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DEVELOPMENT OF PROBABILISTIC RIGID PAVEMENT DESIGN METHODOLOGIES FOR MILITARY AIRFIELDS

by

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December 1983
Final Report

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The current Corps of Engineers design procedure for rigid airfield pavements is based on the Westergaard free edge stress slab theory, and a proposed procedure is based on the multilayer elastic theory. These two design procedures have been expanded to airfield pavement designs expressed in probabilistic and reliability terms. Further developments were required in these procedures to make the analysis more practicable. Two major investigations were conducted:

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20. ABSTRACT (Continued).

a. Evaluation and use of the composite modulus of elasticity for layers beneath the rigid pavement.

b. Evaluation of the maximum tensile stress at the bottom of the slab for different aircraft types.

Derivations obtained from the investigation of the composite modulus and maximum tensile stress are reported and are included in computer programs for probabilistic/reliability analysis of rigid pavements. The approximate closed form (Taylor series expansion) is utilized. Example runs of the computer program are presented.

PREFACE

The work reported herein was performed for the U. S. Army Engineer Waterways Experiment Station (WES) under Purchase Order DACA39-82-M-0074 with the University of Maryland during the period May 1982 to June 1983. The work was funded by the Office, Chief of Engineers, U. S. Army, under the FY 82 RDTE Program, Project: 4A161102AT22, Task AO, Work Unit 009, "Methodology for Considering Material Variability in Pavement Design." OCE Project Monitor was Mr. S. S. Gillespie. The work was conducted and report prepared by Drs. M. W. Witczak and J. Uzan and Mr. M. Johnson.

Dr. W. R. Barker, Pavement Systems Division (PSD), Geotechnical Laboratory (GL), WES, was the Principal Investigator. The study was conducted under the general supervision of Dr. T. D. White, Chief, PSD, and Dr. W. F. Marcuson III, Chief, GL.

Commander and Director of WES during the period of this study was COL Tilford C. Creel, CE. Technical Director was Mr. F. R. Brown.

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VOLUME I

STATE OF THE ART VARIABILITY OF AIRFIELD PAVEMENT MATERIALS

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Chapter 1

INTRODUCTION

At present, pavement design methodologies are deterministic, that is, a unique pavement system is designed for the set of input variables having unique values. The variations of these design parameters have been taken into account either by setting limiting values on the parameters or by using a factor of safety. Such an approach, however, leaves the engineer without any true quantitative estimate of overall design reliability. A probabilistic design method may therefore be developed and used in order to take material variabilities into account within a reliability framework. The use of a probabilistic methodology necessitates the determination of not only the mean (average) value of each parameter; but a measure of the variability of the specific parameter in question (i.e., standard deviation, variance, coefficient of variation).

The purpose of this report is to give a general summary of values or ranges of values, found from a literature search, that describe material variabilities. This report summarizes a state-of-the-art review in variabilities of material properties which are used in pavement design. This study specifically concentrated on normal design input variables commonly used in rigid pavement airfield studies. As such, the wealth of variability information concerning such non-design parameters (i.e., soil classification, gradation, component variability analyses, etc.) was not included.

It should be noted that, in some instances, the standard deviation is the given measure of variability. Since the coefficient of variation may not always be constant, the standard deviation may give a better

indication of the variability of the material. If a standard deviation is given and the coefficient of variation is required, it may be obtained by dividing the standard deviation by the mean ($CV = \frac{\sigma}{\overline{X}} \times 100\%$).

Chapter 2 contains brief discussions of material properties and their variation while Chapter 3 contains the summary and recommendations of values to be used within a probabilistic design method for rigid airfield pavements.

Chapter 2

MATERIAL VARIABILITY

Well defined variability information is more prevalent from highway pavements than for airfield pavements. However, there is little reason to believe that there are major differences in the variability of the two types of pavements. Variabilities, therefore, have been collected for both highways and airfields where there is insufficient airfield information.

PORTLAND CEMENT CONCRETE MATERIALS

Thickness

There is very little variation in the thickness of Portland Cement concrete pavements. Table 1 shows information taken from several highway and airfield pavements. The coefficients of variation are generally below 3% but go as high as 8%. Table 2 shows information from the Louisiana Department of Highways. Here, the coefficients of variation are all less than 5%. Information could only be found for two airfield pavement sections (Table 1). All of the information gathered was for pavement thicknesses of less than 15 inches. An insignificant amount of data was found for pavements of larger thicknesses and therefore no conclusions may be drawn.

Modulus of Elasticity

There was a sufficient amount of information available on the modulus of elasticity for airfields. A summary of the mean values and the

Table 1. Thickness of Portland Cement Concrete Pavements (Kennedy, Hudson and McCullough

	Project Identification	Sample Plan**	Number of Cores	Pavement Thickness* Mean (inches)	CV (%)	Design (inches)
Highways	2-A	ATP	38	8.3	2.6	8.0
	2-E	ATP	50	8.3	2.5	8.0
	17-B	ATP	50	8.2	2.6	8.0
		Cluster	10	7.8	1.2	8.0
	17-M	ATP	47	8.2	2.6	8.0
		Cluster 1	7	7.7	1.0	8.0
		Cluster 2	8	7.6	1.1	8.0
	18-N	ATP	9	8.8	3.7	8.0
	18-0	ATP	24	9.5	4.7	9.0
	19-A	ATP	34	8.2	2.9	8.0
	19-B	ATP	31	8.2	3.4	8.0
		Cluster 1	10	7.6	0.6	8.0
		Cluster 2	9	7.6	1.4	8.0
		Weighted Average excluding clusters		-	2.9	-
		Variation	Limits	-	2.5-4.7	-
			Range	-	2.2	-
Airports	O'Hare Rwy 9R-27L	ATP	12	10.9	8.3	10.0
	O'Hare Rwy 4R-22L	ATP	10	14.8	3.3	14.0
		Weighted Average		-	6.0	-
		Variation	Limits	-	3.3-8.3	-
			Range	-	5.0	-

* Thickness determined by measuring height of core in laboratory.

** Along-the-pavement or cluster samples from thin section where thickness is less than design value.

TABLE 2. Summary of Statistical Results on Thickness of Concrete Pavement (Kennedy, Hudson and McCullough)

Project Number	Number of Samples	Mean (inches)	Variance (inches)	Standard Deviation (inches)	CV (%)
8-In. Uniform Thickness					
1	34	8.66	0.192	0.435	5.0
2	39	8.42	0.171	0.415	4.9
3	48	8.35	0.040	0.200	2.4
4	58	8.36	0.077	0.276	3.3
5	61	8.05	0.035	0.185	2.3
6	66	8.11	0.089	0.300	3.7
7	73	8.06	0.112	0.333	4.1
Pooled Values	--	8.29	0.088	0.300	3.6
9-In. Uniform Thickness					
1	35	9.25	0.0466	0.210	2.3
2	51	9.19	0.121	0.350	3.8
3	58	9.28	0.048	0.220	2.4
4	65	9.18	0.060	0.240	2.6
5	74	9.20	0.185	0.430	4.7
6	88	9.11	0.029	0.170	1.9
Pooled Values	--	9.20	0.083	0.290	3.1
10-In. Uniform Thickness					
1	64	10.38	0.061	0.240	2.3
2	124	10.34	0.079	0.280	2.7
3	132	10.35	0.079	0.230	2.2
4	141	10.28	0.083	0.270	2.8
Pooled Values	---	10.34	0.069	0.270	2.6

coefficients of variation may be found in Table 3. Mean values ranged from 2.90 x 10^6 psi to 3.89 x 10^6 psi with an average of 3.45 x 10^6 psi. Coefficients of variation ranged from 21 to 49 with an average of 34.4. Tables 4a, 4b, and 4c are the results of a series of laboratory tests. These results show that the modulus value reaches a slight peak at an age of 60 days and then decreases slightly with age. The mean modulus value was 4.54 x 10^6 psi with a range of 4.05 x 10^6 psi to 5.16 x 10^6 psi. The standard deviation increased with time until the 60 day mark was reached and then it decreased slightly. The standard deviation ranged from 0.06 x 10^6 psi to 0.22 x 10^6 psi. The coefficient of variation ranged from 1.2% to 4.8%.

Poisson's Ratio

Tables 4, 5, and 6 are a summary of data from Gibeaut (4), who investigated the Poisson's Ratio of PCC. Values tended to decrease with time whereas the standard deviation tended to increase. There was no true trend for the coefficient of variation. The mean value of Poisson's ratio ranged from 0.162 to 0.212 with an average of 0.188. The standard deviation ranged from 0.036 to 0.019 and the coefficient of variation ranged from 9.3% to 20.2%.

TABLE 3. Summary of Test Results for Cored Specimens from PCC Airfield Pavements (Kennedy)

Airport	Project Identification	Number of Tests	Indirect Modulus of Elasticity Mean (10^6 psi)	CV (%)
O'Hare	Runway 9R-27L	48 47	3.39*	34
		15	3.33*	28
	Runway 4R-22L	20	2.90**	26
		39	3.89*	49
Palmdale	Runway 7-25 (Existing Pavement)	11	-	-
	Runway 7-25 (Overlay)	38 39	3.86*	32
	Taxiway A Overlay	7 8	3.39*	40
	Taxiway B Overlay	20	3.09*	21
Midway	Runway 4R-22L 13R-31L	11	3.21**	43
Richmond	Taxiway 5-4 Runway 2	16	3.02**	31
		Weighted Average	3.45	34.4
	Variation	Limits	2.90 - 3.89	21 - 49
		Range	0.99	28

* Assumed Poisson's ratio = 0.20

** Assumed Poisson's ratio = 0.15

Table 4

Individual Sonic E and μ Values

(Expanded from Gibeaut)

	50 Series						
Age Days	$E \times 10^6$ psi	$\overline{E} \times 10^6$ psi	σ_E $\times 10^6$ psi	μ	$\overline{\mu}$	σ_μ	CV %
14	4.71 4.88 4.86 5.07	4.88	0.15	0.176 0.201 0.217 0.215	0.202	0.019	9.41
28	4.72 4.83 4.95 5.10	4.90	0.16	0.147 0.175 0.201 0.211	0.184	0.029	15.76
60	4.74 4.84 5.00 5.16	4.94	0.18	0.153 0.173 0.218 0.229	0.193	0.036	18.65
90	4.68 4.83 4.89 5.14	4.89	0.19	0.140 0.162 0.186 0.224	0.178	0.036	20.22
159	4.71 4.82 4.76 5.08	4.84	0.16	0.143 0.172 0.164 0.208	0.172	0.027	15.75

Table 5

Individual Sonic E and μ Values
(Expanded from Gibeaut)

	60 Series						
Age Days	$E \times 10^6$ psi	$\bar{E} \times 10^6$ psi	$\sigma_c \times 10^6$ psi	μ	$\bar{\mu}$	σ_μ	CV %
14	4.64 4.40 4.69 4.29	4.51	0.91	0.191 0.186 0.223 0.170	0.193	0.022	11.40
28	4.72 4.40 4.81 4.40	4.58	0.21	0.181 0.139 0.204 0.170	0.174	0.027	15.52
60	4.72 4.46 4.80 4.41	4.60	0.19	0.168 0.133 0.190 0.169	0.165	0.024	14.55
90	4.70 4.40 4.77 4.36	4.56	0.21	0.170 0.124 0.193 0.162	0.162	0.029	17.90
152	4.76 4.45 4.76 4.34	4.58	0.22	0.187 0.135 0.184 0.174	0.170	0.024	14.12

Table 6

Individual Sonic E and μ Values
(Expanded from Bibeaut)

	70 Series						
Age Days	$E \times 10^6$ psi	$\overline{E} \times 10^6$ psi	$\sigma_c \times 10^6$ psi	μ	$\overline{\mu}$	σ_μ	CV %
14	4.07 4.14 4.18 4.17	4.14	0.05	0.263 0.203 0.187 0.193	0.212	0.035	16.51
28	4.06 4.18 4.20 4.19	4.16	0.07	0.235 0.190 0.187 0.212	0.206	0.022	10.68
60	4.05 4.24 4.19 4.20	4.17	0.08	0.240 0.193 0.187 0.226	0.212	0.026	12.26
90	4.05 4.24 4.15 4.17	4.15	0.08	0.215 0.191 0.172 0.212	0.198	0.020	10.10
130	4.05 4.20 4.10 4.15	4.13	0.06	0.218 0.191 0.161 0.211	0.195	0.025	12.82

Modulus of Rupture

Current rigid pavement design methods utilize the coefficient of variation in determining the design modulus of rupture. Values of C.V. used are:

< 10% for excellent construction control

10-15% for good control

15-20% for fair control

> 20% for poor control

Tables 7 and 8 show that in most cases excellent construction control may be maintained.

ASPHALT CONCRETE MATERIALS

Thickness

As for the thickness of PCC pavements, there is little information on the wide range of pavement thicknesses. Yoder and Witczak suggest a typical standard deviation range for new highway construction 0.3 to 0.8 inches for asphalt concrete pavements.

Dynamic Modulus

Table 9 is a summary of laboratory tests which were conducted to determine the dynamic modulus of asphalt concrete. It may be observed that the coefficient of variation within a project is a function of temperature and that there is no specific trend of frequency. It should also be noted that the in-situ (field) variability should be greater than the laboratory variability. In general, ranges of the CV values as a function of temperature are: (9%-16%) (11%-19%) and (20%-23%) for temperatures of 40°F, 70°F and 100°F, respectively.

Table 7

Average 7-Day Flexural Strength and Standard Deviations,

Portland Cement Concrete Pavement (Brown)

No. of Samples, n		Strength, psi	Standard Deviation $\hat{\sigma}$, psi	Coefficient of Variation (%)
414		688	73	10.6
74		552	50	9.1
324		713	77	10.8
76		642	75	11.7
8		679	30	4.4
42		645	18	2.8
44		705	64	9.1
170		736	55	7.5
38		680	120	17.6
16		591	52	8.8
41		676	92	13.6
8		640	55	8.6
1255	Avg.	662	63	9.5

Table 8

Average 28-Day Flexural Strength and Standard Deviations, Portland Cement Concrete Pavement (Brown)

No. of Samples, n		Avg Flexural Strength, psi	Standard Deviation $\hat{\sigma}$, psi	of Variation(%)
582		781	61	7.8
146		719	57	5.1
312		862	83	9.6
101		753	70	9.3
82		774	34	4.4
26		734	26	3.5
735		739	66	8.9
67		828	122	4.7
16		688	40	5.8
82		840	68	8.1
8		717	60	8.4
2157	Avg	766	61	8.0

TABLE 9 Lab Variability - Asphalt Concrete

A.C. Dynamic Modulus Tests (E°)(from The Asphalt Institute)

Test Property	Test Conditions		Test Variability		Within Project		Remarks
	Temp.(°F)	Freq.(Hz)	s(X 10^3 psi)	C.V.	S(X 10^3 psi)	C.V.	
A.C. Dynamic Modulus E° (psi)	40	1	53.8	4.1	162.0	12.5	4 Field projects
		4	122.0	7.7	145.5	9.2	3 Cores per project
		16	64.0	3.5	294.0	16.0	2 Replicates per
	70	1	26.4	6.2	71.5	16.7	core
		4	17.8	2.9	117.5	19.0	
		16	28.3	3.6	88.8	11.3	
	100	1	5.8	4.7	27.4	22.3	
		4	4.0	2.2	39.4	22.1	
		16	8.6	3.4	51.4	20.5	
		Average	4.3		16.6		

A.C. Flexural Stiffness Tests (E,) (after Monismith)

Test Property	Test Temp. (°F)	Number of Mixes	S(X 10^3 psi)		s(X 10^3 psi)		C.V.	
			Average	Range	Average	Range	Average	Range
Flexural Stiffness E.(psi) (Lab Prepared Specimens)	68	14	326.0	190.0-656.0	49.0	4.0-92.0	13.1	2.2-23.8

Flexural Stiffness

Table 9 gives a summary of laboratory variability of the flexural stiffness of asphalt concrete. For a test temperature of 68°F, the mean stiffness value was 326.0 x 10^3 psi with a range of 190.0 x 10^3 psi to 656.0 x 10^3 psi. The standard deviation ranged from 4.0 x 10^3 psi to 92.0 x 10^3 psi with an average of 49 x 10^3 psi while the coefficient of variation ranged from 2.2% to 23.8% with an average of 13.1%.

Modulus values of asphalt concrete used in base materials seem to have more variability than the modulus values of asphalt concrete used in surface courses. Test results from blackbase highway projects (Table 10) show that the mean modulus value ranged from 35 x 10^3 psi to 91.5 x 10^3 psi with a weighted average of 58.8 x 10^3 psi. The coefficients of variation ranged from 25% to 52% with a weighted average of 40%. There is no reference made to the temperature used for these tests. Therefore, since the modulus of elasticity is a function of temperature, further research should be conducted before using the values obtained from this report.

Table 11 is a summary of flexural stiffness measurements on field samples. These values were found by using the pulse load method. For a test temperature of 68°F, mean values of flexural stiffness ranged from 1.34 x 10^5 psi to 1.79 x 10^5 psi with an average of 1.58 x 10^5 psi. The coefficient of variation ranged from 23.5% to 27.6% with an average of 25.0%. For a test temperature of 40°F, the flexural stiffness ranged from 5.90 x 10^5 psi to 7.12 x 10^5 psi with an average of 6.71 x 10^5 psi. The coefficient of variation ranged from 18.8% to 27.2% with an average of 22.4%.

Poisson's Ratio

Kennedy, Hudson, and McCullough (5) investigated the variability of the Poisson's ratio of asphalt concrete. Using the results from 15 specimens, they found that the mean value of was 0.40 and that its coefficient of variation was 27%. In Table 10, the mean value of Poisson's ratio varied from 0.16 to 0.34 with an average of 0.25. The coefficient of variation ranged from 38% to 75% with a weighted average of 52%. There is no reference made as to what temperature that was measured at. Therefore, since is a function of temperature, further research should be conducted before using these values.

CEMENT TREATED MATERIALS

Thickness

For cement-treated bases with a thickness of 4 to 8 inches, the standard deviation ranged from 0.60 to 0.72 inches. The coefficient of variation ranged from 7.5% to 18% (9).

Modulus of Elasticity

There are vary large variations in the modulus of elasticity for cement-treated bases. Table 12 shows that mean values of the modulus ranged from 0.6×10^6 psi to 1.90×10^6 psi. The standard deviation ranged from 0.36 to 10^6 psi to 1.19×10^6 psi. The coefficient of variation ranged from 53% to 83%.

SUBGRADE

Modulus of Subgrade Reaction

Treybig, Hudson, and McCullough (8) analyzed data from the AASHO Road Test. This subgrade material had an average modulus of subgrade reaction of 100 pci and a coefficient of variation of 16%. This

TABLE 10. Summary of Test Results for Cored Specimens from Blackbase Highway Projects in Texas (Kennedy)

Project Identification	Number of Specimens	Distance Covered (miles)	Indirect Modulus of Elasticity		Indirect Poisson's Ratio	
			Mean (10^3 psi)	CV (%)	Mean	CV (%)
2-A	76	15.0	38.6	32	0.34	39
8-A	16	3.3	91.5	29	0.28	40
13-A	14	8.0	44.9	46	0.16	58
13-B	28	4.3	87.3	62	0.16	73
13-C	16	3.0	35.0	40	0.26	57
15-A	49	10.9	86.1	59	0.23	47
17-B	100	19.1	55.2	44	0.24	41
18-B	12	0.9	42.2	24	0.20	64
19-A	54	19.3	55.2	33	0.32	38
19-B	36	15.2	64.7	34	0.16	67
Weighted Average			58.8	40	0.25	52
CV of means (%)			36	--	.28	--
Variation	Limits		35-91.5	24-62	.16-.34	38-73
	Range		56.5	38	0.18	35

TABLE 11. Flexural Stiffness Measurements on Field Samples of Asphaltic Concrete Using Pulse Loading Method (Kennedy)

Sample Group	No. of Specimens	Measured Stiffness (psi x 10^5) 68°F Mean	68°F Std. Dev.	68°F CV (%)	40°F Mean	40°F Std. Dev.	40°F CV (%)
Specimens from Surface Course							
1	19	1.79	0.42	23.5	6.80	1.53	22.5
2	20	1.65	0.39	23.6	7.03	1.91	27.2
3	20	1.52	0.41	26.9	7.12	1.41	19.8
4	19	1.34	0.37	27.6	5.90	1.11	18.8
				25.0			22.4
Lab Compacted	26	1.29	0.22	17.0	5.76	0.74	12.8
Specimens from Base Course							
1	12	1.57	0.42	26.6	5.95	1.54	25.9
2	12	1.39	0.26	18.7	5.66	3.14	55.5
3	8	1.47	0.41	27.9	4.40	0.90	20.4
4	10	1.42	0.42	29.6	4.96	1.22	24.6
				25.3			33.0
Lab Compacted	29	1.19	0.19	16.0	5.31	1.50	28.2

TABLE 12. Summary of Properties for Cement-Treated Bases for Airfield and Highway Pavements (Kennedy)

	Identification		Type of Material	Number of Specimens	Distance Covered (miles)	Indirect Modulus of Elasticity*	
						Mean (10^6 psi)	CV (%)
AIRFIELDS	O'Hare Runway 4R-22L	Top	-	15		1.9	53
		Bottom		15		1.8	66
		Combined		30		1.8	59
	DFW		Kingburg Gravel	4		-	-
				Weighted Average			
HIGHWAYS	12-A		Sand Shell	32	-	1.76	72
	19-A		Soil Cement	20	1.4	1.05	83
	19-B		Soil Cement	19	1.2	0.73	57
	20-A		Burned Clay	29	1.5	0.60	60
				Weighted Average		0.09	68
				CV of Means (%)		50	-
				Variation	Limits	0.60-1.76	57-83
					Range	1.16	26

*Assumed Poisson's Ratio-0.22

coefficient of variation should only be applicable to a k of 100 pci. Further information should be gathered before drawing any conclusions about the coefficients of variation for other values of k.

Resilient Modulus

The resilient modulus tended to increase with a decrease in deviator stress. Table 13 shows mean modulus values of 12 x 10^3 psi, 16 x 10^3 psi, and 19 x 10^3 psi for deviator stresses of 8 psi, 5 psi, and 2 psi, respectively. The coefficient of variation ranged from 5.3% to 52%. The average standard deviation was 3.75 x 10^3 psi.

California Bearing Ratio (CBR)

Table 14 shows variability data for the subgrade CBR. Mean CBR values ranged from 4.2 to 26.3. The standard deviation ranged from 0.9 to 8.4 while the coefficient of variation ranged from 17.9% to 36.9%.

TABLE 13. Summary of Resilient Moduli for Subgrade Soils for Airports (Kennedy)

Airport	Project Identification	Number of Tests	Deviator Stress (psi) 8 Mean (10^3 psi)	8 CV (%)	5 Mean (10^3 psi)	5 CV (%)	2 Mean (10^3 psi)	2 CV (%)
Palmdale	Runway 7-25	4	10	52	10	47	8.4	26
		10	24	26	30	29	38	34
O'Hare	Runway 9R-27L	8	7.9	48	-	-	-	-
		3	6.3	21	8.7	12	10	13
	Runway 4R-22L	4	5.7	17	7.0	5.3	8.4	16
Richmond	Taxiway S-4 Runway 2 Taxiway D	3	8.6	8.3	12	20	11	18
Midway	Runway 4R-22L 13R-31L	4	2.4	10	4.0	7.9	5.4	6.6
	Weighted Average		12	29	16	22	19	22
	Variation Limits		2.4-24	8.3-52	4-30	5.3-47	5.4-3.8	6.6-34
	Variation Range		21.6	43.7	26	41.7	32.6	27.4

Table 14

Subgrade CBR Variability
(reduced from Yoder and Witczak)

Mean CBR	S	CV(%)	Remarks
7.1	1.6	22.3	(In-situ; compacted subgrade)
4.2	0.9	21.4	(Estimated; after moisture equil)
26.3	8.4	31.9	(In-situ; compacted subgrade)
20.3	7.6	36.9	(Estimated; after moisture equil)
18.2	4.8	26.2	(In-situ; compacted subgrade)
7.8	1.4	17.9	(Undisturbed samples; soaked)

Chapter 3

SUMMARY AND RECOMMENDATIONS

The purpose of this report was to give a general summary of values, or a range of values, that describe material variabilities. It summarizes a state-of-the-art review in variabilities of material properties which are used in pavement design.

If a probabilistic design methodology has been developed, a measure of the variability of the specific parameter in question must be determined. This variability information, however, is not always readily available. This report, then, is to be used as a guide in determining the level of variability to be used for a design parameter. Table 15 is a table of variability recommendations which may be used for different levels of inherent variability. As variability data becomes available, it should be used to upgrade or to check Table 15. If variability information pertaining to a particular project is available, it should be used in lieu of the information from Table 15.

Table 15. Variability Recommendations

	Coefficient of Variation (%) Inherent Variability		
	Low	Average	High
PCC MATERIALS			
Thickness	1-3	4-6	7-9+
Modulus of Elasticity	20-30	30-40	40-50+
Poisson's Ratio	8-12	13-16	17-20+
Modulus of Rupture	10-13	14-17	18-20+
ASPHALT CONCRETE MATERIALS			
Thickness	1-5	5-10	10-15
Dynamic Modulus			
Temp: 40°F	8-10	11-13	14-16+
70°F	10-12	13-15	16-19+
100°F	18-20	21-22	23-24+
Base Modulus of Elasticity	25-35	35-45	45-55+
Flexural Stiffness			
Temp: 40°F	15-20	20-25	25-28+
68°F	20-23	24-26	27-30+
Poisson's Ratio*	35-48	49-62	63-75+
CEMENT-TREATED MATERIALS			
Thickness	6-10	11-15	16-19+
Modulus of Elasticity	53-63	63-73	73-83
SUBGRADE MATERIALS			
Modulus of Subgrade Reaction*	10-20	20-35	35-50+
Resilient Modulus	10-20	20-35	35-50+
CBR	15-22	23-31	32-40+

* See Text for additional information

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VOLUME II

MATHEMATICAL FORMULATION OF RELIABILITY MODELS UTILIZED IN RIGID PAVEMENT STUDIES

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Chapter 1

INTRODUCTION

For the past several years, probabilistic/reliability analyses have slowly been introduced in pavement design schemes to account for the known variation of the salient design parameters. In existing deterministic design methods, material variability effects on pavement performance may be only indirectly taken into account. Frequently, the design parameters (unique values) are determined from either laboratory or field results. If multiple values are determined, the engineer may select (from the distribution of values) a design percentile level (e.g. . . . 85th) for use in the deterministic solution. The confidence level adopted and reflected by the percentile value is currently based on the engineering judgment and experience of the designer. This approach, however, cannot be easily or directly used to quantitatively evaluate the effect of material variability in design analysis, cost studies or reliability solutions. The probabilistic approach which directly implements the variability distribution of all significant parameters provides the only true, accurate measure of the reliability of the entire pavement system.

The probabilistic analysis of pavement performance is based on concepts and formulations developed in the statistical field dealing with system reliability. This volume presents a summary of

(a) the definitions and mathematical formulation of the general problem applicable for any probability density distribution;

(b) the existing approaches for developing pavement design solutions

in probability and reliability terms. It includes: (i) the interference theory or stress-strength approach, (ii) the simulation technique and (iii) the approximate closed-form probabilistic formulation, based on the Taylor series expansion of the dependent variable.

The various theories are discussed from the viewpoint of their possible implementation in the rigid pavement design procedure of the USACE for airfield systems.

Chapter 2

DEFINITIONS AND MATHEMATICAL FORMULATION

General

This chapter presents a brief overview of basic definitions and theorems related to the probabilistic formulation of design variables applied to the pavement design solution. Further background, details proof of theorem, etc. . . . can be found in most standard textbooks on Probability and Reliability (see, for example, Hines and Montgomery (1))

Probability and Reliability Concepts

Reliability is defined as the probability of success of an event and/or a statement of the error or precision of an estimate. This event, which can take a finite or infinite number of values, is called a random variable. The distribution of these values is related to the probability distribution of the random variable by the concept of probability, i.e.

$$P(X = x_k) = f(x_k) \qquad k = 1, 2, \ldots \tag{2.1}$$

$$\text{with } \int_{-\infty}^{+\infty} f(x_k)\, dx_k = 1$$

where $P(X = x_k)$ represents the probability that the random variable takes the value x_k, and $f(x_k)$ is the density probability distribution of X at x_k. This can also be expressed through the cumulative distribution function defined as

$$P(X \leq x_k) = F(x_k) = \int_{-\infty}^{x_k} f(u)\, du \tag{2.2}$$

When success is defined by X taking all values greater than x_k, reliability takes the form of

$$R = P(X > x_k) = 1 - F(x_k) = 1 - \int_{-\infty}^{x_k} f(u)\, du \qquad (2.3)$$

It can be seen that reliability can be directly computed from the probability distribution of the random variable, directly from its cumulative probability distribution, or from integration of its density probability distribution.

Extension to Multi Variate Expressions

The case of one random variable is the simplest solution. However, in practice, two or more random variables are involved in the process. If one examines the case of two random variables, the joint distribution function of the random variables X and Y is defined as:

$$P(X = x_j,\ Y = y_k) = f(x_j,\ y_k) \qquad (2.4)$$

$$\text{with } \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x_j,\ y_k)\, dx_j\, dy_k = 1$$

The above probability expresses the occurence of two events which can be either dependent or independent. In the case of independent variables, the "product rule" applies, i.e.

$$P(X = x_j,\ Y = y_k) = [P(X = x_j)] \cdot [P(Y = y_k)] \qquad (2.5)$$

$$\text{or} \quad f(x,\ y) = [f_1(x)] \cdot [f_2(y)]$$

where $f_1(x)$ and $f_2(y)$ are called the marginal probabilities of X and Y. The extension of the one random variable to two or more random variables is easily done:

$$P(X \leq x_j,\ Y \leq y_k) = [P(X \leq x_j)] \cdot [P(Y \leq y_k)] \qquad (2.6)$$

$$= [F_1(x)] \cdot [F_2(y)]$$

However, when the random variables are dependent, the product rule does not hold and it is replaced by the so-called conditional probability which is expressed by the following:

$$P(Y = y_k \ / \ X = x_j) = \frac{P(Y = y_k,\ X = x_j)}{P(X = x_j)} = \frac{f(x_j\ ,\ y_k)}{f_1(x_j)} \qquad (2.7)$$

where

$P(Y = y_k / X = x_j)$ is the probability of $Y = y_k$ given that

X is equal to x_j

The joint probability of two dependent random variables is therefore given by:

$$f(x,\ y) \ = \ [f(y/x)] \cdot [f_1(x)] \qquad (2.8)$$

The probability of Y being between the values c and d given that $x < X < x+dx$ is given by:

$$P(c < Y < d / x < X < x+dx) \ = \ \int_c^d f(y/x)\ dy \qquad (2.9)$$

This can be extended for the whole range of X values to bring the marginal distribution for Y as follows:

$$P(Y \leq d / -\infty < X < +\infty) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{d} f(y/x)dy \right] f_1(x)dx \qquad (2.10)$$

$$= \int_{x=-\infty}^{+\infty} \int_{y=-\infty}^{d} f(x,y)\ dx\ dy$$

Change of Variables

If X is a continuous random variable with a probability density function $f(x)$, $U = \phi(X)$ and $X = \Psi(U)$; then the probability density of U is given by $g(u)$ where

$$g(u) = f(x)\left|\frac{dx}{du}\right| = f(\Psi(u))\,|\,\Psi'(u)\,| \qquad (2.11)$$

If X and Y are continuous random variables having a joint density probability function $f(x,y)$, $U = \phi_1(X,Y)$, $V = \phi_2(X,Y)$, $X = \psi_1(U,V)$ and $Y = \psi_2(U,V)$; then the joint density function of U and V is given by $g(u,v)$ where

$$g(u,v) = f(x,y)\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = f[\psi_1(u,v),\ \psi_2(u,v)]\cdot\,|J|$$

where

J = the Jacobian of the transformation (2.12)

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

Probability Distributions of Functions of Random Variables

If X and Y are continuous random variables and $U = \phi_1(X,Y)$, then the density function of U is the marginal density obtained from the joint density of U and V (while V is chosen arbitrarily, V = X or V = Y).

If $f(x,y)$ is the joint density for X and Y, then the density function $g(u)$ of the random variable $U = \phi_1(X,Y)$ is found by differentiating with respect to u the distribution function given by

$$G(u) = P[\phi_1(x,y) \le u] = \int \int_R f(x,y)\,dx\,dy \qquad (2.13)$$

where R is the region for which $\phi_1(x,y) \le u$

The density function of the sum of two continuous random variables X and Y, i.e. of U = X+Y, having joint density function f(x,y) is given by:

$$g(u) = \int_{-\infty}^{+\infty} f(x,\ u-x)\ dx \qquad (2.14)$$

For the special case where X and Y are independent,

$$f(x,y) = [f_1(x)] \cdot [f_2(y)] \qquad (2.15)$$

$$\text{and} \quad g(u) = \int_{-\infty}^{+\infty} [f_1(x)] \cdot [f_2(u-x)\ dx] = f_1 * f_2$$

This is called the convolution of f_1 and f_2.

Expectation

For a continuous random variable X having a density function f(x), the expectation of X is defined as

$$E[X] = \int_{-\infty}^{+\infty} xf(x)\ dx \qquad (2.16)$$

The expection of X is very often called the mean of X (denoted by μ_x) and is a measure of central tendency.

If X is a random variable then Y = g(X), a function of X is also a random variable. The expectation of Y is

$$E[Y] = E[g(X)] = \int_{-\infty}^{+\infty} g(x) \cdot f(x)\ dx \qquad (2.17)$$

For a function of two random variables g(X,Y)

$$E[g(X,Y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y) \cdot f(x,y)\ dx\ dy \qquad (2.18)$$

The variance of a random variable is defined by:

$$VAR[X] = E[(x-\mu)^2] \qquad (2.19)$$

where $g(X) = (x-\mu)^2$ is a function of X. According to the above:

$$E[g(X)] = E[(X-\mu)^2] = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx \qquad (2.20)$$

The variance (or its square root, the standard deviation) is a measure of the dispersion or scatter of the values of the random variable.

The covariance of two random variables is defined similarly as:

$$\begin{aligned} Cov[X,Y] &= E[(X-\mu_x)(Y-\mu_y)] \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x-\mu_x)(y-\mu_y)\, f(x,y)\, dx\, dy \qquad (2.21) \end{aligned}$$

which is equal to zero when the random variables are independent. When X and Y are dependent, the following relation between the covariances, the variances and the correlation coefficient can be found by:

$$\begin{aligned} Cov[X,Y] &= \rho\sqrt{Var[X] \cdot Var[Y]} \\ \sigma_{xy} &= \rho\sigma_x\sigma_y \qquad (2.22) \end{aligned}$$

where ρ is the correlation coefficient.

Higher orders of expectation of the random variable are defined in the literature, to describe mathematically the probability density distributions of the random variable (skewness, kurtosis, etc.)

Chapter 3

PROBABILISTIC THEORIES FOR PAVEMENT DESIGN

General Background

The reliability of a system composed of several units depends upon the reliability of these units and upon the way they are interconnected (in series or in parallel as defined in the reliability theory). The reliability of each unit can be determined by experiment or assessed from the stress-strength method.

At present, the rigid pavement design framework is based on slab failure only, without any consideration of subgrade or joint failure. It is therefore a one unit system with a large number of variables (traffic, material characteristics and slab geometry). A basic approach to the problem would be that of the stress-strength analytical methodology. The second one would be based on simulation, to bypass the complexity of the basic approach, and the third one would be an approximate one based on normal distribution assumption and Taylor series expansion of the functions. These approaches are discussed in detail.

Stress-Strength Approach (Interference Theory)

The unit is assumed to fail where the "stress"(Y) reaches its ultimate value, its "strength" (X). The probability density distribution of both the stress and the strength must be known. If it is assumed that the stress and the strength are independent, the probability of failure is expressed as the probability of X being lower or equal to a given value of Y, i.e.,

$$P\ (X<y/\ Y=y) = \int_{o}^{y} f_1(x)\ dx \qquad (3.1)$$

Multiplying by the probability that Y is in the neighborhood of y, one obtains the joint probability function

$$P\ (X \leq y,\ y < Y \leq y + dy) \ = \ \int_{-\infty}^{y} f_1(x)\ f_2(y)\ dx \qquad (3.2)$$

and extending this to the whole range of Y,

$$P\ (X \leq Y) \ = \ \int_{-\infty}^{+\infty} \int_{-\infty}^{y} f_1(x) \cdot f_2\ (y)\ dx\ dy \qquad (3.3)$$

The reliability of the unit is given by

$$R = 1 - P(X \leq Y) = 1 - \int_{-\infty}^{+\infty} \int_{-\infty}^{y} f_1(x)\ f_2(y)\ dx\ dy$$

$$= 1 - \int_{-\infty}^{+\infty} F(y)\ f_2(y)\ dy \qquad (3.4)$$

where F(y) is the cumulative probability function of x evaluated at y.

It is seen that the reliability can be computed if the density probability functions of the stress and the strength are known. However, only for very simple cases can the above integral be evaluated in a closed form model. Yet the stress itself can be a function of random variables, and its density probability function can be expressed by the above theorem by differentiating

$$G(u) \ = \ \iint_R f(x,y)\ dx\ dy \qquad (3.5)$$

with respect to u.

The problem involving a number of variables is found to be very complex, first to define the probability density distribution of the "stress" and of the "strength", and second, to evaluate the joint probability (or the probability of failure). It should be noted that for the case of the normal density distribution of the stress and the

strength, the probability of failure P(X_Y) is also normally distributed (2). In passing, it should also be noted that log-normal distributions or other statistical distributions (e.g. Beta) can be utilized in the above noted procedure.

Simulation Technique

The technique of simulation has become widely used for estimating distribution parameters of the variable and for providing insight into the cause and effect relationship within a system. In simulation, the system is divided into elements whose behavior can be predicted, at least in terms of probability distributions. They are then combined in their natural order, allowing the computer to present the effect of their interaction on each other. After constructing the model, it is activated by generating appropriate input data to simulate the actual aggregate behavior of the system over time.

The technique of simulation has not been applied widely in estimating pavement performance. Few studies (e.g., Ullitz (3)have advanced reliability based pavement performance schemes. Simulation can be applied for the rigid pavement case to estimate the probability distribution of the overall behavior (performance) of the pavement, given the probability distributions of the individual variables and to estimate the effect of each variable on the overall (aggregate) behavior. Formulation of the model is quite simple and requires the following steps:

(1) Define the random variables and their probability distributions;

(2) Express the rules (equations) that link the different variables to the performance variable - the one whose average value and probability distribution is of interest.

Activation of the model is performed by generating input of the random variables from their probability distribution and deriving the

performance variable. Repeating the process a sufficient number of times gives the description of the probability distribution of the output variable. The appropriate flow chart for simulating rigid pavement performance is shown in Fig. 3.1.

For the case of several variables, and in order to estimate the cause and effect relationships, a factorial simulation experiment must be conducted. The factorial experiment must be designed in such a way to allow to extract the individual effects of the random variables.

It should be noted the number of solutions required (N- in Fig. 3.1) may be quite large in order to accurately reproduce probability distributions of the various variables. Because of this, the simulation technique should be restricted to situations that are too complex to be handled by closed form formulation and/or for cases where there is strong evidence that probability distribution of variables are far from the normal approximation.

Approximate Closed Form Probabilistic Formulation

According to the probability concepts briefly summarized above, the expected value (mean) of a function of variables is expressed as:

$$E\,[g(X_i)] = \int_{-\infty}^{+\infty} g(x_i) \cdot f(x_1,\ldots,x_\eta)\, dx_1 \ldots dx_\eta \qquad (3.6)$$

where $f(x_1, \ldots, x_\eta)$ is the joint probability distribution of all random variables X_i. This is rather difficult to evaluate. Instead of evaluating integrals, the function may be expressed in terms of the Taylor series expansion evaluated near the variable means and its expected value derived:

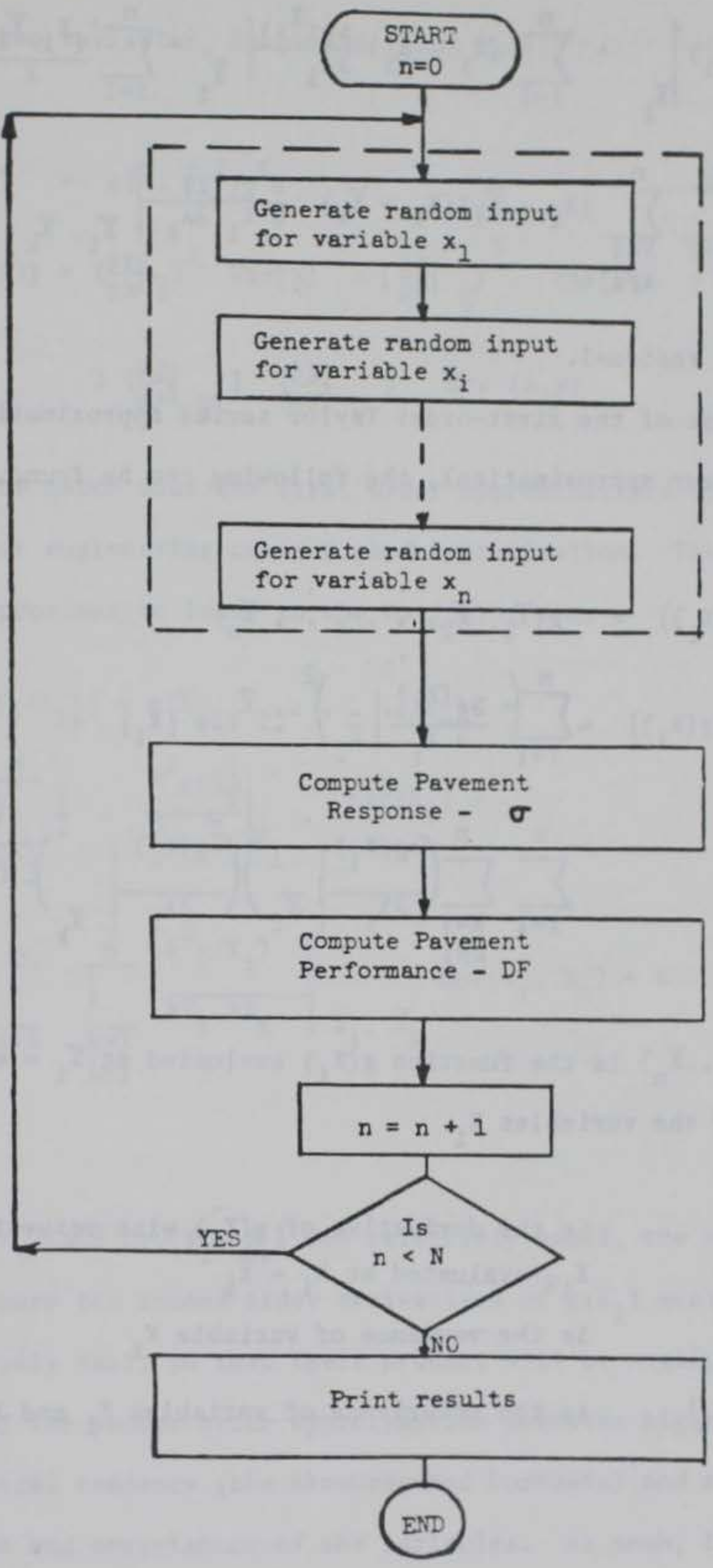

Fig. 3.1 SIMULATION SCHEME

$$g(X_i) = g(X_i)\Big|_{\overline{X}_i} + \sum_{i=1}^{n} (X_i - \overline{X}_i) \frac{\partial g(X_i)}{\partial x_i}\Big|_{\overline{X}_i} + \sum_{i=1}^{n} \frac{(X_i - \overline{X}_i)^2}{2} \frac{\partial^2 g(X_i)}{\partial X_i^2}\Big|_{\overline{X}_i}$$

$$+ \sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} (X_i - \overline{X}_i)(X_k - \overline{X}_k) \frac{\partial^2 g(X_i)}{\partial X_i \partial X_k}\Big|_{\overline{X}_i, \overline{X}_k} + R \qquad (3.7)$$

where R is the residual.

In the case of the first-order Taylor series approximation (corresponding to linear approximation), the following can be found.

$$E[g(X_i)] = g(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)$$

$$\text{Var}[g(X_i)] = \sum_{i=1}^{n} \left(\frac{\partial g(X_i)}{\partial X_i}\Big|_{\overline{X}_i} \right)^2 \cdot \text{Var}[X_i] + \qquad (3.8)$$

$$\sum_{i=1}^{n} \sum_{\substack{k=1 \\ k \neq i}}^{n} \left(\frac{\partial g(X_i)}{\partial X_i}\Big|_{\overline{X}_i} \right) \left(\frac{\partial g(X_k)}{\partial X_k}\Big|_{\overline{X}_k} \right) \cdot \text{Cov}[X_i, X_k]$$

where:

$g(\overline{X}_1, \overline{X}_2, \ldots \overline{X}_n)$ is the function $g(X_i)$ evaluated at $\overline{X}_i$ = mean values of the variables X_i

$\left.\frac{\partial g(X_i)}{\partial X_i}\right|_{\overline{X}_i}$ is the derivative of $g(X_i)$ with respect to X_i, evaluated at $X_i = \overline{X}_i$

$\text{Var}[X_i]$ is the variance of variable X_i

$\text{Cov}[X_i, X_k]$ is the covariance of variables X_i and X_k.

