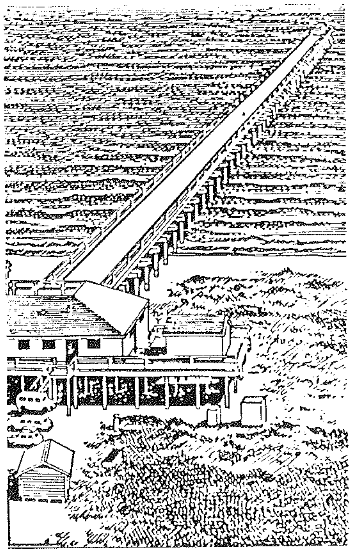
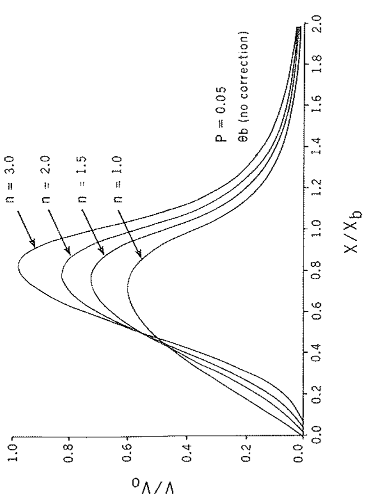
US Army Corps of Engineers





MISCELLANEOUS PAPER CERC-88-3

AN ANALYTICAL MODEL OF WAVE-INDUCED LONGSHORE CURRENT BASED ON POWER LAW WAVE HEIGHT DECAY

by

Jane McKee Smith, Nicholas C. Kraus

Coastal Engineering Research Center

DEPARTMENT OF THE ARMY
Waterways Experiment Station, Corps of Engineers
PO Box 631, Vicksburg, Mississippi 39180-0631

January 1988
Final Report

Approved For Public Release; Distribution Unlimited

Prepared for DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Miscellaneous Paper CERC-88-3	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION USAEWES, Coastal Engineering Research Center 6b. OFFICE SYMBOL (If applicable) CEWES-CR-O	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) PO Box 631 Vicksburg, MS 39180-0631	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION US Army Corps of Engineers 8b. OFFICE SYMBOL (If applicable) DAEN CWH-D	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) Washington, DC 20314-1000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO. 31592

11. TITLE (Include Security Classification)
An Analytical Model of Wave-Induced Longshore Current Based on Power Law Wave Height Decay

12. PERSONAL AUTHOR(S)
Smith, Jane M., and Kraus, Nicholas C.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM Jul 85 TO Oct 86	January 1988	134

16. SUPPLEMENTARY NOTATION
Available from National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22161.

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	Longshore Current	Current Distribution
			Wave Breaking	Power Law
			Wave Decay	Surf Zone

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

An analytical model of the wave-induced longshore current is derived. A unique feature of the model is an empirical power law expression, developed in this study, employed to describe the broken wave height in the surf zone. The model also includes the effect of wave setup, finite incident wave angle, and lateral mixing. The power law expression represents the surf zone wave height decay significantly better than the linear decay profile assumed in previous longshore current models. The exponent in the power law expression is determined to be a function of the beach slope and the wave height to water depth ratio at wave breaking. The longshore current model is derived by balancing the longshore stresses: local wave stress, lateral mixing stress, and bottom friction stress. The solution for the longshore current distribution is an infinite power series which is truncated to second order. The model gives the longshore current as a function of distance offshore, breaking wave conditions (height, depth, and angle), beach slope,

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code) / 22c. OFFICE SYMBOL

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. ABSTRACT (Continued).

friction coefficient, and a parameter, P , that expresses the relative importance of lateral mixing and bottom friction. Estimates of the longshore current distribution using this model agree with previous results in the appropriate limits, but the estimated current varies significantly under certain breaking wave conditions and beach slopes. Wave setup is also derived using the power law expression of the broken wave height. For the limited amount of setup data examined, the setup based on the power law wave height decay represents the data better than the calculated setup based on linear wave height decay.

