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# Estimation of Overtopping Flow Velocities on Earthen Levees Due to Irregular Waves

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**PURPOSE:** This Coastal and Hydraulics Engineering Technical Note (CHETN) provides empirical equations to estimate flow velocity and flow thickness resulting from irregular waves overtopping a trapezoidal-shaped earthen levee. The CHETN summarizes published European research with the goal of providing practicing engineers with design guidance in a concise and understandable form. Worked examples illustrate application of the empirical equations.

**INTRODUCTION AND BACKGROUND:** Erosion protection (armoring) systems placed on the crown and protected-side slope must resist the forces of fast-flowing, turbulent water that has overtopped the levee crest. Figure 1 illustrates the three overtopping cases that might occur: (a) overtopping by wind-generated waves when the still water level is beneath the elevation of the crest, (b) overflow by water levels above the levee crest, but without wave activity, and (c) overtopping by combined waves and storm surge.

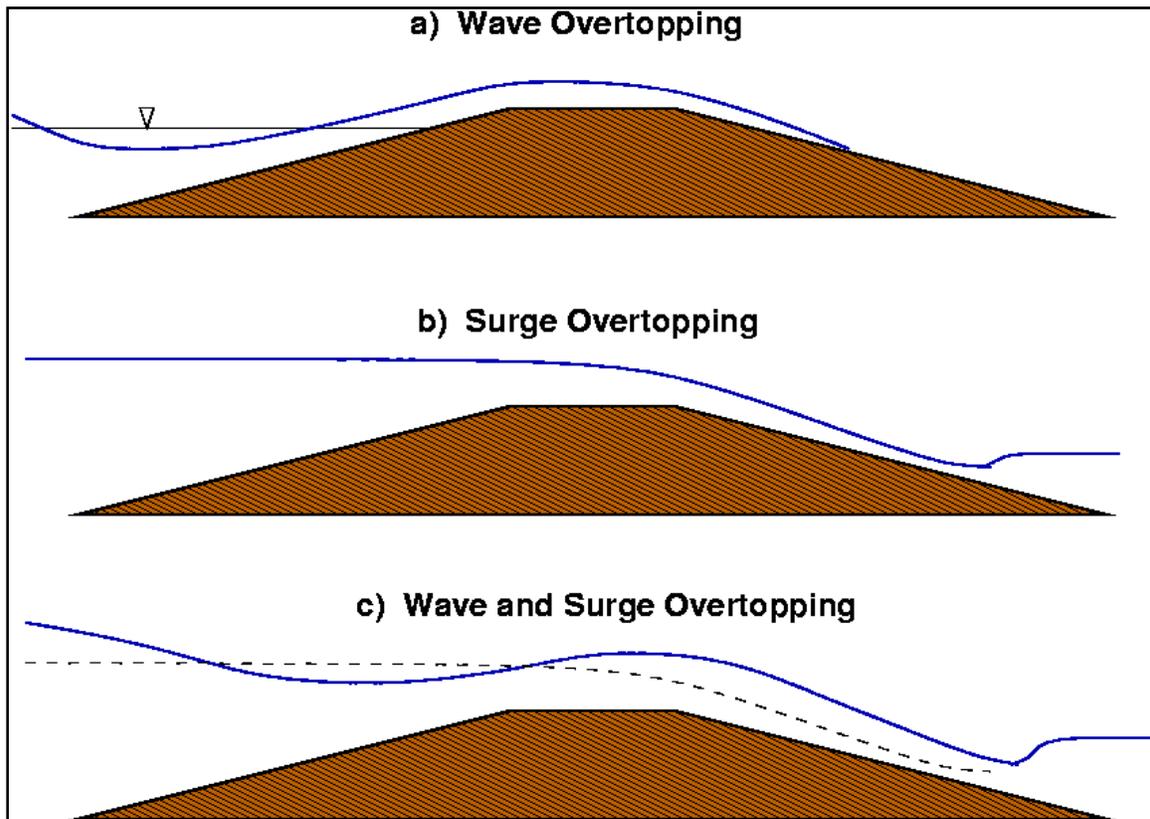


Figure 1. Overtopping scenarios for earthen levees.

The guidance presented in this CHETN pertains only to irregular wave overtopping while the surge level (still water elevation) is lower than the levee crest elevation, as shown in the top sketch of Figure 1. The main difference between wave overtopping and steady flow overtopping (shown in the middle sketch of Figure 1) is the unsteady flow that characterizes periodic wave overtopping. As each wave overtops, it has a velocity across the levee crest that can exceed the velocity of comparable surge overtopping. Thus, unprotected soil on the levee crest that is stable for low levels of surge overtopping may erode when waves overtop.

For typical slopes on the protected side of levees, overtopping waves can create critical flow conditions near the leeward edge of the levee crest, resulting in supercritical wave overtopping flow on the backside slope. However, this flow condition is unsteady, and peak velocities are sustained for only a brief time. In addition, the unsteady discharge over the crest results in a limited overtopping volume. Consequently, any erosion on the backside slope due to wave overtopping is intermittent, and the erosion rate will vary with overtopping intensity.

**SUMMARY OF EUROPEAN EXPERIMENTS:** Experiments have been conducted in Europe at small and large scale with the aim of quantifying the wave overtopping flow parameters on the inner slope of dikes and levees (Schüttrumpf et al. 2002; van Gent 2002; Schüttrumpf and van Gent 2003; Schüttrumpf and Oumeraci 2005). The European researchers developed analytical expressions to represent the velocity and flow depths at the edge of the crest on the flood side ( $u_A, h_A$ ), at the edge of the crest on the protected side ( $u_B, h_B$ ), and down the backside slope ( $u_{sb}, h_{sb}$ ), as illustrated in Figure 2.

