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PROBABILISTIC SOLUTION FOR ONE-DIMENSIONAL PLANE WAVE PROPAGATION IN HOMOGENEOUS BILINEAR HYSTERETIC MATERIALS

by

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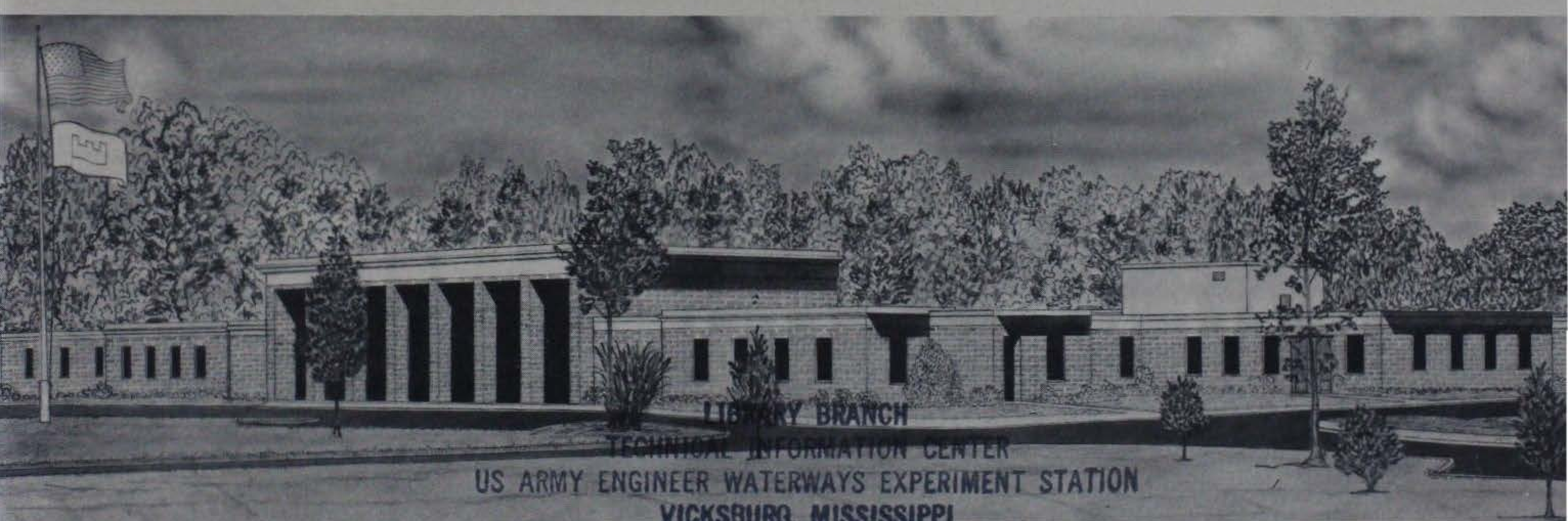

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The ground shock calculations codes currently used to predict the states of stress and ground motions induced in natural earth masses by explosive detonations are deterministic tools; that is, their input parameters (soil stress-strain and strength properties, soil density, applied airblast surface overpressure, etc.) are specified as single-valued quantities. In actuality, however, both the properties of earth materials and the characteristics of the airblast pulse are random variables. Consequently, the resulting states of stress and ground motion in a natural earth mass (Continued)

20. ABSTRACT (Continued)

are also random variables. Therefore, the ground shock calculation problem should be treated probabilistically.

This paper describes the development of a probabilistic solution for stress wave propagation in homogeneous bilinear hysteretic materials subjected to an exponentially decaying surface airblast pulse. The partial derivative method is used to calculate the time histories of the moments of the dependent (output) variables, consisting of particle displacement, particle velocity, and stress, in terms of functions of moments of the independent (input) variables. The partial derivatives are evaluated numerically using finite-difference approximations. Several sensitivity analyses are conducted to illustrate how the solution technique can be used to quantify and rank the relative effects of input variabilities or uncertainties on the dispersion of output quantities. Finally, application of the methodology for conducting probabilistic analyses of airblast-induced vertical ground shock is demonstrated.

PREFACE

The work reported herein was conducted at the U. S. Army Engineer Waterways Experiment Station (WES) under the sponsorship of the Office, Chief of Engineers, Department of the Army, as part of Project 4A762719AT40, Task A0, Work Unit 024, "Ground Shock Prediction Techniques for Earth and Earth-Structure Systems."

This study was conducted by Dr. Behzad Rohani under the direction of Dr. J. G. Jackson, Jr., Chief, Geomechanics Division, Structures Laboratory.

COL T. C. Creel, CE, was the Commander and Director of WES during this investigation. Mr. F. R. Brown was Technical Director.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
feet per second	0.3048	metres per second
inches	0.0254	metres
kips (1000 lb force) per square inch	6.894757	megapascals
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre

PROBABILISTIC SOLUTION FOR ONE-DIMENSIONAL PLANE WAVE PROPAGATION IN HOMOGENEOUS BILINEAR HYSTERETIC MATERIALS

PART I: INTRODUCTION

1. The ground shock calculation techniques currently used to predict the states of stress and ground motions induced in natural earth masses by explosive detonations are deterministic tools--that is, their input parameters (media stress-strain and strength properties, density, surface airblast loading, etc.) are specified as deterministic quantities or functions. In actuality, however, both the properties of earth materials and the characteristics of airblast pulses are random variables. Consequently, the randomness of these input variables indicates that the resulting states of stress and ground motion in natural earth masses are also random variables. Therefore the ground shock calculation problem should be treated probabilistically.