PREFACE

The investigation described in this report was authorized as a part of the Civil Works Research and Development Program by the Office, Chief of Engineers (OCE), US Army Corps of Engineers. Work was performed under Wave Estimation for Design Work Unit 31592, Coastal Flooding Program, at the Coastal Engineering Research Center (CERC) of the US Army Engineer Waterways Experiment Station (CEWES). Messrs. John H. Lockhart, Jr., and John G. Housley were OCE Technical Monitors. Dr. C. Linwood Vincent is CERC Program Manager.

The study was conducted from July 1985 through October 1986 by Ms. Jane McKee Smith, Hydraulic Engineer, and Dr. Nicholas C. Kraus, Research Physical Scientist, CERC. This report is substantially the same as the thesis submitted to Mississippi State University by Ms. Smith in partial fulfillment of the requirements for an M.S. degree in civil engineering. Dr. Kraus was the thesis advisor.

This study was done under general supervision of Dr. James R. Houston and Mr. Charles C. Calhoun, Jr., Chief and Assistant Chief, CERC, respectively; and under direct supervision of Mr. H. Lee Butler, Chief, Research Division; Dr. Edward F. Thompson, Chief, Coastal Oceanography Branch (CR-O); and Dr. Robert E. Jensen, Principal Investigator, Wave Estimation for Design Work Unit, CR-O, CERC.

The author gratefully acknowledges Dr. Kiyoshi Horikawa, Department of Civil Engineering, University of Tokyo, who provided his extensive data on breaking waves, and Dr. Tsuguo Sunamura, Institute of Geosciences, University of Tsukuba, who assisted in obtaining other wave data reported in the Japanese literature.

COL Dwayne G. Lee, CE, was Commander and Director of CEWES during report publication. Dr. Robert W. Whalin was Technical Director.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I: INTRODUCTION

Wind generated or short period waves continually arrive at the coast. The approaching waves become unstable at a certain depth and the tops of their crests spill down or plunge over their forward faces. The wave height decreases as the wave energy is converted into turbulent eddies in the surf zone. If waves break at an angle to the shore, they induce a longshore current in the surf zone. The current acts somewhat analogous to a river, transporting sediment mobilized by the breaking waves. Coastal engineers have long worked to correlate sediment movement and current velocities to predict sediment transport, shoreline evolution, and pollutant transport. This requires accurate estimation procedures, or models, of the longshore current. This report presents an analytical longshore current model for engineering use. The model employs an expression developed in this report to describe the nonlinearity of the wave height decay, and it also includes the effect of wave setup, finite incident wave angles, and lateral mixing. The advantages of an analytical model over a numerical model are the ease of discerning the functional dependencies of the physical parameters and the ease of applying the model.

Waves transfer momentum from offshore to the nearshore. In the nearshore, the waves break when they reach a depth comparable to their height, and the wave energy is dissipated in the surf zone. Waves breaking at an angle to the shoreline induce a current parallel to the shoreline due to changes in the longshore component of momentum. The balance of momentum is conserved in the surf zone by the external forces of bottom and surface shear stresses. The change in the onshore component of momentum also causes a change in the mean water level in the surf zone, known as wave setup. Momentum is also diffused or transported by turbulent eddies.

Water motion in the surf zone is extremely complex. The flow is unsteady and three-dimensional, with dynamic upper and lower boundaries. No adequate theoretical description of water motion in the surf zone presently exists. Therefore, to predict longshore currents it is necessary to simplify the problem by considering an idealized environment and to include a certain amount of empiricism. Applying various degrees of simplification, many investigators have calculated longshore currents analytically and numerically using empirical correlations, continuity of water mass, energy flux, and momentum flux.

