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FLUIDIZATION PHENOMENA IN SOILS DURING VIBRO-COMPACTION AND VIBRO-PILE-DRIVING AND -PULLING

by

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PREFACE

This report was prepared by Dr. R.K. Bernhard (Expert, USA CRREL) for the Applied Research Branch (Mr. A. Wuori, Chief) of the Experimental Engineering Division (Mr. K.A. Linell, Chief). Some of the experiments were conducted at Rutgers University and some at Princeton University.

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SUMMARY

Fluidization, representing the change of soils from a solid into a quasi-fluid state by means of vibratory forces, is discussed. This hydrodynamic state is of particular importance for vibratory compaction and vibratory-pile driving and pulling. Experiments "in situ" and in the laboratory are described. The response of soils to static and dynamic loads was analyzed to determine the prerequisites for liquefaction.

Results indicate that under certain conditions a thin discrete zone in the immediate vicinity of the exciting source exists which shows characteristics similar to those of a viscoelastic fluid.
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\[ \sigma_i \quad \text{soil pressure (subscript } i=1, 2\text{-}) \]

\[ \tau_i \quad \text{soil shear stress (subscript } i=1, 2\text{-}) \]

\( \phi \quad \text{shear strain} \)

\( \phi \quad \text{phase angle} \)

\( \omega \quad \text{frequency (rad sec}^{-1}\text{)} \)

\( \omega_e \quad \text{exciter frequency (rad sec}^{-1}\text{)} \)

\( \omega_{cr} \quad \text{critical frequency (rad sec}^{-1}\text{)} \)

\( \omega_r \quad \text{resonance frequency (rad sec}^{-1}\text{)} - \phi = 90^\circ \)

F=force, L=length
T=time, d=dimensionless

\[ F-L^{-2} \]

\[ F-L^{-2} \]

\[ d \]

\[ d \]

\[ T^{-1} \]

\[ T^{-1} \]

\[ T^{-1} \]

\[ T^{-1} \]
FLUIDIZATION PHENOMENA IN SOILS DURING VIBRO-COMPAC'TION
AND VIBRO-PILE-DRIVING AND-PULLING
by
R. K. Bernhard

INTRODUCTION

The impetus for these investigations emanated from contradictory results
relating to vibro-compaction of soils and vibro-pile-driving, -pulling and
-drilling. The effectiveness of vibratory rollers as compared to static rollers,
and vibro-pile drivers as compared to impact pile drivers, has been under dis-
cussion with opposing views, particularly in reference to loosening instead of
compaction in the uppermost soil strata due to vibratory rollers and to different
concepts concerning the most effective frequency ranges for vibro-pile-driving
and -pulling. An attempt will be made here to explain some of the fundamental
physical phenomena accompanying various vibratory methods. The theoretical
background is covered in the list of references, including further details about
the instrumentation.

"Soil fluidization" might be defined as the change of a soil from its normal
state into a state where quasi-fluid (vibro-viscous) characteristics predominate.
This liquefaction is of primary significance in vibro-soil-compaction, where it
would increase the bearing capacity of deeper strata but might cause loosening
of the upper strata, and for vibro-pile-driving or -pulling, where it would reduce
skin friction.

The purpose of this investigation is to determine:
1) under what conditions fluidization can be obtained, particularly the
correlation between the required magnitude, direction and frequency
of the vibratory surface loads, and
2) the character of the liquefied soil volume with respect to dimensions
and sustainability.

For simplicity's sake, the evaluation is based on the preliminary assump-
tion that the soil can be regarded as an isotropic, homogeneous, elastic half-
space continuum. Furthermore, the experiments had to be limited in number
by reducing the variable parameters. For these reasons the investigation is
to be considered only as a preliminary study.

Six series of experiments dealing with response of soils to sinusoidal
force excitation were conducted. All experiments except Series II were made in the
laboratory. Series I is a study of the volume of a fluidized zone in the vicinity
of the disturbing source; Series II is an investigation of compaction "in situ"
due to normal surface forces, particularly to find the decay of the transmitted
pressure from static and dynamic loads. Series III describes the determination
of shear modulus and viscosity produced by dynamic shear forces. Series IV,
V, and VI are concerned with pile driving: Series IV investigates the effect
of vibratory force on skin friction, Series V deals with frequency response,
particularly response to critical frequencies in the range 20-6000 cps and
Series VI is a study of fluidization effects connected with "floating depth," that
is, a depth of penetration where the pile reaches a final equilibrium position.

The main characteristics of the three soils used in this investigation,
Beach sand, Ottawa sand, and Princeton red clay, are plotted in Figures 1a
and 1b.
The purpose of this small model experiment is to demonstrate the existence of a fluidized zone surrounding a vibrating rod.

Experimental set-up
A cylindrical glass jar (diam, 12 in.; height, 12 in.) was filled with dry Ottawa sand (20-30). A brass tube (o.d. 0.75 in.) with a flat tip was buried 2 in. into the soil, held in this position, and subjected to sinusoidal vibrations (frequency, 30 cps; double displacement amplitude, \( \frac{1}{8} \) in.).
Analysis

The blurred zones surrounding the pile (Fig. 2a, b) indicate the extent to which soil particles participated in the vibratory motion. They comprise the volume (~20 in.³) of the fluidized zone.

The existence of a discrete fluidized zone in the vicinity of the dynamic load has been predicted by various investigators. Krynine (1941) called it a disturbed zone or zone of discontinuity, and Slade (1953) defined it as a zone of high energy state in which the medium changes its statistical characteristics. Winterkorn (1953) likened it to a micromeritic liquid and the author (Bernhard and Finelli, 1953) called it a viscoelastic solid, made visible by slow motion pictures of the soil surface close to the contact plate of the vibration exciter.

Figure 2. Fluidized zone, dry Ottawa sand. Rod vibrating at 30 cps in glass jar (1/3 sec exposure).
Purpose

The main purpose of these compaction studies is the determination of the fluidized zone due to normal surface loads.

Experimental set-up

All experiments were confined to noncohesive, dry soil: beach sand with predominant grain size distribution from 0.1 to 1 mm diam; void ratio 0.3 to 0.4; MC = 3%; density \( \sim 118 \text{ lb ft}^{-3} \) (Fig. 1a).

Differential pressure cells (Bernhard and Finelli, 1953) were located at three different levels below the contact plate of the surface load (Fig. 3). The pressure cells (0.9 in. diam; 1.2 in. high) comprised a linear differential transformer connected to a pressure-sensitive membrane deflecting in proportion to the soil pressure. At each level the pressure cells were oriented in six different axial directions (Fig. 4). Three orthogonal \((\sigma_1, \sigma_2, \sigma_3)\)* and three non-orthogonal \((\sigma_4, \sigma_5, \sigma_6)\) at 45° in the 1-2, 2-3, and 1-3 plane) normal stress components were measured. From these six normal stresses the three shear stresses \((\tau_{12}, \tau_{23}, \tau_{13})\) could be computed (Jaeger, 1956). The output of one cluster of three pressure cells was recorded at a time to avoid any distortion from adjacent measuring points.

*Symbols adopted for use in this paper are listed on pages vii-ix.
FLUIDIZATION PHENOMENA IN SOILS

Figure 3. Experimental set-up.

Figure 4. Orientation of differential pressure cells at the 8-in., 12-in., and 18-in. level.
A static load was produced by lowering slowly onto the soil surface a mechanical oscillator (not operating) including surcharge (Fig. 3).

A dynamic load was generated by operating the vibrator in its lowered position. The oscillator transmitted a vertical sinusoidal force vector into the soil. Its frequency was kept outside the natural frequency range of the vibrating mass-soil system (Bernhard, 1937) and its magnitude below the dead weight of oscillator plus surcharge. This prevented jumping of the vibrator, hence transmission of impact forces into the soil.

The oscillator (Bernhard, 1949) consisted of two rotatable pairs of eccentrically supported weights, \( W \). Each pair of weights excites centrifugal force vectors. The magnitude of these forces depends upon the frequency \( \omega \) and the angle \( \theta \) between the two weights of each pair. If the two pairs of weights rotate with the same velocity, but in opposite directions, a sinusoidal one-directional component remains. The equation for the resultant force component is:

\[
F = 4 \frac{W}{g} r \omega^2 \cos \frac{\theta}{2} \sin(\omega t) = U \cdot \frac{\omega^2}{g} \sin(\omega t) \quad (\text{II-1})
\]

where \( 4W r \cos \frac{\theta}{2} = U \); for \( W = 6 \) lb and \( r = 3 \) in.

\[
F_{\text{max}} = 7.44 n^2 \cos \frac{\theta}{2}. \quad (\text{II-2})
\]

By means of a differential gear coupling, connecting the two pairs of weights, the angle between the two weights of each pair can be changed while the machine is operating. Hence, constant force vectors may be excited at different frequencies.

In a few cases, when the displacement became so large that linearity of the vibrating system could not be maintained, the unbalance \( U \) had to be reduced; in all other cases \( U \) was kept constant.

A special frequency control unit for the oscillator drive was necessary to maintain a preset speed, particularly in the critical frequency ranges. This unit comprised the power supply, consisting of an ac-dc motor generator, two amplidynes, a servo amplifier, a tachometer-generator and a dc motor driving the mechanical oscillator (Fig. 5). One amplidyne supplied a constant voltage to the field of the oscillator motor; the other amplidyne generated a variable voltage in the field of the dc generator. This variable voltage was controlled by a servo-amplifier acting as a closed loop system.

The operating frequency was selected by a helipot. The difference between the selected frequency and the tachometer generator output comprised the error signal which was adjusted to zero by the servo-amplifier. Preset speeds did not deviate more than \( \pm 0.1 \) cps regardless of sudden load changes in the critical frequency ranges.

**Experimental results**

A typical time-pressure record is shown in Figure 6. The three upper oscillograph traces represent the output of differential pressure cells in the \( \sigma_1 \), \( \sigma_3 \), and \( \sigma_6 \) direction at 8-in. depth due to a vibratory load acting upon the soil surface. The three almost sinusoidal traces of the three normal stress components correspond to the sinusoidal force vector. All troughs of these images indicate a pressure increase, the crests a pressure decrease. Peaks in the reference line (R) mark the vertical upward position of the exciting force vector.
In Figure 6 each trough in the reference line, identifying a downward force vector, coincides closely with troughs of all three stress components and discloses a pressure increase. This case is designated as positive. A negative sign signifies a pressure decrease which might be due to an increase of interparticle spacing.

Numerous experiments with force vectors of different frequencies and magnitudes were made. Some of the results are summarized in Table I. All numerical values represent the average normal stress components measured from a series of tests under identical loading conditions. The corresponding shear stresses are computed.

In Figure 7 the stress distributions in the 1-2, 2-3, and 1-3 planes are plotted.

