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Sentinel Satellite Positional Precision Derived From the NAVSTAR Global Positioning System

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Error estimates of position are presented for a variety of symmetric circular satellite constellations when four or five observations are made on NAVSTAR GPS satellites. Results are calculated in terms of minimum PDOP and expected outages.

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PREFACE

This study was performed under DA Project 4A762707A855, "Topographic Mapping Technology."

The research was conducted during the summer and autumn of 1988 under the supervision of Mr. Donald R. Barnes, Chief, Space Concepts Division; and Dr. Joseph J. Del Vecchio, Director, Space Programs Laboratory.

Col. David F. Maune, EN, was Commander and Director, and Mr. Walter E. Boge was Technical Director of the U.S. Army Engineer Topographic Laboratories during the report preparation.

SENTINEL SATELLITE POSITIONAL PRECISION DERIVED FROM THE NAVSTAR GLOBAL POSITIONING SYSTEM

INTRODUCTION

In this report, the NAVSTAR Global Positioning System (GPS) is evaluated for its ability to position platforms of a constellation of hypothetical sentinel system satellites. This study is part of a continuing effort by the U.S. Army Corps of Engineers (USACE) and the Engineer Topographic Laboratories (ETL) to analyze target location and to evaluate feasibility of a near autonomous real-time attitude and positioning system. A portion of the real-time attitude capability was addressed in a previous ETL study.[1] That work was in support of an ETL-initiated, USACE-approved NASA shuttle experiment intended to validate the concept of precision real-time attitude keyed to a digital image stellar camera operating in space.

The subject sentinel system constellations are a subset of those described in the referenced study. Generally, the subset used in this study included constellations of 12 satellites or less, constellations whose satellite heights were less than GPS satellite heights, and finally those constellations that performed well in the referenced study. The constellations are characterized by symmetric circular orbits, wherein every satellite within a specific constellation has a common inclination and a common elevation above the earth. The constellations fall into four groups, depending on whether they are designed to provide single, double, triple, or quadruple ground coverage.

Three GPS configurations were evaluated. The first was the basic 18 circular symmetric satellite configuration organized around 6 regularly spaced planes with 3 satellites per plane, regularly spaced in central angle. The orbital planes are inclined 55 degrees to the equator and the semi-major axis of each satellite is 26,561.144 kilometers. The second configuration was composed of the basic configuration plus three in-plane spares. The third configuration was composed of the basic configuration plus three equatorial spares.

The purpose of this work is to determine minimum Position Dilution of Precision (PDOP) for sentinel system satellites when four or five GPS observations are made. Minimum PDOP was determined by evaluating all possible combinations of four or five GPS satellites out of the total number of GPS satellites that had line of sight (LOS) with the subject vehicle at a given time. Another purpose of this work is to determine GPS outages. Outages occur whenever less than four or five GPS satellites are in view or whenever PDOP > 6.

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[1] M.A. Crombie. *Ground Target Location Errors Derived From Measurements Collected From a Variety of Hypothetical Satellite Sentinel Systems.* U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546. Report ETL-0538, June 1989, AD-A209 523.

EXPERIMENT

Real-time exterior orientation is a requirement of the hypothetical sentinel target tracking system. The attitude component will be derived from a star camera gyro system.[2] The position component will be derived from knowledge of the target tracker's orbit which will be updated from observations on GPS satellites.

All of the computer runs were performed on the Zenith 248-PC desktop computer. Generally, a sentinel system constellation was defined at input, and the system was allowed to operate for the equivalent of 2 days with observations made on one of the three GPS configurations every 15 minutes. A determination was made of N, the number of LOS between each sentinel system satellite and the GPS satellites. Minimum PDOP was determined by evaluating all possible combinations of four or five of the N GPS satellites. The results were written on file for future processing. Outages were also recorded.

