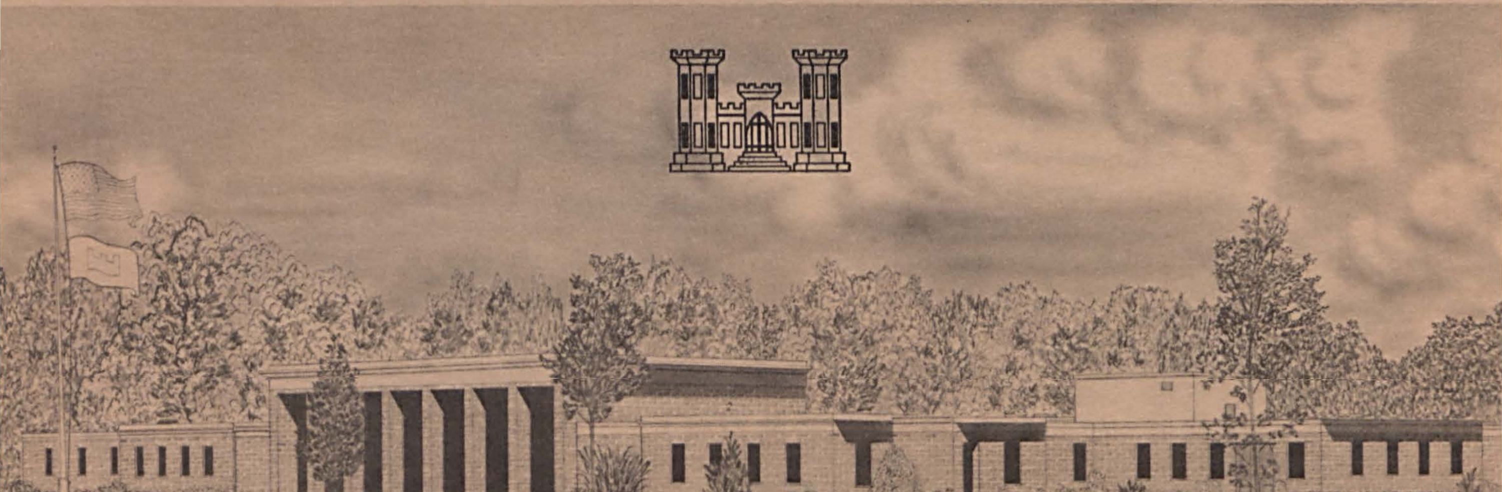
MISCELLANEOUS PAPER C-74-9

A LOW-MODULUS INTERNAL CONCRETE STRAIN METER

by

A. M. Alexander



June 1974

Sponsored by **Assistant Secretary of the Army (R&D), Department of the Army**

Conducted by **U. S. Army Engineer Waterways Experiment Station**
Concrete Laboratory
Vicksburg, Mississippi

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CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
microinches	0.0254	micrometres
mils	0.0254	millimetres
inches	2.54	centimetres
microinches per inch	1.000	millionths
cubic feet	0.02831685	cubic metres
pounds (mass)	0.4535924	kilograms
pounds (force) per square inch	6894.757	pascals
ohms (Ω int-US)	1.000495	ohms (Ω)
Fahrenheit degrees	5/9	Celsius degrees or Kelvins*

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* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: $C = (5/9)(F - 32)$. To obtain Kelvin (K) readings, use: $K = (5/9)(F - 32) + 273.15$.

SUMMARY

Previous work has indicated the need for a low-modulus internal strain meter to define better the behavior of fresh portland-cement concrete. Such a meter was designed and constructed, using the linear variable differential transformer (LVDT). It and two well-known commercially available strain meters were tested for comparison purposes.

The LVDT meter was found to be capable of measuring volume changes in fresh concrete as well as in hardened concrete. Currently available meters are designed to measure strains only in hardened concrete. After the concrete had attained final set, all three of the meters tested gave volume change results that were essentially comparable.

The LVDT meter is recommended to be used for short-term strain measurements in concrete. It is also recommended that further work be performed to fully prove out the meter in long-term use in concrete. It would be advantageous to construct a meter having a temperature correction equal to the linear coefficient of thermal expansion of concrete or even a zero temperature correction. It would also be advantageous to construct a meter with an axial length of 1 in. or less.

A LOW-MODULUS INTERNAL CONCRETE STRAIN METER

PART I: INTRODUCTION

Background

1. The Concrete Laboratory of the U. S. Army Engineer Waterways Experiment Station (WES) became interested in the development of a linear variable differential transformer (LVDT) strain meter in 1968. Difficulties had been encountered with all well-known internal strain meters used in an earlier investigation.*

2. The report on that investigation recommended the following: "...that a low-modulus transducer be developed to measure the expansion of concrete before it hardens and studies be conducted to better define the behavior of fresh concrete. All transducers now available for long-term internal measurements in concrete exhibit a modulus too high to measure the movement of concrete before it hardens. Due to the extremely low modulus and many excellent characteristics of the linear variable differential transformer (LVDT) as a displacement transducer, it is recommended that the possibility of using an LVDT for internal strain measurements be explored."

3. In the previous investigation, three well-known strain meters were employed. They were the Carlson strain meter, the vibrating wire gage, and SR-4 strain gages cemented to reinforcement bars. All of the meters were too stiff to measure early strains. The Carlson strain meter fails to measure these strains due to at least two reasons. The brass housing is too stiff for fresh concrete, and the gage wires are under 100,000-psi** tension, resulting in a high-modulus gage. The

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* A. M. Alexander and D. L. Ainsworth, "Strains Developed in Concrete During and Subsequent to Hardening," Miscellaneous Paper C-71-6, Jun 1971, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.

** A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page ix.

vibrating wire gage has a thin steel housing that must deform before a volume change in the fresh concrete is registered, and obviously reinforcement bars do not follow the concrete until the concrete is completely hardened. Fresh concrete will flow past the steel since very large forces are required to deflect a reinforcement bar. Even the latest internal concrete meter--the Microdot embedment meter--is very stiff due to its steel housing.

The ideal meter

4. Geymayer* commented on meters for use in concrete as follows: "When a meter is embedded in a material, it is obvious that it will have to occupy space normally occupied by the material and will, therefore, disturb the continuity of the material unless the meter and the displaced material have identical elastic or viscoelastic properties, coefficients of thermal expansion, and volume stability, and the meter is perfectly bonded to the surrounding material. For a material such as concrete, whose complicated viscoelastic properties change with time, complete matching of meter and displaced material is practically impossible. Hence, the designer's objective is to construct a meter so that the error introduced in the measurements by the imperfect matching of physical properties is as small as possible. In the case of internal strain measurement, therefore, the deformation of the meter and the deformation which the displaced concrete would have normally undergone should be as nearly equal as possible, or should bear a constant ratio to each other. Similarly, in the case of internal stress measurement the load acting on the meter and the load which would have normally been acting on the concrete displaced by the meter should either be equal or maintain a constant ratio to each other."

5. The ideal strain meter should be small enough to prevent a disturbance of the stress field that would have existed in the displaced concrete. It should be large enough to give a meaningful average

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* H. G. Geymayer, "Strain Meters and Stress Meters for Embedment in Models of Mass Concrete Structures," Technical Report No. 6-811, Mar 1967, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.

of the strains at a point. Although one may consider concrete homogeneous in some situations, it becomes a very heterogeneous substance to a gage that is 1 in. long when it contains aggregate up to 2 in. in diameter. Thus, the size of the aggregate dictates the length of the meter. On the other hand, a meter 1 in. long would be desirable for a small-aggregate or mortar mixture.

6. Loh* has expressed the opinion that the ideal meter would have a modulus of elasticity that is approximately zero. The linear coefficient of thermal expansion should equal that of the surrounding material. Otherwise, lateral stresses will result between the meter and material that are directly related to the difference between the thermal coefficients. Also, it is desirable to have equal Poisson's ratios. Lateral stresses are also produced by the difference in viscoelastic properties. The length-to-diameter ratio should be 5 or larger. The ideal meter should have sensitivity only in the direction of its length or have zero cross-sensitivity. It should require no temperature correction or be easily corrected for temperature changes. It should be capable of resolution of a few units of microstrain with a total range of several thousand. It should be reliable and function for a long time to allow evaluation of time-dependent mechanisms in the concrete, such as creep, etc. The internal gage mechanism should not be subject to creep with age. It should be easy to place and read, inexpensive, and available in various sizes.

Hostile environment of concrete

7. Measurements must be made within concrete in order that the actual stress and deformation within the structure or a model of the structure can be evaluated. It is necessary that the meter be constructed to be resistant to the hostile environment within concrete. One of the primary considerations is waterproofing. A meter must be completely waterproofed to prevent corrosion of the gage system within

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* Y. C. Loh, "Internal Stress Gauges for Cementitious Materials," Contract No. N5ori-07847 NR 064-331, Jul 1951, Massachusetts Institute of Technology, Department of Civil and Sanitary Engineering, Cambridge, Mass.

the casing or housing. The environment will be highly alkaline and may contain aggressive agents that tend to corrode the meter. Consideration must be given to the type of material of which the housing is constructed. The meter must be sufficiently rugged to withstand placement and consolidation of the concrete. Although mortar will tend to flow to relieve large stresses on the meter, large forces may be present in concrete due to varied sizes and shapes of aggregate. A satisfactory strain meter must be capable of withstanding these effects and yet exhibit a low modulus of elasticity in the direction of the sensitive axis.

8. Meters embedded in mass concrete structures should remain operable during and after exposure to a temperature range of approximately -20 F to 150 F. Conditions in modeling tests are usually less severe. However, prestressed-concrete reactor vessels may involve temperatures in excess of 150 F, which would also be reflected in models of these structures.

Characteristics of the sensing unit

9. The LVDT is used extensively in industrial application for the measurement of many physical quantities, such as temperature, humidity, vibration, pressure, flow, radiation, and strain. The LVDT is known as a high-precision transducer with a large signal output and highly stable a-c signal that is linear with displacement; it has a wide strain range, has a large length-to-diameter ratio, and is rugged in harsh environments such as high humidity and vibration. Effects due to temperature change are very small when excitation voltages of higher frequencies are employed in the signal conditioning equipment. Since none of the LVDT components are under stress, hysteresis does not exist in the transducer. Calibration is easily accomplished with a micrometer mounted on a small fixture. It has a wide operating range of temperature from -85 F to 450 F. The output voltage variation of the LVDT is stepless and has infinite resolution limited only by the associated equipment.