Effect of static loads

The main criteria for fluidization are that all normal stress vectors tend to become equal and therefore directionally independent and all shear stresses approach zero. In other words, the normal stress vector diagram degenerates into a sphere, indicating hydrostatic conditions.

8-Inch level. At 8-in. depth the normal stress vectors approximate an almost hyperboloidal distribution (Bernhard, 1967b). The shear stresses do not approach zero nor have the normal stress vectors semicircular characteristics, disclosing that under static loads no fluidization takes place, regardless of the proximity of the surface load.

12-Inch and 18-inch levels. At these depths the normal stress components do not reach significant values and can be neglected. This indicates that below the 8-in. level self-locking or arching phenomena predominate and prevent any noticeable pressure transmission.
Table I. Measured normal and computed shear stress components, Beach sand.

<table>
<thead>
<tr>
<th>Depth (in.)</th>
<th>Depth (in.)</th>
<th>( \sigma ) (lb/in.(^2))</th>
<th>( \tau ) (lb/in.(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Static</td>
<td>+0.3</td>
<td>( \sigma_6 - \frac{1}{2}(\sigma_1 + \sigma_2) )</td>
</tr>
<tr>
<td>8</td>
<td>Dynamic</td>
<td>+0.4</td>
<td>( \sigma_5 - \frac{1}{2}(\sigma_1 + \sigma_3) )</td>
</tr>
<tr>
<td>8</td>
<td>Static + dynamic</td>
<td>+0.7</td>
<td>( \sigma_4 - \frac{1}{2}(\sigma_2 + \sigma_3) )</td>
</tr>
<tr>
<td>8</td>
<td>Dynamic</td>
<td>+0.45</td>
<td>( \sigma_6 - \frac{1}{2}(\sigma_1 + \sigma_2) )</td>
</tr>
<tr>
<td>12</td>
<td>Dynamic</td>
<td>-0.05</td>
<td>( \sigma_5 - \frac{1}{2}(\sigma_1 + \sigma_3) )</td>
</tr>
<tr>
<td>18</td>
<td>Dynamic</td>
<td>0.25</td>
<td>( \sigma_4 - \frac{1}{2}(\sigma_2 + \sigma_3) )</td>
</tr>
</tbody>
</table>

Dynamic load: Force vector of \( \pm 1500 \) lb at 25 cps.
Static load: \( 1700 \) lb.

Effect of dynamic plus static loads

**8-Inch level.** At 8-in. depth the shear stresses approach zero (Table I) and the normal stress vectors form an almost spherical distribution (Fig. 7a). These two criteria show that the predicted fluidized or hydrostatic state prevails.

**12-Inch and 18-inch levels.** At 12-in. and 18-in. depth the form of the vector diagrams changes substantially. \( \sigma_1 \) at 12-in. and \( \tau_{13} \) at 8-in. depths become negative, corresponding to a pressure decrease at the downward stroke of the exciting force vector. The previously mentioned two indications of fluidization have vanished, indicating that the liquefied zone does not reach these two lower levels.

Analysis

The analysis is confined to a noncohesive dry beach sand. The results are summarized as follows:

1) Dynamic surface loads reach greater depth than static loads by reducing interparticle friction or self-locking and arching effects. Some of the static friction is changed into the smaller dynamic friction by vibratory motion of the grains.

2) A discrete layer under vibratory surface loads exists where fluidization of the soil predominates. This liquefied zone may be considered as a coupling agent between load and deeper strata and likened to an impedance matching device.

3) The frequency of the dynamic force required to produce this zone need not necessarily be in the resonance range (Bernhard, 1937) of the vibrating mass-soil system.

Conclusion

In vibratory soil compaction, the transmission of dynamic loads to greater depth is advantageous. However, excessive vibratory forces may cause a loosening of the uppermost stratum, thus requiring an additional final run of a non-vibrating compactor. Reports of contractors using vibratory compactors in the field have verified this requirement.
The purpose of these studies was to determine 1) that particular shear force frequency which produces a substantial decrease in shear modulus and 2) whether vibro-viscosity follows a similar pattern.

**Theoretical considerations**

For the sake of simplicity, the tentative assumption is made that the vibrating exciter-soil-mass system may be represented by a one degree of freedom system with linear spring constant and linear viscous damping. This corresponds to a viscoelastic Kelvin model, consisting of one spring and one dashpot.
The differential equation of motion for sinusoidal excitation can be written in the form (Bernhard and Finelli, 1953):

\[ m \ddot{x} + c \dot{x} + kx = F \sin(\omega t). \]

Let \( x = a \sin(\omega t - \phi) \) represent the displacement, where

\[ a = \frac{F}{[(c\omega)^2 + (k - m\omega^2)^2]^{\frac{1}{2}}} \]

and

\[ \tan \phi = \frac{c\omega}{k - m\omega^2}. \]

Eliminating \((k - m\omega^2)\) yields:

\[ a = \frac{F}{[(c\omega)^2 + (c\omega)^2 \cot^2 \phi]^{\frac{1}{2}}} = \frac{F \sin \phi}{c\omega}. \]

The coefficient of viscosity \( \nu \) of the material may be defined as the tangential force per unit area on either of two horizontal planes a unit distance apart; one plane is fixed while the other moves with unit velocity. The space between the planes is filled with the material. Therefore the damping force is:

\[ c \dot{x} = \nu A \frac{(\ddot{x})}{h} \]

or

\[ c\omega = \frac{\nu A \omega}{h} \]

and

\[ a = \frac{Fh \sin \phi}{\nu A \omega}. \]

The elastic force is:

\[ ka = GA\phi. \]

For small displacements

\[ ka = GA \frac{\ddot{a}}{h} \]

and

\[ G = \frac{kh}{A}. \]
FLUIDIZATION PHENOMENA IN SOILS

From $k = m\omega^2 + c\omega \cot \phi$

and $c\omega = \frac{F \sin \phi}{a}$

follows $k = m\omega^2 + \frac{F \cos \phi}{a}$

and finally:

$$G = \frac{h}{A} \left( m\omega^2 + \frac{F \cos \phi}{a} \right). \quad (III-3)$$

The parameters $h$, $A$, $m$, $\omega$, and $F$ are known and $\phi$ and $a$ are measured. Thus equations III-1, -2, and -3 yield $c$, $v$, and $G$. A computer was used to evaluate these three soil characteristics from the data obtained.

Experimental set-up (Fig. 8, 9)

The experiments were restricted to two distinct types of soil, standard Ottawa sand (20-30) (Fig. 1a) representing a homogeneous, non-cohesive soil, and Princeton red clay (Fig. 1a and 1b) as a cohesive soil.

The grain size distribution of Princeton red clay ranged from 0.002 to 1 mm; maximum density was 122 lb ft$^{-3}$ at 8% M.C., dry weight, Mod. Proctor.

The cylindrical soil samples had a diameter of 23.25 in. The height varied from 1.4 to 3.2 in. The samples were sandwiched between two serrated, circular steel plates with a diameter of 23.25 in. The upper plate was movable; the lower plate was fixed to the foundation. A circular rubber membrane wrapped around these plates restrained the soil. A cross beam, attached to the upper plate, transmitted the static and dynamic forces to the sample.

A dynamic load of sinusoidal wave form was applied horizontally to the upper serrated steel plate. This type of load simulates as closely as possible the shear force action between the skin of a pile and the adjacent soil during vibro-pile driving. The dynamic force vector $F$ was generated by the same mechanical oscillator, including control unit, as used for Series II.

The static surcharge or bias participating in the vibratory motion consisted of the oscillator weight on one end of the cross beam, counterbalanced by additional weights on the opposite end of the beam. This surcharge could be reduced by an overhead crane acting upon the beam. However, to decrease the number of variable parameters, the surcharge was kept constant at ~290 lb for this series of experiments.

Displacement. A piezo-electric transducer responded to the displacement of the upper serrated steel plate with respect to the foundation. The transducer output was amplified and recorded by an oscillograph. No transducer had to be inserted into the soil sample. Thus all the difficulties, particularly discontinuities, due to a measuring device buried in the soil are avoided.

The calibration of the displacements, independent of any electronic device, is based on an optical system consisting of a micro-meter-microscope fixed to the foundation and a transparent scale (subdivision $10^{-1}$ mm) attached to the movable upper steel plate. A stroboscope, adjusted closely to the operating frequency of the force generator, illuminated the scale. The precision of the optical system was to about $\pm 10^{-2}$ mm.
Figure 8. Soil sample on vibration table.

Figure 9. Experimental set-up.

Figure 10. Phase angle (φ) vs frequency (Ne), Princeton red clay.
Phase angle. A transformer (similar to that used in Series II) was attached to the housing of the oscillator and a small permanent magnet rotated with the same speed as the weight in the oscillator. The magnet, passing the transformer, generated a voltage whenever the maximum force vector was in its downward position. The voltage peak could be amplified and recorded by the oscillograph in the form of a narrow "pip," indicating the phase angle, $\phi$.

The advantage of measuring phase angles is that $\phi = 90^\circ$ at the resonance frequency ($\omega_r$) of the vibrating system regardless of damping ($\nu$) and any non-linearity (DenHartog, 1940).

Shear area. Two shortcomings of the present setup should be avoided in future studies.

In the experimental setup the shear area is not well defined because of the use of two circular steel plates with their grooves cut in concentric rings. This was done to facilitate the machining operation and to simplify the restraining of the soil samples by means of a cylindrical rubber membrane wrapped around the circumference of these plates. From the theoretical point of view, it would be better to use two rectangular plates with parallel grooves, all at $90^\circ$ to the motion of the upper plate, thus providing a better defined shear area. However, the shear area enters the computation only to the first power. It may be assumed that the numerical values for $c$ and $G$ would change linearly when using rectangular instead of circular plates.

The soil sample was subjected only to a horizontal force vector. Theoretically a couple comprising vertical force vectors should be added to induce pure shear in the sample.

Experimental results

Only a few of the numerous test runs can be presented (Fig. 10-14). The frequencies are given in rpm (N) following the computer set-up.

Princeton red clay. The first tests were made with Princeton red clay, vibro-precompacted to an average density of \(\sim 100 \text{ lb ft}^{-3}\). The sample height was restricted to 2.0, 2.2, and 3.2 in., the moisture contents to 3.0%, 7.8% and 14.8%.

$\phi$ versus N (Fig. 10): The three curves pass $\phi = 90^\circ$, indicating resonance at 850, 950 and 900 rpm for moisture contents of 3.0%, 7.8% and 14.8% respectively.

$G$ versus N (Fig. 11): Significant $G_{\text{min}}$ values, as low as 58 lb in. $^{-2}$ for 7.8% moisture content, become obvious. However, these minima occur below the corresponding $N_r$ values of Figure 10.