Sentinel Systems. Sentinel system constellations are characterized by symmetric circular orbits, wherein every satellite within a specific constellation has a common inclination and a common elevation above the earth.[3] A specific constellation is defined by six parameters. A set of three integers (T/P/F) is used to define the total number of satellites in the constellation (T), the number of equally spaced orbital planes (P), and the relative phasing parameter (F). There are T/P satellites per plane, all equally spaced in central angle. The relative-phasing parameter F is used to relate satellites in one orbital plane to those in another plane. For example, if there is a satellite at its ascending node in one orbital plane, then the argument of latitude of a satellite in an adjacent plane will be F* 360/T degrees.

Three more parameters are used to define the constellation. The first is *i*, the inclination of the orbital plane. The second is ϵ, the elevation angle of the sentinel system viewing cone. The angle ϵ is the smallest viewing angle a ray from the satellite makes with a plane tangent to the earth at the point of intersection. The viewing angle was set to 5 degrees to avoid ground clutter. The third parameter is Θ, the half angle at the center of the earth subtended by the satellite viewing cone. The satellite height above the earth *h* is determined from the following equation:

$$\cos(\Theta + \epsilon) = \cos \epsilon / (1 + h/R_E)$$

where R_E is the earth radius. The satellite orbits were determined by using the following values for R_E and GM, the product of the constant of gravitation and the mass of the earth:

$$R_E = 6378144.0 \text{ meters}$$
$$GM = 3.986018 \times 10^{14} \text{ meters}^3/\text{seconds}^2$$

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2 Thomas E. Strikwerda and John L. Junkins. *Star Pattern Recognition and Spacecraft Attitude Determination.* U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546, Report ETL-0260, May 1981, AD-A103 806.

3 T.J. Lang. "Symmetric Circular Orbit Satellite Constellations For Continuous Global Coverage," *AAS/AIAA Astrodynamics Specialist Conference*. Kalispell, Montana, August 1987.

Nineteen sentinel systems, evaluated previously, were evaluated in this study.[4] The satellite heights varied from 2,243.74 nm to 10,737.77 nm. Each of the selected constellations met or nearly met coverage requirements in that study, and no constellation had more than 12 satellites. The primary reason for limiting the number of satellites, especially those close to the earth, was the large number of observation sets that must be tested for PDOP. For example, if a particular sentinel satellite had 10 GPS satellites in view, then $C_{10,4} = 210$, plus $C_{10,5} = 252$, combinations must be tested for PDOP.

Global Positioning System. The basic GPS constellation is defined by six parameters where the first three are identical to those described above.[5] In this case, T = 18, P = 6, and F = 2. The other three parameters are inclination (i), eccentricity (e), and semi-major axis (a). Here i = 55 degrees, e = 0.0, and a = 26,561,144.0 meters.

The three in-plane spares are specified in the following way. The first spare is in the first orbital plane with argument of latitude equal to 40 degrees, and the first of the three basic satellites in that plane is at the ascending node. The second spare is in the fifth orbital plane with argument of latitude equal to -40 degrees. The third spare is in the third orbital plane with an argument of latitude equal to 160 degrees.

The three equatorial spares are spaced 120 degrees apart at zero degrees inclination. If the first basic GPS satellite in the first orbital plane is at the ascending node and at zero degrees right ascension, the first equatorial spare is at -30 degrees right ascension, and the third spare is at 90 degrees right ascension.[6]

Line-of-Sight Definition. Consider figure 1 where LOS between a GPS satellite and a sentinel system satellite is defined. If a sentinel satellite (S_1) is in the interior cone and between the earth and the GPS satellite, then LOS is assured. If a sentinel satellite (S_2) is in the shadowed portion of the interior cone, then LOS is impossible. If a sentinel satellite (S_3 or S_4) is within the outer cone and outside of the interior cone, then LOS is assured. Finally, if a sentinel satellite (S_5) is outside of the outer cone, then LOS is impossible. The inner cone is that cone with the vertex at the GPS satellite, so that the conical surface is tangent to the surface of the earth. The outer cone is generated by rotating a line about the local vertical at the GPS satellite. The angle between the local vertical and the generating line is the GPS antenna half-beam-width. The half-beam width is 22 degrees for L_1 frequency and 27 degrees for L_2 frequency. The L_1 half-beam width was used in this study. The inner cone half angle is $\xi = \sin^{-1}(R_E/a) = 13.89425$ degrees, where *a* is the GPS semi-major axis.

