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MODEL ICE HEAT SINK

Roscoe E. Perham

March 1973

CORPS OF ENGINEERS, U.S. ARMY
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

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PREPARED FOR
U.S. ARMY ENGINEER REACTOR GROUP
BY
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PREFACE

This report was prepared by Roscoe E. Perham, Research Mechanical Engineer, Technical Services Division, U.S. Army Cold Regions Research and Engineering Laboratory (USA CRREL). It provides a summary of studies directed towards the design of a model ice heat sink and also gives the resultant model in prototype form. The model has been used for investigations under the project *Heat Sink Studies*, USA Engineer Reactor Group (ERG) Order No. 34-69.

Persons participating in these studies besides the author were Mr. William Quinn and Dr. Y.C. Yen. Their comments and assistance are appreciated.

NOMENCLATURE

Symbol	*Quantity*	*Units*
$a = \dfrac{g \beta \rho^2 C_p}{\mu k}$	A substitute term for natural convection calculations	ft^{-3} $°F^{-1}$
A	Area	ft^2
C_p	Specific heat	Btu lb^{-1} $°F^{-1}$
D	Diameter	ft
g	Gravitational constant $[4.17(10)^8]$	ft hr^{-1}
h	Coefficient of surface heat transfer	Btu hr^{-1} ft^{-2} $°F^{-1}$
H	Latent heat of fusion	Btu lb^{-1}
k	Thermal conductivity	Btu hr^{-1} ft^{-1} $°F^{-1}$
L	Length, characteristic length	ft
N	Number quantity	
P	Perimeter length	ft
q	Heat flow	Btu hr^{-1}
Q	Total heat	Btu
$R_D = D/D_0$ or D_0/D	Diameter ratio	
R_u	Unused ice fraction	
t	Temperature	°F
v	Velocity	ft hr^{-1}
V	Total volume	ft^3
Vol	Volumetric flow	ft^3 hr^{-1}
w	Weight flow	lbm hr^{-1}
W	Total weight	lbm
β	Coefficient of cubic thermal expansion	$°F^{-1}$
θ	Time	hr
μ	Dynamic viscosity	lbm ft^{-1} hr^{-1}
ρ	Weight density	lbm ft^{-3}
ΔD	Diameter difference	ft
Δt	Temperature difference	°F
$\Delta\theta$	Time difference	hr

NOMENCLATURE (Cont'd)

Symbol	*Quantity*	*Units*
Δt_m	Logarithmic mean temperature difference	°F
$N_{Gr} = \dfrac{g\,\beta\,\Delta t\,L^3\,\rho^2}{\mu^2}$	Grashof number where Δt is the temperature excess between the surface and the main body of the fluid.	
$N_{Nu} = \dfrac{h\,L}{k}$ or $\dfrac{h'\,D}{k}$	Nusselt number	
$N_{Pr} = \dfrac{C_p\,\mu}{k}$	Prandtl number	
$N_{Re} = \dfrac{v\,D\,\rho}{\mu}$	Reynolds number	

MODEL ICE HEAT SINK

by

Roscoe E. Perham

Introduction

This report summarizes studies directed towards the design of a model ice heat sink and shows the resultant model in prototype form. The report does not give all the analytical approaches used in attempting to predict the performance of the full scale heat sink. But it does indicate most of the problems involved and presents a method of correlating the performance of a model heat sink with that of the full scale heat sink.

The report first describes methods of utilizing the full scale heat sink and predicting its heat transfer characteristics. These prediction formulas (dimensionless parameters with coefficients determined from tests) are then applied to a reasonable model size. Finally, a sketch of the prototype model is given.

There is a tremendous difference in size between the model ice heat sink that we can effectively handle and the actual heat sink intended for use. It became apparent early in our model calculations that we had to approach the problem from the perspective of the design engineer. It was decided, therefore, to take a look at various aspects and methods of utilizing the actual heat sink under the operating conditions imposed. The knowledge obtained from this phase of the study was then applied to the model effort. The liquid passing through the heat sink is referred to as the coolant. However, it cools only the heat source and not the sink.

Heat sink criteria

The heat sinks are part of a power generation complex. Three equal capacity, ice-water heat sinks are planned for the complex. The waste heat load is assumed to be split three ways. As scheduled, the heat input to one sink starts at $5 \times (10)^6$ Btu hr^{-1}. It increases linearly to $10 \times (10)^6$ Btu hr^{-1} in thirty days. The load then drops to $5 \times (10)^6$ Btu hr^{-1} and remains there for 10 days.

Each heat sink is 65 ft in diameter and 115 ft high. It is to be initially filled with ice. The coolant fluid is to be water. The ice is to be kept free of entrapped mechanical devices.

Further, ERG guidance indicates a total maximum coolant water flow rate of between 3000 and 8000 gpm. Split three ways the expected maximums for each sink will be 1000 to 2667 gpm.

Concepts

Conceptually, two methods of utilizing the heat sink seemed most promising, and a third was being considered by the ERG (see Fig. 1). The first method has several axial and parallel holes through the ice in the heat sink. The coolant water flows through these holes from top to bottom.

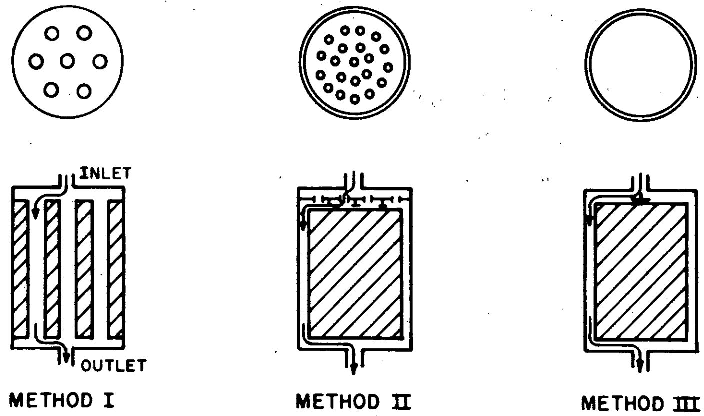

Figure 1. Coolant water flows.

The second method has an array of inlet nozzles at the top of the heat sink. They distribute the coolant water evenly over the upper ice surface plane. This water flows out and down the annulus between the sink wall and the ice to discharge at the bottom. As the upper surface of the ice cylinder melts, the cylinder moves up naturally due to buoyancy. The inlet pipe support structure has pressure pads on its underside to maintain a water flow space between the inlet and the ice. The third method uses just the flow through the annulus between the ice cylinder and the heat sink wall to dump heat to the sink.

Heat transfer background

Calculating the performance of these three methods of utilizing the heat sink is quite difficult. Heat transfer relationships have been found for predicting the performance of the first and third type but not the second. Even then, these relationships are not quite applicable because they have been figured out for flows through solid walled pipes. The mass transfer that occurs at the flow-wall interface when ice melts is not accounted for. It is bound to affect the flow pattern in some way and give a different heat transfer coefficient. This uncertainty means that we can only tentatively predict the performance of the heat sink. This is not a deterrent because even under ideal circumstances these predictions are fairly inaccurate. In this case, the application of heat transfer relationships will give us a rough but useful grasp of the problems involved.

