PILE DRIVING BY MEANS OF LONGITUDINAL AND TORSIONAL VIBRATIONS

Austin Kovacs and Frank Michitti

July 1970

CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

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PREFACE

This report was prepared by Mr. A. Kovacs and Mr. F. Michitti, Research Civil Engineers, of the Construction Engineering Branch (Mr. E.F. Lobacz, Chief), Experimental Engineering Division (Mr. K.A. Linell, Chief), U.S. Army Cold Regions Research and Engineering Laboratory.

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PILE DRIVING BY MEANS OF LONGITUDINAL AND TORSIONAL VIBRATIONS

by

Austin Kovacs and Frank Michitti

INTRODUCTION

Pile driving by longitudinal vibration is not a new idea. Machinery utilizing this principle was in use in Germany and the Soviet Union in the early 1930’s. These vibrators weighed up to 25 tons and had a frequency range up to 50 Hz. They were designed to produce bodily vibratory movement of the pile in respect to the soil and to overcome the resistance of the soil to pile penetration by setting the soil structure in such a state of agitation as to allow the grains to be readily displaced into a closer-knit configuration. Under favorable conditions, these drivers proved to be more effective than conventional impact-type drivers.

The idea of vibratory pile driving was formally introduced to American engineers by D. Barkan, a noted Soviet soil expert, in 1936. Application of this technique in America was slow. Raymond International was perhaps the first to build a prototype vibratory driver in this country but because of its unsatisfactory performance further development was abandoned.

In 1954, A. Bodine, Jr., was granted a U. S. patent for an earth boring drill. His later endeavors led to subsequent patents and the development of a 500-hp high speed longitudinal vibrator called the Bodine Resonant Driver (BRD). Unlike earlier vibratory drivers, this unit was designed to establish a longitudinal resonant standing wave within the pile, i.e. no bodily vibratory movements. The BRD was publicly demonstrated by C. L. Guild Construction Company in 1961. It later showed that it could outperform hammer-type drivers under a wide range of soil conditions and proved that the concept of resonant pile driving was feasible. Subsequent development led to a larger 1000-hp unit weighing up to 11 tons. This driver has been used in America and more recently in England.

Hammer-type pile drivers have the inherent characteristics of producing noise and vibration which on occasion have led to lawsuits. In addition, when large driving forces are required the pile is often damaged. The undesirable shock waves have caused damage to nearby buildings and their contents and the noise is a nuisance. Vibratory drivers on the other hand are generally quieter, produce no shock and drive piles faster. If due caution is not exercised, however, these drivers may cause soil settlement along existing piles, thereby overloading them; and resonance—can be produced in nearby buildings, leading to structural failures. Soil resonance, which is often achieved with vibratory drivers, may cause retaining walls and embankments to settle or slip or the soil under an existing foundation to subside differentially. With resonant pile driving these effects are reduced or eliminated because the higher resonant frequencies generated do not adversely affect the surrounding soil except in the immediate area of the pile being driven. However, pile foundations within this area may lose part of their carrying capacity as a result of the soil being subjected to local high accelerations. In addition, concrete piles can be broken near the center by end-to-end resonance if the tip should pass into very soft material. To avoid this, steel piles are generally used with the BRD.

From a theoretical standpoint, the idea of resonant pile driving by torsional vibration
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would seem to merit some respect. However, only one known company, Sonico Research and Development Company, Inc., has performed field investigation to determine the feasibility of using torsional vibrations for pile driving. Their results were discouraging when compared to longitudinal vibrations in the sonic range.

DISCUSSION OF RELATED THEORY AND APPLIED RESULTS

Driving of piles by vibration is based on the fact that the shear and frictional resistance of soils are significantly reduced when subjected to vibrations. These vibrations are transferred to the soil by the response of the pile to cyclic loading.

For the purposes of discussion, it is convenient to have a mental picture of how a vibrating wave might look in a pile. While it is difficult to represent longitudinal wave motion diagrammatically, one representation might be that shown in Figure 1a, where the successive compressions of the pile, as the wave travels along its length, are shown by the collection of dashes of variable density. However, this is not a very flexible method of representation. It is more convenient to represent the longitudinal wave in the manner of a transverse wave (Fig. 1b). This is accomplished by making the ordinates of the curve proportional to the magnitude of the displacement of any point. It must be kept in mind that this method of representation is merely a convention and the curve does not indicate the direction of particle displacement but the magnitude of displacement.