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[4] M.A. Crombie. *Ground Target Location Errors Derived From Measurements Collected From A Variety of Hypothetical Satellite Sentinel Systems.* U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546. Report ETL-0538, June 1989, AD-A209 523.

[5] Ibid.

[6] Stein, B.E. and Col. E. Wheaton. "Equatorial Sparing of the NAVSTAR/GPS 18 Satellite Constellation," *IEEE Plans '86 Position Location and Navigation Symposium*, Record 86 CH23655-5, Caesar's Palace, Las Vegas, Nevada, November 4-7, 1986.

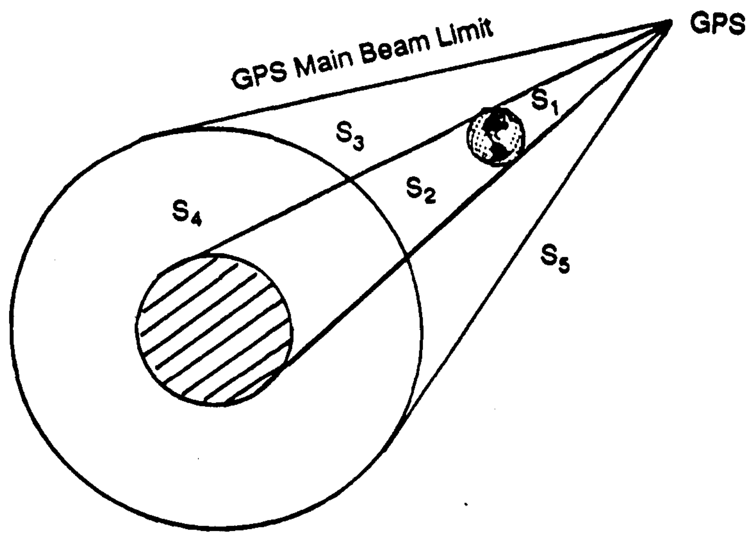

Figure 1. Line of Sight (LOS).

PDOP and Outages. PDOP is defined as the square root of the trace of the sample position covariance matrix derived by least squares adjustment. The basic observation equation is

$$PR_i = \bar{\gamma}_i^T \cdot (\bar{R}_T - \bar{R}_i) + \beta_T \qquad i = 1, M = 4 \text{ or } 5$$

P_{Ri}: i^{th} Pseudo Range

$\bar{R}_T$: Target Tracker Position

$\bar{R}_i$: i^{th} GPS Satellite Position

β_T: Time Bias

$\bar{\gamma}_i$: Direction Cosines of i^{th} GPS Satellite f.om Tar~et Tracker

The relative covariance matrix of R_T and β_T is

$$\sum_{R_T, \beta_T} = \begin{pmatrix} \sum_i^M \bar{\gamma}_i \bar{\gamma}_i^T & \sum_i^M \bar{\gamma}_i \\ \sum_i^M \bar{\gamma}_i^T & M \end{pmatrix}^{-1}$$

The covariance matrix of $\bar{R}_T$, the sentinel system satellite, is obtained by matrix partitioning.

$$\sum \bar{R}_T = \left(\sum_{i}^{M} \bar{y}_i \bar{y}_i^T - \frac{1}{M} \sum_{i=1}^{M} \bar{y}_i \sum_{i=1}^{M} \bar{y}_i^T \right)^{-1}$$

$$PDOP = \left(T_r \Sigma \bar{R}_T \right)^{1/2}$$

PDOP is a measure of the geometrical strength of the least squares adjustment for the three target tracker position coordinates. The 99 percent confidence sphere radius for position estimate can be calculated from the following formula:

$$SP(99\%) = \frac{PDOP}{\sqrt{3}} * 3.367 * SIGR$$

SIGR = Slant Range Measuring Error

An outage was declared in this study whenever PDOP < 6. Outages also occurred when an insufficient number of GPS satellites were in view, i.e. whenever M < 4 or 5.