10. The principle of operation of the LVDT is as follows. A primary and two secondary coils are symmetrically spaced on a hollow cylindrical form (fig. 1). A small iron core, supported by a nonmagnetic rod, moves axially within the cylinder in response to the mechanical

input. When the core is in the center or "null" position, the a-c voltages induced in the secondary windings will be equal due to the symmetry of magnetic coupling to the primary. However, if the core is moved, one voltage will increase and the other will decrease in such a manner that the net difference will be proportional to core displacement (fig. 2). An exciter-demodulator provides excitation to the LVDT and conditions the output from the secondary to power a d-c recorder, oscilloscope, voltmeter, etc.

11. The LVDT has characteristics that approach those of the ideal meter. It has a modulus of elasticity near zero since the core that passes through the center of the small transformer is frictionless.

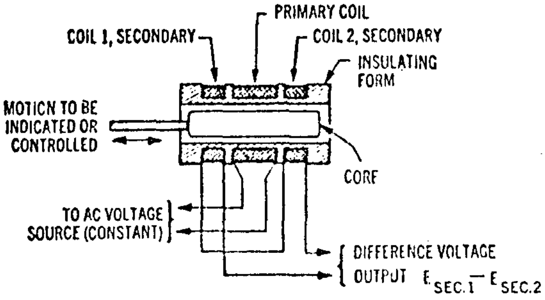


Fig. 1. Operation of the LVDT

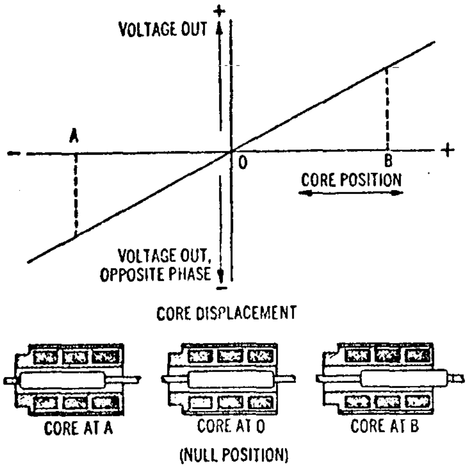


Fig. 2. LVDT output voltage and phase as function of core position

This characteristic is important when attempting to measure volume changes in fresh concrete. Although a sealed housing will necessarily add frictional forces to the meter, it is of utmost importance to develop a housing that will allow a meter to retain the low-friction characteristic.

12. As previously mentioned, a meter for model work should be small. This produces less disturbance of the stress field about the displaced concrete. The LVDT has the characteristic that will allow one to make a smaller meter than now exists. It should be possible to build one having a practical length of 1 in. or less. The LVDT used in these tests was capable of resolving a few microstrains and had a total range of 0.1 in. in either direction. The sensitivity was excellent at 0.1 $mv/\mu\epsilon$ and the output is compatible with most d-c recording instruments. The temperature correction was about 10 $\mu\epsilon/^{O}F$ for a Daytronic DS200 LVDT. The transformer could be constructed of manganin rather than copper, and would, therefore, require a smaller temperature correction. The only possible disadvantage known is that drift may be possible, not from the gage itself, but from the excitation unit. It is not intended to suggest that the excitation unit is unreliable, but to indicate only that the stability of the output is a function of the excitation unit as well as of the sensing unit.

Purpose and Scope

13. The purpose of this investigation was to develop a low-modulus internal concrete meter with a frictionless LVDT sensing unit having the capability of measuring dimensional changes in freshly mixed concrete as well as in hardened concrete. Before designing a housing, a literature search was made to find a sufficiently durable sensing unit that would withstand the rugged environment of concrete and at the same time possess the characteristics necessary for a precise and accurate measurement. Well-known meters available on the market would then be used as references in hardened concrete to verify the potential of the new meter. Although it was anticipated that all meters would

indicate identical dimensional changes in hardened concrete, the worth of the new meter would be gaged by its response in freshly mixed concrete.

PART II: DESCRIPTION OF STUDY

Construction and Operation of LVDT Strain Meter

14. The essential features of the low-modulus LVDT internal strain meter are shown in fig. 3. Only a short portion of the copper rod containing the electrical wires will be outside or external to the concrete. As the concrete is placed about the meter, a bond is established between the coupling plates and the concrete. Cloth tape, or a thin piece of some insulating material, is wrapped about the cylindrical tube, which prevents the concrete from bonding to the body metal. The two plates can move toward each other or apart from each other, depending on whether the material is decreasing or increasing in volume. Any displacement of one plate with respect to the other causes a corresponding displacement of the core rod relative to the transformer. The principle utilized in this meter is the LVDT.

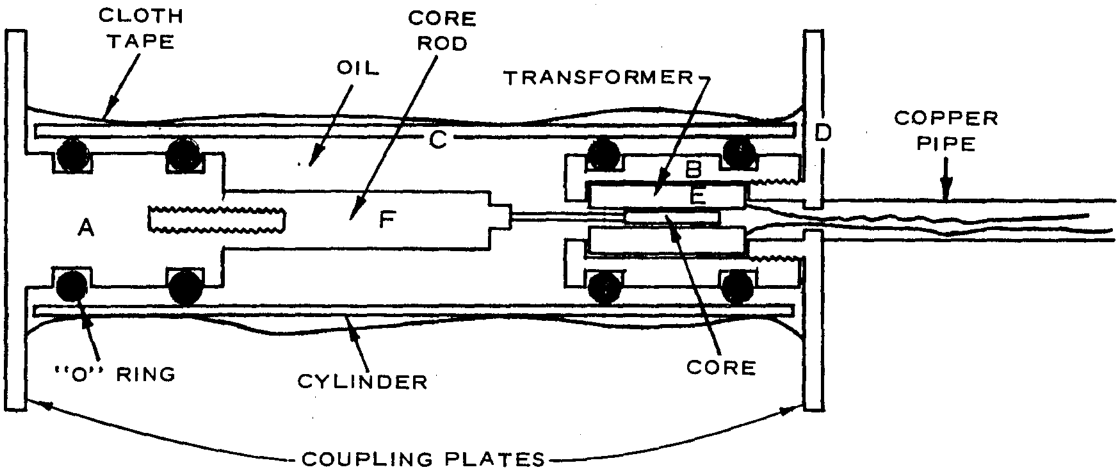
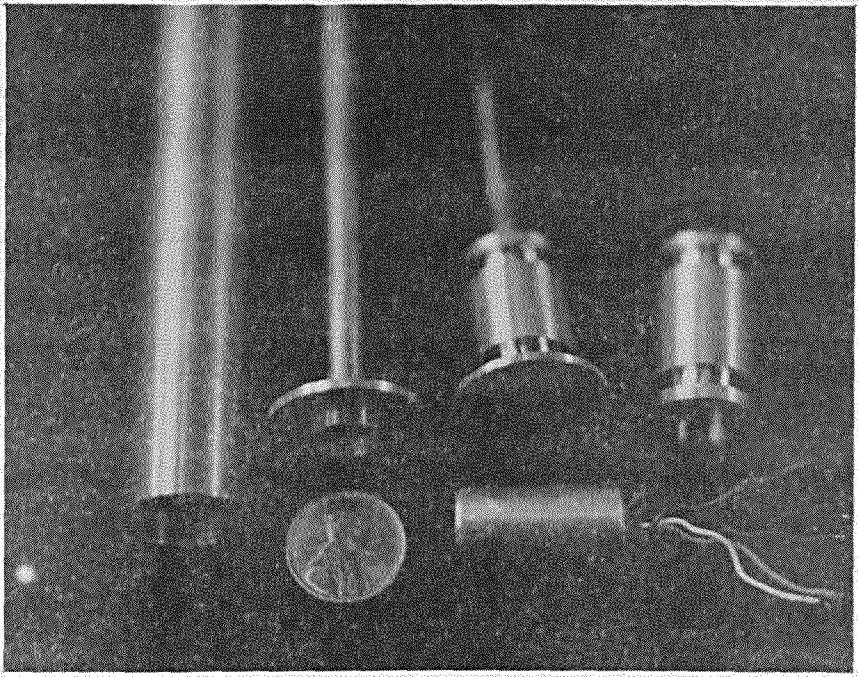
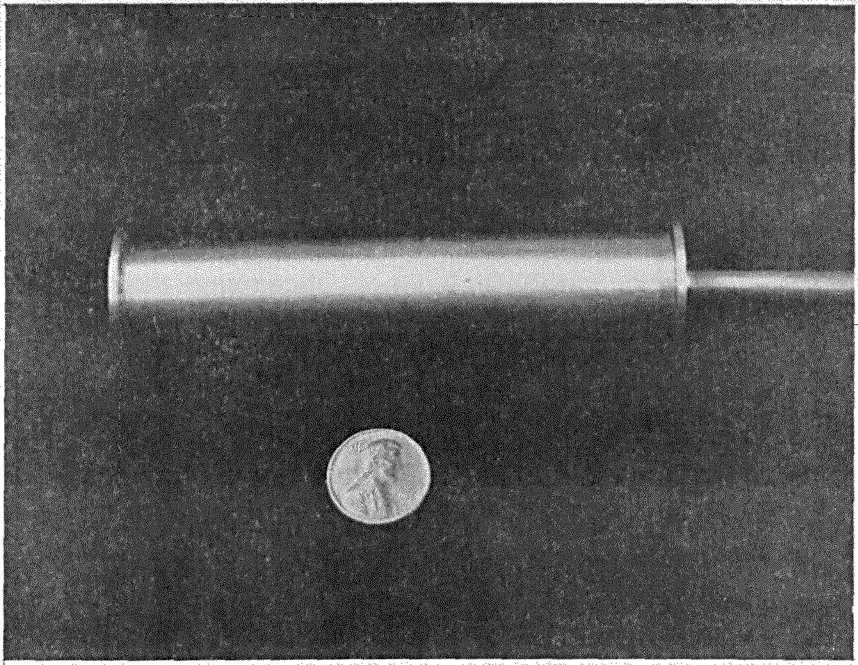


Fig. 3. Low-modulus LVDT strain meter

15. The space within the meter is filled with oil to prevent the accumulation of moisture. The "O" rings that allow the flanges to move have just enough roll and elastic movement to cover the entire range of deformation expected in concrete. Photographs of the meter are shown in fig. 4. The housing was made by the WES Machine Shops, and the dimensions of the components are shown in fig. 5.