In the case of two variables, it can be shown that:

$$E[g(x,y)] = g(\bar{x}, \bar{y})$$

$$\text{Var}[g(x,y)] = \left(\frac{\partial g}{\partial x}\Big|_{\bar{x}}\right)^2 \text{Var}[x] + \left(\frac{\partial g}{\partial y}\Big|_{\bar{y}}\right)^2 \text{Var}[y] + \tag{3.9}$$

$$2\left(\frac{\partial g}{\partial x}\Big|_{\bar{x}}\right)\left(\frac{\partial g}{\partial y}\Big|_{\bar{y}}\right) \text{Cov}(x,y)$$

It should be noted that the first order approximations of the Taylor series is in most engineering cases a good approximation. Taking the second-order approximation leads to the following mean:

$$E[g(X_i)] = g(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n) +$$

$$\sum_{i=1}^{n} \frac{1}{2} \cdot \frac{\partial^2 g(X_i)}{\partial x_i^2}\Big|_{\bar{X}_i} \cdot \text{Var}[X_i] +$$

$$\sum_{i=1}^{n} \sum_{\substack{k=1 \\ k\neq i}}^{n} \frac{\partial^2 g(X_i)}{\partial X_i \, \partial X_k}\Big|_{\bar{X}_i, \bar{X}_k} \text{Cov}[X_i, X_k] + R \tag{3.10}$$

Since $\text{Var}[X_i]$ and $\text{Cov}[X_i, X_k]$ are relatively small, the approximation is a good one where the second order derivatives of $g(X_i)$ evaluated at $\bar{X}_i$ are also relatively small so that their product will be negligible. The variance term of the second-order approximation involves higher order of measures of central tendency (the skewness and kurtosis) and multiplication of the variances and covariances of the variables. As such, the precise

derivation becomes rather complex. The linear approximation is improved in symmetric probability distributions when the skewness coefficient is nil. Therefore, the approximate closed form formulation should be adopted only when the output variable has a symmetric density distribution. For the closed-form approximation to be appropriate, it should satisfy the following conditions: (a) the output variable is normally distributed and (b) the second order terms of the Taylor series expansion can be neglected, in comparison of the first order form.

Chapter 4

SUMMARY

Three basic approaches for expressing rigid pavement design in probabilistic and reliability terms were summarized. From the above, it is noted that

(a) in the stress-strength approach, definition and characterization of the stress component need to be clarified. In the current design method, initial failure is defined as the "point at which the crack, which originates at the bottom of the slab and migrates upward to the surface, commences to spall and ravel, which produces debris on the surface (4). The "stress" that would cause such a deterioration (cracking initiation and propagation, spalling and ravelling) cannot at this stage be given a mechanistic interpretation.

The stress-strength approach assumes that the variables are independent. Moreover, it is based on comparing the values of the random variables, X - the stress, and Y - the strength, which is equivalent to defining the new variable Z = X-Y. The approach currently followed in the rigid pavement design is quite different. The failure is defined through the Design Factor which is the ratio of the strength (the modulus of rupture) and the stress (further reduced by the load transfer coefficient). For this case, the probability distribution of the design factor remains unknown and quite difficult to determine.

While the discussion has denoted the random variables in authentic fasion, it should be understood that other distributions (i.e., log normal, Beta, Gamma) can be assumed to represent the PDF of each variable. Such assumptions would cause further mathematical problems in the "Strength-Stress" approach to the reliability solution.

(b) Simulation techniques are powerful for solving complex problems, which cannot be solved in closed form. However, full factorial experiment analysis may be very computer time consuming, and hence costly. It could be combined with the approximate closed form solution which could be used for evaluating the effect of each variable on the overall aggregate performance.

(c) Where analytical expressions are available, or can be developed by statistical regression predictive techniques, the direct application of the Taylor Series expansion (approximate closed-form) to a probabilistic formulation of reliability analysis appears warranted. It is without question a very powerful mathematical tool that has great potential in reliability studies of pavement performance.

Because of these general considerations, it is noted that the primary method of developing the probabilistic solutions to the rigid pavement performance analysis of the USACE for airfield pavements was the Approximate Closed-Form solution (Taylor Series). This approach was used to formulate both the probabilistic Westergaard-USACE procedure shown in Volume III as well as the Multi-Layered Elastic Design (developed by the USACE) presented in Volume IV. In the latter case, a simulation scheme is also presented to clearly demonstrate that the two approaches are, from an engineering viewpoint, equivalent. Such a conclusion only enhances the utilization of the Taylor Series approach to the reliability problem. Inherent within this suggested probabilistic approach, is the necessity to develop closed-form solutions and models by statistical regression techniques for various variables important in the design/analysis solution. The development of these particular expressions are clearly presented in the appropriate report Volumes.

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VOLUME III

PROBABILISTIC ANALYSIS OF RIGID AIRFIELD DESIGN BY THE WESTERGAARD FREE EDGE THEORY

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Chapter 1

INTRODUCTION

General

The design of rigid pavements (plain and reinforced) for military purposes is currently based upon the classical Westergaard free edge stress slab theory. Present U.S. Army Technical Manuals and U.S. Air Force Manuals use this theory as the basis for their design methods. The Westergaard model consists of a linear elastic slab supported by a dense liquid (equivalent to independent linear springs). While Westergaard developed stress solutions for loads placed at various slab locations (i.e., edge, corner and interior), the military design approach is based upon a (modified) free edge stress condition. (1)

At present, the design method approach is deterministic, i.e., a unique pavement system is designed for the set of input variables which are also unique. The design corresponds to an acceptable performance level (first crack) with an unknown margin of safety. The deterministic design approach, indirectly and only qualitatively, may account for the effect of material variability on pavement performance by judicious selection of the design input values. However, a quantification of these effects is possible and will improve the design procedure by showing the partial effect of each variable.

Study Objective

The aim of this project was to include the design parameter variability in the current USACE design procedure for unreinforced concrete pavements, and to evaluate (quantitatively) their effect on the final design. The final rigid pavement design is then expressed in reliability terms. The

mathematical and analytical development of the probabilistic approach used in this volume is presented in Volume II of the study.

It was also the aim of this project to expand the current design procedure so that systems with more than one foundation layer could be evaluated. This was accomplished through the use of the composite modulus of reaction which considers all layers below the pavement to respond as an "equivalent", "lumped" or "composite" foundation. The layer parameters of this composite section are then used as the foundation material response within the Westergaard model.

Report Organization

This volume has been subdivided into two major chapters. They are:

Chapter 2: Composite Modulus of Subgrade Reaction

Chapter 3: Probabilistic Analysis of PCC Airfield Design

Chapter 2 will discuss the background of the composite modulus of subgrade reaction as well as the development of the equations which are used within the reliability based design method.

Chapter 3 deals with the development of the equation used to predict the maximum stress computations. These equations, in turn, are used to evaluate the variability of the pavement design. The probabilistic solution used in the Westergaard analysis is based upon the Approximate Closed-form Probabilistic Formulation (Taylor Series) discussed in Volume II. The detailed mathematical derivations used to develop this solution are contained in this volume. A computer program has been developed to solve this probabilistic methodology. A user's guide and a program listing have been included in the Appendices.

Chapter 2

COMPOSITE MODULUS OF SUBGRADE REACTION

Introduction

The composite modulus of subgrade reaction was introduced to take into account the combined effect of a layered system beneath the rigid pavement. This composite modulus is a very useful and necessary tool in the evaluation of layered systems. The Westergaard theory is based upon a slab resting on a foundation material. As a consequence, multiple foundation layers cannot be solved for directly. However, the composite modulus is a means of making the layered system into a single equivalent layer which may be used within the Westergaard model. The composite modulus is defined as an equivalent modulus that will lead to the same response from the original layered system (in this case: equal deflections of the subsystem below the slab). The major variables in determining the composite modulus are the layer thickness and modulus. This chapter presents the evaluation of the composite modulus of subgrade reaction for the case of a layered system. All of the composite modulus relationships used in this solution have been based upon those contained in TM 5-824-3 (AFM 88-6, Chap. 3), Rigid Pavements for Airfields Other Than Army, August 1979.

Current USACE Method for Determining the Composite Modulus of Subgrade Reaction (k_c Value)

In the current USACE method (August 1979), several relationships for composite modulus (k_c) are presented for different materials (2). Figure 2.1 shows the k_c relationship for a well-graded crushed material and Figure 2.2 shows the relationship for natural sands and gravels (PI<8). The k_c is developed as a function of the layer thickness, the modulus of

subgrade reaction and the material type. Figure 2.3 shows the k_c relationship for stabilized subbase materials. In this case, the k_c value is dependent on the modulus of elasticity and layer thickness of the stabilized layer as well as the modulus of subgrade reaction.

Figures 2.1 and 2.2 are based on field plate-loading tests while Figure 2.3 is based on computations using the elastic layered theory with corrections (3). These k_c values are based on an equivalent deflection criterion. The deflection of the composite section is equal to the deflection of the layered system under the same 30 inch diameter plate.

Formulation of Composite Modulus Equations

The approximate closed-form probabilistic approach presented in Volume II and implemented in this volume requires the variables to be expressed in equation form. Therefore, the results of the composite modulus from figures 2.1 to 2.3 were analyzed to derive regression equations. This was accomplished by the use of the Statistical Package for the Social Sciences (SPSS). A multiple regression analysis was performed on the information taken from these figures to generate the equations. (The SPSS outputs may be found in Appendix I). The final equations selected resulted in excellent correlation coefficients and agreement between actual and predicted values. The SPSS generated equations are:

For well-graded crushed materials:

$$\log k_c = -1.251182 + 2.219732 \log k_{sg} - 0.2949522 (\log k_{sg})^2 + 0.08901252h - 0.0004425194 h^2 - 0.02901488 h \log k_{sg} \quad (2.1)$$

$R^2 = 0.998$, Standard Error of Estimation SEE = 3%

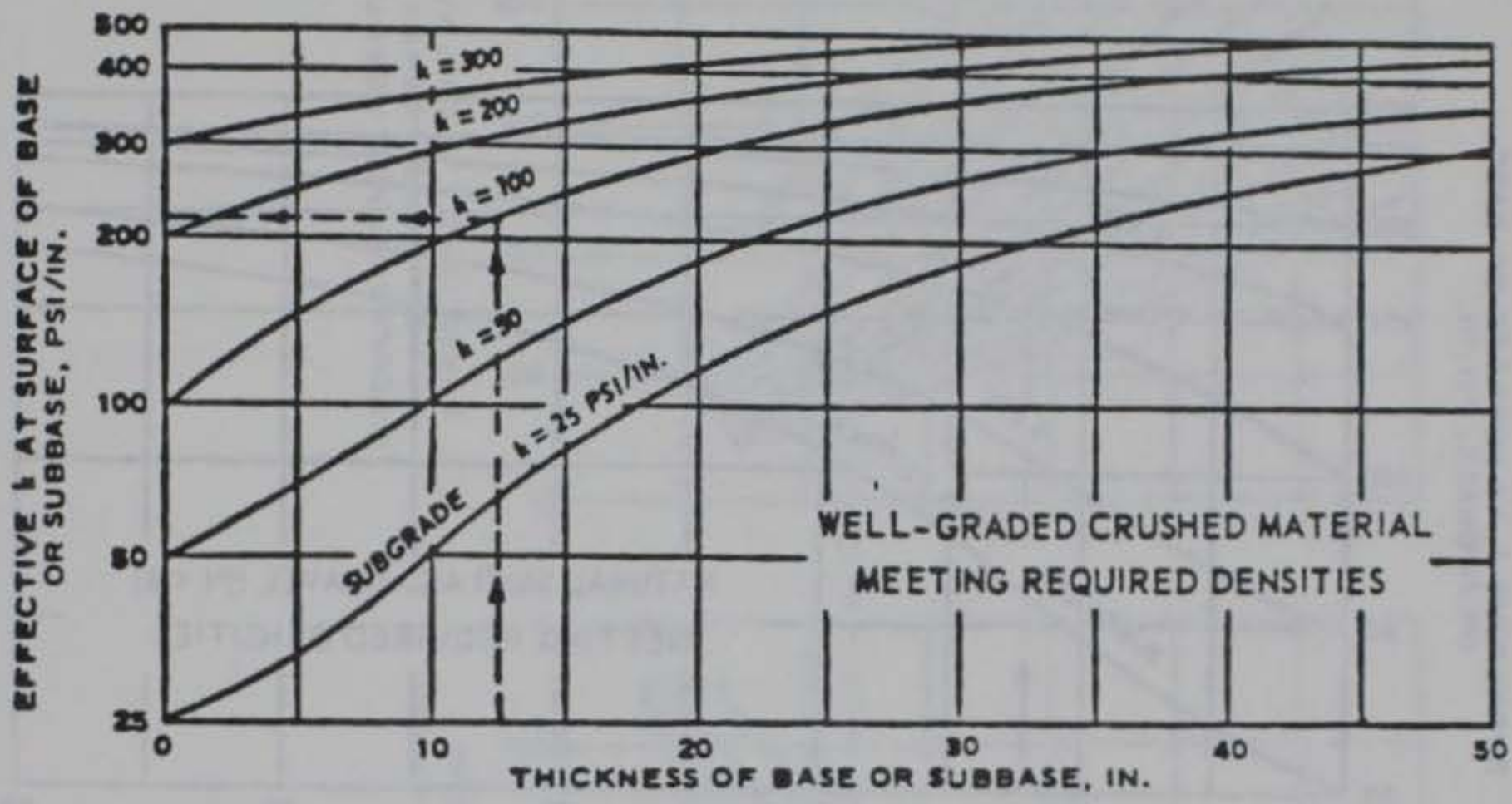

Figure 2.1 COMPOSITE SUBGRADE MODULUS OF REACTION FOR WELL-GRADED CRUSHED MATERIAL

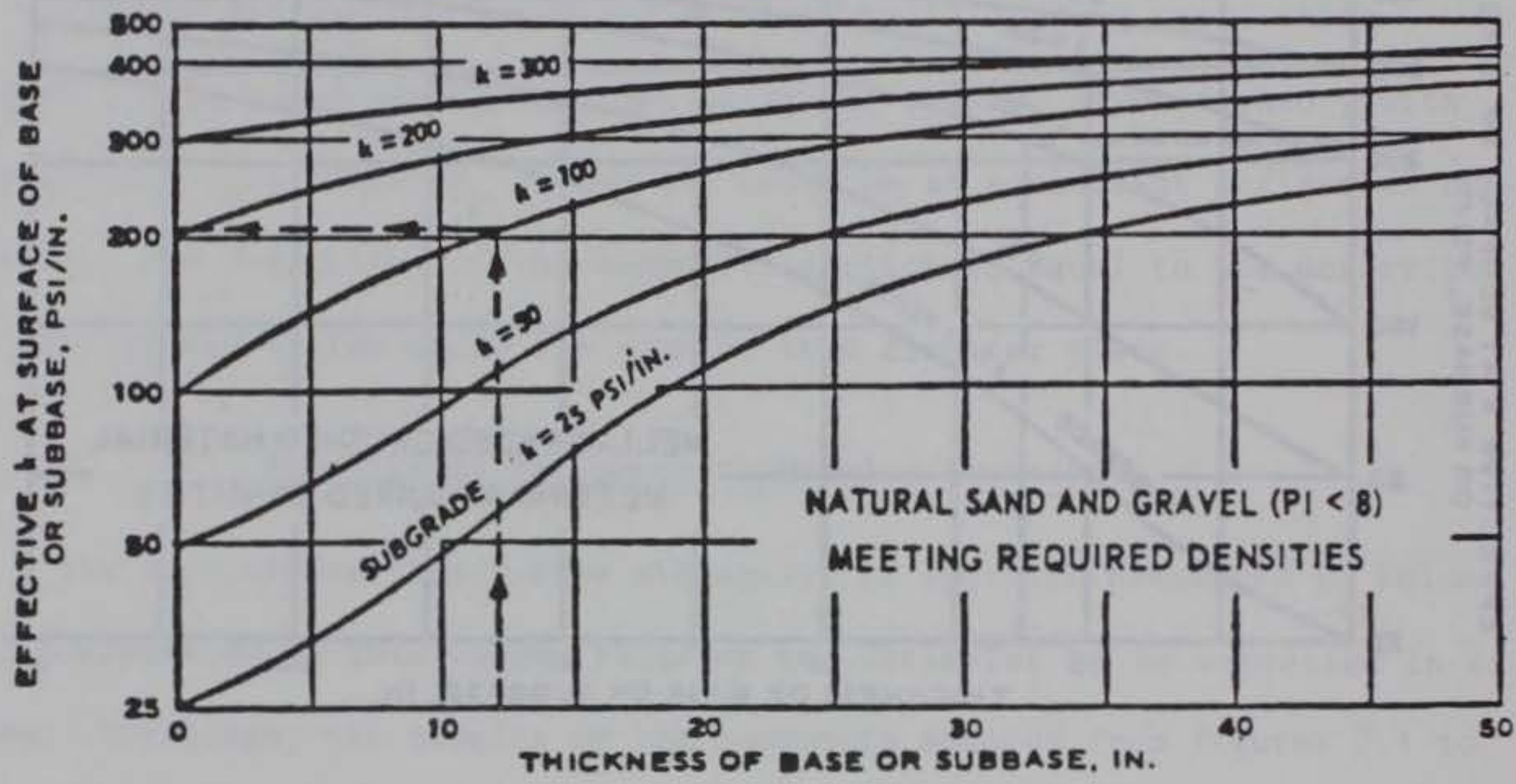

Figure 2.2 COMPOSITE SUBGRADE MODULUS OF REACTION
FOR NATURAL SAND AND GRAVEL

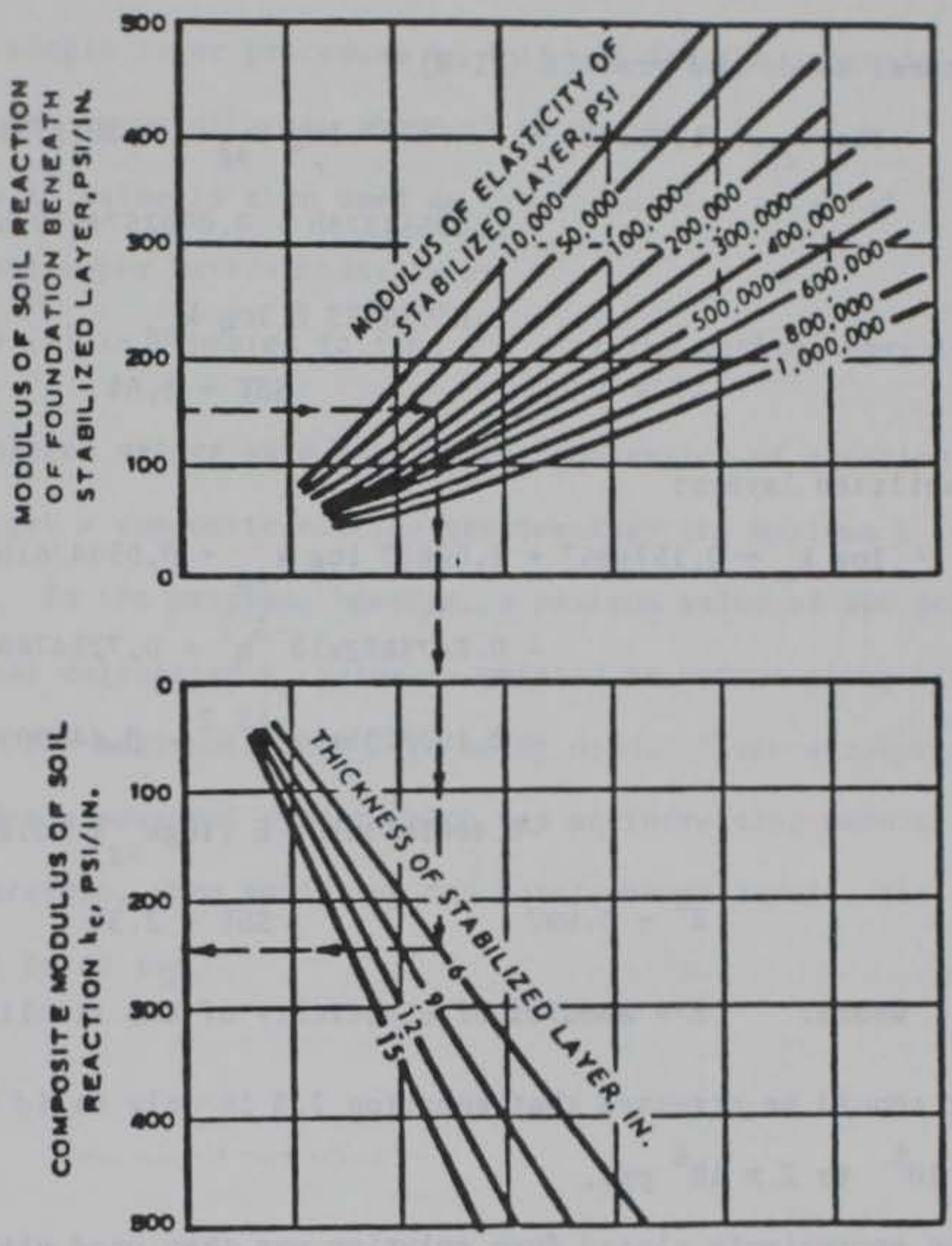

Figure 2.3 COMPOSITE SUBGRADE MODULUS OF REACTION FOR DIFFERENT BASE-SUBBASE MATERIALS

where: h = layer thickness, in.

k_{sg} = modulus of subgrade reaction, pci

For natural sands and gravels (PI<8):

$$\log k_c = -1.296084 + 2.263407 \log k_{sg} - 0.3013741 (\log k_{sg})^2 + 0.08554373h - 0.0002574619 h^2 - 0.03050173 h \log k_{sg} \quad (2.2)$$

$R^2 = 0.997$ SSE = 3.6%

For stabilized layers:

$$\log k_c = -0.1578667 + 1.02813 \log k_{sg} + 0.0544761h - 0.8473852x10^{-3}h^2 + 0.7254749x10^{-6}E - 0.1937293x10^{-12}E^2 - 0.4409096x10^{-2}h \log k_{sg} - 0.4601653x10^{-7}E (\log k_{sg}) - 0.2465638x10^{-8}Eh \quad (2.3)$$

$R^2 = 0.997$ SSE = 2.3%

where: E = modulus of elasticity of the stabilized layer, psi.

It should be stressed that equation 2.3 is only valid within the range of $E = 10^5$ to 2×10^6 psi.

The approximate closed form solution was then used with these equations in order to generate an equation for the $CV^2(k)$ value to be used in the probabilistic analysis. (This value will be discussed in Chapter 3.)

It should be noted that the maximum number of layers for which the program may be used is four (including the slab and the subgrade). The computation of the k_c for a single base/subbase layer is a straightforward solution. The required values (subgrade modulus, layer thickness and the modulus of elasticity, if applicable) are input into the equation for the

composite modulus. If there are two base/subbase layers, the procedure used is as follows:

a. The single layer procedure is followed for the subgrade and the layer directly above it to obtain a k_c' value
b. This k_c' value is then used as the foundation material of the upper base/subbase layer
c. Step (a) is repeated to find a k_c for the entire subsystem.

Since extrapolated values were used in the generation of equation 2.3, it is possible to get a composite modulus greater than the maximum k_c for design of 500 pci. In the program, however, a maximum value of 500 pci is used. The actual calculated k_c value is printed as output along with an indication that the maximum design k is being used. These equations have been set up for a pavement system which has an increasing modulus from the subgrade. Therefore, when analyzing two base/subbase layers, the stiffer layer must be on top.

There are two approaches available for determining the composite modulus of subgrade reaction for stabilized layers. One method accounts for the base material by increasing the modulus of the subgrade while the other method accounts for the base by using a section of PCC with an increased thickness.

For the first method, the composite k-value chart (fig. 2.3) was based on calculations using the elastic layer theory. The values which were obtained by using the elastic layer theory were then corrected based on field data and experience. The value of k_c was determined by using equivalent deflections. The k_{sg} value was raised until the deflection of the "improved" subgrade equalled that of the base-subgrade system.

The other method of accounting for a base material is the use of an increased PCC thickness. This increased thickness is determined by the partially bonded rigid overlay pavement design equation:

$$h_{doc} = \sqrt[1.4]{h_{dc}^{1.4} - (0.0063 \sqrt[3]{E_{fc}}\, h_b)^{1.4}}$$

This equation is based upon equivalent stiffnesses ($D = \frac{Eh^3}{12(1-\mu^2)}$) i.e., the stiffness of a PCC layer for a given modulus of elasticity and thickness is equivalent to the stiffness of a base layer for a given modulus of elasticity and thickness. From this, the increased thickness of the PCC layer may be calculated.

In this probabilistic design methodology, the method of the composite modulus of subgrade reaction has been used. It has been chosen because it yields the best estimate of values to be used in a composite pavement section.

Chapter 3

PROBABILISTIC ANALYSIS OF PCC AIRFIELD DESIGN USING THE WESTERGAARD FREE EDGE THEORY

Introduction

The analysis presented in this chapter is based on the approximate closed-form probabilistic approach which was formulated in Volume II. The use of this closed-form approach requires stress computations which can be made with the H-51 computer program. While the use of the H-51 is justified for the pavement design for a particular aircraft, it can become uneconomical for the probabilistic approach due to the large amount of computer time needed. The analysis was therefore confined to the controlling aircraft of each USAF AGI group. Solutions for other aircraft may be developed by using the user defined option in the program to develop the regression constants to be used in the maximum tensile stress equation.

The probabilistic approach presented in the following paragraphs includes:

(1) Stress computations and derivation of an equation to predict the maximum tensile stress for USAF AGI 1 to USAF AGI 15;

(2) Derivation of the relationship between variances of the dependent and independent variables for the approximate closed-form approach. The linear or first order Taylor series expansion is assumed as presented in Volume II.

Stress Computation

The Corps of Engineers uses the H-51 computer program (4) to compute the bending stress at the edge of a slab supported by a dense liquid. The program, developed by General Dynamics Co., numerically integrates the number of blocks of the influence chart covered by the load. The wheel

load is assumed to be uniformly distributed over an elliptical area shape with any desired ratio of the axes. The results of the computation are generally within 2-3 percent accuracy.

The numerical computation of the stress constitutes a limitation for deriving a closed form probabilistic model. It was therefore necessary to develop an equation for the stress at the edge for the load of interest. The form of the equation was taken from the original Westergaard's work. Westergaard (4,5,) developed an approximate general formula for stresses for the interior and edge cases for any loaded area. The formula for the edge case reads as follows (6):

$$\sigma_e = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[4K - 0.28 - \frac{4}{3}\mu - \mu^2 + (1-\mu)S + 1.18(1+2\mu)\frac{\bar{y}}{\ell}\right] \qquad (3.1)$$

where:

$$K = \frac{-1}{A}\int_A \ln\frac{r}{\ell}\, dA$$

$$S = \frac{-1}{A}\int_A \cos 2\theta\, dA$$

$$\ell = \sqrt[4]{\frac{E h^3}{12(1-\mu^2)k}}$$

σ_e = tensile stress at the bottom of the slab along the edge or joint at x=y=0.

P = load

μ = Poisson's ratio of the concrete

h = thickness of the slab

K and S = area coefficients defined by the equations noted

A = area of contact

r,θ = radius and angle in polar coordinates describing the distance of the infinitesimal area from the origin

ℓ = radius of relative stiffness given by the equation noted

k = modulus of subgrade reaction

$\bar{y}$ = the distance from the edge or joint to the center of gravity of the load.

According to Westergaard's results, Equation 3.1 can be written as follows, for the elliptical contact area:

$$\sigma_e = \frac{3(1+\mu)P}{\pi(3+\mu)h^2} \left[4 \ln \ell + f(x,y) + 1.18(1 + 2\mu) \frac{\bar{y}}{\ell}\right] \qquad (3.2)$$

where f(x,y) is the function of the load configuration and geometry. It should be observed that this function is constant for a given loading (aircraft) condition.

It should be stressed that equation (3.1) was developed using a finite polynomial series and is restricted to the case of a "small distance" of the loaded area to the origin at which the stress is computed. Investigation of the extent of this "small distance" for the particular case of a rectangular loaded area showed that the small distance can be as large as about 1.5 ℓ. Without this restriction, equation (3.2) is a powerful result which simplified the analysis. It is interesting to note that, excluding the load geometry conditions, only two pavement variables: h and ℓ, determine the edge stress.

With the above restriction in mind, the following general equation form for expressing the edge stress as function of h and ℓ was adopted:

$$\sigma_e = \frac{1}{h^2} \left[a_o + a_1 \ln \ell + a_2/\ell\right] \qquad (3.3)$$

where a_o, a_1, a_2 - coefficient dependent upon load, load configuration and geometry.

The steps involved in computing the above coefficients for any aircraft type are as follows:

(1) Compute the maximum stress at the edge induced by load for different ℓ values. This can easily be achieved using the H51 computer program by choosing one set of values of k, E (modulus of reaction of subgrade and modulus of elasticity of concrete) and varying h - the slab thickness.

(2) Find the coefficients of a_o, a_1, a_2 by the least square error method, over the stress values computed in step (1).

The methodology is illustrated for the aircraft designated AGI 1 to AGI 15. Their characteristics are shown in Table 3.1. The results of these computations are shown in Appendix II. The stresses computed using eq. (3.3) are also shown in Appendix II. It can be observed that they are almost identical to the original values computed from the H-51 computer program. In general, correlation coefficients greater than 0.999 were obtained. The errors in computing the stresses with the regression equation have been tabulated as a percentage of the original stress and are also presented in Appendix II. (The maximum error is -0.81 of 1%). These results clearly support the use of equation (3.3) for a closed-form solution of the problem.

If the analysis is to be used for a user defined aircraft (other than AGI type), it is necessary to input at least five slab thickness levels. This is necessary so that the least square error regression has enough data points to predict coefficients which are truly representative of the data.

Table 3.1

Summary of USAF Evaluation
Characteristics for Aircraft Groups

USAF Aircraft Group Index	Controlling Aircraft	Type Gear	Tire Spacing	Tire Contact Area	Max.Gross Weight (kips) Gear	Aircraft
1	C-123	Single Wheel	-	275 in^2	27.0	60.0
2	F-4	Single Wheel	-	100 in^2	27.0	60.0
3	F-111	Single Wheel	-	241 in^2	45.0	100.0
4	C-130	Single Tandem	60"cc	400 in^2	69.75	155.0
5	C-9 (DC-9)	Twin Wheel	26"cc	165 in^2	51.3	108.0
6	T-43 (B-747)	Twin Wheel	30.5"cc	174 in^2	54.625	115.0
7	B-727	Twin Wheel	34.0"cc	237 in^2	90.25	190.0
8	E-3 (B-707)	Twin-Tandem	34.5" x 56.0"cc	218 in^2	159.6	336.0
9	C-141	Twin-Tandem	32.5" x 48.0"cc	208 in^2	155.3	345.0
10	C-5	Twin-Delta-Tandem	34.0" x 53.0"cc x 65.0"cc	285 in^2	361.4	769.0
11	KC-10 (DC-10-30)	Twin Tandem	54.0" x 64.0"cc	294 in^2	212.04	558.0
12	E-4 (B-747)	Twin Tandem	44.0" x 58.0"cc	245 in^2	369.6	778.0
13	B-52	Twin-Twin	37.0" x 62" x 37.0"cc	267 in^2	249.6*	480.0
14	OV-1	Single Wheel	-	70 in^2	8.235	18.3
15	C-54	Dual Wheel	18.0"cc	106 in^2	21.15	47.0

*Gear Load does not include bicycle gear factor

Probabilistic Closed-form Approach

General Formulation

The rigid pavement design is based on computation of the design factor DF defined as:

$$DF = \frac{MR}{\alpha\sigma} \qquad (3.4)$$

where MR = modulus of rupture of concrete

α = load transfer coefficient (=0.75 in the Corps of Engineers procedure)

σ = free edge stress induced by load (Westergaard solution)

The design factor is related to the number of coverages on the basis of USACE test section results and experience. The relationships are shown in Figs. 3-1a to Fig. 3-1c for three levels of failure mode: (i) initial failure, (ii) shattered slab condition and (iii) complete failure. Therefore, knowledge of the design factor and its probability distribution leads to a complete description of the design in probabilistic and reliability terms. In the following, the closed form approach is described.

In equation (3.4) the stress is expressed as function of the material and slab geometry variable, (E, μ , k and h) using equation (3.3). In the analysis, a constant μ of 0.15 was used. It has been held as a constant since there is a relatively insignificant effect of μ on σ. Also, H-51 uses a equal to 0.15 in order to generate the stress predictor coefficients. Therefore, μ is equal to a constant value of 0.15 instead of being stoichastic. Therefore, the average design factor is:

$$\overline{DF} = \frac{\overline{MR}}{\overline{\alpha} \cdot \sigma(\overline{E}, \overline{k}, \overline{h})}$$

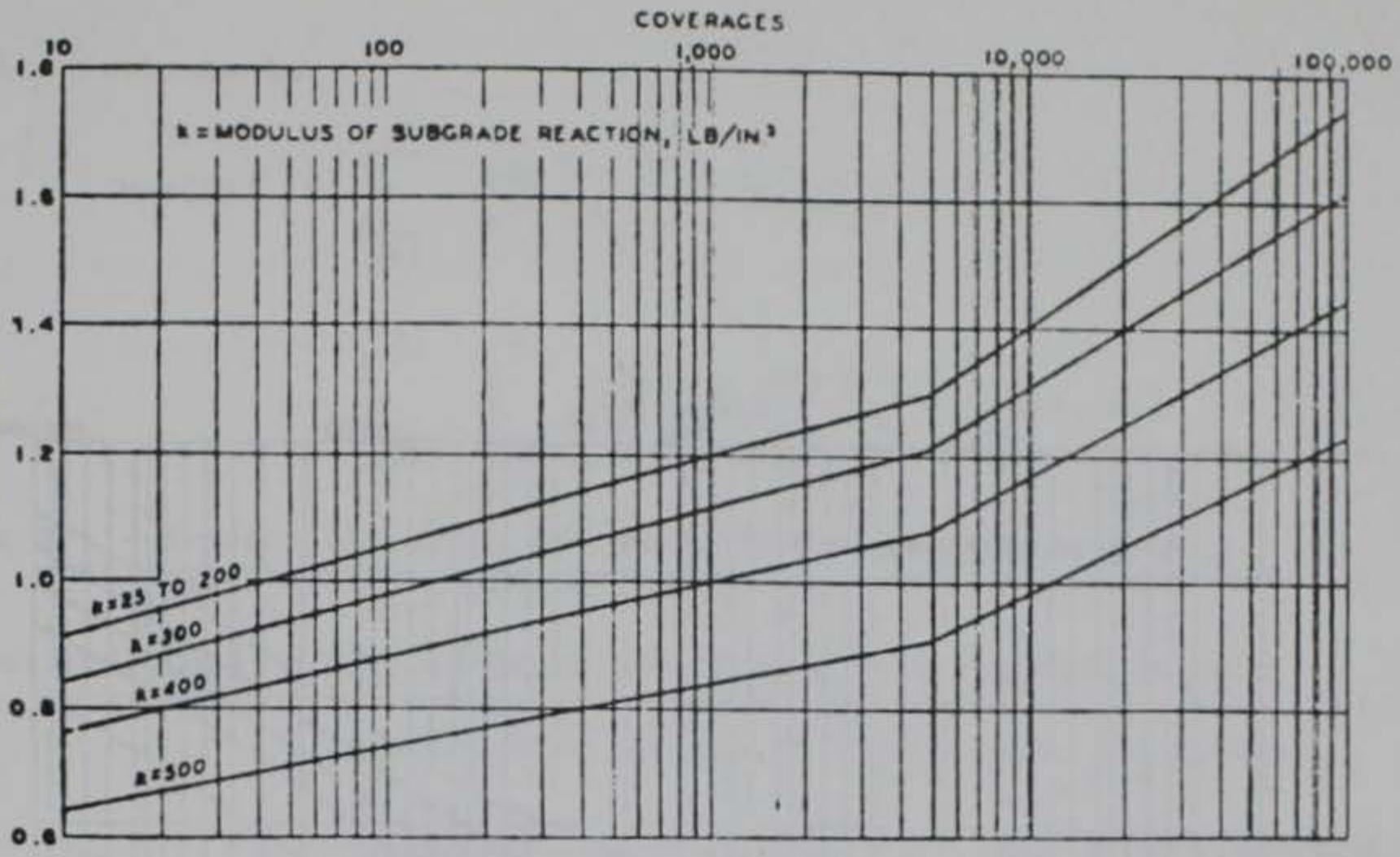

Figure 3.1a Design Factor Versus Coverages for Initial Failure Condition.

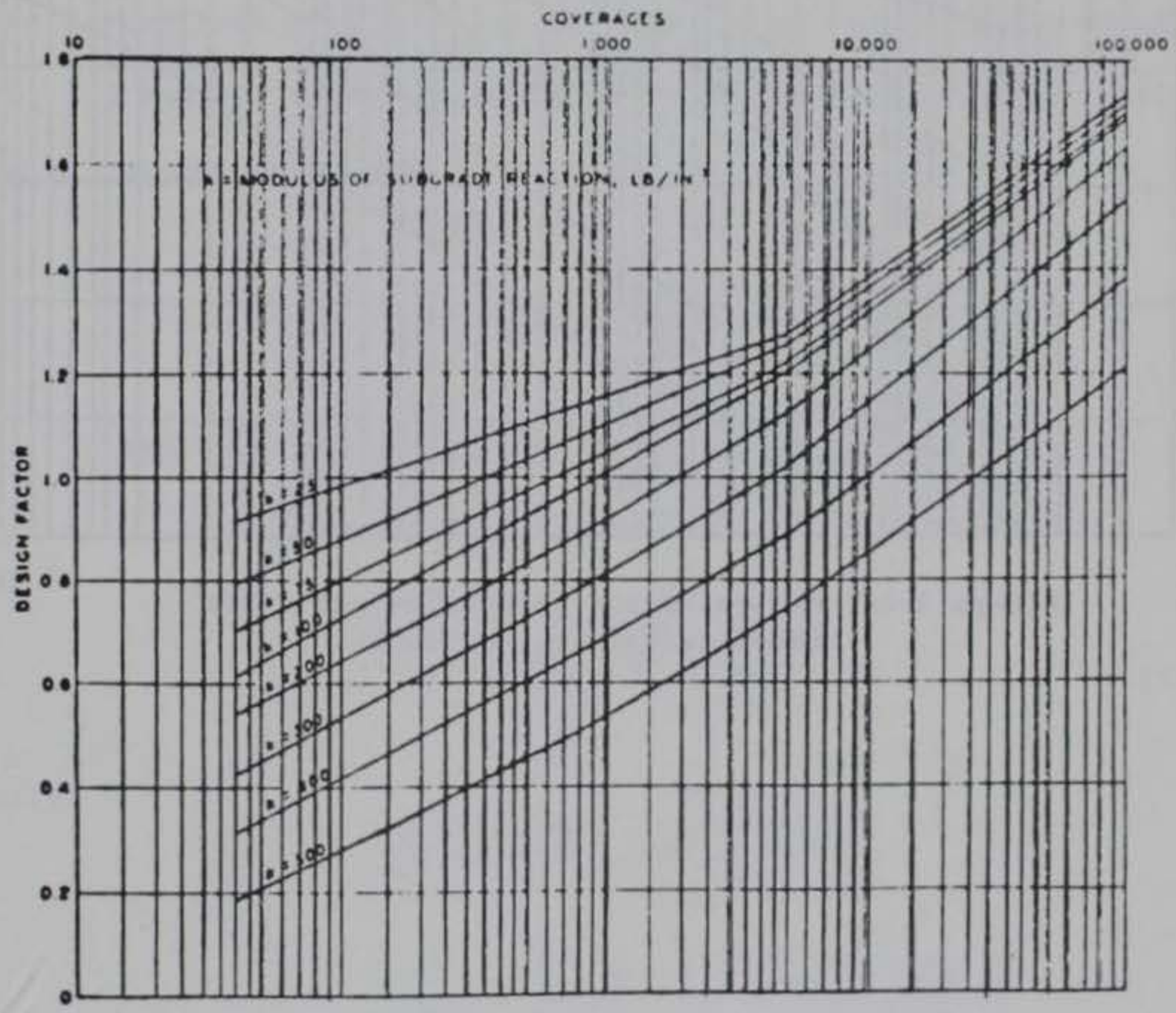

Figure 3.1b Design Factor Versus Coverages for Shattered-Slab Failure.

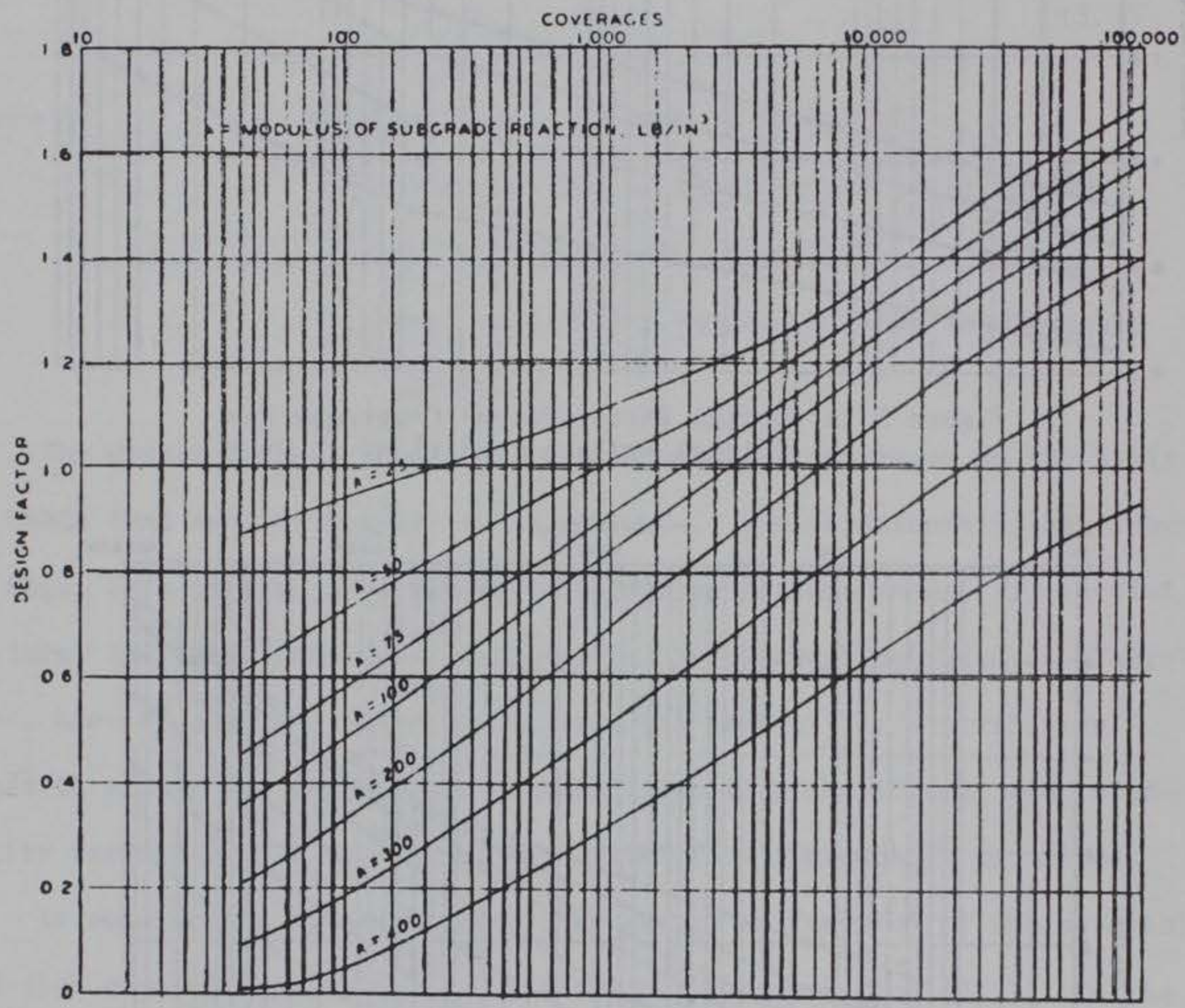

Figure 3.1c Design Factor Versus Coverages for Complete Failure Condition.

and its variance is:

$$\text{Var[DF]} = \sum_{i=1}^{n} \left(\frac{\partial DF}{\partial X_i}\Big|_{\bar{X}_i}\right)^2 \text{Var}[X_i] + \sum_{i=1}^{n} \sum_{\substack{i=1 \\ k\neq 1}}^{n} \left(\frac{\partial DF}{\partial X_i}\Big|_{\bar{X}_i}\right)\left(\frac{\partial DF}{\partial X_k}\Big|_{\bar{X}_k}\right) \text{Cov}[X_i, X_k) \qquad (3.6)$$

where X_i - denotes the different variables, MR, α, E, k, h

- $\text{Var}[X_i]$ - variance of variable X_i

$\text{Covar}\,[X_i, X_k]$ - covariance of variables X_i and X_k which is zero when these variables are independent, and equals

$\rho \,/\, \overline{\text{Var}[x_i] \,.\, \text{Var}\,[x_k]}$ when the variables are dependent, with a correlation coefficient ρ .