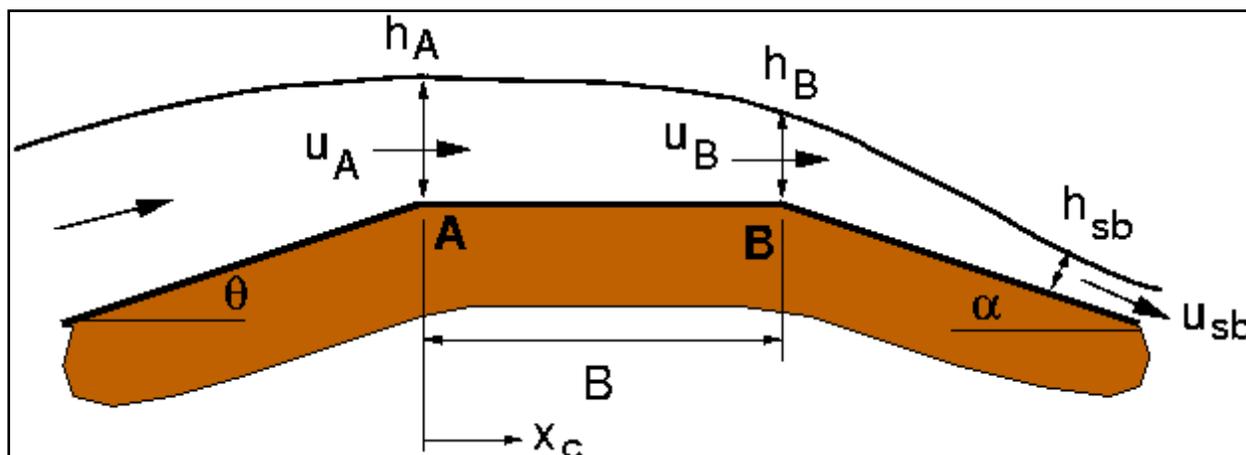


Figure 2. Wave overtopping definition sketch (after Schüttrumpf and Oumeraci 2005).

The key parameters necessary for estimating the flow velocities and depths are the levee freeboard,  $R_c$ , the runup elevation exceeded by 2 percent of the waves,  $R_{u2\%}$ , and a friction factor,  $f_F$ , that accounts for frictional energy loss as the overtopping wave travels across the levee crest and down the protected-side slope.

Independent laboratory experiments were conducted in The Netherlands (van Gent 2002) and in Germany (Schüttrumpf et al. 2002). These two studies produced similar estimation analysis techniques, with only minor differences in the details. A joint paper (Schüttrumpf and van Gent 2003) reconciled the differences to the extent possible.

Van Gent's (2002) small-scale experiments had a 1:100 foreshore slope with a 1:4 slope on the flood side of the dike. Two levee crest widths (0.2 and 1.1 m) were tested with two protected-side slopes (1:2.5 and 1:4) to give four dike geometries using a smooth dike surface. A fifth test series was conducted with a rough surface. Velocity and flow thickness were measured at the flood-side and protected-side edges of the crest and at three locations spaced down the protected-side slope. Micro-impellers were used to measure velocity. Eighteen irregular wave tests were performed for the different dike geometries, ten with single-peaked spectra and eight with double-peaked spectra. Incident wave conditions were determined by measuring the generated waves without the structure in place, and applying the Mansard and Funke (1980) frequency-domain method to remove reflection caused by the dissipating beach profile. Van Gent (2002) used the wave parameter  $H_{1/3}$  in the analysis, but did not indicate how this time-domain parameter was determined from the frequency-domain value of  $H_{m0}$  found from the reflection analysis. Wave period was specified as mean period  $T_{m-1.0}$ , and it was estimated from the moments of the incident wave frequency spectra. The mean period is reported to better represent double-peaked spectra.

The experiments by Schüttrumpf et al. (2002) included both small- and large-scale tests. The small-scale tests used three flood-side slopes (1:3, 1:4, and 1:6), a crest width of 0.3 m, and five protected-side slopes (1:2, 1:3, 1:4, 1:5, and 1:6). A total of 270 small-scale tests were run using regular waves and irregular waves conforming to the JONSWAP spectrum. Flow depths were measured with resistance wave gauges, and overtopping flow velocity was recorded using micro-impellers. For the large-scale tests the flood-side slope was 1:6, the crest width was 2 m, and the protected-side slope was 1:3. A total of 250 large-scale model tests were run using some regular waves, but mostly irregular waves. Flow depth and velocity were measured using wave gauges and micro-impellers.

Wave data from tests by Schüttrumpf et al. (2002) were analyzed in the frequency domain using the reflection method of Mansard and Funke (1980). The time-domain wave height parameter  $H_{1/3}$  was used in their overtopping analysis with the conversion from the frequency domain wave height given as  $H_{1/3} = 0.94 H_{m0}$  (Schüttrumpf 2006, personal communication). This conversion is a little odd because we should expect  $H_{1/3}$  to be greater than  $H_{m0}$  for shallow water waves. Also, the conversion is valid strictly for these tests and not in general because it was determined for wave flume data with a constant water depth for all tests. The wave period was specified as the mean wave period, and it was determined from the calculated incident wave spectra by the simple relationship  $T_m = 0.88 T_p$  (Schüttrumpf 2006, personal communication).

**FLOW PARAMETERS AT THE FLOOD-SIDE LEVEE CREST EDGE:** Flow depth of the runoff on the flood-side slope was assumed to be a linear decrease from the still water level to the highest elevation of runoff. At the flood-side edge of the levee crest (denoted by the subscript letter A in Figure 2), the flow parameters are given by the equations

$$\frac{h_{A2\%}}{H_s} = C_{Ah2\%} \left[ \frac{(R_{u2\%} - R_c)}{H_s} \right] \quad (1)$$

and

$$\frac{u_{A2\%}}{\sqrt{g} H_s} = C_{Au2\%} \sqrt{\frac{(R_{u2\%} - R_c)}{H_s}} \quad (2)$$

where

- $h_{A2\%}$  = peak flow depth exceeded by 2% of the waves
- $H_s$  = significant wave height [ $\approx H_{m0}$ ]
- $C_{Ah2\%}$  = empirical depth coefficient determined from test data
- $R_{u2\%}$  = runup elevation exceeded by 2% of the waves
- $R_c$  = crest freeboard [= crest elevation minus still water elevation]
- $u_{A2\%}$  = flow depth-averaged peak velocity exceeded by 2% of the waves
- $g$  = acceleration of gravity
- $C_{Au2\%}$  = empirical velocity coefficient determined from test data

The values of  $h_{A2\%}$  and  $u_{A2\%}$  were determined from the peaks of the overtopping wave time series, and these parameters represent the levels exceeded by only 2 percent of the total waves during the tests. For example, if a test had 1000 waves, perhaps only 200 waves overtopped the crest. The 2 percent exceedance level would be the level exceeded by 20 of the 1000 waves ( $0.02 \times 1000$ ), but this is 10 percent of the overtopping waves. Schüttrumpf et al. (2002) also provided coefficients for the average overtopping parameters  $h_{A50\%}$  and  $u_{A50\%}$ . All of the equations pertain to the maximum velocity at the leading front of the overtopping wave. Flow velocities and depths associated with a single wave decrease after passage of the wave front.