Heat transfer relationships

Method I. The flow through the Method I heat sink is characteristically tube flow. Turbulent and laminar flow regimes are expected. The ASHRAE equations* seem typical and most suited for predicting its performance.

$$\text{Turbulent flow} \qquad N_{Nu} = 0.023(N_{Re})^{0.8}(N_{Pr})^{0.4} \tag{1}$$

$$\text{Laminar flow} \qquad N_{Nu} = 1.86[(N_{Nr})(N_{Pr})(D/L)]^{1/3}(\mu/\mu_s)^{0.14} \tag{2}$$

where μ = viscosity of fluid at the bulk temperature

μ_s = viscosity of fluid at the heat transfer surface temperature.

Also the fluid properties are based on the bulk temperature t.

For smooth circular tubes, flow is laminar for Reynolds numbers below 2100 and turbulent above 3000.

Method II. Relationships for predicting the performance of the Method II heat sink were not found except in part. Some of the heat is transferred in the annulus which is the subject of Method III.

Method III. The E.S. Davis relationship for annular flow cited by Jakob† seems most applicable.

$$(N_{Nu}) = 0.038(N_{Re})^{0.8}(N_{Pr})^{1/3}(\mu/\mu_1)^{0.14}(D_2/D_1)^{0.15} \tag{3}$$

where the Nusselt and Reynolds numbers apply to the inner annulus diameter.

subscript 1 refers to the inner annulus diameter

subscript 2 refers to the outer annulus diameter (a constant)

other properties are assumed to be at the arithmetic mean temperature of the fluid.

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* ASHRAE Handbook of Fundamentals, American Society of Heating, Refrigerating, Air Conditioning Engineers, Inc., New York, 1967, p. 44.

† Jakob, M. Heat Transfer. Vol. I. John Wiley and Sons, Inc., New York, 1949, p. 522, eq 26-21.

In general eq 3 has been proved to be roughly valid for turbulent flow when

$$7 \leqq N_{Re} \leqq 180{,}000$$

and at the same time

$$1.2 \leqq (D_2/D_1) \leqq 6800.$$

The equation is a representation of the experimental results of various investigators.

Method I

Log mean temperature difference. The following method is related to performance calculations for heat exchangers. The inlet flow is cooled throughout the length of its passage through the circular hole in the ice. The ice wall is assumed to be at 32F. Figure 2 illustrates the temperature pattern. We have the following relations:

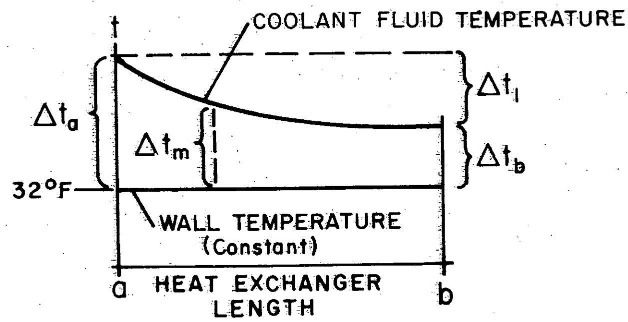

Figure 2. Fluid temperature drop in ice passage.

$$\Delta t_m = \frac{\Delta t_a - \Delta t_b}{\ln(\Delta t_a/\Delta t_b)} \tag{4}$$

$$\Delta t_1 = \Delta t_a - \Delta t_b \tag{5}$$

$$h\,P\,L\,\Delta t_m = w\,C_p\,\Delta t_1. \tag{6}$$

Assuming constant coolant flow, heat load and water properties, we can derive from eq 6

$$\frac{\Delta t_m}{\Delta t_{m_0}} = \frac{h_0\,D_0}{h\,D} \tag{7}$$

where subscript 0 refers to initial conditions.

From eq 1

$$\frac{h_0}{h} = \frac{D}{D_0}\left(\frac{v_0\,D_0}{v\,D}\right)^{0.8} \tag{8}$$

using

$$w = \rho A v = \rho \frac{\pi}{4} D^2 v \qquad (9)$$

with eq 7 and 8 gives

$$\frac{\Delta t_m}{\Delta t_{m_0}} = \left(\frac{D}{D_0}\right)^{0.8} . \qquad (10)$$

We can first establish the initial hole diameter, apportion the heat load and flow rate according to the number of holes in the heat sink and calculate the initial mean temperature difference. As the hole diameter increases from D_0 to D at constant weight flow and heat input, the mean temperature difference increases to Δt_m according to eq 10. In this way we can estimate the diameter at which $t_m = (32F + \Delta t_m) = 160F$.

Melting time estimate. In conjunction with this calculation we can also estimate the time θ it would take to reach a diameter at which $t_m = 160F$. A further assumption must be made that all the heat load goes into melting ice and not increasing its temperature.

For a cylinder of ice

$$Q = W H = \frac{\pi}{4} D^2 L \rho_i H = \text{total latent heat} \qquad (11)$$

where ρ_i = specific weight of ice.

Let $[(\pi/4) L \rho_i H] = C$

$$\therefore \quad Q = CD^2$$

$$\frac{dQ}{d\theta} = q = C\, 2D \frac{dD}{d\theta}$$

let q = constant and integrate from D_0 to D and $\theta = 0$ to θ

$$\int_{D_0}^{D} 2d\, dD = \int_0^{\theta} \left(\frac{q}{C}\right) d\theta$$

or

$$D^2 - D_0^2 = \left(\frac{q}{C}\right)\theta$$

or

$$D^2 \left[1 - \left(\frac{D_0}{D}\right)^2\right] = \left(\frac{q}{C}\right)\theta \qquad (12)$$

$$\therefore\ \theta = \frac{D^2\left[1-\left(\frac{D_0}{D}\right)^2\right]}{\frac{q}{C}} . \tag{13}$$

Note that for small (D_0/D)

$$\theta = \frac{C\,D^2}{q} . \tag{14}$$

For the case that $dq/d\theta$ = constant = C_1, because of the nonlinear nature of the differential equation, i.e.

$$D\,\frac{d^2 D}{d\,\theta^2} + \frac{2d\,D}{d\,\theta} = \frac{C_1}{C} .$$

It seems deficient to obtain an expression for melting θ in terms of heat rate q and cylinder diameter D explicitly.

Number of passage holes. Equations 10, 12 and 14 are useful in predicting performance for the Method I heat sink. The question remaining to be answered is the number of holes we should have in the sink for optimum use of the sink.

It seems that the correct number and spacing would be such that most of the ice is used up (melted) when the holes have enlarged to such an extent that they merge with one another or with the heat sink wall.

Let R_u be the fraction of ice remaining or unused when the hole diameters achieve their interference diameter D_m

$$R_u = \frac{A_R}{A_0} = \frac{\frac{\pi}{4}(D_s)^2 - N\,\frac{\pi}{4}(D_m)^2}{\frac{\pi}{4}(D_s)^2} = 1 - N\left(\frac{D_m}{D_s}\right)^2 \tag{15}$$

where D_s is the heat sink diameter and N is the number of holes. For example, let $N = 2$ holes centered on D_s at the ¼ and ¾ points

$$D_m = \frac{D_s}{2}$$

and

$$R_u = 1 - (2)\left(\frac{\frac{D_s}{2}}{D_s}\right)^2 = \frac{1}{2}$$

or one half of the original ice remains.

