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# HEIGHT VARIATION ALONG SEA ICE PRESSURE RIDGES AND THE PROBABILITY OF FINDING "HOLES" FOR VEHICLE CROSSINGS

W.D. Hibler III and S.F. Ackley

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PREPARED FOR  
ADVANCED RESEARCH PROJECTS AGENCY  
ARPA ORDER 1615

BY

CORPS OF ENGINEERS, U.S. ARMY  
**COLD REGIONS RESEARCH AND ENGINEERING LABORATORY**  
HANOVER, NEW HAMPSHIRE

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## PREFACE

This report was prepared by Dr. W.D. Hibler III, Research Physicist, and Mr. S.F. Ackley, Research Physicist, of the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory (USA CRREL).

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## Introduction

A significant parameter required for the design of a surface vehicle to traverse the sea ice of the Arctic Basin is an estimate of its ability to traverse the major terrain obstacles, sea ice pressure ridges. This problem is postulated: If a portion of a ridge with points or segments higher than the vehicle's traversing capability is encountered, how far along that ridge would the vehicle have to travel to encounter a hole wide enough and low enough to go through? One possible solution is to travel until the ridge dies off. Figure 1, which gives an oblique aerial view of first-year pressure ridges in a segment of the Beaufort Sea, shows that this is a practical solution only if it is possible to cross most ridges either by going through them or over them. Clearly, because of the networks formed by the ridges, it is nearly impossible to travel between two points without crossing some ridges. Given the need for the vehicle to cross ridges, an estimate of the expectation of holes of various heights and widths existing in the ridges is then required for optimum vehicle design. For this reason, the height variations along sea ice pressure ridges were studied to determine whether low spots persisted over long enough distances for a wide vehicle such as the surface effect vehicle to pass through.



Figure 1. Oblique aerial view of pressure ridges in the Beaufort Sea.