Note in Equations 1 and 2 that significant wave height  $H_s$  in the denominator cancels on both sides of the equations. Thus, the flow depth is directly proportional to the difference between the 2-percent runup and levee freeboard, and the depth-averaged flow velocity is proportional to the square root of this difference. Wave parameters enter into the estimation of flow depth and velocity at the flood-side crest edge through the estimation of the 2-percent runup parameter  $R_{u2\%}$ . As noted by van Gent (2002), the calculated  $R_{u2\%}$  is a fictitious value in cases where runup exceeds the structure freeboard. It is the level that would be exceeded by 2 percent of the waves if the front slope were continued upward indefinitely.

The values of the empirical coefficients determined for the two studies are given in Table 1. The superscripts following each number refer to the references given in the footnotes to Table 1.

<b>Table 1. Empirical coefficients for flood-side crest edge flow parameters.</b>		
<b>Coefficient</b>	<b>Schüttrumpf</b>	<b>van Gent</b>
$C_{Ah2\%}$	0.33 <sup>2,3</sup> and 0.22 <sup>4</sup>	0.15 <sup>1,3</sup>
$C_{Au2\%}$	1.55 <sup>2</sup> and 1.37 <sup>3</sup>	1.30 <sup>1,3</sup>
$C_{Ah50\%}$	0.17 <sup>2,4</sup>	—
$C_{Au50\%}$	0.94 <sup>2,4</sup>	—
<sup>1</sup> van Gent (2002). <sup>2</sup> Schüttrumpf et al. (2002). <sup>3</sup> Schüttrumpf and van Gent (2003). <sup>4</sup> Schüttrumpf and Oumeraci (2005).		

The coefficient  $C_{Ah2\%}$  is a constant that is actually equal to a slope-dependent constant,  $C_2$ , divided by  $\tan \theta$ , where  $\theta$  is the flood-side structure slope. Values of  $C_2$  given in the various papers are used in an equation slightly different from Equation 1. The value for  $C_{Ah2\%}$  given by Schüttrumpf was revised from 0.33 to 0.22 in the most recent paper (Schüttrumpf and Oumeraci 2005), and this probably represents a better value—as shown by the data plot given in their paper and the fact that it is closer to the value obtained by van Gent. The value of  $C_{Au2\%} = 1.55$  is derived from a table in Schüttrumpf et al. (2002) that associated this coefficient with large-scale tests. A coefficient associated with the 10-percent exceedance level can also be derived from the same table as  $C_{Au10\%} = 1.37$  for large-scale tests. In Schüttrumpf and van Gent (2003), the value of  $C_{Au2\%} = 1.37$  was reported, and this is thought to be a typographical error. The correct value should have been  $C_{Au2\%} = 1.55$ .

Schüttrumpf and van Gent (2003) attributed differences in empirical coefficients to different dike geometries and instruments, but noted the differences are not too great. Van der Meer et al. (2006) suggested an error in measurement or analysis might have caused the factor-of-two difference seen for the coefficient  $C_{Ah2\%}$ , but the revised value of  $C_{Ah2\%} = 0.22$  brings the results closer. A more probable cause for variation might be in the method each investigator used to estimate the value of 2-percent runup,  $R_{u2\%}$ . Van Gent (2002) estimated  $R_{u2\%}$  using a formula he developed earlier (van Gent 2001), which uses  $H_{1/3}$  and  $T_{m-0.1}$  as the wave parameters. Schüttrumpf estimated  $R_{u2\%}$  using the equations of de Waal and van der Meer (1992), with wave height  $H_{1/3}$  and wave period  $T_m$  instead of spectral peak period  $T_p$ . Both formulas give reasonable estimates that fall within the scatter of the 2-percent runup data; thus, whichever formula is selected for calculating  $R_{u2\%}$ , the estimates for overtopping flow parameters should be similar.

Until further clarification becomes available, it is recommended that values of  $C_{Ah2\%} = 0.22$  and  $C_{Au2\%} = 1.55$  be used in Equations 1 and 2 to estimate the overtopping flow parameters associated with the flow depth and velocity exceeded by 2 percent of the incoming waves.

**FLOW PARAMETERS AT THE PROTECTED-SIDE LEVEE CREST EDGE:** Overtopping waves flowing across the dike or levee crest decrease in height, and the velocity decreases as a function of the surface friction factor,  $f_f$ . Flow depth (or thickness) can be estimated at any location on the crest with the equation

$$h_{B2\%} = h_{A2\%} \exp\left(-C_3 \frac{x_c}{B}\right) \quad (3)$$

where  $C_3$  is an empirical coefficient,  $x_c$  is distance along the crest from the flood-side edge, and  $B$  is the horizontal crest width. The flow thickness at the protected-side crest edge (denoted by the subscript letter B in Figure 2) is given when  $x_c = B$ . Different values of the coefficient were given in the various publications, i.e., based on the 2-percent exceedance levels  $C_3 = 0.89$  for TMA spectra and  $C_3 = 1.11$  for natural spectra (Schüttrumpf et al. 2002);  $C_3 = 0.40$  and  $0.89$  (Schüttrumpf and van Gent 2003); and  $C_3 = 0.75$  for irregular and regular waves (Schüttrumpf and Oumeraci 2005). The factor-of-two difference between van Gent and Schüttrumpf was attributed to the difference in estimating wave runup.

For levee calculations, it is recommended that a value of  $C_3 = 0.75$  be used in Equation 3 on the assumption that earlier values have been superseded by publication of the 2005 journal article. The 2-percent runup elevation should be estimated using the runup formulas of de Waal and van der Meer (1992) or Hughes (2004). If van Gent's (2001) method for estimating wave runup is used, it would be more appropriate to use a value of  $C_3 = 0.40$ . Note that Equation 3 is applicable for estimating  $h_{B50\%}$  if the flow depth  $h_{A50\%}$  is used instead of  $h_{A2\%}$ . In fact, Schüttrumpf and Oumeraci (2005) presented only the 50-percent exceedance values.