2. In general, airblast-induced ground motion from a surface burst can be analyzed as an axisymmetric two-dimensional (2D) problem. And, if the ratio of the propagation velocity of the airblast pulse traversing the ground surface is much greater than the propagation velocity of the stress wave in the medium (superseismic conditions), the near-surface ground motions outside the crater are predominantly vertical, in which case one-dimensional (1D) plane wave calculations are usually adequate for predicting free-field response. It is appropriate, therefore, as well as useful to commence an examination of probabilistic ground shock in terms of the 1D problem before examining the more cumbersome 2D problem.

3. A simple, yet powerful, deterministic model for predicting 1D wave propagation phenomena in earth media is the analytic solution for a bilinear hysteretic material loaded by a decaying surface airblast pulse developed by

Salvadori, et al., in 1960 (Reference 1). Although the real stress-strain properties of soil are only approximated by this bilinear relationship, the solution accounts for the major observed features of stress wave propagation through earth materials--namely, the attenuation of stress and particle motion with depth. This paper is concerned with the conversion of this deterministic solution into a probabilistic solution using the method of partial derivatives (Reference 2). The method of partial derivatives greatly alleviates the burden of conducting brute force Monte Carlo analyses for probabilistic wave propagation problems. Furthermore, the partial derivative method has the advantage that it can be used to quantitatively rank the relative effects of input variabilities (uncertainties) on the dispersion of the output quantities, i.e., particle displacement, particle velocity, and stress.

PART II: DETERMINISTIC SOLUTION FOR ONE-DIMENSIONAL WAVE PROPAGATION IN BILINEAR HYSTERETIC MATERIALS

Description of Bilinear Model

4. The bilinear hysteretic model (Reference 1) was first used to approximate the stress-strain behavior of soils in states of uniaxial strain. The actual soil stress-strain curve during virgin loading is approximated as a straight line which is defined by modulus M_o .* During unloading and subsequent reloading to a previous maximum stress level, the actual stress-strain relation is different from the loading relation and is approximated by another straight line, defined by modulus M_1 . When $M_o = M_1$, the model corresponds to a linear elastic material. The propagation velocity of a virgin loading stress wave is given by

$$C_o = \sqrt{M_o/\rho} \qquad (1)$$

where ρ is the mass density of the material and the propagation velocity of an unloading or reloading stress wave is given by

$$C_1 = \sqrt{M_1/\rho} \qquad (2)$$

The bilinear hysteretic material is therefore completely described by three material constants: M_o , M_1 , and ρ .

Boundary Loading

5. The dynamic boundary load considered in Reference 1 allows the solution to be used for blast-type problems. It is a pulse characterized by

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* For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix A).

an instantaneous rise to peak pressure (a shock front) followed by an exponential decay. Its expression is

$$P(t) = P_o \exp(-t/\tau) \qquad (3)$$

where P_o is the peak applied pressure, t is time, and τ is the exponential time constant (time at which pressure has decayed to 0.3678 P_o). The airblast pulse is therefore completely characterized by two parameters: P_o and τ .

Response of the Medium to Surface Boundary Loading

6. The 1D problem treated by this deterministic solution may be viewed as a semi-infinite body of bilinear hysteretic material uniformly loaded at its free surface by the pressure pulse P(t) described above. Surface motion and stress are assumed to be zero before application of the load. According to Reference 1, the response of the medium in terms of time histories of stress σ and particle velocity $\dot{U}$ in the direction of propagating wave at a generic depth Z is given as

$$\sigma(Z,t) = P_o \exp\left[-\left(t - \frac{Z}{C_1}\right)\Big/\tau\right] + P_o \sum_{n=1}^{\infty} \alpha^n \left\{\exp\left[-\alpha^n\left(t - \frac{Z}{C_1}\right)\Big/\tau\right] - \exp\left[-\alpha^n\left(t + \frac{Z}{C_1}\right)\Big/\tau\right]\right\} \qquad (4)$$

$$\dot{U}(Z,t) = P_o\left(\frac{1}{\rho C_o} - \frac{1}{\rho C_1}\right) + \frac{P_o}{\rho C_1} \exp\left[-\left(t - \frac{Z}{C_1}\right)\Big/\tau\right] - \frac{P_o}{\rho C_1} \sum_{n=1}^{\infty} \alpha^n \left\{2 - \exp\left[-\alpha^n\left(t - \frac{Z}{C_1}\right)\Big/\tau\right] - \exp\left[-\alpha^n\left(t + \frac{Z}{C_1}\right)\Big/\tau\right]\right\} \qquad (5)$$

Integration of Equation 5 results in the following expression for particle displacement

$$U(Z,t) = P_o\left(\frac{1}{\rho C_o} - \frac{1}{\rho C_1}\right)\left(t - \frac{Z}{C_o}\right) + \frac{P_o\tau}{\rho C_1}\left\{\exp\left[-\left(\frac{Z}{C_o} - \frac{Z}{C_1}\right)\Big/\tau\right] - \exp\left[-\left(t - \frac{Z}{C_1}\right)\Big/\tau\right]\right\}$$

$$+ \frac{P_o\tau}{\rho C_1}\sum_{n=1}^{\infty}\left\{2\alpha^n\left(\frac{Z}{C_o} - t\right)\Big/\tau + \exp\left[-\alpha^n\left(\frac{Z}{C_o} - \frac{Z}{C_1}\right)\Big/\tau\right] + \exp\left[-\alpha^n\left(\frac{Z}{C_o} + \frac{Z}{C_1}\right)\Big/\tau\right]\right.$$

$$\left. - \exp\left[-\alpha^n\left(t - \frac{Z}{C_1}\right)\Big/\tau\right] - \exp\left[-\alpha^n\left(t + \frac{Z}{C_1}\right)\Big/\tau\right]\right\} \qquad (6)$$