In 1967 Galvin reviewed the state of the art of longshore current prediction. He concluded that the best approach at the time was the prediction of longshore current velocity through empirical correlation of data, but he cautioned that the available data were not reliable. Much progress has been made in the prediction of longshore currents since the review by Galvin. The progress mainly is due to the introduction of the concept of radiation stress by Longuet-Higgins and Stewart (1962, 1963, 1964). Radiation stress is used to calculate the flux of momentum parallel to the shoreline due to incident waves. Bowen (1969), Longuet-Higgins (1970a, 1970b), and Thornton (1971) were the first to apply radiation stress concepts in the equations of motion to predict longshore currents.

In the radiation stress approach it is necessary to specify the wave height through the surf zone a priori, but the mechanisms that determine the wave height in the surf zone (wave breaking, wave deformation, and energy dissipation) are not well understood. No quantitative, first-principle theoretical model of wave height decay exists; therefore an empirical approach is taken in longshore current modeling. The standard assumption is made that, in the surf zone (after initial wave breaking), the wave height, H, is described as a linear function of water depth, h, in the form

$$H = \gamma h \tag{1-1}$$

where γ is a constant of proportionality. This is known as the spilling breaker assumption because it holds fairly well for waves classified as spilling breakers. However, several investigators show that this is not valid in general (Horikawa and Kuo 1967; Nakamura, Shirashi, and Sasaki 1967; Street and Camfield 1967; Divoky, Le Mehaute, and Lin 1970; Dally, Dean, and Dalrymple, 1985a, 1985b), and it is especially inappropriate for mild bottom slopes, on which waves tend to break by plunging.

Investigators following Bowen, Longuet-Higgins, and Thornton built on the radiation stress approach by eliminating some of the simplifying assumptions and making more general models. But, all have retained the spilling breaker assumption despite its proven invalidity.

This investigation examines the effects of using nonlinear wave height decay, namely a power law decay, on the prediction of the longshore currents distribution. The power law wave height decay is of the form

$$H = H_b \, (h/h_b)^n \tag{1-2}$$

where the subscript b indicates breaking conditions, and the exponent n, to be determined empirically, is assumed to be dependent on the beach slope and the breaking wave conditions. It will be shown in this report that a closed-form solution for the longshore current distribution can still be derived if Equation 1-2 is employed instead of Equation 1-1.

The main body of this report begins with a review of previous longshore current models. Special attention is paid to the Longuet-Higgins model because it has served as the basis for most models that followed. Next, the wave height decay portion of this study is presented. Seven independent data sets are empirically fit to the wave height decay power law, and the exponent of the power law is parameterized. Then, an analytical longshore current model is derived from the equations of motion based on the radiation stress approach. The effects of large angles of wave incidence and of lateral mixing are included in the model. The current model gives the longshore current as a function of distance offshore, incident wave conditions, beach slope, friction coefficient, and a parameter, P, expressing the relative importance of lateral mixing and bottom friction as introduced by Longuet-Higgins (1970b).

Review of Previous Models

The radiation stress approach to modeling longshore currents was developed independently by Bowen (1969), Longuet-Higgins (1970a, 1970b), and Thornton (1971). Although the three models are similar, there are differences in the assumptions made in the bottom shear stress and lateral mixing terms.

The former two authors developed analytical solutions for a plane beach; the latter developed a numerical solution for arbitrary profiles of straight, parallel contours, using a more realistic bottom friction stress. The Longuet-Higgins model is the easiest and most straightforward to use (the solution of Bowen is in terms of Bessel functions and the model of Thornton requires a numerical solution), and appears to give very acceptable results for a plane beach. The Longuet-Higgins model, therefore, has been used as the basis for more recent longshore current models. A review of the Longuet-Higgins model is given, followed by overviews of other momentum-based models. Basco (1982) presents an thorough review of surf zone current literature with an annotated bibliography (Basco and Coleman 1982). Table 1-1 gives an intercomparison of selected models of the longshore current distribution across the surf zone.

