Fig.38 CONTOURS OF VALUES OF HORIZONTAL DISPLACEMENT (u_h) IN THE LONGITUDINAL SECTION CALCULATED BY PLANE STRESS ANALYSES, IN PERCENT OF THE VALUES CALCULATED BY THREE-DIMENSIONAL ANALYSES, FOR THREE DIFFERENT VALLEY WALL SLOPES

section, and with the results of both plane strain and plane stress analyses of the maximum longitudinal section. Both the two-dimensional and the three-dimensional analyses considered in these comparisons were performed incrementally, simulating construction of the dams in eight steps, using linear elastic material properties.

The results of the comparisons made are summarized in Table 1. The comparisons indicate the values of stress or displacement calculated by the plane strain or plane stress analyses, expressed as a percentage of the values calculated by the three-dimensional analyses. The values shown were obtained by averaging the calculated values for each element in the mesh, except for the odd-shaped elements along the dam slopes, which were found not to give accurate results.

The comparisons show that reasonably accurate values of stress and displacement in the transverse section may be calculated using plane strain analyses for dams in valleys with valley wall slopes as flat as 3:1 or flatter. As shown in Table 1, the average values of the stresses σ_1, σ_3, and τ_{max} calculated by plane strain analyses differ by 12 percent or less from the values calculated by three-dimensional analyses for these cases. The values of vertical displacement differ by 6 percent or less and the values of horizontal displacement differ by 20 percent or less. Although the percentage differences in the values of horizontal displacement calculated by the different analyses were relatively large as compared to the differences in the other quantities, the magnitudes of the horizontal displacements were quite small, and the somewhat larger percentage differences are therefore not considered to be of great significance. The comparisons also show that plane strain analyses do not provide as accurate results for the transverse section of dams in valleys with steeper valley wall slopes. The average value of maximum shear stress calculated by the plane strain analysis is 38 percent greater than that calculated by the three-dimensional analysis for the 1:1 valley wall slope, the average vertical displacement is 36 percent greater, and the average horizontal displacement is 168 percent greater. Judging by these comparisons, it may be concluded that plane strain analyses of the transverse section provide reasonably accurate results for dams in V-shaped valleys if the valley-wall slopes are 3:1 or flatter, but significantly less accurate results for dams in valleys with valley wall slopes as steep as 1:1.

The comparisons in Table 1 also show that plane strain analyses of the longitudinal section provides fairly accurate results for all of the valley wall slopes analyzed. The greatest difference noted was for the minor principal stress; the average value of σ_3 calculated by plane strain analyses exceeds the average value calculated by the three-dimensional analyses by 22 to 24 percent for the three cases studied. The average values of the other stresses and the displacements differ by smaller amounts. Plane stress analyses of the longitudinal section were found to give less accurate results than plane strain analyses. As shown

Table 1. Summary of Comparisons of Results of Plane Strain and Plane Stress Analyses with Results of Three-Dimensional Analyses of Dams in V-Shaped Valleys.

Section	Traverse Section			Longitudinal Section					
Comparison	(Plane Strain Values / 3-Dimensional Values) x 100%			(Plane Strain Values / 3-Dimensional Values) x 100%			(Plane Stress Values / 3-Dimensional Values) x 100%		
Quantity \ Valley Wall Slope	1:1	3:1	6:1	1:1	3:1	6:1	1:1	3:1	6:1
σ_1	113	102	101	111	110	110	109	110	111
σ_3	98	96	97	122	124	123	77	84	85
τ_{max}	138	112	108	94	91	90	149	149	149
u_v	136	106	100	97	98	97	160	173	173
u_h	268	120	105	117	118	115	220	228	224

in Table 1, the average values of maximum shear stress calculated by plane stress analyses are 49 percent greater than those calculated by three-dimensional analyses, the average vertical displacements are 60 to 73 percent greater, and the average horizontal displacements are 120 to 128 percent greater. On the basis of these results it may be concluded that plane strain analyses provide fairly accurate values of stress and displacement for the longitudinal sections of dams, whereas plane stress analyses provide considerably less accurate results.