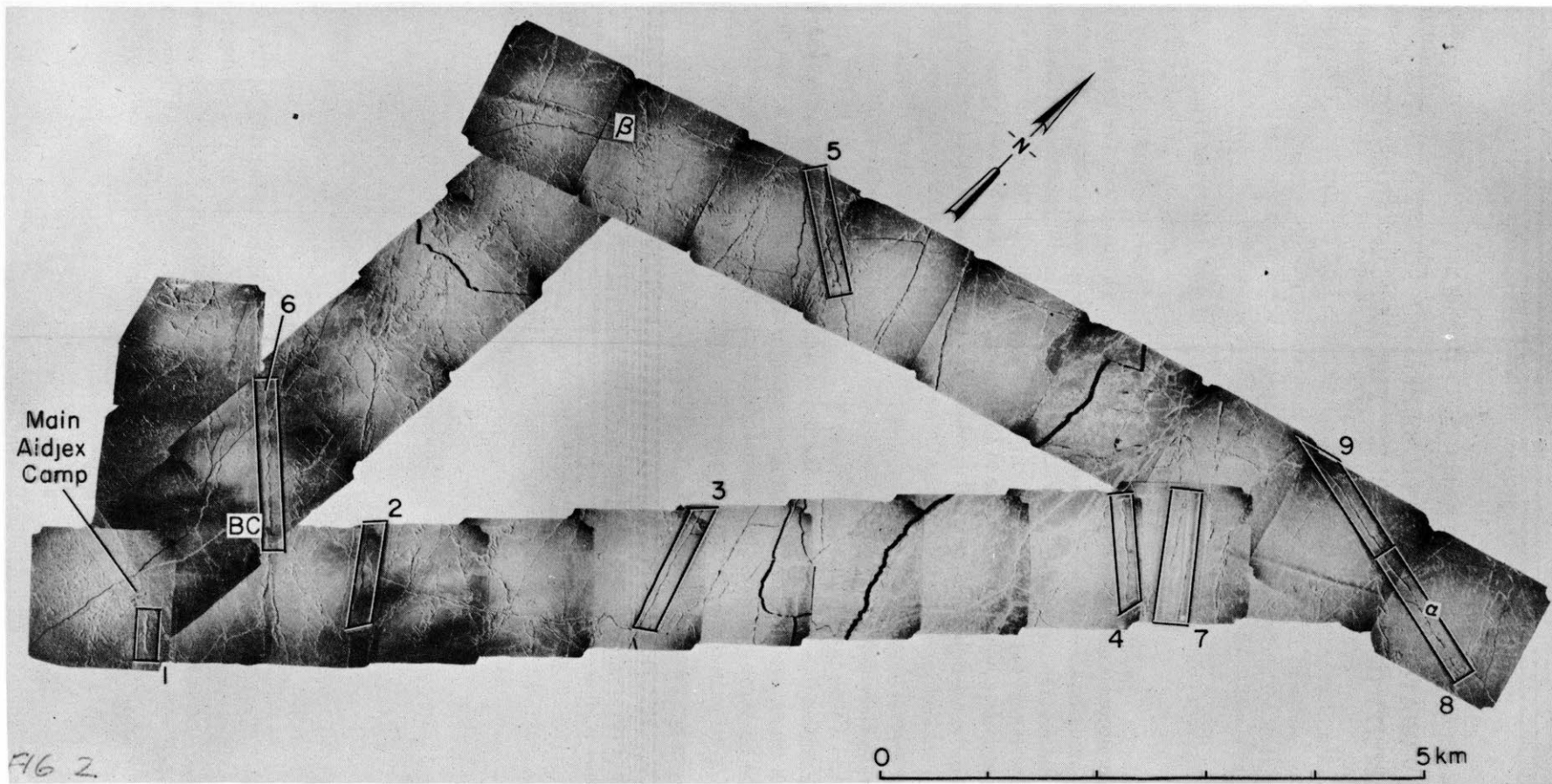


Figure 2. Aerial photo triangle taken at 1067-m altitude.  $\alpha$ ,  $\beta$ , and BC refer to strain measurement line end points. Numbered boxes show the pressure ridge locations investigated for height variations. Ground truth was obtained on ridges 1 and 8 in this figure.

### Experimental Data

To obtain height variations over sufficient ridge lengths for use in the statistical calculation shown in the next section, aerial photography was used. This method was used because ground-based survey measurements were insufficient and obtaining them would be too time consuming in a future field program. However, using aerial photography to obtain the height variations also presented a problem. To resolve small height variations (about 0.2 m), the aerial cameras must be flown at low level (500-m to 1200-m altitudes). In the existing photos taken at low level, the overlap between adjacent photos is usually only 10 to 15%, which does not provide adequate stereo coverage to measure vertical relief by photogrammetric methods.

To overcome this problem, the technique of measuring shadow profiles of ridges to obtain ridge height measurements was employed. Figure 2 shows three strips of sea ice photos of the strain triangle legs (Hibler et al. 1973) measured during the AIDJEX Program in 1971. These photos were obtained by RC-8 aerial camera flown by the NASA CV-990 at an altitude of 1067 m (3500 ft) on 11 March 1971. The pressure ridges can clearly be seen in these photos and upon close inspection the shadows of the ridges also can clearly be seen. The sun angle at this time of year is quite low, so the ridges cast a long shadow. From the altitude of the photography, the date, time of day and latitude of the area, these horizontal shadow lengths can be calibrated to give the vertical height of adjacent points on the ridge. The shadow length  $l$  from a vertical object of height  $h$  is

$$l = \frac{h}{\tan \alpha} \quad (1)$$

where  $\alpha$  is the sun angle above the horizon. By measuring the variations in shadow lengths along a given ridge, the vertical height distribution can therefore be computed. This method does not require stereo aerial photo coverage since the measurement is in the horizontal plane (the shadow length) and is not directly of the vertical height of the ridge.

The original 9-in.  $\times$  9-in. photos were enlarged to approximately three times their original size to obtain a convenient working scale. Comparisons of two profiles obtained by measuring the shadow profiles of ridges with two profiles obtained from ground surveys indicated an absolute error of approximately  $\pm 0.25$  m in any given height. The relative error in heights between any two adjacent shadows was less than this, of the order of  $\pm 0.10$  m. A shadow length was obtained every 5 m along the ridges indicated in Figure 2. The ridges were "first-year" ice ridges, with the exception of ridge 1 which was a multiyear ridge. Details of the structure of this ridge are presented by Kovacs et al. (1972).