a. Disassembled



b. Assembled

Fig. 4. The LVDT meter

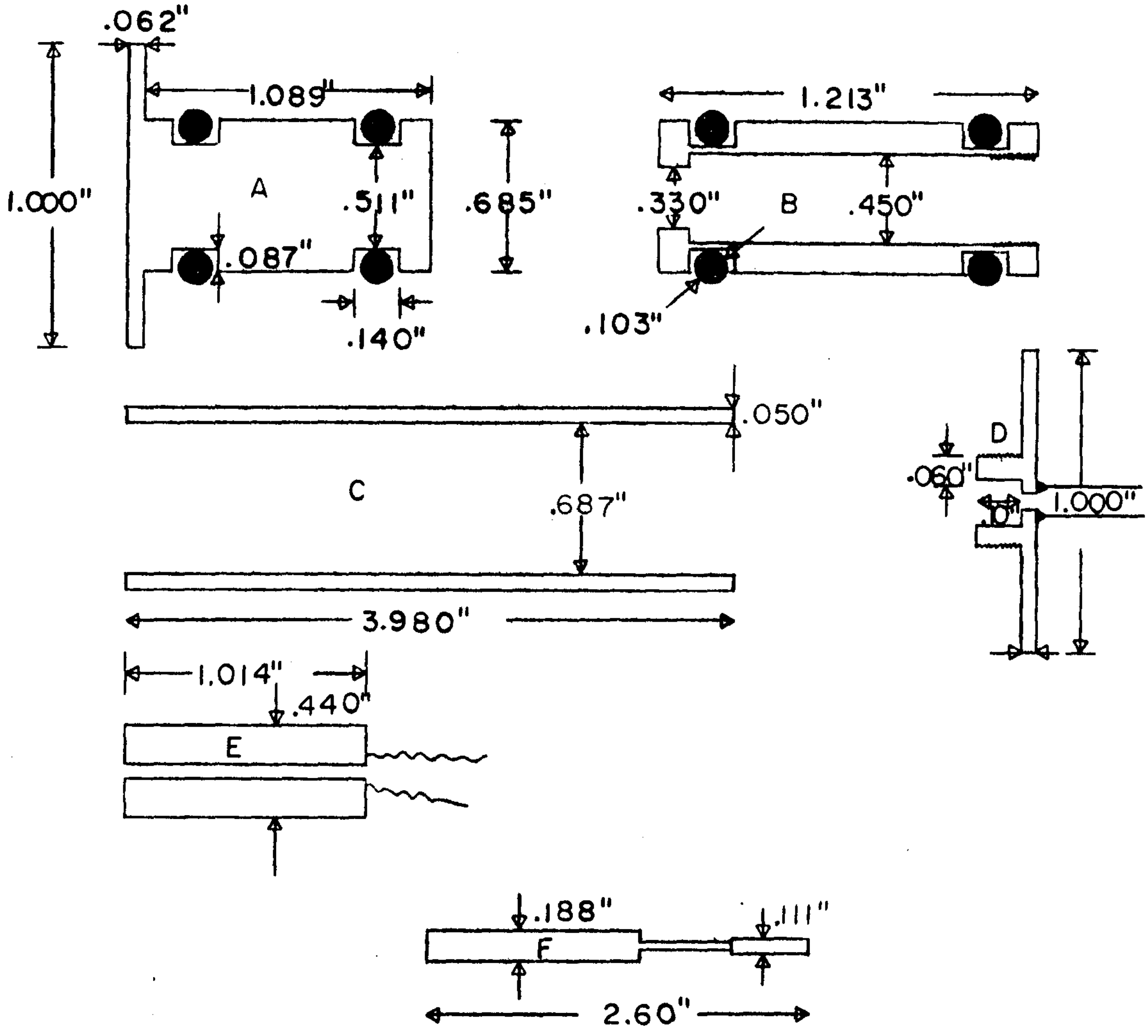


Fig. 5. Meter dimensions

Calibration

16. The calibration of the meter was accomplished by placing it in a fixture which also held a micrometer having a resolution of 0.0001 in. Calibrations were conducted at a constant temperature of 72 F. To determine the temperature correction of the meter, an invar frame was constructed to maintain the meter at a constant length while undergoing temperature changes. By immersing the meter in a water bath, it was possible to get a number of readings at equilibrated

temperatures. The water bath temperature was measured with a 2801A Hewlett-Packard quartz thermometer and recorded on a Model 562A Hewlett-Packard printer. The correction was found to be 9.2 $\mu\epsilon/^{\circ}F$ in a compressive direction; that is, for temperature increases, the output indicated an apparent compression.

Test Description

17. The LVDT meter was tested in a low-strength portland-cement grout. Previous work with this grout indicated that it would shrink for about 15 hr before initial set. The 28-day strength of this grout is about 1000 psi, permitting the meter to be retrieved after testing has been completed. The proportions of the mixture are given in the following tabulation.

Material	Solid Volume cu ft	Saturated Surface Dry Weight lb
Type II portland cement	0.044	8.65
Fly ash	0.039	6.00
ChemStress II	0.046	8.76
Barite	0.052	13.77
Bentonite gel	0.034	5.00
Sand, NTS concrete	0.372	60.00
Cement friction reducer-2	0.02	0.02
Water	0.413	25.70
Total	1.000	127.90

18. A waxed cardboard cylinder 20 in. in diameter was cut to a height of 24 in. and used as a form for the grout. Plaster of paris was placed in the base of the form to a depth of about 1/2 in. to seal the form from leakage. Cloth strings were placed horizontally across the form to support the LVDT meter and two other reference meters. A photograph of the setup is shown in fig. 6. Each of the three meters

was placed 9 in. below the surface of the grout and 120 degrees from each other.

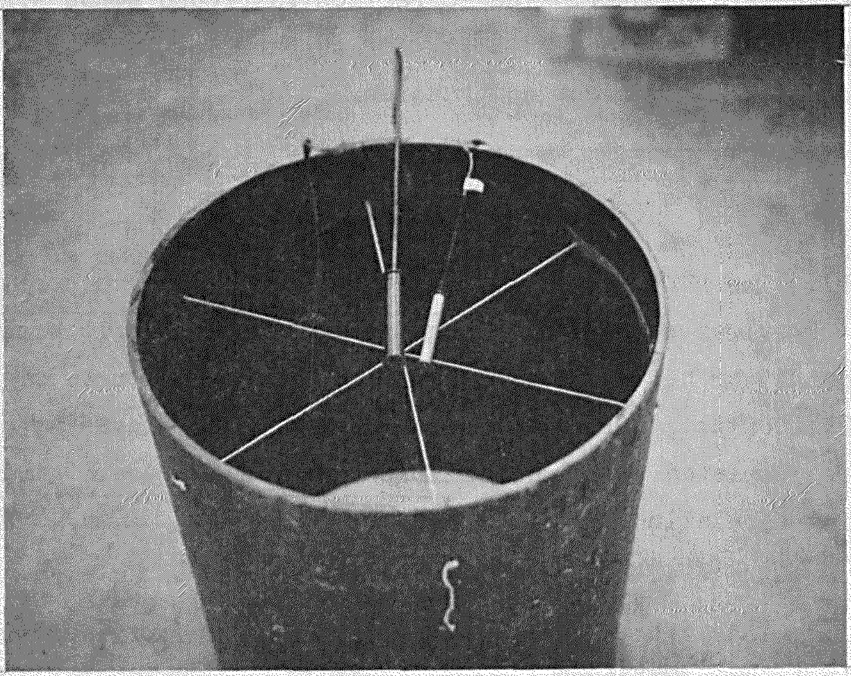


Fig. 6. Waxed cardboard form and meters

19. Strain readings from all three meters were recorded automatically. Instrumentation for the LVDT meter consisted of a 201C Daytronic exciter-demodulator and a 7100B Hewlett-Packard strip-chart recorder.

20. One of the reference meters was a 4-in. Carlson strain meter. A CEC 3-140 power supply powered the meter through a Model SG091 bridge completion network manufactured by Microdot, Inc. Recording was done on the above-mentioned strip-chart recorder. Normally it is required that a small test set be used for manually recording strains from a Carlson strain meter. Techniques to measure strains automatically from a Carlson meter have been developed. The development of the equations that describe the operation of the meter for automatic recording is given in Appendix A. All the details are given in the appendix for conversion from manual to automatic recording.

21. The other reference meter used in the grout was a Microdot

embedment meter. The instrumentation required for the Microdot gage was identical with that used for the Carlson meter: a power supply, a bridge completion network, and a strip-chart recorder. Calibration was accomplished by using the factory-supplied gage factor and shunting resistors across an active arm of the bridge. A diagram of the instrumentation connection is shown in fig. 7.

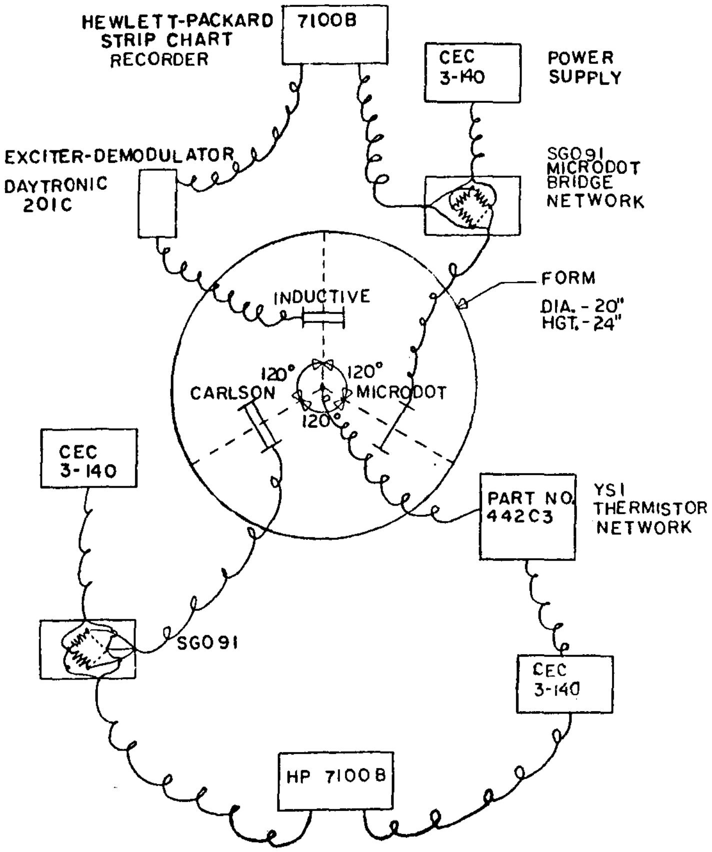


Fig. 7. Measurement system

22. Grout temperature measurements were made with a Yellow Springs thermistor, part No. 44203. Again a CEC 3-140 power supply was used, and measurements were recorded on a 7100B Hewlett-Packard strip-chart recorder. The output of the circuit was related to the temperature by this equation:

$$V_{out}\ [\text{mv}] = (3.77589 \times T_F + 228.10)\ V_{in}\ [\text{volts}]$$

PART III: TEST RESULTS

23. The test was terminated after 80 hr. The test mixture shrank for the first 12 hr until initial set, then expanded for 7 hr. No dimensional changes were noted between the ages of 20 and 60 hr. At the test age of 60 hr, the grout surface was flooded with about 3 in. of water, which produced an expansion during the last 20 hr.