Flow velocity along the dike crest exceeded by 2 percent of the waves is given by a similar equation:

$$u_{B2\%} = u_{A2\%} \exp\left(-\frac{x_c f_F}{2 h_{B2\%}}\right) \quad (4)$$

where  $f_F$  is the Fanning friction factor appropriate for the levee crest surface, and  $h_{B2\%}$  is the flow depth at that location on the crest obtained via Equation 3. At the protected-side crest edge, evaluate Equation 4 with  $x_c = B$ . Van Gent (2002) had a different expression for  $u_{B2\%}$ ; however, in Schüttrumpf and van Gent (2003), both authors agreed on Equation 4. A theoretical derivation for Equation 4 was given in Schüttrumpf and Oumeraci (2005).

**ESTIMATION OF FRICTION FACTOR:** The Fanning friction factor has a significant influence on flow velocity across the crest and down the protected-side slope. The small-scale experiments of Schüttrumpf et al. (2002) had a structure surface constructed of wood fiberboard, and the friction factor on the crest was determined experimentally to be  $f_F = 0.0058$  (Schüttrumpf and Oumeraci 2005). The structure in the companion large-scale experiments was constructed with a bare, compacted clay surface, and experimental results gave the friction factor as  $f_F = 0.01$  (Schüttrumpf et al. 2002). Schüttrumpf and Oumeraci (2005) also list the representative value for friction factor on the protected-side slope as  $f_F = 0.02$  (smooth slopes), and Cornett and Mansard (1994) give  $f_F = 0.1 - 0.6$  (rough revetments and rubble-mound slopes). Grass-covered slopes would probably have a friction coefficient above  $f_F = 0.01$ .

Determination of an appropriate value of friction factor for various grass coverings and armoring alternatives may be difficult because of the lack of published values. As a first approximation, an estimate can be made if a representative value of Manning's  $n$  is known for a particular slope surface

or armoring product. Manning's  $n$  can be related to the Chezy coefficient,  $C_z$ , by the expression (e.g., Henderson 1966)

$$C_z = \frac{R^{1/6}}{n} \quad (5)$$

where  $R$  is the hydraulic radius, and Manning's  $n$  is given in metric units. For wide channels,  $R$  is essentially the same as the depth,  $h$ . The Chezy coefficient can be given in terms of the Darcy friction factor ( $f_D$ ). Because the Fanning friction factor is one fourth of the Darcy friction factor, i.e.,  $f_D = f_F/4$ , the Chezy coefficient can also be given in terms of the Fanning friction factor as (Henderson 1966)

$$C_z = \sqrt{\frac{8g}{f_D}} = \sqrt{\frac{2g}{f_F}} \quad (6)$$

Equating 5 and 6, substituting  $h$  for  $R$ , and rearranging yields an equation (in metric units) for  $f_F$  in terms of Manning's coefficient and flow depth  $h$  in meters, i.e.,

$$f_F = \frac{2g n^2}{h^{1/3}} \quad (7)$$

The validity of Equation 7 has not been proven, and it is based on the assumption that friction factors and Manning's  $n$  associated with steady supercritical overflow that has reached equilibrium (e.g., Chezy or Manning equation) will be the same for unsteady, rapidly varying flows due to wave overtopping. Therefore, caution must be exercised when applying Equation 7.

Hewlett et al. (1987) suggested values of Manning's  $n$  for grass-covered slopes exposed to steady supercritical overflow. They recommended  $n = 0.03$  for slopes of 1:10 ( $\tan \alpha = 1/10$ ), decreasing linearly to  $n = 0.02$  for slopes of 1:3. They recommended using  $n = 0.02$  for slopes steeper than 1:3. This linear relationship can be expressed by a simple formula:

$$n = 0.0343 - 0.043 \tan \alpha \quad (8)$$

for the range  $1/10 < \tan \alpha < 1/3$ . Yong and Stone (1967) recommended  $n = 0.035$  for steep grass slopes. Table 2 presents an example of friction factors calculated using Equation 7 for several flow depths on a 1:3 slope ( $\tan \alpha = 1/3$ ).

<b>Table 2. Fanning friction factors, <math>f_F</math>, for 1:3 slope.</b>			
<b>Depth, <math>h</math> (ft)</b>	<b>Depth, <math>h</math> (m)</b>	<b><math>f_F</math> for <math>n = 0.02</math></b>	<b><math>f_F</math> for <math>n = 0.035</math></b>
0.5	0.152	0.015	0.045
1.0	0.305	0.012	0.036
2.0	0.610	0.010	0.028

As seen in Table 2, the value of friction factor is sensitive to the choice of Manning's  $n$ . This is unfortunate because the friction factor is quite influential for determining flow velocity magnitudes across the levee crown and down the protected-side slope.

**FLOW PARAMETERS ON THE PROTECTED-SIDE LEVEE SLOPE:** Both European investigators derived theoretical expressions for the wave front depth-averaged, slope-parallel flow velocity down the protected-side slope based on simplification of the momentum equation. Schüttrumpf and Oumeraci (2005) presented an iterative solution, whereas van Gent (2002) derived an explicit formula. A comparison between the two solutions reveals only small differences in the result, and both formulations approach the same equation in the limit as distance down the slope becomes large (Schüttrumpf and van Gent 2003). For ease of application, van Gent's formula is preferred, and it was given as

$$u_{sb2\%} = \frac{K_2}{K_3} + K_4 \exp(-3K_2 \cdot K_3^2 \cdot s_b) \quad (9)$$

with

$$K_2 = (g \sin \alpha)^{1/3} \quad (10)$$

$$K_3 = \left[ \frac{f_F}{2} \frac{1}{(h_{B2\%} \cdot u_{B2\%})} \right]^{1/3} \quad (11)$$

$$K_4 = u_{B2\%} - \frac{K_2}{K_3} \quad (12)$$

and  $s_b$  is the distance down the slope from the crest edge,  $\alpha$  is the angle of the protected-side slope, and  $h_{B2\%}$  and  $u_{B2\%}$  are the flow depth and flow velocity, respectively, at the protected-side crest edge. For long distances down slope, the exponential term in Equation 9 vanishes, and the velocity equation reduces to

$$u_{sb2\%} = \frac{K_2}{K_3} = \left[ \frac{2g \cdot h_{B2\%} \cdot u_{B2\%} \cdot \sin \alpha}{f_F} \right]^{1/3} \quad (13)$$