The experience gained during the course of this study indicates that it would be feasible to perform three-dimensional analyses of dams or other structures for practical purposes. With the computer program used in this study (SAP, developed by Wilson, 1970) three-dimensional analyses of dams may be performed using a sufficient number of elements so that accuracy is not impaired, without using unreasonable amounts of computer time. The analyses described in this report were accomplished within an amount of computer time estimated as about two hours of CDC 6400 time or 40 minutes of CDC 6600 time, including the time required for developing the procedures used.

APPENDIX I

COMPARISON OF RESULTS OF COMPUTER PROGRAMS SAP AND LSBUILD FOR PLANE STRAIN ANALYSES OF THE TRANSVERSE SECTION

The computer programs SAP (Wilson, 1970) and LSBUILD (Kulhawy, et. al., 1969) were both used to perform plane strain analyses of the transverse section of the dam analyzed in this study. As shown in Figs. 39 and 40, the values of stress and displacement calculated by both computer programs were plotted together for purposes of evaluating the differences in results. The differences were found to be greatest along the slopes where the brick elements used in SAP were distorted to pentahedral or tetrahedral shapes, and where the quadrilateral elements used in LSBUILD were distorted to triangular shapes. It is believed that virtually all of the differences noted between the results of these two computer programs resulted from the less-than-perfect characteristics of these odd-shaped elements. Therefore, in plotting the contours shown in this report the results for these elements were ignored. The values of stress and vertical displacement shown in Figs. 39 and 40 calculated using SAP and LSBUILD may be seen to agree within about 2% to 3% for the regions away from the slopes. The calculated values of horizontal displacement differed somewhat more, however; the maximum value calculated using LSBUILD is about 10% larger than that calculated using SAP (1.04 ft as compared to 0.95 ft).

To insure that these differences would not influence the conclusions regarding the comparisons of results of two-dimensional and three-dimensional analyses, the plane strain analyses of the transverse section were performed using the same computer program (SAP) used for the three-dimensional analyses. It was not considered to be necessary to use the same program for the two-dimensional analyses of the longitudinal section, because the only odd-shaped elements in the meshes used were adjacent to the valley walls, most of their nodal points were fixed, and they played a small role in determining the behavior of the structure. Therefore, because it was more convenient, the computer program LSBUILD was used for these analyses