Longuet-Higgins. Longuet-Higgins (1970a, 1970b) derives an analytical model for the steady longshore current from the governing equations of water motion. He makes the assumptions given in Table 1-2; in addition he assumes linear wave height decay given by Equation 1-1. The equation of motion for the longshore direction for this idealized case reduces to a balance between the local wave stress, the stress due to horizontal turbulent eddies, and the time-averaged bottom friction stress. The local wave stress is the driving force of the currents, and it is the net stress in the longshore direction exerted by the waves on the water in the surf zone. This stress is calculated from the radiation stress. The bottom shear stress is linearized by assuming the incident wave angle is small and the steady current is weak compared with the wave orbital velocities. These assumptions reduce the bottom shear stress to the product of the orbital velocity and the longshore current speed. The lateral mixing stress is a function of the horizontal eddy coefficient. Longuet-Higgins assumes the horizontal eddy coefficient is proportional to the offshore distance multiplied by a typical velocity, the shallow-water wave celerity. The distances (measured from the mean shoreline) are nondimensionalized by the distance from the mean shoreline to the breaker line. The longshore current velocity is nondimensionalized by the velocity

TABLE 1-1

Comparison of Longshore Current Models

	Plane Beach	Parallel Contours	Numerical	Analytical	Linear Theory	Nonlinear Theory	Lateral Mixing	2-D	Non-lin Bottom Stress	Finite Angle	Strong Current	H = h	$H = h_b(h/h_b)^n$
Bowen (1969)	X			X	X		X					X	
Longuet-Higgins (1970a)	X			X	X							X	
Longuet-Higgins (1970b)	X			X	X		X					X	
Thornton (1971)		X	X			X*	X					X	
James (1974)	X		X			Xo	X		X		X	X	
Jonsson et al. (1975)		X	X		X							X	
Keely and Bowen (1977)	X		X		X			X				X	
Liu and Dalrymple (1978)		X		X	X					X		X	
		X	X		X				X	X	X	X	
Kraus and Sasaki (1979a, 1979b)	X			X	X		X			X		X	
present study (1986)	X			X	X		X			X			X

* solitary wave theory

o hyperbolic wave theory

Table 1-2

Longshore Current Model Assumptions

WAVE FIELD

- Monochromatic waves
- Linear, shallow-water wave theory
- Steady state wave field

BEACH

- Plane, sloping beach
- Impermeable beach
- Hydrostatic pressure distribution

FLUID

- Incompressible, homogeneous fluid

CURRENT

- Current constant through depth and time
- Current homogeneous in the longshore direction
- Current weak relative to the wave orbital velocity

NEGLECTED STRESSES

- No wind stress
- No atmospheric pressure gradient
- No wave-current interaction
- No Coriolis force
- No tide

at the breaker line when the effect of lateral mixing is omitted. The stress balance is described by a second-order differential equation with a closed-form solution. The solution is a function of the relative effects of lateral mixing and bottom friction. Longuet-Higgins does not include wave setup explicitly, but he suggests modifying the beach slope to include the change in water depth due to wave setup. He also does not include refraction because the angle of wave incidence is assumed small.

The strong points of the Longuet-Higgins model are: (a) the model solution is simple and easy to apply and, (b) the model results compare well to available data. The weak points of the model are: (a) the numerous simplifying assumptions, and (b) the spilling breaker assumption in the lateral mixing and bottom stress terms was applied seaward of the breaker line where it is no longer valid.

Bowen. The Bowen (1969) model differs from the Longuet-Higgins model in several ways. Bowen assumes the bottom shear stress is proportional to the longshore current speed, neglecting the contribution of the wave orbital velocity. He also does not account for the effect of variation in depth in the lateral mixing stress. Although Bowen simplifies the stress terms considerably more than Longuet-Higgins, his solution is more complicated. The solution is in terms of Bessel functions and is, therefore, more difficult to use. On the positive side, Bowen explicitly includes wave setup in the surf zone, and he neglects it outside the surf zone where it is negligible compared to the depth.