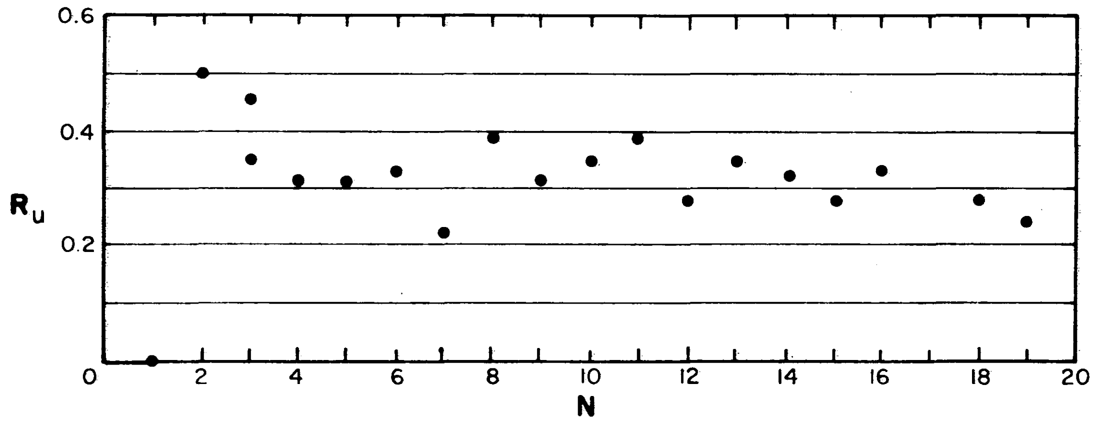

Figure 3. Fraction of ice remaining when passage holes blend from melting vs number of holes used.

Table I. Heat sink summary.
(q = constant, $5(10)^6$ Btu hr^{-1}.)

N	D_m *(ft)*	θ *to* D_m *(hrs)*	w *to reach* D_m *at* t_m *= 160F* *(gpm)*	N_{Re} *at* D_m	N_{Re} *original* D_0 = ½ *ft* (θ = 0)	R_u
1	65					0.00
7	21½	480	3340	4660	200,000	0.22
12	16	499	1910	2344	75,000	0.27
19	13	521	1375	1310	34,000	0.24

Figure 3 is a graph of the ice remaining fraction vs the number of holes used.

Table I summarizes the calculated results for some more promising numbers of holes.

Laminar flow region. From Table I for 12 and 19 holes, N_{Re} is in either the mixed flow or the laminar flow range. This indicates that the turbulent flow equation is not valid over the full range of diameters considered. The laminar flow equation (eq 2) is then used, as was eq 1 before, to correlate the log mean temperature differences in the laminar range. However when this is done one notices a peculiar thing

$$\frac{\Delta t_m}{\Delta t_{m_0}} \quad 1.0. \qquad (16)$$

That is, Δt_m does not change.

This phenomenon is mentioned by Jakob* with respect to laminar flow in a long duct at uniform temperature. The significance of this may be that when the laminar flow regime is reached Δt_m tends to stay constant. Should this occur before t_m becomes 160F, t_m may tend to stay below 160F. This is obviously something more to test for in a model.

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* Op cit., p. 464.

Increasing heat load compensation. The fact that a linearly varying heat rate was not considered in these calculations does not completely invalidate the answers. The time estimates indicate that with Method I we have the opportunity of utilizing as much as ¾ of the heat sink's latent heat in ½ the time allowed (about 20 days), with a $5(10)^6$ Btu hr^{-1} heat input. But much of the increasing portion of the heat rate could be considered as going into heating the water. The sensible heat in heating water from 32 to 160F is nearly the equivalent of melting the same volume of ice. Subsequently, the melting and heating to 160F of ¾ of the volume of the heat sink would mean that ¾ of the total heat load could have been absorbed. Based on the original schedule of heating, ¾ of the total heat load will be applied in 681 hours or 28.4 days which is an appreciably longer period than 20 days. In other words, Method I appears able to accept the required heat without being restricted by a slow rate of ice melting.

Method III (annular flow)

Forced convection. In this method a gap between the ice cylinder and the heat sink wall is initially heated by exterior means. The coolant liquid is then passed through this annular gap to be cooled. To calculate the heat absorptior. potential of the sink, a somewhat different approach was used.

Diameter ratios R_D were used except that they were equal to D_0/D or D_2/D_1 or always greater than 1. Equation 3 was separated into three terms and curves were developed for each (Fig. 4).

Curve 1 values are related to that portion of eq 3 which varies only due to changes in the properties of water. Curve 2 gives just the effect of varying weight flow on eq 3. Curve 3 gives just the effect of annulus geometry in terms of the diameter ratio on eq 3.

$$h_1 = 0.038\left(\frac{K}{D_1}\right)\left(\frac{D_1 v_m}{v}\right)^{0.8}\left(\frac{v}{u}\right)^{1/3}\left(\frac{\mu}{\mu_1}\right)^{0.14}\left(\frac{D_2}{D_1}\right)^{0.15} . \qquad (17)$$

By substituting equivalent values such as

$$w = \rho\frac{\pi}{4}(D_0^2 - D^2)v_m$$

and

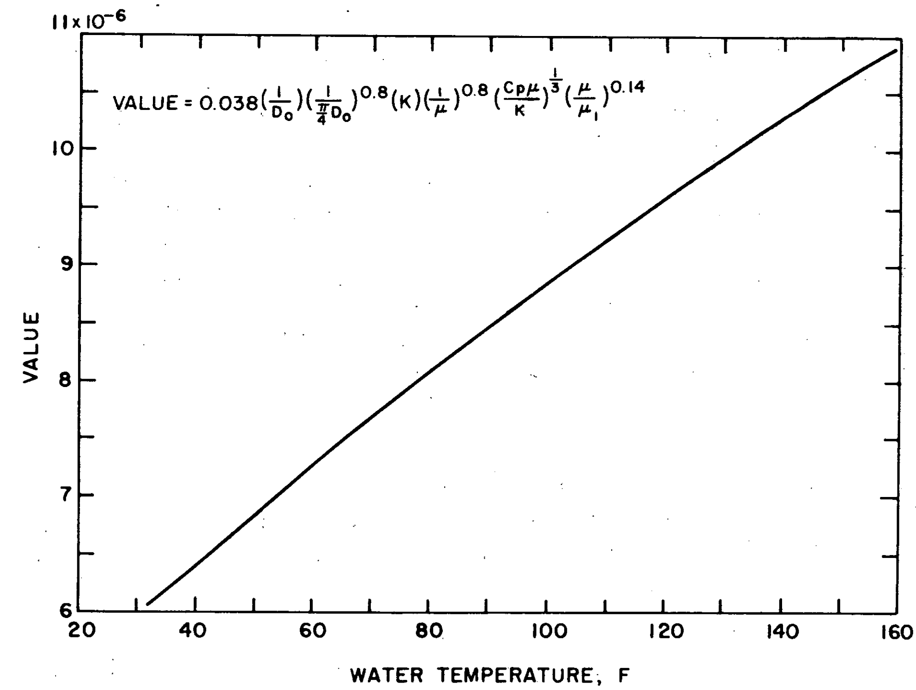
$$R_D = \frac{D_0}{D} = \frac{D_2}{D_1}$$