24. The test results showed that the LVDT meter indicated more shrinkage than the other two meters while the grout was in the unhardened state. After the grout had hardened, all three meters performed essentially the same. The performance of each meter and a temperature curve are shown in figs. 8-12.

25. Figs. 8 and 9 show the performance of the LVDT meter plotted on two different scales. Figs. 8, 10, and 11 show the performances of the LVDT meter, the Microdot meter, and the Carlson meter, all plotted on the same scale. The LVDT meter indicated a shrinkage of 3500 $\mu\epsilon$, while the Microdot and Carlson meters indicated shrinkages of only 558 and 956 $\mu\epsilon$, respectively.

26. The above numbers do not take into account the apparent shrinkage due to the temperature rise of the grout during hydration. All three of the meters require a temperature correction in the same direction, but have different correction factors. The LVDT meter indicates an apparent compression of 9.2 $\mu\epsilon/^{\circ}$F as the temperature rises, while the Microdot and Carlson meters indicate apparent compressions of 6.0 and 6.7 $\mu\epsilon/^{\circ}$F, respectively. In fig. 12, it can be seen that the temperature rose 73°F during hydration. The temperature corrections are 675 $\mu\epsilon$ for the LVDT meter, 438 $\mu\epsilon$ for the Microdot meter, and 489 $\mu\epsilon$ for the Carlson meter, and the actual shrinkage measurements were, therefore, 2828 $\mu\epsilon$, 120 $\mu\epsilon$, and 467 $\mu\epsilon$, respectively. A five- to twenty-fold increase in readings was obtained with the LVDT meter as compared with the other meters in measuring dimensional change of the unhardened grout.

27. The addition of water to the hardened grout surface at the test age of 60 hr produced grout expansion and allowed the LVDT meter

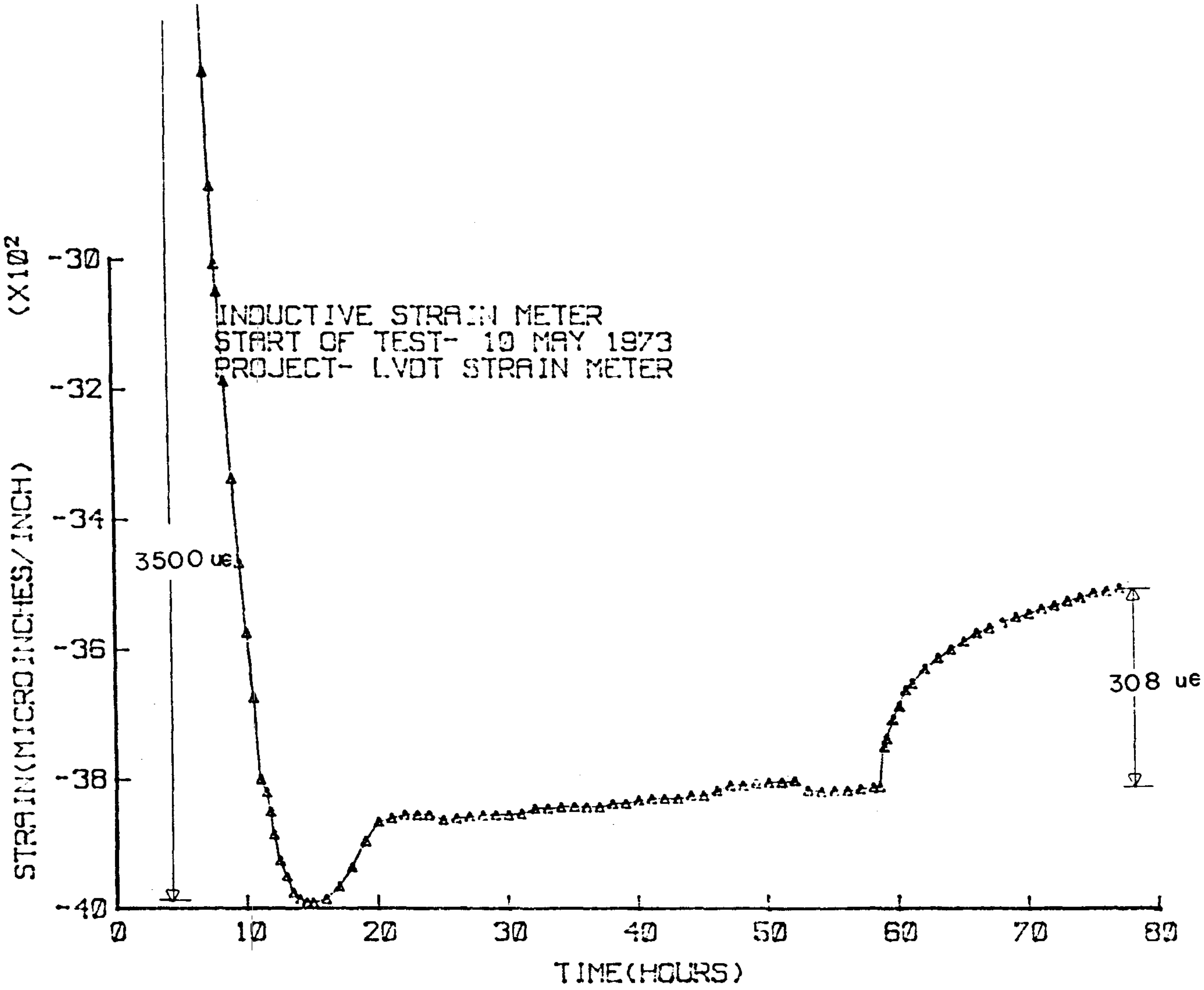

INDUCTIVE STRAIN METER
START OF TEST- 10 MAY 1973
PROJECT- LVDT STRAIN METER
STRAIN(MICROINCHES/INCH)
(X10^2)
-30
-32
-34
-36
-38
-40
3500 ue
308 ue
0
10
20
30
40
50
60
70
80
TIME(HOURS)

Fig. 8

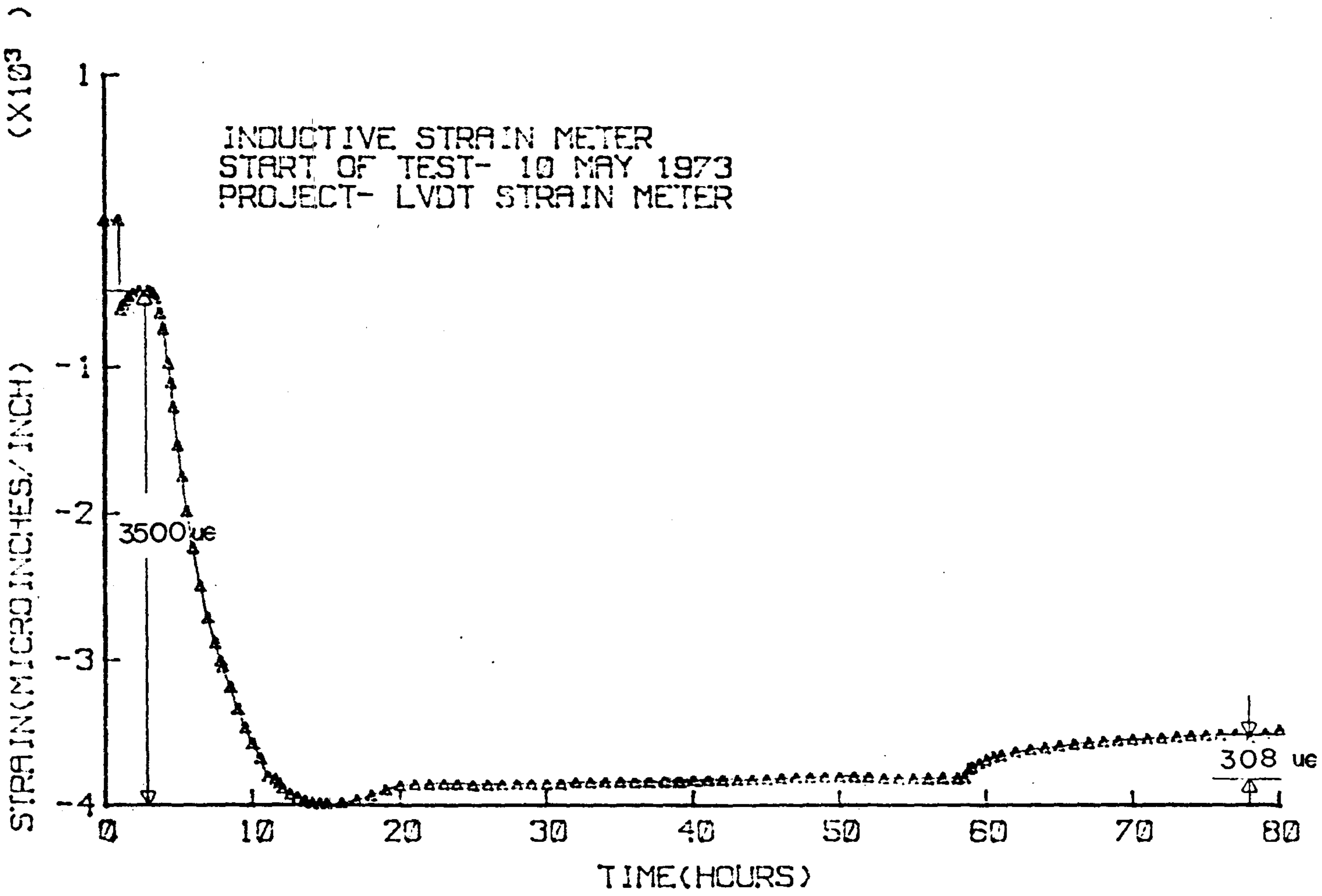

INDUCTIVE STRAIN METER
START OF TEST- 10 MAY 1973
PROJECT- LVDT STRAIN METER
STRAIN(MICROINCHES/INCH)
(X10^3)
1
-1
-2
-3
-4
3500 ue
308 ue
0
10
20
30
40
50
60
70
80
TIME(HOURS)