NUMERICAL RESULTS

Three sets of results were generated, one for each of the three GPS configurations. The tabulated results in tables 1, 2, and 3 were derived by allowing each of the sentinel constellations to operate for the equivalent of 2 days at 15-minute intervals for a total of 192 observation times. The first column in each table is the satellite height above the earth in nautical miles. The next two columns are the average minimum PDOP values for four and five GPS observations, respectively. All average values were taken over time and over satellite. The fourth column is the average number of GPS satellites in view at any given time. The last four columns are the average outage counts. The results in tables 1, 2, and 3 can be related to the results in the previous study[7] by referring to table 4.

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[7] M.A.Crombie. *Ground Target Location Errors Derived From Measurements Collected From A Variety of Hypothetical Satellite Sentinel Systems.* U.S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia 22060-5546. Report ETL-0538. June 1989, AD-A209 523.

Table 1. Basic GPS Constellation

	PDOP			OUTAGES			
				OBS < 4 or 5		PDOP > 6	
h	M=4	M=5	OBS/T	M=4	M=5	M=4	M=5
2,243.74	1.63	1.43	9.28	0	0	0	0
2,296.87	1.65	1.45	8.75	0	0	0	0
2,464.16	1.72	1.51	7.93	0	0	0	0
2,478.66	1.73	1.51	7.81	0	0	0	0
2,567.57	1.75	1.51	7.44	0	0	0	0
2,597.96	1.75	1.52	7.18	0	0	0	0
2,777.83	1.84	1.59	6.63	0	3	0	0
2,941.42	1.94	1.66	6.09	2	12	1	0
3,066.77	2.03	1.73	5.67	4	28	5	0
3,378.62	2.22	1.83	5.24	16	62	9	1
3,769.44	2.49	1.97	4.55	31	93	16	5
3,840.07	2.58	2.01	4.35	35	113	16	5
4,087.39	2.66	2.02	4.09	61	124	16	3
4,754.03	2.93	2.16	3.33	111	162	14	2
5,300.73	2.91	2.24	3.17	117	160	31	10
6,173.37	3.25	2.37	2.42	159	186	20	0
6,834.05	3.61	2.34	2.18	175	191	10	0
7,494.18	3.90	-	2.17	179	192	5	0
10,737.77	-	-	1.46	188	192	4	0

Table 2. Basic GPS Constellation, Plus Three In-Plane Spares

h	PDOP		OBS/T	OUTAGES			
				OBS < 4 or 5		PDOP > 6	
	M=4	M=5		M=4	M=5	M=4	M=5
2,243.74	1.60	1.41	10.86	0	0	0	0
2,296.87	1.62	1.43	10.28	0	0	0	0
2,464.16	1.69	1.47	9.26	0	0	0	0
2,478.66	1.68	1.47	9.10	0	0	0	0
2,567.57	1.72	1.49	8.68	0	0	0	0
2,597.96	1.72	1.50	8.39	0	0	0	0
2,777.83	1.79	1.56	7.73	0	0	0	0
2,941.42	1.88	1.63	7.11	0	4	0	0
3,066.77	1.97	1.71	6.62	0	9	3	1
3,378.62	2.13	1.85	6.11	7	28	12	4
3,769.44	2.39	2.01	5.27	15	53	18	8
3,840.07	2.48	2.08	5.08	18	64	19	8
4,087.39	2.55	2.14	4.76	35	83	24	7
4,754.03	2.90	2.33	3.89	78	132	32	8
5,300.73	2.97	2.43	3.63	91	144	47	17
6,173.37	3.17	2.66	2.83	136	173	38	7
6,834.05	3.70	3.05	2.55	147	184	36	5
7,494.18	4.27	3.25	2.53	149	188	34	2
10,737.77	–	–	1.72	184	191	8	1