and recombining terms, eq 17 becomes

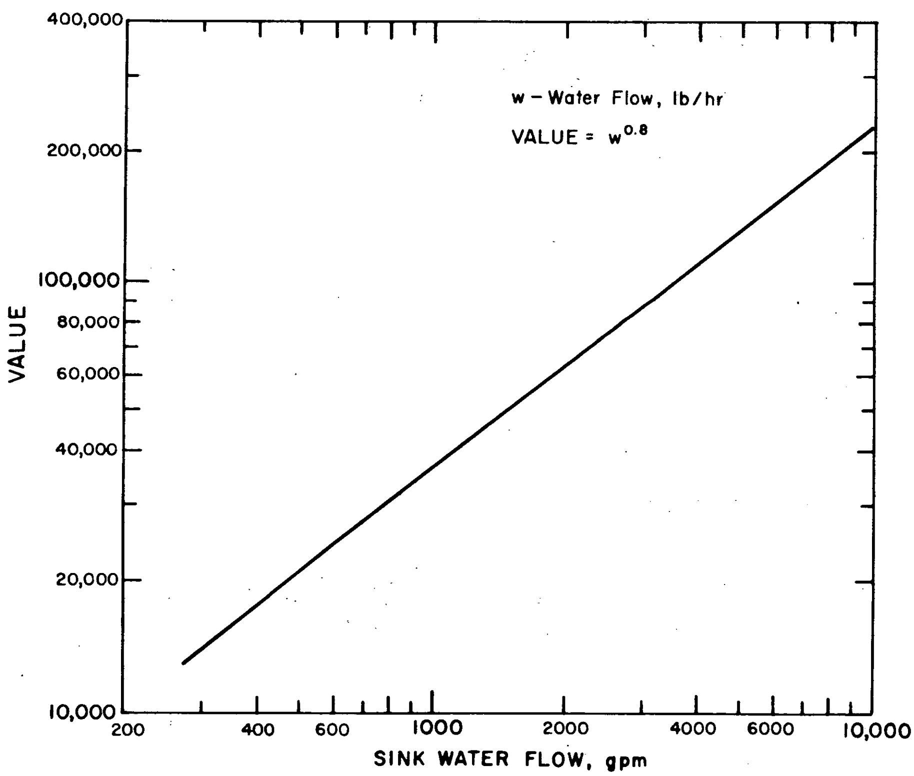
$$h_1 = \left[0.038\left(\frac{1}{D_0}\right)\left(\frac{1}{\frac{\pi}{4}D_0}\right)^{0.8}(K)\left(\frac{1}{\mu}\right)^{0.8}\left(\frac{C_p\,\mu}{K}\right)^{1/3}\left(\frac{\mu}{\mu_1}\right)^{0.14}\right]\left(w^{0.8}\right)\left[R_D^{1.15}\left(\frac{R_D}{R_D^2 - 1}\right)^{0.8}\right] \qquad (18)$$

or

$$h_1 = [\text{Curve 1}]\,[\text{Curve 2}]\,[\text{Curve 3}].$$

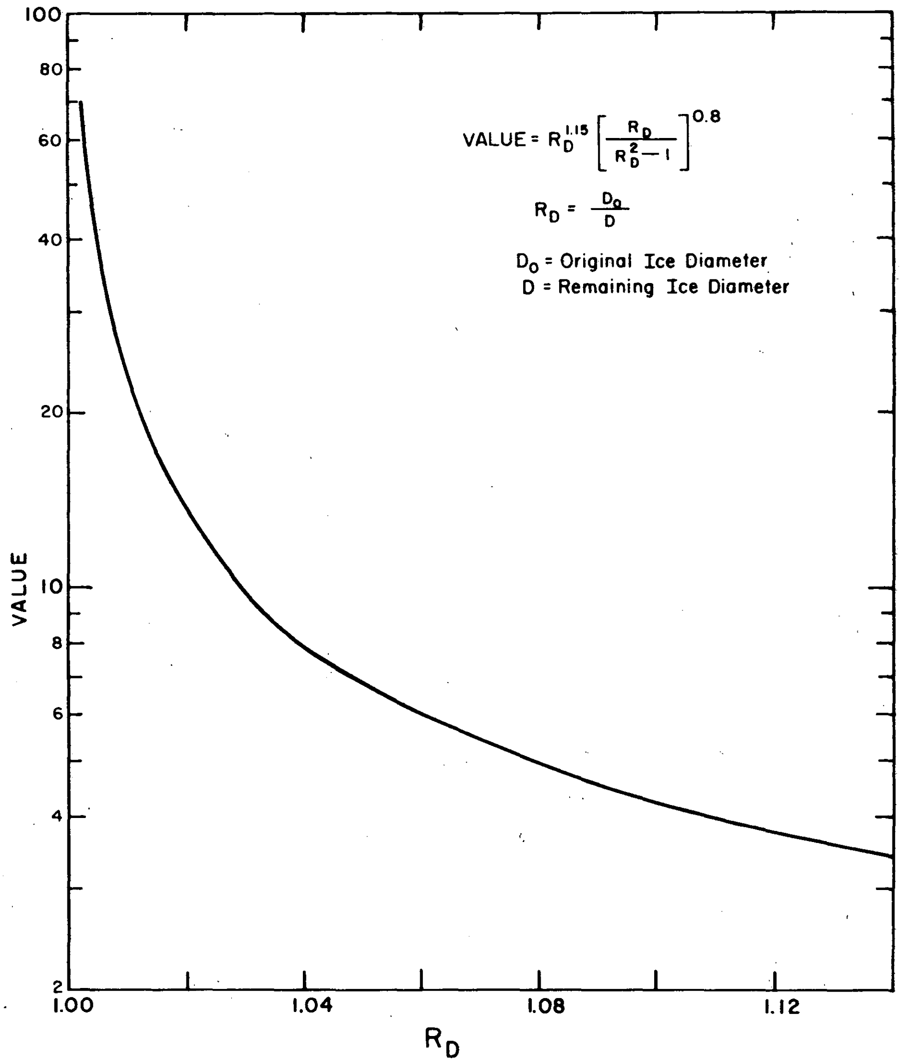


Curve 1. Value of a calculation factor based on water properties and annular flow at various temperatures.



Curve 2. Value of water flow rate term $w^{0.8}$.

Figure 4.



Curve 3. Value of terms related to dimensional changes in annular flow, thin annuli.

Figure 4. (Cont'd).