When a force impulse is applied to the top of a pile, it travels down the pile to the bottom and is "reflected" back at the same speed. If only one impulse is applied, it travels back and forth along the pile until its amplitude finally becomes dissipated, i.e., is damped out. When successive impulses are applied at a given frequency, it can be seen that, for some particular pile length, the reflected wave traveling upward will be exactly superimposed upon (reinforced by) the one traveling downward. In this condition, the wave curve appears to remain stationary. For pile driving it is desirable that the positions of maximum amplitude, the antinodes, be at the tip of the pile where it first touches the soil and at the top where the energy is delivered. An antinode at the top allows maximum transfer of energy to the pile and an antinode at the tip allows maximum transfer of energy to the soil, thus tending to maximize penetration rate.

Bodine based the development of his driver on this idea, along with the concept that a pile initially vibrates as a "free-free" bar, with a velocity antinode at each end and a node (minimum amplitude) at the midpoint. When a vibrating wave fulfills these conditions, it is known as half-wave vibration (Fig. 2). Undergoing such wave action, the pile alternately elongates and contracts.
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The number of cycles created in a standing wave depends upon the resonant frequency. From the standpoint of creating the greatest pressure at the bottom of the pile, first mode half-wave resonant frequencies appear to give best results. Elastic theory suggests that at this frequency, velocity antinodes (maximum displacement) are formed at the pile ends while a velocity node (minimum displacement) is formed at the center. In short, the greater the number of antinodes, the lower the amplitude, i.e., the higher the frequency the lower the displacement amplitude. However, Bernhard determined empirically that the resonant stress waves in a model pile did not assume the idealized shapes (Fig. 2) predicted by elastic theory. His results show that the wave form (Fig. 3) has one node at both the first and second resonant frequencies, that the wavelength at the fundamental resonant frequency is three and not two times the pile length as predicted by elastic theory, that maximum tip displacement occurs at the second

Figure 2. Wave pattern of half-wave and harmonics.
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resonant frequency, and that from the standpoint of maintaining a power level the second resonant frequency is the most desirable. He therefore concludes that maximum efficiency is attained when pile driving is performed at the second resonant frequency.

Resonant pile drivers take advantage of the property of elasticity and the natural characteristics related to elasticity. For example, the velocity of stress wave propagation and the mass of the material determine the "natural-frequency" response to a forced vibration. Therefore every pile, depending upon its own physical characteristics and properties, has its own natural frequency.

<table>
<thead>
<tr>
<th>( \eta_{res} ) (cps)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

| NO. OF NODES | 0 | 1 | 1 | 2 | 3 |

Figure 3. Wave form of model pile (after Bernhard, 1967).

The resilient materials of piles are highly elastic and have large stress wave velocities compared to soils. Steel piles, which have strong crystalline bonds, can oscillate in unison with force impulses over a range of frequencies from zero to several thousand Hertz. In comparison, soil has a weak bond structure which fails just as if its elastic limit were exceeded. At a given stress, most soil bonds fail at frequencies lower than 30 Hz. Once the bond structure fails under cyclic loading, individual grains tend to rearrange themselves in a flowing, fluid-like motion. Continuous cyclic motion keeps the pile-soil interface resistance in a state of "sliding friction" as opposed to static friction. In this "quasi-fluidized state" soil particles are readily displaced and the pile "sinks" as though it were in a highly viscous fluid.

Values for Young's modulus larger than three times the shear modulus have been observed for soils in the quasi-fluidized state. This phenomenon is indicative of a volumetric expansion wherein Poisson's ratio seems to lose its customary physical significance.