The height profiles were computed along the ridges indicated in Figure 2 and are shown in Figure 3.

### Theory

To estimate the expectation of holes of various heights existing in the ridges, the following calculation was carried out. First, the random function  $H(x)$  defined by  $H(x)$  = height of pressure ridge with the variable  $x$  being measured along the length of the pressure ridge was introduced. We assumed that  $H(x)$  is normally distributed and homogeneous. These assumptions are reasonable in view of the variation of heights along the ridges shown in Figure 3.



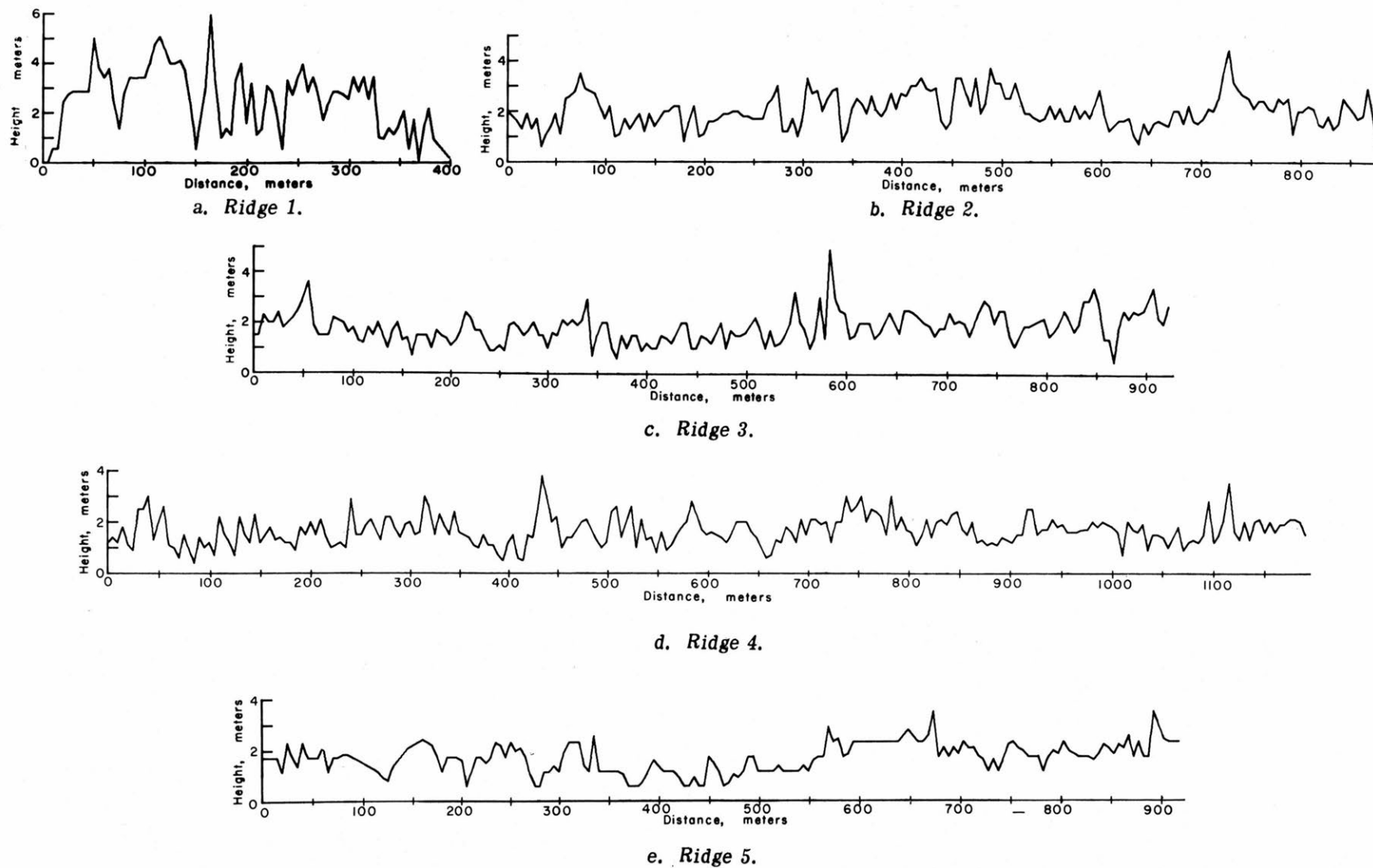


Figure 3. Digitized profiles at 5-m intervals of ridges in Figure 2.

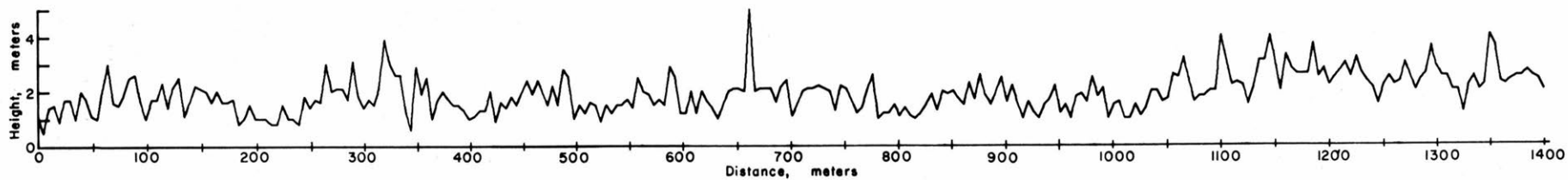
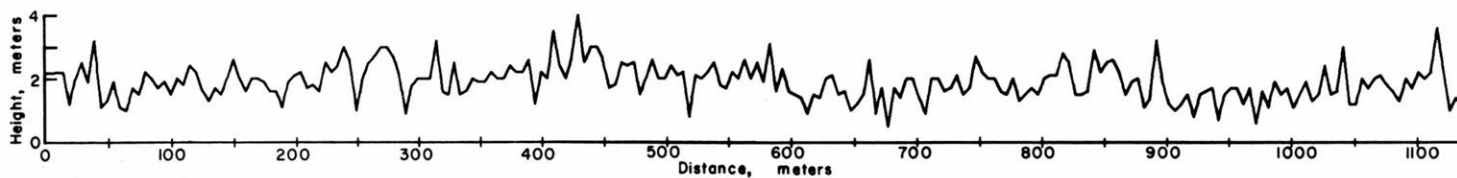
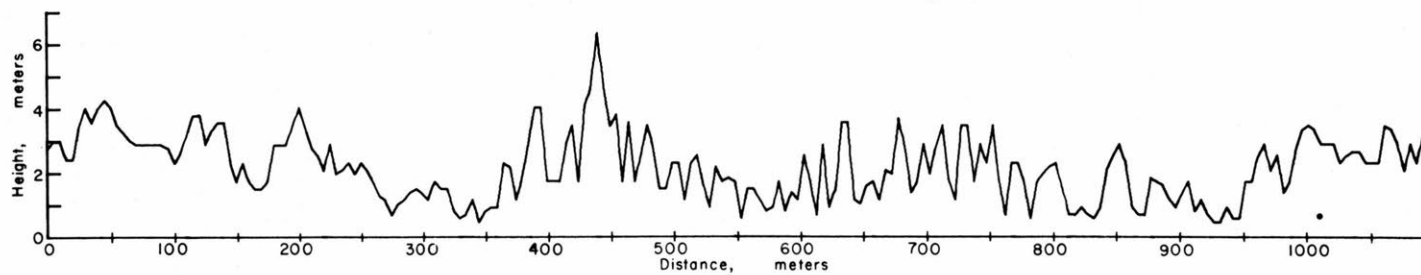
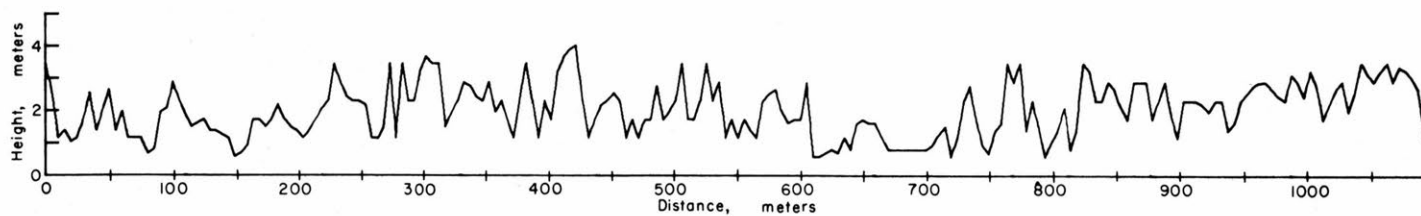
*f. Ridge 6.**g. Ridge 7.**h. Ridge 8.**i. Ridge 9.*