The derivatives of DF with respect to all variables can be found from eq. 3.3 and 3.4 using the chain rule. For example, the derivative of DF with respect to E is given by

$$\frac{\partial DF}{\partial E} = \frac{\partial DF}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial E} = -\frac{MR}{\alpha \sigma^2} \cdot -\frac{\partial \sigma}{\partial E}$$

and $$\frac{\partial \sigma}{\partial E} = \frac{\partial \sigma}{\partial \ell} \cdot \frac{\partial \ell}{\partial E} = \frac{a_1 - a_2/\ell}{h^2 \ell} \cdot \frac{\ell}{4E}$$

leading to

$$\frac{\partial DF}{\partial E} = -\frac{MR}{\alpha \sigma^2} \cdot \frac{(a_1 - a_2/\ell)}{4h^2 E} \qquad (3.7)$$

Similarly:

$$\frac{\partial DF}{\partial k} = -\frac{MR}{\alpha \sigma^2} \cdot \frac{(a_1 - a_2/\ell)}{4h^2 E}$$

$$\frac{\partial DF}{\partial h} = -\frac{MR}{\alpha \sigma^2} \left[-\frac{2\sigma}{h} + \frac{3(a_1 - a_2/\ell)}{4h^2 \cdot h} \right]$$

and:

$$\frac{\partial DF}{\partial MR} = \frac{1}{\alpha\sigma}$$

$$\frac{\partial DF}{\partial \alpha\sigma} = \frac{MR}{\alpha^2\sigma}$$

For the following, only MR and E are assumed to be dependent. If the coefficient of variation, CV, is defined as the ratio of the standard deviation to the mean value of the variable, then substituting eq (3.7) into eq. (3.6) and rearranging the terms lead to:

$$\begin{aligned} CV^2[DF] = {} & CV^2[MR] + CV^2[\alpha] + \\ & \left[\frac{a_1 - a_{2/\ell}}{4\sigma h^2}\right] (CV^2[E] + CV^2[k]) + \\ & \left[-2 + \frac{3(a_1 - a_{2/\ell})}{4\sigma h^2}\right]^2 CV^2[h] + \\ & -2\left[\frac{a_1 - a_{2/\ell}}{4\sigma h^2}\right] \rho[MR,E] \cdot CV[MR] \cdot CV[E] \end{aligned} \qquad (3.8)$$

where $CV[X_i]$ is the coefficient of variation of variable X_i. In this equation, the value of $CV^2[k]$ is either the input value of $CV^2[k]$ or it is the calculated value of $CV^2[k]$ if the composite modulus of subgrade reaction is calculated. These $CV^2[k]$ values may be calculated for the materials found in equations 2.1 to 2.3 by using the first-order Taylor series approximation (from Vol. II).

For well-graded crushed materials:

$$\begin{aligned} CV^2[k_c] = \frac{Var[k_c]}{\bar{k}_c^2} = {} & 5.3018981 \left[h^2(0.08901252 - 0.0008850388h \right. \\ & - 0.02901488 \log k_{sg})^2 CV^2[h] \\ & + (0.9640174 - 0.2561922 \log k_{sg} - \\ & \left. - 0.012601\, h)^2\, CV^2[k_{sg}]\right] \end{aligned} \qquad (3.9)$$

where: CV[i] = coefficient of variation of variable i

k_c = composite modulus of subgrade reaction, pci

h = layer thickness, in.

k_{sg} = modulus of subgrade reaction, pci

For natural sand and gravel:

$$CV^2[k_c] = 5.3018981 \left[h^2(0.08554373 - .0005149238h - 0.03050173 \log k_{sg})^2 CV^2[h] + (0.9829852 - 0.2617702 \log k_{sg} - 0.0132467h)^2 CV^2[k_{sg}]\right] \quad (3.10)$$

For stabilized layers:

$$CV^2[k_c] = 5.3018931\Big[\Big[h^2(0.0544761 - 1.6947704\times10^{-3}h - 0.4409096\times10^{-2}\log k_{sg} - 0.2465638\times10^{-8}E)^2 CV^2[h] + E^2(0.7254749\times10^{-6} - 0.3874568\times10^{-12}E - 0.4601653\times10^{-7}\log k_{sg} - 0.2465638\times10^{-8}h)^2 CV^2[E]$$
$$+ (0.4465112 - 0.1914846\times10^{-2}h - 0.1998463\times10^{-7}E)^2 CV^2[k_{sg}]\Big] \quad (3.11)$$

where E = modulus of elasticity of the stabilized layer, psi

Equations 3.5 and 3.8 are used to compute the average DF and its coefficient of variation, using (i) equation 3.3 and (ii) material characteristics.

These equation can easily be numerically transformed into different reliability-number of coverage - slab thickness relationships. Two possibilities of implementing the above formulation are discussed in the next section.

Reliability-Number of Coverages Relationship

for Different Slab Thicknesses

With the design factor assumed normally distributed, the number of coverages corresponding to DF(1 + k.CV[DF]) can be computed, and the probability of DF $\leq$ DF(1 + kCV[DF]) is taken from the normal distribution.

In order to do this, equations had to be developed for the DF-Number of Coverages curve (Figure 3.1a, since initial failure is assumed).

Since each curve is bilinear, two equations were derived for each. The first equation defines the curve below 5000 coverages and the second equation defines the rest of the curve. Since these equations were developed independent of the modulus of subgrade reaction, ranges had to be assumed for each curve. These ranges are:

Curve	Range of k Assigned to Curve
k = 25 - 200	25 - 250
300	250 - 350
400	350 - 450
500	450 - 500

This part of the design procedure yields the reliabilities for a given slab thickness in terms of the number of coverages it will take to produce that reliability.

Reliability-Slab Thickness Relationship for Different Number of Coverages

This part of the design procedure gives the reliability as a function of the slab thickness for a given number of coverages. This can be done by basically working the previous procedure in the reverse order. If the desired number of coverages is known, then the DF may also be calculated by using the equations described in the previous section. The probability is then taken from the normal distribution. The information obtained from this analysis can be very useful. If a certain reliability is desired for a given coverage level, then the necessary slab thickness can be found.

Study of Effects of the Variation of the Design Parameters

The closed form solution has the advantage to bring an insight of the effect of the design parameters. In eq. (3.8), the coefficient of variation of DF and hence the reliability of the design, is expressed as a function of the coefficient of variation of the design parameters. The effect of these parameters is enhanced if equation (3.8) is written as follows:

$$CV^2[DF] = \sum_i w_i \cdot CV^2[X_i] + \sum_i \xi_i \cdot \rho[X_i, X_k] \cdot CV[X_i] \cdot CV[X_k] \quad (3.12)$$

where: w_i, ξ_i - weighting factors ($w_i = \left(\frac{a_1 - a_2/\ell}{4\sigma h^2}\right)^2$

for the variable E, $w_i = 1$ for the variable MR, etc.)

The effect of each parameter is either increased or reduced by the corresponding weighting factor which comes from the response function. It should be noted that the weighting factor is dependent upon load and material characteristics and slab thickness. An evaluation of the weighting factors for the thirteen Aircraft Group Indices is presented in Appendix II, over the practical range of ℓ values. It is seen that the weighting factors of E and k (the modulus of elasticity of the concrete and the subgrade modulus of reaction) are quite small, resulting in a minor effect of their variability on the reliability of the design. The weighting factor of the slab thickness is the largest one, with the result of amplifying the effect of thickness variability on DF variability. However, it should be remembered that thickness variability is relatively small. It seems that the leading parameters controlling DF variability are thus the

modulus of rupture and load transfer variabilities. With the increase of number of wheels (from light to heavy-load design), the contribution of the variability of E and k increases.

Run Example

The new approach was used to develop reliability-number of coverages curves for different slab thicknesses (11, 12, 13, 14, 15 and 16 inch) for AGI 13. Reliability-slab thickness curves were then developed for different coverage levels (1000, 5000, 10,000, 20,000, 50,000, and 100,000 coverages). The average values of the variables and their coefficients of variation are given in Figure 3.2a. Output may be found in Figure 3.2b. Graphical results (plots of Reliability-Coverages-Slab Thickness relationships) may be found in Figure 3.3 and 3.4.

```
RUN EXAMPLE USAF AGI 13 INPUT DATA
**********************************

13  1       250.        0.1  4000000.       0.15
 3        12.        0.05   500000.        0.1
       11.        16.         1.
      800.         .1        .75          .1        .05        .02        .8
 6 1 1
   1000.   5000.  10000.  20000.  50000. 100000.
```

Figure 3.2a

Input Data for Run Example

USAF AGI-13

```
                    MULTI-LAYERED SYSTEM ANALYSIS

SUBGRADE:
          MODULUS OF SUBGRADE REACTION =  250.00 PCI      CV =    .1000
     BASE:
          TYPE: STABILIZED
          HEIGHT = 12.00 INCHES          CV =    .0500
          MODULUS =  400000.00 PSI       CV =    .1000

COMPOSITE:
  LAYER  :
          MODULUS OF SUBGRADE REACTION =  905.07 PCI      CV =    .1098

     A MAXIMUM DESIGN K OF 500 PCI IS USED
```

Figure 3.2b Run Example Output

```
LSAF AIRCRAFT GROUP INDEX: 1?

      A0            A1            A2            M2          B
  -5.7727.      2.5062.      4.2526P.          1.000       1.000

NO.    THICK     SIGMA       DF       CVDF       FACT1      FACT2
  1     11.00   1182.20      .9027     .1401      .2291    -1.3127

        RELIA          DF            N
        .9987          .5231          1.
        .9974          .5484          1.
        .9953          .5737          2.
        .9918          .5990          4.
        .9861          .6242          7.
        .9772          .6495         12.
        .9681          .6748         22.
        .9452          .7001         40.
        .9192          .7254         71.
        .8849          .7507        127.
        .8413          .7759        228.
        .7881          .8012        408.
        .7257          .8265        730.
        .6554          .8517       1307.
        .5793          .8770       2339.
        .5000          .9023       4184.
        .4207          .9276       7456.
        .3446          .9528       7469.
        .2743          .9781      11989.
        .2119         1.0034      11989.
        .1587         1.0287      18489.
        .1151         1.0539      19748.
        .0808         1.0792      28521.
        .0548         1.1045      30888.
        .0359         1.1298      49133.
        .0228         1.1550      49579.
        .0139         1.1803      62813.
        .0082         1.2056      79579.
        .0047         1.2309     100821.
        .0026         1.2562     127733.
        .0013         1.2814     161828.

NO.    THICK     SIGMA       DF       CVDF       FACT1      FACT2
  2     12.00   1054.78      1.0113     .1401      .2351    -1.3097

        RELIA          DF            N
        .9987          .5284          4.
        .9974          .5547          6.
        .9953          .5810         11.
        .9918          .6073         21.
        .9861          .6336         39.
        .9772          .6599         76.
        .9681          .6862        145.
        .9452          .7125        279.
        .9192          .7388        536.
        .8849          .7651       1028.
        .8413          .7914       1974.
        .7881          .8177       3790.
        .7257          .8440       5827.
        .6554          .8703       7496.
        .5793          .8966       9957.
        .5000          .9229      12909.
        .4207          .9492      14827.
        .3446          .9755      21936.
        .2743         1.0018      28457.
        .2119         1.0281      37779.
        .1587         1.0544      48458.
        .1151         1.0807      63353.
        .0808         1.1070      82488.
        .0548         1.1333     107468.
        .0359         1.1596     141342.
        .0228         1.1859     182966.
        .0139         1.2122     238519.
        .0082         1.2385     310937.
        .0047         1.2648     405783.
        .0026         1.2911     529817.
        .0013         1.3174     688859.
```

Figure 3.2b Run Example Output

NO.	THICK	SIGMA	DF	CVDF	FACT1	FACT2
3	13.00	845.76	1.1271	.1481	.2295	-1.3114

RELIA	DF	N
.6587	.2613	13.
.9974	.4834	27.
.9963	.7141	55.
.9919	.7454	113.
.9841	.7770	234.
.9773	.8185	483.
.9681	.8755	954.
.9452	.8714	2056.
.9197	.9029	4283.
.8980	.9347	8282.
.8613	.9618	16837.
.7881	.9972	31721.
.7257	1.0217	51157.
.6554	1.0412	70833.
.5793	1.0514	87184.
.5193	1.0731	98748.
.4207	1.0946	105258.
.3648	1.1041	108758.
.2743	1.1176	113582.
.2113	1.1249	116799.
.1587	1.1314	119708.
.1151	1.1419	123171.
.0808	1.1513	126852.
.0584	1.1708	129745.
.0359	1.1842	131845.
.0229	1.1977	133845.
.0139	1.2052	134965.
.0082	1.2093	135573.
.0047	1.2121	135987.
.0028	1.2184	136904.
.0013	1.2262	138044.

NO.	THICK	SIGMA	DF	CVDF	FACT1	FACT2
4	14.00	861.85	1.2329	.1401	.2278	-1.3114

RELIA	DF	N
.6587	.2777	40.
.9974	.4529	133.
.9963	.7076	365.
.9919	.8217	855.
.9841	.8718	1654.
.9773	.8911	3015.
.9681	.9258	4758.
.9452	.9635	8022.
.9197	.9751	11059.
.8980	1.0000	15357.
.8613	1.0145	21047.
.7881	1.0393	29196.
.7257	1.0716	40472.
.6554	1.1046	54272.
.5793	1.1281	72855.
.5193	1.2379	107717.
.4207	1.2726	149033.
.3648	1.3077	208194.
.2743	1.3426	284284.
.2113	1.3767	398707.
.1587	1.4114	549100.
.1151	1.4441	755541.
.0808	1.4813	1045423.
.0589	1.5158	1444321.
.0359	1.5511	2101265.
.0229	1.5848	3245595.
.0139	1.6195	4830110.
.0082	1.6543	6299736.
.0047	1.6819	7732454.
.0028	1.7215	11188034.
.0013	1.7582	18173167.

NO.	THICK	SIGMA	DF	CVDF	FACT1	FACT2
5	15.00	786.66	1.3559	.1401	.2256	-1.3232

RELIA	DF	N
.9987	.7959	297.
.9974	.8295	765.

Figure 3.2b (cont'd)

NO.	THICK	SIGMA	DF	CVDF	FACT1	FACT2
6	16.00	722.65	1.8722	.1472	.2250	-1.2210

DELTA	DF	N
.5487	.8560	1882.
.9978	.8978	3761.
.9953	.9360	4662.
.9918	.9813	9456.
.9841	1.0213	18772.
.9777	1.0451	21984.
.9681	1.1(84	37564.
.9453	1.1459	85519.
.9197	1.1873	87054.
.8949	1.2287	98815.
.8411	1.2711	184567.
.7881	1.3115	218519.
.7257	1.3560	311764.
.6556	1.4084	465784.
.5791	1.4538	694726.
.5103	1.4977	1111301.
.4307	1.5486	1489978.
.3484	1.6033	2195845.
.2743	1.6518	3236394.
.2119	1.6938	4766594.
.1567	1.7363	7127862.
.1151	1.7757	10758894.
.0808	1.8171	15284941.
.0549	1.8585	22466792.
.0359	1.8955	33107218.
.0228	1.9311	48787449.
.0139	1.9627	71481284.
.0082	1.9781	115911844.
.0047	2.0134	158(4(834.
.0024	2.0373	2299617718.
.0013	2.0568	3378628228.

M	DELTA	LP	N
11.00	.4894	.8401	1939.
12.00	.8918	.8801	1886.
13.00	.9863	.8801	1939.
14.00	.9991	.8801	1939.
15.00	.9997	.8801	1901.
16.00	.9993	.8801	1939.
17.00	.8734	.5107	5701.
12.00	.7627	.9105	5390.

Figure 3.2b (cont'd)

13.00	.9123	.9100	5000.
14.00	.9707	.9100	5000.
15.00	.9905	.9100	5000.
16.00	.9965	.9100	5000.
11.00	.2599	.9981	10000.
12.00	.5767	.9980	10000.
13.00	.8117	.9980	10000.
14.00	.9278	.9980	10000.
15.00	.9749	.9980	10000.
16.00	.9918	.9980	10000.
11.00	.1089	1.0581	20000.
12.00	.3706	1.0581	20000.
13.00	.6008	1.0581	20000.
14.00	.8502	1.0581	20000.
15.00	.9415	1.0581	20000.
16.00	.9785	1.0581	20000.
11.00	.0224	1.1559	50000.
12.00	.1535	1.1559	50000.
13.00	.4171	1.1559	50000.
14.00	.6818	1.1559	50000.
15.00	.8537	1.1559	50000.
16.00	.9398	1.1559	50000.
11.00	.0048	1.2300	100000.
12.00	.0613	1.2300	100000.
13.00	.2489	1.2300	100000.
14.00	.5183	1.2300	100000.
15.00	.7463	1.2300	100000.
16.00	.8837	1.2300	100000.

Figure 3.2b (cont'd)

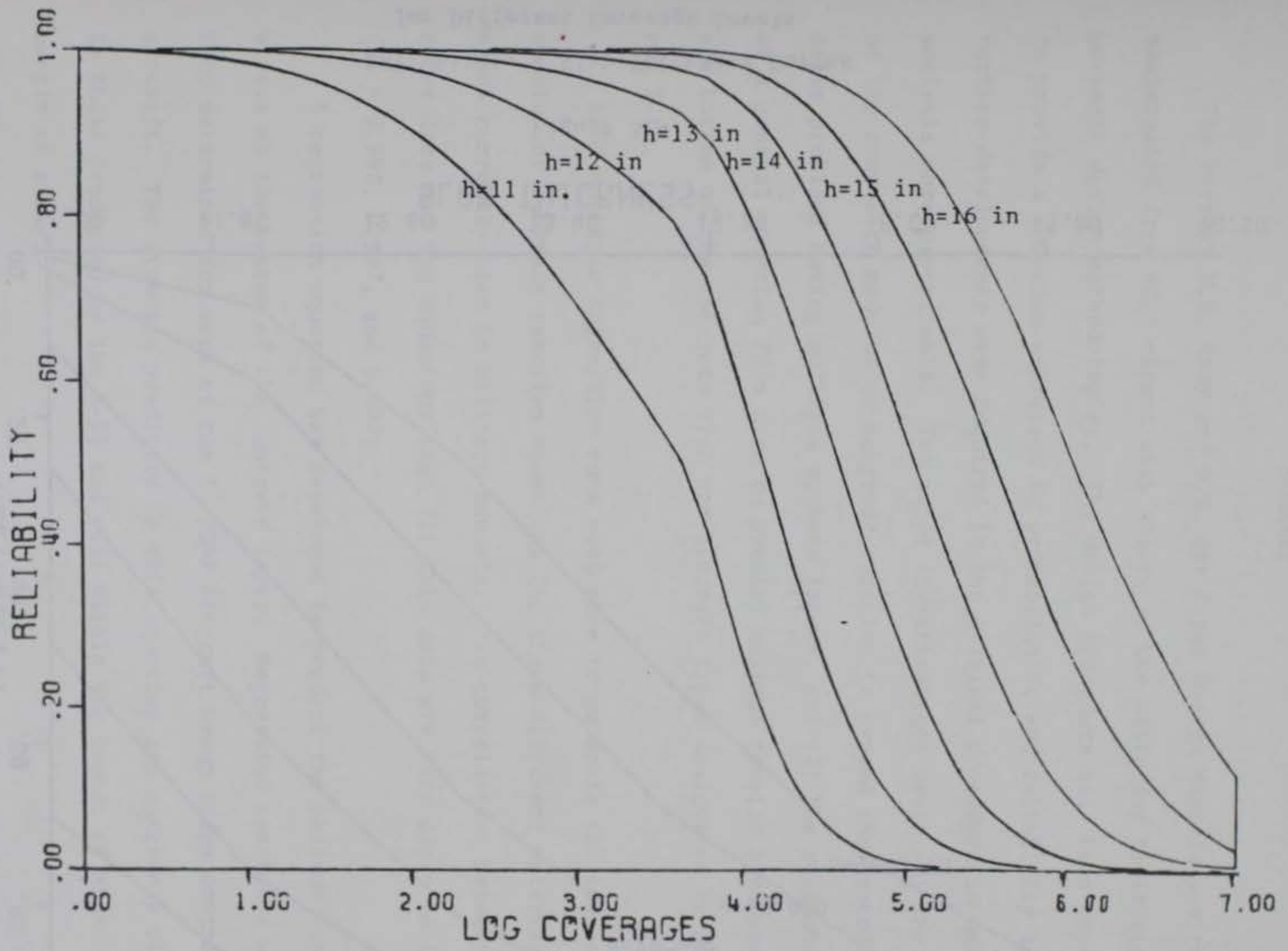

Figure 3.3

Reliability-Log Coverages Curves for
Different Slab Thicknesses

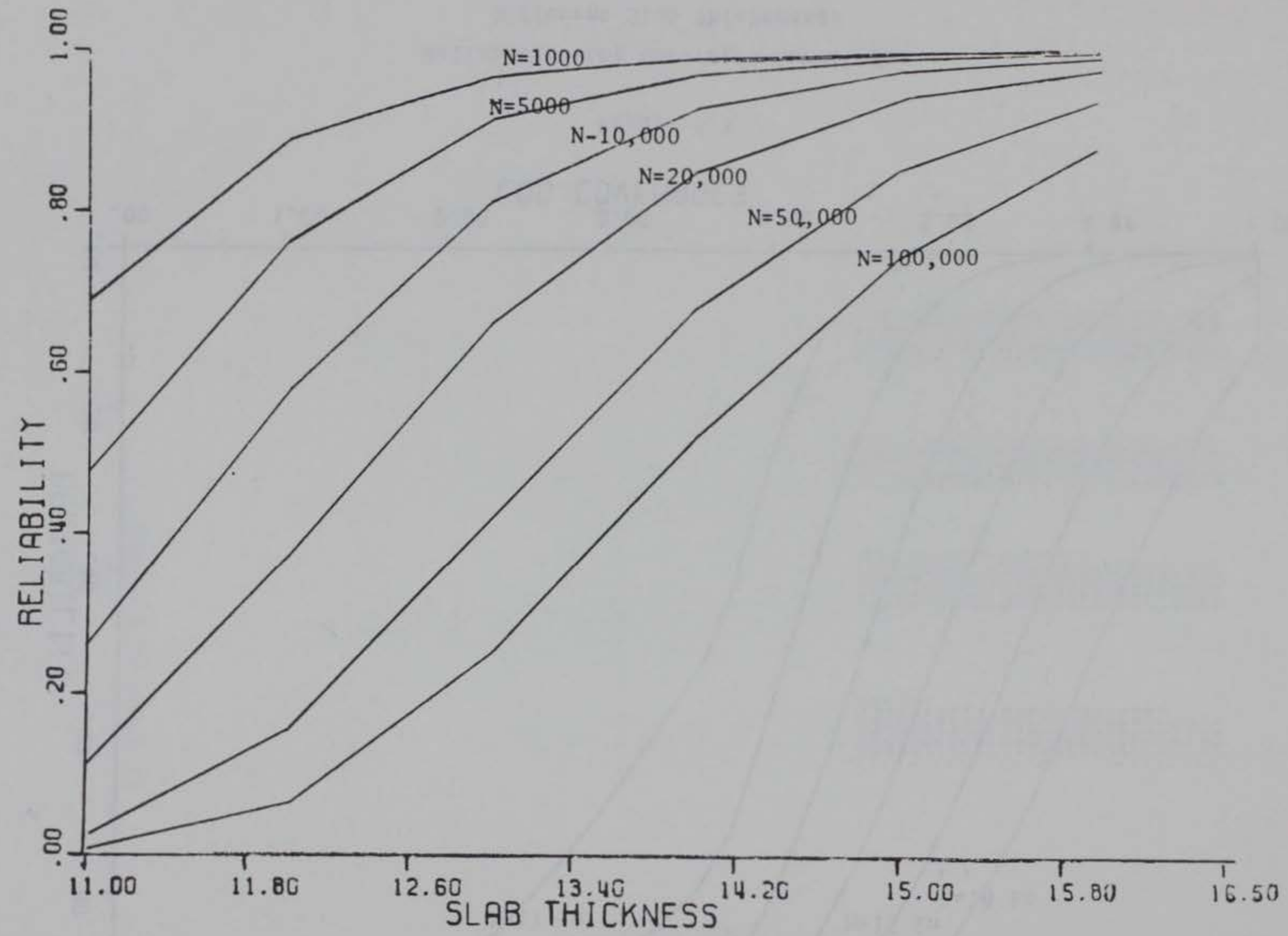

Figure 3.4

Reliability - Slab Thickness Curves
for Different Coverage Levels

Chapter 4

SUMMARY

The current U.S. Army and U.S. Air Force Design Manuals use the Westergaard free edge stress slab theory as the basis for their rigid pavement design methodologies. The design procedure has been expanded to provide a solution expressed in probabilistic and reliability terms. Further developments were required in the original procedure to make the analysis more practicable. Two major investigations were: (1) The use of the composite modulus of subgrade reaction to expand the procedure to solve problems having multiple subbase layers, and (2) The evaluation of a general equation form used to predict maximum tensile stresses at the bottom of the concrete slab for aircraft types designated by USAF AGI 1-15.

SPSS Multiple Regression runs were made to generate the composite modulus of subgrade reaction equations for three different material types currently used in military manuals. The correlations between the curve data and the equations that fit this data are very close to 1 (R^2 = 0.998, 0.997, and 0.997).

A regression equation was developed to predict the maximum tensile stress at the bottom of the concrete layer. Regression constants were then determined for each of the 1 USAF Aircraft Group Index controlling aircraft. The stresses predicted by this equation are extremely close to those predicted by the H-51 and well within the bounds of an acceptable margin of error.

The derivations above have been included in a computer program for the probabilistic/reliability analysis of rigid pavements. The approximate

closed-form probabilistic approach (Taylor Series) has been utilized. The computer program can be used:

(1) in the analysis in probabilistic/reliability terms for a given pavement system and loading aircraft. The means and coefficients of variation of the design parameters serve as the input. The computer program produces values of the number of coverages and their reliability levels.

(2) in the design of a rigid pavement for a given reliability. Here, the number of coverages is also an input so that the slab thickness may be determined for the given reliability and coverage level.

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APPENDIX I

SPSS Outputs for the Composite Modulus Equations

FILE NONAME (CREATION DATE = 09/27/87)

* MULTIPLE REGRESSION * * * * * * * * * * * * * VARIABLE LIST 1
REGRESSION LIST 1

DEPENDENT VARIABLE.. AR1

VARIABLE(S) ENTERED ON STEP NUMBER 5.. M2

MULTIPLE R	.99919	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
R SQUARE	.99838	REGRESSION	5.	7.27740	1.45548	9107.95893
ADJUSTED R SQUARE	.99827	RESIDUAL	74.	.01183	.00016	
STANDARD ERROR	.01264					

VARIABLES IN THE EQUATION

VARIABLE	B	BETA	STD ERROR B	F
AK2	.2319712+001	2.88289	.08753	2181.259
M	.8901252-001	1.35978	.00288	1325.502
AK4	-.2001888-001	-1.00888	.00078	1276.039
AK3	-.2089522+000	-1.89152	.01198	609.828
M2	-.8825198-001	-.17120	.00007	38.998
(CONSTANT)	-.1251182+001			

VARIABLES NOT IN THE EQUATION

VARIABLE	BETA IN	PARTIAL	TOLERANCE	F

MAXIMUM STEP REACHED

STATISTICS WHICH CANNOT BE COMPUTED ARE PRINTED AS ALL NINES.

Figure I-1 - SPSS Multiple Regression Output for the Composite Modulus of Subgrade Reaction of Well-Graded Crushed Materials

```
COMPOSITE SUBGRADE MODULUS - II                                                    09/29/82        PAGE    7
FILE    NONAME    (CREATION DATE = 09/29/82)
* * * * * * * * * * * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * *   VARIABLE LIST  1
                                                                                                                      REGRESSION LIST  1
DEPENDENT VARIABLE..     AK1
VARIABLE(S) ENTERED ON STEP NUMBER  5..    H2

MULTIPLE R              .99876
R SQUARE                .99752
ADJUSTED R SQUARE       .99736
STANDARD ERROR          .01553

ANALYSIS OF VARIANCE      DF      SUM OF SQUARES        MEAN SQUARE          F
REGRESSION                5.            7.18836            1.43687      5961.43448
RESIDUAL                 74.             .01784             .00024

---------------- VARIABLES IN THE EQUATION ----------------

VARIABLE            B            BETA      STD ERROR B        F

AK2          .2243807+001     2.95651        .05837     1503.658
H            .8554773-001     1.71826        .00300      811.656
AK4         -.3050177-001    -1.06697        .00096     1008.221
AK7         -.3013741+000    -1.53317        .01467      422.118
H2          -.2574619-003     -.10021        .00009        7.858
(CONSTANT)  -.1296084+001

MAXIMUM STEP REACHED

STATISTICS WHICH CANNOT BE COMPUTED ARE PRINTED AS ALL NINES.

------------- VARIABLES NOT IN THE EQUATION -------------

VARIABLE       BETA IN      PARTIAL    TOLERANCE          F
```

Figure I-2 - SPSS Multiple Regression Output for the Composite Modulus Subgrade Reaction of Natural Sand and Gravel (PI<8)

FILE NONAME (CREATION DATE = 10/06/83)

* MULTIPLE REGRESSION * * * * * * * * * * * * VARIABLE LIST 1
REGRESSION LIST 1

DEPENDENT VARIABLE.. A1

VARIABLE(S) ENTERED ON STEP NUMBER 7.. X3

		ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
MULTIPLE R	.99796	REGRESSION	7.	2.45040	.35009	2271.42487
R SQUARE	.99593	RESIDUAL	65.	.01001	.00015	
ADJUSTED R SQUARE	.99560					
STANDARD ERROR	.01241					

---------- VARIABLES IN THE EQUATION ----------

VARIABLE	B	BETA	STD ERROR B	F
X1	.1000101E+001	1.11179	.03000	1111.451
X9	-.1195210E-008	-.01899	.00000	.251
X7	-.1191058E-002	-.08500	.00243	.208
X8	-.8845284E-007	-.11784	.00000	1.644
X4	-.1974279E-013	-.20815	.00000	35.416
X5	.7185857E-006	1.02823	.00000	83.447
X3	.2964268E-001	.02370	.00417	23.208
(CONSTANT)	-.1209141E-001			

---------- VARIABLES NOT IN THE EQUATION ----------

VARIABLE	BETA IN	PARTIAL	TOLERANCE	F
X2	-.81477	-.17849	.00075	2.106
X6	-.31131	-.62178	.01600	40.232

* *

VARIABLE(S) ENTERED ON STEP NUMBER 8.. X6

		ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE	F
MULTIPLE R	.99835	REGRESSION	8.	2.45886	.30681	3198.92684
R SQUARE	.99670	RESIDUAL	64.	.00815	.00010	
ADJUSTED R SQUARE	.99719					
STANDARD ERROR	.00980					

---------- VARIABLES IN THE EQUATION ----------

VARIABLE	B	BETA	STD ERROR B	F
X1	.1028130E+001	1.14309	.02819	1820.537
X9	-.2855439E-008	-.01914	.00000	2.388
X7	-.6809023E-002	-.13453	.00218	9.751
X8	-.8601459E-007	-.13571	.00000	3.203
X4	-.1937293E-013	-.20424	.00000	120.615
X5	.7258789E-006	1.03810	.00000	77.872
X3	.5887810E-001	.04700	.00428	76.278
X6	-.8873952E-001	-.31331	.00017	40.232
(CONSTANT)	-.1578687E-000			

---------- VARIABLES NOT IN THE EQUATION ----------

VARIABLE	BETA IN	PARTIAL	TOLERANCE	F
X2	-.03291	-.28610	.00075	5.616

F-LEVEL OR TOLERANCE-LEVEL INSUFFICIENT FOR FURTHER COMPUTATION

STATISTICS WHICH CANNOT BE COMPUTED ARE PRINTED AS ALL NINES.

Figure I-3 - SPSS Multiple Regression Output for the Composite Modulus of Subgrade Reaction of Stabilized Layers

APPENDIX II

Regression Constants for Free Edge Stress Equations

(AGI 1 to AGI 13)

Table II.1 - Summary of Stress and Weighting Factor Computations for AGI-1/C-123

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\xi_{E,MR}$	
6.0	26.47	1212.38	1212.56	1.0000	0.0351	2.0681	-0.3746	+0.01
7.0	29.72	968.94	968.38	1.0000	0.0300	2.1916	-0.3464	-0.06
8.0	32.85	791.94	792.98	1.0000	0.0265	2.2840	-0.3258	+0.13
9.0	35.88	663.21	662.59	1.0000	0.0238	2.3636	-0.3084	-0.09
10.0	38.83	563.20	562.93	1.0000	0.0218	2.4258	-0.2950	-0.05
11.0	41.71	484.50	484.90	1.0000	0.0202	2.4765	-0.2842	+0.08
12.0	44.52	422.35	422.55	1.0000	0.0188	2.5230	-0.2744	+0.05
13.0	47.27	372.17	371.89	1.0000	0,0177	2.5642	-0.2658	-0.07
14.0	49.98	330.12	330.18	1.0000	0.0167	2.5979	-0.2588	+0.02

a_o = - 70026 $\quad R = 1.000$

a_1 = 34374 $\quad R^2 = 1.000$

a_2 = 44166

k = 150 pci

Table II.2 - Summary of Stress and Weighting Factor Computations for AGI-2/F-4

h (in.)	(in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\xi_{E,MR}$	
6.0	26.47	1676.00	1673.95	1.0000	0.0221	2.4165	-0.2970	-0.12
7.0	29.72	1311.20	1312.76	1.0000	0.0179	2.5565	-0.2674	+0.12
8.0	32.85	1056.24	1057.95	1.0000	0.0151	2.6611	-0.2458	+0.16
9.0	35.88	872.14	871.61	1.0000	0.0131	2.7450	-0.2288	-0.06
10.0	38.83	731.12	731.24	1.0000	0.0117	2.8090	-0.2160	+0.02
11.0	41.71	623.84	622.81	1.0000	0.0105	2.8646	-0.2050	-0.17
12.0	44.52	537.68	537.22	1.0000	0.0096	2.9084	-0.1964	-0.09
13.0	47.27	468.09	468.45	1.0000	0.0089	2.9453	-0.1892	+0.08
14.0	49.98	412.12	412.37	1.0000	0.0084	2.9784	-0.1828	+0.06

a_o = 146

a_1 = 22439

a_2 = -354549

k = 150 pci

R = 1.000

R^2 = 1.000

Table II.3 - Summary of Stress and Weighting Factor Computations for AGI-3/F-111

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
6.0	26.47	2122.98	2121.58	1.0000	0.0318	2.1465	-0.3566	-0.07
7.0	29.72	1685.46	1687.93	1.0000	0.0276	2.2560	-0.3320	+0.15
8.0	32.85	1378.84	1378.34	1.0000	0.0244	2.3461	-0.3122	-0.04
9.0	35.88	1150.03	1149.21	1.0000	0.0221	2.4165	-0.2970	-0.07
10.0	38.83	974.10	974.65	1.0000	0.0203	2.4728	-0.2850	+0.06
11.0	41.71	838.07	838.36	1.0000	0.0188	2.5230	-0.2744	+0.03
12.0	44.52	730.33	729.69	1.0000	0.0176	2.5670	-0.2652	-0.09
13.0	47.27	641.42	641.55	1.0000	0.0166	2.6027	-0.2578	+0.02
14.0	49.98	568.94	569.08	1.0000	0.0158	2.6358	-0.2510	+0.03

a_o = -115728 $\quad$ R = 1.000

a_1 = 57671 $\quad$ R^2 = 1.000

a_2 = 84016

k = 150 pci

Table II.4 - Summary of Stress and Weighting Factor Computations for AGI-4/C-130

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\xi_{E,MR}$	
7.0	26.15	762.33	763.86	1.0000	0.0463	1.8344	-0.4304	+0.20
8.0	28.91	641.88	639.96	1.0000	0.0537	1.7028	-0.4634	-0.30
9.0	31.58	549.96	549.82	1.0000	0.0578	1.6353	-0.4808	-0.03
10.0	34.17	481.24	480.68	1.0000	0.0590	1.6162	-0.4858	-0.12
11.0	36.71	425.55	425.94	1.0000	0.0590	1.6162	-0.4858	+0.09
12.0	39.18	380.20	381.17	1.0000	0.0582	1.6292	-0.4824	+0.26
13.0	41.61	343.93	344.03	1.0000	0.0565	1.6569	-0.4752	+0.03
14.0	43.98	312.90	312.56	1.0000	0.0547	1.6864	-0.4676	-0.11
15.0	46.32	285.88	285.71	1.0000	0.0530	1.7145	-0.4604	-0.06

a_o = -332671 $\quad$ R = 1.000

a_1 = 94339 $\quad$ R^2 = 1.000

a_2 = 1626321

k = 250 pci

Table II.5 - Summary of Stress and Weighting Factor Computations for AGI-5/C-9

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
7.0	26.15	1240.10	1238.26	1.0000	0.0496	1.7748	-0.4452	-0.15
8.0	28.91	1033.77	1033.08	1.0000	0.0422	1.9149	-0.4108	-0.07
9.0	31.58	873.87	875.66	1.0000	0.0372	2.0209	-0.3856	+0.21
10.0	34.17	749.80	752.37	1.0000	0.0333	2.1098	-0.3650	+0.34
12.0	39.18	575.56	574.63	1.0000	0.0275	2.2569	-0.3318	-0.16
13.0	41.61	509.95	509.23	1.0000	0.0256	2.3113	-0.3198	-0.14
14.0	43.98	455.69	454.67	1.0000	0.0239	2.3608	-0.3090	-0.22
16.0	48.62	369.76	369.77	1.0000	0.0214	2.4380	-0.2924	+0.004
17.0	50.88	335.62	336.26	1.0000	0.0204	2.4690	-0.2858	+0.19

a_o = -127582 $\quad R = 1.000$

a_1 = 56841 $\quad R^2 = 1.000$

a_2 = 71556

k = 250pci

Table II.6 - Summary of Stress and Weighting Factor Computations for AGI-6/T-43

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\varepsilon_{E,MR}$	
7.0	26.15	1247.20	1247.41	1.0000	0.0476	1.8109	-0.4362	+0.02
8.0	28.91	1041.67	1039.90	1.0000	0.0424	1.9108	-0.4118	-0.17
9.0	31.58	880.58	882.23	1.0000	0.0388	1.9861	-0.3938	+0.19
10.0	34.17	759.96	759.30	1.0000	0.0354	2.0604	-0.3764	-0.09
12.0	39.18	580.79	582.44	1.0000	0.0309	2.1677	-0.3518	+0.28
13.0	41.61	518.12	517.35	1.0000	0.0288	2.2219	-0.3396	-0.15
14.0	43.98	463.69	462.97	1.0000	0.0273	2.2632	-0.3304	-0.16
16.0	48.62	378.62	378.18	1.0000	0.0247	2.3351	-0.3146	-0.12
17.0	50.88	344.08	344.62	1.0000	0.0238	2.3627	-0.3086	+0.16

a_o = -183723 $\quad R = 1.000$

a_1 = 69928 $\quad R^2 = 1.000$

a_2 = 434391

k = 250 pci

Table II.7 - Summary of Stress and Weighting Factor Computations for AGI-7/B-727

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
8.0	26.58	1415.27	1414.53	1.0000	0.0517	1.7366	-0.4548	-0.05
9.0	29.03	1207.37	1209.18	1.0000	0.0480	1.8028	-0.4382	+0.15
11.0	33.75	920.14	919.40	1.0000	0.0415	1.9282	-0.4076	-0.08
13.0	38.25	728.09	726.88	1.0000	0.0368	2.0286	-0.3838	-0.17
15.0	42.58	589.67	591.66	1.0000	0.0336	2.1037	-0.3664	+0.34
17.0	46.77	493.45	492.57	1.0000	0.0304	2.1818	-0.3486	-0.18
19.0	50.84	417.62	417.52	1.0000	0.0282	2.2389	-0.3358	-0.02
20.0	52.84	386.65	386.62	1.0000	0.0272	2.2659	-0.3298	-0.01
21.0	54.81	359.03	359.17	1.0000	0.0263	2.2904	-0.3244	+0.04

a_o = -348726 $\quad$ R = 1.000

a_1 = 121875 $\quad$ R^2 = 1.000

a_2 = 1049552

k = 350 pci

Table II.8 - Summary of Stress and Weighting Factor Computations for AGI-8/E-3

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\xi_{E,MR}$	
8.0	26.58	1006.29	1008.38	1.0000	0.0525	1.7224	-0.4584	+0.21
9.0	29.03	869.08	868.61	1.0000	0.0668	1.5001	-0.5168	-0.05
11.0	33.75	687.93	685.33	1.0000	0.0789	1.3393	-0.5618	-0.38
13.0	38.25	565.64	565.66	1.0000	0.0807	1.3179	-0.5680	+0.004
15.0	42.58	478.17	479.41	1.0000	0.0778	1.3526	-0.5580	+0.26
17.0	46.77	413.21	413.74	1.0000	0.0731	1.4132	-0.5408	+0.13
19.0	50.84	362.05	361.93	1.0000	0.0682	1.4796	-0.5224	-0.03
20.0	52.84	340.47	339.97	1.0000	0.0658	1.5141	-0.5130	-0.15
21.0	54.81	320.04	320.08	1.0000	0.0639	1.5423	-0.5054	+0.01

a_o = -823985

a_1 = 221392

a_2 = 4314471

k = 350 pci

R = 1.000

R^2 = 1.000

Table II.9 - Summary of Stress and Weighting Factor Computations for AGI-9/C-141

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
8.0	26.58	1027.58	1029.53	1.0000	0.0783	1.3463	-0.5598	+0.19
9.0	29.03	902.06	900.45	1.0000	0.0866	1.2479	-0.5886	-0.18
11.0	33.75	722.64	721.84	1.0000	0.0897	1.2133	-0.5990	-0.11
13.0	38.25	598.93	599.54	1.0000	0.0854	1.2614	-0.5846	+0.10
15.0	42.58	509.12	509.24	1.0000	0.0789	1.3393	-0.5618	+0.02
17.0	46.77	439.47	439.61	1.0000	0.0726	1.4197	-0.5390	+0.03
19.0	50.84	384.38	384.35	1.0000	0.0669	1.4987	-0.5172	-0.01
20.0	52.84	360.99	360.87	1.0000	0.0643	1.5364	-0.5070	-0.03
21.0	54.81	339.60	339.58	1.0000	0.0620	1.5708	-0.4978	-0.005

a_o = -802693 $\quad$ R = 1.000

a_1 = 220139 $\quad$ R^2 = 1.000

a_2 = 3893752

k = 350 pci

Table II.10 Summary of Stress and Weighting Factor Computations for AGI-10/C-5A

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\xi_{E,MR}$	
8.0	26.58	838.15	842.22					+0.49
9.0	29.03	719.03	715.91					-0.43
11.0	33.75	556.58	554.84	Weighting factors not given due to the non-symmetric nature of the C-5A gear configuration				-0.31
13.0	38.25	454.00	453.11					-0.20
15.0	42.58	379.20	381.48					+0.60
17.0	46.77	327.63	327.78					+0.05
19.0	50.84	286.07	285.88					-0.07
20.0	52.84	268.25	268.23					-0.01
21.0	54.81	252.59	252.29					-0.12

a_o = -653058

a_1 = 174658

a_2 = 3563166

k = 350 pci

Table II.11 - Summary of Stress and Weighting Factor Computations for AGI-11/KC-10