The resonant frequency of a system, e.g. resonator and pile, is controlled by the spring characteristic, the mass, the impedance, and the damping factor of the system. The spring characteristic of a system determines its ability to respond elastically to vibrations, and its mass is a measure of its inertia. The tendency of a system to resist particle motion is referred to as
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its impedance. In a mechanical system, the impedance is related to stiffness and inertia, or the impedance \( Z \) is equal to the product of the density \( \gamma \) and the velocity of propagation \( v \). The impedance plays an important part in power transmission and the following list illustrates a set of systems classified on the basis of impedance in the order of increasing power loss:

1. Longitudinal waves in a pipe
2. Longitudinal waves in a wire
3. Torsional waves in a pipe.

Impedance is equally important in designing the force generator as it governs the force and frequency requirements of the generator to produce significant displacements.

The internal friction or damping factor of a system such as a pile is a measure of its ability to remain at a particular position once placed there. One way to determine this factor is to observe the system's \( Q \). A mechanical system's \( Q \) is defined as the ratio of the mass reactance (inertia or density) to the resistance. The mass reactance \( M \) can be obtained directly but the resistance \( R \) must be obtained by an indirect method. One way to measure the resistance is to note the frequency for peak resonance and then note the frequency on each side of the peak where the amplitude is down by 30%. The difference between the two frequencies divided into the peak resonance frequency is equal to the \( Q \) of the system.\(^6\) \( Q \) can also be measured by measuring the log decrement, viscosity coefficient, and/or the lag or loss angle between stress and strain.

A question arises as to the effect damping has on the vibration of a pile. A system without damping has its frequency of maximum amplitude at resonant frequency. Bodine uses this concept as an indication that his system is resonating. Damping, however, causes the maximum amplitude to occur at some frequency less than the elastic resonance, as well as decreasing the amplitude.

In any event DenHartog (1956) points out the the error introduced by assuming that resonance is the elastic resonance rather than the actual damped resonance is slight unless the damping force is of considerable magnitude or the mass of the system is extremely small.

Pile driving at resonant frequencies has shown some very encouraging results. However, difficulty was experienced with a compact Aeolian sand.\(^{19} \)\(^{20} \) In this instance a BRD was unable to drive a pile after its tip encountered the sand layer at a depth of 35 ft. The difficulty proved to be entirely due to the sand's near-perfect sorting. As soon as enough of the material was ejected by compressed air and water to allow for the pile's passage, it was easily driven through the problem layer. This experience affords evidence that the effect of vibration in the usual circumstance of relatively poor sorting is to drive the finer grains into the interstitial space between coarser ones, allowing the pile to pass.

An important question that arises when a resonant pile driver is used is at what depths pile driving should be stopped. Piles have gone deeply into adequate bearing materials with little slowdown of penetration. The primary use of the driver may be for installing and extracting sheet piles, and for driving piles to end bearing on known hard strata. Another question might arise as to using resonant frequencies for installing piles and other members into frozen soil. The installation of piles approximately fifty feet in length seems to be quite feasible,\(^{11} \)\(^{12} \) but attempts to install short lengths such as stakes in frozen soil have failed.\(^{17} \) However, the experiments reported were inconclusive.

Load tests on installed piles have indicated that those driven with resonant drivers sustain greater loads than identical piles driven with conventional drivers.\(^{18} \)\(^{31} \) This is reasoned to be due to the rearrangement and tighter packing of soil particles around the pile as a result of vibration. This effect apparently contributes more to static wall-friction than does the displacement of soil associated with conventionally driven piles.
While the preceding remarks have been primarily directed towards longitudinal vibrations associated with resonant pile driving, the related theory and phenomenon will in general apply to torsional vibrations. The only change in the analysis would be that instead of the alternate elongation and shortening caused by longitudinal impulses, there would be an alternate swelling and contracting of the pile's cross section caused by torsional excitation. We must remember, of course, that under longitudinal impulses each elongation is accompanied by a contraction of the cross section and that each shortening is accompanied by a swelling of the cross section. Likewise, under torsional excitation, each swelling of the cross section is accompanied by a shortening of the pile and each contraction of the cross section is accompanied by a lengthening. It would seem that these two processes are similar, differing only in that in one case the cross-sectional movements are secondary to the longitudinal, and in the other, the longitudinal movements are secondary to the cross-sectional. This difference becomes clear once we realize that the primary movements are those caused and accompanied by the applied force impulses and that the secondary movements are accompanied by a very small, if any, part of the applied force impulse. In other words, the applied force impulses are used almost entirely to move the system in the direction in which the force is applied.