$\nu$ and $c$ versus N (Fig. 12): With 3.0% and 7.8% moisture content the $\nu$ and $c$ parameters increase from lower to higher $N_r$. Only the curve for 14.8% moisture content indicates a minimum at $N_{\text{cr}} = 1150$ rpm which is above the $N_r$ of 900 rpm (Fig. 10).

These experiments revealed that a thorough precompaction of the soil is a prerequisite for all further testing. The time elapsed after mixing the sample will gradually decrease the free water by adsorption, and produce higher stiffness ratios. In other words, the history of the sample has to be considered. This is of particular importance for clayey material where environmental conditions after mixing the sample, even without precompaction, have to be taken into account (Hall and Richart, 1963; Hardin and Richart, 1963).
Figure 11. Shear modulus (G) vs frequency (Ne), Princeton red clay.

Figure 12. Damping (c) and viscosity (v) vs frequency (Ne), Princeton red clay.
Ottawa sand. Further experiments were conducted with Ottawa sand, vibro-precompacted to an average density of about 119 lb ft\(^{-3}\). The sample height was restricted to 1.4 and 1.7 in. because thicker samples produced non-stable conditions, particularly at higher moisture contents. The moisture content could be varied from 0 (dry) to 8% (drained) and to 14% (fully saturated).

All results differ substantially from those obtained with Princeton red clay.

\(\phi\) versus \(N\) (Fig. 13): The three curves pass \(\phi = 90^\circ\), indicating resonance at 950, 1125, and 900 rpm for a moisture content of 0, 8.0% and 14.2% respectively.

\(G\) versus \(N\) (Fig. 14): Characteristic minima of the \(G\) values as low as 30 lb in.\(^2\) are discernible at 920 rpm for 14.2% MC, which is above the corresponding resonance frequency of 900 rpm (Fig. 13). This deviates from the results obtained with Princeton red clay where \(G_{\text{min}}\) occurred at lower frequency than resonance. The large range of \(G\) required a logarithmic scale on the Y-axis.

\(v\) versus \(N\): Similar to the results with Princeton red clay, the dry Ottawa sand indicated minimum values for \(v\). However, the data became very erratic. Phenomena of instability have been reported previously for dry beach sand (Bernhard and Finelli, 1953) and were attributed to interparticle slipping action or structural breakdown of the bond between individual grains.

The drained and saturated sand showed a tendency of rather-steadily-increasing \(v\) values without displaying any pronounced \(v_{\text{min}}\).

Comparison between Princeton red clay and Ottawa sand

In Figure 15 the variables are separated to gain some insight into the phenomena. (An attempt is made to retain at least quantitatively the basic features of the problem.) Average values of all data are plotted.

Fig. 15a shows \(G_{\text{min}}\) versus \(h\). For PRC a \(G_{\text{min}}\) of 58 lb in.\(^2\) is reached at \(h = 2.2\) in. For OS a \(G_{\text{min}}\) of 30 lb in.\(^2\) occurs at \(h = 1.7\) in.

It is significant that for a specific \(h\) both soils display low \(G\) values thus approaching a state of liquefaction, overriding all other characteristics within the above mentioned limits.

Figures 15b and c relate \(N_r\) to \(h\) and moisture content. For Princeton red clay and Ottawa sand a steady decrease of \(N_r\) with increase of \(h\) becomes obvious (Figure 15b), approaching a steady value of \(N_r\) at larger \(h\). This seems to indicate that for thicker soil samples only the upper layers take part in the induced vibratory motion, while the lower strata remain at rest. The motion profile between the two plates has to be investigated in more detail.

The maximum \(N_r\) values (Fig. 15c) occur for PRC at 950 rpm and 9% MC and for OS at 1150 rpm and 7% MC.

All three graphs (Fig. 15a, b, c) display somewhat similar curvature.

Empirical equation

Numerous mathematical analogies simulating most of the pertinent soil characteristics have been proposed, starting from a simple "Kelvin" model to a most comprehensive analogue consisting of two "Burger" bodies placed in parallel. For larger displacements several of the model parameters (spring- and dashpot - characteristics) will become nonlinear, particularly when considering dynamic loads. Furthermore, the variables controlling the behavior of soils subjected to vibratory loads must include environmental conditions. Some
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Figure 13. Phase angle ($\phi$) vs frequency ($N_e$), Ottawa sand.

Figure 14. Shear modulus ($G$) vs frequency ($N_e$), Ottawa sand.
of these numerous parameters which are partially interdependent may be frequency, dynamic force vector, static load, viscosity, spring constant, displacement amplitude, grain size distribution, moisture content, temperature, time, depth, etc.

A rigorous mathematical solution based on any type of model analogy including all or most of these variables would be rather complex. For this reason and also because only a few of the above characteristics could be varied during the experiments an empirical equation is suggested as a temporary expedient. The only purpose of this equation, which represents at best a first crude approximation, is to find that exciter frequency ($\omega_0$) which produces the minimum shear modulus. In other words, the maximum amount of fluidization for this particular experimental setup has to be determined. Obviously, the equation must yield a curve that follows closely the experimentally obtained data. All $G$ versus $N$ graphs show a similarity to the response curves of an undamped linear system but with inversed curvature. Such a system may be a mass-spring model (Hookean substance) subjected to an exciter force, ($F$) increasing with the square of the exciter frequency, $(\omega_0)^2$ (Bernhard, 1941).

The application of the empirical equation will be enhanced by introducing dimensionless parameters.

As an arbitrary approach the following form (DenHartog, 1940; Bernhard, 1941) might be used:

$$R_1 = \frac{1 - Z^2}{Z^2},$$  
(III-4)
FLUIDIZATION PHENOMENA IN SOILS

Let \( R_1 = \frac{G^{\text{stat}}}{G^{\text{dyn}}} \) 

and \( Z = \frac{\omega_e}{\omega_r} \)

yielding for:\n\( Z = 0, \quad R_1 = \infty \) (static, \( \omega_e = 0 \))
\( Z = 1, \quad R_1 = 0 \) (resonance, \( \omega_e = \omega_r \))
\( Z \rightarrow \infty, \quad R_1 = 1 \) (\( \omega_e > \omega_r \)).

Experiments have shown that \( R_1 \) will never be zero or reach infinity. In order to prevent \( R_1 = 0 \), a coefficient \( C_1 \) must be added and to eliminate \( R_1 = \infty \) the range of \( Z \) has to be restricted to values close to unity. Finally a shift of \( G_{\text{min}} \) towards smaller or larger values than \( Z = 1 \) takes place and a second coefficient \( C_2 \) has to be introduced.

It is interesting to note that \( G_{\text{min}} \) covers a rather small range:

\[ 30 < G_{\text{min}} < 58 \text{ lb in.}^2 \]

The modified equation III-4 reads:

\[ R_2 = R_1 + C_1 = \frac{1-(Z\pm C_2)^2}{(Z\pm C_2)^2} + C_1 \] (III-5)

where \( R_1 = 0 \) for \( G_{\text{min}} \).

Figures 16 and 17 represent a comparison between experimentally obtained \( G - N_e \) curves and \( R_2 - Z \) graphs for Princeton red clay and Ottawa sand.

The constants are \( C_1 = -0.3, \quad C_2 = +0.05 \) for Princeton red clay (Fig. 16); and \( C_1 = -0.08, \quad C_2 = -0.02 \) for Ottawa sand (Fig. 17).

In both cases the descending branches and in particular \( G_{\text{min}} \) coincide rather satisfactorily. The maximum deviation of \( \sim 5\% \) occurs at the upper end of the ascending branch.

For any type of soil tested in a similar setup, \( G_{\text{min}} \) and the corresponding \( \omega_e \) may be determined by changing the values of \( C_1 \) and \( C_2 \).

No attempt can be made at this time to define the physico-chemical relations of \( C_1 \) and \( C_2 \) with respect to the numerous parameters controlling the various soil characteristics.

Analysis

Two fundamental prerequisites for a linear system are fulfilled:

First: All experimentally obtained curves are smooth and do not display any discontinuities.

Second: All phase angles intersect the resonance frequency ordinate at \( \phi = 90^\circ \) and do not reveal abrupt changes near \( \phi = 90^\circ \), which would prevail, for example, if friction damping were preponderant.
Figure 16. Comparison between experimental results and empirical equation, Princeton red clay.

Figure 17. Comparison between experimental results and empirical equation, Ottawa sand.
A theoretical analysis of these phenomena, including slipping action, has been presented elsewhere (Bernhard and Finelli, 1953).

Pronounced troughs in most G-N graphs are visible. It must be well understood that this is a characteristic of the combined mass-spring-damping system of this particular setup and valid for one particular mode only.

The reproducibility of all graphs seems to indicate that the experimental and simplified theoretical approach is adequate.

Shear modulus. Of primary importance are the G-N curves. A substantial drop of G near resonance occurs in all cases. Again it must be emphasized that G is a characteristic of this setup and therefore will be designated as an apparent shear modulus for a thin layer of soil.

The low G-values manifest a state approaching fluidization. Any strata in the vicinity of the disturbing source are in the range of high energy input.

**Liquefaction.** In an elastic medium

\[
\mu = \frac{E}{2G} - 1
\]

and for \(G < \frac{E}{3} \), \(\mu > 0.5\).

Earlier investigations (Bernhard and Finelli, 1953; Wilson and Dietrich, 1960) yielded values for E larger than 3G and the strange phenomenon of an apparent Poisson's ratio larger than 0.5 would indicate a volumetric phenomenon (dilatancy). The interspace between individual soil particles is increased by their induced vibratory motion and the grains acquire more elbow room regardless of any compressional effort. In other words, \(\mu\) seems to lose its physical significance, at least as far as its customary interpretation is concerned.

An attempt may be made to explain the mechanical part of this process. In Figure 18 idealized conditions are shown, that is, a soil consisting of only four rows of spheres with equal diameter arranged in one plane with densest possible packing (Fig. 18a). If the horizontal shear forces \(S_1\) and \(S_2\) are applied, a dislocation pattern (Fig. 18b) could be produced, assuming that the energy input is only large enough to vibrate the upper two rows. The increase in height from \(h_1\) to \(h_2\) with corresponding volume increases in the direction of \(h\) might account for \(\mu > 0.5\).

Summarizing the analysis it can be stated:

1. The experimental results are reproducible and consistent with previous investigations using similar or other testing methods.
2. The simplified theoretical approach, assuming a linear system, seems adequate as long as the displacement amplitudes are kept very small.
3. Vibratory shear forces applied to a thin layer of soil can produce a drop in the apparent shear modulus approaching a state of a quasi-viscous fluid.
4. Stress history and environmental conditions including physico-chemical changes, in particular of clayey soils, are of major importance. The tests should be extended to soils of different composition and with changing parameters such as moisture contents, soil sample thickness, dynamic and static loads, pre-compaction, etc. A more systematic change of only one of these parameters during each series of experiments is essential.
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HEIGHT: \( h_1 < h_2 \)
SHEAR MODULUS: \( G_1 > G_2 \)
DENSITY: \( \rho_1 > \rho_2 \)

NOTE: Spheres 7 thru 12 do not move

Figure 18. Fluidization process idealized.