Figure 3 (cont'd).

The mean height and autocovariance functions were defined by

$$\langle H(\bar{x}) \rangle = \bar{H} \quad (2)$$

$$C(x_i, x_j) = \langle [H(x_i) - \bar{H}][H(x_j) - \bar{H}] \rangle \quad (3)$$

where brackets denote expectation values and  $C(x_i, x_j) = C(|x_j - x_i|)$  since  $H$  is homogeneous. Following Smith and Nakano (1973), if the covariance matrix  $\gamma_{ij} = C(x_i, x_j)$  is nonsingular, then the multivariate probability density function  $f_n(H_1, H_2 \dots H_n, x_1, x_2 \dots x_n)$  (probability of point  $x_1$  being at a height  $H_1$ , with  $x_2$  at height  $H_2$ , etc.) is given by

$$f_n = \frac{|\Lambda|^{1/2}}{(2\pi)^{n/2}} \exp \left[ -\frac{1}{2} \lambda_{ij} (H_i - \bar{H})(H_j - \bar{H}) \right] \quad (4)$$

where repeated indices are summed over

$$\lambda_{ij} = (\gamma_{ij})^{-1} \quad (5)$$

and  $|\Lambda|$  = determinant of  $\lambda_{ij}$ .

Using the multivariate pdf, the probability  $P$  that there is no point  $\delta$  units above the mean height  $\bar{H}$  in a passage of width  $n\Delta x$  where  $\Delta x$  is spacing between sample points is given by

$$P[H(x_1) \leq \bar{H} + \delta; H(x_2) \leq \bar{H} + \delta; \dots] \\ = \int_{-\infty}^{\bar{H}+\delta} \dots \int_{-\infty}^{\bar{H}+\delta} dH_1 \dots dH_n f(H_1 \dots H_n, x_1 \dots x_n). \quad (6)$$

To carry out the calculation indicated in eq 6, it was necessary to determine values of the autocovariance function  $C(x_i, x_j)$  for typical ridge profiles.

## Results

Using the digitized profiles shown in Figure 3, autocovariance functions were calculated and compared. A summary of the mean heights and variances for ridges 2-9 is given in Table I. Ridge 1 was not included in the calculation because it was of insufficient length for statistical purposes. In general, it was found that the autocorrelation functions [ $C(r)$  normalized to 1 at  $r = 0$ ] of the various ridges were quite similar. For an average probability of finding ridge passage, the  $C(r)$  values for all ridges used were averaged at each value of  $r$  (Table II) and the integral in eq 6 was performed for holes up to 25 m in width. The results are shown in Figure 4. Since the autocorrelation factor rather than the autocovariance factor was used, all heights were normalized to the square roots of the respective ridge height variances. The average variance was used to calculate the ridge passage probabilities. In addition to the probabilities obtained from the average autocorrelation values, the limiting cases, if there were no correlation at all between the adjacent points on the ridge, are shown in Figure 4. These represent the worst possible cases of the probability of finding passage based on the averaged variance of the ridges.