In the first analysis, it would seem that torsional vibrations would have certain advantages over longitudinal vibrations. It not only can be shown mathematically but is a well established fact that any particular mode resonance can be achieved at a lower frequency in torsion. Since power is proportional to the cube of the driving frequency, it would seem more economical to use torsional vibration to drive piling. However, the impedance of a torsionally resonant system is less because the particle velocity is less. Lower impedance means that the system is more sensitive to reactance at the ends and at joints, if present. Being more sensitive to reactance means that the particles in the system are more resistant to propagation and that attenuation is greater, which is highly undesirable.

Another fact that might tend to discredit pile driving by longitudinal vibration is that the vertical component in longitudinal vibration which opposes the desired direction of pile movement will hinder pile penetration or extraction. Russian experiments have demonstrated that by eliminating part of the upward component higher penetration rates are possible. However, in order to significantly overcome the undesirable component, a complicated system of eccentrically rotating weights must be used and the system becomes economically unjustifiable after a point.

In a torsionally resonated system, both components of displacement are utilized in that both contribute directly to horizontal displacement. The horizontal movement caused by torsional vibrations should tend to produce a drilling effect. However, it must be pointed out that torsional vibrations contribute little force displacement in the longitudinal direction and that any movement in this direction must be provided for by additional dead weight.

Insofar as this search could reveal, only one actual comparison has been made of longitudinal versus torsional vibration as applied to resonant pile driving. Mr. W.C. Rockefeller, Vice President of Sonico Research and Development Company Incorporated, San Diego, California, states that Sonico torsionally resonated small pipes for the purpose of coring and found that in order to drive at all, the bottom edge of the open-ended pipe had to be serrated to form teeth. This allowed penetration to some degree. Sonico found that driving in this fashion was inefficient and that the teeth wore rapidly. Driving with longitudinal resonant vibrations proved far more effective.

Sonico also conducted experiments involving larger pipes and found that they could be driven using torsional resonance, but in these cases the torsional vibrations were converted into longitudinal vibrations by means of the configuration of the pipe being driven. Exactly how this conversion was made is not clear.
Mr. Rockefeller further states that even though all their experimental work indicated that torsional vibrations have some of the same characteristics as longitudinal vibrations insofar as the side or shaft resistance is concerned, it proved to be rather impotent with regard to tip penetration. Mr. Rockefeller stated that he would not recommend torsional vibration over longitudinal for pile driving.

EQUATIONS RELATED TO RESONANT PILE DRIVING

Theory predicts that the resonant wave in a pile will assume the shape of the fundamental half wave shown in Figure 1. Under this condition the resonant frequency is (Feather, 1961):

\[ N_L = \frac{n}{2L} \sqrt{\frac{E}{\rho}} \]  

(1)

where

- \( N_L \) = longitudinal resonant frequency
- \( n \) = mode of vibration
- \( L \) = pile length
- \( E \) = Young’s modulus
- \( \rho \) = mass density of pile.

Equation 1 is arrived at through the basic equation of distance equals rate times time

\[ d = rt. \]  

(2)

Then

\[ \frac{L}{t} = \frac{r}{d} \]

or

\[ t = \frac{r}{d} \]  

(3)

where

- \( t \) = frequency
- \( r \) = rate = velocity (v)
- \( d \) = distance = wavelength (\( \lambda \)).

Therefore

\[ t = \frac{v}{\lambda} \]  

(4)
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because

\[ v = \text{velocity of wave propagation} = \sqrt{E/\rho} \]

\[ f = \frac{1}{\lambda} \sqrt{E/\rho} \]

or

\[ N_L = \frac{1}{\lambda} \sqrt{E/\rho} \]  

(5)

And similarly

\[ N_T = \frac{1}{\lambda} \sqrt{G/\rho} \]  

(6)

where

\[ N_T = \text{torsional resonant frequency} \]

\[ G = \text{shear modulus}. \]

By replacing \( \lambda \) in eq 5 and 6 with \( 2L \) we have:

\[ N_L = \frac{1}{2L} \sqrt{E/\rho} \]  