Thornton. Thornton (1971) uses solitary wave theory in the surf zone to specify wave celerity and linear wave theory outside the surf zone. Thornton relaxes the plane beach assumption, but still assumes a beach of straight and parallel contours. He also includes setup and refraction inside and outside the surf zone. Thornton uses Prandtl's mixing length hypothesis to calculate the horizontal eddy coefficient in the lateral mixing stress. He assumes the horizontal eddy coefficient is equal to the amplitude of wave particle motion multiplied by water particle velocity fluctuations due to waves in the shore normal direction. The Jonsson (1967) friction factor for turbulent flow was used in the bottom stress term. Thornton also does not account for the variation in depth in the lateral mixing stress. Thornton's model requires a numerical solution.

James. James (1974) uses hyperbolic wave theory in the surf zone and linear wave theory far outside the surf zone with a transition region in between to calculate the wave stress. Hyperbolic wave theory is an approximation of cnoidal wave theory which is believed to describe the wave form in the surf zone better than linear theory. James includes refraction, setup, and return flows (to insure the mean shoreward mass flux is zero). He also eliminates the weak current assumption. Outside the surf zone, he uses experimental results to define the eddy coefficient to be proportional to the inverse of the depth. James relaxes the plane beach assumption, but requires the beach slope to be mild. The mild slope assumption may invalidate the linear wave height decay assumption (as stated earlier). Also, the model is formulated as a set of differential equations that must be solved numerically. This model is much too complicated for practical engineering use.

Jonsson, Skovgaard, and Jacobsen. Jonsson, Skovgaard, and Jacobsen (1975) return to using linear wave theory throughout the nearshore region. They use a nonlinear bottom shear stress and introduce a friction factor that is an interpolation between the friction factor for waves only and the friction factor for currents only. Jonsson, Skovgaard, and Jacobsen adopt Thornton's (1971) formulation for the lateral mixing stress, but they do account for the variation in depth. The model is a differential equation which is solved numerically.

Keeley and Bowen. Keeley and Bowen (1977) take into account longshore variations in longshore currents, removing the assumption of the current being homogeneous in the longshore direction. Spatial variations in the longshore current, typical in the field, are caused by irregular bathymetry and spatial variations in the wave field. Keeley and Bowen follow the Longuet-Higgins derivation of the longshore current due to obliquely incident waves, but omit the lateral mixing stress. They linearly add the currents due to obliquely incident waves, variations in the wave height in the longshore direction, variations in the wave angle in the longshore direction, and nonlinear effects (due to the advection term in the longshore momentum balance). They also include wave setup. The Keeley and Bowen model must be driven by a refraction model which provides the variation of wave heights and angles in the longshore direction. The contributions of the longshore variation in wave height and the nonlinear effects to the longshore current are small. The model requires a numerical solution.

Liu and Dalrymple. Liu and Dalrymple (1978) present a weak current model and a strong current model. Both models include the effects of large incident wave angle and wave setup, but exclude the lateral mixing stress. In the weak current model, the longshore current velocity is assumed small compared to the wave orbital velocity. In the strong current model, the longshore current is assumed to be of the same order of magnitude or larger than the wave orbital velocity. The absolute value of the total velocity (longshore current plus wave orbital motion) is approximated with a truncated binomial series. For the weak current model, the bottom stress term is simplified to a linear function of the current velocity using the weak current assumption. The solution of the weak current model is in closed form. The strong current model results in a nonlinear ordinary differential equation solved numerically. The solution of the strong current model is found iteratively because the setup is not known a priori. The neglect of lateral mixing limits the use of this model.