SERIES IV: SKIN FRICTION

Purpose

The purpose of this investigation was to determine the rate of penetration \((v_p)\) during pile driving. The primary aim was to study skin friction at frequencies from 10 to 40 cps.

Experimental set-up (Fig. 19a, b)

A model pile (steel tube - diam, 2 in.; length, 60 in.) was subjected to vertical sinusoidal force vectors acting upon the top of the pile. The tube, not supported at its lower end, could be driven through the soil under various pressures acting upon the soil.

A circular pressure tank (ID, 19 in.; height, 36 in.) was filled with dry Ottawa sand (20-30). A 17 in. diam cylindrical rubber sleeve separated the soil from the tank. Air pressure between tank wall and sleeve transmitted up to 30 lb in. \(^{-2}\) to the soil.

The pressure tank was tested hydraulically up to 120 lb in. \(^{-2}\) and showed no leakage. During preliminary experiments the attached strain gages indicated that the stresses in the wall and cover plates were below the allowable limits.

The same mechanical oscillator, including control unit, as used in Series II and III, generated the dynamic force vector. The weight of the oscillator was balanced with counterweights (Fig. 19a) via a pulley arrangement. Static surcharges could be varied by reducing these counterweights to cause a positive bias, and by adding weight to produce a negative bias. A series of springs acting as vibration isolators prevented the weights from participating in the vibratory motion.

At equal positive and negative biases the downward velocity of the tube was lower than the upward velocity. During the downward motion the soil tended to squeeze into the lower seal, thus increasing the frictional resistance. During the upward motion gravity prevented a similar effect at the upper seal. For the computation of skin friction the average values of up and down velocities were used.

At a very small positive bias and low soil pressure in the tank an increased resistance of the lower seal effectively blocked any downward motion. The dynamic force vector \((F_{dyi})\) in its upward position predominated, causing a slow
Figure 19a. Diagram of experimental set-up.

Figure 19b. Photograph of experimental set-up.
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upward motion of the tube. A similar type of upward motion will be discussed in Series VI.

Accelerations, forces, displacements, and penetration velocities were recorded at different exciter frequencies and soil pressures.

Experimental results

The eight experimentally obtained parameters are: \( f(g), v_p, \sigma, n, U, F_{dyn}, K, SC \). In the following four graphs three of these parameters are plotted as variables and one is kept constant.

<table>
<thead>
<tr>
<th>Variables</th>
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<tbody>
<tr>
<td>Fig</td>
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<tr>
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\( n \) versus \( K \) (Fig. 20). The purpose of this graph is to show that a critical frequency \( (n_{cr}) \) of the vibrating system exists. The compliance \( (K) \) reaches a maximum for surcharges \( (SC) \) of 50 lb and 100 lb at about the frequency \( n \) of \( \sim 13 \) and 14 cps, indicating that \( n_{cr} \) represents a resonance range.

The wide spread of experimental values in the critical frequency range is characteristic for friction damping (DenHartog, 1940). A bending of the response curves toward lower frequencies results in discontinuous jumps in amplitude causing unstable conditions near resonance.

Similar non-stability effects were encountered for OS samples of greater height as mentioned in Series III.

\( \sigma \) versus \( v_p \) (Fig. 21). With higher soil pressures \( (\sigma) \) the penetration velocity \( (v_p) \) becomes smaller due to increase in skin friction. For the same reason the effect of magnitude of the force vector \( (F_{dyn}) \) on \( v_p \) becomes less significant.

\( f(g) \) versus \( v_p \) (Fig. 22, log scale). All three straight lines for \( \sigma_{Soil} = 0, 5 \) and 10 lb in. \( ^{2} \) follow similar patterns with the penetration velocities decreasing at higher soil pressures.

\( f(g) \) versus \( n \) (Fig. 23, log scale). The unbalance \( (U) \) of the mechanical oscillator is introduced as variable parameter. Each individual curve shows a steady increase of \( n \) with increase of \( f(g) \). However, a decrease of \( n \) for larger \( U \) is noticeable.

The continuity and sequence of all curves seems to indicate the reliability of the test procedure. The soil pressure is of paramount importance as far as penetration velocity is concerned.

Analysis

Theoretical considerations. As long as the applied frequency remains above the resonance frequency of the vibrating system \( (n_{r} \sim 14 \text{ cps, Fig. 20}) \), it can be assumed that for small damping the force vector opposes the inertia force (phase angle \( \sim 180^\circ \)).
**Figure 20.** Frequency (n) vs compliance (K).

**Figure 21.** Soil pressure (σ) versus penetration velocity (v_p).

**Figure 22.** Acceleration [f(g)] vs penetration velocity (v_p).
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Figure 23. Acceleration \([f(g)]\) vs frequency \((n)\).

Neglecting the mass of the model pile and the friction of the seals at the upper and lower end of the pressure tank, a simplified solution may be used as a first approach. An equation which yields at least the order of magnitude of the skin friction is:

\[
F_{\text{dyn}} - M \ddot{x} = R_p
\]

where \(F_{\text{dyn}}\) = max downward force vector of the oscillator (lb)

\(M = \frac{W}{g}\) = mass of vibrating parts (lb in. \(-1\) - sec\(^2\))

\(\ddot{x}\) = acceleration of \(M\) \([f(g); d]\)

\(R_p\) = total pile resistance (lb)

and \(R_p = \sigma_{\text{soil}} A \eta\)

where \(\sigma_{\text{soil}}\) = applied soil pressure (lb in. \(-2\))

\(A\) = contact area of tube with soil (in.\(^2\))

\(\eta\) = coefficient of friction (d).

Combining eq IV-1 and -2 yields:

\[
F_{\text{dyn}} - W'f(g) = \sigma_{\text{soil}} A \eta
\]

\[
\eta = \frac{F_{\text{dyn}} - W'f(g)}{\sigma_{\text{soil}} A}
\]

*Measured parameters.*
Numerical example. For $\sigma_{\text{soil}} = 5\text{ lb in.}^{-2}$, $v_p = 4\text{ in. - sec}^{-1}$, $f(g) = 8$, $U = 0.648\text{ in. - lb}$, $n = 25\text{ cps}$, and $F_{\text{dy}n} = 900\text{ lb}$, and with vibrating $W' = 100\text{ lb}$; $A = 226\text{ in.}^2$:

$$\eta = \frac{900 - 100 \times 8}{5 \times 226} = 0.09.$$ 

This value for $\eta$ approaches the friction between lubricated smooth surfaces, 0.05, and is much lower than the static friction of earth on earth (dry sand) ranging from 0.38 to 0.75. It indicates that the vibrating particles surrounding the pile act as a quasi-lubricant and reduce the skin friction considerably, as was expected.

**SERIES V: FREQUENCY RESPONSE**

**Purpose**

The purpose of this study was the determination of the frequency response during vibro-pile-driving. The primary aim was to investigate response to critical frequencies in the range from 20 to 6000 cps.

**Experimental set-up (Fig. 24a, b, c)**

A model pile (brass tube - diam, 0.75 in.; length, 36 in.) with flat and inter-changeable conical tips (apex: 30°) was subjected to vertical sinusoidal force vectors acting upon the top of the pile and driving the tube, in contrast to Series IV, into a non-pressurized soil.

The experiments were restricted to dry Ottawa sand (20-30) and "dry" Princeton clay.

The dimensions of the cylindrical soil container were: diam, 18 in.; height, 24 in. Pile penetration never exceeded 12 in.

The electrodynamic force generator (MB-Electronics) had a frequency range from 5 to 10000 cps, a maximum DA of 0.5 in. up to 30 cps and was balanced by counterweights via a pulley arrangement.

The total cathode current from the output stage of two power amplifiers (in series) was limited to 1 amp and was kept constant at this level. The power input ($P$) had to be computed from the measured acceleration, the dynamic force vector and the exciter frequency. All calculations were by computer.

A surcharge or bias was varied from 0 to 15 lb, representing the static load acting upon the pile without participating in the vibratory motion.

The instrumentation was similar to that used in Series IV.

Vibration characteristics and resultant effects on the model pile were recorded in time-penetration, time-force, and time-acceleration diagrams.

**Experimental results**

**Static loads.** In Figure 25 the correlation between static load ($F_{\text{stat}}$ - nonvibrating weights) and depth of penetration ($p_{\text{stat}}$) is plotted. The results are used later in connection with the effect of dynamic loads ($F_{\text{dyn}}$).

$F_{\text{stat}} < 2\text{ lb}$: The steep increase of $p_{\text{stat}}$ is primarily due to the weight of the model pile.
Figure 24a. Diagram of experimental set-up.

Figure 24b. Photograph of experimental set-up.
Figure 24c. Model pile.

Figure 25. Static load ($F_{stat}$) vs penetration ($p_{stat}$), cone tip.
FLUIDIZATION PHENOMENA IN SOILS

**F_{stat} < 13 lb:** The cohesiveness of PRC reduces $p_{stat}$ when compared with the non-cohesive OS.

**F_{stat} > 13 lb:** $p_{stat}$ becomes larger for PRC than for OS. The higher density of OS overrides the cohesive effect of PRC.

**Dynamic loads**

Critical frequencies. The resonance frequencies, nodes and wave forms for the free-free brass tube under steady state conditions are shown in Figure 26a.

Computed and experimentally obtained wavelengths for the brass model pile, as compared in Table II, deviate not more than 4%. The four ratios: $\lambda/L = 3$, $= 1.5$, $= 1$, and $= 0.75$ are of particular significance.

Nodes and phase angles (multiples of 90°) correspond to the resonance frequencies.

Figure 26b shows the excited force vector versus frequency while the lower end of the brass tube with a conical tip was buried 10 in. in dry Ottawa sand. The vibrating system (exciter and pile) remained in this position. No significant changes at higher critical frequencies from those obtained with the free-free conditions (Fig. 26a) could be detected.

In Table III the approximate values of the computed logarithmic damping decrements ($\delta$) are compiled (Bernhard, 1941). They show a steady growth with decrease in critical frequencies ($n_{cr}$). This growth may be due mainly to increased participation in the vibratory motion of the adjacent soil particles when approaching lower frequency ranges. Average values of $\delta \sim 0.6$ have been reported for soils and $\delta \sim 0.06$ for ferrous material (Bernhard, 1965).