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\xi_{E,MR}$	
8.0	26.58	1089.90	1094.80	1.0000	0.0344	2.0846	-0.3708	+0.45
9.0	29.03	935.34	930.83	1.0000	0.0517	1.7374	-0.4546	-0.48
11.0	33.75	726.64	725.47	1.0000	0.0727	1.4182	-0.5394	-0.16
13.0	38.25	596.92	596.81	1.0000	0.0794	1.3331	-0.5636	-0.02
15.0	42.58	505.14	505.94	1.0000	0.0794	1.3338	-0.5634	+0.16
17.0	46.77	436.60	437.35	1.0000	0.0763	1.3722	-0.5524	+0.17
19.0	50.84	383.50	383.43	1.0000	0.0720	1.4275	-0.5368	-0.02
20.0	52.84	360.37	360.58	1.0000	0.0701	1.4542	-0.5294	+0.06
21.0	54.81	340.47	339.88	1.0000	0.0678	1.4862	-0.5206	-0.17

a_o = -968715

a_1 = 254788

a_2 = 5396732

k = 350 pci

R = 1.000

R^2 = 1.000

Table II.12 - Summary of Stress and Weighting Factor Computations for AGI-12/E-4

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_{\alpha}$	$W_{EL}=W_{k}$	W_{h}	$\xi_{E,MR}$	
9.0	26.55	833.52	839.32	1.0000	0.0377	2.0090	-0.3884	+0.70
11.0	30.87	650.71	646.26	1.0000	0.0683	1.4789	-0.5226	-0.68
13.0	34.99	536.25	531.93	1.0000	0.0823	1.2987	-0.5736	-0.81
15.0	38.95	452.55	453.09	1.0000	0.0875	1.2379	-0.5916	+0.12
17.0	42.78	392.51	394.00	1.0000	0.0866	1.2479	-0.5886	+0.38
19.0	46.51	345.76	347.59	1.0000	0.0834	1.2850	-0.5776	+0.53
21.0	50.13	308.94	309.80	1.0000	0.0788	1.3407	-0.5614	+0.28
23.0	53.67	278.86	278.49	1.0000	0.0739	1.4026	-0.5438	-0.13
24.0	55.41	265.82	264.74	1.0000	0.0715	1.4347	-0.5348	-0.41

a_o = -1018918 $\quad$ R = 1.000

a_1 = 266253 $\quad$ R^2 = 1.000

a_2 = 5077764

k = 500 pci

Table II.13 - Summary of Stress and Weighting Factor Computations for AGI-13/B-52

h (in.)	ℓ (in.)	σ - (psi)		Weighing Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
9.0	26.55	1537.84	1542.53	1.0000	0.0476	1.8109	-0.4362	+0.30
11.0	30.87	1185.48	1182.31	1.0000	0.0522	1.7279	-0.4570	-0.27
13.0	34.99	953.73	949.83	1.0000	0.0522	1.7279	-0.4570	-0.41
15.0	38.95	786.10	786.64	1.0000	0.0510	1.7493	-0.4516	+0.07
17.0	42.78	665.87	665.89	1.0000	0.0485	1.7940	-0.4404	+0.003
19.0	46.51	570.58	573.27	1.0000	0.0464	1.8328	-0.4308	+0.47
21.0	50.13	499.64	500.01	1.0000	0.0436	1.8860	-0.4178	+0.07
23.0	53.67	441.16	440.95	1.0000	0.0413	1.9332	-0.4064	-0.05
25.0	57.14	393.27	392.48	1.0000	0.0392	1.9777	-0.3958	-0.20

a_o = -917727

a_1 = 269062

a_2 = 4258863

k = 500 pci

R = 1.000

R^2 = 1.000

Table II - 14 - Summary of Stress and Weighting Factor Computations

For AGI - 14/OV-1

h (in.)	ℓ (in.)	σ - (psi)		Weighting Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
6.0	26.47	561.83	563.00	1.0000	0.0147	2.6758	-0.2420	+0.21
7.0	29.72	437.56	436.77	1.0000	0.0130	2.7510	-0.2276	-0.18
8.0	32.85	350.14	351.02	1.0000	0.0117	2.8060	-0.2166	+0.25
9.0	35.88	287.58	286.81	1.0000	0.0108	2.8514	-0.2076	-0.27
10.0	38.83	239.66	239.94	1.0000	0.0101	2.8839	-0.2012	+0.12
11.0	41.71	203.41	204.00	1.0000	0.0095	2.9135	-0.1954	+0.29
12.0	44.52	175.35	175.76	1.0000	0.0090	2.9412	-0.1900	+0.24
13.0	47.27	153.13	153.13	1.0000	0.0086	2.9670	-0.1850	+0.02
14.0	49.98	135.17	134.78	1.0000	0.0081	2.9908	-0.1804	-0.29

a_o = -9544. R = 0.999

a_1 = 9268. R^2 = 1.000

a_2 = 14558.

k = 150 psi

Table II - 15 - Summary of Stress and Weighting Factor Computations

For AGI - 15/C-54

h (in.)	ℓ (in.)	σ - (psi)		Weighting Factors				Error as % of H-51
		H-51	EQ	$W_{MR}=W_a$	$W_{EL}=W_k$	W_h	$\varepsilon_{E,MR}$	
7.0	26.15	635.12	635.23	1.0000	0.0301	2.1880	-0.3472	+0.02
8.0	28.91	520.93	520.87	1.0000	0.0283	2.2362	-0.3364	-0.01
9.0	31.58	436.59	436.38	1.0000	0.0266	2.2813	-0.3264	-0.05
10.0	34.17	372.55	371.88	1.0000	0.0252	2.3232	-0.3172	-0.18
11.0	36.71	320.77	321.48	1.0000	0.0241	2.3553	-0.3102	+0.22
12.0	39.18	280.57	281.12	1.0000	0.0230	2.3886	-0.3030	+0.20
13.0	41.61	248.55	248.31	1.0000	0.0218	2.4239	-0.2954	-0.10
14.0	43.98	221.79	221.15	1.0000	0.0209	2.4549	-0.2888	-0.29
15.0	46.32	198.15	198.46	1.0000	0.0202	2.4756	-0.2844	+0.16

a_o = 76186. R = 1.000

a_1 = 30235. R^2 = 1.000

a_2 = 225665.

k = 250 psi

Card 1 (I2, I3, 4F10.4)

1-2	IG	- USAF Aircraft Group Index (=0 for user defined aircraft)
3-5	NOLBP	- Number of base (subbase) layers (excludes subgrade, maximum of 2)
6-15	XK	- Modulus of subgrade reaction, pci
16-25	CVK	- Coefficient of variation of XK
26-35	ETEMP	- Modulus of elasticity of the pavement, psi
36-45	XMUTEM	- Poisson's ratio of the pavement

Card 2 (I2, 4F10.4) Repeat NOLBP times *See notes

1-2	MATOPT	- Material of the base (subbase) layer = 1 for well-graded crushed materials = 2 for natural sands and gravels (PI 8) = 3 for stabilized materials
3-12	HB	- Thickness of base (subbase) layer, in.
13-22	CVHB	- Coefficient of variation of HB
23-32	EB	- Modulus of elasticity of base (subbase) layer, psi (only required when MATOPT = 3)
33-42	CVEB	- Coefficient of variation of EB (only required when MATOPT = 3)

Card 3 (10A6) Required if IG = 0 *See notes

1-60		- Comments (Only input if aircraft is user defined)

Card 4 (4F10.0) Required if IG = 0

1-10	G	- Load on gear, lb.
11-20	P	- Tire inflation pressure, psi
21-30	A	- Contact area of one tire, sq. in.
31-40	XNOG	- Number of γ (GAMMA)

Card 5 (6F10.0) Required if IG = 0

1-10 XLA -
11-20 XLB
21-30 XLC
31-40 XLD

Gear spacing parameters a, b, c, d, in.

41-50 XNOD - Number of Δ's (DELTA)

51-60 NXOSG -Number of (σ,G)'s (ASIG, AG)

Card 6 (5F10.0) Required if IG = 0

1-10 XNA
11-20 XNB
21-30 XNC
31-40 XND

Gear spacing parameters n_a, n_b, n_c, n_d

41-50 PHIE - Ratio of length to width of the tire print (optional)

Card 7 (5F10.0) Required if IG = 0

1-10 B - Number of tire approximation points

11-20 XOP3 - Print option. Enter 1.0 to print subtotals for each wheel

21-30 BIGX - Location of stress calculation point relative to gear, in.

31-40 BIGY - Location of stress calculation point relative to gear, in.

41-50 XOP6 - Coordinate option. Enter 1.0 when using optional H-51 input method

Card 8 (10F6.0) Required if IG=0

1-6 GAMMA(1) - Position of gear (angular), deg.

7-12 GAMMA(2)

. .
. .
. .

GAMMA(XNOG)

Card 9 (10F6.0) Required if IG = 0

1-6 DELTA(1)

7-12 DELTA(2)

. .
. .
. .

DELTA(XNOD)

Card 10 (8F10.0) Repeat XNOSG times, Required if IG = 0

1-10 ASIG(I)

Input Pairs

11-20 AG(I)

Card 11 (3F10.2)

1-10 HMIN - Minimum slab thickness to be evaluated, in.

11-20 HMAX - Maximum slab thickness to be evaluated, in.

21-30 HINCR - Increment by which the slab thickness is increased from HMIN to HMAX, in.

Card 12 (7F10.0)

1-10 AMR - Modulus of rupture of concrete, psi

11-20 CUMR - Coefficient of variation of AMR

21-30 ALPHA - Load transfer coefficient

31-40 CVALP - Coefficient of variation of ALPHA

41-50 CVE - Coefficient of variation of ETEMP

51-60 CVH - Coefficient of variation of slab thickness

61-70 ROEMR - Correlation coefficient between AMR and ETEMP

Card 13 (3IZ)

1-2	NON	-	Number of N's (coverage levels) to be input
2-3	JOPPLI	-	Plot option. Enter 1 to plot Reliability vs log coverages
3-4	JOPPLZ	-	Plot option. Enter 1 to plot Reliability vs. Slab thickness

Card 14 (10F8.0)

1-8	BN(1)	-	Number of coverages
9-16	BN(2)		
.	.		
.	.		
.	.		
	BN(NON)		

Notes

1. Card 2 should be omitted if no composite modulus of subgrade reaction is to be calculated.
2. If NOLBP = 2, the data for the layer directly above the subgrade should be input first.
3. In the output, Layer 2 will always be labeled the "BASE", and Layer 3 will always be labeled the "SUBBASE".
4. Cards 4-11 should only be included if the aircraft is user defined (i.e IG=0)
5. The parameters listed on cards 3-10 are defined in the same manner as in the H-51 program. Any additional information concerning these parameters may be found in the H-51 user's manual(4).

APPENDIX IV

PROGRAM LISTING

```
UZAN*NEW(1).WEST(3)
FOR,IS MAIN
C
C                    MS1 - MAIN PROGRAM
C
C     MS1 COMPUTES BENDING STRESS IN CONCRETE
C
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)
      COMMON /BBLK/ BIGX,BIGY,B
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
      COMMON /EBLK/ E,ER(10)
      COMMON /FBLK/ FG(10)
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
      COMMON /HBLK/ H(10)
      COMMON /IBLK/ IC(10),ILM,IPT,IT,ILD,ISG,INDG    ,KBP
      COMMON /NBLK/ NOG,NOH,NPT,NX,NT,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMIST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOB,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10)  ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10)  ,YPZ(4097),YZERO,YZEROP
      COMMON /MS1A1/ KERN(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /UBLK/ UNUM(10,10),UMAX(10)
      COMMON /MPJ/  AO,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
     1              JOPPL1,JOPPL2
      DIMENSION AA(13,3)
      DATA AA / -70626.,146.,-115728.,-332671.,-127562.,-181723.,
     A-348726.,-823985.,-802693.,-653058.,-968715.,-1018918.,-917727.,
     B-9544.,-74186.,34374.,22436.,57621.,94339.,56841.,69928.,121875.,
     C221392.,220139.,174658.,254788.,266253.,269062.,9268.,30235.,
     D44166.,-356549.,84016.,1626321.,71556.,434391.,1049552.,4314471.,
     E3893752.,3563166.,5396732.,5677764.,4258868.,-14558.,225665./
      DATA KPROC/6HMS1/,KNAME/6HMS1A/
   10 CONTINUE
  150 KERN(1) = KPROC
      KERN(2) = KNAME
      KERN(3) = 1
      KERN(4) = 0
      KERN(5) = 0
  200 FORMAT(I2,I3,4F10.4)
      READ(5,200,END=9999)IG,NOLBP,XK,CVK,ETEMP,XMUTEM
      IF(ETEMP.GT.0.)E=ETEMP
      IF(ETEMP.GT.0.)GO TO 400
      E=6000000.
  400 IF(XMUTEM.GT.0.)XMU=XMUTEM
      IF(XMUTEM.GT.0.)GO TO 500
      XMU=0.15
  500 CONTINUE
      IF(NOLBP.EQ.0)GO TO 1000
      CALL KCOMP
 1000 CONTINUE
 1010 IF(IG)2000,2000,2010
 2000 CALL PROGRD
 2010 READ(5,210)HMIN,HMAX,HINCR
  210 FORMAT(3F10.2)
      H(1)=HMIN
      NOH=1
  220 IF(H(NOH).GE.HMAX)GO TO 300
      NOH=NOH+1
      H(NOH)=H(NOH-1)+HINCR
      GO TO 220
  300 CONTINUE
      IF(IG)2100,2100,2110
 2110 AO=AA(IG,1)
      A1=AA(IG,2)
      A2=AA(IG,3)
      GO TO 3000
 2100 DO 100 ILM=1,NOM
      CALL GEOM
      CALL OUTLNE
```

```
      IF(XOP6.GT.0.5)  GO TO  110                                       00000760
      UMAX(ILH)=UNUM(1,1)                                               00000770
      DO 101 IUZ=1,NOD                                                  00000780
      DO 101 IJA=1,NOSG                                                 00000790
      IF(UMAX(ILH).GT.UNUM(IUZ,IJA))  GO TO  101                        00000900
      UMAX(ILH)=UNUM(IUZ,IJA)                                           00000910
  101 CONTINUE                                                          00000820
      GO TO 100                                                         00000930
  110 UMAX(ILH)=PVMTST(1)                                               00000940
  100 CONTINUE                                                          00000850
      A0=0.                                                             00000860
      A1=0.                                                             00000870
      A2=0.                                                             00000880
 3000 CALL RELIAB                                                       00000890
      CALL STORPL                                                       00000900
      GO TO 10                                                          00000910
 9999 STOP                                                              00000920
      END                                                               00000930
C                                                                       00000940
C                                                                       00000950
@FOR,IS RELIAB                                                          00000960
      SUBROUTINE RELIAB                                                 00000970
C                                                                       00000980
C                                                                       00000990
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)              00001000
      COMMON /BBLK/ BIGX,BIGY,B                                         00001010
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)                     00001020
      COMMON /EBLK/ E,ER(100)                                           00001030
      COMMON /FBLK/ FG(10)                                              00001040
      COMMON /GPLK/ G,GAMMA(10),GPRINT(100,10)                          00001050
      COMMON /HBLK/ H(10)                                               00001060
      COMMON /IBLK/ ICA(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP            00001070
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG                          00001080
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE                                00001090
      COMMON /RBLK/ RPD,RZERO                                           00001100
      COMMON /SBLK/ SIGMA(10),S,SR                                      00001110
      COMMON /TBLK/ T,TR                                                00001120
      COMMON /WBLK/ WR,WR2                                              00001130
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,    00001140
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,  00001150
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),     00001160
     3              XZERO,XZEROP                                        00001170
      COMMON /YBLK/ YMN(10),YP(100,10) ,YPZ(4097),YZERO,YZEROP          00001180
      COMMON /H51A1/ KERN(5)                                            00001190
      COMMON /BLK1/ NW,NZ,CV(700)                                       00001200
      COMMON /UBLK/ UNUM(10,10),UMAX(10)                                00001210
      COMMON /MPJ/  AO,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,     00001220
     1              JOPPL1,JOPPL2                                       00001230
      DIMENSION A(3,3),AA(3,4),BB(2,3),DDF(33),PR(33),AN(33),B(3),JC(3),00001240
     &          AL(10),V(3)                                             00001250
      DATA KPROC/3HH51/,KNAME/4HH51A/                                   00001260
      R=1.                                                              00001270
      R2=1.                                                             00001280
      DO 1 I=1,3                                                        00001290
      B(I)=0.                                                           00001300
      DO 1 J=1,3                                                        00001310
      A(I,J)=0.                                                         00001320
    1 CONTINUE                                                          00001330
      READ(5,1000)AMR,CVMR,ALPHA,CVALP,CVE,CVH,ROEMR                    00001340
      READ(5,70)NON,JOPPL1,JOPPL2                                       00001350
   70 FORMAT(3I2)                                                       00001360
 1000 FORMAT(7F10.0)                                                    00001370
      WRITE(6,1001)                                                     00001380
 1001 FORMAT(1H1)                                                       00001390
      IF(IG.EQ.0)GO TO 2000                                             00001400
      DO 2001 ILH=1,NOH                                                 00001410
      AX=E*H(ILH)**3./11.73/XK                                          00001420
      AL(ILH)=AX**0.25                                                  00001430
      UMAX(ILH)=AO+A1*ALOG(AL(ILH))+A2/AL(ILH)                          00001440
 2001 CONTINUE                                                          00001450
      GO TO 210                                                         00001460
 2000 DO 10 ILH=1,NOH                                                   00001470
      AX=E*H(ILH)**3./11.73/XK                                          00001480
      AL(ILH)=AX**0.25                                                  00001490
      X1=ALOG(AL(ILH))                                                  00001500
      X2=1./AL(ILH)                                                     00001510
```

```
152             UMAX(ILH)=0.0006*P*SQRT(AX)*UMAX(ILH)                                   00001520
153             B(1)=B(1)+UMAX(ILH)                                                     00001530
154             A(1,1)=A(1,1)+1.                                                        00001540
155             A(1,2)=A(1,2)+X1                                                        00001550
156             A(1,3)=A(1,3)+X2                                                        00001560
157             B(2)=B(2)+X1*UMAX(ILH)                                                  00001570
158             A(2,2)=A(2,2)+X1*X1                                                     00001580
159             A(2,3)=A(2,3)+X1*X2                                                     00001590
160             B(3)=B(3)+X2*UMAX(ILH)                                                  00001600
161             A(3,3)=A(3,3)+X2*X2                                                     00001610
162             SYY=SYY+UMAX(ILH)*UMAX(ILH)                                             00001620
163          10 CONTINUE                                                              00001630
164             A(2,1)=A(1,2)                                                           00001640
165             A(3,1)=A(1,3)                                                           00001650
166             A(3,2)=A(2,3)                                                           00001660
167             DO 20 I=1,3                                                             00001670
168             AA(I,4)=B(I)                                                            00001680
169             DO 20 J=1,3                                                             00001690
170          20 AA(I,J)=A(I,J)                                                          00001700
171             YAV=B(1)/A(1,1)                                                         00001710
172             V(1)=4.                                                                 00001720
173             CALL GJR(AA,4,3,3,4,$100,JC,V)                                          00001730
174             A0=AA(1,4)                                                              00001740
175             A1=AA(2,4)                                                              00001750
176             A2=AA(3,4)                                                              00001760
177             GO TO 150                                                               00001770
178         100 WRITE(6,1015)                                                           00001780
179             DO 130 I=1,2                                                            00001790
180             BB(I,3)=B(I)                                                            00001800
181             DO 130 J=1,2                                                            00001810
182         130 BB(I,J)=A(I,J)                                                          00001820
183             V(1)=4.                                                                 00001830
184             CALL GJR(BB,3,2,2,3,$999,JC,V)                                          00001840
185             A0=BB(1,3)                                                              00001850
186             A1=BB(2,3)                                                              00001860
187             A2=0.                                                                   00001870
188          150 CONTINUE                                                               00001880
189             Y2EST=A(1,1)*A0*A0+A(2,2)*A1*A1+A(3,3)*A2*A2+2.*A0*A1*A(1,2)+          00001890
190            &      2.*A0*A2*A(1,3)+2.*A1*A2*A(2,3)                                  00001900
191             YAVN=YAV*A(1,1)                                                         00001910
192             Y2AVN=YAVN*YAV                                                          00001920
193             R2=(Y2EST-Y2AVN)/(SYY-Y2AVN)                                            00001930
194             R=SQRT(R2)                                                              00001940
195             IF(IG)200,200,210                                                       00001950
196          210 WRITE(6,211)IG                                                         00001960
197          211 FORMAT(/,5X,'USAF AIRCRAFT GROUP INDEX: ',I2,/)                        00001970
198          200 WRITE(6,1010)  A0,A1,A2,R2,R                                           00001980
199         1010 FORMAT('          A0           A1           A2          R2          R'/ 00001990
200            &      3F12.0,2F10.3)                                                   00002000
201             IF(JOPPL1.LT.1)GO TO 71                                                 00002010
202             CALL PLOTUZ(AN,PR,-1)                                                   00002020
203          71 CONTINUE                                                            00002030
204         2020 DO 50 I=1,NOM                                                      00002040
205             WRITE(6,1011)                                                           00002050
206             SIG=UMAX(I)/H(I)/H(I)                                                   00002060
207             FACT1=(A1-A2/AL(I))/4./UMAX(1)                                          00002070
208             FACT2=-2.+3.*FACT1                                                      00002080
209             DF=AMR/ALPHA/SIG                                                        00002090
210             CVDF=CVMR*CVMR+CVALP*CVALP+FACT1*FACT1*(CVE*CVE+CVK*CVK)+               00002100
211            &      FACT2*FACT2*CVH*CVH-2.*FACT1*RDEMR*CVE*CVMR                      00002110
212             CVDF=SQRT(CVDF)                                                         00002120
213             CVDDF(I)=CVDF                                                           00002130
214             WRITE(6,1012) I,H(I),SIG,DF,CVDF,FACT1,FACT2                            00002140
215         1011 FORMAT(/,1X,'NO.',3X,'THICK',4X,'SIGMA',5X,'DF',5X,'CVDF',5X,          00002150
216            1'FACT1',4X,'FACT2')                                                     00002160
217         1012 FORMAT(I4,2F9.2,4F9.4)                                                 00002170
218             XX=3.                                                                   00002180
219             WRITE(6,1013)                                                           00002190
220             DO 40 J=1,16                                                            00002200
221             ZX=EXP(-XX*XX/2.)/2.5066283                                             00002210
222             TX=1./(1.+0.2316419*XX)                                                 00002220
223             PX=1.-ZX*TX*(0.31938153+TX*(-0.356563782+                               00002230
224            &           TX*(1.781477937+TX*(-1.821255978+TX*1.330274429))))          00002240
225             DDF(J)=DF*(1.-CVDF*XX)                                                  00002250
226             PR(J) =PX                                                               00002260
227             DDF(32-J)=DF*(1.+CVDF*XX)                                               00002270
```

```
      PR(32-J) =1.-PX                                                   00002280
      XX=XX-0.2                                                         00002290
      II=J                                                              00002300
      DO 39 K=1,2                                                       00002310
      IF(XK.LT.250.)  GO TO 31                                          00002320
      IF(XK.LT.350.)  GO TO 33                                          00002330
      IF(XK.LT.450.)  GO TO 35                                          00002340
      IF(DDF(II).GT.0.91)  GO TO 37                                     00002350
      AN(II)=10.**((DDF(II)-0.5401)/0.1)                                00002360
      GO TO 38                                                          00002370
   37 AN(II)=10.**(DDF(II)/0.246)                                       00002380
      GO TO 38                                                          00002390
   35 IF(DDF(II).GT.1.09)  GO TO 36                                     00002400
      AN(II)=10.**((DDF(II)-0.6413)/0.1186)                             00002410
      GO TO 38                                                          00002420
   36 AN(II)=10.**((DDF(II)-0.0565)/0.2767)                             00002430
      GO TO 38                                                          00002440
   33 IF(DDF(II).GT.1.21)  GO TO 34                                     00002450
      AN(II)=10.**((DDF(II)-0.7029)/0.1371)                             00002460
      GO TO 38                                                          00002470
   34 AN(II)=10.**((DDF(II)-0.0445)/0.3151)                             00002480
      GO TO 38                                                          00002490
   31 IF(DDF(II).GT.1.3) GO TO 32                                       00002500
      AN(II)=10.**((DDF(II)-0.7655)/0.1445)                             00002510
      GO TO 38                                                          00002520
   32 AN(II)=10.**((DDF(II)-0.049)/0.3382)                              00002530
   38 II=32-J                                                           00002540
   39 CONTINUE                                                          00002550
   40 CONTINUE                                                          00002560
      WRITE(6,1014)  ((PR(K),DDF(K),AN(K)),K=1,31)                      00002570
      DO 42 J=1,31                                                      00002580
      AN(J)=ALOG10(AN(J))                                               00002590
      IF(AN(J).LT.-0.0001)  AN(J)=-0.0001                               00002600
      IF(AN(J).GT.6.999)  AN(J)=6.999                                   00002610
   42 CONTINUE                                                          00002620
      IF(JOPPL1.LT.1)GO TO 72                                           00002630
      CALL PLOTUZ(AN,PR,1)                                              00002640
   72 CONTINUE                                                          00002650
   50 CONTINUE                                                          00002660
      GO TO 998                                                         00002670
  999 WRITE(6,1015)                                                     00002680
 1013 FORMAT('          RELIA       DF                  N')            00002690
 1014 FORMAT(2F12.4,F15.0)                                              00002700
 1015 FORMAT('    ILL MATRIX !!!!!!!    ')                             00002710
      STOP                                                              00002720
  998 RETURN                                                            00002730
      END                                                               00002740
C                                                                       00002750
C                                                                       00002760
$FOR,IS FINISH                                                          00002770
                    SUBROUTINE FINISH                                   00002780
C                                                                       00002790
C                                                                       00002800
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)              00002810
      COMMON /BBLK/ BIGX,BIGY,B                                         00002820
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)                     00002830
      COMMON /EBLK/ E,ER(100)                                           00002840
      COMMON /FBLK/ FG(10)                                              00002850
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)                          00002860
      COMMON /HBLK/ H(10)                                               00002870
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG,  KBP               00002880
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG                          00002890
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE                                00002900
      COMMON /RBLK/ RPD,RZERO                                           00002910
      COMMON /SBLK/ SIGMA(10),S,SR                                      00002920
      COMMON /TBLK/ T,TR                                                00002930
      COMMON /WBLK/ WR,WR2                                              00002940
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,    00002950
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,  00002960
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),     00002970
     3              XZERO,XZEROP                                        00002980
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP          00002990
      COMMON /H51A1/ KERN(5)                                            00003000
      COMMON /BLK1/ NW,NZ,CV(700)                                       00003010
      WRITE (6,10) (ICM(I),I=1,10)                                      00003020
   10 FORMAT(1H1,10X,10A6)                                              00003030
```

```
      HHH = H(ILH)
      WRITE (6,20) H(ILH), XK, E, XMU, RZERO, DELTA(ILD)
   20 FORMAT (1H0,10X,22HRUNWAY CHARACTERISTICS / 11X,3HH =,F8.3,
     1 4H(IN),5X,3HK =,F8.1,11H(LBF/IN**3),5X,3HE =,F10.1 /
     2 11X,4HMU =,F6.3,5X,4HRO =,F8.2,4H(IN),5X,7HDELTA =,F7.3 )
      WRITE (6,30) XLA,XLB,XLC,XLD
   30 FORMAT(1H0,10X, 6HGEAR /
     1         1H ,10X,3HA =,F7.2,8H(IN) B =,F7.2,8H(IN) C =,F7.2,
     2         8H(IN) D =,F7.2,4H(IN) )
      WRITE (6,40) XNA,XNB,XNC,XND
   40 FORMAT(1H ,10X,4HNA =,F5.1,5X,4HNB =,F5.1,6X,4HNC =,F5.1,6X,4HND =
     1 , F5.1 )
      WRITE (6,50) A,PHIE
   50 FORMAT(1H0,10X,26HCONTACT AREA OF ONE TIRE =,F9.2,8H(SQ.IN.),
     1 10X, 6HPHIE =,F5.3)
      WRITE (6,60) P
   60 FORMAT(1H0, 10X,20HINFLATION PRESSURE =,F7.1,5H(PSI) )
      WRITE (6,70) G,B
   70 FORMAT(1H0, 10X,11HGEAR LOAD =,F10.1,5H(LBF),10H       B =,F8.1//)
      DO 500 INOG = 1,NOG
      GAMMA(INOG) = GAMMA(INOG)*DPR
      WRITE (6,90) INOG,GAMMA(INOG),PVMTST(INOG),YMN(INOG),SIGMA(INOG)
   90 FORMAT(1H ,10X,6HGAMMA(,I2,3H) =,F8.3,9H(DEG) N =,F7.2,5H MN =,
     1 F9.2,8H(IN.LB.) /
     2 1H ,10X,17HPAVEMENT STRESS =,F11.3,5H(PSI)//)
  500 CONTINUE
      IF (XOP3 - 0.5) 160, 160, 110
  110 WRITE (6,120)
  120 FORMAT (1H0,14X,1HZ,14X,1HW,10X,8HSUBTOTAL,10X,5HGAMMA)
      DO 150 J=1,NOG
      DO 150 I=1,NPT
      DISTY=RZERO * ZP(I,J)
      DISTX=RZERO * WP(I,J)
      STRESS = (RZERO*RZERO/10000.)*P*GPRINT(I,J)
      STRESS = 6.*STRESS/(HHH*HHH)
      WRITE(6,130) DISTY,DISTX,STRESS ,GAMMA(J)
  130 FORMAT ((10H             ,2(F10.4,5X),2(F10.3,5X)))
  150 CONTINUE
  160 CONTINUE
      DO 600 INOG = 1, NOG
  600 GAMMA(INOG) = GAMMA(INOG)*RPD
      RETURN
      END
C
BFOR,IS BLOCK
                    BLOCK DATA
C
C
      COMMON /BLK1/ NW,NZ,CV(700)
      DATA NW, NZ / 25, 25 /
      DATA (CV(J),J=1,175) /.0,.1,.2,.3,.4,.5,.6,.8,1.0,1.2,1.4,1.6,1.8,
     1  2.0,2.2,2.4,2.6,2.8,3.0,3.2,3.4,3.6,3.8,4.0,8.0,
     2  0.,.1,.2,.3,.4,.5,.6,.8,1.0,1.2,1.4,1.6,1.8,2.0,2.2,2.4,2.6,2.8,
     3  3.0,3.2,3.4,3.6,3.8,4.0,8.0,
     4 0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
     5  0.,0.,0.,0.,
     6 0.,59.61,96.44,121.67,139.43,151.81,160.10,167.67,166.85,160.54,
     7  150.71,138.77,125.74,112.35,99.14,86.47,74.61,63.70,53.82,45.03,
     8  37.23,30.47,24.65,19.70,19.70,
     9 0.,104.57,175.97,226.34,262.29,287.66,304.92,321.48,321.23,
     A  309.93,291.52,268.79,243.78,217.95,192.40,167.84,144.80,123.58,
     B  104.35,87.18,72.05,58.88,47.55,37.91,37.91,
     C 0.,140.04,241.75,315.90,369.83,408.44,435.13,461.85,463.43,
     D  448.39,422.58,390.18,354.22,316.90,279.84,244.15,210.61,179.69,
     E  151.65,126.60,104.52,85.29,68.75,54.68,54.68,
     F 0.,169.01,296.76,392.57,463.64,515.33,551.83,589.36,593.86,
     G  576.22,544.14,503.13,457.21,409.31,361.58,315.50,272.13,232.10,
     H  195.77,163.30,134.67,109.73,88.28,70.05,70.05/
      DATA (CV(J),J=176,325) /0.,193.18,343.27,458.41,545.37,609.60,
     J  655.41,704.64,712.96,693.75,656.41,607.82,552.89,495.29,437.69,
     K  381.96,329.41,280.86,236.75,197.31,162.52,132.22,106.17,84.03,
     K  84.03,
     L D.,213.63,382.94,515.16,616.59,692.58,747.58,808.45,821.26,
     M  801.36,759.75,704.47,641.42,574.95,508.25,443.57,382.47,325.96,
     N  274.58,228.61,188.05,152.73,122.37,96.58,96.58,
     O D.,246.13,445.42,606.88,732.99,829.74,901.53,984.96,1007.91,
```

```
      B   988.70,940.95,874.88,798.08,716.24,633.53,552.95,476.58,405.78,
      C   561.51,285.57,252.61,188.24,150.11,117.75,117.75,
      D  0.,270.36,494.03,676.25,821.03,935.68,1021.74,1125.54,1159.02,
      E   1142.32,1090.98,1016.97,929.35,834.96,738.97,644.87,555.59,
      F   472.54,396.80,328.91,268.06,216.78,171.96,133.97,133.97,
      G  0.,288.65,530.07,729.03,890.03,1017.39,1115.16,1236.45,1279.87,
      H   1265.56,1213.40,1133.68,1037.67,933.23,826.27,721.09,620.76,
      I   527.33,442.00,365.44,297.82,238.97,188.45,145.68,145.68,
      J  0.,302.51,557.45,769.25,942.15,1080.24,1187.41,1323.31,1375.43,
      K   1365.74,1311.89,1228.14,1125.70,1013.25,897.39,782.99,673.53,
      L   571.39,477.98,394.10,319.96,255.52,200.22,155.45,153.45/
      M DATA (CV(J),J=326,475) /0.,313.02,578.23,796.85,981.92,1128.36,
      N   1242.95,1390.47,1450.14,1443.88,1389.98,1303.40,1196.05,1077.26,
      O   954.22,832.28,715.29,605.92,505.79,415.79,336.27,267.12,207.84,
      P   157.78,157.78,
      Q  0.,320.94,593.85,823.03,1012.11,1164.98,1285.34,1442.14,1507.91,
      R   1504.67,1451.04,1362.45,1251.33,1127.56,998.78,870.73,747.60,
      S   632.29,526.59,431.55,347.56,274.56,212.01,159.27,159.27,
      T  0.,326.92,605.76,840.47,1034.86,1192.63,1317.41,1421.40,1552.01,
      U   1551.27,1498.00,1407.95,1293.94,1166.26,1032.90,899.94,771.82,
      V   651.67,541.43,442.28,354.85,278.52,213.34,158.47,158.47,
      W  0.,331.54,614.56,853.47,1051.83,1213.27,1341.39,1510.85,1585.19,
      X   1586.42,1533.47,1442.32,1326.06,1195.26,1058.28,921.39,789.25,
      Y   665.20,551.29,448.81,358.24,279.59,212.31,155.76,155.76,
      Z  0.,334.62,621.03,863.04,1064.33,1228.49,1359.08,1532.61,1609.74,
      A   1612.43,1559.70,1467.67,1349.63,1215.42,1075.53,936.49,801.13,
      B   673.94,557.09,451.95,359.04,278.40,209.49,151.65,151.65,
      C  0.,336.98,625.71,869.66,1073.37,1239.50,1371.88,1548.36,1627.49,
      D   1631.22,1578.58,1485.82,1366.34,1231.16,1088.66,946.22,808.51,
      E   678.85,559.65,452.41,357.66,275.48,205.33,146.53,146.53/
      F DATA (CV(J),J=476,575) /0.,338.55,629.02,874.85,1079.76,1247.28,
      G   1380.92,1559.47,1639.98,1644.37,1591.69,1498.27,1377.61,1240.86,
      H   1096.82,952.30,812.36,680.71,559.69,450.81,354.64,271.28,
      I   200.25,140.72,140.72,
      J  0.,339.80,631.30,878.22,1084.15,1252.62,1387.11,1567.04,1648.43,
      K   1653.19,1600.37,1506.36,1384.71,1246.68,1101.16,955.06,815.53,
      L   680.34,557.59,447.43,350.48,266.24,194.48,134.53,134.53,
      M  0.,340.56,632.80,880.44,1087.04,1256.12,1391.16,1571.96,1653.86,
      N   1658.75,1605.70,1511.13,1388.64,1249.56,1102.84,955.49,812.71,
      O   678.32,554.77,443.64,345.57,260.69,188.46,128.21,128.21,
      P  0.,341.02,633.72,881.80,1088.81,1258.26,1393.62,1574.90,1657.03,
      Q   1661.89,1608.55,1513.45,1390.24,1250.29,1102.61,954.27,810.51,
      R   675.20,550.81,438.95,340.29,254.97,182.43,122.01,122.01/
      S DATA (CV(J),J=576,675) /0.,341.28,634.23,882.55,1089.78,1259.41,
      T   1394.93,1576.40,1658.55,1663.25,1509.57,1513.97,1390.13,1249.46,
      U   1101.20,951.86,807.33,671.30,546.28,433.88,334.80,249.19,176.46,
      V   115.98,115.98,
      W  0.,341.39,634.45,882.86,1090.16,1259.85,1395.41,1576.88,1658.93,
      X   1663.38,1609.34,1513.27,1389.87,1247.59,1095.69,948.71,803.58,
      Y   667.01,541.53,428.75,329.38,243.57,170.77,110.28,110.28,
      Z  0.,341.39,634.45,882.86,1090.14,1259.81,1395.33,1576.67,1658.52,
      A   1662.68,1608.26,1511.76,1386.88,1245.10,1095.48,945.20,799.60,
      B   662.62,536.79,423.73,324.16,238.24,165.42,104.93,104.93,
      C  0.,341.39,634.45,882.86,1090.14,1259.81,1395.33,1576.67,1658.52,
      D   1662.68,1608.26,1511.76,1386.88,1245.10,1095.48,945.20,799.60,
      E   662.62,536.79,423.73,324.16,238.24,165.42,104.93,104.93/
      F END
@FOR,IS ERN
C
C       C.C.ELLIS
C     **PROGRAM TO PRINT ERROR NO., JOB NO., ETC. OFF AND/OR ON-LINE.
C     **ARGUMENT DEFINITION
C       N1--ERROR NUMBER.
C           FIXED POINT INTEGER OF NOT MORE THAN 3 DIGITS.
C       N2--1 DIMENSIONAL ARRAY CONTAINING THE FOLLOWING.
C           N2(1)--PROCEDURE CODE.
C                  3 HOLLERITH CHARACTERS, LEFT ADJUSTED.
C           N2(2)--NAME OF SUBROUTINE
C                  6 HOLLERITH CHARACTERS OR LESS
C           N2(3)--OFF-LINE, ON-LINE PRINT OPTION
C                  N2(3)=1 PRINT OFF-LINE ONLY
C                  N2(3)=2 PRINT ON-LINE ONLY
C                  N2(3)=3 PRINT OFF-LINE AND ON-LINE
C           N2(4)--CARD NO. OPTION
C                  N2(4)=0 USE CARD NO. IN IS(4)
C                  N2(4) NOT =0 SET IS(4)=N2(4) AFTER CALL STATUS
```

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C
C          N2(5)--ERN ENTRY COUNTER.                                                    00004560
C                  N2(5) IS INCREMENTED BY 1 EACH TIME ERN IS ENTERED.                  00004580
C                  N2(5) MUST BE INITIALIZED BY CALLING PROGRAM.                        00004590
C       N3--STATEMENT NO. OR SOME OTHER NUMERIC TO INDICATE STATEMENT                   00004600
C          FROM WHICH ERN WAS CALLED IN CALLING PROGRAM. N3 MUST BE A                   00004610
C          FIXED POINT INTEGER OF NOT MORE THAN 5 DIGITS.                               00004620
C                                                                                       00004630
      SUBROUTINE ERN (N1, N2, N3)                                                       00004640
C                                                                                       00004650
      DIMENSION  IS(7), N2(5)                                                           00004660
C                                                                                       00004670
      DATA K2, K3 /6HL     , 6HP                                                        00004680
C                                                                                       00004690
      N2(5) = N2(5) + 1                                                                 00004700
      IF (N2(4)) 90,95,90                                                               00004710
   90 IS(4) = N2(4)                                                                     00004720
   95 IF (IS(2) - 2) 100,150,150                                                        00004730
  100 K1 = K2                                                                           00004740
      GO TO 190                                                                         00004750
  150 K1 = K3                                                                           00004760
  190 IF (N2(3) - 2) 200,300,200                                                        00004770
  200 WRITE (6,900) N1, IS(1), K1, IS(3), IS(4), IS(5), IS(6), IS(7),                   00004780
     1                  N2(1), N2(2), N3                                               00004790
      IF (N2(3) - 1) 400,400,300                                                        00004800
  300 PRINT 910, IS(1), K1, IS(3), N2(1)                                                00004810
  400 RETURN                                                                            00004820
  900 FORMAT (11HOERROR NO. ,I3,12H.   JOB NO. ,I6,14H.   DECK TYPE ,A1,                00004830
     1 13H.   DECK NO. ,I3/ 9H CARD SEQUENCE NO. ,I5,14H.   LINE CTR. ,I3              00004840
     2,13H.   PAGE NO. ,I3/ 6H DATE ,A6,15H.   PROCEDURE ,A3,15H.   SUB               00004850
     3OUTINE ,A6,15H STATEMENT NO. ,I5)                                                 00004860
  910 FORMAT (18H ERROR IN JOB NO. ,I6,13H.  DECK TYPE ,A1,12H.  DECK NO               00004870
     1. ,I3,13H.  PROCEDURE ,A3)                                                        00004880
      END                                                                               00004890
C                                                                                       00004900
$FOR,IS GEOM                                                                            00004910
                                SUBROUTINE GEOM                                         00004920
C                                                                                       00004930
C                                                                                       00004940
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)                              00004950
      COMMON /BBLK/ BIGX,BIGY,B                                                         00004960
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)                                     00004970
      COMMON /EBLK/ E,ER(100)                                                           00004980
      COMMON /FBLK/ FG(10)                                                              00004990
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)                                          00005000
      COMMON /HBLK/ H(10)                                                               00005010
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP                            00005020
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG                                          00005030
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE                                                00005040
      COMMON /RBLK/ RPD,RZERO                                                           00005050
      COMMON /SBLK/ SIGMA(10),S,SR                                                      00005060
      COMMON /TBLK/ T,TR                                                                00005070
      COMMON /WBLK/ WR,WR2                                                              00005080
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,                    00005090
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,                 00005100
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),                    00005110
     3              XZERO,XZEROP                                                       00005120
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP                          00005130
      COMMON /M51A1/ KERN(5)                                                            00005140
      COMMON /BLK1/ NW,NZ,CV(700)                                                       00005150
      RZERO =SQRT( SQRT( (E*H(ILH)**3) / ( 12.*(1.-XMU**2)*XK ) ) )                     00005160
      XLAR = XLA / RZERO                                                                00005170
      XLBR = XLB / RZERO                                                                00005180
      XLCR = XLC / RZERO                                                                00005190
      XLDR = XLD / RZERO                                                                00005200
      WR = (XNA-1.) * XLAR + XNA*(XNB-1.)*XLBR                                          00005210
      XLR= (XNC-1.) * XLCR + XNC*(XND-1.)*XLDR                                          00005220
      NX =INT((XNA*XNB+1.) / 2.0 + 0.0001)                                              00005230
      NY =INT((XNC*XND+1.) / 2.0 + 0.0001)                                              00005240
      WR2= WR/2.                                                                        00005250
      ER(2) = XLAR-XLBR                                                                 00005260
      XLR2  = XLR/2.                                                                    00005270
      ER(5) = XLCR-XLDR                                                                 00005280
      N1    =INT(XNB + 0.0001)                                                          00005290
      DO 10 I = 1,NX                                                                    00005300
      ER(3) = I - 1                                                                     00005310
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      ER(I+50) = WR2 - ER(3)*XLBR - ER(2) * FLOAT ((I-1)/N1)                    00005320
   10 CONTINUE                                                                  00005330
      N2 =INT(XND + 0.0001)                                                     00005340
      DO 20 J = 1,NY                                                            00005350
      ER(6) = J - 1                                                             00005360
      ER(J+7) = XLR2 - ER(6)*XLDR - ER(5) * FLOAT ((J-1)/N2)                    00005370
   20 CONTINUE                                                                  00005380
      NX1 = NX                                                                  00005390
      NY1 = NY                                                                  00005400
      IF ( ABS(ER(NX+50)) - 1.E-07 ) 30,30,40                                   00005410
   30 ER(NX+50) = 0.                                                            00005420
      NX1 = NX - 1                                                              00005430
   40 NPT = 0                                                                   00005440
      DO 60 J = 1,NY                                                            00005450
      DO 50 I = 1,NX                                                            00005460
      NPT = NPT + 1                                                             00005470
      XPZ(NPT) = ER(I+50)                                                       00005480
      YPZ(NPT) = ER(J+7 )                                                       00005490
   50 CONTINUE                                                                  00005500
   60 CONTINUE                                                                  00005510
      IF ( ABS(ER(NY+7)) - 1.E-07 ) 70,70,75                                    00005520
   70 ER(NY+7) = 0.                                                             00005530
      NY1 =NY-1                                                                 00005540
   75 IF ( NY1 ) 105,105,80                                                     00005550
   80 DO 100 J = 1,NY1                                                          00005560
      DO 90  I = 1,NX                                                           00005570
      NPT = NPT + 1                                                             00005580
      XPZ(NPT) = ER(I+50)                                                       00005590
      YPZ(NPT) =-ER(J+7)                                                        00005600
   90 CONTINUE                                                                  00005610
  100 CONTINUE                                                                  00005620
  105 IF (NX1) 123,123,107                                                      00005630
  107 DO 120 J = 1,NY                                                           00005640
      DO 110 I = 1,NX1                                                          00005650
      NPT = NPT + 1                                                             00005660
      XPZ(NPT) = -ER(I+50)                                                      00005670
      YPZ(NPT) =  ER(J+7)                                                       00005680
  110 CONTINUE                                                                  00005690
  120 CONTINUE                                                                  00005700
  123 IF (NX1) 145,145,125                                                      00005710
  125 IF (NY1) 145,145,128                                                      00005720
  128 DO 140 J = 1,NY1                                                          00005730
      DO 130 I = 1,NX1                                                          00005740
      NPT = NPT + 1                                                             00005750
      XPZ(NPT) = -ER(I+50)                                                      00005760
      YPZ(NPT) = -ER(J+7)                                                       00005770
  130 CONTINUE                                                                  00005780
  140 CONTINUE                                                                  00005790
  145 IF (DP(1)-0.5) 170,170,150                                                00005800
  150 WRITE (6,160) RZERO,XLAR,XLBR,XLCR,XLDR,WR,XLR                            00005810
  160 FORMAT (1HO,2X,32HRZERO,XLAR,XLBR,XLCR,XLDR,WR,XLR /                      00005820
     1 1H ,10E12.4)                                                             00005830
  170 IF (DP(2)-0.5) 200,200,180                                                00005840
  180 WRITE (6,185) NX,NY,NPT,NX1,NY1,ILH                                       00005850
  185 FORMAT (1HO,35H   NX    NY   NPT   NX1   NY1   ILH      / 1H ,1015 )     00005860
      WRITE (6,190) ER , (XPZ(I),YPZ(1),I=1,NPT)                                00005870
  190 FORMAT(1HO,2X,37HER(1)-ER(100),(XPZ(I),YPZ(I),I=1,NPT) /                  00005880
     1   (1H , 10E12.4 ) )                                                     00005890
  200 CONTINUE                                                                  00005900
      RETURN                                                                    00005910
      END                                                                       00005920
C                                                                               00005930
CC                                                                              00005940
C                                                                               00005950
@FOR,IS OUTLNE                                                                  00005960
                    SUBROUTINE OUTLNE                                           00005970
C                                                                               00005980
C                                                                               00005990
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)                      00006000
      COMMON /BBLK/ BIGX,BIGY,B                                                 00006010
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)                             00006020
      COMMON /EBLK/ E,ER(100)                                                   00006030
      COMMON /FBLK/ FG(10)                                                      00006040
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)                                  00006050
      COMMON /HBLK/ H(10)                                                       00006060
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP                    00006070
```