The parameter α in Equations 4 through 6 is given by

$$\alpha = \frac{\sqrt{M_1/M_o} - 1}{\sqrt{M_1/M_o} + 1} \qquad (7)$$

To account for the travel time of the wave to the depth of interest the initial value of t in Equations 4 through 6 must correspond to the arrival time t_a at that depth (i.e., these equations hold for $t \geq t_a$), which is given by

$$t_a = Z/C_o \qquad (8)$$

Equations 4, 5, 6, and 8 (which relate the four dependent variables σ, $\dot{U}$, U, and t_a and the five independent variables M_o, M_1, ρ, P_o, and τ) provide the complete deterministic solution for 1D stress wave propagation in bilinear hysteretic materials. Note that since the stress and particle velocity maxima of this solution always occur at the wave front, t in Equations 4 and 5 can be replaced by t_a to obtain expressions for the peak stress σ_{max} and peak particle velocity $\dot{U}_{max}$ as a function of depth.

7. The purpose of a probabilistic analysis is to develop a method by which the variability or uncertainties in the independent parameters in a particular problem can be evaluated or estimated in terms of their effects on the dispersion of the dependent (output) variables. A highly useful procedure for implementing such an analysis is to apply the method of partial derivatives, described in References 2 and 3, to a deterministic solution of the problem. The method gives approximations for the moments of the dependent variables in terms of functions of moments of the independent variables. That is if a random variable Y is functionally related to the random variables X_i

$$Y = Y(X_1, X_2, \ldots, X_n) \tag{9}$$

and if the X_i are uncorrelated, then according to the partial derivative method the approximations for the expected value of Y, $E[Y]$ and the variance of Y, $Var[Y]$, are

$$E[Y] \approx Y(\mu_1, \mu_2, \ldots, \mu_n) + \frac{1}{2} \sum_{i=1}^{n} \left. \frac{\partial^2 Y}{\partial X_i^2} \right|_{(\mu_1, \mu_2, \ldots, \mu_n)} Var[X_i] \tag{10}$$

$$Var[Y] \approx \sum_{i=1}^{n} \left(\left. \frac{\partial Y}{\partial X_i} \right|_{(\mu_1, \mu_2, \ldots, \mu_n)} \right)^2 Var[X_i] \tag{11}$$

where $(\mu_1, \mu_2, \ldots, \mu_n)$ denotes the respective mean of the random variables $(X_1, X_2, \ldots, X_n)$. The first term on the right-hand side of Equation 10 corresponds to the mean value of Y , i.e., the value of Y obtained using the mean values of all of the random variables. The second term represents the contributions to the expected value of Y due to uncertainties in the X_i. The second term is negligible if $Var[X_i]$ and the nonlinearity in the function Y itself are not large. As pointed out in Reference 2, Equation 11 "may be interpreted as meaning that each of the n random variables X_i contributes to the dispersion of Y in a manner proportional to its own variance $Var[X_i]$ and proportional to a factor

$$\left(\left. \frac{\partial Y}{\partial X_i} \right|_{(\mu_1, \mu_2, \ldots, \mu_n)} \right)^2,$$ which is related to the sensitivity of changes in Y to changes in X_i." This interpretation can be used to conduct sensitivity analyses to quantify and rank the relative effects of the input variabilities or uncertainties on the dispersion of the output quantities.

8. The partial derivatives in Equations 10 and 11 can be evaluated analytically if an explicit expression is available for the dependent variable Y. However, as pointed out by Mlakar (Reference 4), in many cases, even when an explicit relation for Y does exist, it is often more convenient to evaluate these partial derivatives numerically using finite-difference approximations. Following the method proposed by Mlakar (Reference 4), the partial derivatives may be expressed as

$$\frac{\partial Y}{\partial X_i} = \frac{Y(\mu_1, \ldots, \mu_i + kS_i, \ldots \mu_n) - Y(\mu_1, \ldots, \mu_i - kS_i, \ldots, \mu_n)}{2kS_i} \tag{12}$$

$$\frac{\partial^2 Y}{\partial X_i^2} = \frac{Y(\mu_1, \ldots, \mu_i - kS_i, \ldots, \mu_n) - 2Y(\mu_1, \ldots, \mu_i, \ldots, \mu_n) + Y(\mu_1, \ldots, \mu_i + kS_i, \ldots \mu_n)}{(kS_i)^2} \tag{13}$$

$$i = 1, 2, \ldots, n$$

The first partial derivative (Equation 12) is calculated from the functional values of Y at k standard deviation kS_i above and below the mean value of X_i (or μ_i) where k is a constant and S_i is the standard deviation of X_i. Similarly, the second partial derivative (Equation 13) is calculated from the mean value of Y and the functional values of Y at kS_i above and below μ_i. The term kS_i therefore may be viewed as a finite-difference mesh spacing. The value of k is heuristically taken to be 1.0 in Reference 4. However, if either the standard deviation S_i or the nonlinearity in the function Y is large, smaller values of k should be tried until the results are no longer affected by further reductions in k. The probabilistic calculations reported in this paper (next section) were conducted using $k = 0.5$. In order to determine the sensitivity of the results to variations in k several calculations were conducted for k varying from 1.0 to 0.1. These calculations indicated that an order of magnitude reduction in the value of k decreased the numerical results on the average by 6 percent and at most by 12 percent. Reducing the value of k from 0.5 to 0.1, however, changed the results of the calculations on the average by 1 percent and at most by 3 percent. For most practical problems it would therefore appear that a value of k within the range of 0.1 to 0.5 would be satisfactory.