One typical penetration versus time record for Ottawa sand at the critical frequency of 4150 cps is reproduced in Figure 27. The vibrating system, including its surcharge of 4 lb, moved rapidly downwards during the first 2 sec, reaching a constant equilibrium position at a penetration depth of 5.7 in.

### Table II. Wave length at resonance frequencies.

<table>
<thead>
<tr>
<th>No. of nodes</th>
<th>L (ft)</th>
<th>$v_{brass}$ (ft/sec)</th>
<th>$n_{c}$ (cps)</th>
<th>$\lambda = v/n_{c}$</th>
<th>Theor. (ft)</th>
<th>Exp. (ft)</th>
<th>Deviation (%)</th>
<th>$\lambda/L$ (mode) approximately</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>11,500</td>
<td>1250</td>
<td>9.2</td>
<td>9.3</td>
<td>~1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>11,500</td>
<td>2650</td>
<td>4.35</td>
<td>4.5</td>
<td>~3</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>11,500</td>
<td>4150</td>
<td>2.78</td>
<td>2.9</td>
<td>~4</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>11,500</td>
<td>5250</td>
<td>2.19</td>
<td>2.18</td>
<td>~0.5</td>
<td>0.75</td>
<td></td>
</tr>
</tbody>
</table>
Table III. Logarithmic decrement ($\delta$) - OS - cone tip.

<table>
<thead>
<tr>
<th>$n_{cr}$ (cps)</th>
<th>$\delta$ (dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.6</td>
</tr>
<tr>
<td>1250</td>
<td>0.12</td>
</tr>
<tr>
<td>2650</td>
<td>0.09</td>
</tr>
<tr>
<td>4150</td>
<td>0.07</td>
</tr>
<tr>
<td>5250</td>
<td>0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$n_{res}$ (cps)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 26a. Wave form of model pile.

Figure 26b. Exciter frequency ($n_e$) vs force vector ($F_{dyn}$), cone tip.
FLUIDIZATION PHENOMENA IN SOILS

FREQUENCY: 4150 cps
SURCHARGE: 4 lb
AVERAGE PENETRATION VELOCITY: 4.3 in/sec

Figure 27. Penetration and transmitted force vector vs time, Ottawa sand, cone tip.

Efficiency factor. For further discussion a non-dimensional parameter \( \pi_1 \) is introduced which combines all pertinent experimentally obtained values, that is \( \pi_1 = f(\bar{x}, v_p, F_{\text{stat}}, F_{\text{dyn}}, n_{\text{cr}}, P, p, L) \).

Let \( \pi_1 = \frac{v_p F_{\text{stat}}}{P} \frac{p}{L} \) \hspace{1cm} (V-1)

where \( P = \frac{1}{2} \bar{x} \frac{F_{\text{dyn}}}{\omega e} = \frac{1}{2} f(g) g \frac{F_{\text{dyn}}}{2 \pi n_{\text{cr}}} \) \hspace{1cm} (V-2)

since at resonance \( \phi = 90^\circ \) or \( \sin \phi = 1 \).

Or \( \pi_1 = \frac{4 \pi n_{\text{cr}}}{f(g) g} \frac{v_p F_{\text{stat}}}{F_{\text{dyn}}} \frac{p}{L} \) \hspace{1cm} (V-3)

From an economical point of view the most effective pile driving operation should combine the maximum total penetration \( (p) \) with a maximum penetration velocity \( (v_p) \) at minimum power input \( (P) \).

Equations V-1 and V-3 indicated that \( \pi_1_{\text{max}} \) is obtained when the parameters in the numerator are maximized and those in the denominator minimized. Thus \( \pi_1_{\text{max}} \) represents the highest efficiency factor.

Experiments were made in Ottawa sand and Princeton clay, with cone- and flat-tipped piles and surcharges of 4 and 6 lb. The double displacement amplitudes \( (DA - \text{Fig. 28}) \), the dynamic force vector \( (F_{\text{dyn}} - \text{Fig. 29}) \), the acceleration \( [f(g) - \text{Fig. 30}] \), the power input \( (P - \text{Fig. 31}) \) and the efficiency factor \( (\pi_1 - \text{Fig. 32}) \) are plotted against the five critical frequencies \( (n_{\text{cr}}) \) determined above \( (\text{Figs. 26 a, b}) \).

The similarity of the shapes of all graphs, independent of the type of soil, the pile tip and the bias is an indication of the reliability of the test procedure. The DA for each \( n_{\text{cr}} \) was at almost the same level \( (\text{Fig. 28}) \) so that results for the different soils, tips, and surcharges could be compared.
Figure 28. Critical frequencies ($n_{cr}$) vs double displacement amplitudes (DA).

Figure 29. Critical frequencies ($n_{cr}$) vs dynamic force vector ($F_{dyn}$). See Figure 28 for legend.

Figure 30. Critical frequencies ($n_{cr}$) vs accelerations [$f(g)$]. See Figure 28 for legend.

Figure 31. Critical frequencies ($n_{cr}$) vs power input ($P$). See Figure 28 for legend.
FLUIDIZATION PHENOMENA IN SOILS

Figure 32. Critical frequencies \( n_{cr} \) vs efficiency factor \( \pi_1 \). See Figure 28 for legend.

In Table IV some of the results of these five graphs are summarized for the frequency of 2650 cps which is of special significance.

For OS the cone tip shows higher values than the flat tip at this particular frequency, which was not always the case for PRC, indicating that the shape of the tip is of minor importance for this type of soil.

The first minima for DA, \( F_{dyn} \), \( f(g) \) and \( P \) and the corresponding maximum efficiency factor \( \pi_1 \) appear at the same \( n_{cr} \) of 2650 cps. Such a result seems reasonable. This frequency occurs at the ratio \( \lambda/L = 1.5 \) (Table IId and h). The \( \lambda/L \) ratio, with one node about 3 in. above the center of the pile due to additional mass on top of the pile (accelerometer and moving parts of exciter)(c., Fig. 26a), produces a maximum force at the top as well as the maximum acceleration at the tip of the pile.

Analysis

Comparison between low and high frequency excitation. At low frequencies the pile vibrates longitudinally without any nodes (a., Fig. 26a) and almost equal displacement amplitudes along its length. At high frequencies the pile is excited to longitudinal vibrations with one or more nodes (b. - e., Fig. 26a).

In the low frequency range the critical frequency responds essentially to resonance of the combined "adjacent soil + pile mass system," while in the high frequency ranges the critical frequencies are governed mainly by the natural frequency of the pile alone.

At low as well as high critical frequencies the skeleton structure of the soil surrounding the pile collapses and acquires the physical properties of a viscoelastic fluid, provided enough power input is transmitted. The primary result is a considerable decrease in shear modulus (see Series I and III) combined with a substantial reduction in skin friction (see Series IV).
Table IV. Results from Figures 28-32 at \( n = 2650 \) cps, cone and flat tip.

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Ordinates</th>
<th>1st minimum</th>
<th>Pile tip</th>
<th>((V = \text{cone}; U = \text{flat}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>( n_{cr} )</td>
<td>DA</td>
<td>( V \approx U )</td>
<td>PRC</td>
</tr>
<tr>
<td>29</td>
<td>( n_{cr} )</td>
<td>( F_{dyn} )</td>
<td>( V &gt; U )</td>
<td>( V &gt; U )</td>
</tr>
<tr>
<td>30</td>
<td>( n_{cr} )</td>
<td>( f(g) )</td>
<td>( V &gt; U )</td>
<td>( V \approx U )</td>
</tr>
<tr>
<td>31</td>
<td>( n_{cr} )</td>
<td>( P )</td>
<td>( V &gt; U )</td>
<td>( V \approx U )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1st maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC: 4 lb</td>
</tr>
<tr>
<td>SC: 6 lb</td>
</tr>
<tr>
<td>SC: 6 lb - 4 lb</td>
</tr>
<tr>
<td>( V &gt; U )</td>
</tr>
</tbody>
</table>

As a secondary effect, flexural vibrations of a pile might be considered. If at high frequencies and large power input the resultant horizontal accelerations are of sufficient magnitude, some adjacent soil particles will be pushed away. They cannot recover their original position due to the fast rate of bending, causing an additional decrease in skin friction.

During some model pile experiments, whenever the static and dynamic loads were not perfectly centered, horizontal accelerations up to 30 g could be measured. For \( n_{F} = 2650 \) cps this corresponds to a DA of \( 80 \times 10^{-6} \) in. or \( 2 \times 10^{-3} \) mm. The smallest grain size of PRC (silt particles) is \( \sim 2 \times 10^{-3} \) mm (Fig. 1a) which indicates that the bending effect can be at best only of secondary importance.

A third effect, the lateral expansion and contraction of a pile, has been mentioned (Esson, 1965). The magnitudes of the transverse displacement amplitudes, however, are so small that they seem unlikely to be of any significance.

Comparison with field performances. It must be kept in mind that this discussion is restricted to results from the previous vibrator model pile soil combination with dry Ottawa sand and dry Princeton red clay. All higher frequencies, \( n_{cr} \), (starting with 1250 cps) refer only to longitudinal vibrations of the pile (standing wave patterns).

The simple nomograph in Figure 33 might serve as a first approach to establish a comparison between the model experiments and the behavior of piles as used on actual construction jobs (the mass of the exciter is not included).

From the known parameters such as pile length (\( L \)) and material (steel, concrete, wood), the exciter frequency (\( n_{e} \)) as the unknown variable can be obtained. For example, a steel pile of 100-ft length and the ratio: \( \lambda / L = 1.5 \) (e, Fig. 26a and Table II) would respond to a critical frequency (\( n_{cr} \)) of approximately 110 cps. This frequency can be achieved with high-power oscillators without excessive structural difficulties, (i.e., oversized bearings, housing, etc.).

The operation of high-frequency vibro-pile-drivers reduces the possibilities of detrimental effects to nearby buildings or of causing inconvenience to persons in the neighborhood for the following reasons:

1. Frequencies over 100 cps are much higher than the natural frequencies of buildings vibrating as a unit or of any individual parts such as walls, windows, pictures, etc.
Figure 33. Nomograph for determining critical frequency for longitudinal vibrations.

2. Propagated waves of higher frequencies decay at shorter distances than those of lower frequencies.

3. The required displacement amplitudes to generate the same power are substantially smaller at high than at low frequencies.

Determination of bearing capacity. The following simple approach to determine the bearing capacity of a vibro-driven pile in the field is suggested:

It is assumed that the bearing capacity depends upon the maximum efficiency factor ($\pi_1^{\max}$), that is, the power input ($P$), the penetration velocity ($v_{pav}$), the total penetration ($p$) and the length of the pile ($L$).