```
      COMMON /NBLK/ NOG,NOM,NPT,NX,NT,NOD,NOSG
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE
      COMMON /RBLK/ RPD,RZERO
      COMMON /SBLK/ SIGMA(10),S,SR
      COMMON /TBLK/ T,TR
      COMMON /WBLK/ WR,WR2
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XMA,XMB,XMC,XND,XNOG,XNOM,XNOD,XNOSG,XNT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
      COMMON /H51A1/ KERM(5)
      COMMON /BLK1/ NW,NZ,CV(700)
      COMMON /UBLK/ UNUM(10,10),UMAX(10)
      SAM(PX)=(1.+(PHIE-1.)*PX**3)*(SIN(PI*PX/2.))/PHIE
      S = SQRT(A/(PI*PHIE))
      T = SQRT(A*PHIE/PI)
      SR= S/RZERO
      TR= T/RZERO
      ILD = 1
      IF (NOD .EQ. 0) GO TO 8
    7 DELTAR(ILD) = DELTA(ILD)/RZERO
    8 DO 50 K = 1,10
      PVMTST(K) = 0.0
      DO 51 J = 1,100
      GPRINT(J,K) = 0.0
   51 CONTINUE
   50 CONTINUE
      DO 305 INOG=1,NOG
      IF (XOP6 .GT.0.5) GO TO 10
      IF(ABS(GAMMA(INOG)).LT.0.0001) GO TO 20
      IF ( ABS(GAMMA(INOG) - 90.*RPD) .LT. 1.0E-7 ) GO TO 30
      GETAN = SIN(GAMMA(INOG))/COS(GAMMA(INOG))
      XZEROP = -(XPZ(1)+SR*GETAN/SQRT((GETAN)**2
     1 + PHIE**2)  + DELTAR(ILD)*COS(GAMMA(INOG)))
      YZEROP = -(YPZ(1)+TR*PHIE/SQRT((GETAN)**2 +
     1 PHIE**2) - DELTAR(ILD) * SIN(GAMMA(INOG)))
      GO TO 40
   10 XZEROP = BIGX/RZERO
      YZEROP = BIGY/RZERO
      GO TO 40
   20 XZEROP = -(DELTAR(ILD) + XPZ(NX))
      YZEROP = -(YPZ(1) + TR)
      GO TO 40
   30 XZEROP = -(XPZ(1) + SR)
      YZEROP = (DELTAR(ILD) + YPZ(NY))
   40 M =INT(B/4.0 + 0.99999999)
      KBP= 4*M
C                    START LOOP ON TIRE POINTS
      IE1=0
      IE2=0
      IE3=0
      IE4=0
      DO 250 IPT = 1,NPT
      XZERO=XPZ(IPT) + SR - XZEROP
      YZERO=YPZ(IPT) - YZEROP
      ZZERO=XZERO*COS(GAMMA(INOG))-YZERO*SIN(GAMMA(INOG))
      WZERO=XZERO*SIN(GAMMA(INOG))+YZERO*COS(GAMMA(INOG))
C                    START LOOP ON TIRE PERIMETER
      DO 200 J=1,KBP
      DM =FLOAT(M)
      DJ =FLOAT(J)
      IF(1.LE.J .AND. J.LE.M) GO TO 60
      IF(M+1.LE.J .AND. J.LE.2*M) GO TO 70
      IF(2*M+1.LE.J .AND. J.LE.3*M) GO TO 80
      PX = (4.*DM-DJ)/DM
      XI=XPZ(IPT)-XZEROP+SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP-TR*SAM(PX)
      GO TO 90
   60 PX = (DJ/DM)
      XI=XPZ(IPT)-XZEROP+SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP+TR*SAM(PX)
      GO TO 90
   70 PX = (2.*DM-DJ)/DM
      XI=XPZ(IPT)-XZEROP-SR*SQRT(1.-SAM(PX)**2)
      YI=YPZ(IPT)-YZEROP+TR*SAM(PX)
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      GO TO 90                                                                  00006840
   80 PX = (DJ-2.*DM)/DM                                                        00006850
      XI=XPZ(IPT)-XZEROP-SR*SQRT(1.-SAM(PX)**2)                                 00006860
      YI=YPZ(IPT)-YZEROP-TR*SAM(PX)                                             00006870
   90 ZI=X1*COS(GAMMA(INOG))-YI*SIN(GAMMA(INOG))                                00006880
      WI=X1*SIN(GAMMA(INOG))+YI*COS(GAMMA(INOG))                                00006890
      ZINT = (ZI + ZZERO) / 2.0                                                 00006900
      IF (ZINT .LT. 0.0) GO TO 110                                              00006910
      CALL TABINT (ZINT, TNA, WI, NZ, NW, CV, IND)                              00006920
      IF (IND.LT.2) GO TO 100                                                   00006930
      IF (ABS(WI) .LT. 1.0E-05) GO TO 100                                       00006940
      IE1=IE1+1                                                                 00006950
      IF(IE1.GT.2) GO TO 400                                                    00006960
      WRITE (6,198) IND, ZINT, WI                                               00006970
  198 FORMAT (1HO,32HDATA IS OUTSIDE LIMITS OF TABLE. / 10H      IND =,12       00006980
     1  ,5X,6HZINT =,E15.7,5X,4HWI =,E15.7)                                    00006990
  400 CONTINUE                                                                  00007000
      TNA = 0.0                                                                 00007010
  100 CALL TABINT (ZINT, TNB, WZERO, NZ, NW, CV, IND)                           00007020
      IF (IND .LT. 2) GO TO 130                                                 00007030
      IF (ABS(WZERO) .LT. 1.0E-05) GO TO 130                                    00007040
      IE2=IE2+1                                                                 00007050
      IF(IE2.GT.2) GO TO 500                                                    00007060
      WRITE (6,199) IND, ZINT, WZERO                                            00007070
  199 FORMAT (1HO,32HDATA IS OUTSIDE LIMITS OF TABLE. / 10H      IND =,12       00007080
     1  ,5X,6HZINT =,E15.7,5X,7HWZERO =,E15.7)                                 00007090
  500 CONTINUE                                                                  00007100
      TNB=0.0                                                                   00007110
      GO TO 130                                                                 00007120
  110 AZINT = -ZINT                                                             00007130
      CALL TABINT (AZINT, TSA, WI, NZ, NW, CV, IND)                             00007140
      TNA = -TSA                                                                00007150
      IF (IND.LT.2) GO TO 120                                                   00007160
      IF (ABS(WI) .LT. 1.0E-05) GO TO 120                                       00007170
      IE3=IE3+1                                                                 00007180
      IF(IE3.GT.2) GO TO 600                                                    00007190
      WRITE (6,191) IND, AZINT, WI                                              00007200
  191 FORMAT (1HO,32HDATA IS OUTSIDE LIMITS OF TABLE. / 10H      IND =,12       00007210
     1  ,5X,7HAZINT =,E15.7,5X,4HWI =,E15.7)                                   00007220
  600 CONTINUE                                                                  00007230
      TNA = 0.0                                                                 00007240
  120 CALL TABINT (AZINT, TSB, WZERO, NZ, NW, CV, IND)                          00007250
      TNB = -TSB                                                                00007260
      IF (IND .LT. 2) GO TO 130                                                 00007270
      IF (ABS(WZERO) .LT. 1.0E-05) GO TO 130                                    00007280
      IE4=IE4+1                                                                 00007290
      IF(IE4.GT.2) GO TO 700                                                    00007300
      WRITE (6,192) IND, AZINT, WZERO                                           00007310
  192 FORMAT (1HO,32HDATA IS OUTSIDE LIMITS OF TABLE. /                         00007320
     1 4X,5HIND =,12,5X,7HAZINT =,E15.7,5X,7HWZERO =,E15.7)                     00007330
  700 CONTINUE                                                                  00007340
      TNB = 0.0                                                                 00007350
  130 TANS=TNA-TNB                                                              00007360
      IF (DP(3) - 0.5) 150, 150, 140                                            00007370
  140 WRITE (6,95) XI, YI, ZI, WI, XZERO, YZERO, ZZERO, WZERO,                  00007380
     1 ZINT, GAMMA(INOG), TNA, TNB                                              00007390
   95 FORMAT (1HO,6X,2HXI,13X,2HYI,13X,2HZI,13X,2HWI/    4(E15.7)//            00007400
     1  5X,5HXZERO,10X,5HYZERO,10X,5HZZERO,10X,5HWZERO/    4(E15.7)//         00007410
     2 6X,4HZINT,6X,11HGAMMA(INOG),9X,3HTNA,12X,3HTNB/ 4(E15.7))               00007420
  150 GPRINT(IPT,INOG) = GPRINT(IPT,INOG) + TANS                                00007430
  197 XZERO=XI                                                                  00007440
      YZERO=YI                                                                  00007450
      ZZERO=ZI                                                                  00007460
      WZERO=WI                                                                  00007470
  200 CONTINUE                                                                  00007480
      PVMTST(INOG) = PVMTST(INOG) + GPRINT(IPT,INOG)                            00007490
      IF (XOPS - 0.5) 250,250,230                                               00007500
  230 ZP(IPT,INOG)= COS(GAMMA(INOG))*(XPZ(IPT)-XZEROP) - SIN(GAMMA(INOG))       00007510
     1 )*(YPZ(IPT)-YZEROP)                                                      00007520
      WP(IPT,INOG)= SIN(GAMMA(INOG))*(XPZ(IPT)-XZEROP) + COS(GAMMA(INOG))       00007530
     1 )*(YPZ(IPT)-YZEROP)                                                      00007540
  250 CONTINUE                                                                  00007550
      YMN(INOG)=RZERO*RZERO/10000.0*P*PVMTST(INOG)                              00007560
      SIGMA(INOG)=6.0*YMN(INOG)/H(ILH)**2                                       00007570
      IF (NOSG .EQ. 0) GO TO 305                                                00007580
      DO 280 ISG=1,NOSG                                                         00007590
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760           AP(ISG)= AG(ISG)/(XMT*A)                                                00007600
761           AH(ISG)=RZERO*SQRT(ABS((6.D*AP(ISG)*PVMTST(INOG))/(10000.*ASIG          00007610
762          1 (ISG))))                                                               00007620
763           AK(ISG) = (34.1/AH(ISG))*(10.D*AH(ISG)/RZERO)**4                        00007630
764       280 CONTINUE                                                               00007640
765       290 CALL CURVE                                                             00007650
766       305 CONTINUE                                                               00007660
767           CALL FINISH                                                            00007670
768           DO  330  INOG=1,NOG                                                    00007680
769       330 UNUM(ILD,INOG)=PVMTST(INOG)                                            00007690
770           IF (NOD .EQ. 0) GO TO 320                                              00007700
771           ILD = ILD + 1                                                          00007710
772           IF (ILD .LE. NOD) GO TO 7                                              00007720
773       320 RETURN                                                                 00007730
774           END                                                                    00007740
775     C                                                                            00007750
776     $FOR,IS CURVE                                                                00007760
777                           SUBROUTINE CURVE                                       00007770
778     C                                                                            00007780
779     C                                                                            00007790
780           COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)                   00007800
781           COMMON /BBLK/ BIGX,BIGY,B                                              00007810
782           COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)                          00007820
783           COMMON /EBLK/ E,ER(100)                                                00007830
784           COMMON /FBLK/ FG(10)                                                   00007840
785           COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)                               00007850
786           COMMON /HBLK/ H(10)                                                    00007860
787           COMMON /IBLK/ ICA(10),ILM,IPT,IT,ILD,ISG,INOG,  KBP                    00007870
788           COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG                               00007880
789           COMMON /PBLK/ P,PI,PVMTST(10),PHIE                                     00007890
790           COMMON /RBLK/ RPD,RZERO                                                00007900
791           COMMON /SBLK/ SIGMA(10),S,SR                                           00007910
792           COMMON /TBLK/ T,TR                                                     00007920
793           COMMON /WBLK/ WR,WR2                                                   00007930
794           COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,         00007940
795          1              XMU,XMA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,     00007950
796          2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),        00007960
797          3              XZERO,XZEROP                                           00007970
798           COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP               00007980
799           COMMON /HSIAI/ KERN(5)                                                 00007990
800           COMMON /BLK1/ NW,NZ,CV(700)                                            00008000
801           GAMMA(INOG) = GAMMA(INOG) * DPR                                        00008010
802           WRITE (6,10) E, XMU                                                    00008020
803       10 FORMAT (1H1,4X, 3HE =,F10.1,5X, 5HXMU =,F6.3)                           00008030
804           WRITE (6,30) XLA,XLB,XLC,XLD                                           00008040
805       30 FORMAT(1HO,10X, 4HGEAP /                                                00008050
806          1         1H ,10X,3HA =,F7.2,8H(IN) B =,F7.2,8H(IN) C =,F7.2,          00008060
807          2         8H(IN) D =,F7.2 )                                            00008070
808           WRITE (6,40) XNA,XNB,XNC,XND                                           00008080
809       40 FORMAT(1H ,10X,4HNA =,F5.1,6X,4HNB =,F5.1,6X,4HNC =,F5.1,6X,4HND =     00008090
810          1 , F5.1 )                                                             00008100
811           WRITE (6,50) A,PHIE                                                    00008110
812       50 FORMAT(1HO,10X,26HCONTACT AREA OF ONE TIRE =,F9.2,8H(SQ.IN.),           00008120
813          1 10X, 6HPHIE =,F5.3)                                                  00008130
814           WRITE (6,60) H(ILM), GAMMA(INOG), DELTA(ILD), PVMTST(INOG)             00008140
815       60 FORMAT (1HO,4X, 3HH =,F8.3,5X, 7HGAMMA =,F8.3,5X, 7HDELTA =,F8.3,       00008150
816          1  5X, 3HK= ,F11.3)                                                    00008160
817           DO 100 ISG=1,NOSG                                                      00008170
818           WRITE (6,70) ASIG(ISG), AG(ISG), AP(ISG), AH(ISG), AK(ISG)             00008180
819       70 FORMAT (1HO,2X, 7HSIGMA =,F 8.3,2X, 3HG =,F 8.1,2X, 3HP =F7.1,2X,       00008190
820          1  3HH =F8.3,2X,3HK =,F8.1)                                            00008200
821      100 CONTINUE                                                               00008210
822           GAMMA(INOG) = GAMMA(INOG) * RPD                                        00008220
823           RETURN                                                                 00008230
824           END                                                                    00008240
825     C                                                                            00008250
826     $FOR,IS TABINT                                                               00008260
827           SUBROUTINE TABINT (X,Y,Z,MX,MZ,CV,IND)                                 00008270
828           DIMENSION CV(700)                                                      00008280
829           IND = 1                                                                00008290
830           DO 100 I=26,50                                                         00008300
831           IF(Z.LT.CV(26))GO TO 200                                               00008310
832           IF(Z.LT.CV(I)) GO TO 300                                               00008320
833       100 CONTINUE                                                               00008330
834           IND = 5                                                                00008340
835           IS = 49                                                                00008350
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      IL = 50                                                           00008360
      GO TO 250                                                         00008370
  200 IND = 4                                                           00008380
      IS = 26                                                           00008390
      IL = 27                                                           00008400
      GO TO 250                                                         00008410
  300 IL = I                                                            00008420
      IS = I-1                                                          00008430
  250 DO 800 J = 1,25                                                   00008440
      IF(X.LT.CV(1)) GO TO 900                                          00008450
      IF(X.LT.CV(J)) GO TO 1000                                         00008460
  800 CONTINUE                                                          00008470
      IND = 3                                                           00008480
      JS = 24                                                           00008490
      JL = 25                                                           00008500
      GO TO 950                                                         00008510
  900 JS = 1                                                            00008520
      JL = 2                                                            00008530
      IND=2                                                             00008540
      GO TO 950                                                         00008550
 1000 JL = J                                                            00008560
      JS = J-1                                                          00008570
  950 CONTINUE                                                          00008580
      IZ =IS-25                                                         00008590
      IQ =IL-25                                                         00008600
      K1=50+(IZ-1)*25+JS                                                00008610
      Y1S=CV(K1)                                                        00008620
      K2=K1+1                                                           00008630
      Y1L=CV(K2)                                                        00008640
      K3=50+(IQ-1)*25+JS                                                00008650
      Y2S=CV(K3)                                                        00008660
      K4=K3+1                                                           00008670
      Y2L=CV(K4)                                                        00008680
      IF(IND.GT.1) GO TO 1100                                           00008690
 1010 Y1=(((Y1L-Y1S)/(CV(JL)-CV(JS)))*(X-CV(JS)))+Y1S                   00008700
      Y2=(((Y2L-Y2S)/(CV(JL)-CV(JS)))*(X-CV(JS)))+Y2S                   00008710
      IF(IND.GT.1) GO TO 1050                                           00008720
 1020 Y=(((Y2-Y1)/(CV(IL)-CV(IS)))*(Z-CV(IS)))+Y1                       00008730
      IF(IND.GT.1) GO TO 1070                                           00008740
      GO TO 2000                                                        00008750
 1100 IF(X.LT.CV(1))GO TO 1010                                          00008760
      IF(X.GT.CV(25))GO TO 1040                                         00008770
      GO TO 1010                                                        00008780
 1040 Y1= (((Y1L-Y1S)/(CV(JL)-CV(JS)))*(X-CV(JL)))+Y1L                  00008790
      Y2= (((Y2L-Y2S)/(CV(JL)-CV(JS)))*(X-CV(JL)))+Y2L                  00008800
 1050 IF(Z.LT.CV(26)) GO TO 1020                                        00008810
      IF(Z.GT.CV(50)) GO TO 1060                                        00008820
      GO TO 1020                                                        00008830
 1060 Y = (((Y2-Y1)/(CV(IL)-CV(IS)))*(Z-CV(IL)))+Y2                     00008840
 1070 CONTINUE                                                          00008850
 2000 CONTINUE                                                          00008860
      RETURN                                                            00008870
      END                                                               00008880
C                                                                       00008890
C                                                                       00008900
C                                                                       00008910
@FOR,IS PROBRD                                                          00008920
                                    SUBROUTINE PROBRD                   00008930
C                                                                       00008940
C                                                                       00008950
      COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)              00008960
      COMMON /BBLK/ BIGX,BIGY,B                                         00008970
      COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)                     00008980
      COMMON /EBLK/ E,ER(100)                                           00008990
      COMMON /FBLK/ F6(10)                                              00009000
      COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)                          00009010
      COMMON /HBLK/ H(10)                                               00009020
      COMMON /IBLK/ ICM(10),ILH,IPT,IT,ILD,ISG,INOG    ,XBP            00009030
      COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG                          00009040
      COMMON /PBLK/ P,PI,PVMTST(10),PHIE                                00009050
      COMMON /RBLK/ RPD,RZERO                                           00009060
      COMMON /SBLK/ SIGMA(10),S,SR                                      00009070
      COMMON /TBLK/ T,TR                                                00009080
      COMMON /WBLK/ WR,WR2                                              00009090
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,    00009100
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,  00009110
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      2                XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
      3                XZERO,XZEROP
       COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
       COMMON /HS1A1/ KERN(5)
       COMMON /BLK1/ NW,NZ,CV(700)
       COMMON /MPJ/  A0,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
      1              JOPPL1,JOPPL2
       DATA KNAME/6HPROBRD/
       KERN(2) = KNAME
       PI = 3.1415927
       DPR= 180.0/PI
       RPD= PI/180.0
       READ(5,10,END=999)(ICM(I),I=1,10)
   10  FORMAT(10A6)
       DP(1)=0.
       WRITE (6,20)
   20  FORMAT(1H1 ,19X,35HP R O B L E M  I N P U T  D A T A   )
       WRITE (6,30) (ICM(I),I=1,10)
   30  FORMAT (1HO, 11X, 10A6 )
   15  READ(5,40,END=999)G,P,A,XNOG
   17   CONTINUE
   40  FORMAT(4F10.0)
  211  IF (XNOG) 220, 220, 225
  220  CALL ERN (03, KERN, 220)
       GO TO 8990
  225  IF (XNOG - 10.) 230, 230, 215
  215  CALL ERN (04, KERN, 215)
       GO TO 8990
  230  IF (P.LT.0.0001.AND.A.LT.0.0001) GO TO 240
       WRITE (6,45)
   45  FORMAT (1HO,3X,11HK(LB/CU IN),5X,5HG(LB),8X,6HP(PSI),6X,8HA(SQ IN)
      1,4X,12HNO. GAMMA(I) )
       WRITE (6,50) XK, G, P, A, XNOG
   50  FORMAT (1H ,5X,F7.1,5X,F10.1,5X,F7.1,5X,F9.1,5X,F4.1)
       GO TO 245
  240  CALL ERN(05, KERN, 240)
       GO TO 8990
   41  FORMAT(6F10.0)
  245  READ (5,41) XLA, XLB, XLC, XLD, XNOD, XNOSG
       IF (XNOD) 250, 255, 255
  250  CALL ERN (06, KERN, 250)
       GO TO 8990
  255  IF (XNOD - 10.) 265, 265, 260
  260  CALL ERN (07, KERN, 260)
       GO TO 8990
  265  IF (XNOSG) 270, 275, 275
  270  CALL ERN (08, KERN, 270)
       GO TO 8990
  275  IF (XNOSG - 10.) 285, 285, 280
  280  CALL ERN (09, KERN, 280)
       GO TO 8990
  285  WRITE (6,60)
   60  FORMAT (1HO,3X,50HSMALL A(IN)  SMALL B(IN)  SMALL C(IN)  SMALL D(I
      1N),22H NO. DELTA(I) NO.(S,G) )
       NOG =INT(XNOG + 0.0001)
       NOD=INT(XNOD+0.0001)
       NOSG=INT(XNOSG+0.0001)
       WRITE (6,65) XLA,XLB,XLC,XLD,XNOD,XNOSG
   65  FORMAT (1H ,5X,F7.2,3(6X,F7.2),7X,F4.1,5X,F4.1)
   43  FORMAT(5F10.0)
       READ(5,43)XNA,XNB,XNC,XND,PHIE
       XOP1=0.
       IF (PHIE.LT.0.0001) PHIE = 5./3.
       WRITE (6,70)        XNA,XNB,XNC,XND,PHIE
   70  FORMAT (1HO,5X,3HXNA,9X,3HXNB,9X,3HXNC,9X,3HXND,9X,6HPHI(E)/
      11H ,4X,F4.1,4(8X,F4.1))
       READ (5,43) B, XOP3, BIGX, BIGY, XOP6
       WRITE (6,75) B, XOP3, BIGX, BIGY, XOP6
   75  FORMAT (1HO,8X,1HB,6X,9HPRINT OPT,3X,5HBIG X,7X,5HBIG Y,6X,
      18HCOOR OPT /1H ,5X,F7.1,5X,F4.1,2(5X,F7.2),7X,F4.1 )
   42   FORMAT(10F6.0)
        READ (5,42) (GAMMA(I), I=1,NOG)
        WRITE (6,85) (GAMMA(I), I=1,NOG)
   85  FORMAT (1HO,27X,14HGAMMA(I) (DEG)/
      11H ,2X,6(F7.2,4X) )
        IF (NOD.EQ.0) GO TO 97
```

```
      READ (5,42) (DELTA(I), I=1,NOD)
      WRITE (6,90) (DELTA(I), I=1,NOD)
   90 FORMAT (1HO,30X,8HDELTA(I) /
     1 11H ,2X,6(F7.2,4X) )
   97 IF (NOSG .EQ. 0) GO TO 99
      READ (5,92) (ASIG(I), AG(I), I=1,NOSG)
   92 FORMAT(8F10.0)
      WRITE (6,95) ( ASIG(I),AG(I), I=1,NOSG)
   95 FORMAT (1HO,6X,7HASIG(I),9X,5HAG(I) /    2(5X,F10.1)//)
   99 DO 300 I = 1,NOG
      GAMMA(I) = GAMMA(I)*RPD
  300 CONTINUE
      IF (DP(1)-0.5) 340, 340, 325
  325 READ (5,330) (DP(I), I=1,31)
  330 FORMAT (31F2.0)
      WRITE (6,335) (DP(I), I=1,31)
  335 FORMAT (1HO, 5HDUMPS, 31F4.1)
  340 XNT=XNA*XNB*XNC*XND
      IF (XNT - 4096.001) 410, 400, 400
  400 CALL ERN (10, KERN, 400)
      GO TO 8990
  410 IF (A) 490, 500, 490
  490 P = G/(A*XNT)
      GO TO 510
  500 A = G/(P*XNT)
  510 G = A*P*XNT
      GO TO 540
 8990 CONTINUE
      GO TO 15
  999 CALL EXIT
  540 RETURN
      END
C
C
$FOR,IS KCOMP
      SUBROUTINE KCOMP
C
C
      COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
     1              XMU,XNA,XNB,XNC,XND,XNOG,XNOM,XNOD,XNOSG,XMT,XOP1,
     2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
     3              XZERO,XZEROP
      COMMON /MPJ/  A0,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
     1              JOPPL1,JOPPL2
 1004 FORMAT(1H1,16X,'MULTI-LAYERED SYSTEM ANALYSIS'//)
 1005 FORMAT(4X,'SUBGRADE:')
 1006 FORMAT(14X,'MODULUS OF SUBGRADE REACTION = ',F7.2,' PCI',5X,
     1'CV = ',F7.4//)
 1007 FORMAT(3X'BASE:')
 1008 FORMAT(14X,'TYPE: WELL-GRADED CRUSHED MATERIALS'/)
 1009 FORMAT(14X,'TYPE: NATURAL SAND AND GRAVEL (PI<8)'/)
 1011 FORMAT(14X,'TYPE: STABILIZED'/)
 1012 FORMAT(14X,'HEIGHT = ',F5.2,' INCHES',8X,'CV = ',F7.4/)
 1013 FORMAT(14X,'MODULUS = ',F10.2,' PSI',5X,'CV = ',F7.4//)
 1014 FORMAT(1HO,2X,'COMPOSITE:')
 1015 FORMAT(5X,'LAYER   :')
 1016 FORMAT(5X,'SUBBASE:')
  701 FORMAT(I2,4F10.4)
  451 FORMAT(1H0,20X,'A MAXIMUM DESIGN K OF 500 PCI IS USED')
      WRITE(6,1004)
      WRITE(6,1005)
      WRITE(6,1006)XK,CVK
      DO 702 I=1,NOLBP
      READ(5,701)MATOPT,HB,CVHB,EB,CVEB
      IF(MATOPT.EQ.1)GO TO 101
      IF(MATOPT.EQ.2)GO TO 102
      IF(MATOPT.EQ.3)GO TO 103
  101 Y=ALOG10(XK)
      Y2=(0.08901252+(2.*(-0.0004425194)*HB)+(-0.02901488*Y))
      Y3=(HB**2.)*(Y2**2.)*(5.3018981)*(CVHB**2.)
      Y4=(0.9640174)+(-0.2561922*Y)+(-0.012601*HB)
      Y5=(Y4**2.)*(5.3018981)*(CVK**2.)
      CVKC=Y3+Y5
      CVK=SQRT(CVKC)
      Y1=-1.251182+(2.219732*Y)+(-0.2949522*(Y**2.))
     1+(0.08901252*HB)+(-0.0004425194*(HB**2.))
```

```
     2+(-0.02901488*HB*Y)
      XK=10.**Y1
      GO TO 195
  102 Z=ALOG10(XK)
      Z2=(0.08554373+(2.*(-0.0002574619)*HB)+(-0.03050173*Z))
      Z3=(HB**2.)*(Z2**2.)*(5.3018981)*(CVHB**2.)
      Z4=0.9829852+(-0.2617702*Z)+(-0.013246*HB)
      Z5=(Z4**2.)*(5.3108981)*(CVK**2.)
      CVKC=Z3+Z5
      CVK=SQRT(CVKC)
      Z1=-1.296084+(2.263407*Z)+(-0.3013741*(Z**2.))
     1+(0.08554373*HB)+(-0.0002574619*(HB**2.))
     2+(-0.03050173*HB*Z)
      XK=10.**Z1
      GO TO 195
  103 W=ALOG10(XK)
      W2=0.0544761+(-0.0016948*HB)+(-0.4409096E-2*W)
     1+(-0.2465638E-8*EB)
      W3=(HB**2.)*(W2**2.)*(5.3018981)*(CVHB**2.)
      W4=(0.7254749E-6)+(-0.3874586E-12*EB)+(-0.4601653E-7*W)
     1+(-0.2465638E-8*HB)
      W5=(EB**2.)*(W4**2.)*(5.3018981)*(CVEB**2.)
      W6=0.4465112+(-0.1914846E-2*HB)+(-0.1998473E-7*EB)
      W7=(W6**2.)*(5.3018981)*(CVK**2.)
      CVKC=W3+W5+W7
      CVK=SQRT(CVKC)
      W1=-0.1578667+(1.02813*W)+(0.0544761*HB)+(-0.8473852E-3*(HB**2.))
     1+(0.7254749E-6*EB)+(-0.1937293E-12*(EB**2.))
     2+(-0.4409096E-2*HB*W)+(-0.4601653E-7*EB*W)
     3+(-0.2465638E-8*EB*HB)
      XK=10.**W1
      GO TO 195
  195 CONTINUE
      IF(NOLBP.EQ.1)GO TO 201
      IF(NOLBP.EQ.2)GO TO 202
  201 WRITE(6,1007)
      IF(MATOPT.EQ.1)GO TO 203
      IF(MATOPT.EQ.2)GO TO 204
      IF(MATOPT.EQ.3)GO TO 205
  203 WRITE(6,1008)
      GO TO 206
  204 WRITE(6,1009)
      GO TO 206
  205 WRITE(6,1011)
      GO TO 206
  206 CONTINUE
      WRITE(6,1012)HB,CVHB
      IF(MATOPT.LT.2.5)GO TO 207
      WRITE(6,1013)EB,CVEB
  207 CONTINUE
      WRITE(6,1014)
      WRITE(6,1015)
      WRITE(6,1006)XK,CVK
      IF(XK.LE.500.)GO TO 400
      XK=500.
      WRITE(6,401)
  400 CONTINUE
      GO TO 899
  202 IF(I.EQ.1)GO TO 300
      IF(I.EQ.2)GO TO 301
  300 WRITE(6,1016)
      IF(MATOPT.EQ.1)GO TO 302
      IF(MATOPT.EQ.2)GO TO 303
      IF(MATOPT.EQ.3)GO TO 304
  302 WRITE(6,1008)
      GO TO 305
  303 WRITE(6,1009)
      GO TO 305
  304 WRITE(6,1011)
      GO TO 305
  305 CONTINUE
     -WRITE(6,1012)HB,CVHB
      IF(MATOPT.LT.2.5)GO TO 306
      WRITE(6,1013)EB,CVEB
  306 CONTINUE
      GO TO 700
```

```
      301 WRITE(6,1007)
          IF(MATOPT.EQ.1)GO TO 307
          IF(MATOPT.EQ.2)GO TO 308
          IF(MATOPT.EQ.3)GO TO 309
      307 WRITE(6,1008)
          GO TO 310
      308 WRITE(6,1009)
          GO TO 310
      309 WRITE(6,1011)
          GO TO 310
      310 CONTINUE
          WRITE(6,1012)HB,CVHB
          IF(MATOPT.LT.2.5)GO TO 311
          WRITE(6,1013)EB,CVEB
      311 CONTINUE
          WRITE(6,1014)
          WRITE(6,1015)
          WRITE(6,1006)XK,CVK
          IF(XK.LE.500)GO TO 403
          XK=500.
          WRITE(6,401)
      403 CONTINUE
      899 CONTINUE
      700 CONTINUE
          RETURN
          END
@FOR,IS STORPL
C
C
C
C
          SUBROUTINE STORPL
C
C
C
          COMMON /ABLK/ A,AP(10),AH(10),AK(10),ASIG(10),AG(10)
          COMMON /BBLK/ BIGX,BIGY,B
          COMMON /DBLK/ DP(31),DPR,DELTA(10),DELTAR(10)
          COMMON /EBLK/ E,ER(10)
          COMMON /FBLK/ FG(10)
          COMMON /GBLK/ G,GAMMA(10),GPRINT(100,10)
          COMMON /HBLK/ H(10)
          COMMON /IBLK/ ICR(10),ILH,IPT,IT,ILD,ISG,INOG    ,KBP
          COMMON /NBLK/ NOG,NOH,NPT,NX,NY,NOD,NOSG
          COMMON /PBLK/ P,PI,PVRTST(10),PHIE
          COMMON /RBLK/ RPD,RZERO
          COMMON /SBLK/ SIGMA(10),S,SR
          COMMON /TBLK/ T,TR
          COMMON /WBLK/ WR,WR2
          COMMON /XBLK/ XK,XLA,XLB,XLC,XLD,XLAR,XLBR,XLCR,XLDR,XLR,XLR2,
         1              XMU,XMA,XNB,XNC,XND,XNOG,XNOH,XNOD,XNOSG,XNT,XOP1,
         2              XOP2,XOP3,XOP4,XOP5,XOP6,ZP(100,10) ,XPZ(4097),
         3              XZERO,XZEROP
          COMMON /YBLK/ YMN(10),WP(100,10) ,YPZ(4097),YZERO,YZEROP
          COMMON /H51A1/ KERN(5)
          COMMON /BLK1/ NW,NZ,CV(700)
          COMMON /UBLK/ UNUM(10,10),UMAX(10)
          COMMON /MPJ/  A0,A1,A2,ALPHA,AMR,BN(10),CVDDF(10),CVK,IG,NON,
         1              JOPPL1,JOPPL2
          DIMENSION DDF(10),PR(10)
          READ(5,101)(BN(I),I=1,NON)
      101 FORMAT(10F8.0)
          WRITE(6,103)
      103 FORMAT(//,9X,"H",10X,"RELIA",7X,"DF",15X,"N",/)
          NOH2=NOH+2
          IF(JOPPL2.LT.1)GO TO 71
          CALL PLOTMJ(H,PR,-1,NOH2)
       71 CONTINUE
          DO 50 II=1,NON
          DO 40 JJ=1,NOH
          SIG=UMAX(JJ)/H(JJ)/H(JJ)
          DF=AMR/ALPHA/SIG
          IF(XK.LT.250.)GO TO 31
          IF(XK.LT.350.)GO TO 33
          IF(XK.LT.450.)GO TO 35
```

```
      IF(BN(II).GT.5000.)GO TO 37                                       00012160
      DDF(II)=0.1*ALOG10(BN(II))+0.5401                                 00012170
      GO TO 38                                                          00012180
   37 DDF(II)=0.246*ALOG10(BN(II))                                      00012190
      GO TO 38                                                          00012200
   35 IF(BN(II).GT.5000.)GO TO 36                                       00012210
      DDF(II)=0.1186*ALOG10(BN(II))+0.6413                              00012220
      GO TO 38                                                          00012230
   36 DDF(II)=0.2767*ALOG10(BN(II))+0.0565                              00012240
      GO TO 38                                                          00012250
   33 IF(BN(II).GT.5000.)GO TO 34                                       00012260
      DDF(II)=0.1371*ALOG10(BN(II))+0.7029                              00012270
      GO TO 38                                                          00012280
   34 DDF(II)=0.3151*ALOG10(BN(II))+0.0445                              00012290
      GO TO 38                                                          00012300
   31 IF(BN(II).GT.5000.)GO TO 32                                       00012310
      DDF(II)=0.1445*ALOG10(BN(II))+0.7655                              00012320
      GO TO 38                                                          00012330
   32 DDF(II)=0.3382*ALOG10(BN(II))+0.049                               00012340
   38 CONTINUE                                                          00012350
      XY = (DDF(II)-DF)/(CVDDF(JJ)*DF)                                  00012360
      XX=XY                                                             00012370
      IF(XX.LT.0.)XX=-XX                                                00012380
      ZX=EXP(-XX*XX/2.)/2.5066283                                       00012390
      TX=1./(1.+0.2316419*XX)                                           00012400
      PX=1.-ZX*TX*(0.31938153*TX+(-0.356563782+                         00012410
     &          TX*(1.781477937+TX*(-1.821255978+TX*1.330274429))))     00012420
      PR(JJ)=1.-PX                                                      00012430
      IF(XY.LT.0.)PR(JJ)=PX                                             00012440
   40 CONTINUE                                                          00012450
      WRITE(6,102)((M(JJ),PR(JJ),DDF(II),BN(II)),JJ=1,NOH)             00012460
  102 FORMAT(F12.2,2F12.4,F15.0)                                        00012470
      IF(JOPPL2.LT.1)GO TO 72                                           00012480
      CALL PLOTMJ(M,PR,1,NOH2)                                          00012490
   72 CONTINUE                                                          00012500
   50 CONTINUE                                                          00012510
      RETURN                                                            00012520
      END                                                               00012530
@FOR,IS PLOTUZ                                                          00012540
      SUBROUTINE PLOTUZ(X,Y,NN)                                         00012550
      DIMENSION X(33),Y(33)                                             00012560
      IF(NN) 10,10,20                                                   00012570
   10 DO 11 I=3,33                                                      00012580
      X(I)=0.                                                           00012590
   11 Y(I)=0.                                                           00012600
      X(1)=0.001                                                        00012610
      X(2)=6.999                                                        00012620
      Y(1)=0.001                                                        00012630
      Y(2)=0.9999                                                       00012640
      CALL PLOTC(1.0,2.0,-3)                                            00012650
      CALL SCALE(X,7.0,31,1)                                            00012660
      CALL SCALE(Y,5.0,31,1)                                            00012670
      CALL AXIS(0.0,0.0,13HLOG COVERAGES,-13,7.0,0.0,X(32),X(33))       00012680
      CALL AXIS(0.0,0.0,11HRELIABILITY,11,5.0,90.0,Y(32),Y(33))         00012690
      RETURN                                                            00012700
   20 CALL PLOTC(0.0,0.0,3)                                             00012710
      CALL LINE(X,Y,31,1,0,0)                                           00012720
      RETURN                                                            00012730
      END                                                               00012740
@FOR,IS PLOTMJ                                                          00012750
C                                                                       00012760
C                                                                       00012770
C                                                                       00012780
C                                                                       00012790
      SUBROUTINE PLOTMJ(X,Y,NN,NOH2)                                    00012800
C                                                                       00012810
C                                                                       00012820
C                                                                       00012830
C                                                                       00012840
      DIMENSION X(NOH2),Y(NOH2)                                         00012850
      NOH=NOH2-2                                                        00012860
      NOH1=NOH2-1                                                       00012870
      IF(NN)10,10,20                                                    00012880
   10 DO 11 I=1,NOH2                                                    00012890
   11 Y(I)=0.                                                           00012900
      Y(1)=0.001                                                        00012910
```

```
      V(2)=2.9999                                                                00012920
      CALL PAGEUP                                                                00012930
      CALL PLOTC(1.0,2.0,-3)                                                     00012940
      CALL SCALE(X,7.0,NOH,1)                                                    00012950
      CALL SCALE(Y,5.0,NOH,1)                                                    00012960
      CALL AXIS(0.0,0.0,14HSLAB THICKNESS,-14,7.0,0.0,X(NOH1),X(NOH2))           00012970
      CALL AXIS(0.0,0.0,11HRELIABILITY,11,5.0,90.0,Y(NOH1),Y(NOH2))              00012980
      RETURN                                                                     00012990
   20 CALL PLOTC(0.0,0.0,3)                                                      00013000
      CALL LINE(X,Y,NOH,1,0,0)                                                   00013010
      RETURN                                                                     00013020
      END                                                                        00013030
@FOR,IS GJR                                                                      00013040
      SUBROUTINE GJR(A,NC,NR,N,MC,S,JC,V)                                        00013050
      DIMENSION A(NR,NC),JC(1),V(2)                                              00013060
C     ------------------------------------------------------------------------ 00013070
C      JC IS THE PERMUTATION VECTOR                                              00013080
C     KD IS THE OPTION KEY FOR DETERMINANT EVALUATION                            00013090
C     KI IS THE OPTION KEY FOR MATRIX INVERSION                                  00013100
C     L IS THE COLUMN CONTROL FOR AX=B                                           00013110
C     M IS THE COLUMN CONTOL FOR MATRIX INVERSION                                00013120
C     ------------------------------------------------------------------------ 00013130
C     INITIALIZATION                                                             00013140
C     ------------------------------------------------------------------------ 00013150
      IW=V(1)                                                                    00013160
      M=1                                                                        00013170
      S=1.                                                                       00013180
      L=N+(MC-N)*(IW/4)                                                          00013190
      KD=2-MOD(IW/2,2)                                                           00013200
      IF(KD.EQ.1) V(2)=0.                                                        00013210
      KI=2-MOD(IW,2)                                                             00013220
      GO TO (5,20),KI                                                            00013230
C     ------------------------------------------------------------------------ 00013240
C         INITIALIZE JC FOR INVERSION                                            00013250
C     ------------------------------------------------------------------------ 00013260
    5 DO 10  I=1,N                                                               00013270
   10 JC(I)=I                                                                    00013280
C     ------------------------------------------------------------------------ 00013290
C     SEARCH FOR PIVOT ROW                                                       00013300
C     ------------------------------------------------------------------------ 00013310
   20 DO 91  I=1,N                                                               00013320
      GO TO (22,21),KI                                                           00013330
   21 M=1                                                                        00013340
   22 IF (I.:0.M)  GO TO 60                                                      00013350
      X=-1.                                                                      00013360
      DO 30  J=I,N                                                               00013370
      IF (X.GT.ABS(A(J,I)))  GO TO 30                                            00013380
      X=ABS(A(J,I))                                                              00013390
      K=J                                                                        00013400
   30 CONTINUE                                                                   00013410
      IF(K.EQ.I) GO TO 60                                                        00013420
      S=-S                                                                       00013430
      V(1)=-V(1)                                                                 00013440
      GO TO (35,40),KI                                                           00013450
   35 MU=JC(I)                                                                   00013460
      JC(I)=JC(K)                                                                00013470
      JC(K)=MU                                                                   00013480
C     ------------------------------------------------------------------------ 00013490
C     INTERCHANGE ROW I AND ROW K                                                00013500
C     ------------------------------------------------------------------------ 00013510
   40 DO 50  J=M,L                                                               00013520
      X=A(I,J)                                                                   00013530
      A(I,J)=A(K,J)                                                              00013540
   50 A(K,J)=X                                                                   00013550
C     ------------------------------------------------------------------------ 00013560
C     TEST FOR SINGULARITY                                                       00013570
C     ------------------------------------------------------------------------ 00013580
   60 IF (ABS(A(I,I)).GT.1.E-7) GO TO 70                                         00013590
C     ------------------------------------------------------------------------ 00013600
C     MATRIX IS SINGULAR                                                         00013610
C     ------------------------------------------------------------------------ 00013620
      IF(KD.EQ.1) V(1)=0.                                                        00013630
      JC(1)=1-1                                                                  00013640
      RETURN 6                                                                   00013650
   70 GO TO (71,72),KD                                                           00013660
C     ------------------------------------------------------------------------ 00013670
```