Flow thickness perpendicular to the slope at any point down the protected-side slope is found from the continuity equation as

$$h_{sb2\%} = \left[ \frac{h_{B2\%} \cdot u_{B2\%}}{u_{sb2\%}} \right] \quad (14)$$

Equations 1-14 can be used to estimate the wave overtopping peak velocity and associated flow depth over a levee that is exceeded by only 2 percent of the incoming waves. The main input parameters are the wave height ( $H_{m0}$ ), wave period ( $T_p$ ), and flood-side slope ( $\tan \theta$ ) needed to calculate the 2-percent runup elevation ( $R_{u2\%}$ ); the freeboard ( $R_c$ ); the protected-side slope ( $\tan \alpha$ ); the crest width ( $B$ ); and the Fanning friction factor ( $f_F$ ). The following example illustrates application of the flow parameter estimation technique.

### Example 1: Irregular Wave 2% Overtopping Flow Parameters

**Find:** The overtopping flow parameters exceeded by 2 percent of the waves associated with an average wave overtopping discharge rate of  $q_w = 0.1 \text{ ft}^3/\text{sec}$  per foot of levee. The levee is covered with good-quality grass on the crest and protected-side slope.

**Given:**

$H_{m0} = 6 \text{ ft}$	= zeroth-moment significant wave height [= $H_s$ ]
$T_p = 8 \text{ sec}$	= wave period associated with the spectral peak
$\tan \theta = 1/4$	= flood-side levee slope
$\tan \alpha = 1/3$	= protected-side levee slope
$B = 10 \text{ ft}$	= levee crest width
$g = 32.2 \text{ ft/sec}^2$	= acceleration due to gravity
$f_F = 0.015$	= Fanning friction factor (from Table 2 for $n = 0.02$ and $h = 0.5 \text{ ft}$ )

**Calculate Wave Runup and Levee Freeboard.** Runup and levee freeboard are the key parameters needed to start the calculation procedure. These parameters can be determined using established equations from Part VI of the Coastal Engineering Manual (Burcharth and Hughes 2002).

First, calculate the Iribarren number (surf-similarity parameter) for the given wave condition and levee flood-side slope (CEM Equation VI-5-2). The deepwater wave length based on peak spectral wave period is

$$L_{op} = \frac{g}{2\pi} T_p^2 = \frac{32.2 \text{ ft/sec}^2}{2\pi} (8 \text{ sec})^2 = 328 \text{ ft} \quad (15)$$

and the corresponding Iribarren number is

$$\xi_{op} = \frac{\tan \theta}{\sqrt{H_{m0} / L_{op}}} = \frac{0.25}{\sqrt{(6 \text{ ft}) / (328 \text{ ft})}} = 1.85 \quad (16)$$

Below are the runup equations given by CEM Equation VI-5-6 (de Waal and van der Meer 1992) and the equations for freeboard obtained by inverting the average wave overtopping formulas given by CEM Equations VI-5-24 and VI-5-25 (van der Meer and Janssen 1995). Different equations are used according to the value of Iribarren number.

**For  $\xi_{op} \leq 2$ :**

$$R_{u2\%} = 1.5 \xi_{op} \cdot H_{m0} \quad (17)$$

$$R_c = -\frac{H_{m0} \xi_{op}}{5.2} \ln \left[ \frac{q_w}{\sqrt{g H_{m0}^3}} \cdot \frac{\sqrt{\tan \theta}}{\xi_{op}} \cdot \frac{1}{0.06} \right] \cdot (\gamma_r \gamma_b \gamma_h \gamma_\beta) \quad (18)$$

**For  $\xi_{op} > 2$ :**

$$R_{u2\%} = 3.0 \cdot H_{m0} \quad (19)$$

$$R_c = -\frac{H_{m0}}{2.6} \ln \left[ 5 \cdot \frac{q_w}{\sqrt{g H_{m0}^3}} \right] \cdot (\gamma_r \gamma_b \gamma_h \gamma_\beta) \quad (20)$$

The “gamma factors” in Equations 18 and 20 account for slope roughness, berm effect, shallow depth, and wave direction. Details are in van der Meer and Janssen (1995) or the Coastal Engineering Manual.

Using the set of equations for  $\xi_{op} \leq 2$  and setting all the gammas equal to unity yields

$$R_{u2\%} = 1.5 \xi_{op} \cdot H_{m0} = 1.5 (1.85)(6 \text{ ft}) = 16.6 \text{ ft}$$

$$R_c = -\frac{H_{m0} \xi_{op}}{5.2} \ln \left[ \frac{q_w}{\sqrt{g} H_{m0}^3} \cdot \frac{\sqrt{\tan \theta}}{\xi_{op}} \cdot \frac{1}{0.06} \right] \cdot (\gamma_r \gamma_b \gamma_h \gamma_\beta)$$

$$R_c = -\frac{(6 \text{ ft})(1.85)}{5.2} \ln \left[ \frac{(0.1 \text{ ft}^2/\text{sec})}{\sqrt{(32.2 \text{ ft}/\text{sec}^2)(6 \text{ ft})^3}} \cdot \frac{\sqrt{0.25}}{1.85} \cdot \frac{1}{0.06} \right] \cdot (1.0 \cdot 1.0 \cdot 1.0 \cdot 1.0) = 11.1 \text{ ft}$$

**Parameters at Flood-Side Levee Crest Edge.** Substituting the recommended values of  $C_{Ah2\%} = 0.22$  and  $C_{Au2\%} = 1.55$  into Equations 1 and 2, respectively, rearranging, and noting that  $H_s$  cancels out in both equations gives equations for the flow thickness and flow velocity magnitudes exceeded by 2 percent of the incident waves. Using the above values for  $R_{u2\%}$  and  $R_c$  yields

$$h_{A2\%} = 0.22(R_{u2\%} - R_c) = 0.22(16.6 \text{ ft} - 11.1 \text{ ft}) = 1.21 \text{ ft}$$

and

$$\begin{aligned} u_{A2\%} &= \sqrt{g} (1.55) \sqrt{(R_{u2\%} - R_c)} = \sqrt{(32.2 \text{ ft}/\text{sec}^2)} (1.55) \sqrt{(16.6 \text{ ft} - 11.1 \text{ ft})} \\ &= 20.6 \text{ ft}/\text{sec} \end{aligned}$$