9. For each of the dependent variables σ, $\dot{U}$, U, and t_a (Equations 4, 5, 6, and 8), Equations 12 and 13 can be used to calculate their first and second partial derivatives with respect to the five independent variables M_o, M_1, ρ, P_o, and τ. These partial derivatives can then be used in Equations 10 and 11 to calculate the expected value and the variance of each of the dependent variables.

10. A computer program has been developed for the express purpose of numerically evaluating this complete system of equations. Within this program, computations are made at successive times at selected depths so that

the time histories of the expected value and the variance of the dependent variables σ , $\dot{U}$, and U can be constructed. The program also computes and prints out the explicit contributions of each of the input (independent) random variables to the overall dispersion of the various output quantities (dependent variables).

PART IV: NUMERICAL EXAMPLES

11. In this section we demonstrate the application of the above methodology for conducting probabilistic wave propagation analyses for bilinear hysteretic materials. Two numerical examples are presented. One is referred to as the "stiff soil" problem; the other is referred to as the "soft soil" problem. We consider the former problem first. The input quantities selected for the first problem consisting of the mean values μ_i and the coefficients of variation S_i/μ_i for the five independent variables M_o , M_1 , ρ , P_o , and τ are given in Table 1. The coefficients of variation specified for this problem are reasonable inasmuch as they have been encountered in actual practice. The bilinear material in this example problem is referred to as "stiff soil" because of the high value of the loading modulus M_o . The expected values $E[\sigma_{max}]$ and $E[\dot{U}_{max}]$ and the coefficients of variation $V[\sigma_{max}]$ and $V[\dot{U}_{max}]$ obtained for various depths from the probabilistic analysis of this problem are presented in Tables 1 and 2. These tables also list the individual contributions produced by the constituent random variables X_i to the dispersion of peak stress and peak particle velocity at different depths. These individual contributions are presented in the last four columns of Tables 1 and 2 in terms of the values of $\left|\frac{\partial\sigma_{max}}{\partial X_i}\right|\frac{S_i}{E[\sigma_{max}]}$ (Table 1) and $\left|\frac{\partial\dot{U}_{max}}{\partial X_i}\right|\frac{S_i}{E[\dot{U}_{max}]}$ (Table 2). Note that Equation 11 stipulates that these individual contributions would correspond to the coefficient of variation of peak stress (or peak particle velocity) if for each variable X_i the other four constituent variables were deterministic.

12. The input quantities for the second example problem are given in Table 3. The bilinear material used for this example problem is referred to as

Table 1. Individual Contributions to Dispersion of Peak Stress Due to Uncertainties in Input Variables (Stiff Soil; α = 0.333)

			$\left|\frac{\partial \sigma_{max}}{\partial X_i}\right| \frac{S_i}{E[\sigma_{max}]}$			
X_i	μ_i	S_i/μ_i	Z = 0.0 ft	Z = 10.0 ft	Z = 20.0 ft	Z = 40.0 ft
M_o	63.0 ksi	0.5	0.0	0.01905	0.03757	0.07286
M_1	252.0 ksi	0.5	0.0	0.00856	0.01690	0.03288
ρ	100.0 lb/ft^3	0.1	0.0	0.00209	0.00415	0.00812
P_o	0.1 ksi	0.1	0.1	0.10008	0.10010	0.09994
τ	0.05 sec	0.1	0.0	0.00420	0.00831	0.01627
			$E[\sigma_{max}]$ = 0.1 ksi	$E[\sigma_{max}]$ = 0.09570 ksi	$E[\sigma_{max}]$ = 0.09168 ksi	$E[\sigma_{max}]$ = 0.08441 ksi
			$V[\sigma_{max}]$ = 0.1	$V[\sigma_{max}]$ = 0.10246	$V[\sigma_{max}]$ = 0.10888	$V[\sigma_{max}]$ = 0.12980

Table 2. Individual Contributions to Dispersion of Peak Particle Velocity Due to Uncertainties in Input Variables (Stiff Soil; α = 0.333)

			$\left|\frac{\partial \dot{U}_{max}}{\partial X_i}\right| \frac{S_i}{E[\dot{U}_{max}]}$			
X_i	μ_i	S_i/μ_i	Z = 0.0 ft	Z = 10.0 ft	Z = 20.0 ft	Z = 40.0 ft
M_o	63.0 ksi	0.5	0.23618	0.21932	0.20262	0.17008
M_1	252.0 ksi	0.5	0.0	0.00779	0.01546	0.03033
ρ	100.0 lb/ft^3	0.1	0.04544	0.04760	0.04970	0.05372
P_o	0.1 ksi	0.1	0.09074	0.09123	0.09164	0.09229
τ	0.05 sec	0.1	0.0	0.00384	0.00761	0.01502
			$E[\dot{U}_{max}]$ = 2.99 fps	$E[\dot{U}_{max}]$ = 2.85 fps	$E[\dot{U}_{max}]$ = 2.72 fps	$E[\dot{U}_{max}]$ = 2.48 fps
			$V[\dot{U}_{max}]$ = 0.25706	$V[\dot{U}_{max}]$ = 0.24266	$V[\dot{U}_{max}]$ = 0.22898	$V[\dot{U}_{max}]$ = 0.20447

"soft soil" because of the low value of the loading modulus M_o. The probabilistic output quantities for this problem are presented in Tables 3 and 4.