```
C                COMPUTE THE DETERMINANT                                        00013680
C                ---------------------------------------------------------------00013690
71      IF(A(I,I).LT.0.) S=-S                                                   00013700
        V(2)=V(2)+ALOG(ABS(A(I,I)))                                             00013710
72      X=A(I,I)                                                                00013720
        A(I,I)=1.                                                               00013730
C                ---------------------------------------------------------------00013740
CC               REDUCTION OF THE I-TH ROW                                      00013750
C                ---------------------------------------------------------------00013760
        DO 80 J=M,L                                                             00013770
        A(I,J)=A(I,J)/X                                                         00013780
C                ---------------------------------------------------------------00013790
CCC              TEST OVERFLOW SWITCH. IF ON                                    00013800
CC               RETURN NEGATIVE VALUE OF I IN JC(1)                            00013810
C                ---------------------------------------------------------------00013820
        IF(ABS(A(I,J)).GT.10**75) IFL=1                                         00013830
        IF(IFL.EQ.1) GO TO 150                                                  00013840
80      CONTINUE                                                                00013850
C                ---------------------------------------------------------------00013860
CC               REDUCTION OF ALL REMAINING ROWS                                00013870
C                ---------------------------------------------------------------00013880
        DO 91  K=1,N                                                            00013890
        IF (K.EQ.I)  GO TO 91                                                   00013900
        X=A(K,I)                                                                00013910
        A(K,I)=0.                                                               00013920
        DO 90  J=M,L                                                            00013930
        A(K,J)=A(K,J)-X*A(I,J)                                                  00013940
C                ---------------------------------------------------------------00013950
CCC              TEST OVERFLOW SWITCH. IF ON                                    00013960
CC               RETURN NEGATIVE VALUE OF I IN JC(1)                            00013970
C                ---------------------------------------------------------------00013980
        IF(ABS(A(K,J)).GT.10**75) IFL=1                                         00013990
        IF(IFL.EQ.1) GO TO 150                                                  00014000
90      CONTINUE                                                                00014010
91      CONTINUE                                                                00014020
C                ---------------------------------------------------------------00014030
C                AX=B AND DET.(A) ARE NOW COMPUTED                              00014040
C                ---------------------------------------------------------------00014050
        GO TO (95,140),KI                                                       00014060
C                ---------------------------------------------------------------00014070
C                PERMUTATION OF THE COLUMNS FOR MATRIX INVERSION                00014080
C                ---------------------------------------------------------------00014090
95      DO 130  J=1,N                                                           00014100
        IF (JC(J).EQ.J)  GO TO 130                                              00014110
        JJ=J+1                                                                  00014120
        DO 100  I=JJ,N                                                          00014130
        IF (JC(I).EQ.J)  GO TO 110                                              00014140
100     CONTINUE                                                                00014150
110     JC(I)=JC(J)                                                             00014160
        DO 120  K=1,N                                                           00014170
        X=A(K,I)                                                                00014180
        A(K,I)=A(K,J)                                                           00014190
120     A(K,J)=X                                                                00014200
130     CONTINUE                                                                00014210
140     JC(1)=N                                                                 00014220
        IF(KD.EQ.1) V(1)=S                                                      00014230
        RETURN                                                                  00014240
150     JC(1)=1-I                                                               00014250
        IF(KD.EQ.1) V(1)=S                                                      00014260
        RETURN 6                                                                00014270
        END                                                                     00014280
**