Fig. 9

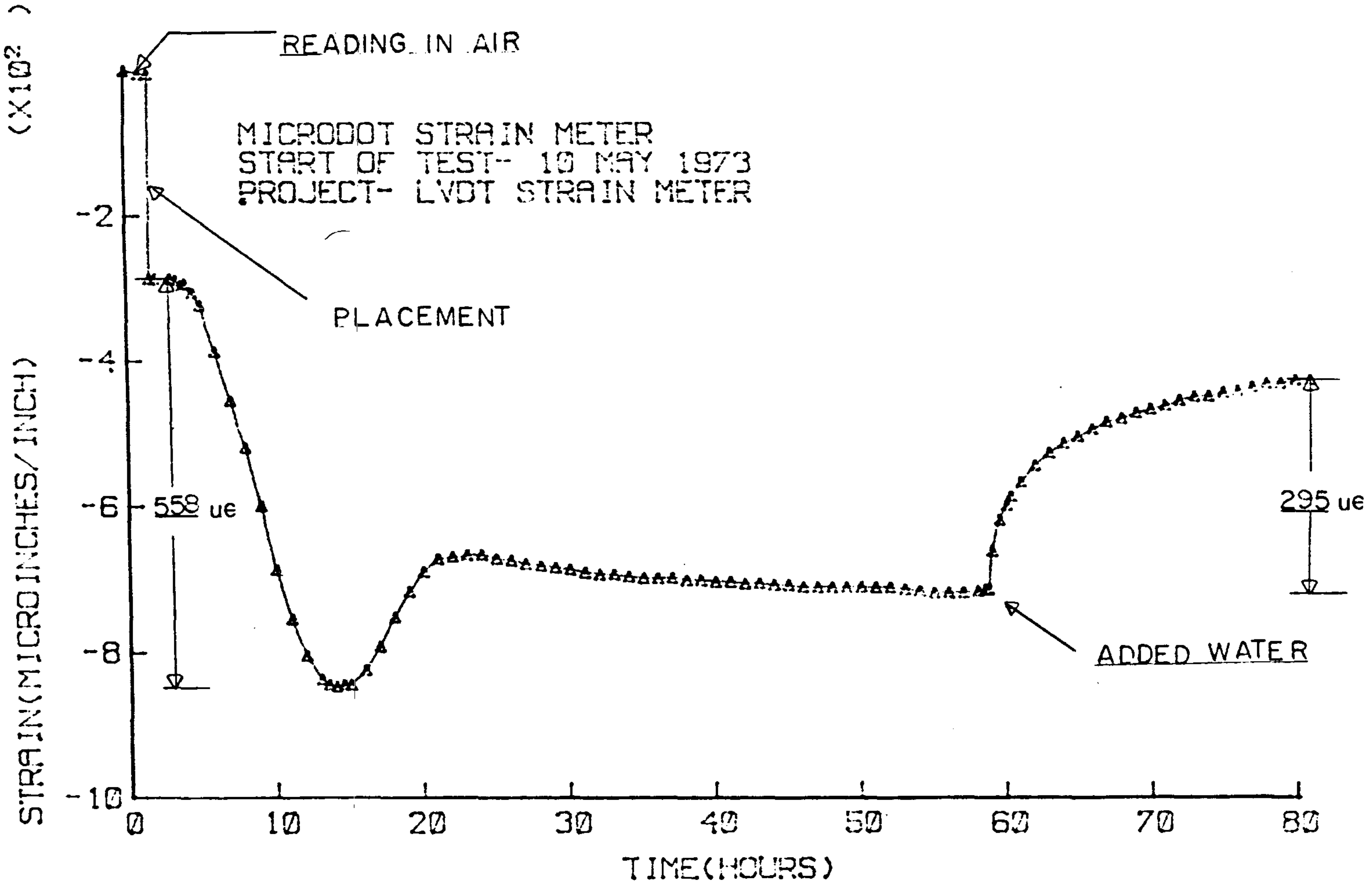

READING IN AIR
MICRODOT STRAIN METER
START OF TEST- 10 MAY 1973
PROJECT- LVDT STRAIN METER
PLACEMENT
558 ue
295 ue
ADDED WATER
(X10²)
STRAIN(MICROINCHES/INCH)
-2
-4
-6
-8
-10
0
10
20
30
40
50
60
70
80
TIME(HOURS)

Fig. 10

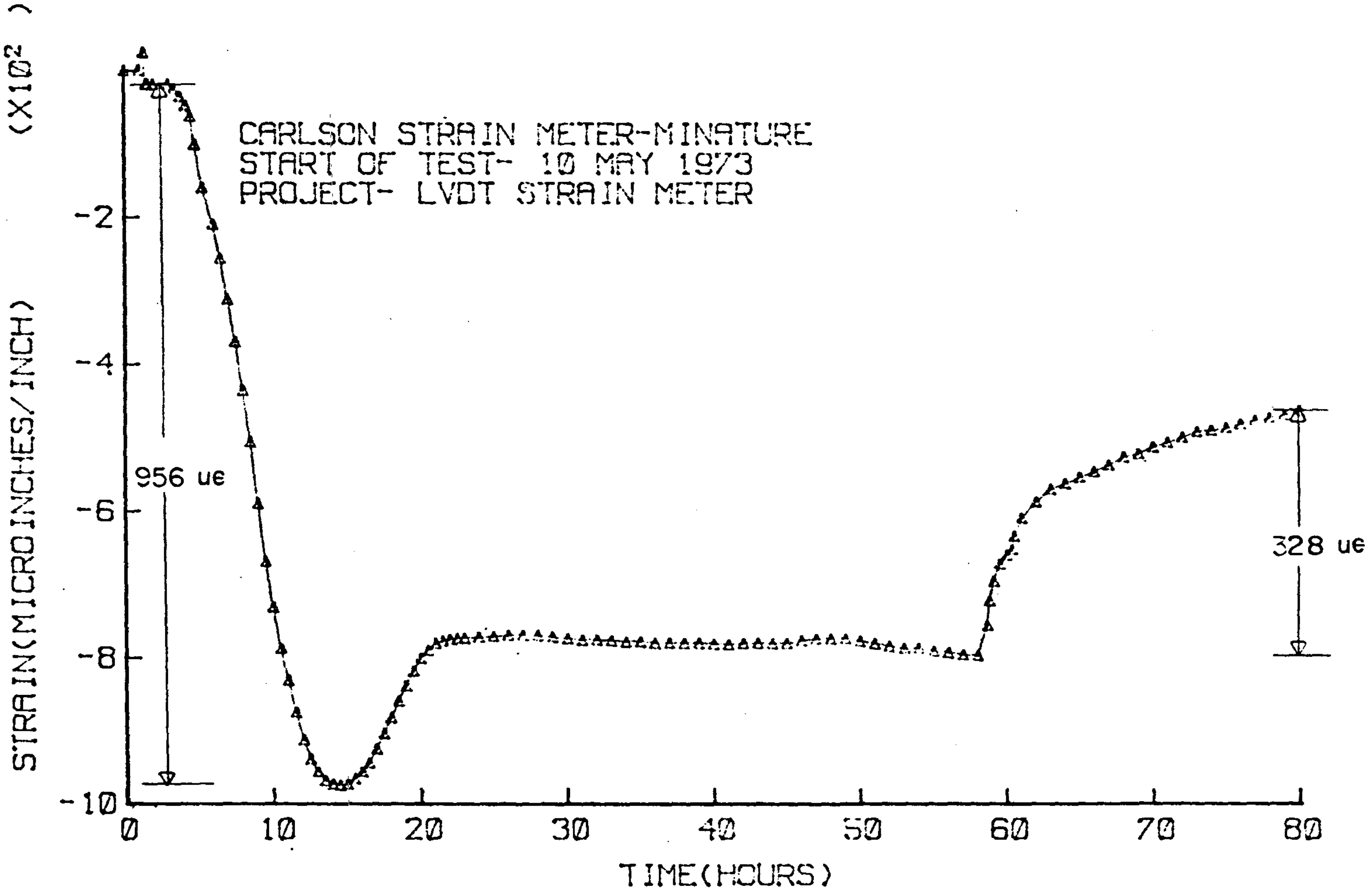

CARLSON STRAIN METER-MINATURE
START OF TEST- 10 MAY 1973
PROJECT- LVDT STRAIN METER
STRAIN(MICROINCHES/INCH)
(X10^2)
-2
-4
-6
-8
-10
0
10
20
30
40
50
60
70
80
TIME(HOURS)
956 ue
328 ue