From eq V-1:

$$F_{\text{stat}} = \frac{\pi_1^{\max} P L}{v_{pav} p}.$$  \hspace{1cm} (V-4)
Numerical example:

Let \( \pi_{\text{max}} = 0.1 \) (average value from Fig. 32)

\[
P = 500 \text{ HP}^* = 27.5 \times 10^4 \text{ ft-lb sec}^{-1} \text{ measured with wattmeter, or}
\]

\[
\text{torque} = 400 \text{ in.-lb at 110 cps (Fig. 33) measured with torque meter.}
\]

\( L = 100 \text{ ft.} \)

\[
\nu_{\text{av}} = \frac{5}{12} \text{ ft sec}^{-1} \text{ (measured with stop watch)}
\]

\( p = 50 \text{ ft (scaled).} \)

Then

\[
F_{\text{stat}} = \frac{0.1 \times 27.5 \times 10^4 \times 100}{\frac{5}{12} \times 50} = 132 \times 10^3 \text{ lb} = 66 \text{ tons.}
\]

With a safety factor of 2 the allowable static design load is 33 tons.

It is of interest to compare eq V-4 with the simple formula for hammers:

\[
F_{\text{stat}} = R = \frac{k \text{Wh}}{S + \frac{c}{2}}
\]

where

- \( k = \) efficiency coefficient
- \( W = \) weight of hammer
- \( h = \) drop height of hammer
- \( S = \) penetration per blow
- \( c = \) energy losses

or

\[
F_{\text{stat}} = k \frac{W h}{(S + \frac{c}{2})/t} = k \frac{P}{\nu_{\text{av}}}
\]

For \( L = p \) this equation becomes identical with eq V-4, where the coefficient \( k \) for heavy pile, light hammer and fresh wooden cushion is 0.15 as compared with \( \pi_1 = 0.1 \) substituted in eq V-4.

SERIES VI: FLOATING DEPTH

Purpose

Special studies were devoted to those fluidization effects which are connected with the appearance of a "floating depth."

Floating depth will be understood to mean that depth of penetration where the pile reaches an equilibrium position, which is not necessarily the maximum depth obtained during the vibration period.

\*P represents the power input minus the losses due to the driving mechanism.

These losses must be predetermined by operating the force generator at the pile driving frequency on a very rigid or very soft support having a natural frequency well above or below this frequency (Bernhard, 1965).
Experimental set up

The experimental set-up was the same as in Series V (Fig. 24a, b, c), but was limited to low frequencies (30 cps) and high power input (~2000 in. -lb sec⁻¹ per 1 in.² of model pile) in a confined space. No floating depth could be observed at higher frequencies and lower power input.

Again dry Ottawa sand (vibro-precompacted to ~119 lb ft⁻³) and dry Princeton clay (vibro-precompacted to ~100 lb ft⁻³) were used. The same tube (diam, 0.75 in.; length, 36 in.) with conical or flat tip was pushed initially to various depths below the soil surface.

Only a limited number of experiments could be made, mainly for the following reasons:

1. The maximum power input driving the electromagnetic force generator yields a double displacement amplitude (DA) of 0.5 in. at a frequency of 30 cps. At higher frequencies the DA decreases substantially and does not lend itself easily to the study of floating depth.

2. The maximum permissible surcharge was 15 lb. Experiments with larger containers are necessary to eliminate any possible effects from wall and bottom.

Experimental results

A few characteristic records of each series are reproduced.

Ottawa sand. The model pile with flat tip was buried initially 1.5 in. in dry Ottawa sand. A surcharge of 4 lb was added. One of the penetration-time records is shown in Figure 34. The two lower traces indicate the magnitude of the applied force vector and acceleration on top of the tube.

Three stages of pile reaction are noticeable during the vibration period.

Stage 1, Penetration period: A rather rapid downward motion of almost constant penetration velocity occurs.

Stage 2, Recovery period: A much slower upward motion. The recovery velocity follows an almost parabolic pattern with time.

Stage 3, Equilibrium position: The pile remains at the same level (3.2 in. floating depth) for an indefinite time.

The pile was then buried initially 6.7 in. into dry Ottawa sand. All other conditions remained the same as before. One of the penetration-time records is shown in Fig. 35. Only stages 2 and 3 are noticeable, indicating that 6.7 in. is below the floating depth of 3.2 in.

Princeton red clay. The tube was buried initially 1.5 in. into dry Princeton clay. Again an exciter frequency of 30 cps and a bias of 4 lb was applied. However, because of the different characteristics of Princeton clay, particularly grain size distribution and cohesiveness, a floating equilibrium similar in magnitude to that obtained with Ottawa sand could be obtained in Princeton clay only by replacing the flat tip of the rod with a cone tip.

A penetration-time record is reproduced in Figure 36. The two lower traces indicate the magnitude of the corresponding accelerations on top and bottom of the tube. The first 25-sec period and the final 64 to 86 sec are reproduced.

All three stages are noticeable, indicating that the maximum penetration of 7.5 in. was below the floating depth of 3.5 in. The long recovery time of 83 sec may be due primarily to the greater cohesiveness of Princeton clay when compared with Ottawa sand.
Figure 34. Penetration-time record, Ottawa sand.

Figure 35. Penetration-time record, Ottawa sand.
Finally the pile was buried initially 2.6 in. in "dry" Princeton clay and the acceleration increased to ± 23 g. All three stages reappear (floating depth of 2.9 in.) (Fig. 37).

Effect of bias. An experiment with dry Ottawa sand was made to determine that surcharge (limiting bias - LB) which is just large enough to prevent any recovery (stage 2).

In Figure 38 the floating depths with a flat- and cone-tipped pile are plotted. The surcharge was varied from 0 to 8 lb, while an acceleration of ± 14 g and of ± 18 g was maintained on top of the tube. Only the 30 cps frequency was kept constant.

The area to the right of the three curves represents the regions where any significant recovery is suppressed, i.e., either the surcharge is too large or the accelerations and displacement amplitudes are too small.

The lines in Figure 38 represent computed values with the limiting bias (LB) determined as a function of ten different parameters:

\[ LB = f(\omega_e, t_c, v_p, a, DA, F_{stat}, \Delta, c_d, \rho, A) \]  

where  
- \( \omega_e \) = exciter frequency (rad sec\(^{-1}\)),  
- \( t_c \) = time of contact between pile tip and soil during each cycle (sec)  
- \( v_p \) = pile penetration velocity (in. sec\(^{-1}\))  
- \( \rho \) = density of soil under pile (in. sec\(^{-1}\))  
- \( DA \) = double displacement amplitude (in.)
Figure 37. Penetration-time record, Princeton red clay.

Figure 38. Floating depth vs surcharge.
FLUIDIZATION PHENOMENA IN SOILS

\[ F_{\text{stat}} = \text{static load-yield resistance (lb)} \]
\[ \Delta = \text{ground quake - elastic soil deformation at } F_{\text{stat}} \text{ (in.)} \text{ (Smith, 1965)} \]
\[ c_d = \text{drag coefficient (d)} \]
\[ \rho = \text{soil-mass density (lb in.}^{-4} \text{- sec}^2 \text{)} \]
\[ A = \text{cross sectional area of pile (in.}^2 \text{).} \]

The theory is based on an approach similar to that used by Bathelt, (1956). A detailed description of the form taken by these parameters in eq IV-1 is beyond the scope of this report. It will suffice here to state that the deviation between the experimental and theoretical values does not exceed ±6% and that for a given DA and \( \omega_e \) the ratios: SC/\( F_{\text{stat}} \) remain constant.

Equal floating depths occur at the following limiting bias ratios \( \frac{L_{B_1}}{L_{B_2}} \):

For cone tipped pile (apex \( \epsilon = 30^\circ \)) and different DA:

Theoretical: \[ \frac{L_{B_1}}{L_{B_2}} = \left( \frac{DA_1}{DA_2} \right)^{3/2} \]

Let \( DA_1 = 0.4 \text{ in. and } DA_2 = 0.3 \text{ in.} \)

Then \[ \frac{L_{B_1}}{L_{B_2}} = \left( \frac{0.4}{0.3} \right)^{3/2} = 1.54. \]

Experimental: at a floating depth of 4 in. (Fig. 38)

\( \frac{L_{B_1}}{L_{B_2}} \sim 1.44. \)

For flat tipped pile (apex \( \epsilon_1 = 180^\circ \)) and cone tipped pile (\( \epsilon_2 = 30^\circ \)) and equal DA:

Theoretical: \[ \frac{L_{B_1}}{L_{B_2}} = \left( \frac{\sin \epsilon_1 / 4}{\sin \epsilon_2 / 4} \right)^{1/2} \]

Let \( \epsilon_1 = 180^\circ \text{ and } \epsilon_2 = 30^\circ \).

Then \[ \frac{L_{B_1}}{L_{B_2}} = \left( \frac{\sin 45^\circ}{\sin 7.5^\circ} \right)^{1/2} = 2.3. \]

Experimental: at a floating depth of 4 in. and DA = 0.4 in. (Fig. 38)

\( \frac{L_{B_1}}{L_{B_2}} \sim 2.4. \)

The plot indicates a lesser floating depth for the flat tipped tube than for the cone tipped pile. This may be due to a stronger turbidity action (dilatancy) under the flat tip.
The curve for the cone tip shows a smaller floating depth when excited with ±18 g than with ±14 g. Thus a smaller dilatancy effect due to reduced acceleration becomes obvious.

The same floating depth occurs if the ratio of

\[
\frac{\text{Bias for 0.4 in. DA}}{\text{Bias for 0.3 in. DA}} = \frac{0.4}{0.3} = 1.54
\]

or if the ratio for the same DA is:

\[
\frac{\text{Bias for flat tip}}{\text{Bias for cone tip}} \sim 2 \text{ to } 3.
\]

In Table V the results of some of the penetration-time records are summarized. Of special interest is the average floating depth of 3.2 in. (maximum deviation ±10%), keeping in mind that the Princeton clay required a cone tipped model pile instead of the flat tip used for Ottawa sand.

This floating depth seems to be almost independent of the starting depth regardless of whether the tube was vibrated initially to the maximum penetration and recovered subsequently or whether the pile was pushed first below or above the final equilibrium position.

**Analysis**

An attempt will be made to explain some of the governing phenomena in a confined space.

**Stage 1. Penetration period (Fig. 39a).** The almost elliptic path of the quasi-liquefied grains is downward along the skin of the pile and upward away from it. This was clearly visible while taking the photographs (Fig 2a, b).

The downward travel of the particles moving along the pile may facilitate somewhat the initially rapid penetration.

**Stage 2. Recovery period (Fig. 39b).** The naturally higher density of the soil with greater depth, if sufficiently increased by vibro-compaction (Fig. 39b) near the tip of the pile, has reduced the fluidized zone to a narrow funnel. The particles move downwards similar to stage 1, but little or no space is left to move upward.