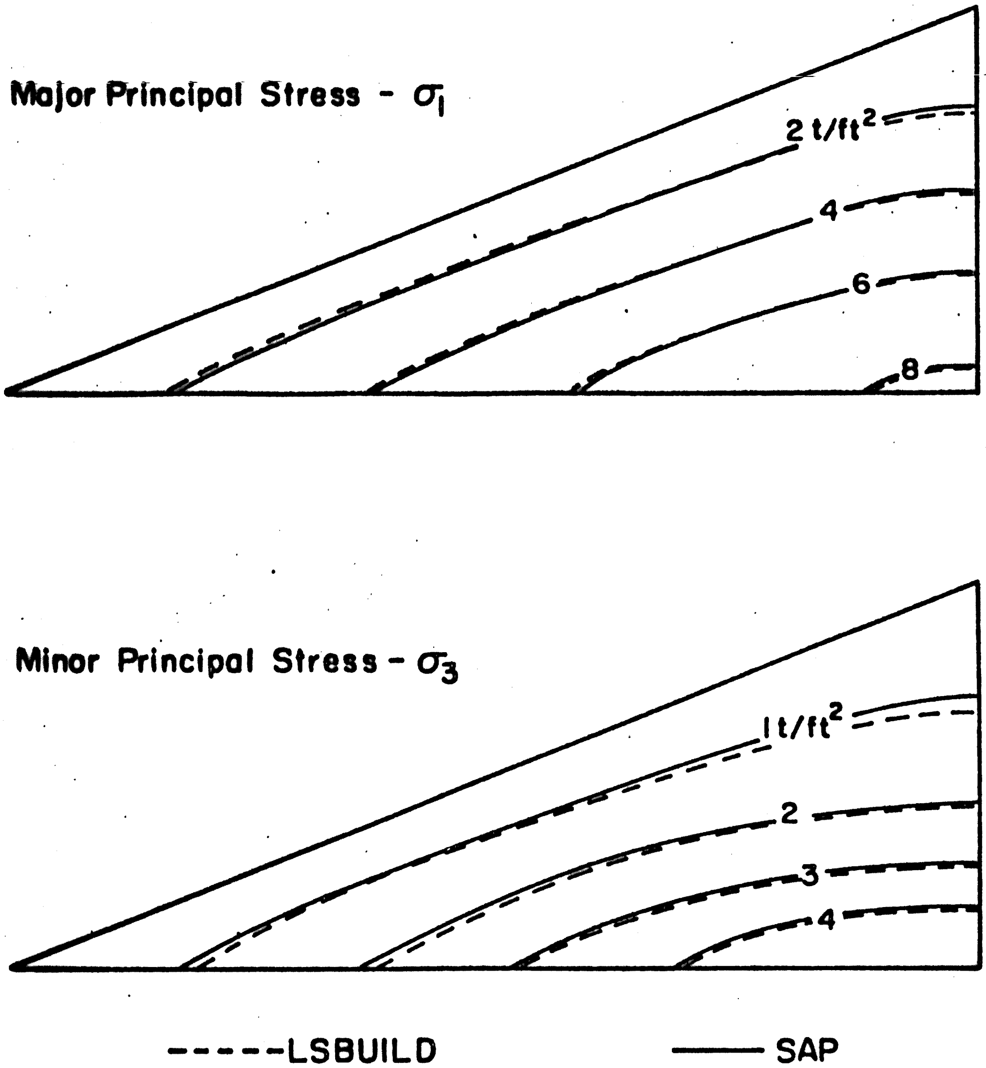

Fig. 39 COMPARISON OF STRESSES IN THE TRANSVERSE SECTION CALCULATED BY INCREMENTAL ANALYSES USING COMPUTER PROGRAMS LSBUILD AND SAP