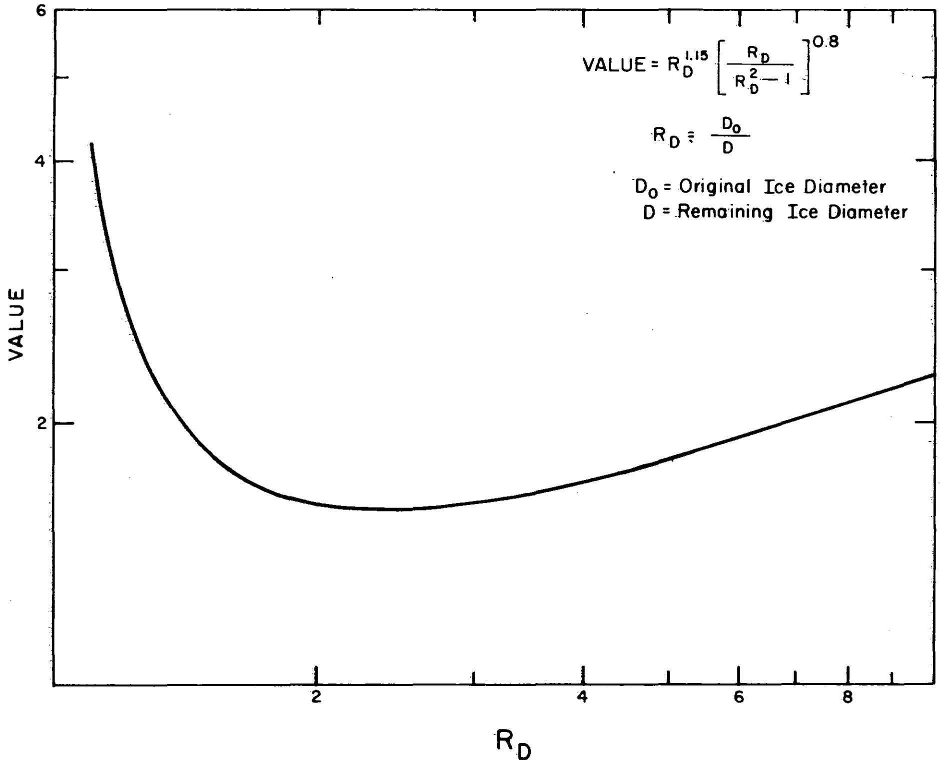
Though the first three terms in the Curve 1 relationship are not a function of water temperature, they are a constant for this sink and can be used in the curve as a calculation expedient.

Curve 1 varies less than 2 to 1 over the range of water temperatures considered. However, Curves 2 and 3 exhibit large variations. One very important factor to notice about Curve 3 is that it achieves a minimum value between $R_D = 2$ and $R_D = 3$. This minimum value is a key point and will strongly indicate whether or not this method of heat transfer will work.

As an example, choose $R_D = 2.0$, the maximum coolant flow rate of 2667 gpm and the maximum water temperature of 160F. Assume also that forced convection is the dominant mode of heat transfer.

$$h = [\text{Curve 1}]\,[\text{Curve 2}]\,[\text{Curve 3}]$$

$$h = [10.92(10)^{-6}]\,[79{,}500]\,[1.615] = 1.405\,\frac{\text{Btu}}{(\text{hr})(\text{ft}^2)(^\circ\text{F})}\,.$$



Curve 3. Value of terms related to dimensional changes in annular flow, thick annuli.

Figure 4. (Cont'd).

Also for a q of $5(10)^6$ Btu hr

$$\text{Coolant } \Delta t = \Delta t_1 = \frac{q}{w\,C_p} = \frac{5(10)^6}{1.327(10)^6(1.0)} = 3.77\,^\circ\text{F}.$$

$$\text{And heat transfer area} = A_1 = \frac{\pi\,D_0\,L}{R_D} = \frac{\pi(65)(115)}{2.0} = 11{,}750\ \text{ft}^2.$$

$$\text{Using eq 6} \quad \frac{\Delta t_1}{\Delta t_m} = \frac{h\,A}{w\,C_p} = \frac{(1.405)(11{,}750)}{1.327(10)^{-6}(1.0)} = 1.243(10)^{-2}.$$

$$\text{Using eq 4 and 5} \quad \frac{\Delta t_1}{\Delta t_m} = \ln\frac{\Delta t_a}{\Delta t_b}$$

$$\text{or} \quad \frac{\Delta t_a}{\Delta t_b} = \exp\left(\Delta t_1/\Delta t_m\right) = \exp\left[1.243(10)^{-2}\right] = 1.01248$$

$$\text{or} \quad \Delta t_b = \frac{\Delta t_a}{1.01248}.$$

$$\text{Substituting into eq 5} \quad \Delta t_1 = \Delta t_a - \frac{\Delta t_a}{1.01240}$$

yields

$$\Delta t_a = \frac{1.01248\ \Delta t_1}{(1.01248-1)} = \frac{1.01248(3.77)}{.01268} = 306°F.$$

$\therefore$ Δt_m must be over 300F to give up $5(10)^6$ Btu hr^{-1} to the sink. Further, by increasing the flow rate through the sink to reduce Δt_1, one can determine the rate needed to keep Δt_m at about 128F. This value is roughly 9000 gpm/sink for the $5(10)^6$ Btu hr^{-1} load.

In summary Method III shows little promise for effectively harnessing the heat sink over the full range of loads and time. However, it will be shown that consideration of natural convection will make annular flow look much better.

Natural convection. The natural convection heat transfer coefficients given by ASHRAE* for vertical planes are as follows.

Laminar flow $N_{Nu} = 0.56(N_{Gr} \cdot N_{Pr})^{1/4}$ (19a)

$(N_{Gr} \cdot N_{Pr})$ between 10^4 and 10^8

Turbulent flow $N_{Nu} = 0.13(N_{Gr} \cdot N_{Pr})^{1/3}$ (19b)

$(N_{Gr} \cdot N_{Pr})$ between 10^8 and 10^{12}.

For surfaces of appreciable height ($L > 0.15$ ft for liquids) the flow will be turbulent and eq 19b can be reduced to the following equation for the heat transfer coefficient:

$$h_c = 0.13K(a\ \Delta t)^{1/3} \qquad \dagger\ (20)$$

wherein the factor L has been cancelled out.

As an example, using the annular flow equation at $R_D = 1.5$, a water temperature of 100F, and a flow of 3000 gpm

Forced convection

$$h = [8.87(10)^{-6}][8.8(10)^4][1.843] = 1.44\ \frac{\text{Btu}}{(\text{hr})(\text{ft}^2)(°\text{F})}.$$

Note that N_{Re} with respect to the ice diameter equals 21,250 and with respect to the annulus equivalent diameter, $N_{Re} = 10.600$ or turbulent conditions.

Natural convection using eq 20

$$h = (0.13)(0.364)[5.2(10)^8(68)]^{1/3} = 155.6\ \frac{\text{Btu}}{(\text{hr})(\text{ft}^2)(°\text{F})}$$

or the natural convection coefficient is over one hundred times larger. That is, the heat transfer due to forced convection at this point in heat sink usage is almost insignificant when compared to natural convection heat transfer. Though puzzling, the effect of natural convection is what would make the annular flow method work. The problem is to understand this effect better and to harness it.

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* ASHRAE, Op. cit., Table 5, p. 42.

† Brown, A.I. and Marco, S.J. Introduction to Heat Transfer, McGraw-Hill Book Company, Inc., New York, 1958, p. 165.

Velocity comparison. Most convective heat transfer people will agree that the velocity of flow nearest the heat transfer surface is one of the most important parameters to look for in predicting relative heat transfer. To this effect Figure 5 was drawn up. It compares the stream flow velocities relative to the heat transfer surface due to forced and to natural convection. No relationship between R_D and water temperature is implied.

The equation:* $v = (g\ \beta\ \Delta t\ L)^{1/2}$ for natural convection comes from the derivation of eq 19a and 19b. The L first used for the curve was 2 ft but indications are that it should have been 0.15 ft because a liquid is being used. Curves for both values are shown in Figure 5. The actual value to use for water under these conditions will have to be found from tests.

Figure 5 indicates that there need not be a very large change in annulus thickness or a very great change in water temperature before natural convection will play an important part in the heat sink operation. At $R_D = 1.02$ the annulus width is only about 8 inches.