Table 3. Basic GPS Constellation, Plus Three Equatorial Spares

				OUTAGES			
	PDOP			OBS < 4 or 5		PDOP > 6	
h	M=4	M=5	OBS/T	M=4	M=5	M=4	M=5
2,243.74	1.60	1.41	10.75	0	0	0	0
2,296.87	1.62	1.42	10.24	0	0	0	0
2,464.16	1.67	1.46	9.26	0	0	0	0
2,478.66	1.67	1.46	9.06	0	0	0	0
2,567.57	1.69	1.47	8.69	0	0	0	0
2,597.96	1.69	1.48	8.34	0	0	0	0
2,777.83	1.77	1.54	7.73	0	2	0	0
2,941.42	1.87	1.61	7.14	0	6	1	0
3,066.77	1.95	1.67	6.67	2	14	2	0
3,378.62	2.10	1.81	6.11	7	30	8	2
3,769.44	2.36	2.00	5.39	13	48	16	3
3,840.07	2.47	2.08	5.18	16	56	13	4
4,087.39	2.56	2.15	4.84	33	81	22	4
4,754.03	2.86	2.36	4.01	68	127	37	4
5,300.73	2.96	2.35	3.60	93	140	45	15
6,173.37	3.32	2.67	2.82	138	176	36	5
6,834.05	3.82	3.02	2.68	145	184	36	4
7,494.18	3.88	3.86	2.54	149	186	35	4
10,737.77	–	–	1.69	184	192	8	0

Table 4. Sentinel System Definitions

h	PER	T/P/F	e	I	COVERAGE
2,243.74	2.989	12/3/1	47.90	50.73	S
2,296.87	3.031	12/3/2	48.30	58.80	S
2,464.16	3.164	12/6/3	49.50	66 70	S
2,478.66	3.176	12/12/2	49.60	48.50	S
2,567.57	3.247	12/12/10	50.20	57.50	S
2,597.96	3.272	12/2/1	50.40	46.50	S
2,777.83	3.419	10/10/7	51.535	47.93	S
2,941.42	3.555	10/10/2	52.50	48.80	S
3,066.77	3.660	10/2/0	53.20	47.70	S
3,378.62	3.926	9/9/7	54.81	70.54	S
3,769.44	4.268	12/3/1	56.60	57.00	D
~~3,840.07~~	~~4.331~~	~~8/2/1~~	~~56.90~~	~~48.20~~	~~S~~
4,087.39	4.554	9/9/2	57.90	61.30	S
4,754.03	5.172	7/7/5	60.26	55.69	S
5,300.73	5.697	9/3/0	61.90	70.50	S
6,173.37	6.571	10/10/2	64.10	61.60	D
6,834.05	7.260	10/10/8	65.50	49.40	D
7,494.18	7.970	6/2/0	66.72	52.24	S
10,737.77	11.670	8/8/2	71.00	57.10	D

Note: The first column of table 4 is, as before, the satellite height above the earth in nautical miles. The second column is the satellite period in hours. The fourth column is the half angle at the center of the earth subtended by the satellite viewing cone. The fifth column is the inclination of the satellite. The sixth column denotes the ground coverage parameter. Note that only 1-fold (S) and 2-fold (D) ground coverages are included in the study.

DISCUSSION

The purpose of this study is to evaluate the NAVSTAR Global Positioning System (GPS) for its ability to determine platform positions associated with a constellation of hypothetical sentinel system satellites. Precision is defined in terms of minimum PDOP and system outages. Minimum PDOP is converted immediately into 99 percent confidence spheres when the slant range measuring error is specified (see error equation in section on PDOP and Outages). The results from this study are conservative since the narrower GPS half-beam width associated with the L_1 = 1575.42 MHz frequency was used. The L_1 frequency was chosen because both the C/A code and the P code are broadcast at this frequency. Orbit position prediction should, in most cases, be precise enough to lock onto the P code, but the C/A code, which is more accessible than the more precise P code, is needed to lock onto the P code whenever the position of a fast moving satellite is in doubt. Both the L_1 and L_2 frequencies are needed to correct for ionospheric delay whenever the LOS between a GPS satellite and a sentinel system satellite approaches the earth closer than 400 nm. At a sentinel system height of 2777 nm and greater, this will occur less than 5 percent of the time. The estimated user slant range measuring error can be derived from the following table of data.[8]