**Parameters at the Protected-Side Levee Crest Edge.** The flow depth and velocity at the protected-side crest edge exceeded by 2 percent of the incident waves are found using Equations 3 and 4. Substituting the recommended value of  $C_3 = 0.75$  into Equation 3, recognizing that the distance  $x_c = B$  at the protected-side crest edge, and using the flow parameters already estimated at the flood-side edge yields the estimates

$$h_{B2\%} = h_{A2\%} \exp\left(-0.75 \frac{B}{B}\right) = (1.21 \text{ ft}) \exp(-0.75) = 0.57 \text{ ft}$$

$$u_{B2\%} = u_{A2\%} \exp\left(-\frac{B f_F}{2 h_{B2\%}}\right) = (20.6 \text{ ft}/\text{sec}) \exp\left(-\frac{(10 \text{ ft})(0.015)}{2(0.57 \text{ ft})}\right) = 18.1 \text{ ft}/\text{sec}$$

The friction of the crest surface resulted in a velocity reduction of just over 12 percent.

**Parameters on the Protected-Side Levee Slope.** The flow velocity and thickness exceeded by 2 percent of the incident waves varies with distance down the protected-side slope until terminal velocity is reached (the supercritical flow condition represented by the Chezy and Manning equations). First, determine the values of  $K_2$ ,  $K_3$ , and  $K_4$  from Equations 10, 11, and 12, respectively.

$$K_2 = (g \sin \alpha)^{1/3} = \left\{ (32.2 \text{ ft/sec}^2) \sin \left[ \tan^{-1}(1/3) \right] \right\}^{1/3} = 2.17 (\text{ft/sec}^2)^{1/3}$$

$$K_3 = \left[ \frac{f_F}{2} \frac{1}{(h_{B2\%} \cdot u_{B2\%})} \right]^{1/3} = \left[ \frac{0.015}{2} \frac{1}{(0.57 \text{ ft}) \cdot (18.1 \text{ ft/sec})} \right]^{1/3} = 0.0899 (\text{sec/ft}^2)^{1/3}$$

$$K_4 = u_{B2\%} - \frac{K_2}{K_3} = (18.1 \text{ ft/sec}) - \frac{2.17 (\text{ft/sec}^2)^{1/3}}{0.0899 (\text{sec/ft}^2)^{1/3}} = -6.038 \text{ ft/sec}$$

Now the flow velocity exceeded by 2 percent of the waves can be found at any location down the slope using Equation 9. For example, at 20 ft from the protected-side edge of the crest (i.e.,  $s_b = 20$  ft) the slope-parallel velocity is estimated to be

$$\begin{aligned} u_{2\%_{(s_b=20)}} &= \frac{K_2}{K_3} + K_4 \exp(-3 K_2 \cdot K_3^2 \cdot s_b) \\ &= \frac{2.17 (\text{ft/sec}^2)^{1/3}}{0.0899 (\text{sec/ft}^2)^{1/3}} \\ &\quad + (-6.038 \text{ ft/sec}) \exp \left[ -3 \left( 2.17 (\text{ft/sec}^2)^{1/3} \right) \cdot \left\{ 0.0899 (\text{sec/ft}^2)^{1/3} \right\}^2 \cdot (20 \text{ ft}) \right] \\ u_{2\%_{(s_b=20)}} &= 22.0 \text{ ft/sec} \end{aligned}$$

And, the corresponding flow depth is given by Equation 14 as

$$h_{2\%_{(s_b=20)}} = \left[ \frac{h_{B2\%} \cdot u_{B2\%}}{u_{2\%_{(s_b=20)}}} \right] = \left[ \frac{0.57 \text{ ft} \cdot 18.1 \text{ ft/sec}}{22.0 \text{ ft/sec}} \right] = 0.47 \text{ ft}$$

Finally, the terminal velocity reached if the slope is long enough is estimated using Equation 13, i.e.,

$$u_{2\%_{(s_b=\infty)}} = \frac{K_2}{K_3} = \frac{2.17 (\text{ft/sec}^2)^{1/3}}{0.0899 (\text{sec/ft}^2)^{1/3}} = 24.1 \text{ ft/sec}$$

(Note in the above calculations the importance of including dimensional units to make sure the units balance in the final answer.)

The flow depths estimated at the protected-side levee crest edge and at a location 20 ft down the protected-side slope bracket the 0.5-ft depth assumed when the friction factor was selected from Table 2. If the depths are substantially different from the depth assumed for the friction factor, it would be prudent to repeat the computation using a revised friction factor.

Table 3 contains the results for the above calculations along with similar results for average wave overtopping rates of  $q_w = 0.2$  and  $0.5 \text{ ft}^3/\text{sec}$  per foot of levee. The table also gives estimates for calculations using the value of friction factor derived assuming  $n = 0.035$ . Doubling the average wave overtopping rate from  $0.1 \text{ ft}^3/\text{sec}$  per foot to  $0.2 \text{ ft}^3/\text{sec}$  per foot increased the terminal velocity on the protected-side slope by only 13 percent. However, the increased friction factor reduced the terminal velocity on the protected-side slope by about one third. Thus, the value of the friction factor is very influential in this estimation procedure.

<b>Table 3. Additional parameter estimates for Example Problem 1.</b>								
$q_w$ ( $\text{ft}^3/\text{s}$ )	$R_c$ (ft)	$h_{A2\%}$ (ft)	$u_{A2\%}$ (ft/sec)	$h_{B2\%}$ (ft)	$u_{B2\%}$ (ft/sec)	$h_{2\%(sb=20)}$ (ft)	$u_{2\%(sb=20)}$ (ft/sec)	$u_{2\%(sb=\infty)}$ (ft/sec)
<b><math>f_F = 0.015</math></b>								
0.10	11.1	1.21	20.6	0.57	18.1	0.47	22.0	24.1
0.20	9.7	1.54	23.2	0.73	21.0	0.62	24.6	27.4
0.50	7.7	1.97	26.3	0.93	24.3	0.82	27.5	31.3
<b><math>f_F = 0.045</math></b>								
0.10	11.1	1.21	20.6	0.57	13.9	0.52	15.2	15.3
0.20	9.7	1.54	23.2	0.73	17.0	0.70	17.6	17.8
0.50	7.7	1.97	26.3	0.93	20.6	0.93	20.6	20.6

Figure 3 plots the velocity of the 2-percent runup wave leading edge as it progresses down the protected-side slope. Curves are shown for the three values of average wave overtopping. The horizontal lines are the estimated terminal velocities (last column in Table 3) associated with each overtopping rate. All curves are for the wave and levee parameters given for the example problem. Depending on the levee cross section, the flow down the protected-side slope may not reach terminal velocity before the slope transitions into a flatter berm.