The effect of an adjacent boundary on natural convection was not considered in these calculations. It would tend to reduce this heat transfer coefficient for a while until a wide annulus was reached and then have no effect. It seems reasonable to assume that forced convection must be relied upon at first to provide the necessary heat transfer coefficient. Later, and for most of the heat sink usage, the forced flow will be mainly concerned with drawing off the naturally cooling water.

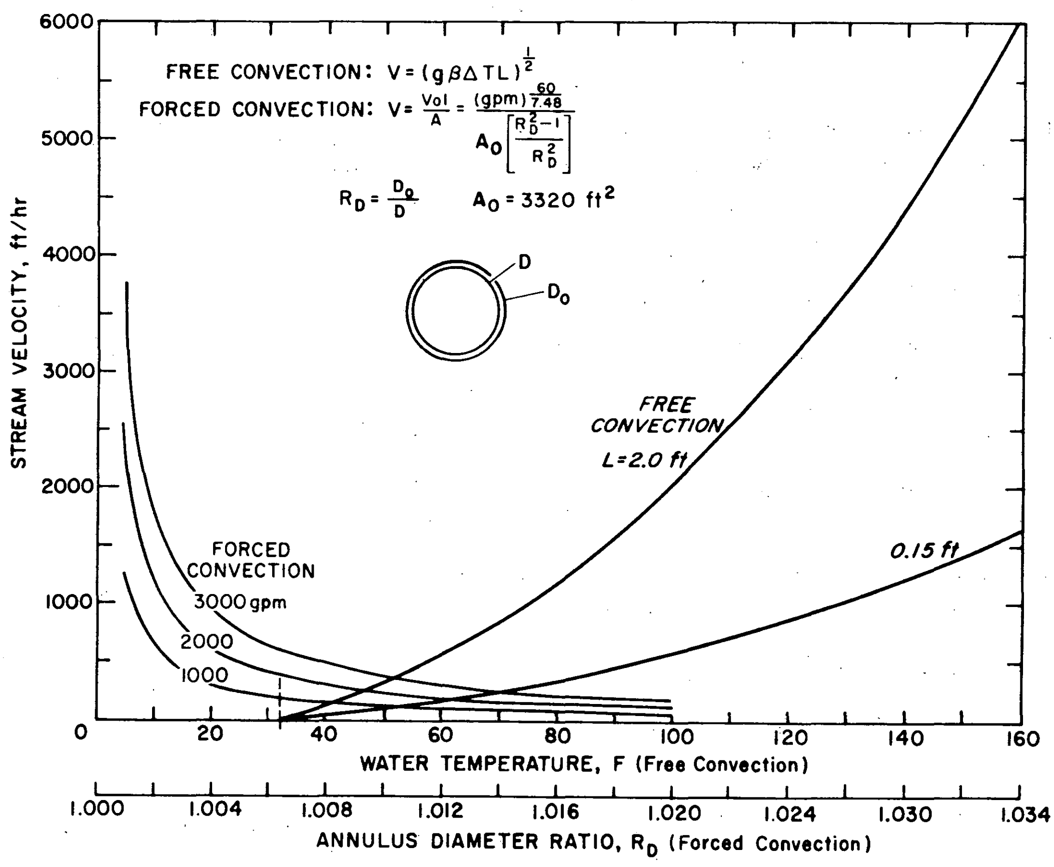


Figure 5. Estimated convective velocities in annular flow (vertical cylinder).

* Ibid., p. 162.

Model heat sink

Correlation problems. Dynamic similarity can not be established because the same heat transfer fluid has to be used in the model and the prototype. In modelling ships or pumps or pipes the properties of the fluids may be changed to aid in keeping the models small, i.e., increased density, viscosity, etc.

The primary shortcoming of the scaling factors is the unknown effect of relative mixing of the water in the heat sink as compared with the model. In a model at some particular diameter ratio the flowing water may pass from the top plane of the ice cylinder into an annulus of, say, six inches width. In the heat sink at that diameter ratio the water may flow into a gap ten feet wide. It may be that the scaling factors account for the required mixing adjustment but this is doubtful.

Three more factors will affect the results:

1. Wall proximity has an effect on natural convection.
2. Heat transfer is accomplished by mixed forced and natural convection.
3. Liquid water is formed at the heat transfer boundary due to melting.

The resolution of the effect that each of the above factors has on the correlation between model and heat sink performance would be a tremendously ambitious testing program. The state of the art with respect to the above factors seems to be that some investigative and theoretical work has been done but none applies directly to this ice heat sink problem.

Direction. The time and money allotted for this model study do not permit excursions into the investigation of lateral phenomena. The only course is to investigate the performance of a model based on existing criteria. A cylindrical model used similarly as the heat sink (Method III) seems most advisable. It should be mentioned that a segment of a cylinder could be tested to achieve a larger relative model size. However the effect of added boundaries would be an added unknown, especially to the flow pattern.

Figure 6 shows basic components of the model. Its sizing relationships are given below. Some functional features follow later on.

Size limit. A tank four feet in diameter and six feet high seems to be about the largest that we can effectively handle.

Correlation. The model is correlated with the heat sink primarily in terms of the diameter ratio R_D (see Annular flow) and the ratio of the coolant fluid temperature drop to the log mean temperature difference $\Delta t_1/\Delta t_m$. Heat transfer from natural and forced convective flows are combined in the following correlation. These primary factors utilize basically the more common modelling parameters such as the Nusselt Number, Prandtl Number and the Reynolds Number. The analysis further considers that the time of melting can be proportioned such that a particular combination of $\Delta t_1/\Delta t_m$ and R_D can be achieved in the model at the same relative time as in the heat sink operated at its design conditions under real time.

Temperature ratio – annular flow.

Forced convection: Combining eq 3, 6 and 21 below

$$v = \frac{w}{\rho A} = \frac{w}{\rho \frac{\pi}{4} D_0^2 \left(\frac{R_0^2 - 1}{R_D^2}\right)} \tag{21}$$

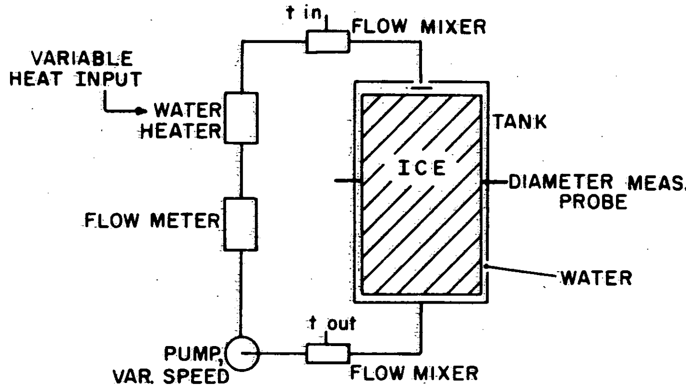


Figure 6. Schematic of ice heat sink melt system.

yields

$$\frac{\Delta t_1}{\Delta t_m} = \frac{(\pi L)(0.038)(K)\left[\frac{w}{(\pi/4) D_0 \mu}\left(\frac{R_D}{R_D^2 - 1}\right)\right]^{0.8}\left(\frac{C_p \mu}{K}\right)^{1/3}\left(\frac{\mu}{\mu_1}\right)^{0.14} R_D^{0.15}}{w\, C_p} \quad . \tag{22}$$

Let the subscript S denote heat sink and the subscript m denote model except that Δt_m always means log mean temperature difference in either case.