13. The information presented in Tables 1 through 4 reveals several

Table 3. Individual Contributions to Dispersion of Peak Stress Due to Uncertainties in Input Variables (Soft Soil; $\alpha = 0.714$)

			$\left\lvert \frac{\partial \sigma_{max}}{\partial X_i} \right\rvert \frac{S_i}{E[\sigma_{max}]}$			
X_i	μ_i	S_i/μ_i	Z = 0.0 ft	Z = 10.0 ft	Z = 20.0 ft	Z = 40.0 ft
M_o	7.0 ksi	0.5	0.0	0.04475	0.08463	0.14888
M_1	252.0 ksi	0.5	0.0	0.00246	0.00451	0.00751
ρ	100.0 lb/ft^3	0.1	0.0	0.00819	0.01559	0.02777
P_o	0.1 ksi	0.1	0.1	0.10153	0.10263	0.10377
τ	0.05 sec	0.1	0.0	0.01640	0.03122	0.05555
			$E[\sigma_{max}]$ = 0.1 ksi	$E[\sigma_{max}]$ = 0.0834 ksi	$E[\sigma_{max}]$ = 0.0706 ksi	$E[\sigma_{max}]$ = 0.0525 ksi
			$V[\sigma_{max}]$ = 0.1	$V[\sigma_{max}]$ = 0.11249	$V[\sigma_{max}]$ = 0.13760	$V[\sigma_{max}]$ = 0.19195

Table 4. Individual Contributions to Dispersion of Peak Particle Velocity Due to Uncertainties in Input Variables (Soft Soil; $\alpha = 0.714$)

			$\left\lvert \frac{\partial \dot{U}_{max}}{\partial X_i} \right\rvert \frac{S_i}{E[\dot{U}_{max}]}$			
X_i	μ_i	S_i/μ_i	Z = 0.0 ft	Z = 10.0 ft	Z = 20.0 ft	Z = 40.0 ft
M_o	7.0 ksi	0.5	0.23618	0.19938	0.16526	0.10806
M_1	252.0 ksi	0.5	0.0	0.00211	0.00391	0.00657
ρ	100.0 lb/ft^3	0.1	0.04544	0.05403	0.06186	0.07470
P_o	0.1 ksi	0.1	0.09074	0.09295	0.09479	0.09723
τ	0.05 sec	0.1	0.0	0.01497	0.02878	0.05195
			$E[\dot{U}_{max}]$ = 8.98 fps	$E[\dot{U}_{max}]$ = 7.43 fps	$E[\dot{U}_{max}]$ = 6.23 fps	$E[\dot{U}_{max}]$ = 4.57 fps
			$V[\dot{U}_{max}]$ = 0.25706	$V[\dot{U}_{max}]$ = 0.22703	$V[\dot{U}_{max}]$ = 0.20240	$V[\dot{U}_{max}]$ = 0.17162

interesting trends regarding the dispersion of peak stress and peak particle velocity:

a. For the two soil materials considered in this paper, the coefficient of variation of peak stress $V[\sigma_{max}]$ increases with depth. On the other hand, the coefficient of variation of peak particle velocity $V[\dot{U}_{max}]$ decreases with depth. The rates of change of the coefficients of variation with depth, however, are more pronounced for the soft soil case.

b. As expected, at the ground surface only the variability in peak pressure P_o contributes to the dispersion of peak stress. With increasing depth, however, the variabilities in the other input variables also contribute to the dispersion of peak stress. Moreover, the contribution of the variability in P_o is (for all practical purposes) the same for all depths whereas the influence of the other input variables becomes more significant with increasing depth. These influences are stronger for the soft soil case. For both soils, the variabilities in peak pressure P_o and loading modulus M_o contribute most to the dispersion of peak stress. The combined variability in unloading modulus M_1, density ρ, and decay constant τ has a very small influence on the dispersion of peak stress.

c. The variability in loading modulus M_o contributes most to dispersion of peak particle velocity. Its influence, however, decreases with increasing depth. The variability in unloading modulus M_1 has a negligible effect on the dispersion of peak particle velocity but its influence increases with increasing depth. The combined variability in density ρ, peak pressure P_o, and decay constant τ has a small effect on the dispersion of peak particle velocity but its influence increases with increasing depth and is more pronounced for the soft soil case.

14. It is worth pointing out that for the two example problems considered, the contributions of the second partial derivatives in Equation 10 to the expected values of the dependent variables σ, $\dot{U}$, U, and t_a are

not very significant. In the case of stress, the contribution is negligible. For the other three dependent variables the contributions on the average are about 10 percent of the expected values and as high as 25 percent for the particle velocity at late times. The contributions are more pronounced for the soft soil problem.

15. To provide a feeling for the effects of input variability on the dispersion of particle displacement for the two problems considered, the results of the probabilistic calculations for surface displacement at $t = 0.15$ second are presented in Table 5. This tabulation shows that the variability in loading modulus M_o obviously contributes most to dispersion of particle displacement for both of the example problems. The combined effect of the variabilities in unloading modulus M_1, density ρ, peak pressure P_o, and decay constant τ on the dispersion of particle displacement is small.

Table 5. Individual Contributions to Dispersion of Particle Displacement Due to Uncertainties in Input Variables (Z = 0.0 ft; t = 0.15 sec)

	Soft Soil, $\alpha = 0.714$			Stiff Soil, $\alpha = 0.333$		
X_i	μ_i	S_i/μ_i	$\left\lvert\frac{\partial U}{\partial X_i}\right\rvert \frac{S_i}{E[U]}$	μ_i	S_i/μ_i	$\left\lvert\frac{\partial U}{\partial X_i}\right\rvert \frac{S_i}{E[U]}$
M_o	7.0 ksi	0.5	0.29385	63.0 ksi	0.5	0.29738
M_1	252.0 ksi	0.5	0.05906	252.0 ksi	0.5	0.05999
ρ	100.0 lb/ft^3	0.1	0.04459	100.0 lb/ft^3	0.1	0.04595
P_o	0.1 ksi	0.1	0.08905	0.1 ksi	0.1	0.09175
τ	0.05 sec	0.1	0.02215	0.05 sec	0.1	0.04232
			E[U] = 10.92 in.			E[U] = 2.73 in.
			V[U] = 0.31661			V[U] = 0.32303