To use Figure 4 to calculate the probability of finding at least one "hole" in a given ridge, an encountered ridge can be broken into adjacent sections of length equal to the desired hole

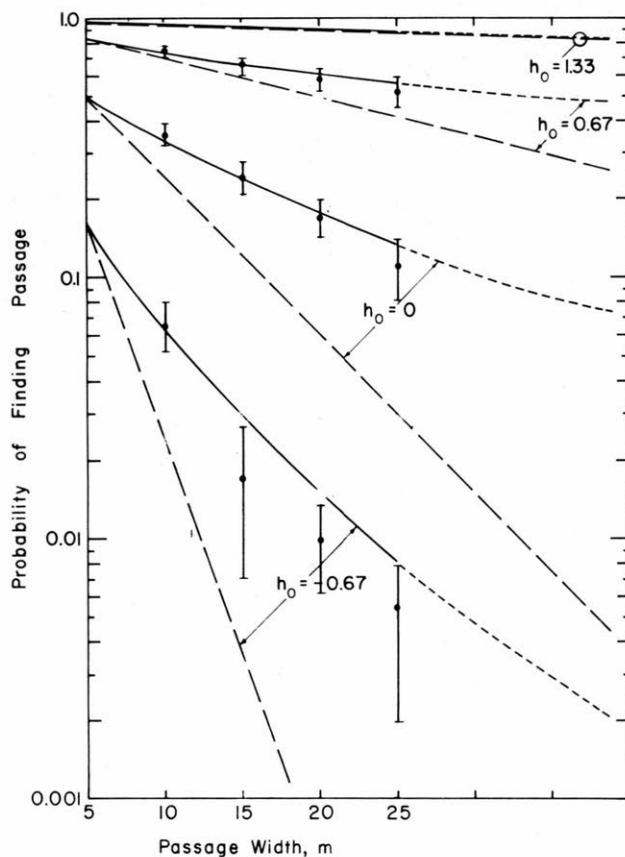


Figure 4. Average probability of finding passage vs passage width. The  $h_0$  values refer to height of passage relative to the mean ridge height; i.e.,  $h_0 = -0.67$  refers to holes 0.67 m below the mean ridge height. The large dashed curves were calculated assuming no correlation between adjacent points on the ridge and the solid curve was calculated from the average autocorrelation function. The data points indicate the probabilities of finding passage obtained by actually sampling the ridges. The points are the mean value of the probability with error bars given by the standard error of the mean for the seven ridges sampled.

Table I. Mean heights and variances for ridges 2-9.\*

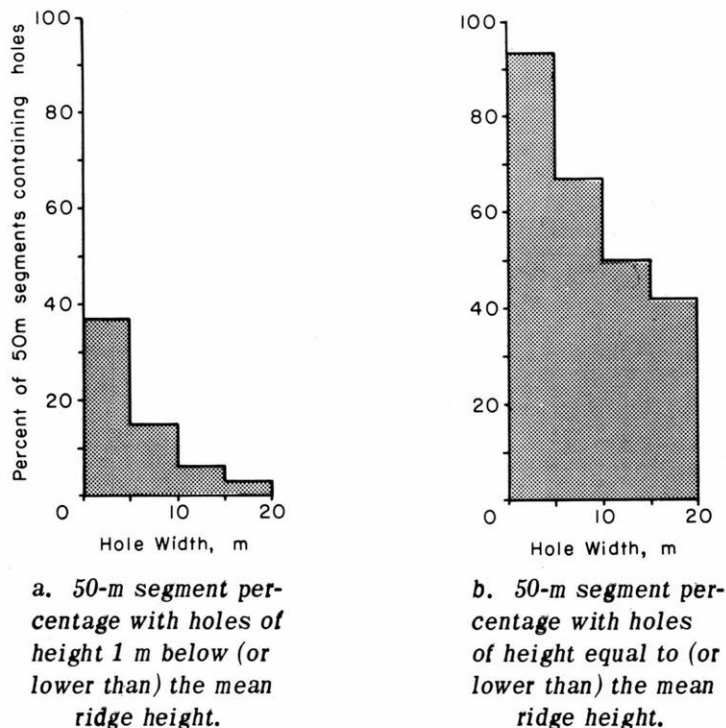
Ridge	$\sigma(m^2)$	$\bar{H}(m)$	Length (m)
2	0.43	2.03	885
3	0.38	1.83	930
4	0.32	1.66	1200
5	0.33	1.69	925
6	0.47	1.91	1410
7	0.32	1.90	1145
8	0.87	2.09	1102
9			1103
Avg	0.45	1.87	

\* Ridges 8 and 9 are two parts of the same ridge.