For the same R_D in the heat sink and in the model

$$\frac{\left(\frac{\Delta t_1}{\Delta t_m}\right)_S}{\left(\frac{\Delta t_1}{\Delta t_m}\right)_m} = \left(\frac{L_S}{L_m}\right)\left(\frac{K_S}{K_m}\right)\left(\frac{D_{0_m}}{D_{0_S}}\right)^{0.8}\left(\frac{\mu_m}{\mu_S}\right)^{0.8}\left(\frac{w_m}{w_S}\right)^{0.2}\left(\frac{C_{p_m}}{C_{p_S}}\right)\left[\frac{\left(\frac{C_p \mu}{K}\right)_S}{\left(\frac{C_p \mu}{K}\right)_m}\right]^{1/3}\left(\frac{\mu_S}{\mu_m}\right)^{0.14} \quad . \tag{23}$$

Note that μ_1 is the same for both the model and the heat sink.

Through a further combining of terms

$$\left[\frac{\left(\frac{\Delta t_1}{\Delta t_m}\right)_S}{\left(\frac{\Delta t_1}{\Delta t_m}\right)_m}\right]_{\text{Forced convection}} = \left(\frac{L_S}{L_m}\right)\left(\frac{D_{0_m}}{D_{0_S}}\right)^{0.8}\left(\frac{w_m}{w_S}\right)^{0.2}\left[\frac{\left(\frac{C_p \mu}{K}\right)_m}{\left(\frac{C_p \mu}{K}\right)_S}\right]^{2/3}\left(\frac{\mu_S}{\mu_m}\right)^{0.34} \quad . \tag{24}$$

Natural convection: Assume $L^3\,\Delta t \geq 3.33$ in both the model and the heat sink i.e. $(N_{Gr} \cdot N_{Pr}) \geq 10^9$. Also let $\Delta t = \Delta t_m$ using eq 6 and 20

$$\left(\frac{\Delta t_1}{\Delta t_m}\right) = \frac{C_1\, K\, (a)^{1/3}\, (\Delta t_m)^{1/3}\, \pi \left(\frac{D_0}{R_D}\right) L}{w\, C_p} \quad . \tag{25}$$

For the same R_D

$$\left[\frac{\left(\frac{\Delta t_1}{\Delta t_m}\right)_S}{\left(\frac{\Delta t_1}{\Delta t_m}\right)_m}\right]_{\substack{\text{Natural}\\ \text{convection}}} = \left(\frac{K_S}{K_m}\right)\left(\frac{a_S}{a_m}\right)^{1/3}\left(\frac{\Delta t_{m_S}}{\Delta t_{m_m}}\right)^{1/3}\left(\frac{C_{p_m}}{C_{p_S}}\right)\left(\frac{w_m}{w_S}\right)\left(\frac{D_{0_S}}{D_{0_m}}\right)\left(\frac{L_S}{L_m}\right) . \tag{26}$$

Combined mode: Because both types of convection occur simultaneously at the ice-water interface

$$\left[\frac{\left(\frac{\Delta t_1}{\Delta t_m}\right)_S}{\left(\frac{\Delta t_1}{\Delta t_m}\right)_m}\right]_{\substack{\text{Forced}\\ \text{convection}}} = \left[\frac{\left(\frac{\Delta t_1}{\Delta t_m}\right)_S}{\left(\frac{\Delta t_1}{\Delta t_m}\right)_m}\right]_{\substack{\text{Natural}\\ \text{convection}}} . \tag{27}$$

Using eq 27, 26 and 24 and combining terms

$$\frac{[\quad]_{\text{Natural}}}{[\quad]_{\text{Forced}}} = 1 = \left(\frac{\Delta t_{m_S}}{\Delta t_{m_m}}\right)^{1/3}\left(\frac{w_m}{w_S}\right)^{0.8}\left(\frac{D_{0_S}}{D_{0_m}}\right)^{1.8}\left(\frac{a_S}{a_m}\right)^{1/3}\left(\frac{K_S}{K_m}\right)\left(\frac{C_{p_m}}{C_{p_S}}\right)\left[\frac{\left(\frac{C_p\,\mu}{K}\right)_S}{\left(\frac{C_p\,\mu}{K}\right)_m}\right]^{2/3}\left(\frac{\mu_m}{\mu_S}\right)^{0.34} \tag{28}$$

or

$$1 = \left(\frac{w_m}{w_S}\right)^{0.8}\left(\frac{D_{0_S}}{D_{0_m}}\right)^{1.8}\left(\frac{\Delta t_{m_S}}{\Delta t_{m_m}}\right)^{1/3}\left[\frac{(g\,\beta\,\rho^2\,C_p)_S}{(g\,\beta\,\rho^2\,C_p)_m}\right]^{1/3}\left(\frac{K_m}{K_S}\right)^{1/3}\left[\frac{\left(\frac{C_p\,\mu}{K}\right)_m}{\left(\frac{C_p\,\mu}{K}\right)_S}\right]^{1/3}\left(\frac{\mu_S}{\mu_m}\right)^{0.33} \tag{29}$$

Dimensional scale

For conditions in which the coolant water temperatures are about the same for the model as for the heat sink at a particular R_D the water property ratios are near or equal to 1.0. Also the properties that vary the most with temperature, i.e. μ and N_{Pr} , are raised to a small fractional power which reduces the effect of their variations. Equation 29 becomes

$$\therefore \left(\frac{w_m}{w_S}\right)^{0.8} = \left(\frac{D_{0_m}}{D_{0_S}}\right)^{1.8}$$

or

$$\frac{w_m}{w_s} = \left(\frac{D_{0_m}}{D_{0_S}}\right)^{1.8/0.8} = \left(\frac{D_{0_m}}{D_{0_S}}\right)^{2.25} . \qquad (30)$$

For a model diameter of four feet

$$\left(\frac{D_{0_m}}{D_{0_S}}\right) = \left(\frac{4}{65}\right) \quad \text{and} \quad \left(\frac{D_{0_m}}{D_{0_S}}\right)^{2.25} = \left(\frac{1}{530}\right) .$$

For a heat sink flow of 2667 gpm, the model equivalent would be (2667/530) = 5.04 gpm. For 1000 gpm the equivalent would be 1.89 gpm.

Time basis

The time needed to achieve a particular R_D in both the model and the heat sink is calculated below. As in Method I it is based on a constant heat input and the addition of the latent heat of fusion only to ice. It ignores the fact of an increasing heat load and the fact that heat is absorbed by the water.