Kraus and Sasaki. Kraus and Sasaki (1979a, 1979b) add still another improvement to the lineage of momentum-based longshore current models. Their model includes the effects of large incident wave angles and the lateral mixing stress (omitted by Liu and Dalrymple). They assume that the magnitude of the longshore current is small compared to the wave orbital velocity. Setup is approximated by modifying the beach slope as suggested by Longuet-Higgins. Similar to the Liu and Dalrymple strong current model, Kraus and Sasaki approximate the absolute value of the total velocity (wave orbital plus longshore current) with a truncated binomial expansion. Inside the surf zone, they also apply the approximation

$$\cos\theta = (1 - h/h_b \sin^2\theta_b)^{1/2}$$

derived from a trigonometric identity, shallow-water approximations for the wave celerity, and Snell's law, where θ is the angle of wave incidence. The model has an analytic solution in the form of an infinite series of successively smaller terms. Kraus and Sasaki verified the model with laboratory data (Mizuguchi et al. 1978) and their own field data.

CHAPTER II: WAVE HEIGHT DECAY

The derivation of the wave-induced longshore current requires knowledge of the wave height and the gradient of the wave height in the surf zone. Historically, the wave height in the surf zone has been estimated as a linear function of the water depth,

$$H = \gamma h \tag{2-1}$$

Longuet-Higgins and Stewart (1964) and Bowen et al. (1968) suggest the similarity between the decrease in wave height and the decrease in water depth shoreward of breaking as the motivation for Equation 2-1. Bowen et al. support the assumption with laboratory data on a slope of 1/12. The γ-values ranged from 0.9 to 1.3. This empirical expression is attractive because of its simplicity, but the surf zone wave height decay is not linear in general as has been noted, for example, by Horikawa and Kuo (1967), Street and Camfield (1967), and Van Dorn (1977) on the basis of their carefully performed laboratory experiments. Figure 2-1 shows idealized curves fit to laboratory wave height decay data. The curves are increasingly concave upward with decreasing beach slope. The purpose of this chapter is to develop an empirical power law decay model to describe the wave height decay more accurately than the linear model, but still retain the useful simple form of the linear model. The simple form will allow the longshore current model to be solved analytically.

The dissipation of wave energy in the surf zone is due primarily to turbulence (Horikawa and Kuo 1967; Sawaragi and Iwata 1975; Mizuguchi 1981; Dally, Dean, and Dalrymple 1985a, 1985b; and others). The power law decay model is entirely empirical. The model is not meant to replace more sophisticated models based on the physics of the turbulent energy dissipation. These more sophisticated models solve the energy flux equation in the surf zone,

$$\partial(ECg)/\partial x = \varepsilon \tag{2-2}$$

where EC_g is the energy flux and ε is the energy dissipation rate.