$RESUME,E
```

VOLUME IV

PROBABILISTIC ANALYSIS OF RIGID AIRFIELD DESIGN BY ELASTIC LAYERED THEORY

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* Copyrighted program. Information on the program can be obtained from the authors of this report.

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Chapter 1

INTRODUCTION

General

The design of military rigid (plain and reinforced) pavements is currently based upon the classical Westergaard free edge stress slab theory. It is this method that has historically been the basis for design methods found in present US Army Technical Manuals and U.S. Air Force Manuals. In addition to this approach, recent research conducted by Parker et al. (1) at the USACE Waterways Experiment Station has demonstrated that an equivalent, if not superior, design methodology based upon multi-layer linear elastic theory is also another feasible design approach for rigid pavements.

At present, both of these design methods are deterministic in that a unique pavement system is designed for the specific set of input variables necessary to solve the problem. In a deterministic model, all of these input variables are also unique in terms of their input magnitude.

Study Objective

The objective of the research study reported in this volume was to include the design parameter variability into the USACE design procedure based upon multi-layer elastic theory. This probabilistic analysis is based upon the design procedure developed by Parker et al. (1). The probabilistic methodology for the Westergaard based design method is presented in Volume III of this overall research study. The mathematical and analytical development of both probabilistic approaches used in this Volume and Volume III (Westergaard) are presented in Volume II of the research study.

Report Organization

The method of development used to establish the probabilistic multi-layer elastic theory design approach necessitated the investigation of determining an elastic theory based "Composite Modulus" for a multi-layered foundation system (i.e. a subbase layer over an existing subgrade soil). Because, this in itself, required a significant effort and investigation, this volume is subdivided into two major chapters. They are:

Chapter 2: Composite Modulus

Chapter 3: Probabilistic Analysis of PCC Airfield Design

In essence, Chapter 2 presents the solution to the "composite modulus" problem, which is an integral part of the overall probabilistic analysis presented in Chapter 3. The "composite modulus" has been evaluated from the BISAR multi-layered elastic theory computer code. As will be noted, this parameter is expressed not only in terms of the layer thickness and material properties but the specific loading conditions (aircraft) as well.

Chapter 3 dealing with the probabilistic analysis presents the development of the regression equation used for maximum stress computations used to evaluate the variability of the pavement design. Two solution approaches have been developed: (a) the approximate closed - form using first order Taylor series expansion and (b) Monte Carlo simulation. A computer program has been developed to solve this probabilistic approach and run example problems, a User's Guide and program listing are presented in Appendices.

Chapter 2

COMPOSITE MODULUS

Introduction

The concept of a composite modulus (of elasticity) was introduced to take into account the combined effect of a layered (subbase/subgrade) system underneath the rigid pavement on its performance. This parameter is commonly used in pavement design and evaluation and several methods were developed for computing the composite modulus as a function of layer thicknesses and moduli. It should be noted that the composite modulus, like the Equivalent Single Wheel Load, is defined as an equivalent modulus that will ultimately lead to the same correct response as with the original system of layers. Therefore, like the ESWL parameter, the composite modulus may be expected to depend upon: (1) the pavement response (maximum stress, strain or deflection) chosen for equivalency computations; (2) the load configuration and (3) the pavement geometry (layer thickness and moduli). It is stressed that all variables should be initially evaluated. Deleting anyone variable from the relation can be made only if its effect is found to be negligible. This chapter presents the evaluation of the composite modulus of elasticity for the case of a layered system. It is subdivided into the following sections:

(1) A brief description and discussion of the method for deriving the composite subgrade modulus of reaction (Westergaard model) in the current rigid pavement design system;

(2) A study of the effect of the composite modulus on rigid pavement performance in order to assess the degree of accuracy needed in the composite modulus evaluation;

(3) A comprehensive study of the effect of all design variables on the composite modulus of elasticity. All variables of the pavement geometry are included in a regression equation relating the composite modulus for one single wheel load to the pavement layer thicknesses and moduli. The load configuration effect is included in a separate equation relating the composite modulus for the given gear configuration to the composite modulus for one single wheel load.

Although, the composite modulus is not mandatory for the elastic layered systems (it is possible to compute the stress for the original system), it is of practical and economic interest to use the composite modulus in order to assess the value of additional layers and to simplify the design procedure.

Current U.S. Corp of Engineers Method for Determining the Composite Subgrade Modulus of Reaction (k_c Value)

In the current U.S.A.C.E. method (2) separate diagrams were developed for different materials. Figures 2.1 and 2.2 relate the composite subgrade modulus of reaction (denoted k_c) for specific unbound granular materials (well-graded crushed material and natural sand and gravel (PI < 8), respectively) to the base layer thickness and subgrade modulus of reaction. In Figure 2.3, the relationship is given for a broad range of subbase materials which are characterized through their modulus of elasticity. As explained in Volume III, the partially bonded rigid overlay pavement design equation for h_{doc} could also have been used.

Results of Figures 2.1 and 2.2 are based on field plate tests, while those of Figure 2.3 are based on computations using the elastic layered theory and correction procedures (3). In both diagrams, the equivalency criterion is deflection or stiffness, i.e., the composite

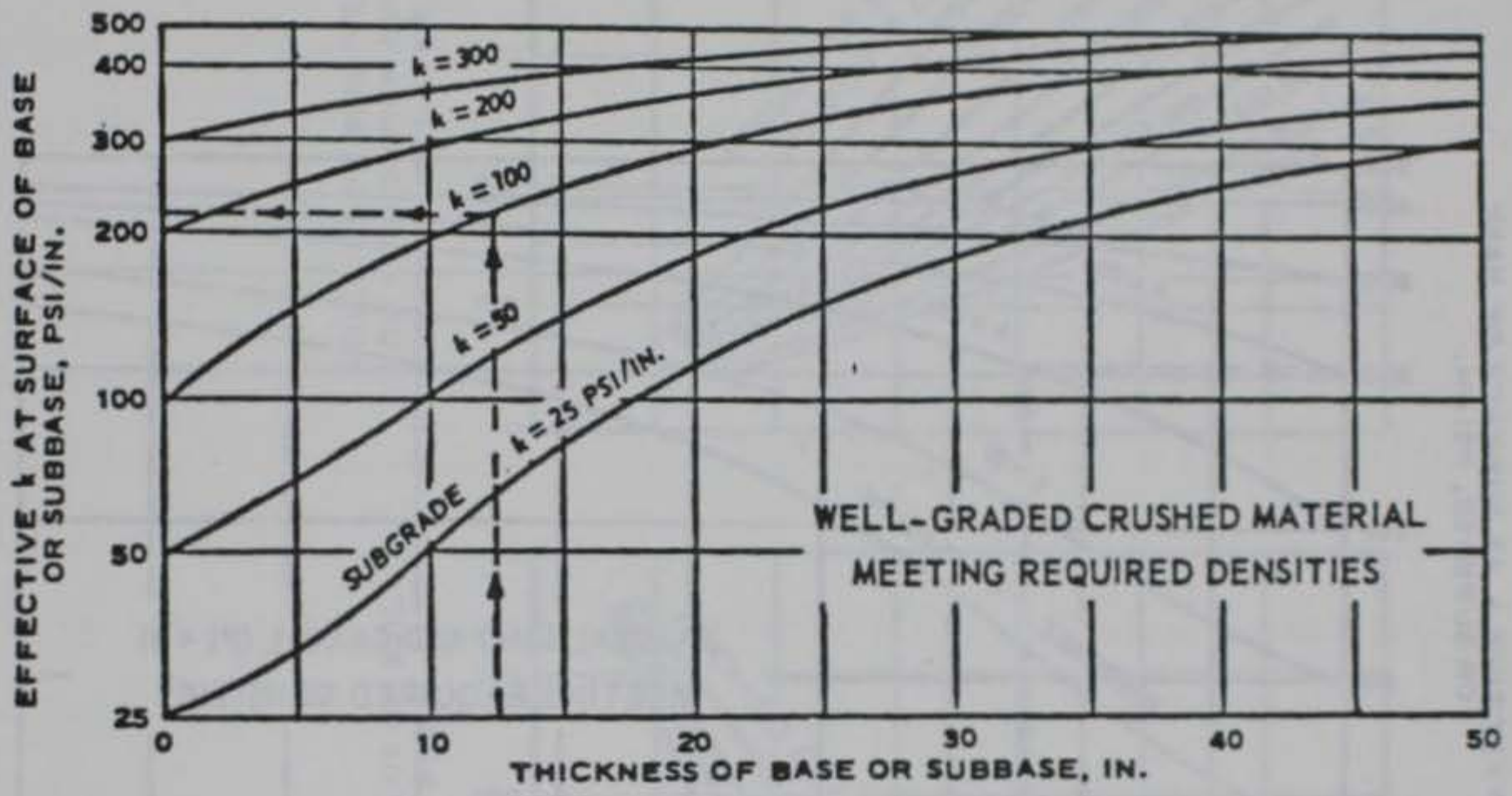

Figure 2.1 COMPOSITE SUBGRADE MODULUS OF REACTION FOR WELL-GRADED CRUSHED MATERIAL

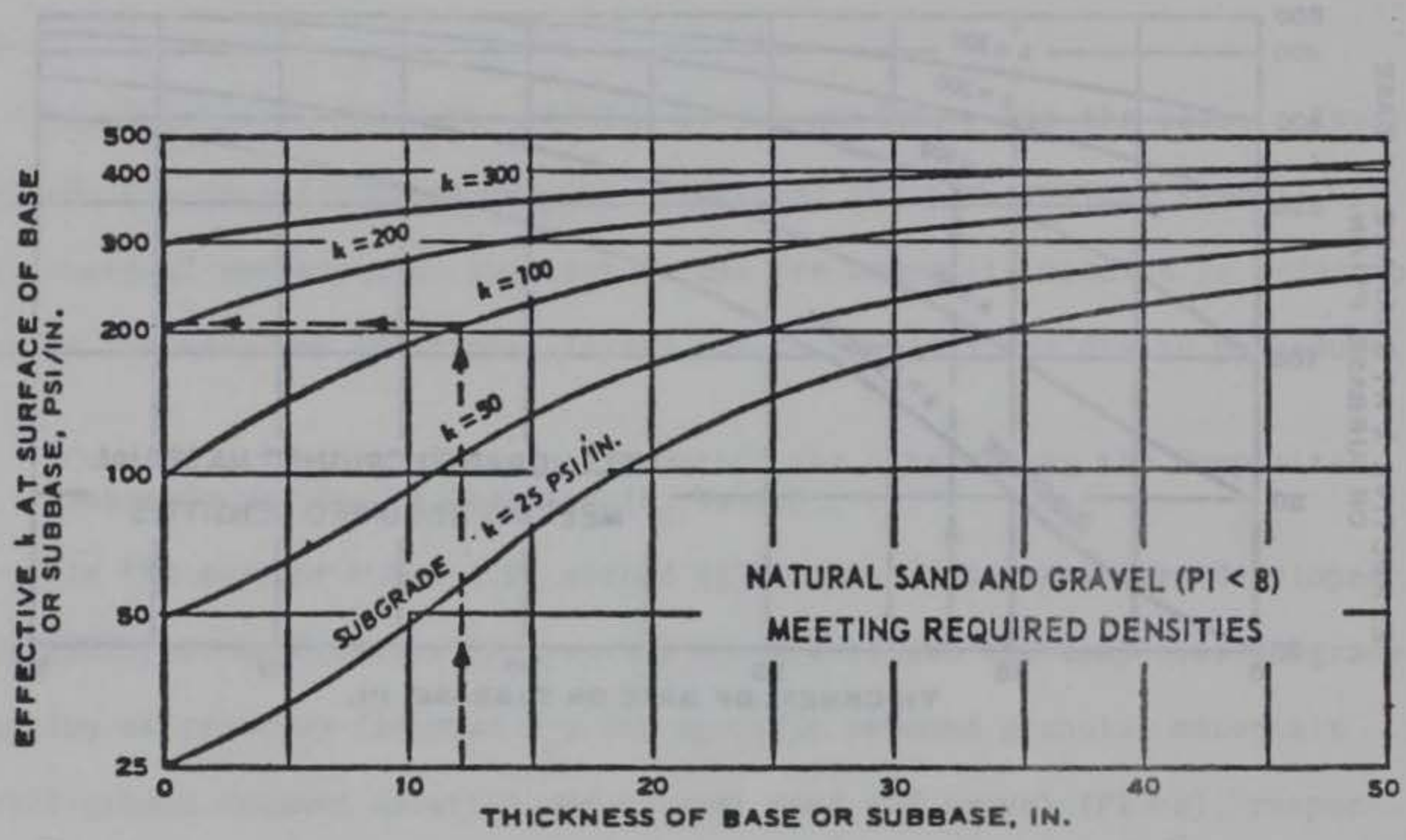

Figure 2.2 COMPOSITE SUBGRADE MODULUS OF REACTION FOR NATURAL SAND AND GRAVEL

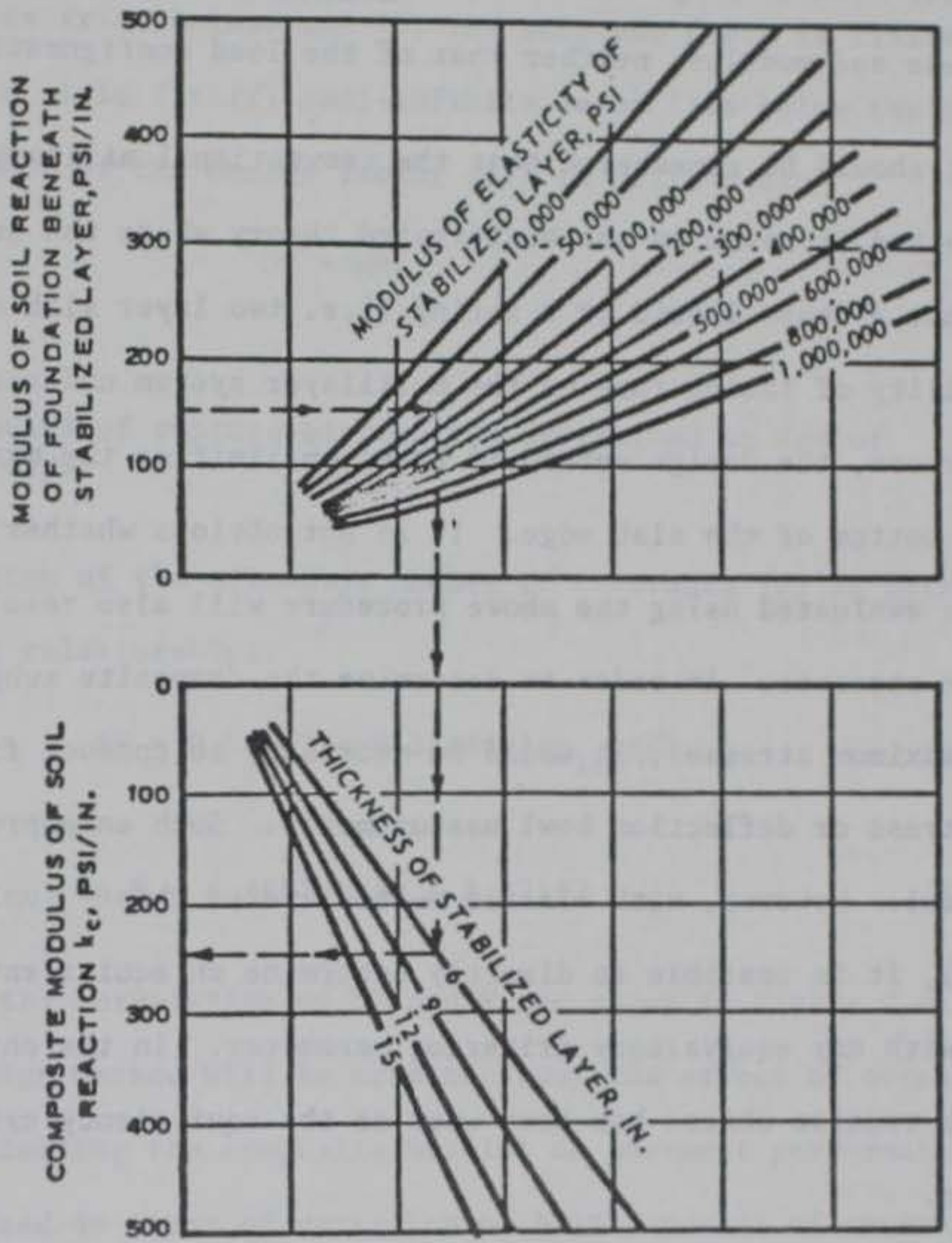

Figure 2.3 COMPOSITE SUBGRADE MODULUS OF REACTION FOR DIFFERENT BASE-SUBBASE MATERIALS

modulus of subgrade corresponds to equal stiffnesses of the equivalent subgrade and that of the original base-subgrade system. Therefore, the composite modulus computation scheme does not include the effect of slab thickness and modulus, neither that of the load configuration.

It should be remembered that the conventional military rigid pavement design method is based on the Westergaard theory where the subgrade is represented by either a dense liquid or a spring (i.e. two layer slab system). There is no possibility of taking care of the multilayer system underneath the pavement slab. Furthermore, the design method is based on limiting the maximum tensile stress at the bottom of the slab edge. It is not obvious whether the composite subgrade modulus evaluated using the above procedure will also result in equal maximum tensile stresses. In order to determine the composite subgrade modulus for equal maximum stresses, it would be necessary to conduct field tests on slabs, with stress or deflection bowl measurements. Such an approach would not be practical. However, with elastic multi-layered theory (unlike the Westergaard theory), it is possible to directly determine an equivalent or composite modulus, with any equivalency criterion parameter. In the ensuing analysis, the maximum tensile stress has been used as the equivalency criterion for obvious reasons.

Study of the effect of Composite Modulus on Rigid Pavement Performance

An alternative design procedure, using multi-layered elastic systems and center loading, to the existing one using Westergaard theory and edge loading has been developed by the U.S.A.C.E. (1). The salient features of this design analysis include:

(1) Computation of the maximum tensile stress (σ) at the bottom of the concrete slab, using elastic layered system theory. In these computations, it was assumed that (a) the interface between the concrete slab and the base or subgrade layer is frictionless and (b) the subgrade depth is finite and equals 20 ft; (c) a rigid (stiff) semi-infinite layer lies below the subgrade.

(2) Computation of the design factor (DF) is defined as:

$$DF = MR/\sigma \tag{2.1}$$

where

MR = modulus of rupture of concrete determined at age of 90 days.

(3) Computation of the allowable number of coverages (N) is obtained from the following relationships:

$$DF = 0.58901 + 0.35486 \log_{10} (N) \tag{2.2}$$

or

$$\ell n\ N = 6.4887\ MR/\sigma - 3.82192 \tag{2.3}$$

The data and the correlation of DF and N are shown in Figure 2.4.

The above design method will be used to study the effect of overestimating or underestimating the composite modulus on pavement performance. This can be expressed in terms of variation of N (the number of coverages) as a function of the variation of E_{comp} (the composite modulus). Substituting the function of σ (a function of the slab thickness and modulus, the composite or subgrade modulus and the loading conditions) into Equation 2.3 leads to the relationship between N and E_{comp}.

Expressing σ in its general form as:

$$\sigma = \sigma (h_1\ ,\ E_1/E_{comp},\ LOAD) \tag{2.4}$$

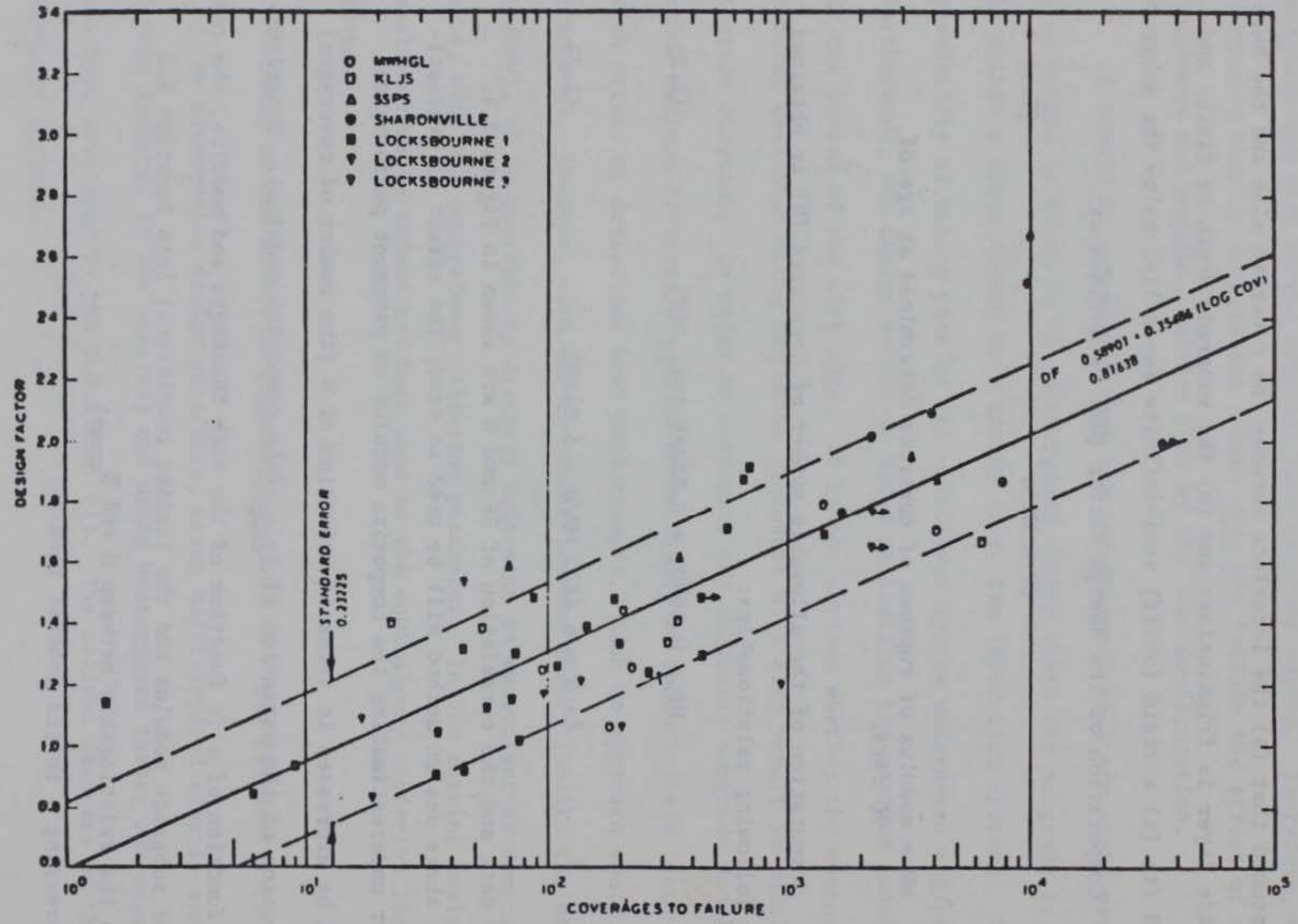

Figure 2.4 PERFORMANCE CRITERIA BASED ON DESIGN FACTOR

where

h_1 = slab thickness

E_1 = concrete modulus

E_{comp} = composite modulus subgrade modulus

LOAD = loading parameters

and combining equations 2.3 and 2.4 lead to:

$$\ell nN = 6.4887 \frac{MR}{\sigma(h_1, E_1/E_{comp}, LOAD)} - 3.82192 \qquad (2.5)$$

It is seen that the effect of varying E_{comp} on N is not a simple function. It will vary with all pavement and load variables. Equation 2.5 is used to compute the deviation of N due to a deviation of E_{comp} in the following way:

(a) For a given aircraft, slab thickness, modulus and E_{comp}, compute N_o;

(b) Compute N_1 and N_2 at $(0.8\ E_{comp})$ and $(1.2\ E_{comp})$ corresponding to 20% deviation is E_{comp};

(c) Compute the relative deviation $100\ (N_2 - N_1) \div (\frac{N_o}{2})$;

(d) Repeat computations (b) and (c) for different percentage 15, 10 and 5% of E_{comp} deviations;

(e) Repeat computations for different sets of pavement variables.

(f) Repeat computations for different aircrafts.

Results of this analysis are given in Tables 2.1.a to 2.1.e for 5 different aircrafts AG-2, AG-9, AG-10, AG-12 and AG-13 where E_1 and MR are assumed equal to 4,000,000 and 700 psi respectively. It can be seen that the relative variation of N for N values between 100 and 100000 is 1.25 to 5.0

times that of E_{comp}. The magnifying factor increases with increasing N (corresponding also to increasing slab thickness and/or to increasing E_{comp}). It is larger for the heavier aircrafts (AG-9 to 13) than for the lighter aircraft (AG-2). Therefore, it is concluded that the design procedure is very sensitive to E_{comp} and more attention should be given to the determination of E_{comp}. Care should be taken in the use of E_{comp} since there is no maximum value of E_{comp} which may be used as there is with the composite modulus of subgrade reaction (k_{max} = 500 pci). It should be noted, however, that h is still the major governing factor. For example, if the computation of N is kept within 50% accuracy, determination of E_{comp} should be limited within less than 40% accuracy for the light aircraft and within less than 10% accuracy for the heavier aircrafts. The above conclusion must be taken into account in the design procedure and any applications of the procedure as discussed in the following paragraphs.

TABLE 2.1a: Effect of Composite Modulus Deviation on Number of Coverages for AG-2

Slab Thickness in.	Composite or Subgrade Modulus, psi	Number of Coverages-N	Relative Deviation of N for Different Relative Derivations of E_{comp} %			
			20	15	10	5
6	50,000	25	31	23	16	8
8	50,000	980	42	31	21	10
10	50,000	82,620	55	41	27	14
8	20,000	190	31	23	15	8
10	20,000	9500	41	31	20	10
12	20,000	1,070,000	54	40	27	13
8	10,000	73	25	18	12	6
10	10,000	2,650	33	25	16	8
12	10,000	203,000	43	32	21	11
8	5,000	34	19	14	10	5
10	5,000	967	25	19	13	6
12	5,000	54,950	32	24	16	8

TABLE 2.1b: Effect of Composite Modulus Deviation on Number of Coverages for AG-9

Slab Thickness, in.	Compute or Subgrade Modulus, psi	Number of Coverage-N	Relative Deivation of N for Different Relative Deviations of E_{comp} %			
			20	15	10	5
8	50,000	200	64	48	32	16
10	50,000	2,100	84	61	40	20
12	50,000	2,450	105	76	49	24
14	50,000	322,000	130	91	58	28
8	20,000	16	47	35	23	11
10	20,000	95	57	42	28	14
12	20,000	630	67	49	33	16
14	20,000	4,700	78	57	37	19
16	20,000	41,000	90	65	42	21
18	20,000	427,000	102	73	47	23
10	10,000	18	41	30	20	10
12	10,000	92	48	36	24	12
14	10,000	520	56	41	27	14
16	10,000	3,500	63	46	31	15
18	10,000	28,200	69	51	33	17
20	10,000	292,000	74	54	36	18
12	5,000	22	37	28	18	9
14	5,000	100	42	32	21	11
16	5,000	560	47	35	23	12
18	5,000	3930	50	37	24	12
20	5,000	38,400	49	36	24	12

TABLE 2.1c: Effect of Composite Modulus Deviation on Number of Coverages for AG-10

Slab Thickness, in	Composite or Subgrade Modulus, psi	Number of Coverages N	Relative Deviation of N for Different Relative Deviations of E_{comp}, %			
			20	15	10	5
8	50,000	5,000	89	65	43	21
10	50,000	102,500	119	85	55	27
8	20,000	170	64	48	31	16
10	20,000	1,580	80	59	39	19
12	20,000	16,400	99	71	46	23
14	20,000	188,000	120	85	55	27
8	10,000	26	48	36	24	12
10	10,000	157	60	45	29	15
12	10,000	1,040	73	54	35	18
14	10,000	7,470	87	63	41	20
16	10,000	59,200	101	73	47	23
18	10,000	520,000	116	82	53	26
10	5,000	26	46	35	23	11
12	5,000	121	55	41	27	13
14	5,000	630	63	46	31	15
16	5,000	3,800	68	50	33	16
18	5,000	26,600	70	51	34	17
20	5,000	236,000	66	48	31	15

TABLE 2.1d: Effect of Composite Modulus Deviation on Number of Coverages for AG-12

Slab Thickness, in.	Composite or Subgrade Modulus, psi	Number of Coverages N	Relative Deviation of N for Different Relative Deviations of E_{comp}, %			
			20	15	10	5
10	50,000	1,350	71	53	35	17
12	50,000	16,800	92	67	44	22
14	50,000	243,000	117	84	54	26
10	20,000	82	53	39	26	13
12	20,000	550	65	48	32	16
14	20,000	4,060	79	58	38	19
16	20,000	33,000	94	68	45	22
18	20,000	294,000	112	80	52	25
10	10,000	17	40	30	20	10
12	10,000	80	50	37	24	12
14	10,000	410	60	44	29	15
16	10,000	2,300	71	52	34	17
18	10,000	13,600	83	61	40	20
20	10,000	87,000	96	70	45	22
12	5,000	18	39	29	20	10
14	5,000	68	47	35	23	12
16	5,000	276	55	41	27	14
18	5,000	1,200	64	47	31	16
20	5,000	5,700	72	53	35	17
22	5,000	29,200	79	58	38	19
24	5,000	166,000	85	62	40	20

TABLE 2.1e: Effect of Composite Modulus Deviation on Number of Coverages for AG-13

Slab Thickness in.	Composite or Subgrade Modulus, psi	Number of Coverages N	Relative Deviation of N for Different Relative Deviation of E_{comp}, %			
			20	15	10	5
11	50,000	52	52	39	26	13
13	50,000	277	62	46	30	15
15	50,000	1,550	72	53	35	17
17	50,000	9,400	82	60	39	20
19	50,000	63,300	94	68	44	22
13	20,000	28	41	31	21	10
15	20,000	112	48	35	23	12
17	20,000	488	54	40	26	13
19	20,000	2,330	60	45	30	15
21	20,000	12,500	68	50	33	16
23	20,000	77,700	76	55	36	18
15	10,000	27	35	26	18	9
17	10,000	100	40	30	20	10
19	10,000	400	45	33	22	11
21	10,000	1,780	50	37	24	12
23	10,000	9,080	55	40	27	13
25	10,000	55,300	60	44	29	15
17	5,000	30	31	23	16	8
19	5,000	103	35	26	17	9
21	5,000	402	38	28	19	9
23	5,000	1,800	41	31	20	10
25	5,000	9,750	43	32	22	11

Determination of the Composite Modulus

The composite modulus is determined on the basis of equal maximum tensile stress, i.e. the same maximum tensile stress will occur when analyzing either the original system composed of a concrete layer on a base-subbase-subgrade system or an equivalent system composed of the same concrete layer on the top of the "equivalent" composite subgrade. Like the ESWL, all pavement variables (h_i, E_i) and the loading (gear) variables may be expected to affect the composite modulus. In the following,it was assumed that the E_{comp} function was of the following mathematical form.

$$E_{comp} = E_3 \, f_1^{\,f_2} \qquad (2.6)$$

where

f_1 = a function of the pavement geometry only, under the application of one wheel load only

f_2 = a function of the loading (gear) conditions only

Composite Modulus for One Wheel Load

The following range of variables were included in the computation of the maximum tensile stress and computation of the E_{comp} for one wheel load.

(a) Wheel load of AG-2, AG-4, AG-6, AG-9, AG-10, AG-12 and AG-13;

(b) h_1 - thickness of concrete layer varying from 6 in. to 25 in.; for each aircraft group, four thicknesses covering the whole range of expected design thicknesses (see Table 2.2);

(c) h_2 - thickness of base-subbase layer of 6 and 12 in;

(d) E_2 - modulus of base-subbase layer of 50,000, 200,000 and 800,000 psi to include stabilized material, as well as unbound materials;

(e) E_3 - modulus of subgrade of 4000, 12,000 and 35,000 psi.

The form of the function f_1 was chosen to include the limiting case of $h_2=0$ and $E_2=E_3$ where no base-subbase layer exists. Several functions of the pavement geometry variables were assumed. The following equation gave the most satisfactory correlation coefficient and standard error of estimate:

$$\ell n(E_{comp}/E_3)_1 = a_1 \ell n\frac{E_2}{E_3} +$$

$$\ell n\alpha \{a_2\ell nh_1 + a_3\ell n\frac{E_1}{E_3} + a_4\ell n(h_2+1) + a_5\ell n\frac{E_2}{E_3}$$

$$+ a_9(\ell nh_1)^2 + a_{10}\ell nh_1\ell n\frac{E_1}{E_3} + a_{11}(\ell n\frac{E_1}{E_3})^2 \} +$$

$$(\ell n\alpha)^2 \{a_6\ell nh_1 + a_7\ell n(h_2 + 1) + a_8\ell n\frac{E_2}{E_3}\} \qquad (2.7)$$

where

$$\alpha = \sqrt[3]{\frac{E_2}{E_3}} \cdot (h_2 + 1)$$

$(E_{comp}/E_3)_1$ = the ratio between E_{comp} and E_3 for one wheel load

$a_1 \ldots a_{11}$ - regression coefficients

$a_1 = 0.434783$, $a_2 = -0.134317$, $a_3 = 0.0816432$,

$a_4 = -0.0807450$, $a_5 = -0.190090$, $a_6 = -0.164512$,

$a_7 = 0.119533$, $a_8 = 0.0550536$, $a_9 = 0.0943027$,

$a_{10} = 0.0102537$, $a_{11} = -0.0118395$

The R^2 of the multiple correlation was 0.994 and the standard error of estimate 0.0776 corresponded to a standard deviation of 8.0% in the E_{comp} evaluation. It should be noted that the equation has a zero intercept and

$\ell n\alpha$, $\ell n\frac{E_2}{E_3}$ equal 0 when $h_2 = 0$ and $E_2 = E_3$.

A simpler equation, with 5 terms only was also derived:

$$\ell n(f_1/E_3) = (\ell n\alpha)^2 \{a_1' \ell nh_1 + a_2' \ell n(h_2+1) + a_3' \ell n\frac{E_2}{E_3}\} + \qquad (2.8)$$

$$\ell n\alpha \quad \{ a_4'(\ell nh_1)^2 + a_5'(\ell n\frac{E_1}{E_3})^2 \}$$

where

$$a_1' = -0.161317, \quad a_2' = 0.0977117, \quad a_3' = 0.0358696$$
$$a_4' = 0.0802967, \quad a_5' = -0.00196301$$

with $R^2 = 0.991$ and standard error of estimate = 0.095 corresponding to a standard deviation of 10% in the E_{comp} evaluation. It should be noted that (1) the contribution of the terms with h_1 and E_1/E_3 to the regression was relatively substantial, partly due to the wide range of h_1 and E_1/E_3 included in the analysis; (2) the general trend of the effect of pavement geometry on the E_{comp} parameter was as follows: (a) E_{comp}/E_3 increases as h_2 and/or E_2/E_3 increases; (b) E_{comp}/E_3 decreases as h_1 increases and/or E_1/E_3 decreases; (3) within the range of the wheel loads used, it seems that the relationship is independent of the radius of the contact area; (4) for the 378 data points used in the regression the deviation in evaluating E_{comp} with the regression equation exceeded 25% in some but few cases. Therefore, depending on the accuracy level required for predicting the number of coverages, equation 2.7 may or may not be adequate for design purposes. According to the above analysis of the effect of composite modulus deviation on number of coverages, a deviation of 10 percent in E_{comp} evaluation resulted in about 25 and 40 percent deviation in the expected number of coverages for the light and heavy loads respectively. A procedure for improving the composite modulus evaluation was developed,

based on only one run of the BISAR program. The procedure, presented in Chapter 3 must be used at least for all heavy aircrafts; (5) equation 2.8 predicts the dependence of E_{comp} upon the pavement geometry variables quite well. Because it is easier to differentiate, this equation was used in the probabilistic approach, in the evaluation of the variance.

Composite Modulus for Different Aircrafts

The effect of loading (gear) conditions, that is, the number of wheels and wheel configuration is expressed in Equation 2.6 by the f_2-function. For one wheel load, $f_2 = 1$ (i.e. for AG-1, 2 and 3). The loading conditions seem to have a similar effect on E_{comp} as h_1, the concrete layer thickness. From previous paragraphs, it was stated that while all other variables are kept constant, increasing the h_1 resulted in a decrease of the E_{comp} term. This can be interpretated as follow: increasing h_1 increases the pavement stiffness, the radius of relative stiffness (ℓ) and the size of the deflection bowl; The relative base-subbase layer thickness h_2/ℓ decreases, entraining a reduction of its effect on E_{comp}. The same relation between E_{comp} and number of wheels exists: when the number of wheel loads increases, the deflection bowl size increases, and the relative base-subbase layer thickness decreases. Therefore, the f_2-function may logically be expected to decrease as the number of wheels increases. Instead of expressing f_2 for each aircraft group, it was decided to simplify the analysis of f_2 and confine it to three types of wheel loading configuration, (namely two, four and six gear wheels). Figures 2.5 to 2.9 show the relationship between the E_{comp}/E_3 parameter for one wheel load and E_{comp}/E_3 for AG-4, 6, 9, 13 and 10. A simple log-log function without intercept was fitted to the computation results for the two, four and six wheel assemblies, (AG-4 & 6, 9 & 13 and 10 respectively):

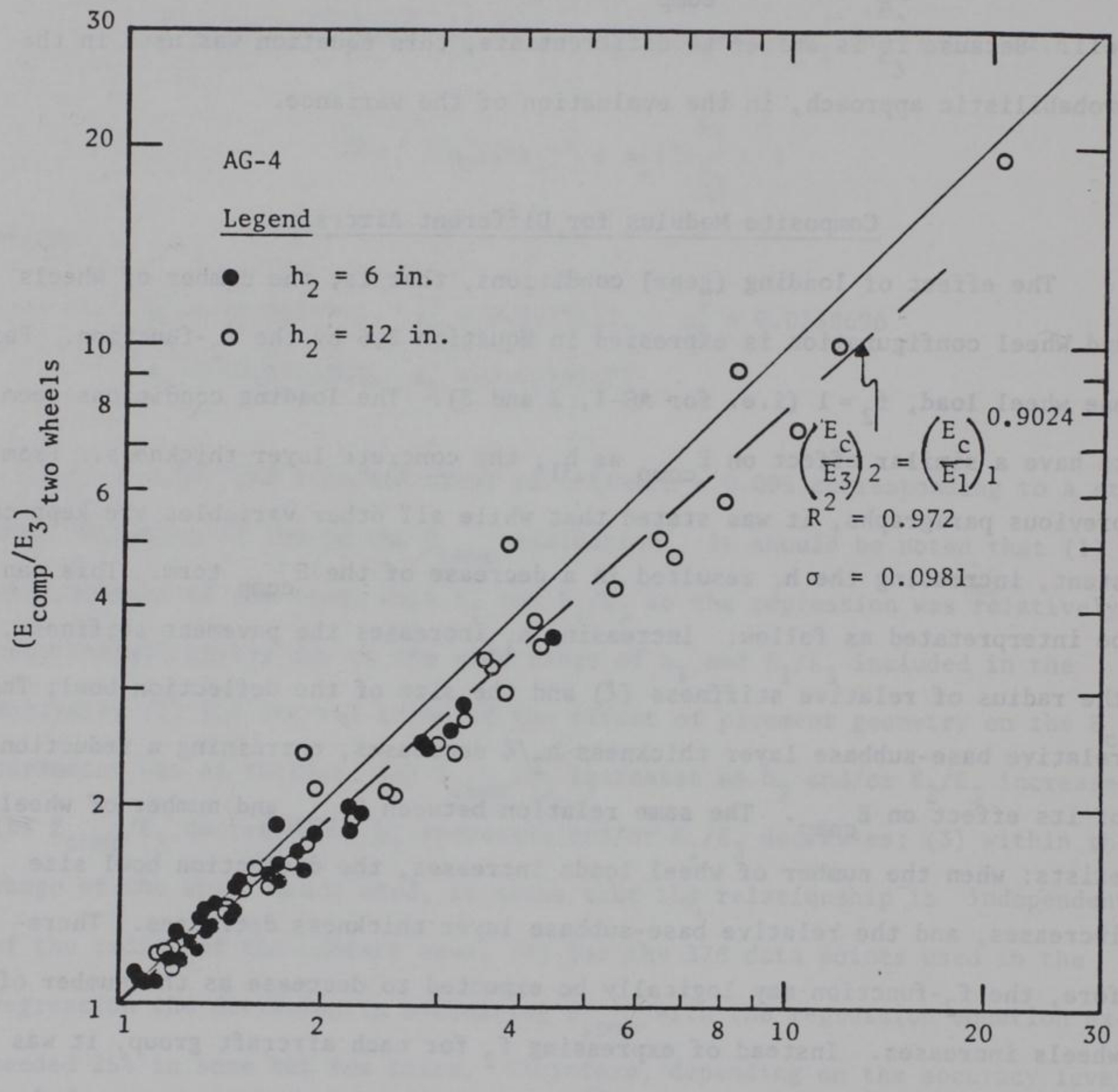

Figure 2.5 THE f_2 - FUNCTION FOR AG-4

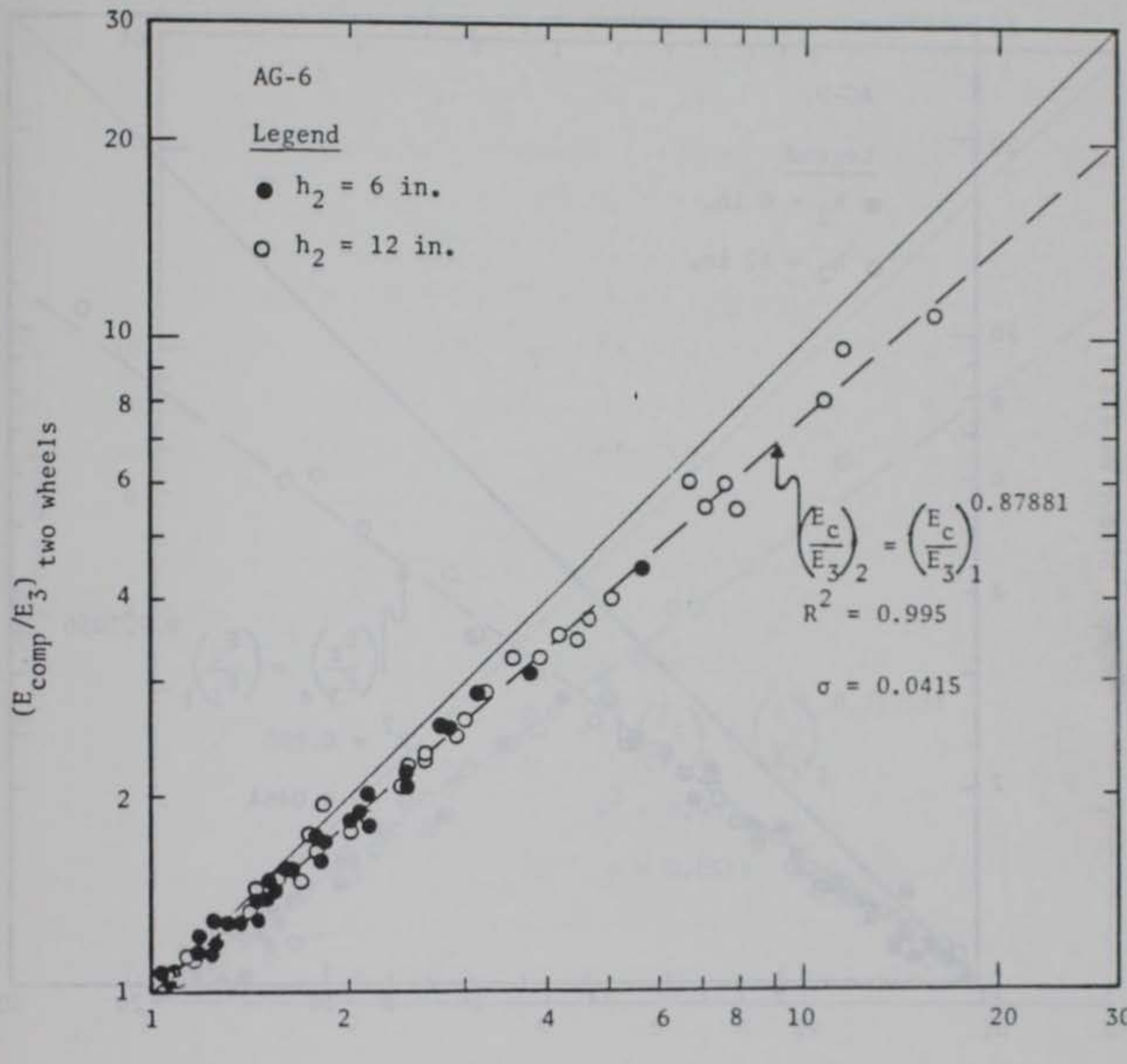

Figure 2.6 THE f_2 - FUNCTION FOR AG-6

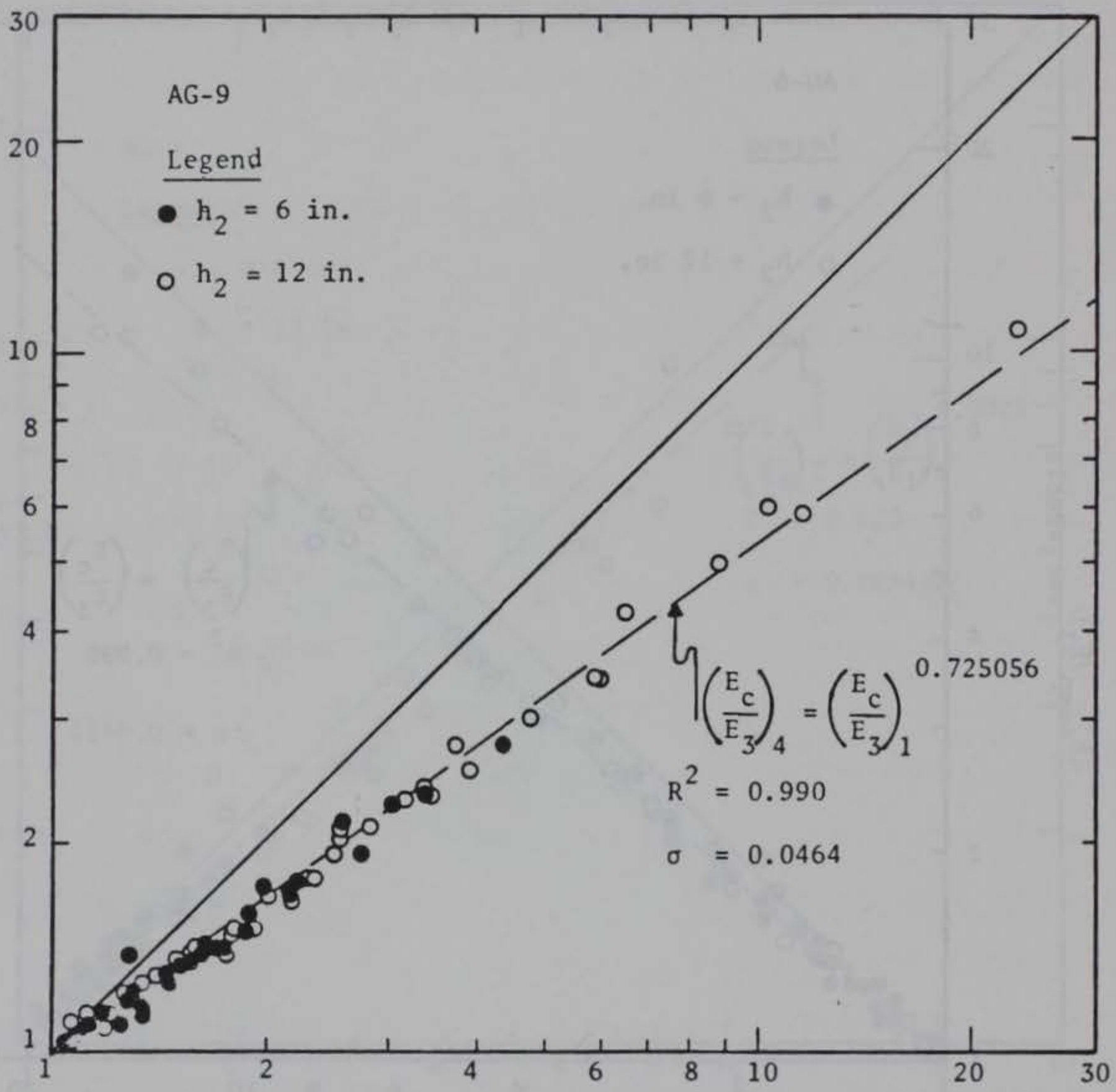

Figure 2.7 THE f_2 - FUNCTION FOR AG-9

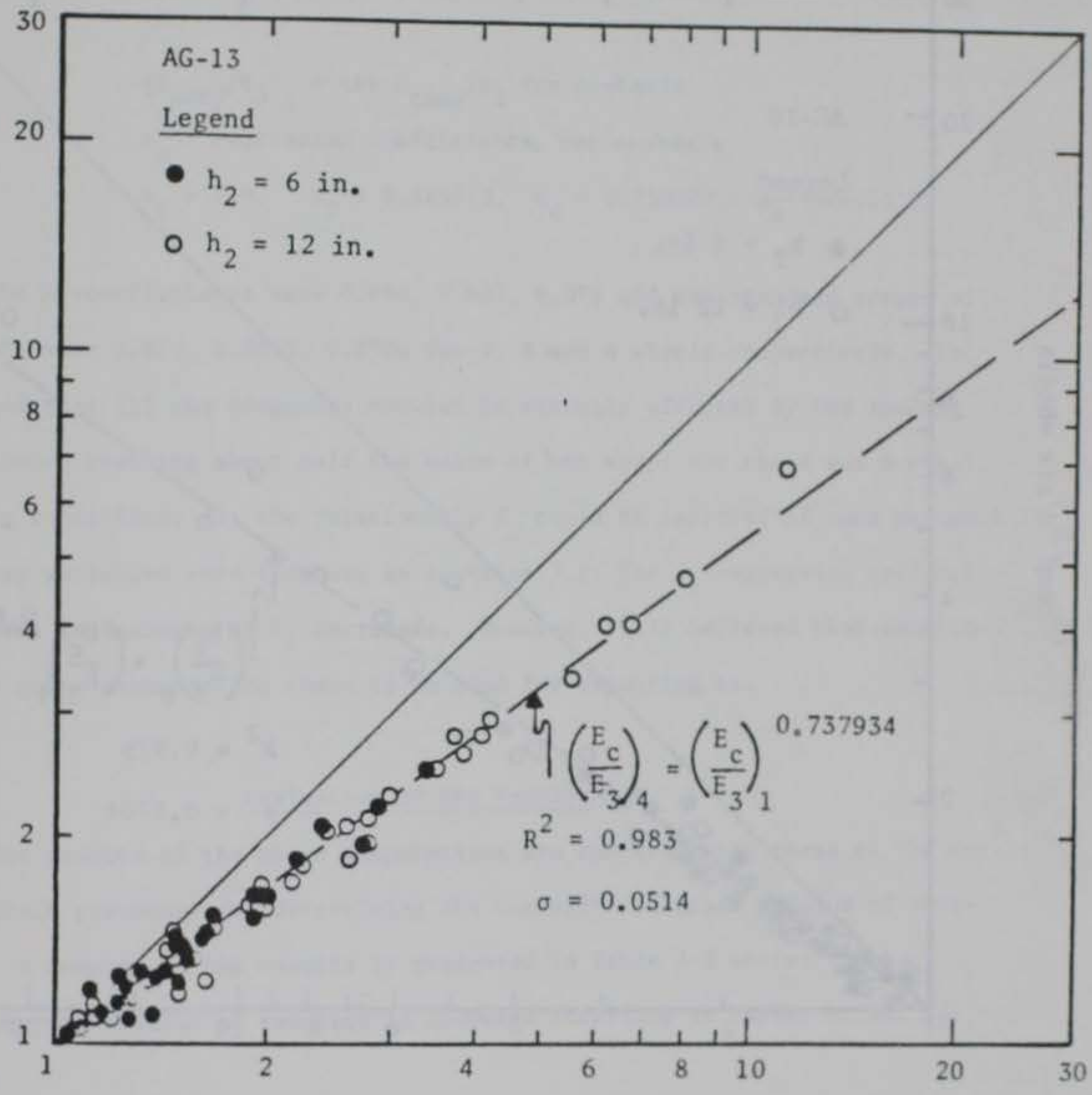

Figure 2.8 THE f_2 - FUNCTION FOR AG-13

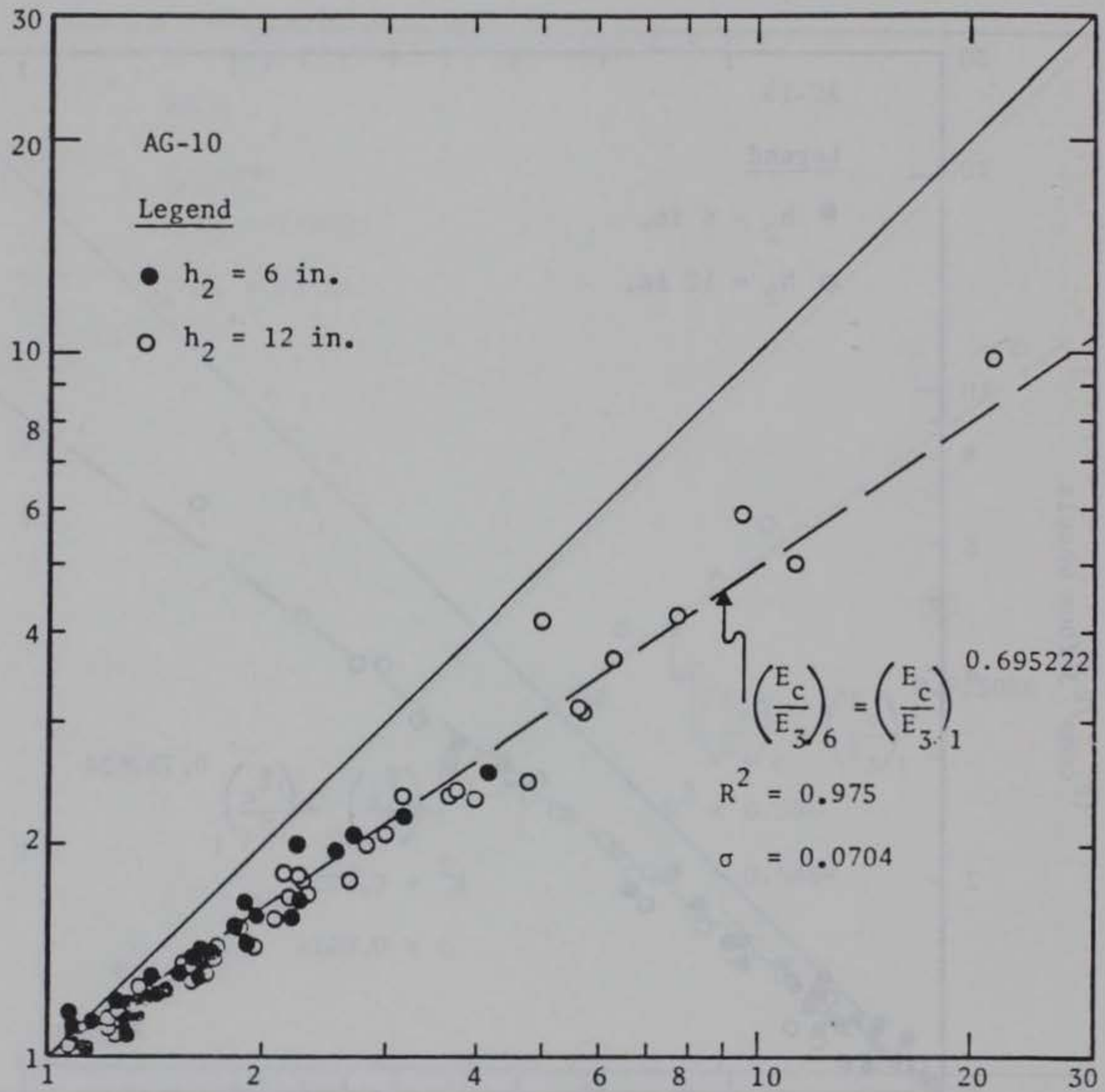

Figure 2.9 THE f_2 - FUNCTION FOR AG-10

$$\left(\frac{E_c}{E_3}\right)_n = \left(\frac{E_c}{E_3}\right)_1^{a_n}$$

$$\ell n(E_{comp}/E_3)_n = a_n \ell n(E_{comp}/E_3)_1 \qquad (2.9)$$

where

$(E_{comp}/E_3)_n$ = the E_{comp}/E_3 for n-wheels

a_n = regression coefficients, for n-wheels

$a_1 = 1.0, \quad a_2 = 0.889813, \quad a_4 = 0.730507, \quad a_6 = 0.695222$

The R^2-coefficients were 0.984, 0.987, 0.975 and the standard errors of estimate were 0.073, 0.0493, 0.0704 for 2, 4 and 6 wheels respectively. It is noted that (1) the composite modulus is strongly affected by the loading conditions, reaching about half the value of one wheel for the 4 and 6-wheel loading conditions; (2) the relationship f_2 could be improved if some pavement geometry variables were included in equation 2.9: The a_n-regression coefficient seems to increase as h_1 increases. However, it is believed that equation 2.9 is quite accurate and there is no need for expanding it.

Evaluation of the Results

The results of the above computations are compared with those of the current USACE procedure for determining the composite subgrade modulus of reaction. A summary of the results is presented in Table 2.3 where: (1) k_{sg}-the subgrade modulus of reaction is computed according to Parker et al. (1), using:

$$\log k_{sg} = (\log E_3 - 1.415)/1.284 \qquad (2.10)$$

(2) k_{comp} - the composite subgrade modulus of reaction is evaluated from Figure 2.3; (3) $k(E_{comp})$ is evaluated using equation 2.10 and substituting E_{comp} for E_3; (4) the range of $k(E_{comp})$ includes all aircrafts and concrete layer thicknesses given in Table 2.2. It should be noted that the upper limit corresponds to the light aircraft (AG-2) and very thin concrete layer (6") while

TABLE 2.2: Summary of Concrete Layer Thickness Used in the Analysis of Composite Modulus for One Wheel

Aircraft Group	Concrete Layer Thickness, in.
2	6
4	7,10,12,15
6	7,10,13,17
9	8,12,15,19
10	8,12,15,20
12	10,15,19,24
13	11,16,20,25

TABLE 2.3: Comparison of Composite Subgrade Moduli of Reaction

h_2,in	E_2,psi	E_3,psi	$k_{sg}(E_3)$,pci	k_{comp},pci	$k(E_{comp})$,pci
6	50,000	35,000	275	360	278-295
		12,000	120	160	124-154
		4,000	50	50	53- 75
	200,000	35,000	275	490	307-389
		12,000	120	220	134-207
		4,000	50	60	65-113
	800,000	35,000	275	(970)	315-584
		12,000	120	390	165-349
		4,000	50	120	78-226
12	50,000	35,000	275	(550)	282-317
		12,000	120	240	134-202
		4,000	50	65	63-120
	200,000	35,000	275	(720)	342-577
		12,000	120	340	174-397
		4,000	50	105	78-285
	800,000	35,000	275	(1270)	428-1352
		12,000	120	(580)	242-1125
		4,000	50	200	133-984

Note: Numbers in parentheses are obtained from extrapolation in Figure 2.3 . The value should be 500 pci

the lower limit corresponds to heavy aircrafts and thick concrete layer.

The values of k_{comp} for weak subgrade were evaluated by extrapolating the results of Figure 2.3. For the case of $E_3 = 4000$ psi ($k_{sg} = 50$ pci) and $E_2 = 50{,}000$ psi, higher k_{comp} values are obtained from Figure 2.2 than from extrapolating Figure 2.3.

Comparison of the k_{comp} and $k(E_{comp})$ in Table 2.3 shows that: (1) for weak subgrades, the $k(E_{comp})$ is either in the range of or higher than the k_{comp} evaluated with the current USACE procedure. However, if the extreme cases of very thin concrete layer were excluded from $k(E_{comp})$ and the k_{comp} were derived from Figure 2.2, the discrepancy would be quite substantially reduced; (2) for relatively strong subgrades, the $k(E_{comp})$ is lower than the k_{comp}, and exceeds 500 pci in few cases only. This result seems to support the current USACE procedure where the subgrade modulus of reaction (or the composite) is limited to a maximum value of 500 psi. It appears that the computation results of the composite modulus are in general agreement with the current USACE k_{max} value, ahthough more research studies are necessary to fully support this general observation. Furthermore, the present derivation takes into account all of the design variables which appear to have different effects on the pavement performance.

Summary and Conclusions

A composite modulus computation scheme has been presented that includes all pavement geometry and loading condition variables. It was found that the composite modulus of elasticity is not only a function of the base-subbase and subgrade layers, but also a function of the concrete layer and load configuration.

The study of the effect of E_{comp} on allowable number of coverages (or pavement performance) clearly showed that the degree of accuracy achieved in

the design (predicted number of coverages) is strongly related to that achieved in E_{comp} evaluation. The relative deviation in the allowable number of coverages is 1.2 to 5 times the relative deviation in E_{comp}.

These two results- of the function E_{comp} being dependent upon all variables and that of the sensitivity analysis, emphasize the importance of the evaluation of E_{comp}, and give quantitative relationships to determine the required degree of accuracy.

Chapter 3

PROBABILISTIC ANALYSIS OF PCC AIRFIELD DESIGN USING ELASTIC LAYERED THEORY

Introduction

The analysis presented in this chapter is based on the mathematical formulation given in Volume II, namely the approximate closed-form probabilistic approach and the probabilistic simulation approach. Both methods require stress computations which can be handled with computer programs such as the Shell BISAR code. However, while the use of a computer program is justified for the design of a particular pavement, it does become unrealistic and uneconomical for the probabilistic approach, due to the large computer time needed. The analysis was therefore confined to specific critical aircraft type included in the military (USAF) aircraft classification scheme (AG-1 to 13). Solutions for other aircraft type may be developed by following the procedure noted below and developing an equation for the maximum tensile stress.

The probabilistic approach presented in the following paragraphs includes:

(1) Stress computations and derivation of an equation for the maximum tensile stress for each of 13 aircraft types: USAF classification AG-1 to 13;

(2) Derivation of the relationships between variances of the dependent and independent variables for the approximate closed-form approach. The linear or first order Taylor series expansion is assumed as presented in Volume II.

(3) Formulation of a simulation model which bypasses the linear assumption in the closed-form approach.

(4) Run examples, to compare between the closed-form approach and the simulation one, at different variation levels of the design parameters.

Stress Computations

In the "normal" linear elastic layered system, the stress at any point is a function of the layer thicknesses (expressed in terms of the radius of contact area) and of the modular ratios of the layers and the subgrade. However, because of the assumptions made by Parker et al. (1) concerning the depth of the subgrade, the modulus of the rigid layer underneath the subgrade (chosen to be equal to 1,000,000 psi), and the value of the friction coefficient at the interface between the concrete and base on subgrade layers, it is not possible to use dimensionless variables. Therefore stress computation were conducted for all 13 aircrafts of the USAF classification scheme (AG-1 to 13) for the following ranges of variables (see Table 3.1):

(1) Modulus of concrete - E_1 = 4,000, 000 psi and Poisson's ratio of 0.2;

(2) Concrete layer thickness - h_1, choosen to cover a wide range of design possibilities. Four different values were assigned for each aircraft group, as shown in Table 3.1;

(3) Subgrade moduli or composite subgrade moduli of 4,000, 12,000, 35,000 and 100,000 psi and Poisson's ratio of 0.4.

(4) Subgrade thickness of 20 ft = 240 in.

(5) Elastic modulus of the stiff infinite layer beneath the subgrade of 1,000,000 psi.

(6) Friction coefficient (interface compliance) at the interface of the concrete and subgrade layer of 1,000. (defined in the BISAR program).

Table 3.1 Summary of Input Data for Stress Computations

AG No.	Radius of Contact area in.	Contact Pressure psi	Wheel Coordinates			Max.Stress Under Wheel No	Concrete Layer Thicknesses in.
			Wheel No	X	Y		
1	9.3560	98.1818	1	0.0	0.0	1	6 9 11 14
2	5.6419	270.0	1	0.0	0.0	1	6 9 11 14
3	8.7586	186.722	1	0.0	0.0	1	7 10 12 15
4	11.2838	87.1875	1	0.0	0.0	1	7 10 12 15
			2	60.0	0.0		
5	7.2471	155.4545	1	0.0	0.0	1	7 10 13 17
			2	26.0	0.0		
6	7.4422	156.9684	1	0.0	0.0	1	7 10 13 17
			2	30.5	0.0		
7	8.6856	190.4008	1	0.0	0.0	1	8 11 14 18
			2	34.0	0.0		
8	8.3302	183.0275	1	0.0	0.0	1	8 12 15 19
			2	34.5	0.0		
			3	0.0	56.0		
			4	34.5	56.0		
9	8.1369	186.6587	1	0.0	0.0	1	8 12 15 19
			2	32.5	0.0		
			3	0.0	48.0		
			4	32.5	48.0		
11	9.6738	180.3061	1	0.0	0.0	1	8 12 15 20
			2	54.0	0.0		
			3	0.0	64.0		
			4	54.0	64.0		
12	8.8310	188.5714	1	0.0	0.0	4	10 15 19 24
			2	44.0	0.0		
			3	0.0	58.0		
			4	44.0	58.0		
13	9.2189	233.7079	1	-37.0	0.0	2	11 16 20 25
			2	0.0	0.0		
			3	62.0	0.0		
			4	99.0	0.0		
10	9.5246	105.6728	1	34.0	0.0	2	8 12 15 20
			2	0.0	0.0		
			3	-53.0	0.0		
			4	-87.0	0.0		
			5	-2.50	65.0		
			6	-50.5	65.0		

The general form of the maximum tensile stress at the bottom of the concrete layer is given by:

$$\sigma = \sigma(h_1, E_1, E_{comp}, \text{loading conditions}) \quad (3.1)$$

where E_{comp} = the composite modulus of subgrade is equal to the subgrade modulus when no base-subbase layer exists.

The computation results for each aircraft were used to develop a regression equation for prediction the maximum tensile stress for any combination of pavement geometry variables. The following model fitted all thirteen aircrafts:

$$\sigma = a_o + a_H h + a_E E + a_{EH} Eh + a_{E2H} E^2 h + a_1 \frac{\ell n\beta}{h} + a_2/\beta + a_3 \frac{\ell n\beta}{\beta h} + a_4 \left(\frac{\ell n\beta}{h}\right)^2 + a_5/\beta^2 \quad (3.2)$$

where

- $h = h_1$
- $E = E_1/E_{comp}$
- $\beta = h\sqrt[3]{E}$
- h_1 = concrete layer thickness, inch.
- E_1 = modulus of concrete, psi.
- E_{comp} = composite or subgrade modulus, psi.
- σ = maximum tensile stress, psi.
- a_i = regression coefficients.

The values of the regression coefficient, R^2 and the relative error of estimate are given in Table 3.2. It is seen that the regression equation is an excellent one, giving a maximum standard error of 0.8%.

Table 3.2 Regression Coefficients, R^2 and Standard Error of Estimate for Stress Computations

AG	a_o	a_H	$a_{H'}$	a_{EH}	a_{E2H}	a_1	a_2	a_3	a_4	a_5	R^2	σ-%
1	47.13551	-3.114693	-0.02416481	0.0009148971	0.	0.	1640.099	-10162.07	1849.660	7426.127	.99999	0.21
2	-29.09671	-0.8760680	-0.03832524	0.001772930	0.	0.	1456.306	-808.1055	2288.042	-21056.89	.99999	0.17
3	284.7195	-11.03868	-0.04843520	0.	0.0000017	-415.1382	0.	-12672.95	3433.536	20821.21	.99997	0.29
4	51.76199	-2.852370	0.06405583	-0.002479894	-0.0000010724	217.1736	0.	-20064.09	2127.983	72434.43	.99997	0.28
5	84.40505	-3.809808	0.	-0.00071885	0.	0.	628.4619	-18569.88	2751.819	55943.80	.99994	0.44
6	103.4132	-4.370129	0.	-0.00078082	0.	0.	0.	-19993.78	2785.334	75051.80	.99994	0.44
7	158.8361	-6.001916	0.	-0.001021292	0.	0.	0.	-35657.54	4331.092	137800.8	.99995	0.38
8	-49.08532	0.	0.1624640	-0.008325638	0.	949.1228	-5345.551	-41370.78	2769.408	310403.1	.99981	0.78
9	-66.52055	0.	0.1370117	-0.007595463	0.	1027.890	-3937.352	-52771.66	3098.378	342184.9	.99983	0.75
10	-167.6606	2.671640	0.2175972	-0.005508302	-0.0000029737	1103.715	-1893.149	-30801.11	1564.013	185562.8	.99985	0.67
11	-394.8007	7.124734	0.2578348	-0.006082276	-0.0000040731	1798.772	0.	-34739.42	1875.246	193755.1	.99980	0.80
12	-164.3000	2.081288	0.1425741	-0.002873140	-0.0000010815	1188.027	-2294.815	-46476.85	2808.230	298345.3	.99998	0.24
13	224.6386	-4.946335	0.07221752	-0.003123284	0.	0.	-5139.840	-75798.53	7112.995	460427.8	.99991	0.54

Although equation 3.2 contains quite a large number of terms in order to achieve the high degree of accuracy needed, it is quite easily programmable on micro-computers and calculators. It should be noted that: (a) the maximum tensile stress was found to be higher for a 6-wheel gear than for the 12-wheel gear of the C-5 (AG-10); (b) maximum tensile stress was found to correspond to a 4-wheel gear in some combinations of h and E and to an 8-wheel gear in other combinations for the B-747 (AG-12). Equation 3.2 expresses the maximum stress corresponding to either 4 or 8-wheel gear.

Equation 3.2 covers a quite wide range of values of the pavement design parameters and design analysis. It should be remembered that equation 3.2 applies for a two-layer system only. When the layered system underneath the concrete contains a base-subbase layer, it should be transformed into an equivalent subgrade and its composite modulus evaluated according to Chapter 2 of this volume.

The Approximate Closed Form Probabilistic Approach

In the approximate closed-form probabilistic approach, the average and the variance of the random variables are given by the following equations, assuming linear or first order TAylor series expansion of the variable function:

$$E[g(x_i)] = g(x_i)\Big|_{\bar{x}_i} \tag{3.3a}$$

$$E[(g(x_i)-\mu)^2] = Var[g(x_i)] = \sum_{i=1}^{n}\left(\frac{\partial g(x_i)}{\partial x_i}\Big|_{\bar{x}_i}\right)^2 Var[x_i] +$$

$$\sum_{i=1}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}\left(\frac{\partial g(x_i)}{\partial x_i}\Big|_{\bar{x}_i}\right)\left(\frac{\partial g(x_k)}{\partial x_k}\Big|_{\bar{x}_k}\right) Covar\,[x_i,x_k] \tag{3.3b}$$

where

$g(x_i)$ = random variable function of x_i variables

$\bar{x}_i$ = average of x_i

Higher orders of expectation of the random variable can also be computerd, for a complete description of the probability density distribution. In the following, it was assumed that the random variable is normally distributed and therefore, the higher order of expectations are not needed. Pavement performance, expressed in terms of number of coverages, is not likely normally distributed but rather closer to a log-normal distribution. The number of coverages can not be chosen as the independent variable. The next variable in the design sequence is the design factor whose distribution can be approximated by the normal one. Through a change of variable, this will lead to log-normal distribution for the number of coverages. Therefore, $g(x_i)$ in equation 3.3 represents DF, and x_i are the design variables namely MR, h_1, E_1, h_2, E_2, E_3 and LOAD. The design factor is expressed, for each aircraft:

$$DF = \frac{MR}{\sigma} \tag{3.4a}$$

$$\sigma = \sigma\ (h_1,\ E_1/E_{comp}) \tag{3.4b}$$

$$\left(\frac{E_{comp}}{E_3}\right)_n = f_1\left(\ h_1, h_2, \frac{E_1}{E_3},\ \frac{E_2}{E_3}\ \right)^{f_2} \tag{3.4c}$$

where the functions σ, f_1 and f_2 are given in equations 3.2, 2.7 and 2.9 respectively.

Assuming that only E_1 and MR are correlated and substituting equations

3.4 into 3.3b lead to the following expression of the coefficient of variation of DF:

$$CV^2[DF] = CV^2[MR] + \sum_{i=1}^{n} \left(\frac{x_i}{\sigma} \frac{\partial \sigma}{\partial x_i}\Big|_{\bar{x}_i} \cdot CV[x_i] \right)^2 +$$

$$-2 \frac{E_1}{\sigma} \frac{\partial \sigma}{\partial E_1} \rho[E_1, MR] . CV[MR] \cdot CV[E_1] \qquad (3.5)$$

where $x_i = h_1, h_2, E_1, E_2, E_3$

Computation of the derivatives of DF with respect to x_i for a given aircraft are as follows:

$$\frac{\partial DF}{\partial h_1} = - \frac{MR}{\sigma^2} \frac{\partial \sigma}{\partial h_1} \qquad (3.6a)$$

$$\frac{\partial DF}{\partial h_2} = - \frac{MR}{\sigma^2} \cdot \frac{\partial \sigma}{\partial h_2} \qquad (3.6b)$$

$$\frac{\partial DF}{\partial E_1} = - \frac{MR}{\sigma^2} \cdot \frac{\partial \sigma}{\partial E_1} \qquad (3.6c)$$

$$\frac{\partial DF}{\partial E_2} = - \frac{MR}{\sigma^2} \frac{\partial \sigma}{\partial E_{comp}} \cdot \frac{\partial E_{comp}}{\partial E_2} \qquad (3.6d)$$

$$\frac{\partial DF}{\partial E_3} = - \frac{MR}{\sigma^2} \cdot \frac{\partial \sigma}{\partial E_{comp}} \cdot \frac{\partial E_{comp}}{\partial E_3} \qquad (3.6e)$$

Using equations 3.2 and 2.8, the following final equations are obtained:

$$\frac{\partial\sigma}{\partial h_1} = \frac{\partial\sigma}{\partial h_1}\Big|_{E_{comp}} - \frac{\partial\sigma}{\partial(E_1/E_{comp})}\left[\frac{E_1E_3}{E_{comp}^2}\cdot\frac{\partial(E_{comp}/E_3)}{\partial h_1}\right] \quad (3.7a)$$

$$\frac{\partial\sigma}{\partial h_2} = -\frac{E_1E_3}{E_{comp}}\cdot\frac{\partial\sigma}{\partial(E_1/E_{comp})}\cdot\frac{\partial(E_{comp}/E_3)}{\partial h_2} \quad (3.7b)$$

$$\frac{\partial\sigma}{\partial E_1} = \frac{1}{E_{comp}}\frac{\partial\sigma}{\partial(E_1/E_{comp})}\left[1 - \frac{E_1}{E_{comp}}\frac{\partial(E_{comp}/E_3)}{\partial(E_1/E_3)}\right] \quad (3.7c)$$

$$\frac{\partial\sigma}{\partial E_2} = \frac{-E_1}{E_{comp}^2}\frac{\partial\sigma}{\partial(E_1/E_{comp})}\cdot\frac{\partial(E_{comp}/E_3)}{\partial(E_2/E_3)} \quad (3.7d)$$

$$\frac{\partial\sigma}{\partial E_3} = -\frac{E_1}{E_{comp}^2}\cdot\frac{\partial\sigma}{\partial(E_1/E_{comp})}\left\{1 - \frac{1}{E_3}\left[E_1\frac{\partial(E_{comp}/E_3)}{\partial(E_1/E_3)} + E_2\frac{\partial(E_{comp}/E_2)}{\partial(E_2/E_3)}\right]\right\} \quad (3.7e)$$

and

$$\frac{\partial\sigma}{\partial h_1}\Big|_{E_{comp}} = a_H + a_{EH}(E_1/E_{comp}) + a_{E2H}(E_1/E_{comp})^2 +$$

$$-\frac{\ell n\beta}{h_1^2}\left[a_1 + \frac{2a_3}{\beta} - \frac{2a_4}{h_1} + 2a_4\frac{\ell n\beta}{h_1}\right] + \frac{1}{h_1}\left[\frac{a_1}{h_1} - \frac{a_2}{\beta} + \frac{a_3}{h_1\beta} - \frac{2a_5}{\beta^2}\right] \quad (3.8a)$$

$$\frac{\partial\sigma}{\partial(E_1/E_{comp})} = a_E + a_{EH}\, h_1 + {}^2a_{E2H}\, h_1\, E_1/E_{comp} +$$

$$\frac{1}{3h_1(E_1/E_{comp})}\left[a_1 - \frac{a_2 h_1}{\beta} + \frac{a_3}{\beta}(1-\ell n\beta) + \frac{2a_4\,\ell n\beta}{h_1} - \frac{2a_5 h_1}{\beta^2} \right] \quad (3.8b)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial h_1} = a_n \left(\frac{E_{comp}}{E_3}\right)_n \cdot \frac{\ell n\alpha}{h_1}\ [a_1'\ell n\alpha + 2a_4'\ \ell n h_1] \quad (3.8c)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial h_2} = a_n \left(\frac{E_{comp}}{E_3}\right)_n \cdot \left\{ (\ell n\alpha)^2 \left(\frac{a'_2}{h_2+1}\right) + \right.$$

$$2\ell n\alpha \left[a_1'\ell n h_1 + a_2'\ell n(h_2+1) + a_3'\ell n \frac{E_2}{E_3} \right] +$$

$$\left. \frac{1}{h_2+1} \left[a_4'(\ell n h_1)^2 + a_5'\left(\ell n \frac{E_1}{E_3}\right)^2 \right] \right\} \quad (3.8d)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial(E_1/E_3)} = a_n \left(\frac{E_{comp}}{E_3}\right)_n \left[2a_5'\ell n\alpha \ell n \frac{E_1}{E_3} \,/\, (E_1/E_3) \right] \quad (3.8e)$$

$$\frac{\partial(E_{comp}/E_3)}{\partial(E_2/E_3)} = a_n \left(\frac{E_{comp}}{E_3}\right)_n \left\{ (\ell n\alpha)^2 \left(\frac{a_3'}{E_2/E_3}\right) + \right.$$

$$\frac{2}{3}\ \ell n\alpha \left[a_1'\ell n h_1 + a_2'\ell n(h_2+1) + a_3\ell n \frac{E_2}{E_3} \right] +$$

$$\left. \frac{1}{3E_2/E_3} \left[a_4' (\ell n h_1)^2 + a_5'\left(\ell n \frac{E_1}{E_2}\right)^2 \right] \right\} \quad (3.8f)$$

The probabilistic analysis of PCC airfield designs, based upon multilayered elastic theory, relies upon the determination of the probability density distribution of N for a given aircraft type and given pavement geometry. The general solution sequence is summarized below:

(1) Computer average stress ($\overline{\sigma}$) using equation 3.2 and average values of the design variables. (BISAR could have been used in the calculation of $\overline{\sigma}$; however, equation 3.2 has been used within the program in order to reduce the additional computer time which would have been required to run BISAR. BISAR is used later in the program if a corrected composite modulus is required.)

(2) Compute average DF (DF) as:

$$\overline{DF} = \overline{MR}/\overline{\sigma} \tag{3.9}$$

(3) Compute the variance of DF using equation 3.5 and the derivations in equations 3.6 to 3.8.

(4) Using $\overline{DF}$ and Var[DF], compute the cumulative distribution of DF from the normal distribution. Subdivide the interval of DF $\pm$ 3σ into say 30 DF_y values and compute the cumulative distribution P_y ($DF \leq DF_y$) corresponding to each DF_y.

(5) Compute N_y corresponding to DF_y using equation 2.3. The N_y is related to P_y, giving the cumulative distribution of number of coverages.

This scheme is implemented in a computer program (see Appendices I and II), and makes use of the regression equations developed for this purpose. However, it was shown that the degree of accuracy achieved in the prediction of the number of coverages depends upon the degree of accuracy in the composite modulus and stress computations. Because most of the error is due to the E_{comp} evaluation, a correction option for reducing the error was included in the program. The correction scheme, which corrects the value of E_{comp} to give an exact tensile stress is as follows:

(1) Input average design variables h_1, h_2, E_1, E_2 and E_3 and the design load using the format of the BISAR program.

(2) Compute maximum tensile stress $-\sigma'$ for the original layered system, using BISAR program.

(3) Using Newton-Raphson method for determination of roots of equation, compute $\overline{E}_{comp}$ corresponding to

$$\sigma - \sigma' = 0 \tag{3.10a}$$

where σ is given by equation 3.2.

(4) Correct E_{comp} as computed from equation 2.7 by adding a constant term

$$a_o = \overline{E}_{comp} - E_{comp} \tag{3.10b}$$

The correction scheme corresponds to a translation of the E_{comp} -function. This procedure does not affect the derivative of E_{comp} with respect to design variables. It is simple and requires only one run of the BISAR program.

The application of the probabilistic analysis and correction schemes will be illustrated through the example runs.

The Simulation Approach

The probabilistic analysis with the simulation approach includes 300 computation runs where the independent variables are randomly generated and the dependent variables DF and N, computed using equation 2.7, 2.3, 2.1 and 3.2. The number of runs (300) was choosen in order to insure a good description of the probability density distribution of both independent and dependent variables. In the following computations, it was assumed that the independent variables are normally distributed.

It should be noted that this assumption is not mandatory, and any given density distribution can be easily simulated. The generation of the random variables was made in the following steps, for each independent variable:

(1) generate n = 12 random decimal numbers with a uniform distribution from 0.0 to 1.0, a mean of 0.5 and a standard deviation of $\sigma = 1/\sqrt{12}$.

(2) From the Central Limit Theorem, the random variable defined as:

$$k = \frac{\sigma}{\sqrt{n/12}} \sum_{i=1}^{n} r_i + \left(\mu - \frac{n}{2}\frac{\sigma}{\sqrt{n/12}}\right) \qquad (3.11a)$$

is a random observation from an approximately normal distribution with mean μ and standard deviation σ. Choosing n = 12 for computation convenience, and $\mu = 0.0$ and $\sigma = 1.0$ for the standard normal distribution, leads to:

$$k = \left(\sum_{i=1}^{12} r_i\right) - 6 \qquad (3.11b)$$

(3) In order to insure closeness to a normal distribution, any set of 300-k whose standard deviation, skewness and kurtosis coefficients is outside the range of 0.95 to 1.05, -0.15 to 0.15 and 2.5 to 3.5 respectively is disregarded, and a new number set is generated.

(4) generate the independent random variables x_j from:

$$x_j = \bar{x}\,(1. + k_j \cdot CV[x]) \qquad (3.12)$$

where k_j = given by equation 3.11b

CV[x] = coefficient of variation of x

The above generated variables are substituted into equation 2.7, 2.3, 2.1 and 3.2 to give 300 random variables of DF_j and N_j. They are then

analyzed to derive their mean, standard deviation, and frequency distribution. Results of these analyses will be presented in the next paragraph and compared with those of the approximate closed-form approach.

It should be noted that simulation approach uses the same option for correction E_{comp} as in the approximate closed-form one. In addition, whenever h_2 (the base/subbase layer thickness) equals zero and E_2/E_3 (the ratio of base and subgrade moduli) equals one, the E_{comp} takes the values of the subgrade modulus.

Run Examples

Composite Modulus Cqrrection

Figures 3.1a to 3.1e show the input data and output results of a run example, illustrating the composite modulus correction methodology. The case dealt with is: AG-4 loading and a combination of pavement geometry variables corresponding to 20 percent deviation in the composite modulus evaluation. The exact computed stress from the BISAR program using the original pavement is 479.62 psi. The composite modulus computed from equation 2.7 for the given pavement geometry is 34777 psi, while a value of 28980 psi is necessary in order to get the correct stress of 479.62 from equation 3.2. Therefore a correction factor of a_o = -5796 psi is used to reduce the composite modulus evaluated from equation 2.7. When this correction factor is applied in the approximate closed-form approach, it leads to exact values of the average stress, design factor and number of coverages.

The corrected modulus replaces that evaluated from equation 2.7 in all subsequent computations. It should be noted that the correction has little effect on the derivatives evaluated from the regression equations.