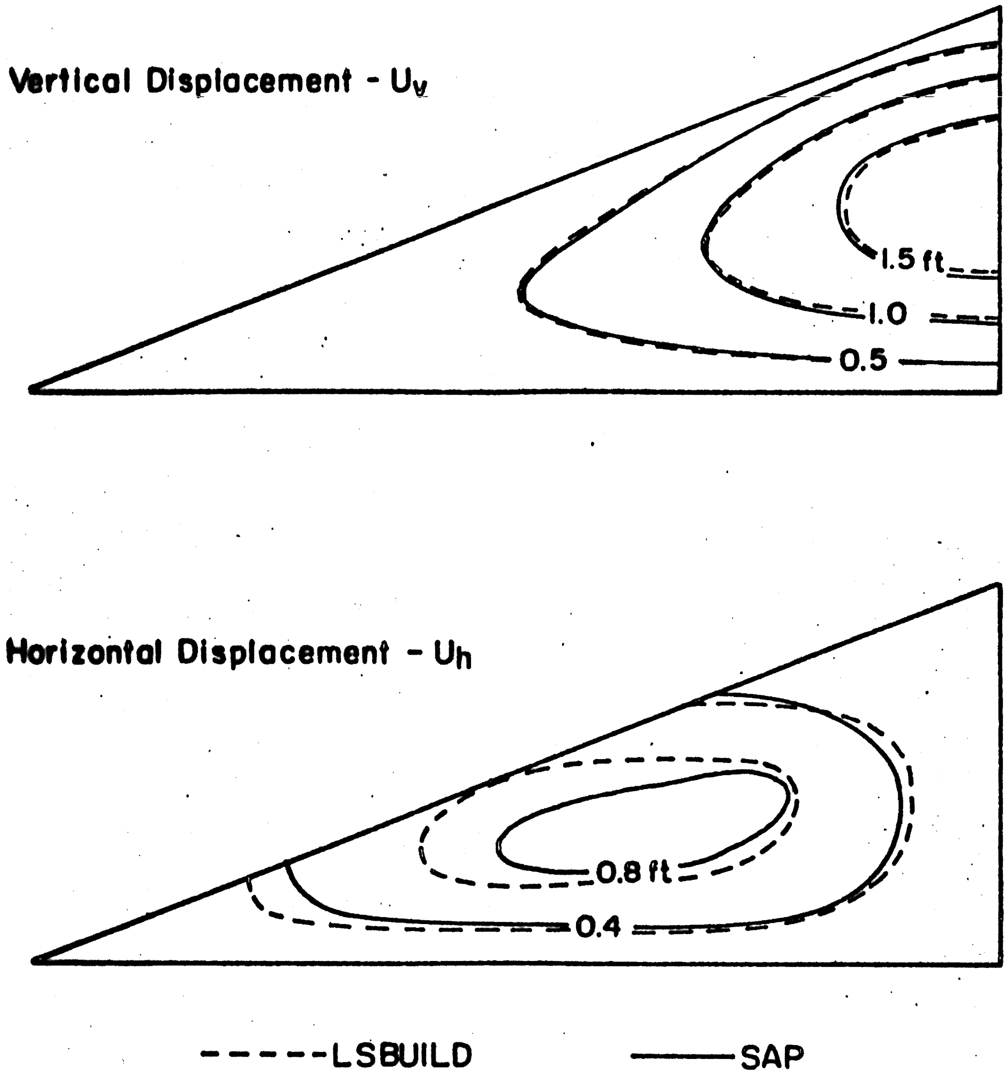

Fig. 40 COMPARISON OF DISPLACEMENTS IN THE TRANSVERSE SECTION CALCULATED BY INCREMENTAL ANALYSES USING COMPUTER PROGRAMS LSBUILD AND SAP

APPENDIX II

REFERENCES

Casagrande, A., and Covarrubias, S. W. (1970) "Tension Zones in Embankments Caused by Conduits and Cutoff Walls," Report No. S-70-7, Harvard University, Cambridge, Mass.

Clough, R. W. and Woodward, Richard J., III (1967) "Analysis of Embankment Stresses and Deformations," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM4, Proc. Paper 5329, July, 1967, pp. 529-549.

Covarrubias, S. W. (1969) "Cracking of Earth and Rockfill Dams, a Theoretical Investigation by Means of the Finite Element Method," Report No. S-69-5, Harvard University, Cambridge, Mass.

Finn, W. D. L. (1967) "Static and Dynamic Behavior of an Earth Dam," Canadian Geotechnical Journal, Vol. 4, No. 1.

Kulhawy, F. H. and Duncan, J. M. (1970) "Nonlinear Finite Element Analysis of Stresses and Movements in Oroville Dam," Report No. TE 70-2, Office of Research Services, University of California, Berkeley, January.

Kulhawy, F. H., Duncan, J. M. and Seed, H. B. (1969) "Finite Element Analyses of Stresses and Movements in Embankments During Construction," Report No. TE 69-4, Office of Research Services, University of California, Berkeley, November.

Wilson, E. L. (1970) "SAP, a General Structural Analysis Program," Report No. UCSESM 70-20, University of California, Berkeley.

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13. ABSTRACT

Three-dimensional analyses of dams were performed using a computer program developed recently by Wilson (1970). Three dams in V-shaped valleys were analyzed using incremental analysis procedures, in which construction of the dams was simulated in eight steps, and the results were compared with the results of two-dimensional incremental analyses.

The comparisons show that the stresses and displacements in the transverse section calculated by plane strain analyses agree closely with the results of three-dimensional analyses of dams in valleys having valley wall slopes as flat as 3:1 or flatter. The values of stress and displacement in the longitudinal section calculated by plane strain analyses agreed closely with the results of three-dimensional analyses for all three valley wall slopes analyzed (1:1, 3:1, and 6:1), but the results of plane stress analyses of the longitudinal section were found to differ significantly from the results of the three-dimensional analyses.

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Earth movements						
Earth stresses						
Finite element analysis						
Incremental analysis						
Plane strain analysis						
Plane stress analysis						
Three-dimensional analysis						
Two-dimensional analysis						

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