Fig. 11

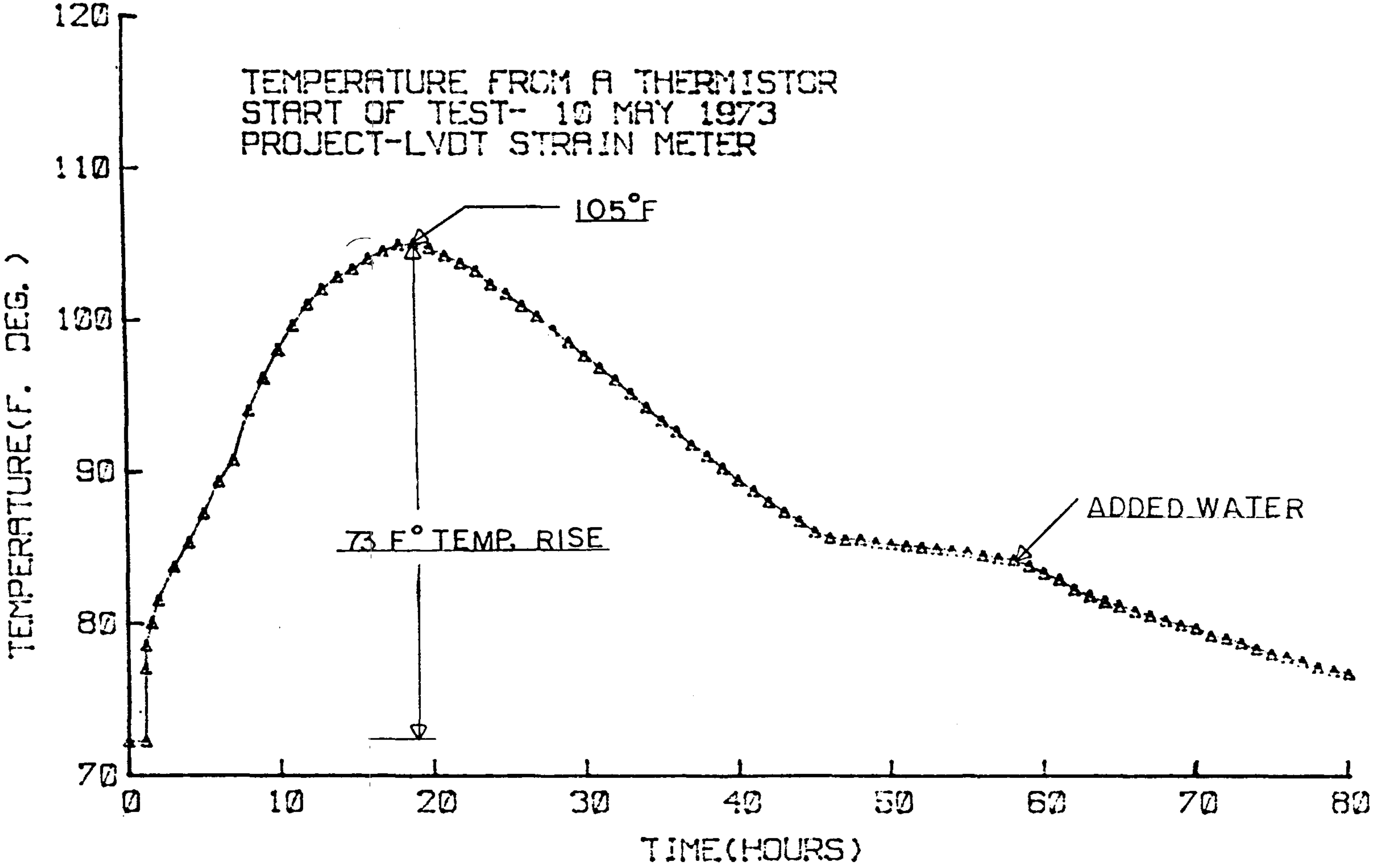

TEMPERATURE FROM A THERMISTOR
START OF TEST- 10 MAY 1973
PROJECT-LVDT STRAIN METER
105°F
73 F° TEMP. RISE
ADDED WATER
TEMPERATURE(F. DEG.)
120
110
100
90
80
70
0
10
20
30
40
50
60
70
80
TIME(HOURS)

Fig. 12

to be tested in hardened grout. This meter performed similarly to the other two meters. The indicated strains were 308, 295, and 328 $\mu\epsilon$ for the LVDT meter, the Microdot meter, and the Carlson meter, respectively.

28. In order to give a quantitative idea of the modulus of elasticity of the LVDT meter, a test was run to compare it with the Microdot meter and the Carlson meter. The meters were not embedded in grout for this test. Each meter was set in a fixture and loaded with a 500-g weight. The LVDT meter indicated a strain of 4985 $\mu\epsilon$, while the Microdot and Carlson meters indicated only 152 and 139 $\mu\epsilon$, respectively. The LVDT meter has a modulus of elasticity 30 times less than that of the other two meters.

PART IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

29. The results obtained in this study indicate that an LVDT strain meter capable of measuring dimension changes in unhardened grout can be constructed. The LVDT meter tested proved to have a very low modulus of elasticity and to be waterproof for the duration of the test. It is felt that there is sufficient evidence to warrant consideration of this meter for immediate short-term strain measurements in concrete.

30. The low-modulus LVDT strain meter has all the advantages of the other well-known meters with one additional distinct advantage which makes it unique. This meter will measure dimensional changes in grout or concrete while the material is still in the unhardened stage. The construction of the meter allows the "O" rings to roll before they start sliding. The construction allowed the meter to be compressed or expanded about 40 mils in either direction. Any movement past 40 mils would cause the "O" rings to start sliding, increasing significantly the modulus of elasticity of the meter. Of course, a range of movement as large as ±40 mils is not needed in concrete or grout.

Recommendations

31. It is recommended that further work be performed with this meter to fully prove out its long-term use in concrete. In addition, it may be possible to construct an LVDT meter having a temperature correction equal to the average linear coefficient of thermal expansion of hardened concrete. It also may be possible to construct a meter having a zero temperature correction. It is, therefore, recommended that additional work be performed to produce such a meter. Finally, it is frequently advantageous to have a meter with an axial length of 1 in. or less. It is recommended that work be performed along these lines.

APPENDIX A: ADAPTATION OF THE CARLSON STRAIN METER FOR AUTOMATIC RECORDING OF STRAIN MEASUREMENTS

1. In order to adapt the Carlson strain meter for automatic recording, it is necessary to understand the physical construction of the meter, its principle of operation, and measurement techniques.

Physical Construction of the Meter*

2. The meter is in the general form of a long cylinder with anchors on the end to engage the surrounding concrete. Within the flexible brass cover tube, a steel framework supports porcelain spools around which are wound, under 100,000-psi tension, two equal coils of very fine steel music wire, 0.0025 in. in diameter.

Principle of Operation

3. The instrument is designed to take advantage of two electrical properties of steel wire: resistance of the wire varies directly with temperature and with the tension on the wire. When the ends of the strain meter are pulled apart by an expansion in concrete, the outer or expansion coil elongates and increases in tension and, consequently, in resistance as well. At the same time, the inner or contraction coil decreases in resistance as it shortens. The ratio of the resistance of the expansion coil to the resistance of the contraction coil, which at all times is very near unity, is used as a sensitive measure of length change in the strain meter. A typical meter usually calibrates to about 4 millionths of an inch per inch per 0.01 percent change in ratio. Resistance ratio changes are not affected by simultaneous temperature changes of the wire since the temperature change affects both coils by an equal percentage.

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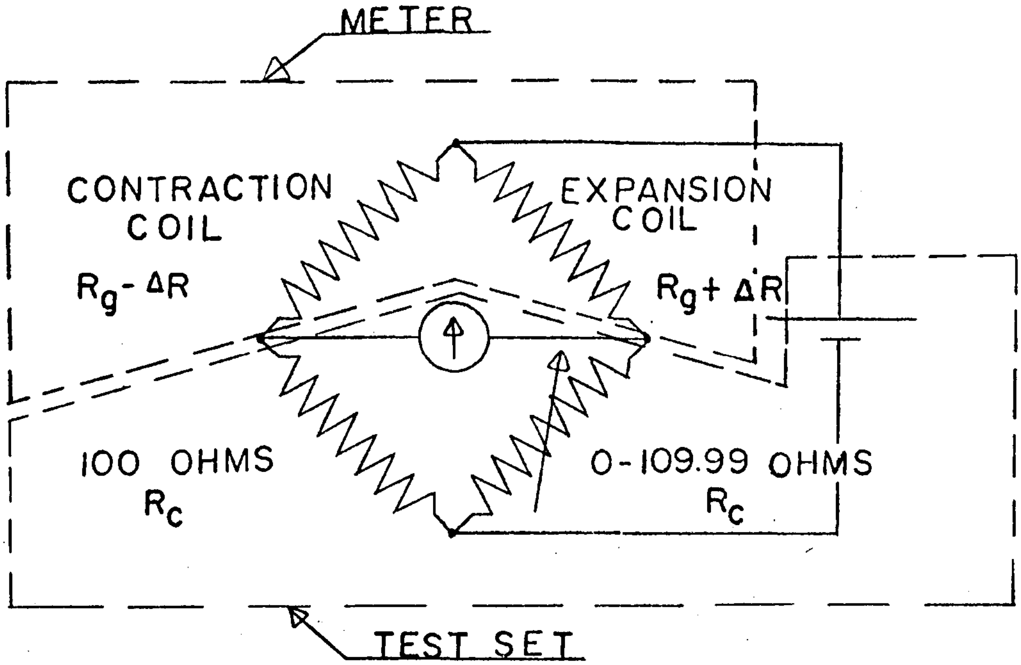
* J. M. Raphael and R. W. Carlson, "Measurement of Structural Action in Dams," 1954, p 5, James J. Gillich & Co., Berkeley, Calif.

4. Temperature is measured by taking the sum of the resistance of the expansion and contraction coils. This sum is not affected materially by variation in resistance due to length changes, as these plus and minus values very nearly cancel each other in the sum. The resistance of the miniature meter is about 60 ohms at 72 F, and increases about 1 ohm for every 9°F rise in temperature. A correction of 6.7 millionths of an inch per inch for each 1°F change of temperature must be made for the expansion of the strain meter frame. Actual calibration data are provided for each instrument.

Measurement Technique--Wheatstone Bridge

5. Strains are determined in the Carlson strain meter by a measurement of the resistance change in the expansion and contraction coils. There is a direct relation between the resistance change and the strain of the coils of the meter. This strain is measured manually with a small test set that employs the Wheatstone bridge technique in a balanced configuration. Two of the four arms required to make up the bridge circuit are in the meter itself. The other two arms are in the test set. The two arms in the miniature meter are about 30 ohms each at room temperature. The two arms in the test set are about 100 ohms each. One of the arms is fixed at 100 ohms, and the other is variable from zero to 109.99 ohms. The diagram below illustrates the electrical connections that are made when making a strain measurement.

6. Each coil in a new meter will be approximately equal in resistance. It can be assumed without error that each arm is equal in resistance and that any subsequent strain after placement in concrete will cause the expansion arm to increase by the amount of ΔR and simultaneously produce a decrease of ΔR in the contraction arm. Later calculations will show the miniature meter to have a gage factor of about 6. This means that the percentage change in resistance ($\Delta R/R \times 100$) is six times the percentage change in the length of the meter ($\Delta L/L \times 100$). In contrast, a bonded SR-4 strain gage has a factor of about 2.



Definition of a Least Reading

7. A least reading on the Carlson test set represents the minimum strain that can be resolved. It is defined in terms of the resistance of the adjustable arm. The smallest change that can be made is 0.01 ohm. This will be shown later to represent a change in ratio of the contraction and expansion coils of 0.0001, which represents approximately a 1.5 mΩ change (ΔR) in each coil. For the SM-4 meter, this corresponds to 8.18 microstrains, the smallest unit of strain that can be resolved. For a larger length meter, a least reading corresponds to about 4 microstrains.

Factory Calibration Constants

8. Two calibration constants are given on the factory calibration sheet. One is the relationship of strain to the least reading and the other the relationship of the resistance change with temperature. Typical calibration constants for the SM-4 meter are 8.18 microstrains/

least reading and 12.27°F/ohm. Also, the resistance of the meter is given on the calibration sheet for a temperature of 0 F. It is then possible to calculate the temperature for any known resistance or vice versa.