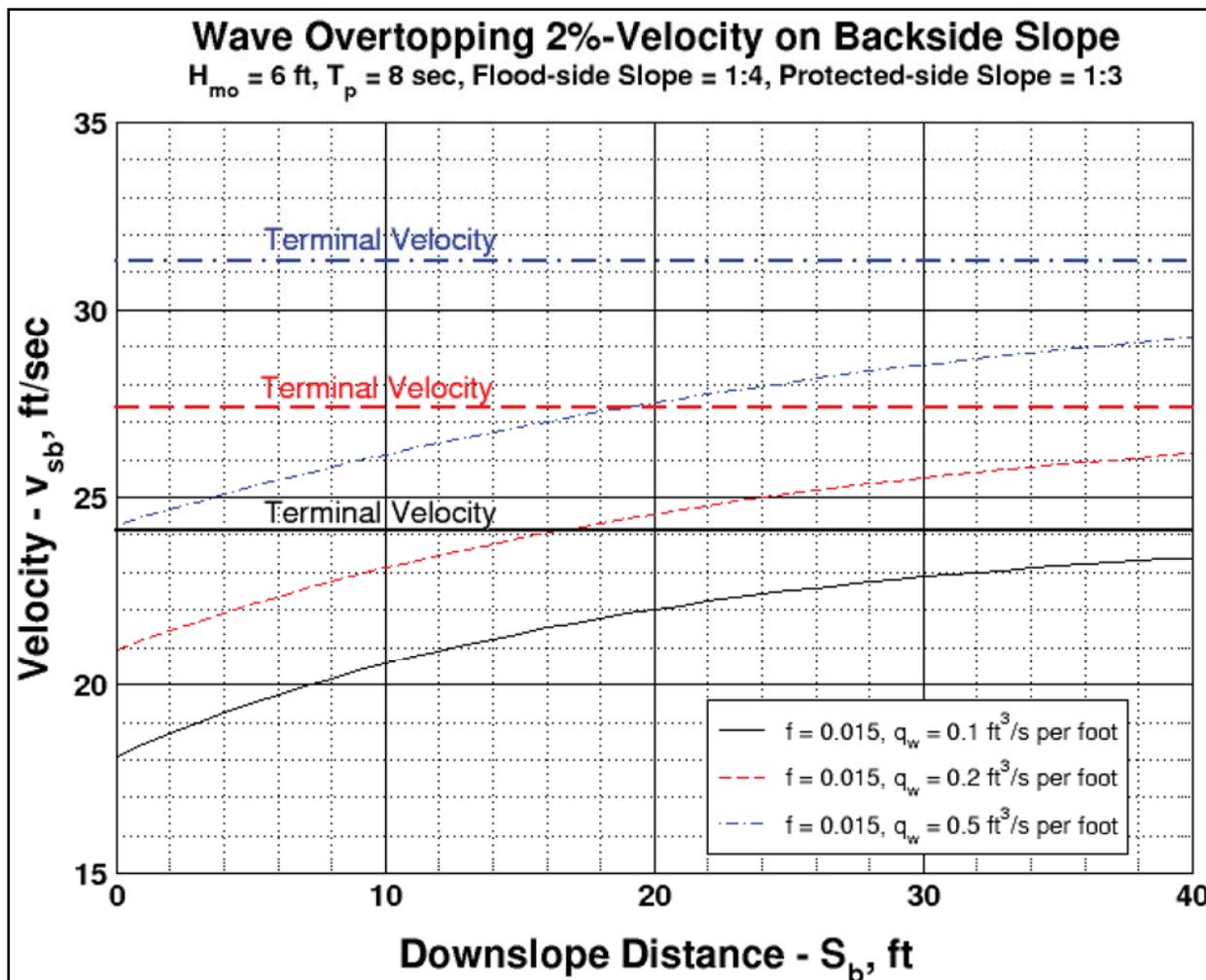


Figure 3. Wave overtopping flow velocity on protected-side slope.

**REMARKS:** The equations in this Technical Note primarily are used to solve for the velocity and flow depth peaks exceeded by only 2 percent of the incident waves. The levee surface is subjected to the peak velocities only momentarily, with lower velocities for the rest of the wave passage. Thus, duration of maximum flow is fleeting, and erosion might be relatively minor for each wave unless the erosion velocity threshold is quite a bit lower than the peak velocity. The relationship between average wave overtopping rates and corresponding soil erosion rates has not yet been established. Finally, it is apparent that a better understanding is needed for specifying an appropriate value for the friction factor for various slope surfaces. The methodology is sensitive to the Fanning friction factor; therefore, until better guidance becomes available, it is advisable to use lower estimates of  $f_F$  in the calculations.

## Example 2: Irregular Wave 50% Overtopping Flow Parameters

**Find:** The overtopping flow parameters exceeded by 50 percent of the waves associated with an average wave overtopping discharge rate of  $q_w = 0.1 \text{ ft}^3/\text{sec}$  per foot of levee. The levee is covered with good-quality grass on the crest and protected-side slope, and the wave parameters are the same as Example 1, i.e.,

**Given:**

$H_{m0} = 6 \text{ ft}$	=	Zeroth-moment significant wave height [= $H_s$ ]
$T_p = 8 \text{ sec}$	=	Wave period associated with the spectral peak
$\tan \theta = 1/4$	=	Flood-side levee slope
$\tan \alpha = 1/3$	=	Protected-side levee slope
$B = 10 \text{ ft}$	=	Levee crest width
$g = 32.2 \text{ ft/sec}^2$	=	Gravitational acceleration
$f_F = 0.015$	=	Fanning friction factor (from Table 2 for $n = 0.02$ and $h = 0.5 \text{ ft}$ )

**Calculate Wave Runup and Levee Freeboard.** Runup and levee freeboard will be the same as calculated in Example 1, i.e.,  $R_{u2\%} = 16.6 \text{ ft}$  and  $R_c = 11.1 \text{ ft}$ .