Table 5. GPS System Error Model

Error Source	**System Responsibility (meters)**	**P Code Error Model (meters)**	**C/A Code Error Model (meters)**
Satellite Clock and Ephemeris Errors	3.9	3.9	3.9
Ionospheric Delay Compensation	2.3	3.1	9.0
Tropospheric Delay Compensation	2.0	2.0	2.0
Receiver Noise and Resolution	1.5	1.1	11.1
Multipath	1.2	1.2	12.0
Other	0.5	--	--
Selective Availability	--	--	30.0
Total System User Error	**5.3**	**5.6**	**35.6**

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[8] D.G. Brown and A. Brown. *Integrating GPS With Inertial Navigation Systems,* Course No. 336, Navigation Technology Seminars, Inc., Arlington, Virginia, November 1987.

The total system error is given in meters and is expressed as 1 standard deviation. Consider the C/A code error model. The 11.1 meter receiver noise can be reduced to 1.1 meters by filtering, and the 12.0 meter multipath error can be nearly eliminated for ground receivers and should be almost nonexistent for satellite-borne receivers. The 30.0 meter error under selective availability is a deliberate error designed to decrease the usefulness of GPS for certain users. The degradation is accomplished by including additional errors in the pseudo ranges. The method for doing this is classified. This error can be cancelled using differential GPS, however, the selected error is deliberately changed over time.

The P code slant range error is the error of interest in this study. The error ranges from SIGR = 5.6 meters to SIGR = 4.2 meters when the ionospheric and tropospheric delays have been discounted. These values translate into the following 99 percent confidence sphere estimates: SP (99%) = 8.164 * PDOP when the LOS is outside of the ionosphere, and SP (99%) = 10.886 * PDOP when the LOS intersects the ionosphere. These results are combined with the PDOP results in tables 1, 2, and 3 to produce the 99 percent confidence sphere results shown in table 6.

From table 6, the 99 percent confidence sphere radii are seen to be an increasing function of satellite height. The table also indicates that there is not a significant difference among precision results taken from the three GPS configurations. There is, however, a large difference in percent of outages among results taken from the three GPS configurations. The percent of outages shown in table 7 was calculated directly from results in tables 1, 2, and 3. ~~Note that there were no outages for 2243 nm < h < 2777 nm.~~ Finally, there is a noticeable improvement in precision when minimum PDOP is calculated from five GPS observations rather than from four GPS observations; however, the percent of outages is much larger for five GPS observations.