Table II. Average autocorrelation function  $C(r)$  for ridges 2-9.

1 lag = 5 m.

Lag no.	$C(r)$
0	1.0
1	0.508192
2	0.348886
3	0.294383
4	0.234582
5	0.192785
6	0.187736
7	0.187511
8	0.185677



*Figure 5. Percentage of 50-m ridge segments containing holes of various widths. Total number of 50-m segments examined was 172. The percentage of segments with at least one hole up to 20 m in width was 100% for hole heights 1 m above the mean ridge height.*

width which, to a first approximation, may be considered independent. The probability of a section's being a hole is given in Figure 4; if the vehicle finds no hole then the next section may be looked at. Figure 4 shows that if the probability of an acceptable hole of length  $\ell$  is  $P$  then the probability of finding at least one hole in a length, say  $10\ell$ , is  $1 - (1 - P)^{10}$ .

The results in Figure 4 generally indicate that the probability of finding wide holes, both from the theoretical curves and from the data points, is not good. This is apparent from the way the curves fall off and their good agreement with the mean value obtained by sampling the profile as indicated by the data points. As another check on this conclusion, the eight ridge profiles were broken into 50-m segments and the number of segments with holes of different widths catalogued. The results, as shown in Figure 5, generally verify the conclusion that there are few wide holes in ridges. Note that Figures 4 and 5 are not directly comparable since Figure 5 shows the number of 50-m segments having holes of a given width, whereas Figure 4 gives the average probability of a given number of random selected adjacent points being below a given height.

### Conclusions

Based on the results shown in Figures 4 and 5, vehicle trafficability can broadly be classified in relationship to the terrain in the following manner. Vehicles, in their ability to negotiate ridges,

fall into two classes. If a vehicle is able to negotiate only small heights (below the mean ridge height), it must be quite narrow since the availability of holes drops off rapidly with the width necessary for passage. This means that a vehicle like a snowmobile with good maneuverability could probably negotiate the pack ice although considerable time for hole-searching would be necessary. If a vehicle has considerable beam width ( $> 10\text{-}15$  m), the probability of finding a hole is so low that the vehicle must *go around* the ridges. The ramifications of this vis-à-vis a more complete trafficability analysis are discussed by Hibler and Ackley (1973). The general nature of the complete trafficability analysis is what one would intuitively expect: since ridges often extend beyond 2.5 km, the presence of on-the-average more than 0.5 impassable ridges/km makes it difficult for the vehicle to proceed rapidly and economically. This may be translated into the necessary height clearance ability of a vehicle by using ridge statistics (Hibler et al. 1972) compiled for different regions of the Arctic during different seasons (Mock et al. 1972).

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13. ABSTRACT Height variations along sea ice pressure ridges were studied to determine whether low spots in the ridges persisted over long enough distances for a wide vehicle such as the surface effect vehicle to pass through them. Heights along eight pressure ridges varying in length from 0.9 km to 2.2 km were measured at 5-m intervals using aerial photographic data obtained over the Beaufort Sea. The probabilities of finding holes at the mean heights of the ridges and 0.67 m above and below the mean heights were calculated as a function of the hole widths using an autocorrelation function obtained from the height data. The curves obtained were in good agreement with passage probabilities calculated directly from the data. The results indicate that the probability of finding passage through a hole at least 0.67 m below the mean ridge height is less than 0.1 for holes greater than 10 m wide. It is concluded that a large vehicle such as a surface effect vehicle that cannot cross ridges at their mean heights will have to go around such ridges because low, wide holes do not appear with sufficiently high probability to make hole-searching a useful method.			
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