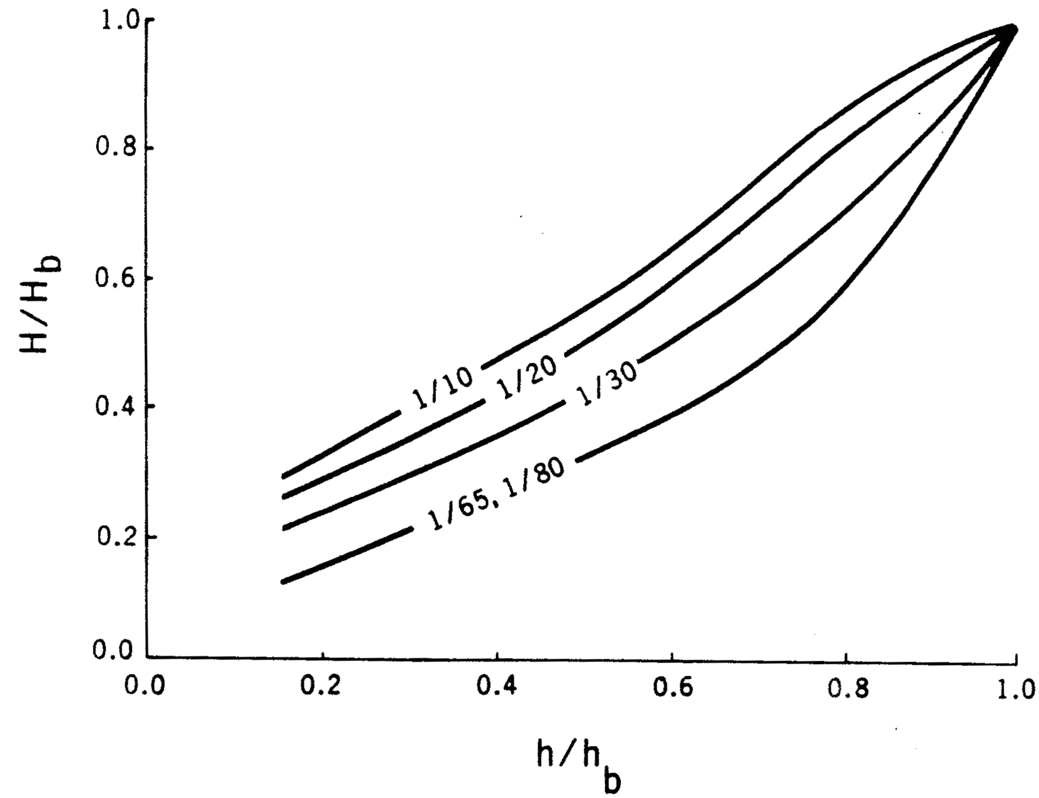


Figure 2-1. Idealized profiles of experimentally determined wave height decay after initial breaking on plane beaches of different slope

For completeness, some of these models are briefly described. Le Mehaute (1963) approximates a breaking wave as a hydraulic jump, substituting the energy dissipation of a hydraulic jump for ε in Equation 2-2. The same approach with some variations is applied to periodic laboratory waves by Divoky et al. (1970), Hwang and Divoky (1971), and Svendsen (1984, 1985). Battjes and Janssen (1979) also use the hydraulic jump model, but apply it to random laboratory waves. Thornton and Guza (1983) refine the approach of Battjes and Janssen and apply it to both laboratory and field data.

Although the hydraulic jump model appears to give the best explanation of the physics of wave breaking, three other approaches are mentioned because of their uniqueness and insight. Horikawa and Kuo (1967) model surf zone energy dissipation due to bottom friction and turbulence using solitary wave theory. The turbulence is assumed to decay exponentially with distance from the break point. The results are good for a horizontal bed, but poor for a plane sloping bed. Mizuguchi (1981) models the surf zone energy dissipation by replacing the molecular viscosity with the turbulent eddy viscosity in the solution for internal energy dissipation due to viscosity. Mizuguchi's model allows more complex beach profiles (step-type beaches) and reformation and second breaking of waves. The model gives good results when tested with laboratory data for wave breaking on a horizontal beach, a 1/10 slope plane beach, and a step-type beach. But, Mizuguchi admits that the eddy viscosity assumption is "obscure." The model requires a numerical solution. Dally et al. (1985a, 1985b) propose what they call an intuitive approach. The dissipation, ε , in Equation 2-2 is assumed to be proportional to the difference between the local energy flux, EC_g, and the "stable" energy flux, EC_{gs}, or

$$\varepsilon = -(k/h)\,(EC_g - EC_{gs}) \tag{2-3}$$

where k is a dimensionless decay coefficient and h is the local still-water depth. The stable energy flux is found to be associated with a wave height equal to approximately 0.35 to 0.40 times the local depth. This approach allows a breaking wave to stabilize or reform and break again. The formulation also allows for an arbitrary beach profile and the inclusion of wave setup, but this requires a numerical solution. Analytical solutions are derived for simple profiles (horizontal bottom, sloping bottom, and Dean's (1977) equilibrium profile). Results are good in comparison to laboratory data. Since this approach is so successful, the power law decay model will be compared to it.