(7)

and

\[ N_T = \frac{1}{2L} \sqrt{G/\rho} \]  

(8)

From eq 7 and 8 we see that:

\[ \frac{N_T}{N_L} = \sqrt{G/E} \]  

(9)

For steel having \( E = 29 \times 10^6 \text{ psi} \) and \( G = 11.5 \times 10^6 \text{ psi} \) we find that \( N_T/N_L \) in eq 9 is 0.63. This implies that torsional resonance will occur at 60% of the corresponding longitudinal resonant frequency. Similarly because power \( P \) is proportional to the frequency squared, the relationship between the torsional and longitudinal power requirements at resonance is:

\[ \left( \frac{P_T}{P_L} \right) = \left( \frac{N_T}{N_L} \right)^2 = 0.40 \]  

(10)

or the power required to resonate torsionally should be 40% of that required for the corresponding longitudinal mode.

In short, the above equations imply that torsional resonance has certain advantages over longitudinal resonance. However, it will be shown that eq 7 and presumably eq 8 cannot be used to accurately predict the resonant frequency of piles driven in the field.

Bernhard (1967) has determined experimentally that at the first resonant frequency \( \lambda = 3L \) whereas theory predicts \( \lambda = 2L \). Thus, by replacing \( \lambda \) in eq 5 and 6 with \( 3L \) we have:

\[ N_L = \frac{1}{3L} \sqrt{E/\rho} \]  

(11)
and
\[ N_T = \frac{1}{3L} \sqrt{G/\rho} \quad (12) \]

Bernhard has also determined experimentally that for optimum driving efficiency in terms of tip displacement and the maintaining of a power level a pile should be resonated at the second resonant frequency. At this frequency \( \lambda = 1.5L \) and eq 11 and 12 become:
\[ N_T = \frac{1}{1.5L} \sqrt{E/\rho} \quad (13) \]
and
\[ N_T = \frac{1}{1.5L} \sqrt{G/\rho} \quad (14) \]

C.L. Guild Construction Co. (1965), which later became Resonant Pile Corp., modified eq 7 to the following empirical relationship for determining the first resonant frequency of a steel pile when driving with the BRD:
\[ N_L = \frac{16,500}{2L_e} \quad (15) \]
\[ = \frac{16,500}{2 (L+W)} \]

where
- \( 16,500 \) = velocity of wave propagation in steel
- \( L_e \) = effective pile length
- \( L \) = actual pile length
- \( W \) = weight of driver and clamp* \( \div \) weight per foot of pile.

It should be obvious that the only difference between eq 7 and 15 is the substitution of \( L_e \) for \( L \) and that the use of eq 15 is restricted to determining the first resonant frequency of steel piles.

Rayleigh (1926) presents a relationship for longitudinal resonance which takes into consideration the spring constant of the pile and the weight of the vibrator as follows:
\[ N_L = \frac{n}{2\pi} \sqrt{SEg/L (W_1 - W_2/3)} \quad (16) \]

where
- \( n \) = mode of vibration
- \( S \) = cross-sectional area of pile
- \( g \) = gravitational acceleration

* Although the total weight of the BRD can be as much as 11 tons, the weight of the driver or the resonator part of the unit physically attached to the pile is only 1450 lb. The motor and additional housing weight are supported on a cushion of air. This weight although contributing to pile load is not a physical part of the pile and is therefore not taken into consideration for determining the pile's frequency. The weight of the clamp varies with the size and type of pile driver.
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\[ W_1 = \text{weight of vibrator} \]
\[ W_2 = \text{weight of pile}. \]

Since
\[ SE/L = \text{spring constant of pile (K)} \]
and
\[ g/W = 1/m \]
where
\[ m = \text{mass} \]
then
\[ N_L = \frac{n}{2\pi} \sqrt{K/m} \] \hspace{1cm} (17)

where
\[ K/m = \omega = \text{angular frequency} = 2\pi f. \]