```
UZAN*LL(1).DAWORK(3)
      4        RUN  EXAMPLE  WITH AG-4
      3
            7.0        0.02  4000000.        0.10
           6.00        0.10  600000.0        0.20
        12000.0        0.20
           700.        0.10        0.00
      0 1
       RUN EXAMPLE  COMPOSITE MODULUS CORRECTION
      1
      4  11
          4.0E+06      0.2E+00      0.7E+01      1.0E+03
          8.0E+05      0.2E+00      0.6E+01
          1.2E+08      0.4E+00      2.4E+02
          1.0E+06      0.4E+00
      2
     STRS  0.871875E+02 11.2336E+00      6.0E+01      0.0E+00
     STRS  0.871875E+02 11.2338E+00      0.0E+00      0.0E+00
              SRRSTTSZZSFTSRZSTZ
      1
      1      0.0E+00      0.0E+00      0.7E+01
      4        RUN  EXAMPLE  WITH AG-4
      3
            7.0        0.02  4000000.        0.10
           6.00        0.10  600000.0        0.20
        12000.0        0.20
           700.        0.10        0.00
      1 1
       RUN EXAMPLE  COMPOSITE MODULUS CORRECTION
      1
      4  11
          4.0E+06      0.2E+00      0.7E+01      1.0E+03
          8.0E+05      0.2E+00      0.6E+01
          1.2E+08      0.4E+00      2.4E+02
          1.0E+06      0.4E+00
      2
     STRS  0.871875E+02 11.2336E+00      6.0E+01      0.0E+00
     STRS  0.871875E+02 11.2338E+00      0.0E+00      0.0E+00
              SRRSTTSZZSFTSRZSTZ
      1
      1      0.0E+00      0.0E+00      0.7E+01
**

@COPY,4 LL.
 1 APS

@XQT,FO LL.APS
  AG- 4        RUN  EXAMPLE  WITH AG-4
```

Figure 3.1a INPUT DATA FOR RUN EXAMPLE AG-4

SYSTEM NUMBER 1

LAYER NUMBER	CALCULATION METHOD	YOUNG'S MODULUS	POISSON'S RATIO	THICKNESS	REDUCED SPRINGCONST
1	SMOOTH	.8[illegible]+07	.2[illegible]+00	.7000+01	.1000+08
2	SMOOTH	.8[illegible]+08	.2[illegible]+00	.6000+01	.0000
3	SMOOTH	.1[illegible]+15	.8[illegible]+00	.2800+03	.0000
4		.1[illegible]+07	.8[illegible]+00		

LOAD NUMBER	NORMAL STRESS	SHEAR STRESS	RADIUS OF LOADED AREA	LOAD - POSITION X	Y	SHEAR DIRECTION
1	.8719+03	.0000	.1123+02	.4000+02	.0000	.0000
2	.8719+03	.0000	.1123+02	.3000	.3033	.0000

Figure 3.1b SYSTEM DESCRIPTION - OUTPUT FOR RUN EXAMPLE

```
                                    POSITION NUMBER  1
                                      LAYER NUMBER   1
                                       COORDINATES
                                  X          Y          Z
                                .0000      .0000      .7000+01
          DISTANCE TO LOAD-AXIS( 1)                                  THETA
                  .6000+02                                             .3142+01
STRESSES
 RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
-.8310+02          .2992+02          -.6931+03            .0000             -.2103+01             .0000
STRAINS
 RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
-.1224-04          .9669-05           .4856-06                             -.6310-06
          DISTANCE TO LOAD-AXIS( 2)                                  THETA
                  .0000                                                .0000
STRESSES
 RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
 .4497+03          .4497+03          -.1951+02            .0000              .0000              .0000
STRAINS
 RADIAL          TANGENTIAL          VERTICAL          RAD./TANG.          RAD./VERT.          TANG./VERT.
 .9092-04          .9092-04          -.4985-04                              .0000
                  XX          YY          ZZ          YZ          XZ          XY          UX          UY          UZ
TOTAL STRESS     .407+03     .480+03    -.202+02     .000        .210+01     .000
TOTAL STRAIN     .787-04     .101-03    -.494-04     .000        .631-06     .000
TOTAL DISPLACEMENT
PRINCIPAL VALUES AND DIRECTIONS OF TOTAL STRESSES AND STRAINS
                NORMAL      NORMAL      SHEAR       SHEAR       X           Y           Z
                STRESS      STRAIN      STRESS      STRAIN      COMPONENT   COMPONENT   COMPONENT
MAXIMUM          .480+03     .101-03                             .000        1.000       .000
MINIMAX          .407+03     .787-04                             1.000       .000       .005
MINIMUM         -.202+02    -.494-04                             -.005       .000       1.000
MAXIMUM                                  .250+03     .750-04     .003       .707       -.707
                 .230+03                                         -.003       .707       .707
MINIMAX                                  .213+03     .640-04     .711       .000       -.704
                 .193+03                                          .704       .000       .711
MINIMUM                                  .365+02     .110-04    -.707       .707       -.003
                 .443+03                                          .707       .707       .003
                STRAIN ENERGY   .4062-01
STRAIN ENERGY OF DISTORTION     .2187-01
```

Figure 3.1c STRESS RESULTS - OUTPUT FOR RUN EXAMPLE

```
PREVIOUS ECOMP=     28777. CORRECTED ECOMP=     28587. CORRECTION =     -5796.
 CORRECTED AVERAGE STRESS FROM BISAR =          879.410

CONCRETE  MODULUS  =        4000000. PSI     CV=              .100
BASE      MODULUS  =         [illegible]. PSI     CV=              .200
SUBGRADE  MODULUS  =          12000. PSI     CV=              .200
SLAB   THICKNESS   =            7.00  IN.    CV=              .020
BASE   THICKNESS   =            6.00  IN     CV=              .100
MODULUS OF RUPTURE =             700. PSI     CV=              .100
REGRESSION COEFFICIENT=                                    .000

AVERAGE  STRESS                 =          879.46 PSI
AVERAGE  DESIGN FACTOR          =            1.86
COEFFICIENT OF VARIATION OF DF  =           .1229
AVERAGE NUMBER OF COVERAGES     =             2FB.
SQRT WEIGHTING FACTOR FOR H1    =          -.09252
SQRT WEIGHTING FACTOR FOR H2    =          -.76[illegible]
SQRT WEIGHTING FACTOR FOR E1    =           .33[illegible]
SQRT WEIGHTING FACTOR FOR E2    =          -.15[illegible]
SQRT WEIGHTING FACTOR FOR ES    =          -.18575

RELIABILITY     DESIGN FACTOR     N. OF COVERAGES
     .9997          .9219               9.
     .9976          .9572              11.
     .9957          .9871              14.
     .9919         1.0290              17.
     .9881         1.0648              22.
     .9772         1.1007              28.
     .9681         1.1365              35.
     .9582         1.1728              44.
     .9405         1.2097              56.
     .9289         1.2481              70.
     .9117         1.2900              89.
     .8981         1.3356             113.
     .8757         1.3918             141.
     .8558         1.4374             179.
     .8202         1.4975             225.
     .7900         1.5558             283.
     .7477         1.6052             358.
     .7088         1.6711             452.
     .6787         1.7178             570.
     .6119         1.7902             715.
     .5597         1.8397             903.
     .5151         1.8784            1140.
     .4578         1.9178            1446.
     .4068         1.9603            1825.
     .3550         1.9929            2303.
     .3028         2.0180            2907.
     .2530         2.0520            3669.
     .2082         2.0858            4630.
     .1687         2.1256            5843.
     .1324         2.1615            7378.
     .1013         2.1978            9307.
AE- 8 RLR EXAMPLE WITH AE-8
```

Figure 3.1d OUTPUT RESULTS FROM APPROXIMATE CLOSED FORM PROBABILISTIC SOLUTION

```
PREVIOUS ECOMP=     *8777. CORRECTED ECOPP=     28980. CORPECTION =    -5796.
 CORRECTED AVERAGE STRESS FROP EISAR =          479.619

CONCRETE  MODULUS  =        4000000. PSI     CV=              .100
BASE      MODULUS  =         800000. PSI     CV=              .200
SUBGRADE  MODULUS  =          12000. PSI     CV=              .200
SLAB    THICKNESS  =           7.00  IN.     CV=              .020
BASE    THICKNESS  =           6.00  IN     CV=              .100
MODULUS OF RUPTURE =            700. PSI     CV=              .100
PEERLESSION COEFFICIENT=                                    .000

STRESS FREQUENCY  DISTRIPLTION

MINIMUM                          =        *78.2829
MAXIMUM                          =        A*7.8769
AVERAGE                          =        4R*.7879
VARIANCE                         =        15*9.*7A8
COEFFICIENT  OF  VARIATION       =            .0818

     X          I     X          I     X          I     X          I     X          I     X          I
  .3787+02      2   .3875+02      5   .3962+02      5   .4050+02      1   .4139+02      6
  .4226+02      8   .4314+02     12   .4402+02     21   .4490+02     13   .4578+02     38
  .4665+02     21   .4753+02     27   .4841+02     20   .4929+02     22   .5017+02     25
  .5105+02     19   .5193+02     15   .5280+02      9   .5368+02     11   .5456+02      7
  .5548+02      7   .5632+02      2   .5720+02      5   .5808+02      2   .5896+02      0
  .5983+02      1   .6071+02      5   .6159+02      5   .6247+02      0   .6335+02      1

 OF  FREQUENCY  DISTRIBUTION

MINIMUM                          =          1.0144
MAXIMUM                          =          2.0929
AVERAGE                          =          1.4542
VARIANCE                         =           .0337
COEFFICIENT  OF  VARIATION       =           .1242

     X          I     X          I     X          I     X          I     X          I     X          I
  .1072+01      2   .1160+01      4   .1248+01     20   .1336+01     23   .1424+01     15
  .1212+01      9   .1288+01     11   .1376+01     21   .1464+01     21   .1552+01     21
  .1352+01     23   .1808+01     21   .1464+01     21   .1552+01     25   .1534+01     21
  .1572+01     15   .1660+01     14   .1648+01     19   .1679+01     13   .1715+01     14
  .1751+01      5   .1797+01      1   .1885+01      4   .1859+01      0   .1895+01      1
  .1971+01      0   .1967+01      1   .2055+01      1   .2079+01      0   .2075+01      1

RELIABILITY  DESIGN FACTOR  N.OF COVERAGES

      .9997          1.0503                  20.
      .9987          1.0843                  25.
      .9900          1.1222                  32.
      .9800          1.1582                  40.
      .9667          1.1981                  51.
      .9467          1.2301                  68.
      .9200          1.2660                  91.
      .7933          1.3020                 102.
      .7167          1.3379                 120.
      .6667          1.3739                 147.
      .5900          1.4098                 206.
      .5200          1.4458                 240.
      .4500          1.4817                 329.
      .3667          1.5177                 414.
      .2967          1.5536                 527.
      .2467          1.5896                 660.
      .2000          1.6255                 828.
      .1367          1.6615                1057.
      .0933          1.6974                1320.
      .0467          1.7334                1678.
      .0300          1.7693                2119.
      .0267          1.8053                2678.
      .0133          1.8412                3379.
      .0133          1.8772                4267.
      .0100          1.9131                5388.
      .0100          1.9491                6803.
      .0067          1.9850                8591.
      .0033          2.0210               10847.
      .0033          2.0569               13697.
      .0000          2.0929               17256.
```

Figure 3.1e OUTPUT RESULTS FROM SIMULATION SOLUTION

When the correction factor is used in conjunction with the simulation approach - i.e., all 300 composite moduli evaluated are shifted by 5796 psi, regardless of their generated value, the average stress generated is 483.74 psi, higher than the one computed from BISAR program by 0.9 percent. It seems that the composite modulus correction scheme is adequate for reducing to minimum the errors associated with the use of regression equations.

Comparison of the Approximate Closed-Form and the Simulation Approaches

The errors involved in the approximate closed-form approach are due to the linear assumption used in the Taylor series expansion. Whenever the dependent variable is related to the independent variables through a non-linear function, some error will result in the evaluation of the average and of the variance of the variable. Therefore, the results should be checked and compared with other results obtained from a different approach which does not assume linearity. In this paragraph, the results of the approximate closed-form approach are compared to those of the simulation scheme which assumes that the independent variables are normally distributed, as achieved by the generation scheme presented above.

From Figures 3.1d and 3.1e, it is seen that the average and the coefficient of variation values of the design factor are very close: 1.46 and 0.1229 in the approximate closed-form solution, and 1.456 and 0.1262 in the simulation process. These values were obtained for realistic values of standard deviations of the independent variables. Figures 3.2a and 3.2b show the input data and the relationship between reliability and number of coverages of AG-13. It is seen that a slight deviation between the results exist at the distribution tails. The deviation which can be attributed to the number generation in the simulation process is considered negligible

```
13        RUN  EXAMPLE  WITH AG-13
 3
      20.0        0.02   4000000.        0.10
     10.00        0.10    50000.0        0.20
   12000.0        0.20
      700.        0.10        0.00
 0 1
  RUN EXAMPLE  COMPOSITE MODULUS CORRECTION
 1
 4  11
     4.0E+06      0.2E+00      2.0E+01      1.0E+03
     5.0E+04      0.2E+00      1.2E+01
     1.2E+04      0.4E+00      2.4E+02
     1.0E+06      0.4E+00
 4
STRS  2.337075E+02  9.2189E+00    -3.7E+01      0.0E+00
STRS  2.337079E+02  9.2189E+00     0.0E+00      0.0E+00
STRS  2.337075E+02  9.2189E+00     6.2E+01      0.0E+00
STRS  2.337079E+02  9.2189E+00     9.9E+01      0.0E+00
        SRRSTTSZZSRTSRZSTZ
 1
 1      0.0E+00      0.0E+00      2.0E+01
13        RUN  EXAMPLE  WITH AG-13
 3
      20.0        0.02   4000000.        0.10
     10.00        0.10    50000.0        0.20
   12000.0        0.20
      700.        0.10        0.00
 1 1
  RUN EXAMPLE  COMPOSITE MODULUS CORRECTION
 1
 4  11
     4.0E+06      0.2E+00      2.0E+01      1.0E+03
     5.0E+04      0.2E+00      1.2E+01
     1.2E+04      0.4E+00      2.4E+02
     1.0E+06      0.4E+00
 4
STRS  2.337075E+02  9.2189E+00    -3.7E+01      0.0E+00
STRS  2.337079E+02  9.2189E+00     0.0E+00      0.0E+00
STRS  2.337075E+02  9.2189E+00     6.2E+01      0.0E+00
STRS  2.337079E+02  9.2189E+00     9.9E+01      0.0E+00
        SRRSTTSZZSRTSRZSTZ
 1
 1      0.0E+00      0.0E+00      2.0E+01
```

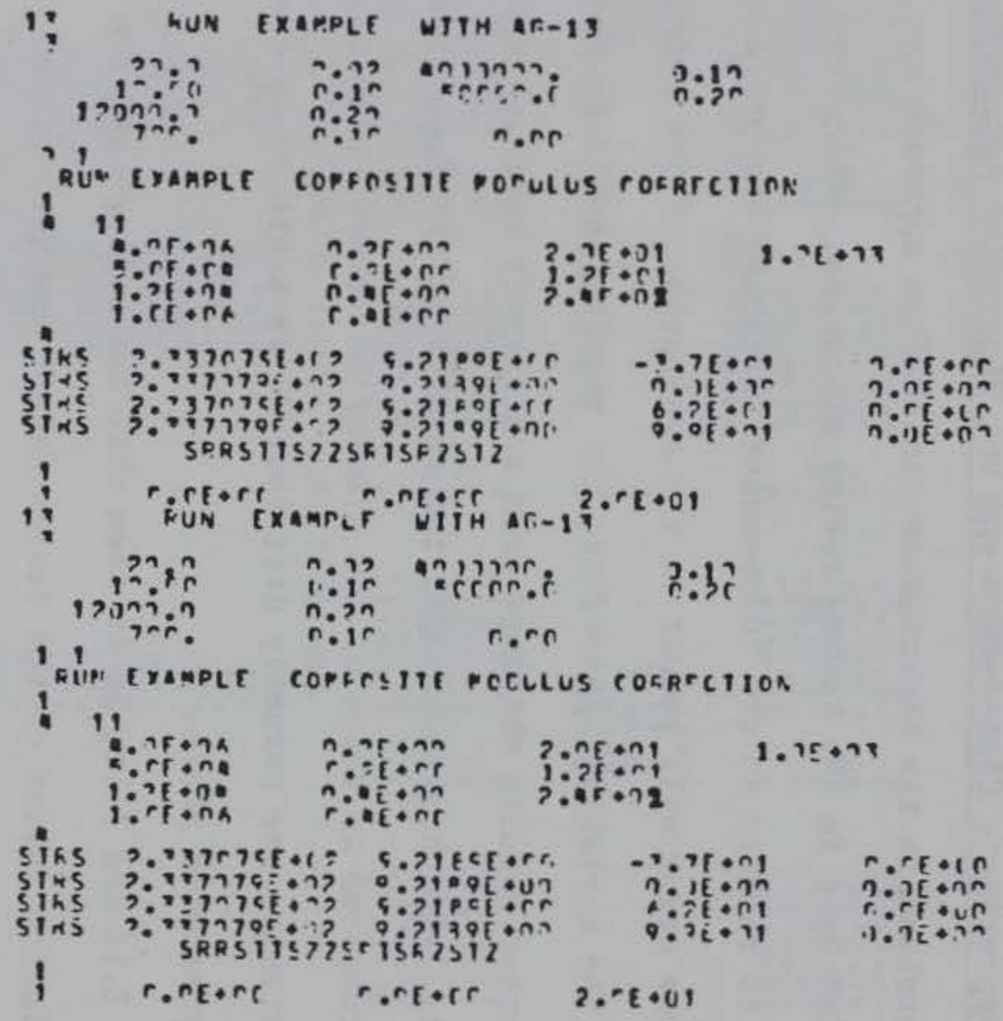

Figure 3.2a INPUT DATA FOR RUN EXAMPLE AG-13

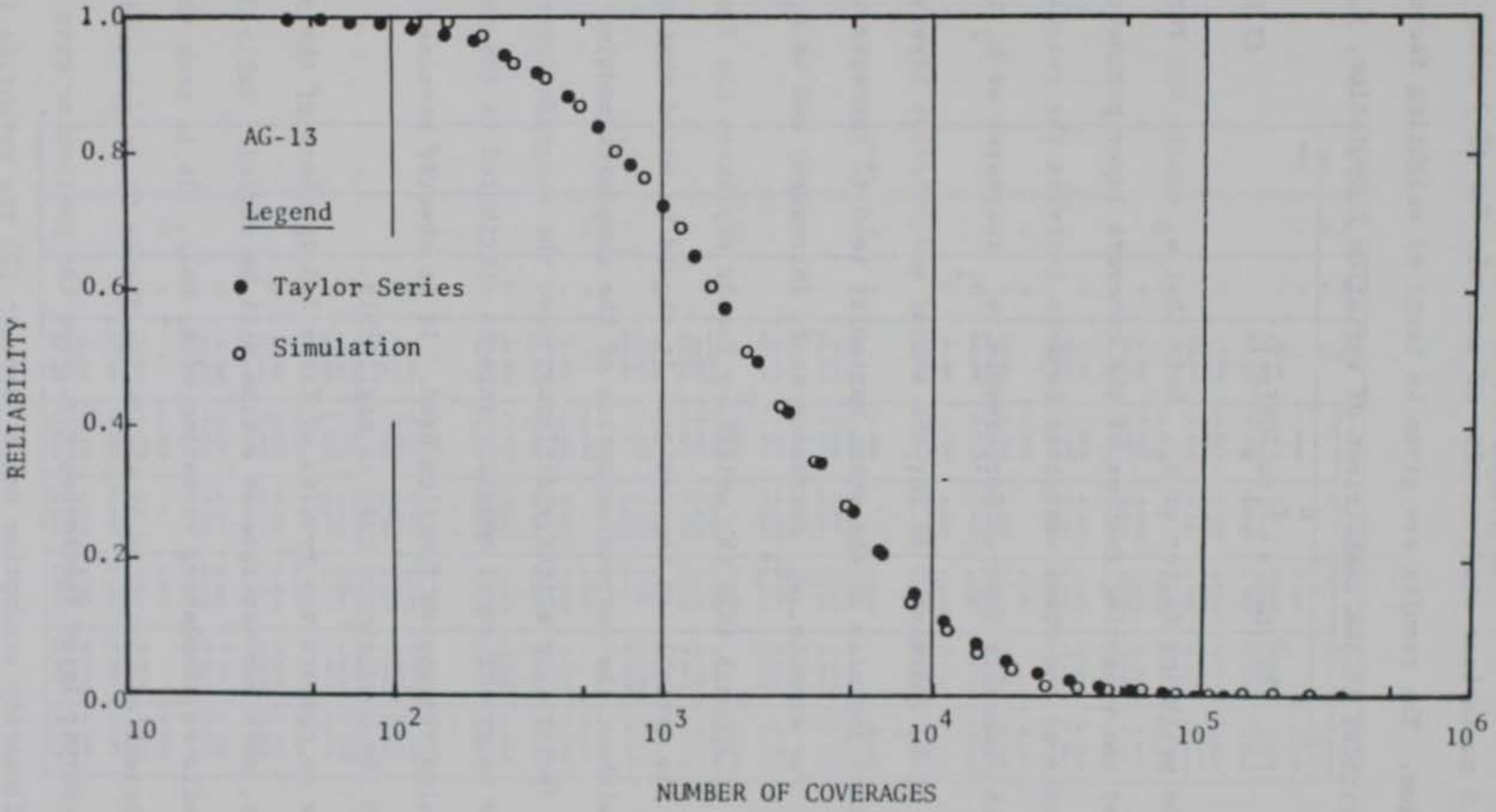

Figure 3.2b COMPARISON OF APPROXIMATE CLOSED FORM AND SIMULATION SOLUTIONS

from an engineering design point of view.

Table 3.3 summarizes study results of analysis for AG-2 using Taylor series expansion. The results are given in terms of weighting factors of the design variables in the coefficient of variation computation, i.e., when

$$CV^2[DF] = \sum_{i=1}^{n} (w_i \; CV[x_i])^2 \qquad (3.13)$$

w_i is the weighting factor of x_i. Note that w_i equals one for x_i = MR. It is seen that the weighting factors of the concrete layer parameters (h_1 and E_1) increases as the other variables tend to increase the relative stiffness of the concrete layer. For example, w_{h_1} increases as h_1 increases, and as E_2, h_2 or E_3 decrease. As for the second and subgrade layers, their weighting factors increase as the other variables tend to increase the stress in the layer. For example, w_{E_3} increases as E_3 increases, and as h_1, h_2 or E_2 decrease. It appears that the weighting factor expresses the functional importance of the variable in the design. It should be noted that the results were derived without the correction option of the composite modulus and that the weighting factors are slightly different when the subgrade layer is subdivided in two layers of equal moduli. This is attributed to the form of the composite modulus regression function used. It is however stressed that the deviation from the homogeneous case is negligible.

In order to compare the results of Table 3.3 to those of the simulation process, only one variable at a time will be varied. Table 3.4a shows the results of simulating variation of h_1 only. It is seen that: (1) the coefficient of variation of the design factor DF is proportional to that of the concrete layer thickness, i.e., for the particular case studied, the linearity assumption seems correct; (2) the weighting factors

Table 3.3 Summary of Results for AG-2 using Taylor Series Expansion

E psi	h_1 in	h_2 in	E_2 psi	w_1 -weighting factors of				
				h_1	h_2	E_1	E_2	E_3
5000	8	0	-	-1.6444	-	0.1309	-	-0.1309
		6	20,000	-1.6212	-0.0420	0.1425	-0.0189	-0.1236
			50,000	-1.5872	-0.0789	0.1506	-0.0333	-0.1172
			200,000	-1.5153	-0.1525	0.1658	-0.0644	-0.1014
		12	20,000	-1.5519	-0.1256	0.1559	-0.0497	-0.1061
			50,000	-1.4911	-0.1907	0.1690	-0.0749	-0.0944
			200,000	-1.3620	-0.3302	0.1941	-0.1276	-0.0665
	10	0	-	-1.7200	-	0.1179	-	-0.1179
		6	20,000	-1.7112	-0.0280	0.1280	-0.0132	-0.1148
			50,000	-1.6847	-0.0566	0.1356	-0.0251	-0.1105
			200,000	-1.6279	-0.1200	0.1498	-0.0513	-0.0985
		12	20,000	-1.6574	-0.0960	0.1398	-0.0386	-0.1012
			50,000	-1.6092	-0.1533	0.1525	-0.0597	-0.0928
			200,000	-1.5078	-0.2682	0.1744	-0.1044	-0.0700
	12	0	-	-1.7980	-	0.1097	-	-0.1097
		6	20,000	-1.7981	-0.0192	0.1193	-0.0099	-0.1097
			50,000	-1.7789	-0.0437	0.1265	-0.0199	-0.1065
			200,000	-1.7348	-0.0990	0.1397	-0.0428	-0.0969
		12	20,000	-1.7571	-0.0768	0.1295	-0.0314	-0.0980
			50,000	-1.7198	-0.1254	0.1417	-0.0501	-0.0917
			200,000	-1.6404	-0.2290	0.1625	-0.0897	-0.0728
10,000	8	0	-	-1.6043	-	0.1519	-	-0.1519
		6	20,000	-1.5968	-0.0282	0.1606	-0.0134	-0.1472
			50,000	-1.5662	-0.0600	0.1668	-0.0265	-0.1403
			200,000	-1.5020	-0.1289	0.1784	-0.0549	-0.1235
		12	20,000	-1.5372	-0.1035	0.1694	-0.0415	-0.1279
			50,000	-1.4833	-0.1621	0.1797	-0.0639	-0.1159
			200,000	-1.3653	-0.2884	0.2017	-0.1119	-0.0897
	10	0	-	-1.6868	-	0.1406	-	-0.1406
		6	20,000	-1.6814	-0.0166	0.1484	-0.0087	-0.1398
			50,000	-1.6677	-0.0428	0.1538	-0.0196	-0.1342
			200,000	-1.6173	-0.1006	0.1636	-0.0435	-0.1201
		12	20,000	-1.6449	-0.0784	0.1553	-0.0321	-0.1233
			50,000	-1.6026	-0.1277	0.1641	-0.0510	-0.1133
			200,000	-1.5121	-0.2320	0.1815	-0.0908	-0.0907
	12	0	-	-1.7765	-	0.1337	-	-0.1337
		6	20,000	-1.7896	-0.0091	0.1414	-0.0057	-0.1358
			50,000	-1.7712	-0.0317	0.1462	-0.0151	-0.1311
			200,000	-1.7315	-0.0824	0.1547	-0.0362	-0.1186
		12	20,000	-1.7520	-0.0621	0.1469	-0.0260	-0.1209
			50,000	-1.7189	-0.1055	0.1549	-0.0427	-0.1122
			200,000	-1.6482	-0.1969	0.1698	-0.0777	-0.0922
20,000	8	0	-	-1.5651	-	0.1695	-	-0.1695
		6	20,000	-1.5717	-0.0140	0.1764	-0.0076	-0.1688
			50,000	-1.5464	-0.0418	0.1807	-0.0191	-0.1616
			200,000	-1.4867	-0.1041	0.1911	-0.0449	-0.1462
		12	20,000	-1.5304	-0.0782	0.1781	-0.0319	-0.1462
			50,000	-1.4731	-0.1327	0.1907	-0.0528	-0.1379
			200,000	-1.3625	-0.2487	0.2115	-0.0971	-0.1144
	10	0	-	-1.6555	-	0.1576	-	-0.1576
		6	20,000	-1.6727	-0.0047	0.1640	-0.0038	-0.1602
			50,000	-1.6530	-0.0271	0.1673	-0.0132	-0.1543
			200,000	-1.6068	-0.0789	0.1755	-0.0347	-0.1408
		12	20,000	-1.6377	-0.0573	0.1861	-0.0241	-0.1619
			50,000	-1.5960	-0.1071	0.1748	-0.0414	-0.1334
			200,000	-1.5124	-0.1967	0.1898	-0.0775	-0.1122
	12	0	-	-1.7560	-	0.1505	-	-0.1505
		6	20,000	-1.7809	-0.0012	0.1565	-0.0014	-0.1551
			50,000	-1.7651	-0.0177	0.1598	-0.0094	-0.1504
			200,000	-1.7286	-0.0627	0.1664	-0.0282	-0.1381
		12	20,000	-1.7503	-0.0435	0.1585	-0.0190	-0.1395
			50,000	-1.7168	-0.0824	0.1654	-0.0340	-0.1314
			200,000	-1.6531	-0.1641	0.1775	-0.0655	-0.1118

Table 3.4a Results of Simulation for AG-2 and Concrete Layer Thickness Weighting Factor

E_3 psi	h_1 in.	h_2 in.	E_2 psi	Values of CV[DF] for		Ratio	w_{h_1}
				$CV[h_1]=$ 0.02 (1)	$CV[h_1]=$ 0.05 (2)	$\frac{(2)}{(1)}$	
20,000	8	6	20,000	0.0321	0.0802	2.50	1.604
			50,000	0.0315	0.0787	"	1.574
			200,000	0.0302	0.0754	"	1.508
		12	20,000	0.0321	0.0802	"	1.604
			50,000	0.0299	0.0747	"	1.494
			200,000	0.0275	0.0687	"	1.374
	10	6	20,000	0.0339	0.0849	2.50	1.698
			50,000	0.0337	0.0843	"	1.686
			200,000	0.0327	0.0818	"	1.636
		12	20,000	0.0339	0.0849	"	1.698
			50,000	0.0325	0.0812	"	1.624
			200,000	0.0307	0.0768	"	1.536
	12	6	20,000	0.0360	0.0901	2.50	1.802
			50,000	0.0361	0.0902	"	1.804
			200,000	0.0353	0.0883	"	1.766
		12	20,000	0.0360	0.0901	"	1.802
			50,000	0.0351	0.0877	"	1.754
			200,000	0.0337	0.0843	"	1.686

are slightly higher, by about 3 percent in the simulation process than in the approximate closed-form solution. This is negligible for engineering practical purposes.

Table 3.4b shows the results of the simulation for the subgrade modulus weighting factor. It is seen that (1) increasing the coefficient of variation of the subgrade modulus from 0.10 to 0.25 increases the coefficient of variation of the design factor by 2.60, a slight deviation from linearity; (2) the weighting factors are higher in the simulation than in the Taylor series expansion results (up to 15 percent). While this discrepancy is quite high for the individual weighting factor, its effect on the coefficient of variation of the design factor with variability of all design parameters will be attenuated in the summation of the squared contribution (see equation 3.3).

It appears from the above that the results of reliability will be similar for the Taylor series expansion and simulation solution. The assumed linearity (or first order expansion) of the function seems to be supported by the simulation approach.

Example

The approximate approach was used to develop reliability-number of coverages curves for different thicknesses of the concrete layer (18, 20, 22 and 24 inch), see Figure 3.3a. Thickness - number of coverages curves were then developed for different reliability levels (0.9, 0.8, 0.7, 0.6 and 0.5) and shown in Figure 3.3b. This run example will serve to find out the reliability level used in the current design. The analysis is made for two values of percentile (0.85 and 0.90) for computing the design parameters in the current "deterministic" method. The results are summarized in Table 3.5 where:

Table 3.4b Results of Simulation for AG-2 and Subgrade Modulus Weighting Factor

E_3 psi	h_1 in	h_2 in	E_2 psi	Values of CV[DF] for		Ratio $\frac{(2)}{(4)}$	w_{E_3}
				$CV[E_3]=$ (1)	$CV[E_3]=$ (2)		
10,000	8	6	20,000	0.0141	0.0367	2.60	0.1468
			50,000	0.0143	0.0370	2.59	0.1480
			200,000	0.0136	0.0353	2.60	0.1412
		12	20,000	0.0127	0.0331	2.61	0.1324
			50,000	0.0121	0.0315	2.60	0.1260
			200,000	0.0101	0.0262	2.59	0.1048
	10	6	20,000	0.0134	0.0347	2.59	0.1388
			50,000	0.0136	0.0352	2.59	0.1408
			200,000	0.0131	0.0340	2.60	0.1360
		12	20,000	0.0122	0.0318	2.61	0.1272
			50,000	0.0118	0.0305	2.58	0.1220
			200,000	0.0101	0.0261	2.58	0.1044
	12	6	20,000	0.0130	0.0336	2.58	0.1344
			50,000	0.0132	0.0342	2.59	0.1368
			200,000	0.0128	0.0333	2.60	0.1332
		12	20,000	0.0120	0.0310	2.58	0.1240
			50,000	0.0116	0.0301	2.59	0.1204
			200,000	0.0101	0.0261	2.58	0.1044

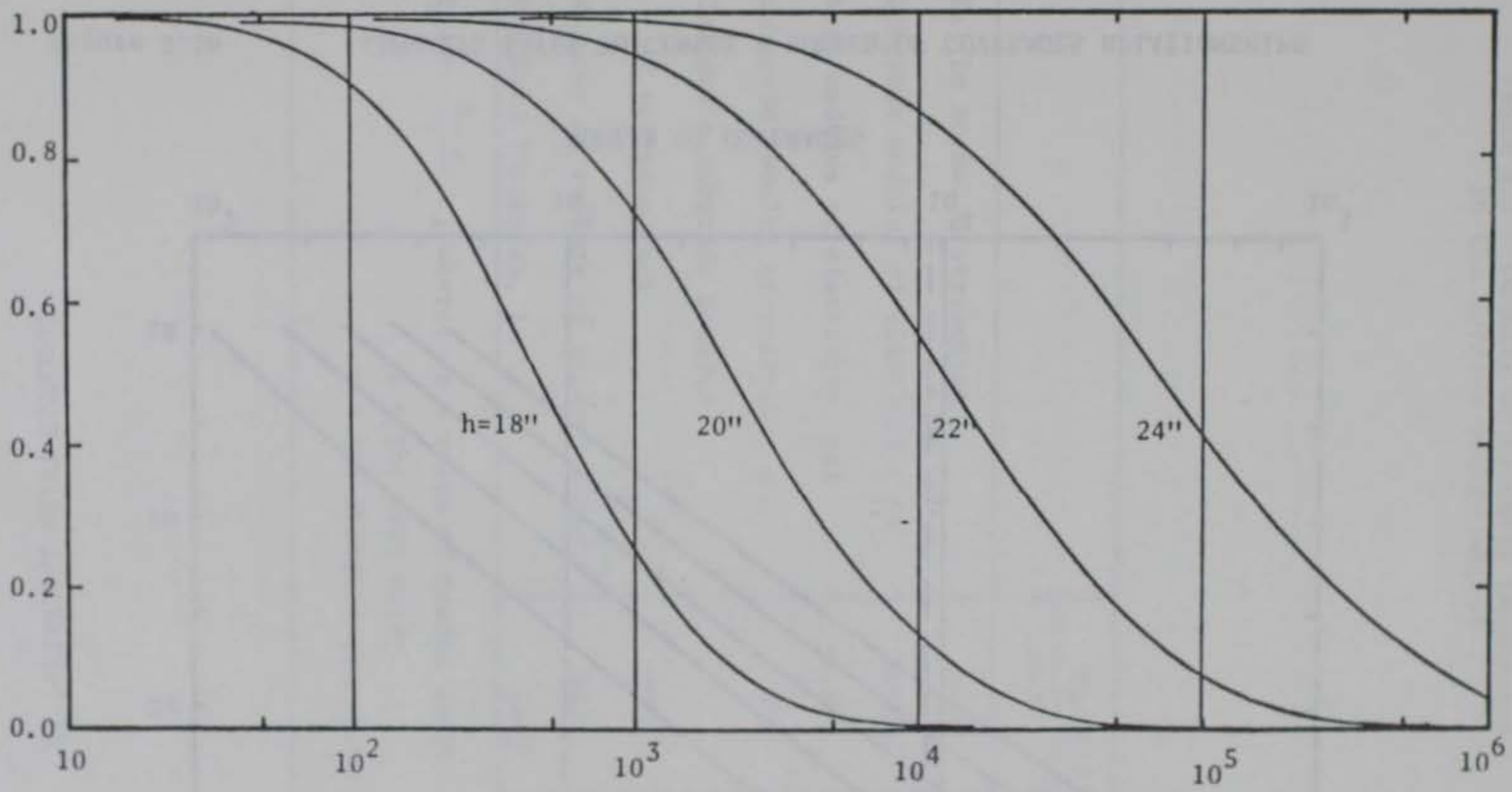

Figure 3.3a RELIABILITY - NUMBER OF COVERAGES RELATIONSHIPS FOR AG-13 AND DIFFERENT CONCRETE LAYER THICKNESSES

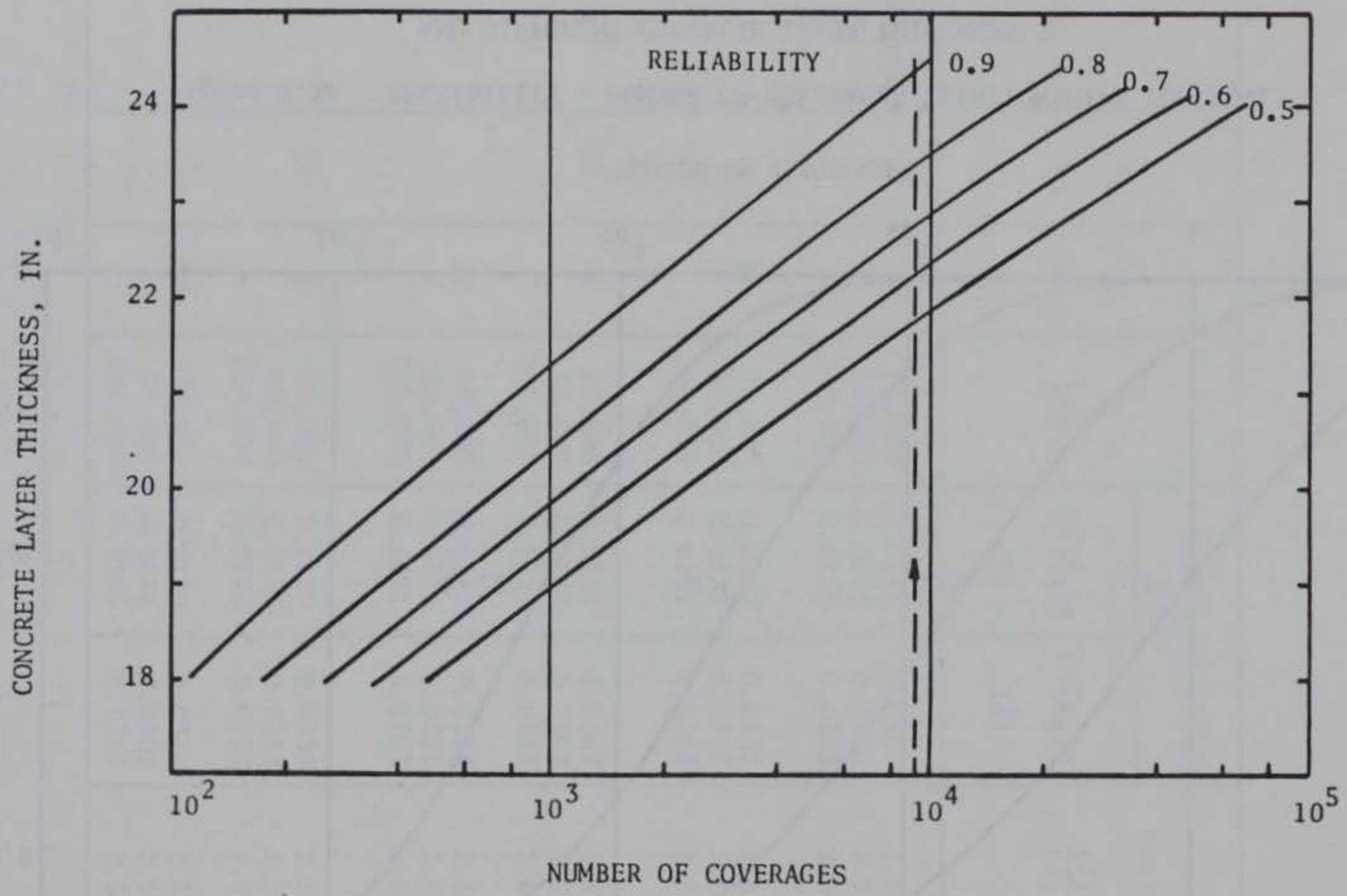

Figure 3.3b CONCRETE LAYER THICKNESS - NUMBER OF COVERAGES RELATIONSHIPS FOR AG-13 AND DIFFERENT RELIABILITY LEVELS

Table 3.5 Summary of Pavement Parameters in the Current Design Method

	Percentile Value	
	0.85	0.90
K_α in normal distribution	1.03	1.30
Subgrade modulus of elasticity, psi	9,600.	8,900.
Base modulus of elasticity, psi	40,000.	37,000.
Subgrade modulus of reaction, pci	100.	94.
Composite subgrade modulus of reaction, pci	200.	180.
Modulus of rupture of concrete, psi	630.	610.
Required Thickness, in	24.2	25.1

$K_\alpha = \frac{a - \mu}{\sigma}$, where: a = value of random width
μ = the mean value
σ = the standard deviation

(a) The subgrade and the base modulus of elasticity, and the modulus of rupture are computed using:

$$x_i = \bar{x}_i \ (1 - K_\alpha \ CV[x_i]) \qquad (3.14)$$

(b) The average values of the variables and their coefficient of variation are given in Figure 3.2a.

(c) The subgrade modulus of reaction is computed from (Parker et al., (1)):

$$\log k = (\log M_R - 1.415)/1.284 \qquad (3.15)$$

(d) The composite subgrade modulus of reaction is derived from Figure 2.3.

(e) The required thickness is calculated using design curves in (1). The thickness corresponds to traffic area A and 9200 coverages.

Entering Figure 3.3b with N = 9200 coverages and h = 24.2 and 25.1 in., the corresponding reliability levels of 0.88 and 0.95 are derived. These values give an idea of the reliability levels used today in the design procedure.

Chapter 4

SUMMARY

The design procedure of military rigid airfield pavements developed by Parker et al (1) is expressed in this volume in probabilistic and reliability terms. The procedure, based on the multi-layer elastic theory necessitated further developments in order to make the analysis feasible. Two major investigations were conducted: (1) Evaluation of the Composite Modulus of Elasticity of the layered (subbase/subgrade) system underneath the rigid pavement and (2) Evaluation of the maximum tensile stress at the bottom of the concrete layer for each of 13 aircraft types: USAF Classification AG-1 to 13.

Valuable results were derived from the investigation of the composite modulus. It was found that:

(1) The composite modulus depends not only upon the design parameters of the layered (subbase/subgrade) system underneath the rigid pavement, but also upon the pavement thickness (for constant elastic properties of the concrete) and the loading conditions (number of wheels in the gear).

(2) The effect of deviations in the composite modulus evaluation on pavement performance (expressed in terms of predicted allowable number of coverages) is quite substantial. The error in the evaluation of number of coverages is about 1.2 to 5 times that achieved in the evaluation of the composite modulus. The lower range corresponds to light load aircraft and traffic whereas the upper range corresponds to heavy load aircraft and traffic. This result emphasizes the need for an accurate determination of the composite modulus.

(3) The accuracy achieved in a correlation between the composite modulus and the design parameters is excellent (R^2 = 0.994 and SE = 8%). However, in the case of heavy load aircraft and larger number of coverages, this (accuracy) is insufficient for both design purposes and probabilistic/reliability analyses. Therefore, a correction methodology is developed to reduce the error to minimum.

A regression equation of the maximum tensile stress at the bottom of the concrete layer is derived for each of 13 aircraft types. The equation is highly accurate, allowing its use without any correction for both design purposes and probabilistic/reliability analyses.

The above derivations of the composite modulus and of the maximum tensile stress are included in a computer program for probabilistic/reliability analysis of rigid pavements. Both the approximate closed form (Taylor series expansion) and the simulation solutions are implemented. The computer program can be used:

(1) In the analysis in probabilistic/reliability terms of a specific pavement structure and loading aircraft, for a given material properties and variabilities. The design parameters (means and coefficient of variations) serve as input to the computer program which produces values of number of coverages and their corresponding reliability levels.

(2) In the design of a rigid pavement at a specific reliability level, given all design parameters. Several solutions of the above under (1) for different pavement structures must be conducted, and the requested design is derived.

Several example runs of the computer program are presented, illustrating (a) the use of the correction procedure in the evaluation of the composite modulus. It is suggested to call the procedure in the

case of medium and heavy load and traffic, (b) the simultude of the results obtained with the approximate closed form and the simulation solutions and (c) the interpretation of the current deterministic design procedure in probabilistic/reliability terms.

LIST OF REFERENCES

1. Parker, F., Jr., Barker, W.R., Gunkel, R.C. and Odom, E.C., "Development of a Structural Design Procedure for Rigid Airport Pavements", U.S. Army Eng., W.E.S. Final Report, April 1979.
2. Army T.M., "Rigid Pavement fer Airfields Other than Army", Army T.M. 5.824-3, and Air Force AFM 88-6, Chap. 3, Aug. 1979.
3. Ulery, H.H., Letter to M.W. Witczak, August 1982.

APPENDIX I

USER'S GUIDE

Card 1 (I2, 19A4)

1-2	IG	- aircraft group number
3-78	HED	- text for title

Card 2 (I2)

1-2	NLA	- number of layers (=3 when base-subbase layer exists)

Card 3 (4F10.0)

1-10	H1AV	- average thickness of concrete layer, inch.
11-20	H1STD	- coefficient of variation of concrete layer thickness
21-30	E1AV	- average modulus of elasticity of concrete, psi.
31-40	E1STD	- coefficient of variation of concrete modulus of elasticity

Card 4 (4F10.0) required if NLA = 3

1-10	H2AV	- average thickness of base-subbase layer, inch.
11-20	H2STD	- coefficient of variation of base-subbase layer thickness
21-30	E2AV	- modulus of elasticity of base-subbase material, psi.
31-40	E2STD	- coefficient of variation of base-subbase modulus of elasticity

Card 5 (2F10.0)

1-10	ES	- subgrade modulus of elasticity, psi.
11-20	ESSTD	- coefficient of variation of subgrade modulus of elasticity

Card 6 (3F10.0)

1-10	MR	- average concrete modulus of rupture, psi.
11-20	MRSTD	- coefficient of variation of concrete modulus of rupture
21-30	ROEMR	- regression coefficient between modulus of elasticity and modulus of rupture of concrete

Card 7 (2I2)

1-2 IRAN - flag ≤ 0 for running Taylor series expansion approach

> 0 for running simulation approach

3-4 IOPT - flag = 0 no correction for composite modulus computations

≠ 0 run BISAR program for correcting composite modulus

Card 8 required if IOPT ≠ 0

Input data required by BISAR program

Note Several problems (Cards 1 - 7) can be run in sequence.

APPENDIX II

PROGRAM LISTING

Appendix II is a copyrighted program listing. Information on the program can be obtained from the authors of this report.