Power Law Model of Wave Height Decay

In this study, the wave height decay is expressed as the power law

$$H = \gamma h_b \, (h/h_b)^n \tag{2-4}$$

This form was chosen because it is similar to the linear wave height decay model, and it reduces to the linear decay model (Equation 2-1) for an exponent, n, equal to 1.0. Equation 2-4 is applicable from the breaker line to the mean shoreline. Two constants, γ and n, must be specified in Equation 2-4. It is noted that the formulation of Dally et al. (1985a, 1985b) also requires specification of two parameters through empirical considerations. The importance of beach slope in the decay profile is clearly shown in Figure 2-1. Horikawa and Kuo also suggest the importance of the wave steepness, H_o/L_o, where H_o is the deepwater wave height and L_o is the deepwater wavelength, and the breaking wave conditions (H_b/h_b) on the decay profile. Following a description of the wave height decay data, the procedures used to analyze the data and quantify γ and n are explained.

Seven independent sets of laboratory and prototype scale data comprising 135 experimental runs on slopes of 1/90 to 1/10 are used to quantify the wave height decay. These data sets were obtained through a comprehensive search of the literature in English and Japanese. Table 2-1 summarizes the data. The breaking wave heights (of monochromatic waves) range from 4.67 cm to 1.37 m, and the wave periods range from 1.2 s to 9.0 s. The wave steepnesses (H_o/L_o) are between 0.0031 and 0.091. The data are listed in Appendix A.

Table 2-1

Data Summary for Wave Height Decay

Source	Slope	Number of Runs
Horikawa and Kuo (1967) and Kuo (1965)	1/80	57
	1/65	16
	1/30	19
	1/20	21
Maruyama et al. (1983)	1/62.5	1
	1/45.5	1
	1/29.4	1
	1/22.2	1
Mizuguchi (1981)	1/10	1
Saeki and Sasaki (1973)	1/50	2
Sasaki and Saeki (1974)	1/90	1
Stive (1985)	1/40	2
Van Dorn (1977)	1/12	4
	1/25	4
	1/45	4

Horikawa and Kuo performed their experiment in two parts. The 1/20 and 1/30-slope data were collected in a flume 17 m long, 0.7 m wide, and 0.6 m deep. The slope was covered with a smooth rubber mat. The 1/65 and 1/80-slope data were collected in a flume 75 m long, 1.0 m wide, and 1.2 m deep. The slope was concrete. The Maruyama et al. data were collected in a prototype-scale flume 250 m long, 3.4 m wide, and 1.2 m deep. The initial slopes (1/22.2, 1/29.4, 1/45.5, and 1/62.5) were formed of sand and were, therefore, not constant throughout each run. The Mizuguchi data were collected in a wave basin 15 m long and 15 m wide, but the width was truncated to 9 m. The 1/50-slope Saeki and Sasaki data were collected in a flume 24 m long, 0.8 m wide, and 0.8 m deep. The 1/90-slope Sasaki and Saeki data were collected in a flume 24 m long, 0.6 m wide, and 1 m deep. The slope in both cases was formed of smooth plastic. The Stive data were collected at two scales to compare scale effects. The large flume was 233 m long, 5 m wide, and 7 m deep. The slope was sand with an initial slope of 1/40. The small flume was 55 m long, 1 m wide, and 1 m deep, with a concrete slope of 1/40. The Van Dorn data were collected in a flume 24 m long, 0.5 m wide, and at a still-water depth of 36 cm. The slopes were formed of plate glass.

The parameter γ is defined as the ratio of the wave height to the local water depth at breaking,

$$\gamma = H_b/h_b \qquad (2\text{-}5)$$

by solving Equation 2-4 for γ with $H=H_b$ and $h=h_b$. This ratio is very significant because it specifies where a wave will break. This is important in the design of coastal structures, so the specification of γ has stimulated much interest.

McCowan (1891) calculates the critical H/h ratio for wave breaking from solitary wave theory. His value,

$$H_b/h_b = 0.78$$

gives a reasonable average of the measured γ-values from the data summarized