Table 6. 99% Confidence Sphere Estimates For Sentinel System Positions

h	Basic				Basic Plus 3 In-Plane Spares				Basic Plus 3 Equatorial Spares			
	LOS Out		LOS In		LOS Out		LOS In		LOS Out		LOS In	
	M=4	M=5	M=4	M=5	M=4	M=5	M=4	M=5	M=4	M=5	M=4	M=5
2,243.74	13.3	11.7	17.7	15.6	13.1	11.5	17.4	15.3	13.1	11.5	17.4	15.3
2,296.87	13.5	11.8	18.0	15.8	13.2	11.7	17.6	15.6	13.2	11.6	17.6	15.5
2,464.16	14.0	12.3	18.7	16.4	13.8	12.0	18.4	16.0	13.6	11.9	18.2	15.9
2,478.66	14.1	12.3	18.8	16.4	13.7	12.0	18.3	16.0	13.6	11.9	18.2	15.9
2,567.57	14.3	12.3	19.1	16.4	14.0	12.2	18.7	16.2	13.8	12.0	18.4	16.0
2,597.96	14.3	12.4	19.1	16.5	14.0	12.2	18.7	16.3	13.8	12.1	18.4	16.1
2,777.83	15.0	13.0	20.0	17.3	14.6	12.7	19.5	17.0	14.5	12.6	19.3	16.8
2,941.42	15.8	13.6	21.1	18.1	15.3	13.3	20.5	17.7	15.3	13.1	20.4	17.5
3,066.77	16.6	14.1	22.1	18.8	16.1	14.0	21.4	18.6	15.9	13.6	21.2	18.2
3,378.62	18.1	14.9	24.2	19.9	17.4	15.1	23.2	20.1	17.1	14.8	22.9	19.7
3,769.44	20.3	16.1	27.1	21.4	19.5	16.4	26.0	21.9	19.3	16.3	25.7	21.8
3,840.07	21.1	16.4	28.1	21.9	20.2	17.0	27.0	22.6	20.2	17.0	26.9	22.6
4,087.39	21.7	16.5	29.0	22.0	20.8	17.5	27.8	23.3	20.9	17.6	27.9	23.4
4,754.03	23.9	17.6	31.9	23.5	23.7	19.0	31.6	25.4	23.3	19.3	31.1	25.7
5,300.73	23.8	18.3	31.7	24.4	24.2	19.8	32.3	26.5	24.2	19.2	32.2	25.6
6,173.37	26.5	19.3	35.4	25.8	25.9	21.7	34.5	29.0	27.1	21.8	36.1	29.1
6,834.05	29.4	19.1	39.3	25.5	30.2	24.9	40.3	33.2	31.2	24.7	41.6	32.9
7,494.18	31.8	-	42.5	-	34.9	26.5	46.5	35.4	31.7	31.5	42.2	42.0
10,737.77	-	-	-	-	-	-	-	-	-	-	-	-

Table 7. Outage Percentages

h	Basic M=4	Basic M=5	Basic Plus 3 In-Plane Spares M=4	Basic Plus 3 In-Plane Spares M=5	Basic Plus 3 Equatorial Spares M=4	Basic Plus 3 Equatorial Spares M=5
2,777.83	0	1.60	0	0	0	0
2,941.42	1.6	6.3	0	2.1	0.1	3.1
3,066.77	4.7	14.6	1.6	5.2	2.1	7.3
3,378.62	13.0	32.8	9.9	16.7	7.8	16.7
3,769.44	24.5	51.0	17.2	31.8	15.1	26.6
3,840.07	26.6	61.5	19.3	37.5	15.1	31.3
4,087.39	40.1	66.1	30.7	46.9	28.6	44.3
4,754.03	65.1	85.4	57.3	72.9	54.7	68.2
5,300.73	77.1	88.5	71.9	83.9	71.9	80.7
6,173.37	93.2	96.9	90.6	93.8	90.6	94.3
6,834.05	96.4	99.5	95.3	98.4	94.3	97.8
7,494.18	95.8	100.0	96.9	99.0	95.8	99.0
10,737.77	100.0	100.0	100.0	100.0	100.0	100.0

In table 6, the confidence sphere radii, or equivalently PDOP, increases with satellite height. This is not true for satellites closer to the earth. Consider table 8, which is similar to tables 1, 2, and 3. Note that there are no outages for 50 nm $\leq$ h $\leq$ 2777 nm. The results in table 8 were calculated exactly as those in tables 1, 2, and 3, except that in each case $\epsilon = 0$ and Θ was chosen so that the satellite height was as indicated in table 8. In each case, i = 60 degrees, T = 6 satellites, P = 3 planes, and F = 1. The results in table 8 show that PDOP decreases as satellite height increases from h = 50 nm to a value of h between 1920 nm and 1950 nm; then PDOP increases as h increases. The breakpoint can be noted best by reviewing the change in OBS/T, the average number of GPS observations at any instant. The PDOP results in table 8 can be converted into ninety-nine percent spherical errors about the location estimate exactly as in table 6.