His equation for torsional resonance is:
\[ N_T = \frac{n}{2\pi} \sqrt{K/(I + I_s/3)} \] \hspace{1cm} (18)

where
\[ K = \frac{G I_p}{L} = \frac{n}{32} G (D^4 - d^4) /L \]
\[ I = \text{inertia of end mass (vibrator)} \]
\[ I_s = \text{inertia of pile} \]
\[ I_p = \text{polar moment of inertia} \]
\[ D = \text{outside pile diameter} \]
\[ d = \text{inside pile diameter}. \]

Kovacs (1966) modified the Rayleigh equation for determining longitudinal resonance to take into consideration the mass of soil brought into play and the additional column length being resonated as a result of the physical attachment of the vibratory unit to the pile (approximately three feet for the BRD). Kovacs set the mass of soil brought into play at 1/3 the mass of the pile. This was predicated upon determinations made by Barkan (1940). The equation is:
\[ N_L = \frac{n}{2\pi} \sqrt{K/L_e} \left[ \frac{(m_1 + m_2)}{A} \right] \] \hspace{1cm} (19)
where

\[ K = \frac{SE}{L_E} \]  
pile spring constant

\[ L_E = \text{effective pile length} = \text{actual pile length plus length of attached vibrator} \]

\[ m_1 = \text{mass of soil brought into play} = \frac{1}{3} m_2 \]

\[ m_2 = \text{mass of pile} = \frac{W_2}{g} \]

\[ A = \text{empirical constant} = 7 \text{ in.} \]

When \( n = 1 \) (first resonant mode) eq 19 can be rewritten:

\[ N_L = \frac{1}{2\pi} \sqrt{\frac{A SEg}{L_E^2} (1.3W_2)} \]  \hspace{1cm} (20)

**COMPARISON BETWEEN OBSERVED AND EQUATED RESONANT FREQUENCIES**

Field data\(^{15, 23}\) pertaining to eight piles driven by the BRD are listed in Table I. The longitudinal resonant frequencies observed at these piles are presented in Table II along with those predicted by the equations discussed. Here it is seen that the Bernhard and Kovacs equations come closest to predicting the observed frequency of each pile. This is similarly shown in Figures 4-10 which, along with the observed frequency, give the resonant frequency of each pile over an extended length as predicted by the equations presented herein.

![Graph showing resonant frequencies](image)

*Figure 4. First mode longitudinal resonant frequency vs length of 8.625 in. O.D., 0.50 in. walled steel pile.*
Figure 5. First mode longitudinal resonant frequency vs length of 10.75 in. O.D., 0.203 in. walled steel pile.

Figure 6. First mode longitudinal resonant frequency vs length of 10.75 in. O.D., 0.250 in. walled steel pile.
Figure 7. First mode longitudinal resonant frequency vs length of 12.75 in. O.D., 0.50 in. walled steel pile.

Figure 8. First mode longitudinal resonant frequency vs length of 10 BP 57 H-pile.
Figure 9. First mode longitudinal resonant frequency vs length of 12 BP 53 H-pile.

Figure 10. First mode longitudinal resonant frequency vs length of 12 BP 74 H-pile.
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Table I. Field data on piles driven with BRD

<table>
<thead>
<tr>
<th>Pile type (steel)</th>
<th>Pile length (ft)</th>
<th>Resonant freq (Hz)</th>
<th>Resonator wt (lb)</th>
<th>Clamp wt (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.625 in. OD, 0.500 in. thick</td>
<td>52*</td>
<td>112</td>
<td>1450</td>
<td>150</td>
</tr>
<tr>
<td>10.750 in. OD, 0.203 in. thick</td>
<td>75*</td>
<td>68</td>
<td>1450</td>
<td>315</td>
</tr>
<tr>
<td>10.750 in. OD, 0.250 in. thick</td>
<td>117†</td>
<td>53</td>
<td>1450</td>
<td>315</td>
</tr>
<tr>
<td>12.750 in. OD, 0.500 in. thick</td>
<td>54*</td>
<td>117</td>
<td>1450</td>
<td>315</td>
</tr>
<tr>
<td>10 BP 57 H-section</td>
<td>50*</td>
<td>113</td>
<td>1450</td>
<td>464</td>
</tr>
<tr>
<td>12 BP 53 H-section</td>
<td>60*</td>
<td>95</td>
<td>1450</td>
<td>655</td>
</tr>
<tr>
<td>12 BP 53 H-section</td>
<td>65*</td>
<td>95</td>
<td>1450</td>
<td>655</td>
</tr>
<tr>
<td>12 BP 74 H-section</td>
<td>50*</td>
<td>118</td>
<td>1450</td>
<td>655</td>
</tr>
</tbody>
</table>

* Open ended
† Closed ended

Table II. Comparison between calculated and observed longitudinal resonant frequencies for eight piles driven with the BRD.