The latent heat of fusion residing in a cylindrical heat sink is

$$Q = \frac{\pi}{4} d^2 L \rho_i H . \qquad (11)$$

By differentiation

$$dQ = \left(\frac{\pi}{4} L \rho_i H\right) 2D \, dD .$$

For q = constant

$$dQ = q \, d\theta$$

or

$$2D \, dD = \left(\frac{q}{\frac{\pi}{4} L \rho_i H}\right) d\theta . \qquad (31)$$

Integrating eq 31 from $D = D_0$ to D and $\theta = 0$ to θ we find

$$D^2 - D_0^2 = \left[\frac{q}{\frac{\pi}{4} L \rho_i H}\right] \theta . \qquad (32)$$

Substituting $D = (D_0/R_D)$ into the expression gives

$$-\left[\frac{R_D^2 - 1}{R_D^2}\right] = \left[\frac{q}{D_0^2 \frac{\pi}{4} L \rho_i H}\right] \theta . \qquad (33)$$

For the same R_D plus combining of terms

$$\frac{\left[\frac{R_D^2 - 1}{R_D^2}\right]_S}{\left[\frac{R_D^2 - 1}{R_D^2}\right]_m} = 1 = \left(\frac{D_{0_m}}{D_{0_S}}\right)^2 \left(\frac{L_m}{L_S}\right)\left(\frac{q_S}{q_m}\right)\left(\frac{\theta_S}{\theta_m}\right) . \tag{34}$$

Complete scale factor

For the 4 ft diam × 6 ft long model

$$1 = \left(\frac{4}{65}\right)^2 \left(\frac{6}{115}\right)\left(\frac{q_S}{q_m}\right)\left(\frac{\theta_S}{\theta_m}\right) = \frac{1}{5.07(10)^3}\left(\frac{q_S}{q_m}\right)\left(\frac{\theta_S}{\theta_m}\right)$$

or

$$q_m = \frac{q_S}{5.07(10)^3}\left(\frac{\theta_S}{\theta_m}\right). \tag{35}$$

Assuming $q_S = 5(10)^6$ Btu hr^{-1} and some time ratios the following heat loads and Δt_1's have been calculated for the model.

	q_m (kw)	Δt_1 1.89 gpm (°F)	Δt_1 5.04 gpm (°F)
1 (real time)	.289	1.1	.4
10:1	2.89	10.6	4.0
20:1	5.78	21.1	7.9

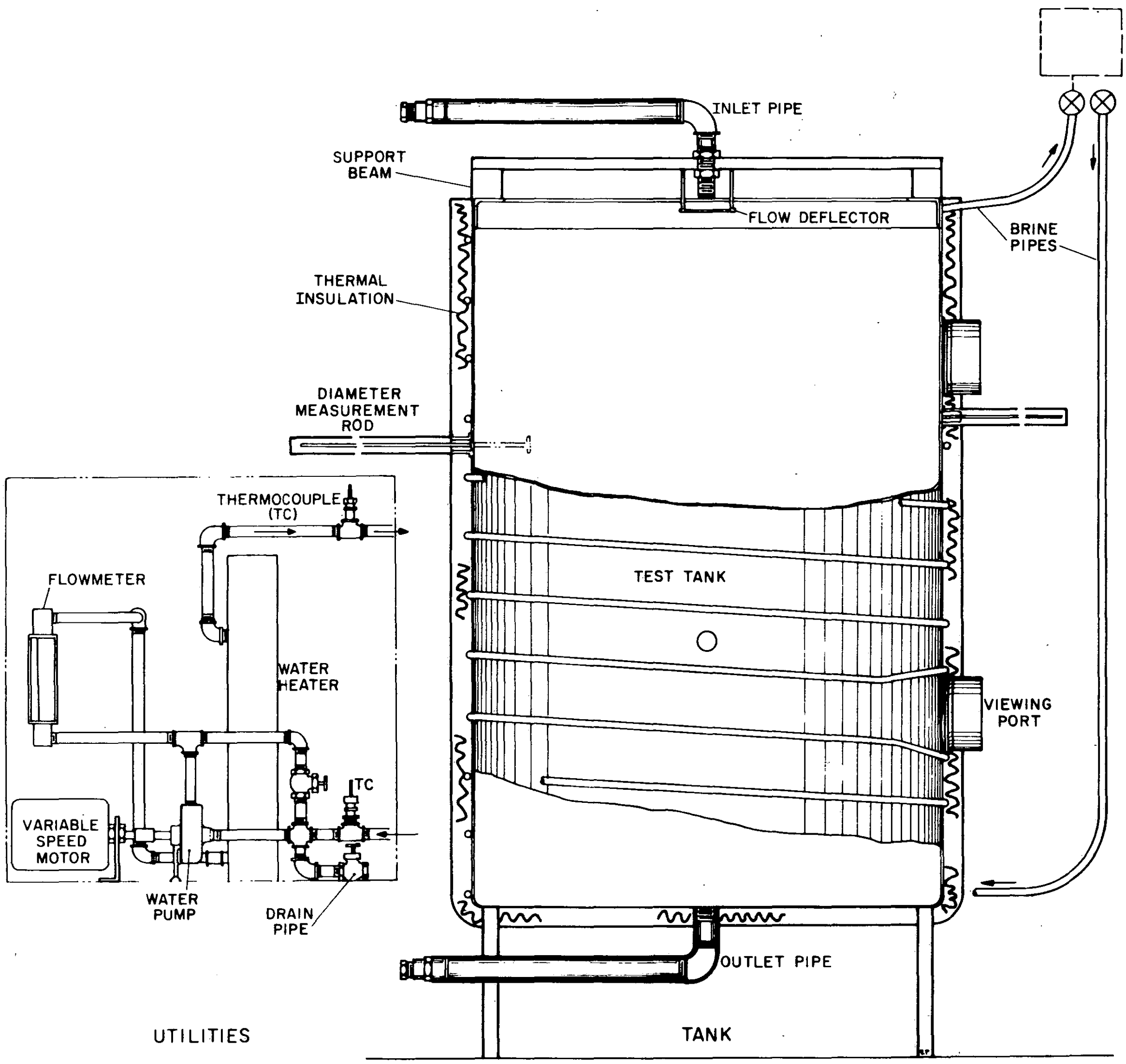
Design features

In general the operation of the model (Fig. 7) should simulate the operation of the heat sink in that its environment is room temperature. The flow and measurement apparatus can be separable from the model ice tank. Water can be frozen in the tank in a cold room and the tank can be wheeled out for testing. An alternative would be to run the coolant pipes through the cold room walls and enclose the sink during the test. The model sink walls and base need warming prior to starting the test. Means must be incorporated to keep the tank from bursting during freezeup.

The inlet and outlet temperatures of the model should be measured under a well mixed condition (turbulent or mechanically imposed equivalent). Thermocouples or possibly thermometers can be used.

The coolant pump can be driven by a speed controlled motor to provide various yet constant flows.

An electric hot water heater can be used in conjunction with a variable transformer voltage control to provide the desired heat input to the coolant, i.e. simulate the coolant load. A watt meter can measure the power input.



MODEL HEAT SINK

Figure 7.

The flow can be measured by an area meter type instrument with visual readout. The test operator can make periodic adjustments of pump speed as required due to temperature changes to keep the fluid weight flow constant.

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13. ABSTRACT

The concept of using a model ice heat sink as a device for indicating the feasibility of large ice container heat sinks was investigated from a conceptual and parametric viewpoint. The main problem addressed was the assurance that heat could be transferred to the sink quickly enough to prevent overheating of the coolant water. To avoid the need for scaling the properties of water, normally used heat transfer relationships were applied to both the model and to the large heat sink. The resultant equations were then converted to ratio and reduced to combinations of terms. These included a geometrical ratio heat flow ratio, temperature difference ratio and a time ratio. The report also includes preliminary model design and test parameter information for time ratios up to 20:1.

14. Key Words

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