A cavity of strong turbidity is created beneath the pile tip. Most of the grains press upward against the lower end of the tube, which acts like a sand pump, and overrides the effect, if any, of the downward moving particles.

The weight of the soil displaced by the pile (approx. 1 lb) is only about 25% of the surcharge. Hence the buoyancy forces in the fluidized zone are not a predominant factor contributing to the upward motion.

Only a combination of dilatancy and quasi-buoyancy can explain the recovery period.

**Stage 3. Equilibrium position (Fig. 39c).** All downward and upward forces are equal. The equilibrium position or floating depth, for the same soil characteristics, depends primarily upon the power input.

This depth remains constant, regardless of the length of time the vibrations are sustained and also after any excitation has been stopped.

**Fundamental characteristics of the three stages**

A few typical penetration-time curves, simplified in the form of straight lines, are sketched in Table VI.
Table V. Results from Figures 34-37.

<table>
<thead>
<tr>
<th>Series</th>
<th>Fig.</th>
<th>Type of soil (dry)</th>
<th>Tip of pile</th>
<th>Acceleration ( f(g) ) top of pile</th>
<th>Depth pushed into soil</th>
<th>Below surface (in.)</th>
<th>Start of vibrations</th>
<th>Max. penetration</th>
<th>Floating equilibrium</th>
<th>Penetration period</th>
<th>Recovery period</th>
<th>Penetration (average)</th>
<th>Recovery (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VI</td>
<td>34</td>
<td>Ottawa sand</td>
<td>flat</td>
<td>( \pm 18g )</td>
<td>1.5</td>
<td>1.5 5</td>
<td>3.2</td>
<td>1 18.5</td>
<td></td>
<td></td>
<td>3.5 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>35</td>
<td>Ottawa sand</td>
<td>flat</td>
<td>( \pm 18g )</td>
<td>6.6</td>
<td>6.6 6.6</td>
<td>3.2</td>
<td>- 19</td>
<td></td>
<td></td>
<td>- 0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>36</td>
<td>Princeton cone clay</td>
<td>( \pm 18g )</td>
<td>1.5</td>
<td></td>
<td>1.5 7.5</td>
<td>3.5</td>
<td>1.5 83</td>
<td></td>
<td></td>
<td>4 0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>37</td>
<td>Princeton cone clay</td>
<td>( \pm 23g )</td>
<td>2.6</td>
<td></td>
<td>2.6 3.8</td>
<td>2.9</td>
<td>3 8</td>
<td></td>
<td></td>
<td>0.4 0.11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*For the average recovery velocity a straight line from the maximum penetration to the beginning of the floating depth is assumed.*
Figure 39. Pile motions at large power input.

Table VI. Typical time-penetration diagrams.
FLUIDIZATION PHENOMENA IN SOILS

Case B seems to govern most practical field applications occurring in an unconfined space, since no reports on recovery periods have come to the attention of the writer. The large surcharge, comprising the heavy weight of force generators and their driving mechanisms, suppresses any recovery effects.

However, a confined space can evolve whenever a cluster of closely spaced piles is required. If the outer piles are driven first they may act as a confining barrier and produce a densified soil volume inside this group. (Incidentally, this gives a reason why such a cluster of piles sometimes has a higher combined bearing capacity than the same number of piles further apart (Kezdi, 1957). For any piles driven later within this densified soil volume, a floating depth might develop which will be less than the maximum penetration of the outer piles.

ZONE DISTRIBUTION DURING VIBRO-COMPACTION AND VIBRO PILE DRIVING AND PULLING

The region affected by vibratory forces acting upon a soil, considered as a homogeneous, isotropic, viscoelastic medium in a half-space continuum, may be subdivided into three zones (Fig. 40 and Table VII).

The borderlines of these zones depend primarily upon the energy input generated by the exciter.

Zone I is a rather thin discrete region adjacent to the disturbing source, which receives a high energy input causing an unstable state of equilibrium. This zone can be considered as an impedance-matching device.

Zone II, surrounding Zone I, receives a medium energy input reduced by attenuation. An irreversible state prevails.

Zone III is subjected to a further decreased energy input and reaches over parts or most of the remainder of the half space. The phenomena in Zone III are cyclic.

Zone I

In Zone I, the high energy input forces the individual soil grains to vibrate with rather large displacement amplitudes, large enough to break most of the existing interparticle bonds. The shear modulus is reduced to very small values thus producing a quasi-fluid state. Volumetric expansion results in an apparent Poisson's ratio $\mu$ larger than 0.5 (whenever $E>3G$) and $\mu$ loses its physical significance, at least as far as the customary interpretation is concerned.

The soil particles attain their maximum displacement amplitudes at frequencies ranging from ~20 to 40 cps regardless of whether the exciter is coupled to the medium via a contact plate or via a pile. A maximum reduction of the shear modulus to 30 lb in. $^{-2}$ can be obtained.

In the field vibro-compaction of soil has produced a loosening effect of the uppermost strata to about 3 in. Reports on vibro-pile driving in Russia (Gumenski and Komanov, 1961) and Germany (Kohler and Ramspeck, 1933) indicate that within the abovementioned frequency range fluidization could be created.

Substantially higher frequency ranges also generate fluidization effects, which are induced by vibrating a pile at its natural longitudinal frequency, including harmonics (1st, 2nd, 3rd...mode). In this case the skin of the pile and in particular the lower end of the pile is vibrating, thus improving the wedge action and hence the penetration. Excitation of a pile in such a mode that its lower end vibrates with maximum displacement amplitudes might result in partial fluidization also around the lower end of the pile.

$^*The lower as well as the high frequency ranges are both in the "sonic" spectrum.
Table VII. Effect of zone distribution.

<table>
<thead>
<tr>
<th>Zones</th>
<th>I High</th>
<th>II Intermediate</th>
<th>III Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy input</td>
<td>Quasi fluid-Liquefaction-Unstable-Impedance matching</td>
<td>Irreversible</td>
<td>Elastic-Cyclic-Reversible</td>
</tr>
<tr>
<td>State of medium</td>
<td>Loosening-Reduction of inter-particle friction</td>
<td>Compaction-Partial breaking of particle bonds</td>
<td>Propagation of waves</td>
</tr>
<tr>
<td>Effect upon medium</td>
<td>Close to disturbance-Max. several inches</td>
<td>Between Zone I and II-Max. several feet</td>
<td>Surrounding Zone II-Thousands of miles</td>
</tr>
<tr>
<td>Extension of zones</td>
<td>Advantageous-Advantageous-Requires static rollers for surface layer,</td>
<td>Low: ~ ±25 cps Exciter + soil</td>
<td>Possible effects on comfort of inhabitants and in some cases safety of structures</td>
</tr>
<tr>
<td>Vibro compaction</td>
<td>Dedensification</td>
<td>Disadvantageous-Facilitates penetration Reduction of skin friction</td>
<td>Disadvantageous</td>
</tr>
</tbody>
</table>
At high energy input at low frequencies in a confined space a recovery period, ending with a floating depth, can occur. This floating depth, depending on the bias, is not necessarily the preceding maximum penetration of the pile.

**Zone II**

In Zone II enough energy input has been dissipated to reduce the bond between soil particles; however, only far enough to permit their relocation into a denser state, that is producing compaction.

Vibro-compaction of highway subsoils, dam layers, etc. has shown that this effect may reach a greater depth - down to several feet under favorable conditions - than compaction by static rollers.

**Zone III**

The energy input has been attenuated to such an extent that only elastic action can occur. This effect, pertaining primarily to the field of geophysics, is indicated by the transmission of seismic (propagation) waves of various characteristics over large distances. For explosive forces (nuclear blasts) and earthquakes, Zone III might extend over thousands of miles.

In summarizing it can be stated that between Zones I and II and Zones II and III some kind of energy thresholds or boundary limits exist so that no fluidization will occur in Zone II and no compaction will occur in Zone III.

**RECOMMENDATIONS**

It must be emphasized that only a limited number of experiments have been made. Substantially more studies are necessary before definite conclusions can be drawn and an adequate theory developed.

Investigations with other soil types (particularly more cohesive) should be made. Torsional vibrations in a horizontal plane ought to be considered. Series IV to VI included dry soils only and must be extended to soils with various moisture contents. Finally, larger dimensions with reference to soil samples, model piles, and in particular containers are essential.
LITERATURE CITED


(1965) Bibliography on soil dynamics, USA CRREL Special Report 89.

(1967a) Resonance curve analysis, USA CRREL Special Report 97.

(1967b) Stress and wave patterns in soils subjected to vibratory loads, USA CRREL Research Report 120.


LITERATURE CITED (Cont'd)


APPENDIX A. VIBRATORY DRIVING AND PULLING OF PILES BY MULTI-
FREQUENCY MECHANICAL OSCILLATORS

Principle of vibro-pile-driving and -pulling machines

The simplest way to generate vertical force vectors of sinusoidal characteristics is by means of rotating eccentric weights, producing centrifugal force vectors.

The equation for the centrifugal force vector (C₁ in lb) is:

\[ C₁ = Me \omega² \]

where \( M \) = rotating mass (lb sec² in.⁻¹), \( e \) = eccentricity (in.) and \( \omega \) = angular velocity (rad/sec).

\[ C = \frac{W}{g} e (2\pi n)^2 = We \frac{4\pi²}{g} n² = U \frac{4\pi²}{g} n² \]

where \( W \) = rotating weights (lb), \( n \) = rotational velocity (cps) and \( U \) = unbalance (in. -lb).

Or \[ C = U \frac{4 \times 1.4²}{386} n² \sim 0.1 U n² . \] (A1)

The correlation between \( C \), \( U \) and \( n \) is shown in Figure A1.

Wave form. The series can be written to include two modes as:

\[ X = \sum_{m=1}^{m=2} \left( \frac{3}{2} - \frac{m}{2} \right) \cos (m\omega t) (-1)^m \] (A2a)

and to include three modes as:

\[ X = \sum_{m=1}^{m=3} \left( 2 - \frac{m}{2} \right) \cos (m\omega t) (-1)^m \] (A2b)

where \( X \) = amplitude, \( m \) = number of mode and \( \omega t \) = time angle.

The equation for one force vector (\( x \) in lb) is

\[ x = C \cos (m\omega t) . \] (A3)

Ratio of unbalances. The ratio of the unbalances \( U_{2} / U_{1} \) for various modes follows for eq A1 and A3:

\[ \frac{U_{2}}{U_{1}} = \frac{C_{2}}{C_{1}} \left( \frac{m_{1}}{m_{2}} \right)^2 . \] (A4)

One-system oscillator. A "one-system" oscillator generating only the fundamental mode consists of two unbalanced masses rotating in opposite directions with the same frequency and producing a pure sinusoidal force vector. This well-known principle requires no further explanation. The disadvantage of this "one system" is that the maximum upward and downward forces are equal in magnitude. For pile-driving, only the dead weight of the machinery increases the downward force. For pile-pulling, an external pulling device must be added.
Two-system oscillator. A 'two-system' oscillator generating the first and second mode comprises two pairs of unbalanced masses, system I and system II.