16. Typical time histories (wave forms) of the expected value and the coefficient of variation of stress, particle velocity, and particle displacement at $Z = 40.0$ feet* are portrayed in Figures 1 through 3, respectively. The arrival times for these graphs correspond to $E[t_a] = 0.0257$ second for the stiff soil and $E[t_a] = 0.0770$ second for the soft soil. The coefficient of variation of the arrival time is a function only of the variability in the independent variables M_o and ρ and is equal to $V[t_a] = 0.2416$ for all depths and for both problems considered. The individual contributions to the dispersion of t_a due to uncertainties in M_o and ρ are, respectively, 0.2373 and 0.0456. Therefore, the variability in M_o contributes most to the dispersion of the arrival time. It is interesting to note from Figures 2 and 3 that the coefficients of variation of particle velocity and particle displacement, $V[\dot{U}]$ and $V[U]$, respectively, also increase with increasing time. In the case of stress (Figure 1), the coefficient of variation $V[\sigma]$ initially decreases with time and then increases at late times (the soft soil history eventually turns upward at $t > 0.2$ second). The increase in $V[U]$ with time is important since the expected value of particle displacement $E[U]$ also increases with time.

17. Information of the type presented in Tables 1 through 5 and Figures 1 through 3 can be used to perform probabilistic analysis of airblast-induced ground shock and to construct reliability based safety factors for blast-resistant design purposes. Assuming that the probability distribution functions for each of the dependent random variables are roughly bell-shaped, the normal distribution function can be used to conduct the desired probability analysis. For example, applying the rule-of-thumb that the probability that a variable lies within two standard deviation bounds of its mean is approximately 95 percent to the surface displacement predictions in

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* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