Predicted frequencies (Hz)

<table>
<thead>
<tr>
<th>Type of pile (steel)</th>
<th>Length (ft)</th>
<th>Theory</th>
<th>Bernhard Const. Co.</th>
<th>Rayleigh</th>
<th>Kovacs</th>
<th>Field observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.625 in. OD 0.50 in. thick open-ended</td>
<td>52</td>
<td>162</td>
<td>108</td>
<td>93</td>
<td>50</td>
<td>113</td>
</tr>
<tr>
<td>10.75 in. OD 0.203 in. thick, open-ended</td>
<td>75</td>
<td>113</td>
<td>75</td>
<td>55</td>
<td>31</td>
<td>80</td>
</tr>
<tr>
<td>10.75 in. OD 0.250 in. thick, closed-ended</td>
<td>117</td>
<td>72</td>
<td>48</td>
<td>47</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>12.75 in. OD 0.50 in. thick, open-ended</td>
<td>54</td>
<td>156</td>
<td>104</td>
<td>106</td>
<td>56</td>
<td>108</td>
</tr>
<tr>
<td>10 BP 57 H-section</td>
<td>50</td>
<td>169</td>
<td>112</td>
<td>100</td>
<td>54</td>
<td>117</td>
</tr>
<tr>
<td>12 BP 53 H-section</td>
<td>60</td>
<td>140</td>
<td>93</td>
<td>83</td>
<td>45</td>
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<tr>
<td>12 BP 74 H-section</td>
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<td>130</td>
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<td>79</td>
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<tr>
<td>12 BP 74 H-section</td>
<td>50</td>
<td>169</td>
<td>112</td>
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<td>57</td>
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CONCLUSIONS

While several attempts have been made to arrive at a rigid mathematical analysis of a pile responding to resonant vibrations, it has been shown that actual field results differ from the predicted results. Kovacs' and Bernhard's equations for predicting pile resonance are shown to equate longitudinal resonant frequencies in close agreement with those observed at piles driven with the BRD. It must be pointed out, however, that the equations should be subjected to a more rigorous evaluation to determine whether they can predict the resonant frequency of all force generator - column systems.

Torsionally vibrated systems have the seeming advantages over longitudinally vibrated systems that they reach resonance at lower frequencies and they lack the undesirable upward force-impulse component which accompanies longitudinal vibration. However, the advantages appear to be heavily outweighed by the disadvantages of lower impedance, more attenuation, and less tip penetration. A considerable amount of written material substantiates the fact that longitudinal resonant systems have many advantages over conventional hammer-type drivers over a wide range of field conditions. Those advantages are increased speed, quietness, efficiency and load-bearing capacity.
PILE DRIVING BY MEANS OF LONGITUDINAL AND TORSIONAL VIBRATIONS

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PILE DRIVING BY MEANS OF LONGITUDINAL AND TORSIONAL VIBRATIONS

This report discusses vibratory pile driving with particular emphasis on pile driving at resonance where maximum driving efficiency can be expected. The theories and concepts associated with longitudinal and torsional pile driving are presented to show that torsional resonance does not appear to be as effective a method as longitudinal resonance and that considerable variations can exist between calculated and observed resonant frequencies. While it is pointed out that equations by Bernhard and Kovacs predict pile resonance in close agreement with that observed during actual pile driving, it is also suggested that these equations be subjected to a more rigorous evaluation to determine whether they can predict the resonant frequency of all force generator - column systems.

14. KEYWORDS

Pile driving  Torsional vibration
Pile drivers  Soil dynamics
Pile foundations  Longitudinal vibration