**Parameters at the Flood-Side Levee Crest Edge.** Substituting the recommended values for mean flow parameters of  $C_{Ah50\%} = 0.17$  and  $C_{Au50\%} = 0.94$  into Equations 1 and 2, respectively, rearranging, and noting that  $H_s$  cancels out in both equations gives equations for the flow thickness and flow velocity magnitudes exceeded by 50 percent of the incident waves. Using the above values for  $R_{u2\%}$  and  $R_c$  yields

$$h_{A50\%} = 0.17(R_{u2\%} - R_c) = 0.17(16.6 \text{ ft} - 11.1 \text{ ft}) = 0.94 \text{ ft}$$

and

$$u_{A50\%} = \sqrt{g} (0.94) \sqrt{(R_{u2\%} - R_c)} = \sqrt{(32.2 \text{ ft/sec}^2)} (0.94) \sqrt{(16.6 \text{ ft} - 11.1 \text{ ft})} = 12.5 \text{ ft/sec}$$

**Parameters at the Protected-Side Levee Crest Edge.** The flow depth and velocity at the protected-side crest edge exceeded by 50 percent of the incident waves are found using Equations 3 and 4. Substituting the recommended value of  $C_3 = 0.75$  into Equation 3, recognizing that the distance  $x_c = B$  at the protected-side crest edge, and using the flow parameters already estimated at the flood-side edge yields the estimates

$$h_{B50\%} = h_{A50\%} \exp\left(-0.75 \frac{B}{B}\right) = (0.94 \text{ ft}) \exp(-0.75) = 0.44 \text{ ft}$$

and

$$u_{B50\%} = u_{A50\%} \exp\left(-\frac{B f_F}{2 h_{B50\%}}\right) = (12.5 \text{ ft/sec}) \exp\left(-\frac{(10 \text{ ft})(0.015)}{2(0.44 \text{ ft})}\right) = 10.5 \text{ ft/sec}$$

**Parameters on the Protected-Side Levee Slope:** The flow velocity and thickness exceeded by 50 percent of the incident waves varies with distance down the protected-side slope until terminal velocity is reached (the supercritical flow condition represented by the Chezy and Manning equations). First, determine the values of  $K_2$ ,  $K_3$ , and  $K_4$  from Equations 10, 11, and 12, respectively.

$$K_2 = (g \sin \alpha)^{1/3} = \left\{ (32.2 \text{ ft/sec}^2) \sin \left[ \tan^{-1}(1/3) \right] \right\}^{1/3} = 2.17 (\text{ft/sec}^2)^{1/3}$$

$$K_3 = \left[ \frac{f_F}{2} \frac{1}{(h_{B50\%} \cdot u_{B50\%})} \right]^{1/3} = \left[ \frac{0.015}{2} \frac{1}{(0.44 \text{ ft}) \cdot (10.5 \text{ ft/sec})} \right]^{1/3} = 0.1175 (\text{sec/ft}^2)^{1/3}$$

$$K_4 = u_{B50\%} - \frac{K_2}{K_3} = (10.5 \text{ ft/sec}) - \frac{2.17 (\text{ft/sec}^2)^{1/3}}{0.1175 (\text{sec/ft}^2)^{1/3}} = -7.968 \text{ ft/sec}$$

Now, the flow velocity exceeded by 50 percent of the waves can be found at any location down the slope using Equation 9. For example, at 20 ft from the protected-side edge of the crest (i.e.,  $s_b = 20 \text{ ft}$ ), the slope-parallel velocity is estimated to be

$$\begin{aligned} u_{50\%(s_b=20)} &= \frac{K_2}{K_3} + K_4 \exp(-3 K_2 \cdot K_3^2 \cdot s_b) \\ &= \frac{2.17 (\text{ft/sec}^2)^{1/3}}{0.1175 (\text{sec/ft}^2)^{1/3}} \\ &\quad + (-7.968 \text{ ft/sec}) \exp \left[ -3 \left( 2.17 (\text{ft/sec}^2)^{1/3} \right) \cdot \left\{ 0.1175 (\text{sec/ft}^2)^{1/3} \right\}^2 \cdot (20 \text{ ft}) \right] \\ u_{50\%(s_b=20)} &= 17.1 \text{ ft/sec} \end{aligned}$$

And, the corresponding flow depth is given by Equation 14 as

$$h_{50\%} = \left[ \frac{h_{B50\%} \cdot u_{B50\%}}{u_{50\%}(s_b=20)} \right] = \left[ \frac{0.44 \text{ ft} \cdot 10.5 \text{ ft/sec}}{17.1 \text{ ft/sec}} \right] = 0.27 \text{ ft}$$

Finally, the terminal velocity reached if the slope is long enough is estimated using Equation 13, i.e.,

$$u_{50\%} = \frac{K_2}{K_3} = \frac{2.17 (\text{ft/sec}^2)^{1/3}}{0.1175 (\text{sec/ft}^2)^{1/3}} = 18.5 \text{ ft/sec}$$

**SUMMARY:** This CHETN has summarized European research and technical publications describing a method for estimating wave overtopping flow velocities and flow thicknesses on an earthen levee. The empirical equations are based on small- and large-scale laboratory experiments, and application is limited to irregular wave overtopping of trapezoidal levee cross sections when the still water level elevation (i.e., surge elevation) is lower than the elevation of the levee crest. The incident wave parameters are used to estimate the elevation of the 2-percent runup (hypothetical runup elevation if the flood-side levee slope continued indefinitely). Once the 2-percent runup value is known, flow velocity and flow thickness can be estimated over the levee crest and down the protected-side slope as a function of levee geometry (slopes and crest width) and an appropriate friction factor. Resulting maximum velocities and flow thicknesses are those values that will be exceeded by only 2 percent of the waves. Different coefficients are used to estimate the flow depth and velocity exceeded by 50 percent of the waves. Two example problems illustrate application. These estimates are intended for use in preliminary design.

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An electronic copy of this CHETN is available from <http://chl.erdc.usace.army.mil/chetn>.

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