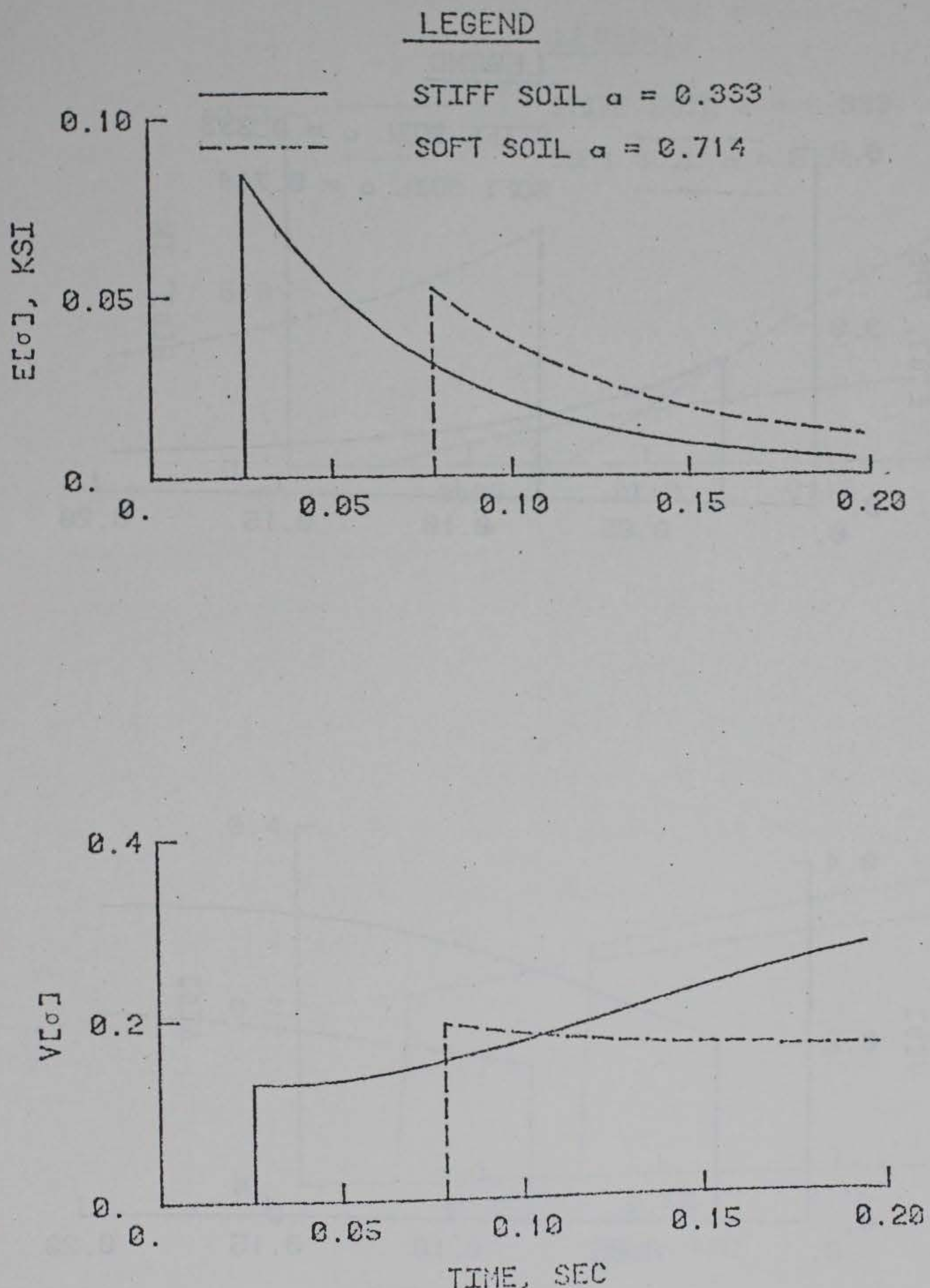

Figure 1. Time histories of E[σ] and V[σ] at Z = 40 feet

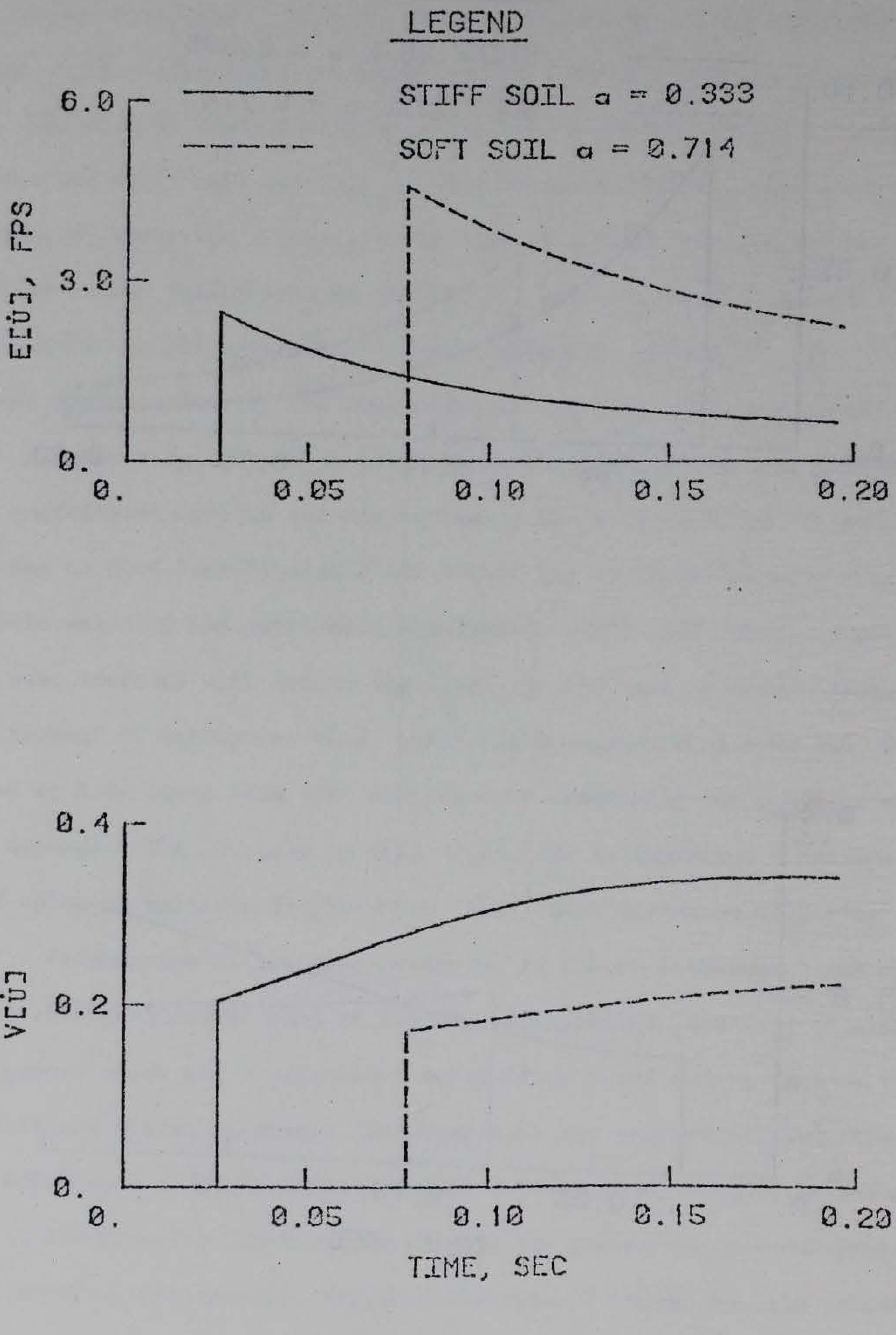

Figure 2. Time histories of $E[\dot{U}]$ and $V[\dot{U}]$ at Z = 40 feet

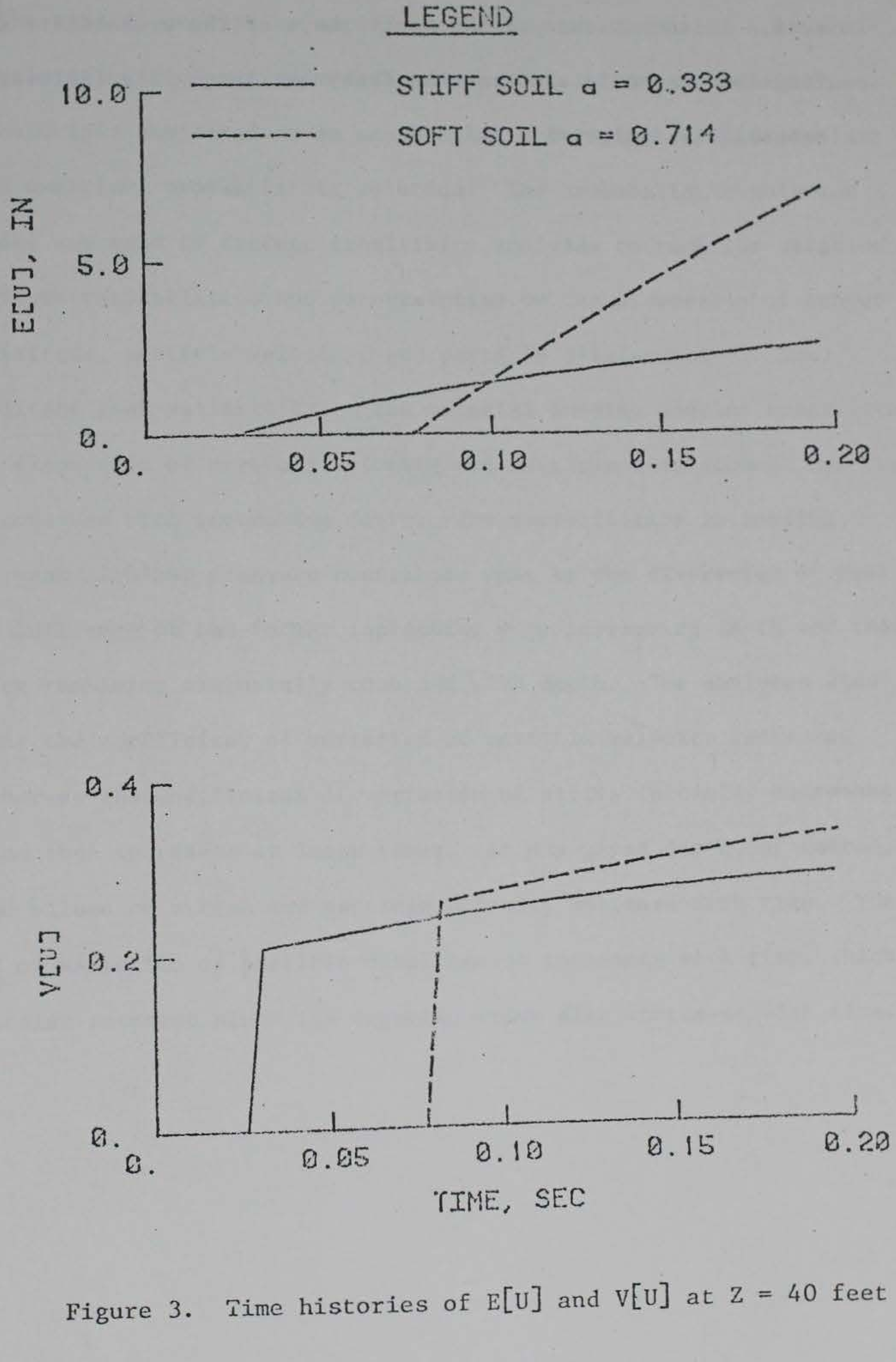

Figure 3. Time histories of E[U] and V[U] at Z = 40 feet

Table 5, it can be stated $P[4.006 \text{ in.} \leq U \leq 17.834 \text{ in.}] = 0.95$ for the soft soil and $P[0.966 \text{ in.} \leq U \leq 4.493 \text{ in.}] = 0.95$ for the stiff soil. In the absence of the exact shape of the probability distribution function one could also use the Chebyshev inequality (Reference 2) for probability analysis.

PART V: CONCLUSIONS

18. The partial derivative method has been used to convert a deterministic analytic solution for stress wave propagation in homogeneous bilinear hysteretic materials subjected to an exponentially decaying surface airblast pulse into a numerical probabilistic solution. The probabilistic solution has been coded and used to conduct sensitivity analyses to rank the relative effects of input variabilities and uncertainties on the dispersion of output quantities (stress, particle velocity, and particle displacement). The analyses indicate that variability in the material loading modulus contributes most to the dispersion of particle velocity and particle displacement but its influence decreases with increasing depth. The variabilities in loading modulus and peak airblast pressure contribute most to the dispersion of peak stress; the influence of the former increasing with increasing depth and that of the latter remaining essentially constant with depth. The analyses also indicate that the coefficient of variation of particle velocity increases with time whereas the coefficient of variation of stress initially decreases with time and then increases at later times. At any given depth, of course, the expected values of stress and particle velocity decrease with time. The coefficient of variation of particle displacement increases with time, which is of particular interest since its expected value also increases with time.