Calculation of the Meter Gage Factor

9. To adapt the Carlson meter to an automatic recording system, it is necessary that the gage factor be known. The definition of the gage factor is the ratio of the percentage change in resistance to the percentage change in the length of the meter [G.F. = $(\Delta R/R)/(\Delta L/L)$]. Assuming a least reading of 8.18 μin./in. ($\Delta L/L = 8.18 \times 10^6$ in./in.) and that the resistance of the meter at 0 F is 54.03 ohms, and the calibration factor for the temperature is 12.27°F/ohm, then the resistance at room temperature can be calculated as follows:

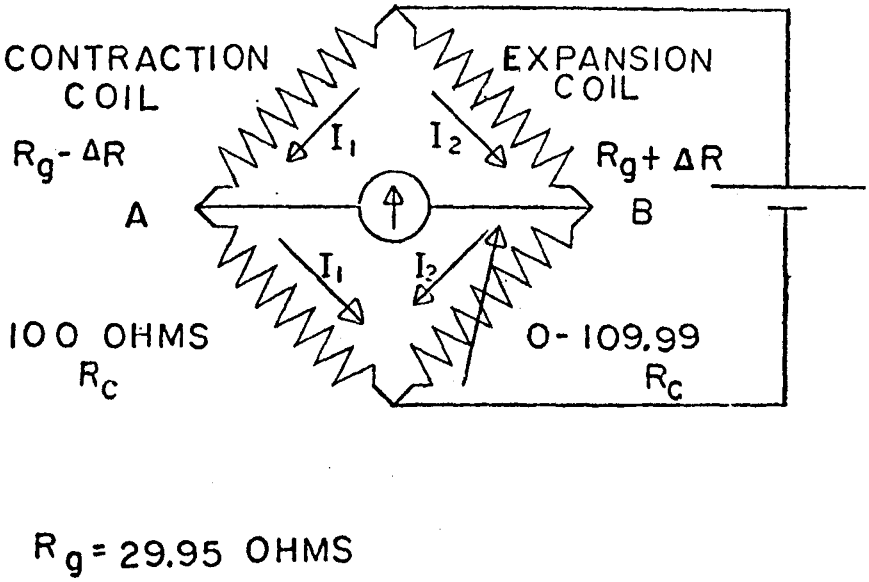
$$Rg = Rg_o + \frac{T}{\alpha} \quad \text{where} \quad Rg_o = 54.03 \quad \text{and} \quad \alpha = 12.27 \qquad (A1)$$

$$Rg = 54.03 + \frac{72.0}{12.27}$$

$$Rg = 59.90 \text{ ohms}$$

10. Since 59.90 ohms represents the resistance of both the expansion and the contraction coil, this resistance must be halved to obtain the resistance of each coil. Except for slight differences in manufacturing of the meter, one can assume that $Rg = 29.95$ ohms in the rest or unplaced position. The only parameter lacking on the right side of the gage factor equation is ΔR. Assume that the flanges of the meter are displaced by 8.18 microstrains. According to the calibration sheet, this represents a least reading and represents a 0.01-ohm change in the adjustable completion resistor in the test set. The following is the schematic diagram of a strain measurement.

11. As noted above, the two coils are assumed equal. It is, therefore, obvious that the adjustable resistor in the diagram would have to read 100.00 ohms to obtain a balance. The ratio of the expansion coil to the contraction coil is 1.0000. At balance, the



current through the meter is zero, which forces the current I_1 in the contraction coil to equal the current in the 100-ohm resistor. The same is true of the other two arms. The current I_2 in the expansion coil will be equal to the current in the adjustable completion resistor. Also since the meter has zero current, the voltage from A to B is zero. This means the voltage drop across the expansion coil will be equal to the voltage across the contraction coil or:

$$I_1(R - \Delta R) = I_2(R + \Delta R) \tag{A2}$$

The same is true for the other two arms.

$$I_1(100) = IR_x \tag{A3}$$

Dividing equation A3 by equation A2, we get:

$$\frac{100}{R - \Delta R} = \frac{R_x}{R + \Delta R}$$

Rearranging

$$R_x = 100\,\frac{R + \Delta R}{R - \Delta R} \tag{A4}$$

If the meter flanges are displaced 8.18 $\mu\epsilon$, a 0.01-ohm change in the adjustable arm is required to rebalance the bridge and the equation would be:

$$100.01 = 100\,\frac{R + \Delta R}{R - \Delta R}$$

$$100.01(R - \Delta R) = 100(R + \Delta R)$$

$$100.01R - 100.01\Delta R = 100R + 100\Delta R$$

$$-200.01\Delta R = -0.01R$$

$$\Delta R = \frac{0.01}{200.01}\,(29.95) = 0.001497\Omega$$

$$\Delta R = 1.497\text{m}\Omega$$

Substituting this into the gage factor equation:

$$\text{G.F.} = \frac{\Delta R/R_g}{\Delta L/L} \qquad \frac{\Delta L}{L} = \epsilon \tag{A5}$$

$$\text{G.F.} = \frac{0.001497/29.95}{8.18 \times 10^{-6}\ \text{in./in.}}$$

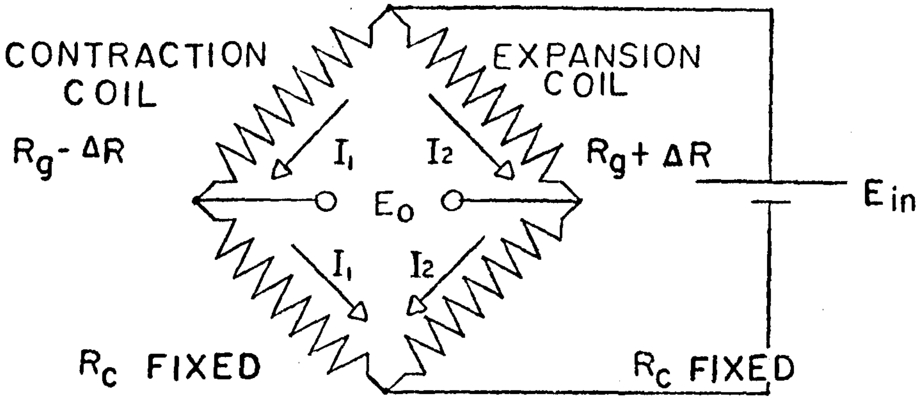
$$\text{G.F.} = 6.11$$

The gage factor is 12.22 effectively since there are two active arms. The gage factor will prove useful below, where it will be used in calculations to simulate a strain for calibration purposes.

Mathematics of Meter Output Voltage in an Unbalanced Circuit

12. It is a common practice to employ Wheatstone bridge circuits in an unbalanced configuration for automatic data recording. The following calculations will deal with the calculation of the output voltage expected from the meter in terms of the excitation voltage to the meter, the value of the completion resistors, and the simulated

strain appropriated to the meter. Consider the following unbalanced configuration for a bridge circuit:



13. Assume that the input impedance of the recorder is infinite. This means that the bridge will have no load on its output. Actually, a recorder with 100K ohms input impedance would be satisfactory with the following calculations in the no-load condition.

14. The difference in the voltage drops across the completion resistors is equal to the output voltage.

$$E_o = I_1R_c - I_2R_c$$

or

$$E_o = R_c(I_1 - I_2) \tag{A6}$$

But the currents can also be expressed in terms of the known resistances and known excitation voltage.

$$I_1 = \frac{E_{in}}{Rg - \Delta R + R_c} \qquad I_2 = \frac{E_{in}}{Rg + \Delta R + R_c}$$

Combining with equation A6,

$$E_o = R_cE_{in}\left(\frac{1}{Rg - \Delta R + R_c} - \frac{1}{Rg + \Delta R + R_c}\right) \tag{A7}$$

Rearranged, equation A5 is:

$$\Delta R = \epsilon G.F. \cdot Rg$$

Combining with equation A2:

$$E_o = R_c E_{in} \left(\frac{1}{Rg - \epsilon G.F.Rg + R_c} - \frac{1}{Rg + \epsilon G.F.Rg + R_c} \right) \qquad (A8)$$

Equation A8 gives the relationship between the output voltage and strain. For a typical circuit, the following constants may be assumed:

R_c = 120 ohms
Rg = 29.95 ohms
G.F. = 12.22
E_{in} = 3.0 volts

$$E_o = 360 \left(\frac{1}{149.95 - 182.99\epsilon} - \frac{1}{149.95 + 182.99\epsilon} \right)$$

As an example, 500 $\mu\epsilon$ would produce a 5.86-mv output. It is, therefore, possible to record strain on a 10-mv multipoint recorder without any signal conditioning equipment other than a stable power supply, a balance network, and the direct-current recorder. The input impedance of the recorder could be as low as 100K ohms without any loss of accuracy.

Simulation of Strain

15. To check calculations or voltage measurements or to have an alternative form of calibration, it is advantageous to use shunt resistors across an active arm. Although the meter has two active arms, only one arm can be shunted in the calibration procedure. A shunt resistance always produces a lower resistance than the arm shunted, which is satisfactory for the contraction coil but unsatisfactory for the expansion coil. A correction by a factor of two in the following

equation is required to make up the output lacking from a single shunt. First the ratio of the change in resistance to the gage resistors is derived. The total resistance (R_T) obtained from shunting the gage resistance (Rg) by the shunt resistance (Rs) is:

$$R_T = \frac{RgRs}{Rg + Rs} \tag{A9}$$

The change in resistance is:

$$\Delta R = Rg - R_T \tag{A10}$$

Combining equations A9 and A10 and solving for Rs:

$$Rs = Rg \left(\frac{Rg}{\Delta R} - 1 \right) \tag{A11}$$

From the gage factor equation:

$$G.F. = \frac{\Delta R/Rg}{\epsilon}$$

and rearranging:

$$\frac{\Delta R}{Rg} = (G.F.)\epsilon \tag{A12}$$

Combining equations A11 and A12:

$$Rs = Rg \left(\frac{1}{G.F.\epsilon} - 1 \right) \tag{A13}$$

Equation A13 gives the relation between simulated strain and the shunt resistance. Because there are two active arms and only one arm is being shunted, a correction is made to the equation and it becomes:

$$Rs = Rg \left(\frac{1}{2G.F.\epsilon} - 1 \right) \tag{A14}$$

Corrections for Lead Resistance

16. For measurements remote from the recording system, it is necessary to adjust the gage factor to correct for lead resistance. The resistance of the lead wires will be in the arms themselves rather than outside the bridge circuit. First, it is important to note that a given strain (ϵ) will always produce the same change in resistance (ΔR), with or without the effects of lead wire resistance. A reduction in output is produced because the bridge is sensitive to the percentage change in resistance rather than the change in resistance. In other words, the difference in output is a function of $\Delta R/(Rg + rl)$ rather than ΔR. Whereas it took a change of one least reading or 0.01 ohm in the variable completion resistor on the Carlson test set to offset a strain of 8.18 $\mu\epsilon$ (factory calibration constant), it will require a different value of resistance to offset the same strain when there is lead resistance. The equation for the balanced configuration is as follows:

$$\frac{Rx}{100} = \frac{Rg + \Delta R + rl}{Rg - \Delta R + rl}$$

For a resistance of a single lead wire of 2.085 ohms, a gage resistance of 29.95 ohms, and a change of resistance equal to that in the earlier calculations (0.001497 ohm), Rx can be calculated:

$$Rx \doteq 100 \left(\frac{29.95 + 0.001497 + 2.085}{29.95 - 0.001497 + 2.085} \right) = 100.0088 \text{ ohms}$$

The change in Rx is 0.0088 ohm, which is somewhat less than a least reading of 0.01 ohm. Therefore, if 8.18 $\mu\epsilon$ corresponds to 0.0088 ohm, the new calibration constant will be:

$$\frac{8.18}{0.0088} \cdot 0.01 = 8.74 \ \mu\epsilon/\text{least reading}$$

The new gage factor is as follows:

$$G.F. = \frac{\Delta R/(Rg + rl)}{\epsilon}$$

$$G.F. = \frac{0.001497/(29.95 + 2.085)}{8.18 \times 10^{-6}} = 5.71$$

In the output voltage equation or the shunt resistor equation, the gage factor would need to be replaced with the corrected value and the gage resistance would be the gage resistance plus the resistance of the lead wire in that arm.