The equations for the two force vectors are:

for system I: \( x_1 = -1 \cos (1\omega t) \) -- fundamental \hspace{1cm} (A3a)

for system II: \( x_2 = +0.5 \cos (2\omega t) \) -- 1st harmonic. \hspace{1cm} (A3b)

Figure A1 indicates such a set-up with the second system operating at double the frequency, but half the maximum force vector, of the first system. The ratio of maximum downward to upward force is 2:1 for driving, and 1:2 for pulling, piles.

Three-system oscillator. A 'three-system' oscillator generating the first, second and third modes and comprising three pairs of unbalanced masses is shown in Figure A2.
The equations for the three force vectors, according to eq A3, are:

for system I: \[ x_1 = -1.5 \cos(1\omega t) \] -- fundamental (A3c)
for system II: \[ x_2 = +1 \cos(2\omega t) \] -- 1st harmonic (A3d)
for system III: \[ x_3 = -0.5 \cos(3\omega t) \] -- 2nd harmonic. (A3e)

System II has \( \frac{1}{3} \) of the maximum force vector and twice the frequency of system I. System III has \( \frac{1}{3} \) of the maximum force vector and three times the frequency of system I. The ratio of maximum downward to upward force is 3:1 for driving, and 1:3 for pulling, piles, which is better than for the two-system oscillator.

The frequency ratio of 3:1 (system III to system I) has the following advantages: the higher frequency might be used to excite the natural frequency of the pile and thus reduce skin friction; the lower frequency with the large downward force amplitude will increase the penetrating power or wedge effect at the lower end of the pile.
With more than three systems, still higher ratios are obtainable; however, the mechanical setup becomes too complicated and uneconomical.

Curves for the three systems, based on the computed data of Tables AI, AII and AIII, are shown in Fig. A1 and A2.

Suggested design and two-system and three-system vibratory pile drivers

The following numerical data represent possible examples of a two- and a three-system vibratory-pile-driving machine. Both machines are based on maximum unbalanced weights of 100 lb.

Data for "two-system" pile-driver.
Rating: 3.75 tons from 6 to 60 cps
Weights: I = 4 x 100 lb; II = 2 x 100 lb
Maximum force vector:

<table>
<thead>
<tr>
<th>Driving-down</th>
<th>-Up</th>
<th>7500 lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulling-up</td>
<td>-Down</td>
<td>3750 lb</td>
</tr>
</tbody>
</table>

Ratio of unbalances from eq A4:

\[ \frac{U_2}{U_1} = \frac{1}{2} \left( \frac{1}{3} \right)^2 = \frac{1}{8}. \]

Data for "three-system" pile-driver.
Rating: 7.50 tons from 6 to 60 cps
Weights: I = 4 x 100 lb; II = 2 x 100 lb; III = 2 x 50 lb
Maximum force vector:

<table>
<thead>
<tr>
<th>Driving-down</th>
<th>-Up</th>
<th>15,000 lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulling-up</td>
<td>-Down</td>
<td>5000 lb</td>
</tr>
</tbody>
</table>

Ratio of unbalances from eq A4:

\[ \frac{U_2}{U_1} = \frac{1}{1.5} \left( \frac{1}{3} \right)^2 = \frac{1}{6} \]

\[ \frac{U_3}{U_1} = \frac{1}{2} \left( \frac{1}{4} \right)^2 = \frac{1}{27}. \]

The numerical data are evaluated in Tables AIV and AV and in Figure A3 the corresponding values for each system are plotted.

The possible arrangements of the mechanism for two- and three-system pile drivers are shown in Figures A4, A5 and A6.

Conclusions

For vibratory pile driving, the downward force vectors can be made substantially larger than the upward force vectors by superposition of various frequencies. For pile-pulling the reverse is obtainable. A "three-frequency" mechanical oscillator generating the first, second and third modes seems to present the most efficient driving mechanism. With more than three frequencies, the mechanical setup becomes complicated and uneconomical.
### Table AI. Numerical evaluation of two-system oscillator.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
<th>105°</th>
<th>120°</th>
<th>135°</th>
<th>150°</th>
<th>165°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1 ( \cos (1\theta) )</td>
<td>-1</td>
<td>-0.966</td>
<td>-0.866</td>
<td>-0.707</td>
<td>-0.5</td>
<td>-0.259</td>
<td>0</td>
<td>+0.259</td>
<td>+0.5</td>
<td>+0.707</td>
<td>+0.866</td>
<td>+0.966</td>
<td>+1</td>
</tr>
<tr>
<td>+0.5 ( \cos (2\theta) )</td>
<td>+0.5</td>
<td>+0.433</td>
<td>+0.25</td>
<td>0</td>
<td>-0.25</td>
<td>-0.433</td>
<td>-0.5</td>
<td>-0.433</td>
<td>-0.25</td>
<td>0</td>
<td>+0.25</td>
<td>+0.433</td>
<td>+0.5</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>-0.5</td>
<td>-0.533</td>
<td>-0.616</td>
<td>-0.707</td>
<td>-0.75†</td>
<td>-0.692</td>
<td>-0.5</td>
<td>-0.174</td>
<td>+0.25</td>
<td>+0.707</td>
<td>+1.116</td>
<td>+1.399</td>
<td>+1.5*</td>
</tr>
</tbody>
</table>

*1st maximum-positive
†2nd maximum-negative
**3rd maximum-negative
### APPENDIX A

#### Table AIII. Amplitudes and angles of all maxima- and zero-values for one complete cycle.

<table>
<thead>
<tr>
<th>System</th>
<th>No.</th>
<th>Angles</th>
<th>Amplitude</th>
<th>Zero angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-system oscillator</td>
<td>1st</td>
<td>180°</td>
<td>+1.5</td>
<td>111°30'</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>60°, 300°</td>
<td>-0.75</td>
<td>248°30'</td>
</tr>
<tr>
<td>Three-system oscillator</td>
<td>1st</td>
<td>180°</td>
<td>-3</td>
<td>124°29'</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>0°, 360°</td>
<td>-1</td>
<td>226°31'</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>90°, 270°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Table AIV. Numerical data for two-system pile-driver.

<table>
<thead>
<tr>
<th>Range</th>
<th>System no.</th>
<th>Frequency (cps)</th>
<th>Weights (lb)</th>
<th>Eccentricities (in.)</th>
<th>Unbalances (in.-lb)</th>
<th>Force vectors (eq A3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>I</td>
<td>6</td>
<td>4 100</td>
<td>3.5</td>
<td>1400</td>
<td>0.1 x 1400 x 6² ~5000</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>12</td>
<td>2 100</td>
<td>0.87</td>
<td>1400 / 8 =175</td>
<td>0.1 x 175 x 12² ~2500</td>
</tr>
<tr>
<td>High</td>
<td>I</td>
<td>30</td>
<td>4 100</td>
<td>0.139</td>
<td>55.5</td>
<td>0.1 x 55.5 x 30² ~5000</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>60</td>
<td>2 100</td>
<td>0.0348</td>
<td>55.5 / 8 =6.95</td>
<td>0.1 x 6.95 x 60² ~2500</td>
</tr>
</tbody>
</table>

#### Table AV. Numerical data for three-system vibro-pile-driver.

<table>
<thead>
<tr>
<th>Range</th>
<th>System no.</th>
<th>Frequency (cps)</th>
<th>Weights (lb)</th>
<th>Eccentricities (in.)</th>
<th>Unbalances (in.-lb)</th>
<th>Force vectors (eq A3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>I</td>
<td>6</td>
<td>4 100</td>
<td>5.25</td>
<td>2100</td>
<td>0.1 x 2100 x 6² ~7500</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>12</td>
<td>2 100</td>
<td>1.75</td>
<td>2100 / 6 =350</td>
<td>0.1 x 350 x 12² ~5000</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>18</td>
<td>2 50</td>
<td>0.77</td>
<td>2100 / 27 = 77</td>
<td>0.1 x 77 x 18² ~2500</td>
</tr>
<tr>
<td>High</td>
<td>I</td>
<td>20</td>
<td>4 100</td>
<td>0.47</td>
<td>189</td>
<td>0.1 x 189 x 20² ~7500</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>40</td>
<td>2 100</td>
<td>0.16</td>
<td>189 / 6 =31.5</td>
<td>0.1 x 31.5 x 40² ~5000</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>60</td>
<td>2 50</td>
<td>0.07</td>
<td>189 / 27 = 7</td>
<td>0.1 x 7 x 60² ~2500</td>
</tr>
</tbody>
</table>
Figure A3. Capacity chart of mechanical oscillators.

Figure A4. Diagram for two-system vibro-pile-driver. (Eccentrics shown for max downward force vector.)

Figure A5. Diagram for three-system vibro-pile-driver. (Eccentrics shown for max downward force vector.)
Figure A6. Exploded diagram for two-system pile driver.

1-4 Rotating eccentrics (system I)
5, 6 Shafts (system I)
7, 8 Meshing gears, 1:1 (system I)
9, 10 Rotating eccentrics (system II)
11, 12 Shafts (system II)
13-16 Meshing gears, 2:1 (system II)
17 Drive
18 Housing
19 Pile

SELECTED BIBLIOGRAPHY


FLUIDIZATION PHENOMENA IN SOILS DURING VIBRO-COMPACTION AND VIBRO-PILE-DRIVING AND-PULLING

Abstract

Investigations dealing with response of soils to sinusoidal force excitation were conducted to determine: 1) under what conditions fluidization can be obtained, particularly the correlation between the required magnitude, direction and frequency of the vibratory surface loads, and 2) the character of the liquefied soil volume with respect to dimensions and sustainability. Fluidization, representing the change of soils from a solid into a quasi-fluid state by means of vibratory forces, is discussed. The hydrodynamic state is of particular importance for vibratory compaction and vibratory-pile driving and pulling. Experiments "in situ" and in the laboratory are described. The response of soils to static and dynamic loads was analyzed to determine the prerequisites for liquefaction. Results indicate that under certain conditions a thin discrete zone in the immediate vicinity of the exciting source exists which shows characteristics similar to those of a viscoelastic fluid.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ROLE</td>
<td>WT</td>
<td>ROLE</td>
</tr>
<tr>
<td>Pile driving (Vibratory)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Pilings -- Testing equipment</td>
<td></td>
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<tr>
<td>Soils -- Compaction effects</td>
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<tr>
<td>Soils -- Elasticity</td>
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