REFERENCES

1. Salvadori, M. G., Skalak, R., and Weidlinger, P., "Waves and Shocks in Locking and Dissipative Media," Transactions, American Society of Civil Engineers, Vol 126, Part I, April 1960.

2. Benjamin, J. R. and Cornell, C. A., Probability, Statistics and Decision for Civil Engineers, McGraw-Hill, Inc., New York, 1970.

3. Haugen, E. B., Probabilistic Approaches to Design, John Wiley & Sons, Inc., 1968.

4. Mlakar, P. F., "Application of an Implicit Linear Statistical Analysis to the Estimation of the Resistance of a Reinforced Concrete Beam-Column," Miscellaneous Paper N-78-7, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss., 1978.

APPENDIX A

NOTATION

C_o	Loading wave speed
C_1	Unloading wave speed
E[ ]	Expectation of a random variable
k	Number of standard deviations above and below mean at which the dependent random variable is evaluated
M_o	Initial loading modulus
M_1	Unloading modulus
P(t)	Applied surface pressure-time history
P_o	Peak applied pressure
P[ ]	Probability of a random variable
S_i	Standard deviation of X_i
t	Time
t_a	Arrival time = Z/C_o
U	Particle displacement
$\dot{U}$	Particle velocity
$\dot{U}_{max}$	Peak particle velocity
V[ ]	Coefficient of variation of a random variable
Var[ ]	Variance of a random variable
X_i	Functionally independent random variable
Y	Functionally dependent random variable
Z	Depth
α	Hysteretic parameter = $\dfrac{\sqrt{M_1/M_o} - 1}{\sqrt{M_1/M_o} + 1}$

τ Exponential time constant (time at which the applied surface pressure has decayed to 0.3678 P_o)

μ_i Mean of X_i

ρ Mass density

σ Stress

σ_{max} Peak stress