Apparatus

18. Very little additional equipment is needed to adapt the Carlson strain meter to automatic recording. If thermocouples can be used to make the temperature measurement, then all that is required is a stable d-c power supply with a range from 0-3 volts and a rating of at least 0.1 amp. A bridge completion network with a balance adjustment such as the Microdot SG091 would be excellent. A multipoint recorder with an input range of 0-10 mv and an input impedance of at least 100,000 ohms will suffice for recording purposes. This would allow either 12 or 24 meters to be recorded at the same time. Many other combinations of equipment would work as the system can be very flexible.

Computer Program

19. A program that computes all the parameters needed based on the factory calibration sheet that arrives with the meter is given below. The equal signs on the program are the numbers fed to the computer and the other numbers are calculated and printed by the computer. Some of the equations in the program are of a different form than in the discussion above, but yield the same answers. Possibly the logic is more straightforward in the discussion and more easily

understood. The program does not contain the equations that allow one to calculate the temperature from the Carlson meter. These equations are simple to derive.

RSHUNT

```
110C     AUTOMATIC DATA RECORDING FROM A CARLSON STRAIN METER
120C     ----------------------------------------------------
130C    GIVEN THE ORIGINAL CALIBRATION CONSTANT OF A STRAIN
140C    METER, THE METER RESISTANCE AT O DEG. F.,AND THE RESISTANCE
150C    OF A PAIR OF LEADS; THE NEW CALIB. CONST. IS CALCULATED.
160C    THIS IS IN UNITS OF UIN./IN./LEAST READING. THIS CONSTANT
170C    IS THEN CONVERTED TO UIN./IN./MILIOHM WHERE THE CHANGE
180C     IN RESISTANCE CORRESPONDS TO THE EXPANSION OR
190C    CONTRACTION GAGE RATHER THAN THE DECADE CONTROL ON THE TEST
200C    BOX. THEN THE TEMP. COEFF. OF THE STRAIN METER IS INPUTTED
210C    AND THE RESISTANCE OF THE GAGE IS CALCULATED FOR 72.0
220C    F. DEG. WHICH IS NORMAL LAB TEMP. FOR CALIBRATION. THE
230C    GAGE FACTOR OF THE METER IS THEN CALCULATED. IT WILL NORMALLY
240C    BE ABOUT 6 TIMES THAT OF AN SR-4 STRAIN GAGE SINCE THE FRAME
250C    OF THE METER IS LONG AND HENCE MULTIPLIES THE GAGE FACTOR
260C    THEN A SIMULATED STRAIN IS INPUTTED AND THE CORRESPONDING SHUNT
270C    RESISTANCE  REQUIRED IS CALCULATED. THE CORRESPONDING
280C    CHANGE IN RESISTANCE OF THAT ARM IS ALSO CALCULATED SINCE
290C    THE STRAIN IS DIRECTLY PROPORTIONAL TO THIS CHANGE  IN
300C    RESISTANCE. THEN KNOWING THE EXCITATION VOLTAGE AND THE
305C    VALUE OF THE COMPLETION RESISTORS THE OUTPUT VOLTAGE CAN BE
310C    CALCULATED WHICH RESULTS FROM STRAIN OR SIMULATED STRAIN.
320C    -------------------------------------------------------------
330$NDM
340     PRINT,"ORIGINAL CALIB. CONST.(UIN./IN./LEAST READING)"
350     READ,CC1
360     PRINT,"METER RESISTANCE AT 0 DEG. F.(OHMS)"
370     READ,RZERO
380     PRINT,"RESISTANCE OF LEAD PAIR(OHMS)"
390     READ,RLEAD
400     CC2=CC1+RLEAD*CC1*0.89/RZERO
410     PRINT,"TEMP. COEFF. OF METER(F. DEG./OHM)"
420     READ,ALPHA
430     RMETER=RZERO+72.0/ALPHA
440C    -----------------------------------------------------
450     PRINT,"RESISTANCE OF METER AT 72.0 F. DEG."
460     PRINT,RMETER
470     PRINT,
480     RGAGE=RMETER/2.0
490     PRINT,"GAGE RESISTANCE(OHMS)"
500     ~~PRINT,RGAGE~~
501     PRINT,
510     PRINT,"NEW CALIB. CONST.(UIN./IN./L.R.)"
520     PRINT,CC2
521     PRINT,
530     CC4=(.01*RGAGE/200.01)*1000.
540     PRINT,"NEW CALIB. CONST.(MILIOHMS/L.R.)"
550     PRINT,CC4
551     PRINT,
560     CC3=CC2/CC4
```

RSHUNT CONTINUED

```
570       PRINT,"NEW CALIB. CONST.(UIN./IN./MILIOHM)"
580       PRINT,CC3
581       PRINT,
590       CC3=CC3/1000.
600C      WELL KNOWN RELATIONSHIP:GAGE FACTOR=DELTA RGAGE/RGAGE/STRAIN
610C      BUT CARLSON'S   CC3=STRAIN/DELTA RGAGE
620C      -------------------------------------------------------
630       GF=1.0/(RGAGE*CC3)
640       PRINT,"GAGE FACTOR(OHM/OHM/IN./IN.)"
650       PRINT,GF
651       PRINT,
660       PRINT,"SIMULATED STRAIN(UIN./IN.)"
670       READ,STRAN1
680C      -------------------------------------------------------
690C      CONVERT FROM UIN./IN. TO IN./IN.
700C      -------------------------------------------------------
710       STRAN2=STRAN1/1000000.0
720C      -------------------------------------------------------
730C      ALSO WELL KNOWN:
740C      -------------------------------------------------------
750       RSHUNT=RGAGE*(1.0/(GF*2.0*STRAN2)-1.0)
760       PRINT,"SHUNT RESISTANCE(OHMS)"
770       PRINT,RSHUNT
780       PRINT,
790C      NEXT CALCULATE THE VOLTAGE OUTPUT FOR THE PARTICULAR
800C      SHUNT.
810C      -------------------------------------------------------
820       DELTR=RGAGE↑2.0/(RGAGE+RSHUNT)
830       PRINT,"CHANGE IN GAGE RESISTANCE BY SHUNTING(MILIOHMS)"
840       DELTRR=DELTR*1000.
850       PRINT,DELTRR
851       PRINT,
860       PRINT,"EXCITATION VOLTAGE(VOLTS)"
870       READ,EIN
880       PRINT,"COMPLETION RESISTOR VALUE(OHMS)"
890       READ,R
900       A=2.0*R/(R+RGAGE-DELTR)
910       B=(R*(2.0*R+2.0*RGAGE))/((R+RGAGE)↑2.0-(DELTR)↑2.0)
920       EO=EIN*(A-B)
930       PRINT,"OUTPUT VOLTAGE FOR SHUNT(MILIVOLTS)"
940       EO=EO*1000.
950       PRINT,EO
960       END
```

```
*RUN
ORIGINAL CALIB. CONST.(UIN./IN./LEAST READING)
=3.71
METER RESISTANCE AT 0 DEG. F.(OHMS)
=63.52
RESISTANCE OF LEAD PAIR(OHMS)
=1.0
TEMP. COEFF. OF METER(F. DEG./OHM)
=8.13
RESISTANCE OF METER AT 72.0 F. DEG.
  0.72376083E 02

GAGE RESISTANCE(OHMS)
  0.36138044E 02

NEW CALIB. CONST.(UIN./IN./L.R.)
  0.37619821E 01

NEW CALIB. CONST.(MILIOHMS/L.R.)
  0.18093117E 01

NEW CALIB. CONST.(UIN./IN./MILIOHM)
  0.20792333E 01

GAGE FACTOR(OHM/OHM/IN./IN.)
  0.13290202E 02

SIMULATED STRAIN(UIN./IN.)
=1000
SHUNT RESISTANCE(OHMS)
  0.26867235E 04

CHANGE IN GAGE RESISTANCE BY SHUNTING(MILIOHMS)
  0.48094640E 03

EXCITATION VOLTAGE(VOLTS)
=5.0
COMPLETION RESISTOR VALUE(OHMS)
=120.0
OUTPUT VOLTAGE FOR SHUNT(MILIVOLTS)
  0.23658270E 02

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13. ABSTRACT

Previous work has indicated the need for a low-modulus internal strain meter to define better the behavior of fresh portland-cement concrete. Such a meter was designed and constructed, using the linear variable differential transformer (LVDT). It and two well-known commercially available strain meters were tested for comparison purposes. The LVDT meter was found to be capable of measuring volume changes in fresh concrete as well as in hardened concrete. Currently available meters are designed to measure strains only in hardened concrete. After the concrete had attained final set, all three of the meters tested gave volume change results that were essentially comparable. The LVDT meter is recommended to be used for short-term strain measurements in concrete. It is also recommended that further work be performed to fully prove out the meter in long-term use in concrete. It would be advantageous to construct a meter having a temperature correction equal to the linear coefficient of thermal expansion of concrete or even a zero temperature correction. It would also be advantageous to construct a meter with an axial length of 1 in. or less.

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