Table 8. Low Earth Orbit PDOP Results

	Basic			Basic Plus 3 In-Plane Spares			Basic Plus 3 Equatorial Spares		
h	M=4	M=5	OBS/T	M=4	M=5	OBS/T	M=4	M=5	OBS/T
50	1.92	1.69	8.3	1.88	1.65	9.6	1.81	1.64	9.7
100	1.83	1.61	9.0	1.79	1.58	10.5	1.78	1.57	10.5
200	1.72	1.51	9.9	1.69	1.49	11.6	1.69	1.49	11.5
500	1.61	1.42	11.5	1.59	1.41	13.4	1.60	1.41	13.3
1,000	1.57	1.39	12.9	1.55	1.38	15.1	1.56	1.38	15.0
1,250	1.56	1.38	13.4	1.55	1.37	15.6	1.55	1.37	15.6
1,500	1.56	1.38	13.8	1.54	1.37	16.1	1.55	1.37	16.1
1,750	1.56	1.37	14.1	1.54	1.37	16.4	1.55	1.37	16.4
1,900	1.56	1.37	14.3	1.54	1.37	16.6	1.55	1.37	16.6
1,910	1.56	1.37	14.3	1.54	1.37	16.6	1.55	1.37	16.6
1,920	1.56	1.37	14.3	1.54	1.37	16.6	1.55	1.37	16.6
1,950	1.56	1.38	12.9	1.55	1.37	15.1	1.56	1.37	15.0
2,000	1.57	1.39	11.8	1.56	1.38	13.9	1.56	1.38	13.8
2,100	1.59	1.40	10.6	1.58	1.39	12.4	1.57	1.39	12.3
2,200	1.62	1.42	9.5	1.60	1.41	11.2	1.59	1.40	11.1

The exact minimum PDOP was calculated in this and previous studies.[9] This is not a procedure that we recommend in any fashion. For example, consider table 9 which shows the number of possible combinations one must test as a function of N, the total number of GPS observations. The values in table 9 indicate that an approximate method for finding the best subset is in order. An appreciation for the task that the Zenith 248-PC desktop computer was asked to perform can be gained by comparing the values in table 9 with the average number of GPS observations shown in tables 1, 2, 3, and 8.

Table 9. Exact Number of PDOP Calculation Combinations

N	$C_{N,4}$	$C_{N,5}$
21	5,985	20,349
20	4,845	15,504
19	3,876	11,628
18	3,060	8,568
17	2,380	6,188
16	1,820	4,368
15	1,365	3,003
14	1,001	2,002
13	715	1,287
12	495	792
11	330	462
10	210	252
9	126	126
8	70	56
7	35	21
6	15	6
5	5	1
4	1	-

[9] M. A. Crombie. *Spatial Targ^t Location Errors Derived from Measurements Collected from Sixteen Satellite Constellations*. Fort Belvoir, Virginia 22060-5546. Report ETL-0532, March 1989, AD-A206 950.

CONCLUSIONS

1. Sentinel system position precision is not dependent on a particular GPS configuration.

2. GPS outages occur when satellite heights are greater than 2,700 nautical miles. The basic GPS configuration plus 3 equatorial spares is superior to the basic GPS configuration plus 3 in-plane spares, which in turn is superior to the basic GPS configuration.

3. GPS outages begin at satellite heights greater than 2,700 nautical miles and approach 100 percent at 10,000 nautical miles.

4. Five GPS observations provide a noticeable improvement in sentinel system position precision over four GPS observations, but the percent of outages is much larger for five GPS observations.

5. Results of this study provide estimates of precision of satellite position when the satellite position is calculated from four or five GPS observations. Satellite heights range from 50 nautical miles to 10,000 nautical miles.

6. Minimum PDOP decreases as satellite heights increase from 50 nautical miles to approximately 1925 nautical miles, where minimum PDOP then increases as satellite heights increase.

7. An evaluation should be made of the loss of accuracy in minimum PDOP when approximate methods are used to determine the best GPS observational set.

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